1 Electric Charges and Fields

Facts that Matter

• Electric Charge

Charge is the property associated with matter due to which it produces and experiences electrical and magnetic effects.
Electric charge interacts another electric charge.

- Accelerated charge radiates energy in the form of electromagnetic waves.
- It is invariant. Like phase it is independent of frame of reference.

• Properties of Electric Charge

- Electric charge is of two types (*i*) Positive charge and (*ii*) Negative charge.
- Like charges repel and unlike charges attract each other.
- Electric charge does not exist without mass.
- Electric charge on a body does not depend on its mass and speed.
- Electric charge is quantized. It means the charge on a body is in the form of integral multiple of least possible charge *i.e.*, the charge of electron (elementary charge). Charge on a body can be expressed as

$$q = ne$$

where e = charge of electron, and n is an integer.

- Electric charge is conserved. It means the total charge on a body is constant.

$$\Sigma q = \text{constant}$$

Electric charge neither be created nor be destroyed. It is always conserved.

- Electric charge is additive. The total charge on a body is algebric sum of all the charges present on the body.

$$Q = q_1 + q_2 + q_3 + ... + q_n$$

or

$$Q = \sum_{i=1}^{n} q_i$$

- Positive and negative charges can be produced simultaneously. It is called pair production. A photon (Y) can produce electron ($\bar{e}$) and position (e)

$$Y = \bar{e} + e$$

- Two equal and opposite charges can be annihilated simultaneously. Electron and proton can be annihilated to produce photon.

$$\bar{e} + e = Y.$$

- **Unit of Electric Charge**
  - The SI unit of electric charge is coulomb (C).
  - The C.G.S. unit of electric charge is stat. coulomb (e.s.u.) or electrostatic unit.

$$1 \text{ C} = 3 \times 10^9 \text{ e.s.u.}$$

  - The electro magnetic unit of charge is e.m.u.

$$1 \text{ e.m.u.} = 10 \text{ C} = 3 \times 10^{10} \text{ e.s.u.}$$

  - The largest unit of electric charge is Faraday (F)

$$1 \text{ F} = 96500 \text{ C}$$

- **Production of Electric Charges**

Electric charge can be produced by three methods (*i*) friction, (*ii*) conduction and (*iii*) induction.

- **Frictional Electricity**

The electric charge produced by friction is called frictional electricity. When two different bodies are rubbed, the transfer of electrons takes place from one body to another. The body which loses electrons becomes positive and the body which gains electron, becomes negative. The number of electron lost by one body is equal to the number of electron gained by another body. Thus, charge appeared on both the bodies equal in magnitude and opposite in nature.

- **Electrostatic Conduction**

When two differently charged bodies different in magnitude and nature or a charged body and uncharged body are kept in contact, the redistribution of charge takes place. The uncharged body gets charged and differently charged bodies acquire charge in the ratio of their capacities (for spherical bodies in the ratio of their radii).

If two charged identical bodies having charges q_1 and q_2 are touched or kept in contact, the charge acquired by each of the body will be equal.

$$q_1' = q_2' = \frac{q_1 + q_2}{2}.$$

- **Electrostatic Induction**

When an uncharged body is placed near to a charged body, it acquired an opposite nature of charge at its surface neared to the charged body and same nature of charge at its farther surface as shown in the diagram.

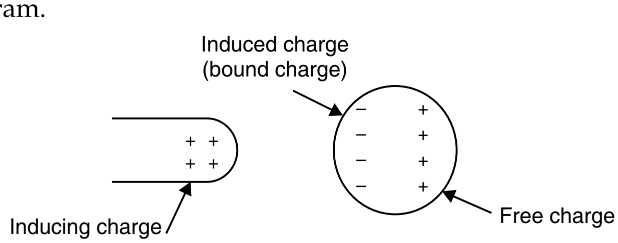


Fig. 1.1

In electrostatic induction the induced charge is given as

$$q = q_0 \left(1 - \frac{1}{K}\right)$$

where q_0 is inducing charge and K is dielectric constant.

• Continuous Distribution Charge

Continuous distribution of charge is of three types:

(i) Linear charge distribution. When charge is distributed along the length it is called linear distribution of charge. The distributed per unit length is called linear charge density (λ).

$$\therefore \qquad \lambda = \frac{q}{l}$$

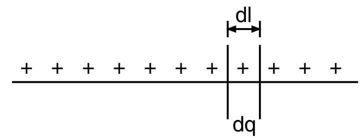


Fig. 1.2

For elementary length dl having charge dq,

$$\lambda = \frac{dq}{dl}$$

$$\text{or } dq = \lambda \, dl$$

The total charge

$$q = \int_L \lambda \; dl.$$

(ii) Surface distribution charge. When charge is distributed over a surface, it is called surface distribution of charge. The charge distributed per unit surface area is called surface charge density (σ).

$$\therefore \qquad \sigma = \frac{q}{S}$$

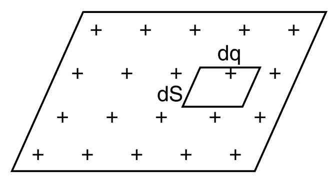


Fig. 1.3

For elementary surface dS having charge dq,

$$\sigma = \frac{dq}{dS}$$

$$\text{or} \qquad dq = \sigma \, dS$$

The total charge

$$q = \int_S \sigma \, dS.$$

(iii) Volume charge distribution. When charge is distributed in a volume, the distribution of charge is called volume charge distribution. The charge distributed per unit volume is called volume charge density (ρ).

$$\therefore \qquad \rho = \frac{q}{V}$$

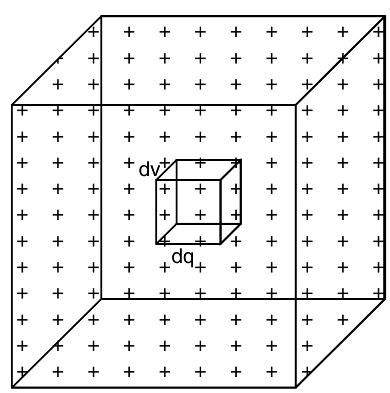


Fig. 1.4

For elementary volume dV having charge dq,

$$\rho = \frac{dq}{dV}$$

$$\text{or} \qquad dq = \rho \, dV$$

The total charge,

$$q = \int_V \rho \; dV.$$

Thus, total charge for linear charge distribution is the line integral of linear charge density. For surface charge distribution the total charge in surface integral of surface charge density. For volume charge distribution, the total charge is volume integral of volume charge density.

• Coulomb's Law

It states that the electrostatic force between two point charges is directly proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance between the charges along the line joining them.

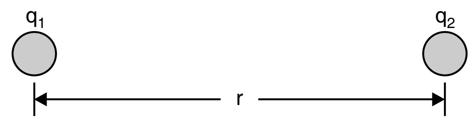


Fig. 1.5

$$F \propto q_1 q_2$$

$$\propto \frac{1}{r^2}$$

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = \frac{1 \,.\, q_1 q_2}{4\pi\varepsilon\, r^2}$$

In vector form

$$\vec{F} = \frac{1}{4\pi\varepsilon} \cdot \frac{q_1 q_2\, \hat{r}}{r^2}$$

where $\hat{r}$ is the unit vector along the line joining them.

$$\because \qquad \hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

$$\therefore \qquad \vec{F} = \frac{1}{4\pi\varepsilon} \cdot \frac{q_1 q_2\, (\vec{r})}{|\vec{r}|^3}$$

ε is the electrostatic property of the medium called absolute permittivity. In free space, vacuum or air.

$$\varepsilon = \varepsilon_0$$

If a medium has permittivity ε_r times the permittivity in free space, then

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

or $$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}.$$

It is called relative permittivity or directric constant.

- Electric charges exert electric force only when they are at rest with respect to each other. However, charges in relative motion exert electric as well as magnetic force on each other.
- Coulomb's force law is in accordance with the Newton's third law of motion. The force exerted by q_1 on q_2 is equal and opposite to the force exerted by q_2 on q_1.

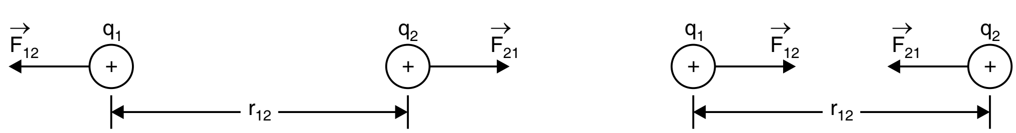


Fig. 1.6

From the Fig. 1.6,

$$\vec{F}_{12} = -\vec{F}_{21}$$

- The force acting on one charge due to other charge is independent of the presence of third charge.

• Principle of Superposition

Let point charges $q_1, q_2, q_3 \ldots q_N$ are placed at the position $\vec{r}_1, \vec{r}_2, \vec{r}_3 \ldots \vec{r}_N$ respectively. According to principle of supperposition the net force one charge is the vector some of all the forces due to other charges.

Now force on q_1 due to q_2,

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2 \vec{r}_{21}}{|\vec{r}_{21}|^3}$$

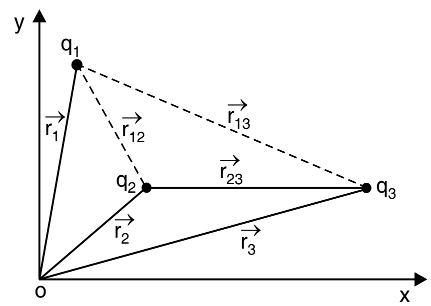


Fig. 1.7

The force on q_1 due to q_3

$$\vec{F}_{13} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2 \vec{r}_{31}}{|\vec{r}_{31}|^3}$$

Similarly,

$$\vec{F}_{1N} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_N \vec{r}_{N1}}{|\vec{r}_{N1}|^3}$$

And the net force on q_1,

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \ldots\ldots + \vec{F}_{1N}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_j (\vec{r}_{j1})}{|\vec{r}_{j1}|^3}$$

Similarly, force on q_2,

$$\vec{F}_2 = \frac{1}{4\pi\varepsilon_0} \frac{q_2 q_j (\vec{r}_{j2})}{|\vec{r}_{j2}|^3}$$

force on q_3,

$$\vec{F}_3 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_3 q_j (\vec{r}_{j3})}{|\vec{r}_{j3}|^3}$$

and

$$\vec{F}_N = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_N q_j (\vec{r}_{jN})}{|\vec{r}_{jN}|^3}$$

The net force on the system of charges

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ldots + \vec{F}_N$$

$$= \frac{1}{4\pi\varepsilon_0} \sum_{\substack{i,j=1, \\ i \neq j}}^{N} \frac{q_i q_j (\vec{r}_{ji})}{|\vec{r}_{ji}|^3}$$

• Electric Field

It is the region in which an electric charge experiences a force. The intensity of electric field is the force experienced by unit positive charge. If a test charge q_0 experiences a force F in electric field, then the intensity of electric field,

$$\vec{E} = \frac{\vec{F}}{q_0}$$

$$\lim_{q_0 \to 0}$$

If q_0 is placed at the distance of r from a source charge q, then

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{qq_0\,(\vec{r})}{|\vec{r}|^3}$$

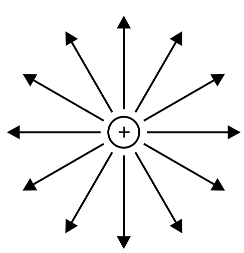
and $$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q\,(\vec{r})}{|\vec{r}|^3}$$

or $$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

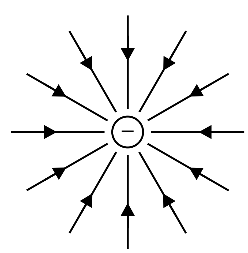
- Electric field intensity is vector quantity and its direction is the direction of force experienced by test charge.

• Electric Field Lines

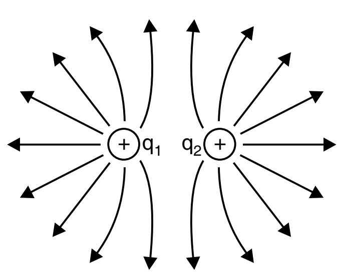
The imaginary path followed by test charge in electric field is called electric field line. The electric field lines of different type 1 of charge distribution can be drawn as given Fig. 1.5.



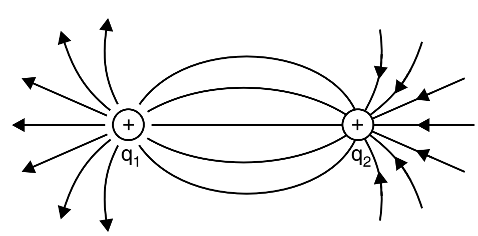
q > 0
(a)



q < 0
(b)



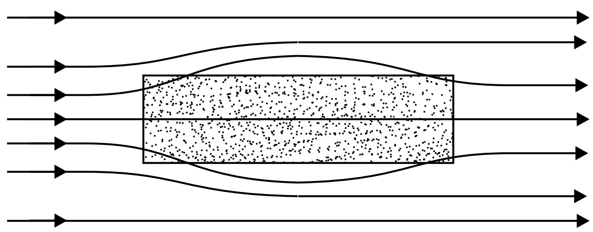
q_1 q_2 > 0
q_1 is +ve
(c)



q_1 q_2 < 0
q_1 is +ve
(d)

Uniform electric field
(e)

Metal sphere placed in uniform electric field
(f)



Dielectric slab placed in uniform electric field
(g)

Fig. 1.8

- **Properties of Electric Field Lines**
  - The electric field lines originate from positive charge and terminate into negative charge.
  - The tangent drawn at any point on field line gives the direction of electric field at that point.

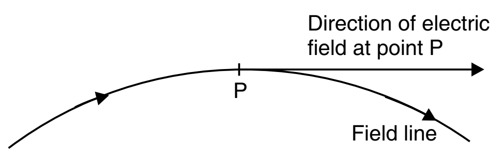


Fig. 1.9

  - Electric field lines do not intersect, because two direction of electric field at a point are not possible.
  - Number of electric field lines is directly proportional to the magnitude of charge.

$$\frac{N_1}{N_2} = \frac{q_1}{q_2}$$

  - Concentration of field lines is more in strong electric field and less in weak electric field.
  - Electric field lines do not exist in metals.
  - Electric field lines move apart when passes through dielectrics.
  - Electric field lines are parallel and equidistant in uniform electric field.
  - The idea of field lines was introduced by Michel Faraday. The electric field can never form a closed loop, as a line never starts and ends on the same charge.
  - If an electric field line is a closed curve, work done round a closed path will not be zero and electric field will not remain conservative.
  - Field lines have tendency to contract longitudinally.
  - Field lines start or end normal on the surface of a conductor.

- **Electric Field Intensity Due to a Point Charge**

Let there be a point P at the distance r from the source charge q where electric field intensity is to be determined.

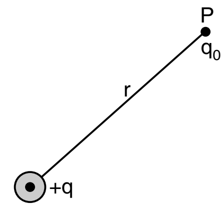


Fig. 1.10

The force experienced by a test charge placed at P,

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q q_0}{r^2}$$

Thus, electric field intensity at point P,

$$E = \frac{F}{q_0}$$

or $$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

In vector form $$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r_2} (\hat{r})$$

$\hat{r}$ is the unit vector along $\vec{r}$ and $\hat{r} = \frac{\vec{r}}{|\vec{r}|}$

$\therefore$ $$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q (\vec{r})}{|\vec{r}|^3}$$

- **Electric Dipole**

It is a system of two equal and opposite charges separated by an infinitely small separation.

- The dipole moment of an electric dipole is the product of the either charge and length of the dipole. It is vector quantity. Its direction is from –ve charge to +ve charge. Therefore dipole moment,

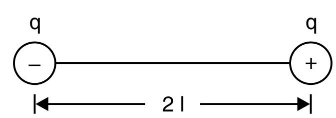


Fig. 1.11

$$\vec{P} = (\vec{q})(2l)$$

or $$\vec{P} = 2\vec{q}\, l$$

- When dipole is placed in uniform electric field, its positive charge experiences a force in the direction of field and negative charge in opposite direction of field in equal magnitude. Hence, there is no net force on the dipole. However, it experiences a torque. The torque on the dipole is equal to the product of force and perpendicular length of the dipole.

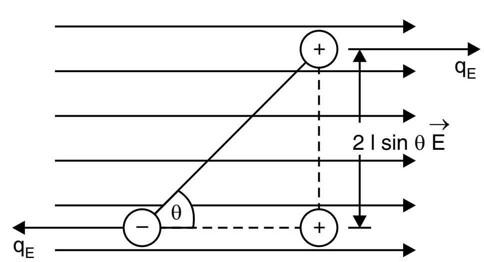


Fig. 1.12

$$\begin{aligned} \tau &= F (2l \sin\theta) \\ &= q\, E\, 2l \sin\theta \\ &= 2ql\, E \sin\theta \\ &= PE \sin\theta \end{aligned}$$

or $$\vec{\tau} = \vec{P} \times \vec{E}.$$

- **Potential Energy of Electric Dipole**
  - Self potential energy of the dipole is the amount of work done during the formation of dipole. It is given as

$$U_1 = -\frac{1}{4\pi\varepsilon_0}\frac{(q)(q)(2l)}{(2l)^2}$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{q}{(2l)^2}(q)\ (2l)$$

$$= -E.P$$

or $$U_1 = -PE.$$

  - When dipole is rotated in electric field the amount of work done is stored in the form of potential energy of the dipole. For small angular displacement the amount of work done.

$$dW = \tau.d\theta$$

$$= PE \sin\theta.\ d\theta$$

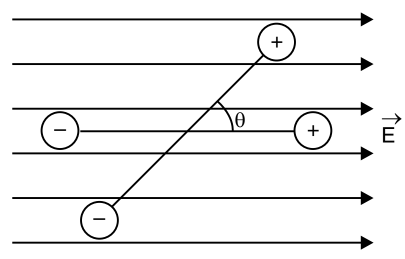


Fig. 1.13

If dipole in rotated from an angle θ_1 to θ_2, the amount of work done or potential energy stored,

$$U_2 = PE\int_{\theta_1}^{\theta_2}\sin\theta\ d\theta$$

$$= PE\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$= PE(\cos\theta_1 - \cos\theta_2)$$

For $$\theta_1 = 0 \text{ and } \theta_2 = 0$$

$$U_2 = PE(1 - \cos\theta)$$

  - Thus, total potential energy of the dipole in electric field is the sum of self potential energy and work done in rotating the dipole

$$U = U_1 + U_2$$

$$= -PE + PE(1-\cos\theta)$$

$$= -PE\cos\theta.$$

- **Electric Field Due To Dipole**

(*i*) **At axial position.** Let there be a point P on the axial position of electric dipole of dipole moment $\vec{P}$ at the distance of r from its mid-point.

