10 Mechanical Properties of Fluids

Facts that Matter

- Fluids are the sustances which can flow *e.g.,* liquids and gases. It does not possess definite shape.
- When an object is submerged in a liquid at rest, the fluid exerts a force on its surface normally. It is called thrust of the liquid.

• Pressure

The thrust experienced per unit area of the surface of a liquid at rest is called pressure.

$$P = \frac{F}{A}$$

In CGS system, unit of pressure is dyne cm^{-2}. In SI, unit of pressure is Nm^{-2} or Pascal (Pa).

$$1 \text{ Pa} = 1 \text{ Nm}^{-2}.$$

- When a liquid is in equilibrium, the force acting on its surface is perpendicular everywhere. The pressure is the same at the same horizontal level.
- The pressure at any point in the liquid depends on the depth (h) below the surface, density of liquid and acceleration due to gravity.

• Pascal's Law

According to Pascal's Law, the pressure applied to an enclosed liquid is transmitted undiminished to every portion of the liquid and the walls of the containing vessel.

- Hydraulic system works on Pascal's law. Force exerted to area, ratio will be same at all cross-sections.

$$\frac{F_1}{a_1} = \frac{F_2}{a_2}$$

> **Note:** A large force is experienced in larger cross-section if a smaller force is applied in smaller 4cross-section.

- A column of height h of a liquid of density ρ exerts a pressure P given by the relation

$$P = h\rho g$$

- If P_a be the atmospheric pressure then pressure in a liquid at a depth h from its free surface is given by $P = P_a + h\rho g$. Relation is true for incompressible fluids only.
- The gauge pressure (P_g), is the difference of the absolute pressure (P) and the atmospheric pressure (P_a).

$$\text{Absolute pressure } (P) = \text{Gauge pressure } (P_g) + \text{Atmospheric pressure } (P_a)$$

$$\Rightarrow \quad P_g = P - P_a.$$

• Archimedes Principle

When a body is partially or completely immersed in a liquid, it loses some of its weight. The loss in weight of the body in the liquid is equal to the weight of the liquid displaced by the immersed part of the body.

• The upward force excerted by the liquid displaced when a body is immersed is called buoyancy. Due to this, there is apparent loss in the weight experienced by the body.

• Law of Floatation

"A body floats in a liquid if weight of the liquid displaced by the immersed portion of the body is equal to the weight of the body."

When a body is immersed partially or wholly in a liquid, then the various forces acting on the body are

(*i*) upward thrust (T) acting at the centre of buoyancy and whose magnitude is equal to the weight of the liquid displaced and

(*ii*) the weight of the body (W) which acts vertically downward through its centre of gravity.

(*a*) When $W > T$, the body will sink in the liquid;

(*b*) When $W = T$, then the body will remain in equilibrium inside the liquid;

(*c*) When $W < T$, then the body will come upto the surface of the liquid in such a way that the weight of the liquid displaced due to its immersed portion equals the weight of the body. Thus the body will float with only a part of it immersed inside the liquid.

• The flow of a liquid is said to be **steady or stream line flow** if such particle of the fluid passing through a given point travels along the same path and with same speed as the preceding particle passing through that very point.

• If the liquid flows over a horizontal surface in the form of layers of different velocities, then the flow of the liquid is called **laminar flow.**

• The flow of fluid in which velocity of all particles crossing a given point is not same and the motion of fluid becomes irregular or disordered is called **turbulent flow.**

• Equation of Continuity

According to equation of continuity, if there is no fluid source or sink along the length of a pipe, then mass of the fluid crossing any section of the pipe per unit time remains constant.

i.e., $$a_1 v_1 \rho_1 = a_2 v_2 \rho_2$$

For incompressible liquids (*i.e.,* fluids) $\rho_1 = \rho_2$ and hence the equation is given as

$$a_1 v_1 = a_2 v_2$$

It means that speed of flow of liquid is more where the pipe is narrower and speed of flow is less where the cross-section of pipe is more.

• Energy of a liquid

A liquid can possess three types of energies: (*i*) kinetic energy, (*ii*) potential energy and (*iii*) pressure energy

The energy possessed by a liquid due to its motion is called kinetic energy *i.e.,* $\frac{1}{2}mv^2$.

The potential energy of a liquid of mass m at a height h is given by

$$\text{P.E.} = mgh$$

The energy possessed by a liquid by virtue of its pressure is called pressure energy.