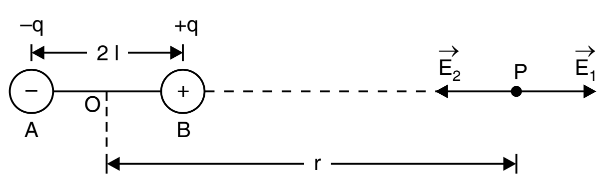


Fig. 1.14

The electric field intensity at P due $+q$,

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-l)^2}(\vec{OP})$$

and electric field intensity at point P due to

$$-q,\ \vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+l)^2}\,(\vec{PO})$$

Thus, net electric field at point P,

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{1}{4\pi\varepsilon_0}\, q\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right](\vec{OP})$$

$$= \frac{1}{4\pi\varepsilon_0}\, q\left[\frac{r^2+l^2+2rl-r^2-r^2+2rl}{(r^2-l^2)^2}\right]\vec{OP}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q(4rl)}{(r^2-l^2)^2}\,(\vec{OP})$$

$$= \frac{1}{4\pi\varepsilon_0}.\frac{2(q\,2l)\,r\,(\vec{OP})}{(r^2-l^2)^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2\vec{P}(r)}{(r^2-l^2)^2}$$

$\because$ $r >> l$ $\therefore$ l^2 can be neglected.

and $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{P}}{r^3}$$

(*ii*) **At equitorial point.** Electric field intensity due to + q at point P,

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+l^2)}\,(\vec{BP})$$

and electric field intensity due – q at P,

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+l)^2}\,(\vec{PA})$$

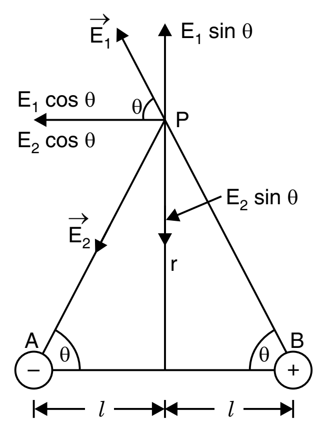


Fig. 1.15

$\vec{E}_1$ and $\vec{E}_2$ can be resolved into two rectangular components *i.e.*, E_1 sin θ, E_1 cos θ and E_1 cos θ_1, E cos θ_2. The magnitude of E_1 and E_2 is same therefore $E_1 \sin\theta_1$ and $E_2 \sin\theta_2$ cancel each other and net electric field at point P,

$$E = E_1\cos\theta + E_2\cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\, q\left[\frac{1}{(r^2+l^2)} + \frac{1}{(r^2+l^2)}\right]\cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\, q\left[\frac{2}{(r^2+l^2)}\right]\frac{l}{\left(\sqrt{r^2+l^2}\right)}$$

or $$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{2ql}{(r^2 + l^2)^{3/2}}$$

$$\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|\vec{P}|}{r^3}$$ [$r >> l$ and l is neglected]

- **Electric Flux**

Electric flux through an elementary area dS is the scalar product of area and electric field intensity.

$$d\phi = \vec{E} \cdot \vec{dS} = E\, dS \cos\theta$$

or $$\phi = \int \vec{E} \cdot \vec{dS} = \int E\, dS \cos\theta$$

where θ is the angle between electric field and normal drawn to the surface.

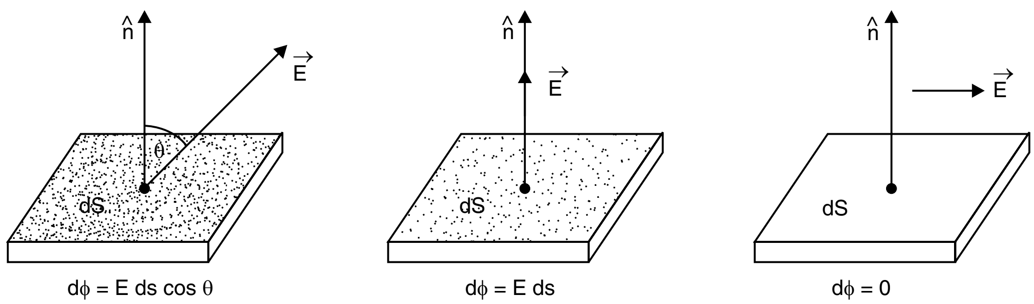


Fig. 1.16

- Electric flux is scalar quantity and its unit is $NC^{-1}\, m^2$ or Vm.
- Normal to the surface is always drawn outward.

- **Gauss's Theorem**

It states that the total electric flus linked with a closed surface is $\left(\frac{1}{\varepsilon_0}\right)$ times the total charge enclosed within the surface.

or $$\oint E\, dS \cos\theta = \frac{1}{\varepsilon_0}(q)$$

- **Electric field intensity due to a point charge**

To find the electric field intensity at P at a distance r from the source charge q a Gaussian surface of radius r can be drawn. Applying Gauss theorem,

$$\oint E\, dS \cos\theta = \frac{1}{\varepsilon_0}(q)$$

$\because$ $$\theta = 0°$$

$\therefore$ $$E\,(4\,\pi r^2) = \frac{1}{\varepsilon_0}(q)$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

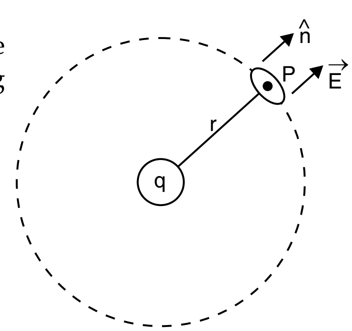


Fig. 1.17

- **Electric field intensity due to a linear charge**

Let there be linear charge of charge density λ and a point *P* at the shortest distance *r* from the linear charge where electric field intensity is to be determine. A Gaussian surface in the form of cylinder of radius can be drawn.

Applying Gauss theorem for surface S_1, S_2 and S_3

$$\oint E\ dS \cos\theta = \frac{1}{\varepsilon_0}(\lambda l)$$

or $$\oint_{S_1} E\ dS \cos\theta + \oint_{S_2} E\ dS \cos\theta + \oint_{S_3} E\ dS \cos\theta = \frac{1}{\varepsilon_0}(\lambda l)$$

$$0 + E\,(2\pi r l) + 0 = \frac{1}{\varepsilon_0}(\lambda l)$$

or $$E = \frac{1}{2\pi\varepsilon_0}.\frac{\lambda}{r}$$

$\because$ $$\lambda = \frac{q}{l}$$

$\therefore$ $$E = \frac{1}{2\pi\varepsilon_0}.\frac{q}{rl}$$

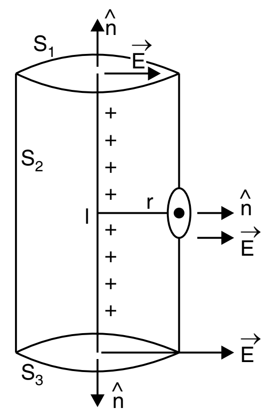


Fig. 1.18

- **Electric field intensity due a surface charge distribution**

Let there be an infinitely large plane sheet of charge of surface charge density σ and a point *P* near to the sheet where electric field intensity is to be determine. A Gaussian surface enclosing the charge of area of cross-section in the form of cylinder can be drawn.

Applying Gauss theorem for surface S_1, S_2 and S_3

$$\oint E\ dS \cos\theta = \frac{1}{\varepsilon_0}(\sigma A)$$

$$\oint_{S_1} E\ dS \cos\theta + \oint_{S_2} E\ dS \cos\theta + \oint_{S_3} E\ dS \cos\theta = \frac{1}{\varepsilon_0}(\sigma A)$$

$$EA + 0 + EA = \frac{1}{\varepsilon_0}(\sigma A)$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

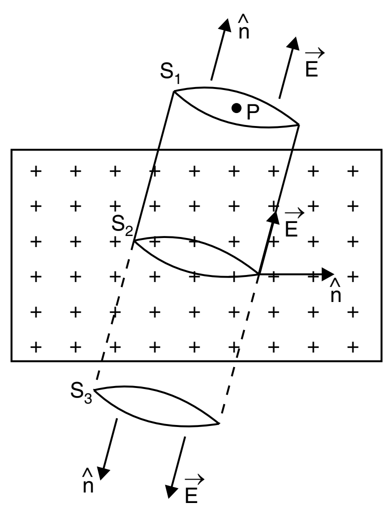


Fig. 1.19

- **Electric field intensity due to a uniformly charged shell**

Let there be a shell of radius *R* having charge *q* uniformly distributed over its surface. The charge exist only at the outer surface of shell. To find the electric field intensity at point *P* at the distance of *r* from the centre of the shell a Gaussian surface of radius *r* can be drawn.

Applying Gauss's theorem for $r < R$, charge enclosed with the Gaussian surface is zero.

$$\therefore \quad \oint E\, dS \cos\theta = \frac{1}{\varepsilon_0}(0)$$

$$\Rightarrow \quad E = 0 \qquad ...(i)$$

for $r = R$

$$\oint E\, dS \cos\theta = \frac{1}{\varepsilon_0}(q)$$

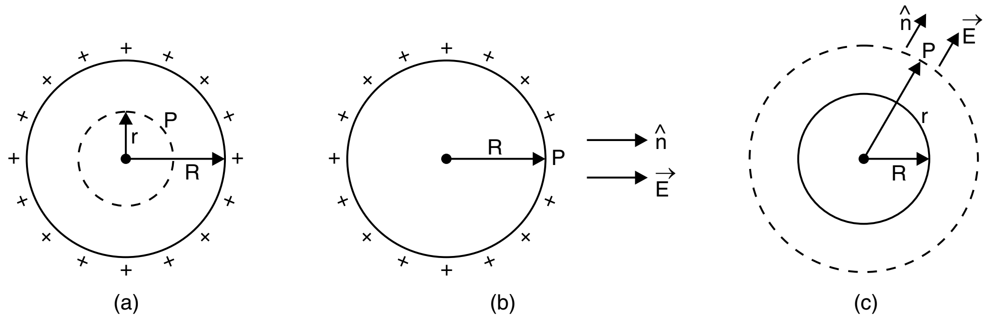


Fig. 1.20

$$E\,(4\pi R^2) = \frac{1}{\varepsilon_0}(q)$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad ...(ii)$$

for $r > R$

$$\oint E\, dS \cos\theta = \frac{1}{\varepsilon_0}(q)$$

$$E\,(4\pi r^2) = \frac{1}{\varepsilon_0}(q)$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad ...(iii)$$

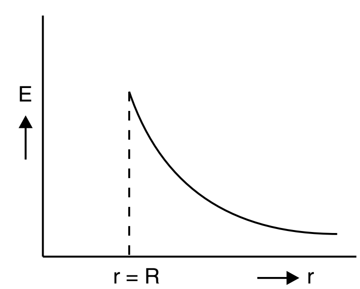


Fig. 1.21

from eqns. (*i*), (*ii*) and (*iii*) the variation of electric field intensity with respect to the distance r measured from the centre of the shell is shown in Fig. 1.21.

• Electric field intensity due to a sphere of charge

Let there be a sphere of charge q and radius R and a point P at the distance of r from the centre of the sphere where electric field intensity is to be determined.

Applying Gauss Theorem for $r < R$.

The charge enclosed within the Gaussian surface.

$$= \frac{q}{4/3\,\pi R^3}\left(\frac{4}{3}\pi r^3\right)$$

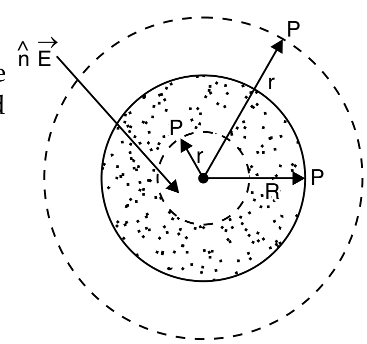


Fig. 1.22

$$= \frac{qr^3}{R^3}$$

$$\therefore \quad \oint E\, dS \cos\theta = \frac{1}{\varepsilon_0}\left(\frac{qr^3}{R^3}\right)$$

$$E\,(4\pi r^2) = \frac{1}{\varepsilon_0}\frac{qr^3}{R^3}$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{qr}{R^3} \qquad ...(i)$$

for $r = R$

$$\oint E\, dS \cos\theta = \frac{1}{\varepsilon_0}(q)$$

$$E\,(4\pi R^2) = \frac{1}{\varepsilon_0}(q)$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2} \qquad ...(ii)$$

for $r > R$

$$\oint E\, dS \cos\theta = \frac{1}{\varepsilon_0}(q)$$

$$E\,(4\pi r^2) = \frac{1}{\varepsilon_0}(q)$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad ...(iii)$$

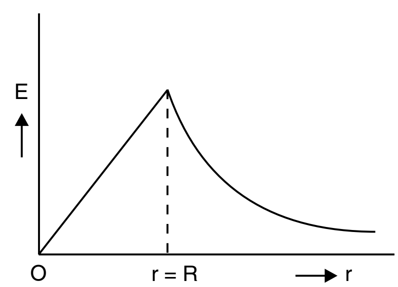


Fig. 1.23

From eqns. (*i*), (*ii*) and (*iii*) the variation of electric field with respect to distance measured from the centre of the sphere of charge is shown in Fig. 1.23.

QUESTIONS FROM TEXTBOOK

1.1. *What is the force between two small charged spheres having charges of 2×10^{-7} C and 3×10^{-7} C placed 30 cm apart in air?*

Sol. Here, $q_1 = 2 \times 10^{-7}\,C, \quad q_2 = 3 \times 10^{-7}\,C$

$r = 30$ cm $= 0.3$ m

$$\therefore \quad F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

$$= \frac{9\times10^9 \times 2\times10^{-7} \times 3\times10^{-7}}{(0.3)^2}$$

$= 6 \times 10^{-3}$ N (Repulsive)

1.2. *The electrostatic force on a small sphere of charge 0.4 μC due to another small sphere of charge –0.8 μc in air is 0.2 N.*

(a) What is the distance between the two spheres?

(b) What is the force on the second sphere due to the first?

Sol. (*a*) Here,

$$F = 0.2 \text{ N}$$

$$q_1 = 0.4 \ \mu\text{C} = 0.4 \times 10^{-6} \text{ C}$$

$$q_2 = 0.8 \ \mu\text{C} = 0.8 \times 10^{-6} \text{ C}$$

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$

Thus,

$$\boxed{r^2 = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{F}}$$

$$r^2 = \frac{9 \times 10^9 \times 0.4 \times 10^{-6} \times 0.8 \times 10^{-6}}{0.2}$$

$$r^2 = 36 \times 4 \times 10^{-4} = 144 \times 10^{-4}$$

$$r = 12 \times 10^{-2} \text{ m} = 12 \text{ cm.}$$

(*b*) Force on the second sphere due to the first is same, *i.e.,* 0.2 N and force is attractive as charges are unlike.

1.3. *Check that the ratio* $\frac{ke^2}{Gm_e m_p}$ *is dimensionless. Look up a table of physical constants and determine the value of this ratio. What does the ratio signify?*

$$F = K\frac{q_1 q_2}{r^2}$$

$$F = G\frac{m_1 m_2}{r^2}$$

Sol.

$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$m_p = 1.67 \times 10^{-27} \text{ kg}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

and

$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \ \frac{\text{Nm}^2}{\text{C}^2}$$

Now,

$$\frac{ke^2}{Gm_e m_p} = \frac{9 \times 10^9 \dfrac{\text{Nm}^2}{\text{C}^2} \times 1.6 \times 10^{-19}\text{C} \times 1.6 \times 10^{-19}\text{C}}{6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2} \times 9.1 \times 10^{-31}\text{kg} \times 1.67 \times 10^{-27}\text{kg}}$$

$$= 2 \times 27 \times 10^{39} \text{ which is dimensionless.}$$

It also establishes that the electrostatic force is about 10^{39} times stronger than the gravitational force.

1.4. *(a) Explain the meaning of the statement electric charge of a body is quantised.*

(b) Why can one ignore quantisation of electric charge when dealing with macroscopic i.e., large scale charges?

Sol. (*a*) **Quantisation of Electric Charges:** Electric charge is due to transfer of electron. The electric charge is always an integral multiple of *e* which is termed as quantisation of charge.

i.e., $q = \pm ne$

Here $+e$ is taken as charge on a proton while $-e$ is taken as charge on an electron. The charge on a proton and an electron are numerically equal *i.e.,* 1.6×10^{-19} C but opposite in sign.

"Quantisation is a property due to which charge exists in discrete packets in multiple of $\pm 1.6 \times 10^{-19}$ rather than in continuous amounts."

(*b*) Based on many practical phenomena, we may ignore quantisation of electric charge and consider the charge to be continuous. Large scale electric charge may be considered as integral multiple of the basic unit '*e*'. The "graininess" of charge can be ignored and it can be imagined that this large scale charge can be charged continuously and its quantisation is insignificant and can be ignored.

1.5. *When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservation of charge.*

Sol. Total charge of an isolated system of objects is always conserved.

As a consequence of conservation of charge, when two charged conductors of same size and same material carrying charges Q_1 and Q_2 respectively are brought in contact and separated, the charge on each conductor will be $\frac{Q_1 + Q_2}{2}$. This condition, however, does not hold true if the conductors are of different sizes or of different material. In that case the charges on the conductors will be Q_1' and Q_2' respectively,

where $Q_1 + Q_2 = Q_1' + Q_2'$.

Example. When a glass rod is rubbed with silk cloth, glass rod becomes positively charged while silk cloth becomes negatively charged. The amount of positive charge on the glass rod is found to be exactly the same as negative charge on silk cloth. Thus, the system of glass rod and silk cloth, which was neutral before rubbing, still possesses no net charge after rubbing.

1.6. *Four point charges $q_A = 2\ \mu C$, $q_B = -5\ \mu C$, $q_C = 2\ \mu C$, and $q_D = -5\ \mu C$ are located at the corners of a square ABCD of side 10 cm. What is the force on a charge of 1 μC placed at the centre of the square?*

Sol. Suppose a square ABCD with each side of 10 cm and centre 0. At the centre, the charge of 1 μC is placed.

$$q_D = -5\ \mu C$$
$$q_C = 2\ \mu C$$
$$q_A = 2\ \mu C$$
$$q_B = -5\ \mu C$$

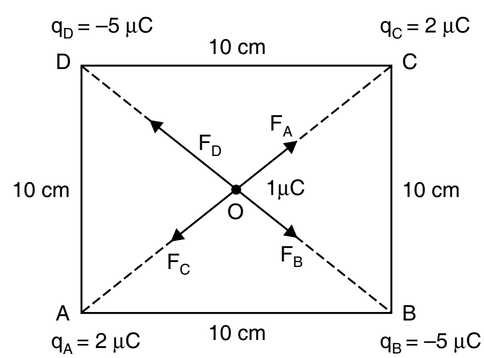


Fig. 1.24

As $q_A = q_C$, the charge of 1 μC experiences equal and opposite forces F_A and F_C due to charges q_A and q_C.

At the same time, the charge 1 μC experiences equal and opposite forces. F_B and F_D due to equal charges q_B and q_D at B and D. Thus, the net force on charge of 1 μC due to the given charges is zero.

1.7. *(a) An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not?*

(b) Explain why two field lines never cross each other at any point?

Sol. (*a*) Lines of forces are the path of a small +ve change in electric field and path of test charge cannot be in breaking. It is always point to point or continuous. The direction of electric field at a point is displayed by the tangent at that point on a line of force. Generally the direction of electric field changes from point to point. Therefore, the lines of force are generally, curved lines. Further, they are continuous curves and cannot have sudden breaks. Even if it is so, the absence of electric field at the break points will be indicated by it.

(*b*) The path of small +ve charge at a point of intersection of field lines must be in two directions which is never possible. So electric field lines can never intersect. At the point of intersection, we can draw two tangents to the lines of force. That is why two field lines never cross each other at any point.

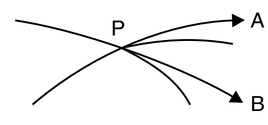


Fig. 1.25

This would mean two directions of electric field intensity at the point of intersection, which is not possible.

1.8. *Two point charges q_A = 3 μC and q_B = –3 μC are located 20 cm apart in vacuum.*

(a) What is the electric field at the midpoint O of the line AB joining the two charges?

(b) If a negative test charge of magnitude 1.5 × 10^{-9} C is placed at this point, what is the force experienced by the test charge?

Sol.

$$q_A = 3\ \mu C = 3 \times 10^{-6}\ C$$
$$q_B = -3\ \mu C = -3 \times 10^{-6}\ C$$

and d = 20 cm

(*a*)

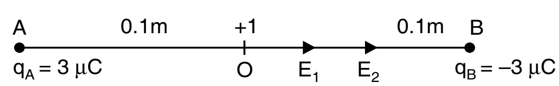


Fig. 1.26

Let us assume that a unit positive test charge is placed at 0. q_A will repel this test charge while q_B will attract. Hence, $\vec{E}_1$ and $\vec{E}_2$ both are directed towards $\overrightarrow{OB}$.

$$\therefore \quad E = \vec{E}_1 + \vec{E}_2$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_A}{r^2} + \frac{1}{4\pi\varepsilon_0}\frac{q_B}{r^2} = \frac{1}{4\pi\varepsilon_0 r^2}[q_A + q_B]$$

$$= \frac{9\times10^9}{(0.1)^2}\ [3 \times 10^{-6} + 3 \times 10^{-6}]$$

$$= 5.4 \times 10^6\ NC^{-1} \text{ along } OB.$$

(*b*) As a negative test charge of q_0 = – 1.5 × 10^{-6} C is placed at 0. q_A will attract it while q_B will repel. Therefore, the net force

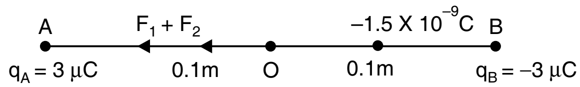


Fig. 1.27

$$F = F_1 + F_2$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

$$F = |F_1| + |F_2|$$

$$= \frac{Kq_Aq_0}{r^2} + \frac{Kq_Bq_0}{r^2}$$

$$= \frac{Kq_0}{r^2}[q_A + q_0]$$

$$= \frac{9\times10^9 \times 1.5\times10^{-19}\,[3\times10^{-6} + 3\times10^{-6}]}{(0.1)^2}$$

$$= \frac{9\times1.50\times6\times10^{-6+9-9}}{0.1\times0.1}$$

$$= \frac{9\times10^9 \times 3\times10^{-6}\times1.5\times10^{-9}}{(0.1)^2} + \frac{9\times10^9\times3\times10^{-6}\times1.5\times10^{-9}}{(0.1)^2}$$

$$= \frac{9\times10^9\times3\times10^{-6}\times1.5\times10^{-9}\times2}{(0.1)^2}$$

$$= 8.1 \times 10^{-3}\text{ N.}$$

1.9. *A system has two charges q_A = 2.5 × 10^{-7} C and q_B = –2.5 × 10^{-7} C located at points A: (0, 0, –15 cm) and B: (0, 0, + 15 cm), respectively. What are the total charge and electric dipole moment of the system?*

Sol. Total charge, $q = q_A + q_B$

$= 2.5 \times 10^{-7} - 2.5 \times 10^{-7} = 0$

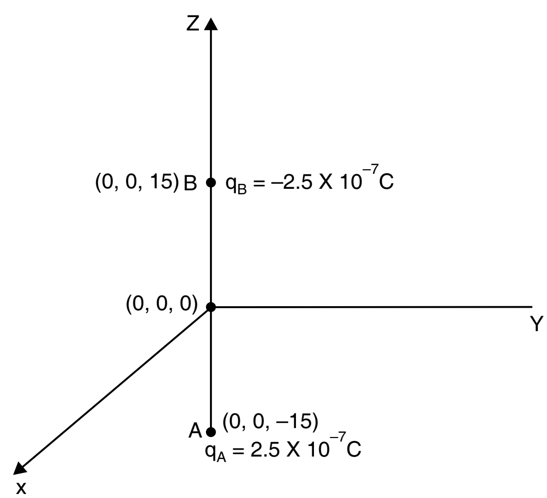


Fig. 1.28

$$a = AB = 15 - (-15) = 30 \text{ cm} = 0.3 \text{ m}$$

Electric dipole moment

$$\boxed{p = q \cdot a}$$

$$= 2.5 \times 10^{-7} \text{ (0.3 m)}$$

$$= 7.5 \times 10^{-8} \text{ Cm (along } -Z\text{-axis).}$$

1.10. *An electric dipole with dipole moment 4×10^{-9} C m is aligned at 30° with the direction of a uniform electric field of magnitude 5×10^{4} NC^{-1}. Calculate the magnitude of the torque acting on the dipole.*

Sol. Given, $P = 4 \times 10^{-9}$ Cm; $\theta = 30°$

$E = 5 \times 10^{4}$ NC^{-1}

Torque, $\boxed{\tau = p \times \vec{E}} = p.E \sin\theta$

$$= 4 \times 10^{-9} \times 5 \times 10^{4} \times \sin 30°$$

or $\tau = 4 \times 10^{-9} \times 5 \times 10^{4} \times \frac{1}{2}$

or $\tau = 10^{-4}$ Nm

1.11. *A polythene piece rubbed with wool is found to have a negative charge of 3.2×10^{-7} C.*

(a) Estimate the number of electrons transferred (from which to which?)

(b) Is there a transfer of mass from wool to polythene?

Sol. (*a*) Given, $q = -3.2 \times 10^{-7}$ C

$e = -1.6 \times 10^{-19}$ C

∴ number of electrons transferred

$$n = \frac{q}{e} = \frac{-3.2 \times 10^{-7}}{-1.6 \times 10^{-19}} = 2 \times 10^{12}$$

Electrons are transferred from wool to polythene during rubbing as polythene has negative charge.

(*b*) From wool to polythene, certainly there is a transfer of mass.

Mass of an electron $= 9.1 \times 10^{-31}$ kg

Thus, amount of mass transferred $= 2 \times 10^{12} \times 9.1 \times 10^{-31}$ kg

$= 18.2 \times 10^{-19}$ kg.

1.12. *(a) Two insulated charged copper spheres A and B have their centres separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion if the charge on each is 6.5×10^{-7} C ? The radii of A and B are negligible compared to the distance of separation.*

(b) What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved?

Sol. (*a*) $q_1 = 6.5 \times 10^{-7}$ C

$q_2 = 6.5 \times 10^{-7}$ C

$r = 50$ cm $= 0.50$ m

$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^{9} \text{ Nm}^2 \text{ C}^{-2}$$

$F = ?$

Coulomb's law

$$F = k\frac{q_1 q_2}{r^2}$$

$$= \frac{9\times10^9 \times 6.5\times10^{-7} \times 6.5\times10^{-7}}{(0.50)^2}\text{ N}$$

$$F = 1.5 \times 10^{-2}\text{ N}$$

(*b*) Now, if each sphere is charged double, and the distance between them is halved then the force of repulsion is:

$$F = k.\frac{2q_1\,2q_2}{(r/2)^2}$$

$$F = 16\,k.\frac{q_1 q_2}{r^2}$$

$$= 16 \times 1.5 \times 10^{-2} = 24 \times 10^{-2}$$

$$F = 0.24\text{ N.}$$

1.13. *Suppose the spheres A and B in question 1.12 have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B?*

Sol. Charge on each of the sphere A and B

$$= q = 6.5 \times 10^{-7}\text{ C}$$

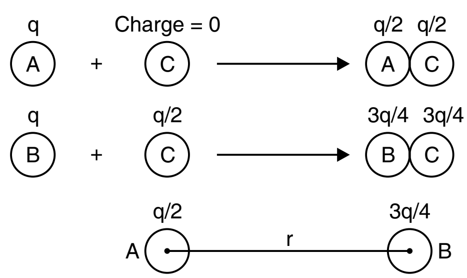


Fig. 1.29

When a similar but uncharged sphere C is placed in contact with sphere A, each sphere shares a charge $q/2$, equally.

Now, if the sphere C is placed in contact with sphere B, the charge is equally redistributed, so that

Charge on sphere B or $C = \frac{1}{2}(q + q/2) = 3q/4$

Thus, the force of repulsion between A and B is

$$F = \frac{1}{4\pi\varepsilon_0}.\frac{\frac{3q}{4}.q/2}{(r/2)^2}$$

$$= \frac{3}{8}.\frac{1}{4\pi\varepsilon_0}.\frac{q^2}{r^2}$$

$$= \frac{3}{8} \times 1.5 \times 10^{-2} \text{ N}$$
$$= 0.5625 \times 10^{-2} \text{ N}$$
$$= 5.7 \times 10^{-3} \text{ N.}$$

1.14. *Figure shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio?*

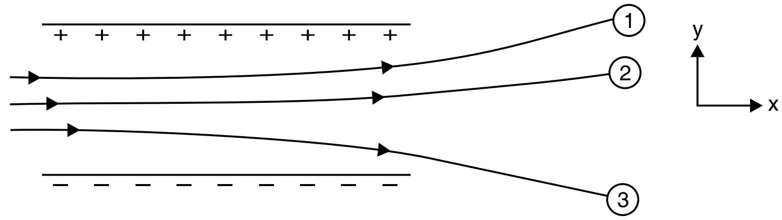


Fig. 1.30

Sol. Particles (1) and (2) are negatively charged and particle (3) is positively charged, since the charged particles are deflected towards oppositely charged plates.

Further, as the displacement $y \propto (e/m)$ therefore, particle (3) having maximum value of y has the highest charge to mass ratio.

1.15. *Consider a uniform electric field* $E = 3 \times 10^3\ \hat{i}$ *N/C. (a) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the yz plane? (b) What is the flux through the same square if the normal to its plane makes a 60° angle with the x-axis?*

Sol. Given $\vec{E} = 3 \times 10^3\ \hat{i}\ \text{NC}^{-1}$

(*a*) ΔS (Area of the square) = 10 × 10 = 100 cm^2 = 10^{-2} m^{-2}

The area of a surface can be represented as a vector along normal to the surface. Since normal to the square is along x-axis, we have

$$\Delta \vec{S} = 10^{-2}\ \hat{i}\ \text{m}^{-2}$$

Electric flux through the square

$$\phi = \vec{E} \cdot \overrightarrow{\Delta S}$$

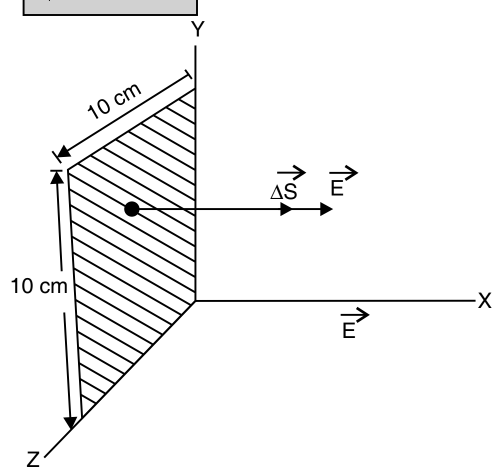


Fig. 1.31

$$= \left(3 \times 10^3 \,\hat{i}\right).\left(10^{-2}\,\hat{i}\right)$$
$$= 30 \text{ Nm}^2 \text{ C}^{-1}$$

(*b*) Given, the angle between area vector and the electric field is 60°. Therefore,

$$\phi = \vec{E}\,.\,\Delta\vec{S}$$
$$= E\,.\,dS \cos 60°$$
$$= 3 \times 10^3 \times 10^{-2} \times \frac{1}{2}$$
$$= 15 \text{ Nm}^2 \text{ C}^{-1}$$

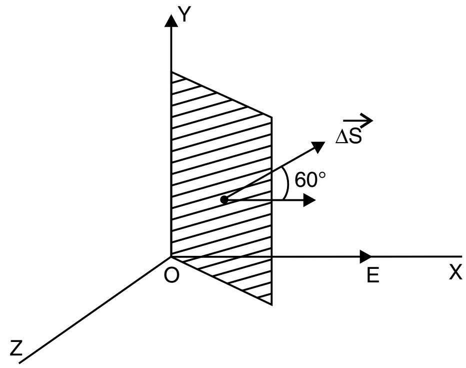


Fig. 1.32

1.16. *What is the net flux of the uniform electric field of question 1.15 through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes?*

Sol. Given, $\vec{E} = 3 \times 10^3 \,\hat{i}\text{ NC}^{-1}$

The area of each face out of the six faces of the cube = 20 × 20 = 400 cm^2 = 4 × 10^{-2} m^2.
The area vector of four faces of cube *ABGH, OBGF*, *OCDF* and *ACDH* is along +Y axis, –*Z* axis, –Y axis and +Z axis. The direction of E (+*X* axis). $\overline{\Delta S}$ is perpendicular to each other so flux through there surfaces are zero.

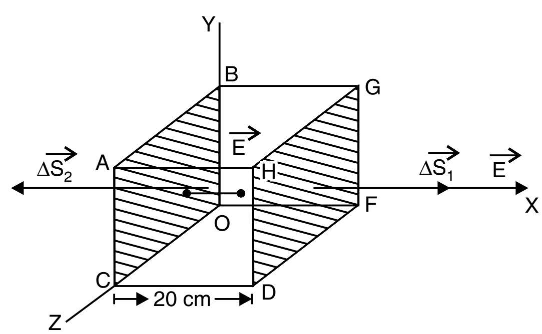


Fig. 1.33

Hence, the net electric flux through the cube

$$\phi = \vec{E}\,.\,\Delta\vec{S}_1 + \vec{E}\,.\,\Delta\vec{S}_2$$

Now, $\left|\Delta\vec{S}_1\right| = \left|\Delta\vec{S}_2\right| = 4 \times 10^{-2}$ m^2 and the angle between $\vec{E}$ and $\Delta\vec{S}_1$ is 0°, whereas the angle between $\vec{E}$ and $\Delta\vec{S}_2$ is 180°.

Thus,
$$\begin{aligned}\phi &= E\,\Delta S_1 \cos 0° + E\,\Delta S_2 \cos 180° \\ &= E \times 4 \times 10^{-2} \times 1 + E \times 4 \times 10^{-2} \times (-1) \\ &= 0\end{aligned}$$

Now, it is established that if some electric flux enters the cube the same amount of flux leaves through the other face, so that the net flux is zero.

1.17. *Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is 8.0 × 10³ Nm²/C. (a) What is the net charge inside the box? (b) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or why not?*

Sol. Given, $\phi = 8.0 \times 10^3$ Nm^2/C

(*a*) As
$$\boxed{\phi = \frac{q}{\varepsilon_0}}$$

Hence, $q = \phi\,.\,\varepsilon_0$

or
$$\begin{aligned}q &= 8.0 \times 10^3 \times 9 \times 10^{-12} \\ &= 0.07 \times 10^{-6}\end{aligned}$$

i.e., $q = 0.07\ \mu$C

(*b*) No, a conclusion cannot be made that there was no charge in the box. Perhaps, the net charge inside the box is zero.

1.18. *A point charge + 10 μC is a distance 5 cm directly above the centre of a square of side 10 cm, as shown in Fig. What is the magnitude of the electric flux through the square?*

(Hint: Think of the square as one face of a cube with edge 10 cm).

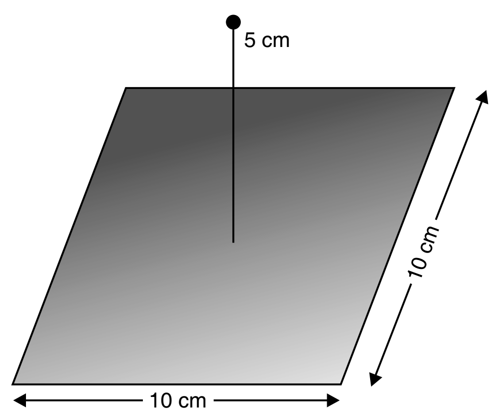


Fig. 1.34

Sol. Let us assume that the charge $q = \pm\, 10\ \mu\text{C} = 10^{-5}$ C is placed at a distance of 5 cm from the square *ABCD* of each side 10 cm. The square *ABCD* can be considered as one of the six faces of a cubic Gaussian surface of each side 10 cm.

Now, the total electric flux through the faces of the cube as per Gaussian theorem

$$\boxed{\phi = \frac{q}{\varepsilon_0}}$$

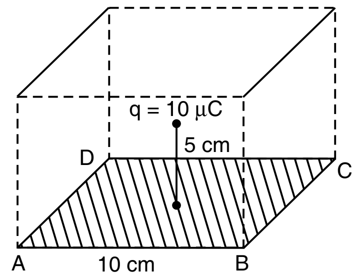


Fig. 1.35

Therefore, the total electric flux through the square *ABCD* will be

$$\phi_E = \frac{1}{6}\times\phi = \frac{1}{6}\times\frac{q}{\varepsilon_0}$$

$$= \frac{1}{6}\times\frac{10^{-5}}{8.854\times10^{-12}}$$

$$= 1.88 \times 10^5 \text{ Nm}^2 \text{ C}^{-1}$$

1.19. *A point charge of 2.0 μC is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface?*

Sol. Given, $q = 2.0\ \mu\text{C} = 2.0 \times 10^{-6}$ C

The total flux through the surface of the cube (using Gaussian theorem) is given by

$$\boxed{\phi = \frac{q}{\varepsilon_0}}$$

$$= \frac{2.0\times10^{-6}}{8.854\times10^{-12}}$$

$$= 2.26 \times 10^5 \text{ Nm}^2 \text{ C}^{-1}$$

1.20. *A point charge causes an electric flux of –1.0 × 10³ Nm²/C to pass through a spherical Gaussian surface of 10.0 cm radius centred on the charge. (a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface? (b) What is the value of the point charge?*

Sol. Given, $\phi = -1.0 \times 10^3$ Nm²/C

$r_1 = 0.1$ m, $r_2 = 0.2$ m

(*a*) Doubling the radius of Gaussian surface will not affect the electric flux since the charge enclosed is the same in the two cases.

Thus, the flux will remain be the same *i.e.*, -1.0×10^3 Nm²/C

(*b*) $$\boxed{\phi = \frac{q}{\varepsilon_0}}$$

∴ $q = \phi \,.\, \varepsilon_0$

or, $q = -1.0 \times 10^3 \times 8.8 \times 10^{-12}$

$= -8.8 \times 10^{-9}$ C

$q = -8.8$ NC

1.21. *A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is 1.5 × 10³ N/C and points radially inward, what is the net charge on the sphere?*

Sol. Given, $r = 10 \text{ cm} = 0.1 \text{ m}$

$E = 1.5 \times 10^3 \text{ N/C at } d = 0.2 \text{ m}$

As, $$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$$

then, $q = E \cdot 4\pi\,\varepsilon_0 \cdot r^2$

or, $$q = 1.5 \times 10^3 \times 4\pi \times \left(\frac{1}{4\pi \times 9 \times 10^9}\right) \times (0.2)^2$$

or, $$q = \frac{6}{9} \times 10^{-8} = \frac{60}{9} \times 10^{-9}$$

$= 6.67 \times 10^{-9} \text{ C}$

Here, q is negative since electric field is directed inward.

Thus, $q = 6.67 \times 10^{-9} \text{ C}$

$= -6.67 \text{ nC.}$

1.22. *A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of 80.0 μC/m². (a) Find the charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?*

Sol. Given, $$r = \frac{2.4}{2} = 1.2 \text{ m}$$

$\sigma = 80 \times 10^{-6} \text{ C/m}^2$

(*a*) Charge on sphere

$$\boxed{q = \sigma \cdot A} = \sigma \cdot 4\pi r^2$$

or, $q = 80 \times 10^{-6} \times 4 \times 3.14 \times (1.2)^2$

$q = 1.45 \times 10^{-3} \text{ C}$

(*b*) The total electric flux leaving the surface of the sphere

$$\boxed{\phi = \frac{q}{\varepsilon_0}}$$

$$= \frac{1.45 \times 10^{-3}}{9 \times 10^{-12}} = 1.6 \times 10^8 \text{ Nm}^2\text{/C}$$

1.23. *An infinite line charge produces a field of 9 × 10⁴ N/C at distance of 2 cm. Calculate the linear charge density.*

Sol. Given, $E = 9 \times 10^4 \text{ N/C}$

$r = 2 \times 10^{-2} \text{ m}$

As, $$\boxed{E = \frac{\lambda}{2\pi r\varepsilon_0}}$$

∴ $\lambda = E \cdot 2\pi r \cdot \varepsilon_0$

$$\lambda = \frac{E \cdot 2\pi r}{4\pi K}$$

or, $$\lambda = \frac{9 \times 10^4 \times 2\pi \times 2 \times 10^{-2}}{4\pi \times 9 \times 10^9} = \lambda = \frac{E \cdot 2\pi r}{4\pi\, k}$$

$$= 10^{-7} = 10 \times 10^{-6}$$

or, $\lambda = 10\,\mu C/m.$

1.24. *Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude 17.0×10^{-22} C/m^2. What is E: (a) in the outer region of the first plate, (b) in the outer region of second plate, and (c) between the plates?*

Sol. Given, $\sigma = 17.0 \times 10^{-22}$ C/m^2

(*a*) To the left of the plates, electric fields are equal and opposite as plates are close to each other electric field is zero as surface charge density in outer side is zero.

(*b*) To the right of the plates, electric fields are equal and opposite as plates are close to each other electric field is zero.

(*c*) Electric fields between the plates are in same direction as total E.F. on both sides of plate due to σ surface charge density = $\dfrac{\sigma}{\varepsilon_0}$

So EF. of inner side of plate = $\dfrac{\sigma}{2\varepsilon_0}$

and for both plate $E = \dfrac{\sigma}{2\varepsilon_0} + \dfrac{\sigma}{2\varepsilon_0}$

$$E = \frac{\sigma}{\varepsilon_0} = \sigma \times 4\pi \times 9 \times 10^9$$

or, $E = 17.0 \times 10^{-22} \times 4 \times 3.14 \times 9 \times 10^9$

or, $E = 1921.7 \times 10^{-13} = 1.92 \times 10^{-10}$ N/C.

1.25. *An oil drop of 12 excess electrons is held stationary under a constant electric field of 2.55×10^4 NC^{-1} in Millikan's oil drop experiment. The density of the oil is 1.26 g cm^{-3}. Estimate the radius of the drop.*
($g = 9.81$ ms^{-2} : $e = 1.60 \times 10^{-19}$ C).

Sol. Given, $E = 2.25 \times 10^4$ NC^{-1}

$n = 12$

$\rho = 1.26$ gm cm^{-3} or 1.26×10^3 kg m^{-3}

Since, the droplet is stationary weight of the droplet = force due to the electric field

$$\therefore \quad \frac{4}{3}\pi r^3 \rho g = Ene \qquad \boxed{mg = Eq}$$

or, $$r^3 = \frac{3Ene}{4\pi \rho g}$$

or $$r^3 = \frac{3 \times 2.55 \times 10^4 \times 12 \times 1.6 \times 10^{-19}}{4 \times 3.14 \times 1.26 \times 10^3 \times 9.81}$$

$$= 0.9 \times 10^{-18}$$

or $r = (0.9 \times 10^{-18})^{1/3}$

$r = 9.81 \times 10^{-7}$ m.

1.26. *Which among the curves shown in the figure cannot possibly represent electrostatic field lines?*

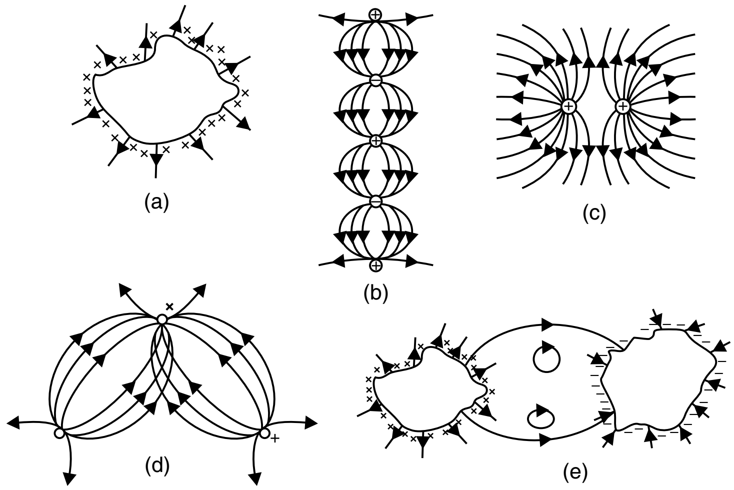


Fig. 1.36

Sol. (*a*) Figure (*a*) cannot represent electrostatic field lines since electrostatic field lines start or end only at 90° to the surface of the conductor.

(*b*) Figure (*b*) too cannot represent electrostatic field lines as electrostatic field lines do not start from a negative charge.

(*c*) Electrostatic field lines are represented by figure (*c*).

(*d*) Figure (*d*) cannot represent electrostatic field lines since no two such lines of force can intersect each other.

(*e*) As electrostatic field lines cannot form closed loop, therefore figure (*d*) also does not represent electrostatic field lines.

1.27. *In a certain region of space, electric field is along the z-direction throughout. The magnitude of electric field is, however, not constant but increases uniformly along the positive z-direction, at the rate of 10^5 NC^{-1} per metre. What are the force and torque experienced by a system having a total dipole moment equal to 10^{-7} cm in the negative z-direction?*

Sol. Force acting on an electric dipole in the positive z-direction which is placed in a non-uniform electric field.

$$F = p_x \frac{\partial E}{\partial x} + p_y \frac{\partial E}{\partial y} + p_z \frac{\partial E}{\partial z}$$

As, the electric field changes uniformly in the positive z-direction, only,

Thus,

$$\frac{\partial E}{\partial z} = +10^5 \text{ NC}^{-1} \text{ m}^{-1}$$

$$\frac{\partial E}{dy} = 0 \quad \text{and} \quad \frac{dE}{dx} = 0$$

As, the system has the total dipole moment equal to 10^{-7} Cm in the *negative* z-direction,

Thus,

$$p_x = 0, \quad p_y = 0, \quad p_z = -10^{-7} \text{ cm}$$

$$\therefore \quad F = 0 + 0 - 10^{-7} \times 10^5 = -10^{-2} \text{ N}$$

It is indicated by the negative sign that the force 10^{-2} N acts in the negative z-direction.

In an electric field $\vec{E}$, the torque on dipole moment $\vec{P}$ is given by

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$\left|\vec{\tau}\right| = pE \sin \theta$$

As $\vec{P}$ and $\vec{E}$ are acting in opposite direction,

$$\theta = 180°,$$

so, $$\left|\vec{\tau}\right| = pE \sin 180° = 0.$$

1.28. *(a) A conductor A with a cavity as shown in figure (a) is given a charge Q. Show that the entire charge must appear on the outer surface of the conductor. (b) Another conductor B with charge q is inserted into the cavity keeping B insulated from A. Show that the total charge on the outside surface of A is Q + q [Fig. (b)]. (c) A sensitive instrument is to be shielded from the strong electrostatic fields in its environment. Suggest a possible way.*

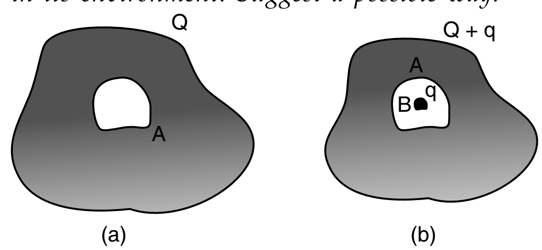


Fig. 1.37

Sol. (*a*) Let us take a Gaussian surface which is lying completely within the conductor and enclosing the cavity. According to the Gaussian theorem the charge enclosed by Gaussian surface must be zero as *electric field vanishes everywhere inside a conductor.* Thus, electric field vanishes inside the cavity. Therefore, charges which are supplied to the conductor reside on its outer surface.

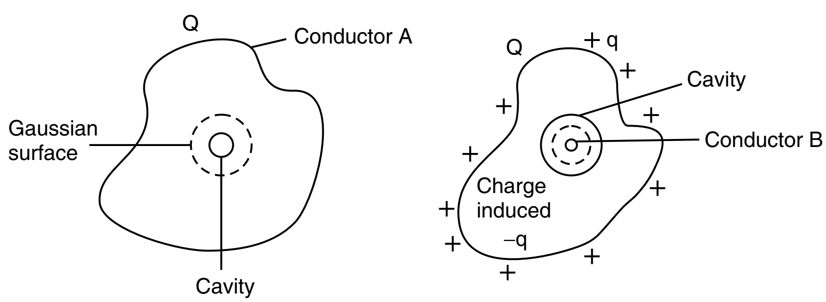


Fig. 1.38

(*b*) Let us take a Gaussian surface inside the conductor which is quite close to the cavity. According to the Gaussian theorem

$$\phi_E = \int E \, . \, ds = \frac{\text{total charge}}{\varepsilon_0}$$

(as the electric field inside the conductor is zero)

The total charge enclosed by the Gaussian surface must be zero. This requires a charge of $-q$ units to be induced on the inner surface of the hollow conductor A. But an equal and opposite charge $+q$ units must appear on the outer surface of conductor A, so that the total charge on the outer surface of A is $Q + q$.

(*c*) Use a metallic surface to enclose the sensitive instrument fully safe. Because of the electrostatic shielding, the electric field inside the metal surface vanishes to zero and all charge reside on outer surface.

1.29. *A hollow charged conductor has a tiny hole cut into its surface. Show that the electric field in the hole is* $(\sigma/2\varepsilon_0)\ \hat{\boldsymbol{n}}$*, where* $\hat{\boldsymbol{n}}$ *is the unit vector in the outward normal direction, and* σ *is the surface charge density near the hole.*

Sol. Let us take a charged conductor with the hole filled up, as shown by shaded portion in the figure.

We find with the application of Gaussian theorem that field inside is zero and just outside is $\frac{\sigma}{\varepsilon_0}\hat{n}$. This field can be viewed as the superposition of the field E_2 due to the filled up hole plus the field E_1 due to the rest of the charged conductor.

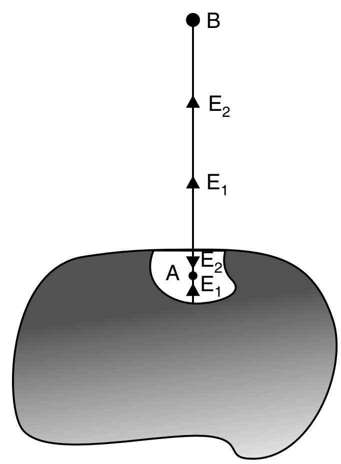


Fig. 1.39

The two fields (E_1 and E_2) must be equal and opposite as the field vanishes inside the conductor.

Thus, $E_1 - E_2 = 0$

Now, the field outside the conductor is given by

$$E_1 + E_2 = \frac{\sigma}{\varepsilon_0}$$

$$\therefore \quad 2E_1 = \frac{\sigma}{\varepsilon_0}$$

or

$$E_1 = \frac{\sigma}{2\varepsilon_0}$$

Therefore, field in the hole (due to the rest of the conductor) is given as:

$$E_1 = \frac{\sigma}{2\varepsilon_0}\hat{n} \quad (\hat{n} \rightarrow \text{unit vector in the outward normal direction})$$

1.30. *Show that electric field due to line charge at any plane is same in magnitude and directed radially upward.*

Sol. A thin long straight line L of charge having uniform linear charge density λ is shown by figure. By symmetry, it follows that electric field due to line charge at distance r in any plane is same in magnitude and directed radially upward.

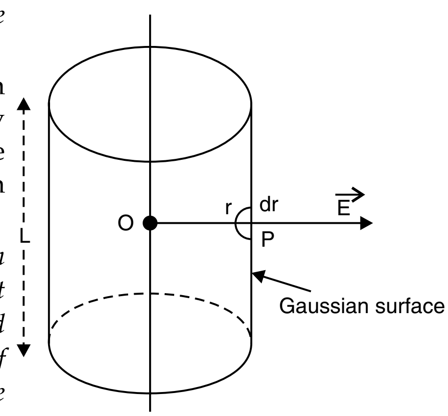


Fig. 1.40

1.31. *It is now believed that protons and neutrons (which constitute nuclei of ordinary matter) are themselves built out of more elementary units called quarks. A proton and a neutron consist of three quarks each. Two types of quarks, the so called 'up' quark (denoted by u) of charge + (2/3) e, and the 'down' quark (denoted by d) of charge*

(–1/3) e, together with electrons build up ordinary matter. (Quarks of other types have also been found which give rise to different unusual varieties of matter.) Suggest a possible quark composition of a proton and a neutron.

Sol. Charge on 'up' quark, $(u) = +\frac{2}{3}e$

Charge on 'down' quark, $(d) = -\frac{1}{3}e$

Charge on a proton = e

Charge on a neutron = 0

Let a proton contains x 'up' quarks and $(3 - x)$ 'down' quarks. Then total charge on a proton is

$$ux + d(3 - x) = e$$

or, $$+\frac{2}{3}ex - \frac{1}{3}e(3-x) = e$$

or, $$+\frac{2}{3}x - 1 + \frac{x}{3} = 1$$

or, $$x = 2$$

and $$3 - x = 3 - 2 = 1$$

i.e., proton contains 2 'up' quarks and 1 'down' quark. Its quark composition should be *uud*.

Let a neutron contains y 'up' quarks and $(3 - y)$ 'down' quarks.

Then total charge on a neutron is

$$ny + d(3 - y) = 0$$

or, $$+\frac{2}{3}ey - \frac{1}{3}e(3-y) = 0$$

or, $$+\frac{2}{3}y - 1 + \frac{y}{3} = 1$$

or, $$y = 1$$

and $$3 - y = 3 - 1 = 2$$

i.e., neutrons contain 1 'up' quark and 2 'down' quarks. Its quark composition should be *udd*.

1.32. *(a) Consider an arbitrary electrostatic field configuration. A small test charge is placed at a null point (i.e., where* **E** *= 0) of the configuration. Show that the equilibrium of the test charge is necessarily unstable.*

(b) Verify this result for the simple configuration of two charges of the same magnitude and sign placed a certain distance apart.

Sol. (*a*) It can be proved by contradiction. Assume that the test charge placed at null point be in stable equilibrium. The test charge displaced slightly in any direction will experience a restoring force towards the null-point as the stable equilibrium requires restoring force in all directions. That is, all field lines near the null point should be directed inwards towards the null point. This indicates that there is a net inward flux of electric field through a closed surface around the null point. But, according to Gauss law, the flux of electric field through a surface enclosing no charge must be zero. This contradicts our assumption. Therefore, the test charge placed at null point must be necessarily in unstable equilibrium.

(*b*) On the mid-point of the line joining the two charges, the null point lies. The test charge will experience a restoring force if it is displaced slightly on either side of the null point along this line. While the net force takes it away from the null point if it is displaced normal to this line. That is no restoring force acts in the normal direction. But restoring force in all directions is demanded by stable equilibrium, therefore, test charge placed at null point will not be in stable equilibrium.

1.33. *A particle of mass m and charge –q enters the region between the two charged plates initially moving along x-axis with speed v_x. The length of plate is L and a uniform electric field E is maintained between the plates. Show that the vertical deflection of the particle at the far edge of the plate is $qEL^2/(2m\, v_x^2)$.*

Sol. The particle is moving along x-axis in a uniformly charged electric field E between two oppositely charged metallic plates of length L. The motion of a charge particle in an electric field is analogous to the motion of a projectile in the gravitational field. The only difference is that here the constant electric field is upward direction and is limited to the region between the plates of length L. Since x-component of the electric force is zero therefore, acceleration along x-axis is zero. So, the velocity v_x along x-axis is constant. If x is the horizontal distance covered in time t, then

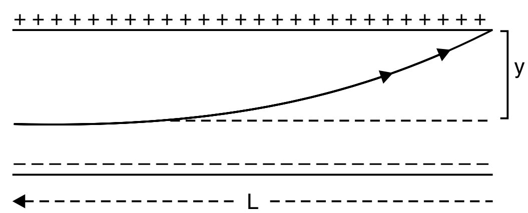


Fig. 1.41

$$x = v_x t \quad \text{or} \quad t = \frac{x}{v_x}$$

Force acting along y-axis, $F_y = qE$

Acceleration along y-axis, $a_y = \dfrac{qE}{m}$

where m is the mass of charged particle (electron)

If y is the vertical distance covered by the particle in time t, then

$$y = u_y + a_y\, t^2$$

$$y = \frac{1}{2} a_y t^2 \qquad [\because \text{ Initial velocity is zero}]$$

or, $$y = \frac{1}{2} \frac{qE}{m} \left(\frac{x}{v_x}\right)^2$$

$\therefore$ $$y = \frac{qE}{2mv_x^2} x^2$$

So, within the electric field, the particle follows a parabolic path.

Let y_1 be the vertical deflection suffered by the particle inside the electric field.

When $x = L$, then $y = y_1$

$\therefore$ $$y_1 = \frac{q\,E\,L^2}{2m\,v_x^2}$$

1.34. *Suppose that the particle in Question 1.33 is an electron projected with velocity $v_x = 2.0 \times 10^6$ m s^{-1}. If E between the plates separated by 0.5 cm is 9.1×10^2 N/C, where will the electron strike the upper plate? ($|e| = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg.)*

Sol. Acceleration, $a = \frac{qE}{m}$

$= \frac{1.6 \times 10^{-19} \times 9.1 \times 10^{2}}{9.1 \times 10^{-31}} = 1.6 \times 10^{14}$ m/s^2

Using formula $y = ut + \frac{1}{2}at^2$,

We get, $0.005 = 0 + \frac{1}{2} \times 1.6 \times 10^{14} \times t^2$

Simplifying for value of t, we get

$t = 8 \times 10^{-9}$ s

The electron covers vertical distance is shown as

$$y = v_x\, t$$
$$= 2.0 \times 10^6 \times 8 \times 10^{-9}$$
$$= 1.6 \times 10^{-2} \text{ m}$$
$$= 1.6 \text{ cm}$$

MORE QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *Which orientation of an electric dipole in a uniform electric field would correspond to stable equilibrium?*

Ans. The dipole is in stable equilibrium when direction of electric dipole moment of electric dipole is in the direction of electric field.

Q. 2. *If the radius of the Gaussian surface enclosing a charge is halved, how does the electric flux through the Gaussian surface change?*

Ans. Electric flux ϕ_E is given by

$$\phi_E = \oint \vec{E} \cdot \vec{ds} = \frac{Q}{\varepsilon_0}$$

where θ is total charge inside the closed surface.

∴ On changing the radius of sphere, the electric flux through the Gaussian surface remains same.

Q. 3. *How many electrons make up one Coulomb of negative charge?*

Ans. From $q = ne, \quad n = q/e$

or $n = \frac{1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}$

Q. 4. *How does the force between two point charges change, if the dielectric constant of the medium in which they are kept decreases?*

Ans. As the dielectric constant of a medium is the ratio of forces between two charges placed in vacuum and medium without no change in magnitude of charges and their distance. The force will increase.

$$\therefore \qquad K = \frac{F_o}{F}$$

$$\text{or} \qquad F = \frac{F_o}{K}$$

$$\text{or} \qquad \left[F \propto \frac{1}{k}\right]$$

Q. 5. *Is the electric force between two electrons greater than the gravitational force between them? If so by what factor?*

Ans. Yes, electric force between two electrons is greater than gravitational force between them by a factor of 10^{42}.

Q. 6. *Two electric field lines never cross each other. Why?*

Ans. Two electric field lines never cross each other. If they intersect, then there will be two directions of electric field at the point of intersection which is not possible.

Q. 7. *How does a torque affect the dipole in an electric field?*

Ans. The torque tends to align the dipole in the direction of the electric field.

Q. 8. *A glass rod rubbed with silk acquires a charge + 1.6×10^{-12} C. What is the charge on the silk ?*

Ans. Charge on silk is equal and opposite to charge on glass *i.e.,* $q = -1.6 \times 10^{-12}$ C.

Q. 9. *An electrostatic field line cannot be discontinuous. Why?*

Ans. An electrostatic field line cannot be a discontinuous curve *i.e.,* it cannot have breaks. If it is so, it will indicate the absence of electric field at the break points. But the electric field vanishes only at infinity.

Q. 10. *What is the cause of charging?*

Ans. The actual transfer of electrons from one body to the other is the basic cause of charging.

Q. 11. *What is the electrostatic potential due to an electric dipole at an equatorial point?*

Ans. Electric potential at any point in the equatorial plane of dipole is zero.

Q. 12. *Can a body have a charge of 0.8×10^{-19} C?*

Ans. The given charge is one-half of the charge of an electron. A fraction of e is not possible.

Q. 13. *What is the basic cause of quantisation of charge?*

Ans. Transfer of only integral number of electrons from one body to the other is the basic cause of quantisation of charge.

Q. 14. *Why does an ebonite rod get negatively charged on rubbing with fur?*

Ans. Electrons are lost by fur while ebonite rod gains them. Electrons in fur are less tightly bound than electrons in ebonite.

Q. 15. *Write the SI unit of (i) electric field intensity and (ii) electric dipole moment.*

Ans. (*i*) *SI* unit of electric field intensity is NC^{-1}.

(*ii*) *SI* unit of electric dipole moment is C-m (coulomb-metre).

Q. 16. *Define the term electric dipole moment. Is it a scalar or a vector quantity?*

Ans. The product of either charge and separation between two charges is termed as electric dipole moment. It is a vector quantity in the direction of the dipole axis from $-q$ to $+q$.

$$\vec{p} = q\left(2\vec{a}\right) \quad \text{or} \quad \vec{p} = (q)\, 2\,\vec{a}$$

Q. 17. *Is the Coulomb force that one charge exerts on another changes if other charges are brought nearby?*

Ans. No, the Coulomb force due to one charge is not changed.

Q. 18. *Two point charges of + 3 μC each are 100 cm apart. At what point on the line joining the charges will the electric intensity be zero?*

Ans. The electric intensity will be zero at a point mid-way between the two charges.

Q. 19. *What is the dimensional formula for* $\in_0$*?*

Ans. $[M^{-1} L^{-3} T^4 A^2]$.

Q. 20. *What is the relevance of large value of K (= 81) for water?*

Ans. It makes water a great solvent. This is because binding force of attraction between oppositely charged ions of the substance in water becomes 1/81 of the force between these ions in air.

Q. 21. *What does* $q_1 + q_2 = 0$ *signify in electrostatics?*

Ans. $q_1 = -q_2$. So, the charges q_1 and q_2 are equal and opposite.

Q. 22. *Does motion of a body affect its charge?*

Ans. No, charge on a body does not change with motion of the body.

Q. 23. *What is the net force on an electric dipole placed in a uniform electric field?*

Ans. Zero.

Q. 24. *Two electrically charged particles having charges of different magnitudes, when placed at a distance 'd' from each other, experiences a force of attraction 'F'. These two particles are put in contact and again placed at the same distance from each other. What is the nature of new force between them? Is the magnitude of the force of interaction between them now more or less than F?*

Ans. Nature of the new force will be repulsive and the magnitude of interaction between the charges will be less than *F*.

Q. 25. *Find the amount of work done in rotating a dipole of dipole moment* 3×10^{-3} *cm from its position of stable equilibrium to the position of unstable equilibrium, in a uniform electric field of intensity* 10^4 NC^{-1}.

Ans. In rotating the dipole from the position of stable equilibrium by an angle θ, the amount of work done,

$$W = PE\,(1 - \cos\theta)$$

For unstable equilibrium θ = 180°

$$\therefore \quad W = PE\,(1 - \cos 180°)$$
$$= 2\,PE$$
$$= 2 \times 3 \times 10^{-3} \times 10^4 \text{ J}$$
$$= 60 \text{ J}.$$

Q. 26. *Electrostatic forces are much stronger than gravitational forces. Give one example.*

Ans. A charged glass rod can lift a piece of paper against the gravitational pull of the earth on this piece.

Q. 27. *How is force between two charges affected when dielectric constant of the medium in which they are held increases?*

Ans. As $F = F_0/K$, therefore, force decreases, when K increases.

Q. 28. *At what points, dipole field intensity is parallel to the line joining the charges?*

Ans. The dipole field intensity is parallel to the line joining the charges at points on the axial line or equatorial line.

Q. 29. *Can a charged body attract another uncharged body? Explain.*

Ans. Yes, a charged body can attract another uncharged body. When the charged body is placed near the uncharged body, the induced charges of opposite kind are produced on the uncharged body and the uncharged body is attract by charged body.

Q. 30. *Consider three charged bodies P, Q and R. If P and Q repel each other and P attracts R, what is the nature of force between Q and R?*

Ans. Attractive. This is because Q and R are oppositely charged.

Q. 31. *Is torque on an electric dipole a vector or scalar?*

Ans. Torque is vector quantity.

Q. 32. *In Coulomb's law, on what factors the value of electrostatic force constant K depends ?*

Ans. It depends on the nature of medium between the two charges and also on the system of units.

Q. 33. *Does Coulomb's law of electric force obey Newton's third law of motion?*

Ans. Yes. Coulomb forces are equal in magnitude, opposite in direction and act on different bodies.

Q. 34. *How does a free electron at rest move in an electric field?*

Ans. When the electron is released, it will move in a direction opposite to the direction of electric field.

Q. 35. *Is the mass of a body affected on charging?*

Ans. Yes, very slightly. The negatively charged body gains mass also along with electrons.

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *A thin conducting spherical shell of radius R has charge +q spread uniformly over its surface. Using Gauss's law, derive an expression for an electric field at a point outside the shell.*

Draw a graph of electric field E(r) with distance r from the centre of the shell for $0 \le r \le \infty$.

Ans. *Electric field intensity at any point outside a uniformly charged spherical shell:*

Assume a thin spherical shell of radius R with centre O. Let charge $+q$ is uniformly distributed over the surface of the shell.

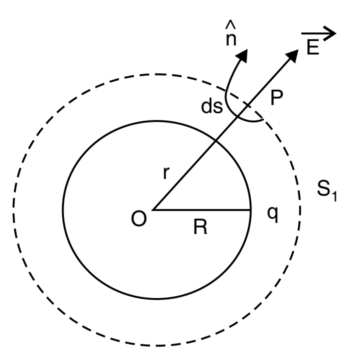


Fig. 1.42

Let P be any point on the Gaussian surface sphere S_1 with centre O and radius r ($r > R$). According to Gauss's law

$$\oint_S \vec{E}\cdot\vec{ds} = \frac{q}{\varepsilon_0} \Rightarrow \oint_E \vec{E}\cdot\hat{n}\,ds = \frac{q}{\varepsilon_0}$$

$\therefore$ $$E\oint ds = \frac{q}{\varepsilon_0} \Rightarrow E\,.\,4\pi r^2 = \frac{q}{\varepsilon_0}$$

$\therefore$ $$E_r = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

At any point on the surface of the shell,

$$r = R$$

$\therefore$ $$E_R = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2}$$

If σ is charge density

$\therefore$ $$q = 4\pi R^2\sigma$$

$\therefore$ $$E_R = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4\pi R^2\sigma}{R^2}$$

Therefore, $$E_R = \frac{\sigma}{\varepsilon_0}$$

Graph: As charge on shell reside on outer surface so there is no charge inside shell so electric field by Gauss's law will be zero.

So inside shell $r < R$

$$q = 0 \quad \text{or} \quad \sigma = 0$$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

$$E = 0$$

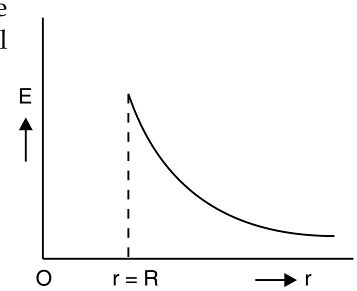


Fig. 1.43

The variation of the electric field intensity $E(r)$ with distance r from the centre for shell $0 \le r < \infty$ is shown below.

Q. 2. *Define electric field intensity. Write its SI unit. Write the magnitude and direction of electric field intensity due to an electric dipole of length 2a at the mid-point of the line joining the two charges.*

Ans. The force experienced by a unit positive charge placed at that point is termed as the electric field intensity.

$$\boxed{E = \frac{F}{q_0}}$$

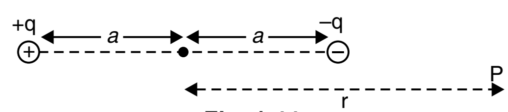


Fig. 1.44

The *SI* unit of electric field intensity = NC^{-1}

Electric field at any equatorial point of a dipole is

$$\boxed{\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{P}}{\left(r^2 + a^2\right)^{3/2}}}$$

At the mid-point of the dipole, $r = 0$,

Hence, $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{P}}{a^3}$$

The direction of $\vec{E}$ is from positive charge to negative charge.

Q. 3. *What is the dimensional formula for ε_0?*

Ans.
$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

or,
$$[\varepsilon_0] = \frac{[q_1 q_2]}{[Fr^2]} = \frac{[A^2 T^2]}{[MLT^{-2}][L^2]}$$
$$= [M^{-1} L^{-3} T^4 A^2]$$

Q. 4. *What is an electric line of force? What is its importance?*

Ans. An electric line of force is an imaginary straight or curved path along which a small positive test charge is supposed to move when free to do so.

The tangent at a point on an electric line of force gives the direction of the resultant electric field at that point.

The relative closeness of electric lines of force in a certain region provides us an estimate of the electric field strength in that region.

Q. 5. *Dielectric constant of a medium is unity. What will be its permittivity?*

Ans. We know that dielectric constant of a medium is

$$K = \in_r = \frac{\in}{\in_0}$$

$$\therefore \quad \in = K \in_0 = 1 \times 8.854 \times 10^{-12} = 8.854 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}.$$

Q. 6. *Two small balls having equal positive charge q coulomb are suspended by two insulating strings of equal length l metre from a hook fixed to a stand. The whole set up is taken in a satellite into space where there is no gravity. What is the angle between the two strings and the tension in each string?*

Ans. In a satellite, there is condition of weightlessness. Therefore, $mg = 0$. On account of electrostatic force of repulsion between the balls, the strings would become horizontal. Therefore, angle between the strings = 180°.

Also, tension in each string = force of repulsion

$$T = \frac{1}{4\pi\in_0}\frac{q^2}{(2l)^2} N$$

Q. 7. *The electric field E due to a point charge at any point near it is defined as $E = \lim_{q \to 0} \frac{F}{q}$, where q is the test charge and F is the force acting on it. What is the physical significance of $\lim_{q \to 0}$ in this expression? Draw the electric field lines of a point charge Q when (i) $Q > 0$ and (ii) $Q < 0$*

Ans. It is indicated by the $\lim_{q \to 0}$ that the test charge q is so small that its presence does not disturb the distribution of source charge and therefore, its electric field. The electric fields of the point charge Q are shown below.

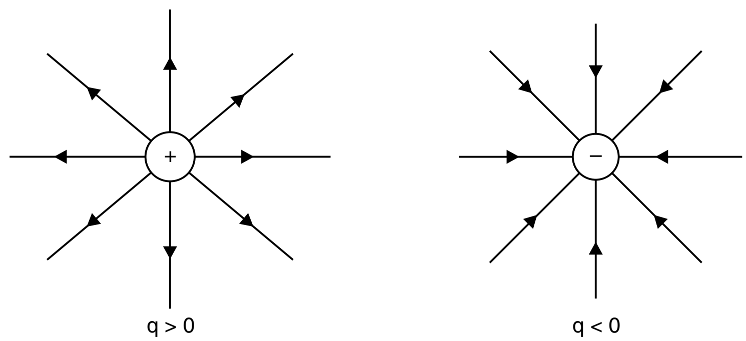


Fig. 1.45

Q. 8. *Define electric flux. Write its SI units. A spherical rubber balloon carries a charge that is uniformly distributed over its surface. As the balloon is blown up and increases in size, how does the total electric flux coming out of the surface change? Give reason.*

Ans. The total number of electric lines of force passing normally through that area. It is given by:

$$\phi_E = \vec{E} \cdot \overrightarrow{\Delta S}$$

SI unit of electric flux is $Nm^2\ C^{-1}$. As the balloon is blown up, the total charge on the balloon surface remains unchanged, so the total electric flux coming out of its surface remains unchanged.

Q. 9. *Sketch lines of force due to two equal positive charges placed at a small distance apart in air.*

Ans. Refer to Q.No. 4 at page No. 45

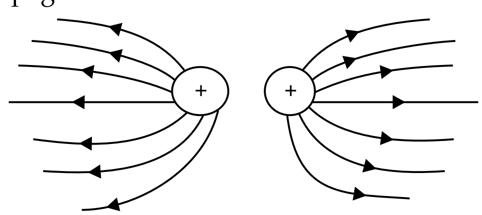


Fig. 1.46

Electric lines of force of two equal positive charges.

Q. 10. *Two point charges qA = + 3 μC and qB = – 3 μC are located 20 cm apart in vacuum.*

(i) Find the electric field at the mid-point of the line AB joining the two charges.

(ii) If a negative test charge of magnitude 1.5×10^{-9} *C is placed at the centre, find the force experienced by the test charge.*

Ans. See Textbook Question 1.8.

Q. 11. *A copper sphere of mass 2 g contains nearly* 2×10^{22} *atoms. The charge on the nucleus of each atom is 29 e. What fraction of the electrons must be removed from the sphere to give it a charge of + 2μC ?*

Ans. Total number of electrons in the sphere = $29 \times 2 \times 10^{22}$

No. of electrons removed = $\dfrac{q}{e} = \dfrac{2 \times 10^{-6}}{1.6 \times 10^{-19}} = 1.25 \times 10^{13}$

Fraction of electrons removed = $\frac{1.25 \times 10^{13}}{29 \times 2 \times 10^{22}}$ = 2.16×10^{-11}

Q. 12. *What is meant by the statement that the electric field of a point charge has spherical symmetry whereas that of an electric dipole is cylindrically symmetrical?*

Ans. Consider a charge q at the centre of a sphere of radius r. The electric field at all points on the surface of the sphere is given by

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}.$$

So, the electric field due to a point charge is spherically symmetric.

In the case of an electric dipole, the electric field at a distance r, from the mid-point of the dipole, on the equatorial line is given by

$$E = \frac{1}{4\pi\varepsilon_0} \frac{p}{\left(r^2 + l^2\right)^{3/2}}$$

Now, imagine a cylinder of radius r drawn with electric dipole as axis. The electric field, due to dipole, at all points on the surface of the cylinder will be the same. So, the electric field due to dipole has cylindrical symmetry.

Q. 13. *Calculate the Coulomb force between 2 α particles separated by 3.2×10^{-15} m.*

Ans. $$F = \frac{9 \times 10^9 \times 2 \times 1.6 \times 10^{-19} \times 2 \times 1.6 \times 10^{-19}}{3.2 \times 10^{-15} \times 3.2 \times 10^{-15}} \text{N} = 90 \text{ N}$$

Q. 14. *What kind of charges are produced on each when (i) a glass rod is rubbed with silk and (ii) an ebonite rod is rubbed with wool?*

Ans. (*i*) Positive charge will be produced on glass rod and negative charge will be produced on silk.

(*ii*) Negative charge will be produced on ebonite rod and positive charge will be produced on wool.

Q. 15. *State Gauss's law in electrostatics. Use this law to derive an expression for the electric field due to an infinitely long straight wire of linear charge density λ cm^{-1}.*

Ans. Gauss's law in electrostatics: It states that total electric flux over the closed surface S in vacuum is $\frac{1}{\varepsilon_0}$ times the total charge (q) contained in side S.

$$\therefore \qquad \phi_E = \oint_S \vec{E} . \vec{dS} = \frac{q}{\varepsilon_0}$$

Electric field due to an infinitely long straight wire.

Let an infinitely long line charge having linear charge density λ. Assume a cylindrical Gaussian surface of radius r and length l coaxial with the line charge to determine its electric field at distance r.

By symmetry, the electric field E has same magnitude at each point of the curved surface S_1 and is directed radially outward. So angle at surfaces between $\vec{dS}$ and $\vec{E}$ is zero, and angle of $\vec{dS}_2$, $\vec{dS}_3$ with $\vec{E}$ at S_2 and S_3 are 90°.

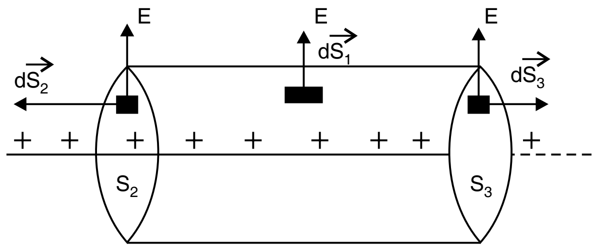


Fig. 1.47

Total flux through the cylindrical surface,

$$\oint \vec{E}\,.\,\vec{ds} = \oint_{S_1} \vec{E}\,.\,\vec{dS}_1 + \oint_{S_2} \vec{E}\,.\,\vec{dS}_2 + \oint_{S_3} \vec{E}\,.\,\vec{dS}_3$$

$$= \oint_{S_1} E\,dS_1\,.\cos 0° + \oint_{S_2} E\,dS_2\,.\cos 90° + \oint_{S_3} E\,dS_3\,.\cos 90°$$

$$= E \oint dS_1 = E \times 2\pi r l$$

Since λ is the charge per unit length and l is the length of the wire,

Thus, the charge enclosed

$$q = \lambda l$$

According to Gaussian law,

$$\oint_S \vec{E}\,.\,\vec{dS} = \frac{q}{\varepsilon_0}$$

or, $$E \times 2\pi r l = \frac{\lambda l}{\varepsilon_0}$$

$$\therefore \quad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Q. 16. *The electrostatic force between charges of 200 μC and 500 μC placed in free space is 5 gf. Find the distance between the two charges. Take g = 10 ms⁻².*

Ans.

$$q_1 = 200 \times 10^{-6}\text{ C} = 2 \times 10^{-4}\text{ C},$$

$$q_2 = 500 \times 10^{-6}\text{ C} = 5 \times 10^{-4}\text{ C},$$

$$F = 5\text{ gf} = 5 \times 10^{-3}\text{ kgf}$$

$$= 5 \times 10^{-3} \times 10\text{ N} = 5 \times 10^{-2}\text{ N},$$

$$r = ?$$

Using $$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\text{, we get}$$

$$5 \times 10^{-2} = \frac{9 \times 10^9 \times 2 \times 10^{-4} \times 5 \times 10^{-4}}{r^2}$$

or $$r = 1.34 \times 10^2\text{ m}$$

Q. 17. *A free pith ball of mass 8 g carries a positive charge of 5×10^{-8} C. What must be the nature and magnitude of charge that should be given to a second pith ball fixed 5 cm vertically below the former pith ball so that the upper pith ball is stationary?*

Ans. Let m be the mass of the upper ball. Let q_1 represent the charge on the upper ball. For equilibrium of upper ball, it must experience an upward electric force F_e. This is possible if the lower ball has positive charge, say, q_2.

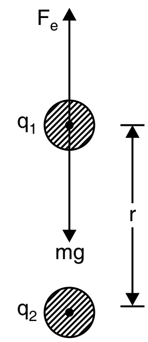


Fig. 1.48

For equilibrium, $\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = mg$

Substituting values,

$$\frac{9\times 10^9 \times 5\times 10^{-8} \times q_2}{\left(5\times 10^{-2}\right)^2} = 8 \times 10^{-3} \times 9.8$$

On simplification,

$$q_2 = 4.356 \times 10^{-7}\ C$$

Q. 18. *A particle of mass m carrying charge + q_1 is revolving around a fixed charge – q_2 in a circular path of radius r. Calculate the period of revolution.*

Ans. Electrostatic force = Centrifugal force

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = mr\omega^2 = \frac{4\pi^2 mr}{T^2}$$

or,
$$T^2 = \frac{(4\pi\varepsilon_0)\, r^2\, (4\pi^2 mr)}{q_1 q_2}$$

or,
$$T = 4\pi r\sqrt{\frac{\pi\varepsilon_0 mr}{q_1 q_2}}$$

Q. 19. *State Gauss's theorem. Using Gauss's theorem, derive an expression of electric field intensity at any point inside a hollow charged conducting sphere.*

Ans. Gauss's theorem given in Q. 15 page no. 41.

Electric field inside a hollow charged conducting sphere:

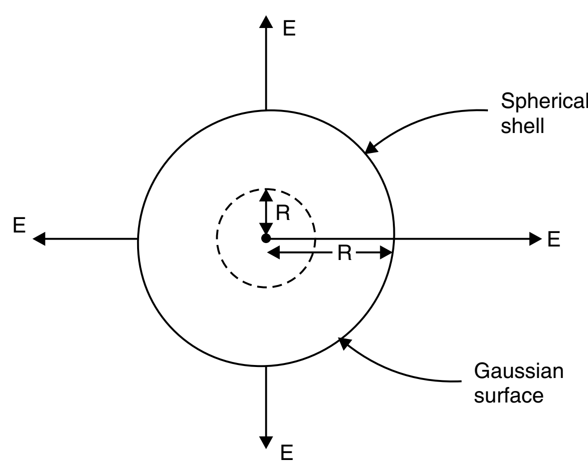


Fig. 1.49

Let R be the radius of the hollow sphere and σ be the uniform surface charge density on it. Consider a Gaussian surface of radius $r < R$.

Charge enclosed by the Gaussian surface, $q = 0$.

Using Gauss's theorem

$$\phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

Since $q = 0$, thus, E_0

i.e., electric field inside a hollow sphere considered, is zero.

Q. 20. *Two charged particles having charge 2.0 × 10–8 C each are joined by an insulating string of length 1 m and the system is kept on a smooth horizontal table. Find the tension in the string.*

Ans. Here $q_1 = q_2 = 2 \times 10^{-8}$ C

$r = 1$ m

Tension in the string is the force of repulsion (F) between the two charges.

According to Coulomb's law,

$$F = \frac{q_1 q_2}{4\pi \in_0 r^2}$$

$$= \frac{9 \times 10^9 \left(2 \times 10^{-8}\right)\left(2 \times 10^{-8}\right)}{1^2}$$

$$F = 3.6 \times 10^{-6} \text{ N}$$

Q. 21. *A charge q is placed at the centre of the line joining two equal charges Q. Show that the system of three charges will be in equilibrium if q = – Q/4.*

Ans. Let two equal charges Q each, be held at A and B, where $AB = 2x$. C is the centre of AB, where charge q is held, figure below.

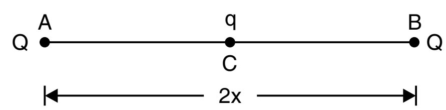


Fig. 1.50

Net force on q is zero. So q is already in equilibrium.

For the three charges to be in equilibrium, net force on each charge must be zero.

Now, total force on Q at B is

$$\frac{1}{4\pi \in_0} \frac{Q\,q}{x^2} + \frac{1}{4\pi \in_0} \frac{Q\,Q}{(2x)^2} = 0$$

or $$\frac{1}{4\pi \in_0} \frac{Q\,q}{x^2} = -\frac{1}{4\pi \in_0} \frac{Q^2}{4x^2}$$

or $$q = -\frac{Q}{4}$$

which was to be proved.

Q. 22. *The electrostatic force of repulsion between two positively charged ions carrying equal charge is 3.7 × 10⁻⁹ N, when they are separated by a distance of 5 Å. How many electrons are missing from each ion?*

Ans. Here, $F = 3.7 \times 10^{-9}$ N,

$q_1 = q_2 = q$, say

$$r = 5 \text{ Å} = 5 \times 10^{-10} \text{ m},$$
$$n = ?$$

From Coulomb's law,

$$F = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r^2}$$

$$3.7 \times 10^{-9} = 9 \times 10^9 \frac{q\,q}{\left(5 \times 10^{-10}\right)^2}$$

$$q^2 = \frac{3.7 \times 10^{-9} \times 25 \times 10^{-20}}{9 \times 10^9} = 10.28 \times 10^{-38}$$

$$q = 3.2 \times 10^{-19} \text{ coulomb}$$

As $q = ne$

$\therefore$ $$n = \frac{q}{e} = \frac{3.2 \times 10^{-19}}{1.6 \times 10^{-19}} = 2$$

III. LONG ANSWER TYPE QUESTIONS

Q. 1. *Point charges having values + 1 μC, – 5 μC and + 2 μC are placed at the corners A, B and C respectively of an equilateral triangle of side 2 m in free space. Determine the magnitude of intensity at the point D midway between A and C.*

Ans. The intensity E_A at D due to charge at A is given by

$$E_A = 9 \times 10^9 \frac{10^{-6}}{1^2} \text{ N C}^{-1} \qquad \left[\because AD = \frac{1}{2} AC = \frac{1}{2} \times 2 \text{ m} = 1 \text{ m}\right]$$

or, $E_A = 9 \times 10^3 \text{ N C}^{-1}$ along DC

The intensity E_C at D due to charge at C is given by

$$E_C = 9 \times 10^9 \frac{2 \times 10^{-6}}{1^2} \text{ N C}^{-1}$$

or, $E_C = 18 \times 10^3 \text{ N C}^{-1}$ along DA

The magnitude of the resultant of E_C and E_A is given by

$$E_C - E_A = (18 \times 10^3 - 9 \times 10^3) \text{ N C}^{-1}$$
$$= 9 \times 10^3 \text{ N C}^{-1} \text{ along } DA$$

The intensity E_B at D due to charge at B is given by

$$E_B = 9 \times 10^9 \frac{5 \times 10^{-6}}{3} \text{ N C}^{-1}$$

or, $E_B = 15 \times 10^3 \text{ N C}^{-1}$ along DB

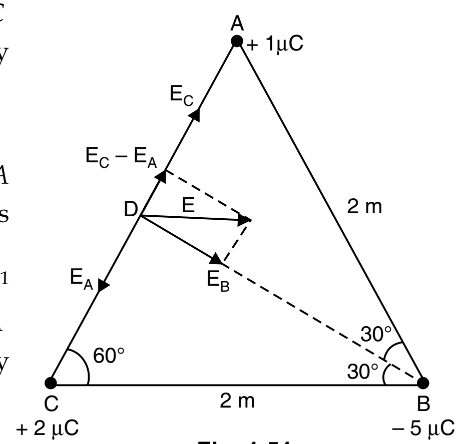


Fig. 1.51

In rt. $\angle d$ ΔCDB,

$$\cos 30° = \frac{BD}{2}$$

or $$BD = \frac{\sqrt{3}}{2} \times 2\,m = \sqrt{3}\,m$$

If E be the magnitude of the resultant intensity, then

$$E = \sqrt{\left(9 \times 10^3\right)^2 + \left(15 \times 10^3\right)^2} \text{ N C}^{-1}$$

$$= 1.749 \times 104 \text{ NC}^{-1}$$

Q. 2. *Two fixed point charges + 4e and + e units are separated by a distance a. Where should the third point charge be placed for it to be in equilibrium?*

Ans. Let a point charge q be held at a distance x from the charge + 4e, figure given below

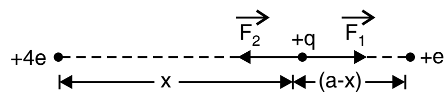


Fig. 1.52

$\therefore$ Distance of q from charge + e = $(a - x)$

Force on this charge exerted by the charge + 4e is

$$F_1 = \frac{q\,(4e)}{4\pi \epsilon_0\, x^2} \text{ directed away from } (4e)$$

Force on this charge exerted by the charge + e

$$F_2 = \frac{q(e)}{4\pi \epsilon_0\, (a-x)^2}, \text{ directed away from } (e)$$

For the charge q to be in equilibrium $F_1 = F_2$

i.e.,
$$\frac{q\,(4e)}{4\pi \epsilon_0\, x^2} = \frac{q\,(e)}{4\pi \epsilon_0\, (a-x)^2}$$

or,
$$\frac{4}{x^2} = \frac{1}{(a-x)^2} \quad \text{or} \quad \frac{2}{x} = \frac{1}{a-x}$$

$\therefore$
$$x = 2a - 2x$$

or,
$$3x = 2a \quad \text{or} \quad x = 2a/3$$

Hence the charge q should be held at a distance $2a/3$ from charge (+ 4e).

Q. 3. *Define the term electric field intensity. Write its SI unit. Derive an expression for the electric field intensity at a point on the axis of an electric dipole.*

Ans. The force experienced by a unit positive charge placed at a point is termed as the electric field intensity at that point. It is vector quantity it's direction is in the force acting on +ive charge.

The *SI* unit of electric field intensity is NC^{-1}.

Electric field at an axial point of electric dipole:

Assume point P is located at distance r from the centre of an electric dipole as shown in the figure.

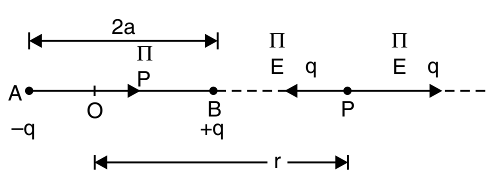


Fig. 1.53

Electric field at point P is $\vec{E} = \vec{E}_{+q} + \vec{E}_{-q}$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(r-a)^2} - \frac{q}{(r+a)^2}\right]\hat{p}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+a)^2-(r-a)^2}{\left(r^2-a^2\right)^2}\right]\hat{p}$$

or $$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{4ra}{\left(r^2-a^2\right)^2}\right]\hat{p}$$

or, $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{\left(r^2-a^2\right)^2}\hat{p} \qquad [p = q(2a)]$$

or, $$\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{2p}{r^3}\hat{p}$$

or, $$E = \frac{1}{4\pi\varepsilon_o}\frac{2p}{r^3} \qquad [\text{For } r >> a]$$

Q. 4. **(*a*)** *Using Gauss's law, derive an expression for the electric field intensity at any point outside a uniformly charged thin spherical shell of radius R and charge density σ C/m². Draw the field lines when the charge density of the sphere is (i) positive, (ii) negative.*

(*b*) *A uniformly charged conducting sphere of 2.5 m in diameter has a surface charge density of 100 μC/m². Calculate the*

(i) charge on the sphere

(ii) total electric flux passing through the sphere.

Ans. (*a*) Electric field intensity at any point outside a uniformly charged spherical shell:

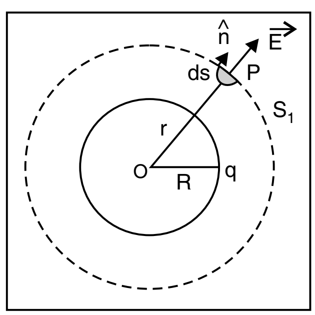


Fig. 1.54

Consider a thin spherical shell of radius R with centre O. Let charge $+q$ is uniformly distributed over the surface of the shell.

Let P be any point on the sphere S_1 with centre O and radius r.

According to Gauss's Law

$$\oint_s \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0} \Rightarrow \oint_E \vec{E}\cdot\hat{n}\,ds = \frac{q}{\epsilon_0}$$

$\therefore \qquad E \oint ds = \frac{q}{\epsilon_0} \Rightarrow E.\ 4\pi r^2 = \frac{q}{\epsilon_0}$

$$\therefore \qquad E = \frac{1}{4\pi \epsilon_0} \cdot \frac{q}{r^2}$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{4\pi R^2 \sigma}{r^2}$$

$$E = \frac{\sigma}{\varepsilon_0} \frac{R^2}{r^2}$$

If σ is charge density,

$\therefore \qquad q = 4\pi R^2 \sigma$

Electric field lines when the charged density of the sphere:

(*i*) Positive (*ii*) Negative

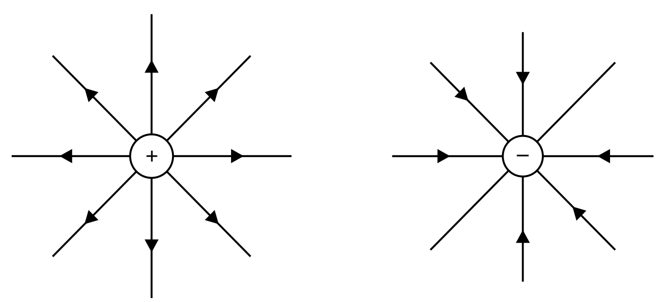


Fig. 1.55

(*b*) Here diameter $= 2.5$ m $\quad \therefore R = \frac{25}{2} = 1.25$ m

Charge density $\sigma = 100\ \mu c/m^2 = 100 \times 10^{-6} = 10^{-4}\ c/m^2$

(*i*) $q = 4\pi\ R^2\sigma$

$= 4 \times 3.14\ (1.25)^2 \times 10^{-4}$

$= 19.625 \times 10^{-4}$

$= 1.96 \times 10^{-3}$ C

(*ii*) Total electric flux

$$\phi_E = \frac{q}{\epsilon_0}$$

$$\therefore \qquad \phi_E = \frac{1.96 \times 10^{-13}}{8.85 \times 10^{-12}} = 0.221 \times 10^9 = 2.21 \times 10^8\ \text{Nm}^2\ \text{C}^{-1}.$$

Q. 5. **(*a*)** *Define electric flux. Write its SI units.*

(*b*) *The electric field components due to a charge inside the cube of side 0.1 m are as shown:*

$E_x = \alpha x$, *where* $\alpha = 500$ N/C-*m*

$E_y = 0, \quad E_z = 0.$

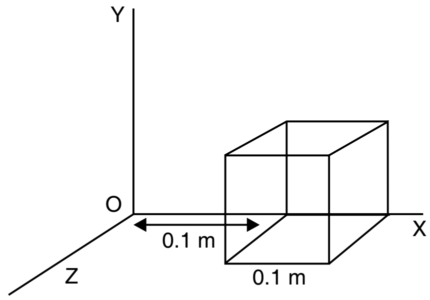


Fig. 1.56

Calculate (i) the flux through the cube and

(ii) the charge inside the cube.

Ans. (*a*) *Electric flux:*

The total number of electric field lines crossing an area in an electric field is termed as electric flux.

It is denoted by ϕ_E.

Electric flux is a scalar quantity, its SI unit is $Nm^2\ C^{-1}$.

(*b*) Here, $E_X = \alpha x,\ E_Y = 0,\ E_Z = 0$

$\alpha = 500$ N/C–m, side of a cube $a = 0.1$ m

As the electric field has only X-component then,

$$\phi_E = \vec{E} \cdot \Delta\vec{S} = 0$$

for each of four faces of cube 1 to *Y*-axis and *Z*-axis.

∴Electric flux is only for left and right face along X-axis of cube.

(*i*) Electric field at the left face,

$X = a$ is

∴ $E_L = \alpha\, a$

$\phi_L = \vec{E}_L \cdot \Delta\vec{S} = \alpha\, aa^2 \cos 180°$

$= -\alpha\, a^3$ [$\because E = \alpha\, x$]

and electric field at the right face, $x = a + a = 2a$ is

∴ $E_R = \alpha\,(2a)$

$\phi_R = \vec{E}_R \cdot \Delta\vec{S} = \alpha\,(2a)\, a^2 \cos 0^0$

$= 2\,\alpha\, a^3$

∴Net flux through the cube $= \phi_L + \phi_R$

$= -\alpha\, a^3 + 2\alpha\, a^3$

$= \alpha\, a^3 = 500 \times (0.1)^3 = 0.5\ Nm^2\ C^{-1}$

(*ii*) According to Gaussian law

$q = \varepsilon_0 \phi$

$= 8.85 \times 10^{-12} \times 0.5$

or, $q = 4.425 \times 10^{-12}$C.

Q. 6. *Point charges having values + 0.1 μC, + 0.2 μC, – 0.3 μC and –0.2 μC are placed at the corners A, B, C and D respectively of a square of side one metre. Calculate the magnitude of the force on a charge of + 1 μC placed at the centre of the square.*

Ans. $$AC^2 = AB^2 + BC^2 = 1 + 1 = 2$$

or, $$AC = \sqrt{2} \text{ m}$$

$$AO = \frac{1}{2}\sqrt{2} \text{ m} = 0.5\sqrt{2} \text{ m}$$

Also, $$AO = CO = BO = DO = 0.5\sqrt{2} \text{ m}$$

Let F_A be the force exerted by the charge of '+ 0.1μC' at A on the charge of '+ 1 μC' at the centre O of the square.

Then $F_A = 9 \times 10^9 \text{ Nm}^2\text{ C}^{-2} \dfrac{(0.1\times10^{-6}\text{C})(1\times10^{-6}\text{C})}{2(0.5)^2\text{m}^2}$

or, $$F_A = \frac{9\times10^9\times0.1\times10^{-12}}{2\times0.25} \text{ N} = 0.0018 \text{ N}$$

If F_C is the force exerted by charge at C on charge at O, then

$$F_C = 9\times10^9 \frac{(0.3\times10^{-6})(1\times10^{-6})}{2(0.5)^2} \text{N}$$

$$= \frac{9\times0.3\times10^{-3}}{2\times0.25} \text{ N} = 0.0054 \text{ N}$$

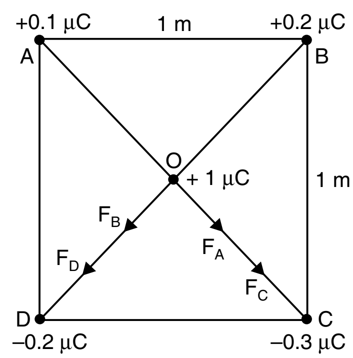


Fig. 1.57

Both F_A and F_C act in the same direction. The resultant of F_A and F_C,

$$F_1 = (0.0018 + 0.0054) \text{ N} = 0.0072 \text{ N}$$

Force exerted by the charge at B on the charge at O,

$$F_B = 9\times10^9 \frac{(0.2\times10^{-6})(1\times10^{-6})}{2(0.5)^2} \text{ N}$$

$$= 3.6 \times 10^{-3} \text{ N} = 0.0036 \text{ N}$$

Force exerted by the charge at D on the charge at O,

$$F_D = 9\times10^9 \frac{(0.2\times10^{-6})(1\times10^{-6})}{2(0.5)^2} \text{ N} = 0.0036 \text{ N}$$

Both F_B and F_D act in the same direction. Resultant of F_B and F_D,

$$F_2 = (0.0036 + 0.0036) \text{ N} = 0.0072 \text{ N}$$

The angle between F_1 and F_2 is clearly 90°. So, the resultant F of F_1 and F_2 is given by

$$F = \sqrt{(0.0072)^2 + (0.0072)^2} \text{ N} = 0.0072\sqrt{2} \text{ N}$$

$$= 0.0072 \times 1.414 \text{ N} = 0.01018 \text{ N}$$

Q. 7. *Find the magnitude of the resultant force on a charge of 1 μC held at P due to two charges of* $+2 \times 10^{-8}$ *C and* -10^{-8} *C at A and B respectively.*

Given $AP = 10$ *cm*

and $BP = 5$ *cm.*

$\angle APB = 90°$,

Ans. Here,

$F = ?$

Charge at P, $q = 1\ \mu C = 10^{-6}$ C

Charge at A, $q_1 = +2 \times 10^{-8}$ C

Charge at B, $q_2 = -10^{-8}$ C

$AP = 10$ cm $= 0.1$ m,

$BP = 5$ cm $= 0.05$ m

$\angle APB = 90°$

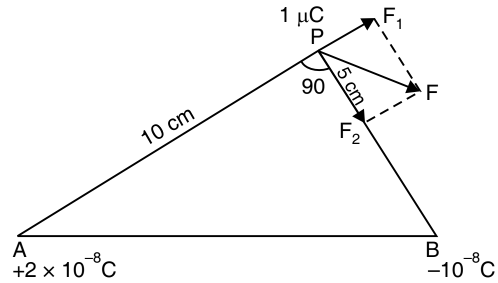


Fig. 1.58

Force at P due to q_1 charge at A,

$$F_1 = \frac{1}{4\pi \in_0} \frac{q_1 q}{AP^2}, \text{ along } AP \text{ produced}$$

$$= \frac{9 \times 10^9 \times 2 \times 10^{-8} \times 10^{-6}}{(0.1)^2}$$

$$= 18 \times 10^{-3} \text{ N}$$

Force at P due to q_2 charge at B,

$$F_2 = \frac{1}{4\pi \in_0} \frac{q_2 q}{BP^2}, \text{ along } BP$$

$$= \frac{9 \times 10^9 \times 10^{-8} \times 10^{-6}}{(0.05)^2}.$$

$$= 36 \times 10^{-3} \text{ N}$$

As angle between $\vec{F}_1$ and $\vec{F}_2$ is 90°,

$\therefore$ Resultant force $F = \sqrt{F_1^2 + F_2^2}$

$$F = \sqrt{\left(18 \times 10^{-3}\right)^2 + \left(36 \times 10^{-3}\right)^2}$$

$$= 18 \times 10^{-3} \times 2.236$$

$$F = 4.024 \times 10^{-2} \text{ N}$$

Q. 8. *Consider three charges q_1, q_2, q_3, each equal to q at the vertices of an equilateral triangle of side l. What is the force on a charge Q (with the same sign as q) placed at the centroid of the triangle, as shown in the figure on the next page?*

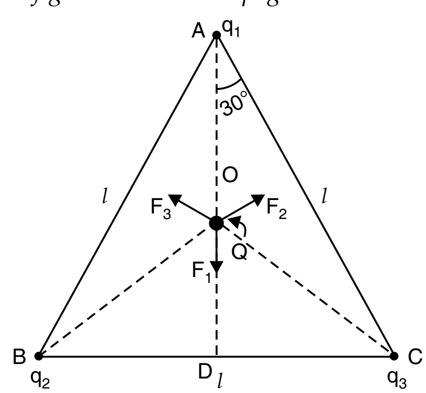


Fig. 1.59

Ans. In the given equilateral triangle ABC of sides of length l, if we draw a perpendicular AD to the side BC,

$$AD = AC \cos 30° = \left(\frac{\sqrt{3}}{2}\right) l$$

and the distance AO of the centroid O from A is

$$\left(\frac{2}{3}\right) AD = \left(\frac{1}{\sqrt{3}}\right) l.$$

Therefore, force F_1 on Q due to charge q at $A = \frac{3}{4\pi \in_0} \frac{Qq}{l_2}$ along AO

Force F_2 and Q due to charge q to $B = \frac{3}{4\pi \in_0} \frac{Qq}{l^2}$ along BO

Force F_3 and Q due to charge q at $C = \frac{3}{4\pi \in_0} \frac{Qq}{l^2}$ along CO

The resultant of forces F_2 and F_3 is $\frac{3}{4\pi \in_0} \frac{Qq}{l^2}$ along OA, by the parallelogram law.

Therefore, the total force on $Q = \frac{3}{4\pi \in_0} \frac{Qq}{l^2} (\hat{r} - \hat{r}) = 0.$

where $\hat{r}$ is the unit vector along AO

QUESTIONS ON HIGH ORDER THINKING SKILLS (HOTS)

Q. 1. *S_1 and S_2 are two hollow concentric spheres enclosing charges 2Q and 4Q respectively as shown in the figure.*

(i) What is the ratio of electric flux through S_1 and S_2?

(ii) How will the electric flux through the sphere S_1 change, if a medium of dielectric constant 6 is introduced in the space inside S_1 in place of air?

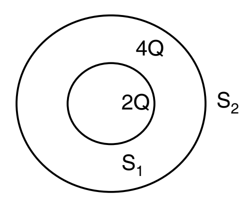


Fig. 1.60

Sol. (*i*)

$$\phi_1 = \frac{2Q}{\varepsilon_0}, \quad \phi_2 = \frac{2Q + 4Q}{\varepsilon_0} = \frac{6Q}{\varepsilon_0}$$

$$\frac{\phi_1}{\phi_2} = \frac{2Q}{\varepsilon_0} \times \frac{\varepsilon_0}{6Q} = \frac{1}{3}$$

(*ii*) Using Gauss's theorem,

$$\phi_1 = \oint \vec{E} \cdot \vec{ds} = \frac{2Q}{\varepsilon_0}$$

and

$$\phi'_1 = \frac{1}{k} \oint \vec{E} \cdot \vec{ds}$$

$$= \frac{1}{k} \frac{2Q}{\varepsilon_0} = \frac{1}{6} \frac{2Q}{\varepsilon_0} = \frac{Q}{3\varepsilon_0}$$

Q. 2. *Two tiny spheres, each having mass m kg and charge q coulomb, are suspended from a point by insulating threads each l metre long but negligible mass. When the system is in equilibrium, each string makes an angle θ with the vertical. Prove that* $q^2 = (4\ mg\ l^2 \sin^2\theta \tan\theta)\ 4\pi\varepsilon_0$.

Sol. Consider the equilibrium of sphere *A*.

Following forces act on the sphere *A*.

(*i*) Force *F* of repulsion on *A* due to *B*.

(*ii*) Weight *mg* acting vertically downwards.

(*iii*) Tension *T* in the string towards the point of suspension 0.

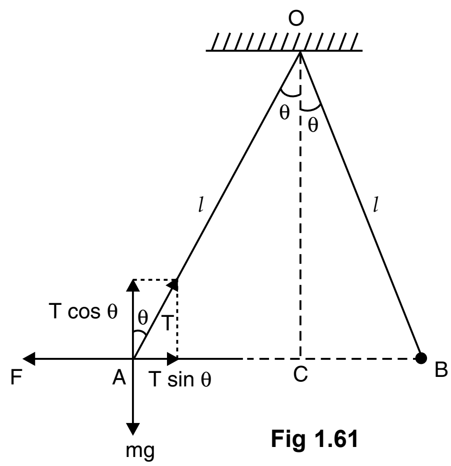


Fig 1.61

Resolve the tension *T* into two rectangular components:

$T\cos\theta$ and $T\sin\theta$

For equilibrium of *A*,

$$T\sin\theta = F$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{AB^2}$$

and

$$T\cos\theta = mg$$

Dividing,

$$\tan\theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{AB^2 mg}$$

But

$$AB = 2AC$$

$$= 2l\sin\theta$$

$$\therefore \quad \tan\theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2l\sin\theta)^2 mg}$$

$$q^2 = (4\ mg\ l^2 \sin^2\theta \tan\theta)\ 4\pi\varepsilon_0.$$

Q. 3. *A uniform line charge of linear charge density λ coulombs per metre exists along the x-axis from x = – a to x = + a. Find the electric field intensity E at a point P at a distance r along the perpendicular bisector.*

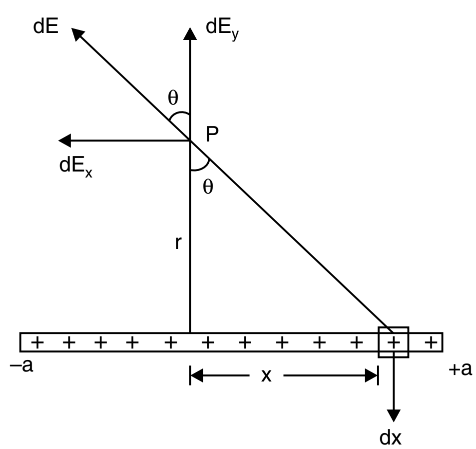


Fig. 1.62

Sol. In all the problems, which involve distribution of charge, we choose an element of charge dq to find the element of the field dE produced at the given location. Then we sum all such dEs to find the total field E at that location.

We must note the symmetry of the situation. For each element dq located at positive x-axis. There is a similar dq located at the same negative value of x. The dE_x produced by one dq is cancelled by the dE_x in the opposite direction due to the other dq. Hence, all the dE_x components add to zero. So, we need to sum only the dE_y components, a scalar sum since they all point in the same direction.

The element charge is $dq = \lambda\, dx$

$$\therefore \quad dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2 + x^2} = \frac{\lambda\, dx}{4\pi\varepsilon_0\left(r^2 + x^2\right)}$$

$$\text{So,} \quad E = \int dE_y = \int dE\cos\theta$$

$$= \int \frac{\lambda\, dx}{4\pi\varepsilon_0\left(r^2 + x^2\right)}\frac{r}{\sqrt{r^2 + x^2}}$$

$$E = \frac{\lambda r}{4\pi\varepsilon_0}\int \frac{dx}{\left(r^2 + x^2\right)^{3/2}}$$

The integral on the right hand side can be evaluated by substituting $x = r\tan\alpha$ and $dx = r\sec^2\alpha\, d\alpha$

$$\therefore \quad \int \frac{dx}{\left(r^2 + x^2\right)^{3/2}} = \int \frac{r\sec^2\alpha\, d\alpha}{r^3\sec^3\alpha} = \int \frac{\cos\alpha}{r^2}d\alpha = \frac{\sin\alpha}{r^2}$$

$$\Rightarrow \quad \int \frac{dx}{\left(r^2 + x^2\right)^{3/2}} = \frac{1}{r^2}\frac{x}{\sqrt{x^2 + r^2}}$$

$$\therefore \quad E_y = \frac{\lambda r}{4\pi\varepsilon_0}\left|\frac{x}{r^2\sqrt{x^2 + r^2}}\right|_{-a}^{+a} = \frac{\lambda r}{4\pi\varepsilon_0}\frac{2a}{r^2\sqrt{a^2 + r^2}}$$

The net field at P is $E = E_y$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}\frac{a}{\sqrt{a^2 + r^2}}}$$

Q. 4. *An electric dipole consists of charges of 2.0×10^{-8} C separated by a distance of 2 mm. It is placed near a long line charge of density 4.0×10^{-4} cm^{-1} as shown in the figure below, such that the negative charge is at a distance of 2 cm from the line charge. Calculate the force acting on dipole.*

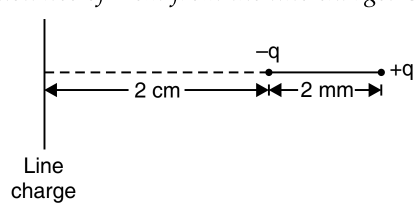


Fig. 1.63

Sol. Electric field intensity at a distance r from line charge of density λ is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$\therefore$ Field intensity on negative charge (r = 0.02 m)

$$E_1 = \frac{4\times10^{-4}\times9\times10^{9}\times2}{0.02} = 3.6\times10^{8} \text{ N/C.}$$

Force on negative charge

$$F_1 = qE_1 = 2\times10^{-8}\,(3.6\times10^{8}) = 7.2 \text{ N}$$

It is directed towards the line charge.

Similarly field intensity at positive charge (r = 0.022 m)

$$E_2 = \frac{4\times10^{-4}\times9\times10^{9}\times2}{0.022} = 3.27\times10^{8} \text{ N/C}$$

Force on positive charge

$$F_2 = qE_2 = 2\times10^{-8}\,(3.27\times10^{8}) = 6.54 \text{ N.}$$

It is directed away from the line charge.

$\therefore$ Net force on the dipole,

$$F = F_1 - F_2 = (7.2 - 6.54)\,N = 0.66 \text{ N.}$$

F is towards the line charge.

Q. 5. *A rigid insulated wire frame in the form of a right-angled triangle ABC, is set in a vertical plane as shown in fig. Two beads of equal masses m each and carrying charges q_1 and q_2 are connected by a cord of length l and can slide without friction on the wires.*

Considering the case when the beads are stationary, determine

(i) the angle α

(ii) the tension in the cord and

(iii) the normal reaction on the beads.

If the cord is now cut, what are the values of the charges for which the beads continue to remain stationary?

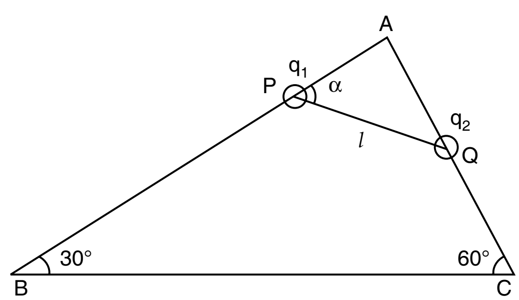


Fig. 1.64

Sol. The forces acting on the bead P are shown in the figure below.

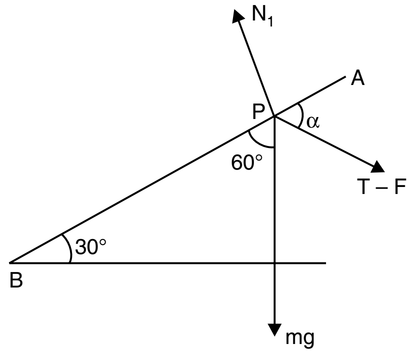


Fig. 1.65

Assuming that the charges are similar, the repulsive force F is

$$F = k\frac{q_1 q_2}{l^2}$$

where, k is a constant. Let T be the tension in the string and N_1, the normal reaction on the bead at P. Considering the components of forces perpendicular and parallel to AB. We have, for equilibrium,

$$mg \cos 60° = (T - F) \cos \alpha \quad ...(i)$$

$$N_1 = mg \cos 30° + (T - F) \sin \alpha \quad ...(ii)$$

For the bead at θ, we have

$$mg \sin 60° = (T - F) \sin \alpha \quad ...(iii)$$

$$N_2 = mg \cos 60° + (T - F) \cos \alpha \quad ...(iv)$$

Squaring and adding equation (*i*) and (*iii*) we get

$$m^2g^2 = (T - F)^2$$

$$T - F = \pm\, mg \quad ...(v)$$

Assuming that the charges are similar, we have

$$T - F = +mg$$

$$T = F + mg = k\frac{q_1 q_2}{l^2} + mg \quad ...(vi)$$

From equation (*i*) we have

$$mg \cos 60° = mg \cos \alpha$$

$$\therefore \quad \alpha = 60°$$

Equation (*ii*) gives

$$N_1 = mg\,[\text{cis } 30° + \cos 30°] = mg \cos 30° + mg \sin 60°$$

$$= 2\, mg \cos 30°$$

$$= mg\sqrt{3}$$

From equation (*iv*) we have

$$N_2 = mg \cos 60° + mg \cos 60° = mg$$

If the string is cut, $T = 0$ and we get, from equation (*v*)

$$F = \pm mg$$

$$F = k\frac{q_1 q_2}{l^2} = \pm mg$$

or, $$q_1 q_2 = \frac{\pm mg l^2}{k}$$

Thus q_1 and q_2 may have same or opposite signs.

Q. 6. *A thin insulating rod of length L carries a uniformly distributed charge Q. Find the electric field strength at a point along its axis at a distance 'a' from one end.*

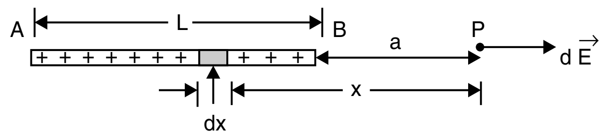


Fig. 1.66

Sol. Let us consider an infinitesimal element of length dx at a distance x from the point P. The charge on this element is $dq = \lambda dx$, where $\lambda \left(= \frac{Q}{L}\right)$ is the linear charge density.

The magnitude of the electric field at P due to this element is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{(\lambda dx)}{x^2}$$

and its direction is to the right since λ is positive. The total electric field strength E is given by

$$E = \frac{1}{4\pi\varepsilon_0}\lambda\int_a^{a+L}\frac{dx}{x^2}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[-\frac{1}{x}\right]_a^{a+L}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{a+L}\right]$$

$$= \frac{Q}{(4\pi\varepsilon_0)\, a\,(a+L)} \quad (\because\ Q = \lambda L)$$

Q. 7. *Two pieces of copper, each weighing 0.01 kg are placed at a distance of 0.1 m from each other one electron from per 1000 atoms of one piece is transferred to other piece of copper. What will be the coulomb force between two piece after the transfer of electrons? Atomic weight of copper is 63.5 g/mole. Avagadro's number = 6 × 10^{23}/gram mole.*

Sol. Number of atoms in each piece of copper

$$= \frac{6 \times 10^{23} \times 10}{63.5} = 9.45 \times 10^{22}$$

Number of electrons transferred = $\frac{1}{1000} \times 9.45 \times 10^{22}$

$$n = 9.45 \times 10^{19}$$

∴ Charges on the each piece after transfer

$$q_1 = q_2 = \pm ne = \pm 9.45 \times 10^{19} \times 1.6 \times 10^{-19}$$
$$= \pm 15.12 \text{ C}$$
$$r = 0.1 \text{ m}$$
$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} = 9 \times 10^9 \frac{(15.12)^2}{(0.1)^2}$$
$$= 2.06 \times 10^{14} \text{ N.}$$

Q. 8. *Two equally charged particles, held 3.2 × 10^{-3} m apart, are released from rest. The initial acceleration of the first particle is observed to be 7.0 m/s² and that of the second to be 9.0 m/s². If the mass of the first particle is 6.3 × 10^{-7} kg, what are (a) the mass of the second particle and (b) the magnitude of the charge of each particle?*

Sol. Given, $a_1 = 7.0 \text{ m/s}^2, \quad a_2 = 9.0 \text{ m/s}^2$

$m_1 = 6.3 \times 10^{-7} \text{ kg}, \quad m_2 = ?$

As $F_1 = F_2$

∴ $m_1 a_1 = m_2 a_2$

$$m_2 = \frac{m_1 a_1}{a_2} = \frac{6.3 \times 10^{-7} \times 7.0}{9.0} = 4.9 \times 10^{-7} \textit{ kg}$$

As $F_1 = F_2 = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = m_1 a_1 = 6.3 \times 10^{-7} \times 7.0 = 44.1 \times 10^{-7}$

∴ $$\frac{9 \times 10^9 q^2}{(3.2 \times 10^{-3})^2} = 44.1 \times 10^{-7}$$

∴ $q = 7.1 \times 10^{-11}$ C.

Q. 9. *Two negative charges of unit magnitude and a positive charge q are placed along a straight line. At what position and for what value of q will the system be in equilibrium? Check whether it is a stable or neutral equilibrium.*

Sol. Let the charge q be at distances r_1 and r_2 from the two charges P and Q respectively, as shown in the figure below:

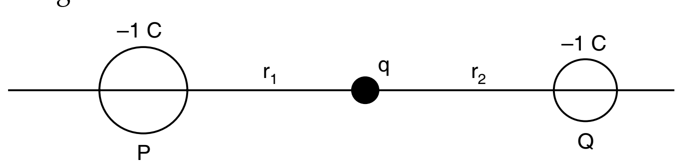


Fig. 1.67

For equilibrium of q, the forces on it exerted by P and Q must be co-linear, equal and opposite.

Force on q by P

$$F_{qP} = \frac{q}{4\pi\varepsilon_0 r_1^2} \text{ towards } P$$

Force on q by Q

$$F_{qQ} = \frac{q}{4\pi\varepsilon_0 r_2^2} \text{ towards } Q$$

$$\therefore \quad |F_{qP}| = |F_{qQ}| \quad \text{or} \quad \frac{q}{r_1^2} = \frac{q}{r_2^2}$$

$$\therefore \quad r_1 = r_2 = r$$

Hence charge q should be equidistant from P and Q.

For the system to be in equilibrium, the charges P and Q must also be in equilibrium. Now,

$$F_{pq} = \text{force on } P \text{ by } q = \frac{q}{4\pi\varepsilon_0 r^2} \text{ (towards } q)$$

$$F_{PQ} = \text{force on } Q \text{ by } P = \frac{1}{4\pi\varepsilon_0 (2r)^2} \text{ (away from } P \text{ and away from } q)$$

Since F_{pq} and F_{PQ} are oppositely directed along the same line, we have, for equilibrium,

$$\frac{q}{4\pi\varepsilon_0 r^2} = \frac{1}{4\pi\varepsilon_0 \left(4r^2\right)}$$

or,
$$q = \frac{1}{4}$$

Similarly for the equilibrium of Q, we would get $q = \frac{1}{4}$. Thus $q = \frac{1}{4}$ in magnitude of either charge P or Q.

Stability: A slight displacement of q towards P increases the magnitude of F_{qP} and decreases the magnitude of F_{qQ}. Consequently, the displacement of q is increased. Thus the three charges are no longer in equilibrium. Hence the original equilibrium is unstable for displacement along the axis on which the charges are located. For a displacement of q along a direction normal to the line PQ, the resultant of the two forces of attraction F_{qP} and F_{qQ} will bring the charge q back to its original position. Thus the equilibrium is stable for displacement in the vertical direction.

Q. 10. *A 12 μC charge is placed at the distance of 10 cm from a linear charge of 100 μC uniformly distributed once the length of 10 cm as shown in figure. Find the force on 12 μC charge.*

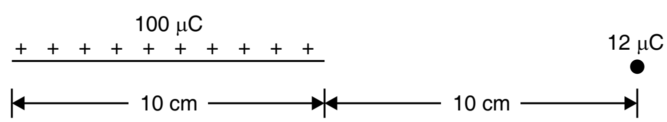


Fig. 1.68

Sol. Force on 12 μC charge due to an elementary part of the linear charge,

$$dF = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(\lambda dx)\, 12 \times 10^{-6}}{x^2}$$

$$= 9 \times 10^9 \times \frac{100 \times 10^{-6}}{10 \times 10^{-2}} \times 12 \times 10^{-6} \times \frac{dx}{x^2}$$

$$= 1.08 \frac{dx}{x^2}$$

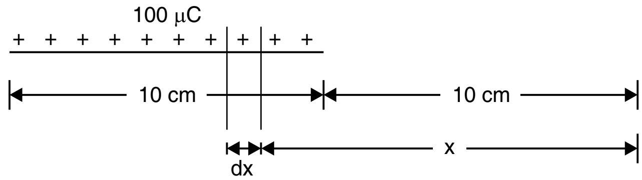


Fig. 1.68

Net force on 12 μC charge

$$F = \int dF = 1.08 \int_{0.10}^{0.20} \frac{dx}{x^2} = 1.08 \left[-\frac{1}{x} \right]_{0.01}^{0.20}$$

$$= 1.08 \left[\frac{1}{0.10} - \frac{1}{0.20} \right] = 10.8 \left[1 - \frac{1}{2} \right] = 5.4 \ N.$$

Q. 11. *Two identical charged bodies have 12 μC and – 18 μC charge respectively. These bodies experience a force of 48 N at certain separation. The bodies are touched and placed at the same separation again. Find the new force between the bodies.*

Sol. When two identical bodies having different magnitude of charge are touched, the redistribution of charge takes place and both the bodies acquire same charge.

∴ Charge on each body after touching

$$= \frac{12 - 18}{2} = -3 \ \mu C$$

The new force between the bodies

$$F = \frac{1}{4\pi\varepsilon_0} \frac{3 \times 10^{-6} \times 3 \times 10^{-6}}{x^2}$$

but

$$48 = \frac{1}{4\pi\varepsilon_0} . \frac{12 \times 10^{-6} \times 18 \times \mu C}{x^2}$$

$$\therefore \ F = \frac{48 \times 3 \times 10^{-6} \times 3 \times 10^{-6}}{12 \times 10^{-6} \times 18 \times 10^{-6}} = \frac{48 \times 3 \times 3}{12 \times 18} = 2 \text{ N.}$$

MULTIPLE CHOICE QUESTIONS

1. Two equal charges are separated by a distance 'd'. A third charge placed on a perpendicular bisector at x distance will experience maximum coulomb force when

(*a*) $x = d\sqrt{2}$ (*b*) $x = d/2$

(*c*) $x = \frac{d}{2\sqrt{2}}$ (*d*) $x = \frac{d}{2\sqrt{3}}$

2. A point charge Q is placed at the mid-point of a line joining two charges, $4q$ and q. If the net force on charge q is zero, then Q must be equal to

(*a*) $-q$ (*b*) $+q$

(*c*) $-2q$ (*d*) $+4q$

3. A force of repulsion between two point charges is F, when these are at a distance 0.1 m apart. Now the point charges are replaced by spheres of radii 5 cm each having the same charge as that of the respective point charges. The distance between their centres is again kept 0.1 m, then the force of repulsion will:

(*a*) increase (*b*) decrease

(*c*) remain F (*d*) become $\frac{10\,F}{9}$

4. Two equal negative charges $-q$ are fixed at points $(0, a)$ and $(0, -a)$. A positive charge Q is released from rest at the point $(2a, 0)$ on the x-axis. The charge Q will

(*a*) execute SHM about the origin (*b*) move to origin and remain at rest

(*c*) move to infinity (*d*) execute oscillation but not SHM

5. Two vertical metallic plates carrying equal and opposite charges are parallel to each other. A small spherical metallic ball is suspended by a long insulating thread such that it hangs freely in the centre of the two metallic plates. The ball, which is uncharged is taken slowly towards the positively charged plate and is made to touch the plate. Then ball will

(*a*) stick to the positively charged plate.

(*b*) come back to the original position and will remain there.

(*c*) oscillate between the two plates touching each plate in turn.

(*d*) oscillate between the plates without touching them.

6. A charge Q is divided into two parts q and $Q - q$ and separated by a distance R. The force of repulsion between them will be maximum when:

(*a*) $q = Q/4$ (*b*) $q = Q/2$

(*c*) $q = Q$ (*d*) none of these

7. A conducting sphere of radius 10 cm is charged with 10 μC. Another uncharged sphere of radius 20 cm is allowed to touch it for some time. After that if the spheres are separated, then surface density of charges on the spheres will be in the ratio of

(*a*) 1 : 4 (*b*) 1 : 3

(*c*) 1 : 2 (*d*) 1 : 1

8. Two equal unlike charges placed 3 cm apart in air attract each other with a force of 40 N. The magnitude of each charge in micro coulombs is

(*a*) 0.2 (*b*) 2

(*c*) 20 (*d*) 200

9. If a charge q is placed at the centre of the line joining two equal like charges Q. The systems or three will be in equilibrium if q is

(*a*) $-Q/2$ (*b*) $-Q/4$

(*c*) $Q/2$ (*d*) $4Q$

10. Hollow spherical conductor with a charge if 500 C is acted upon by a force 562.5 N. What is E at its surface?

(*a*) zero (*b*) 1.125 NC^{-1}

(*c*) 2.25×10^{6} NC^{-1} (*d*) 4.5×10^{-4} NC^{-1}

11. A wire is bent into a ring of radius R is given a charge q. The magnitude of the electrical field at the centre of the ring is

(*a*) two (*b*) 1/2

(*c*) zero (*d*) 3/2

12. The electric field due to an infinitely long thin wire at a distance R varies as

(*a*) $\frac{1}{R}$ (*b*) $1/R^2$

(*c*) R (*d*) R^2

13. A charged particle q is placed at the centre O of a cube of length L (ABCDEFGH). Another same charge q is placed at a distance L from O. Then the electric flux through $ABCD$ is

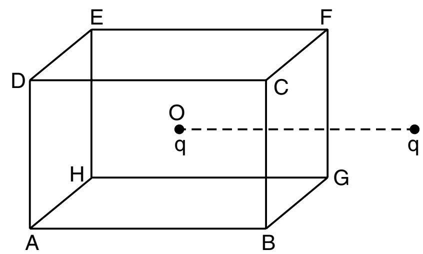


Fig. 1.70

(*a*) $\dfrac{q}{4\pi\varepsilon_0 L}$ (*b*) Zero

(*c*) $\dfrac{q}{2\pi\varepsilon_0 L}$ (*d*) $\dfrac{q}{3\pi\varepsilon_0 L}$

14. A hollow insulating conducting sphere is given a positive charge of 10 μC. What will be the electric field at the centre of the sphere if its radius is 2 metres?

(*a*) Zero (*b*) 5 NC^{-1}

(*c*) 20 NC^{-1} (*d*) 8 NC^{-1}

15. If electric field in a region is radially outward with magnitude $E = Ar$. The charge contained in a sphere of radius r_o centred at the origin is

(*a*) $\dfrac{4\pi\varepsilon_0 A}{r_0^3}$ (*b*) $\dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{A}{r_0^3}$

(*c*) $4\pi\varepsilon_0 A r_0^3$ (*d*) $\dfrac{1}{4\pi\varepsilon_o} A r_0^3$

Answers

1. (*a*)	**2.** (*a*)	**3.** (*c*)	**4.** (*c*)	**5.** (*d*)
6. (*d*)	**7.** (*a*)	**8.** (*a*)	**9.** (*c*)	**10.** (*d*)
11. (*d*)	**12.** (*b*)	**13.** (*d*)	**14.** (*c*)	**15.** (*c*).

TEST YOUR SKILLS

1. What do you understand by the additive property of electric charge?
2. What is meant by 'electrostatic induction'?
3. Why do we say that 'induction always precedes attraction'?
4. Why do vehicles carrying inflammable material and running on rubber tyres, always drag a chain along the ground?
5. Write the complete explicit vector form of Coulomb's law.
6. Estimate the ratio of the electric force of repulsion between two proton to the gravitational force of attraction between them.
7. Sketch the electric field lines due to point charge (*i*) $q > 0$ and (*ii*) $q < 0$.
8. What is an electric field line? Sketch lines of field due to two equal positive charges placed at a small distance apart in air.
9. A uniform electric field, parallel to the x-axis, is present in a certain region. In what direction can a charge be displaced in this field without any external work being done on it?
10. Two identical point charges Q are kept at a distance r from each other. A third point charge is placed on the line joining these two charges such that all the three charges are in equilibrium. What is the magnitude, sign and position of the third charge?
11. In a medium the force of attraction between two point electric charges, distance d apart, is F. What distance apart should these be kept in the same medium so that the force between them becomes (*i*) 3 F (*ii*) $F/3$?

12. Define electric dipole moment. Derive an expression for the electric field intensity at any point along the equatorial line of a dipole.

13. Derive an expression for the maximum torque acting on an electric dipole, when held in a uniform electric field.

14. Two point electric charges of unknown magnitude and sign are placed a distance d apart. The electric field intensity is zero at a point not between the charges but on the line joining them. Write two essential conditions for this to happen.

15. An electric dipole is held in a uniform electric field.

(*i*) Show that no translation force acts on it.

(*ii*) Derive an expression for the torque acting on it.

(*iii*) The dipole is aligned parallel to the field. Calculate the work done in rotating it through 180°.

16. Define electric flux. Write its S.I. unit. A spherical rubber balloon carries a charge that is uniformely distributed over its surface. As the balloon is blown up and increase in size, how does the electric flux coming out of the surface change? Give reason.

17. The electric field due to a point charge at any point near it is defined as $E = \lim_{q \to 0} F/q$ where q is the test charge and F is the force acting on it. What is the physical significance of $\lim_{q \to 0}$ in this expression?

18. Using Gauss's theorem, calculate the field due to a thin plane infinite sheet of charge, having a uniform surface charge density σ.

19. Show that for a uniformly charged hollow sphere, the electric field has a maximum value at the surface of the sphere.

20. Calculate the force between an alpha particle and a proton separated by 5.12×10^{-15} m.

21. If an oil drop of weight 3.2×10^{-13} N is balanced in an electric field of 5×10^{5} Vm^{-1}, find the charge on oil drop.

22. Two insulated charged copper spheres A and B have their centres separated by a distance of 50 cm. What is mutual force of repulsion if charge on each sphere is 6.5×10^{-7} C. The radii of A and B are negligible compared to the distance of separation.

What is force of repulsion if

(*i*) each sphere is charged double the above amount and distance between them is halved?

(*ii*) the two spheres are placed in water of dielectric constant 80?

23. Two point charges q_1 = +0.2 C and q_2 = +0.4 C are placed 0.1 m apart. Calculate the electric field at (*a*) the mid-point between the charges and (*b*) at a point on the line joining q_1 and q_2 such that it is 0.05 m from q_2 and 0.15 m away from q_1.

24. Two point charges q_A = +2 μC and q_B = –3 μC are located 20 cm apart in vacuum.

(*i*) Find the electric field at the midpoint of the line AB joining the two charges, (*ii*) If a negative test charge of magnitude 1.5×10^{-9} C is placed at the centre, find the force experienced by the test charge.

25. Two point electric charges of values q and $2q$ are kept at a distance d apart from each other in air. A third charge Q is to be kept along the same line in such a way that the net force acting on q and $2q$ is zero. Calculate the position of change Q in terms of q and d.

26. Two charges one +5 μC and another –5 μC are kept 1 mm apart. Calculate the dipole moment.

27. An electric dipole, when held at 30° with respect to a uniform electric field of 10^4 N/C experiences a torque of 9×10^{-26} Nm. Calculate the dipole moment of the dipole.

❑❑❑