Pressure energy of liquid in volume $dV = PdV$

Pressure energy per unit mass of the liquid $= \dfrac{PdV}{\rho dV} = \dfrac{P}{\rho}$ $(m = \rho dV)$

• Bernoulli's Theorem

For an incompressible, non-viscous, irrotational liquid having streamlined flow, the sum of the pressure energy, kinetic energy and potential energy per unit mass is a constant *i.e.,*

$$\frac{P}{\rho} + \frac{v^2}{2} + gh = \text{constant} \quad \text{or} \quad \frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{constant}$$

• For steady flow of a non-viscous fluid along a horizontal pipe, Bernoulli's equation is simplified as

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

• Viscosity

Viscosity is the property of the fluid (liquid or gas) by virtue of which an internal frictional force comes into play when the fluid is in motion in the form of layers having relative motion. It opposes the relative motion of the different layers. Viscosity is also called as fluid friction.

• The viscous force directly depends on the area of the layer and the velocity gradient.

$$F = -\eta A \frac{dv}{dx}$$ (*–ve* sign shows the opposing nature)

• Coefficient of Viscosity

Coefficient of viscosity of a liquid is equal to the tangential force required to maintain a unit velocity gradient between two parallel layers of liquid each of area unity.

$$\eta = \frac{F}{A\left(\dfrac{dv}{dx}\right)}$$

The SI unit of coefficient of viscosity is poiseuille (*Pl*) or *Pa* – s or Nm^{-2} s or kg m^{-1} s^{-1}. Dimensional formula of η is $[ML^{-1} T^{-1}]$.

• Stoke's Law

According to Stokes' law the backward dragging force acting on a small spherical body of radius r moving with a velocity v through a viscous medius of coefficient of viscosity η is given by

$$F = 6\pi\eta r v$$

• Terminal Velocity

It is maximum constant velocity acquired by the body while falling freely in a viscous medium. This is attained when the apparent weight is compensated by the viscous force.

It is given by

$$v = \frac{2r^2(\rho - \sigma)\,g}{9\eta}$$

where ρ be the density of the material of the body of radius r and σ be the density of the medium.

- **Poiseuille's Equation**

According to Poiseuille, if a pressure difference (P) is maintained across the two ends of a capillary tube of length 'l' and radius 'r', then the volume of liquid coming out of the tube per second is

(*i*) directly proportional to the pressure difference (P).

(*ii*) directly proportional to the fourth power of radius (r) of the capillary tube.

(*iii*) inversely proportional to the coefficient of viscosity (η) of the liquid.

(*iv*) inversely proportional to the length (l) of the capillary tube.

It is given as

$$V = \frac{\pi p r^4}{8\eta l}$$

- **Reynold's Number**

Reynold number R_e is a dimensionless number whose value gives an approximate idea whether the flow of a fluid will be streamline or turbulent. It is given by

$$R_e = \frac{\rho v d}{\eta},$$

where ρ = density of fluid flowing with a speed v, d stands for the diameter of the pipe and η is the viscosity of the fluid. Value of R_e remains same in any system of units.

- It is observed that flow is streamline or laminar for $R_e \leq 1000$ and the flow is turbulent for $R_e \geq 2000$. The flow becomes unsteady for R_e between 1000 and 2000. The critical value of R_e, at which turbulence sets, is same for the geometrically similar flows.
- R_e may also be expressed as the ratio of inertial force (force due to inertia *i.e.*, mass of moving fluid or due to inertia of obstacle in its path) to viscous force *i.e.*,

$$R_e = \frac{\rho A v^2}{\left(\frac{\eta A v}{d}\right)}$$

- **Critical Velocity**

The critical velocity is that velocity of liquid flow, upto which its flow is streamline and above which its flow becomes turbulent.

It is given by

$$v_c = \frac{K\eta}{\rho r}$$

where K is a dimensionless constant, η is coefficient of viscosity of liquid, ρ is density of liquid and r is the radius of tube.

- **Surface Tension**

It is the property of the liquid by virtue of which the free surface of liquid at rest tends to have minimum area and as such it behaves as a stretched elastic membrane.

- The force acting per unit length of line drawn on the liquid surface and normal to it parallel to the surface is called the force of surface tension.

It is given by

$$T = \frac{F}{l}$$

The SI unit of surface tension is Nm^{-1} and its dimensional formula is $[MT^{-2}]$.

• Surface Energy

Energy possessed by the surface of the liquid is called surface energy. Change in surface energy is the product of surface tension and change in surface area under constant temperature.

• The height to which water rises in a capillary tube of radius r is given by

$$h = \frac{2T \cos \theta}{r\rho g}$$

where T is the surface tension of the liquid and θ is the angle of contact.

Due to surface tension there is excess pressure on the concave side of a surface film of a liquid over the convex side and is equal to $\frac{2T}{r}$. For a soap bubble the excess pressure is $\frac{4T}{r}$ where, r denotes the radius of the surface.

• Angle of Contact

The angle which the tangent to the liquid surface at the point of contact makes with the solid surface inside the liquid is called angle of contact.

• Intermolecular force amongst molecules of the same material is called the force of cohesion. However, force amongst molecules of different materials is called the force of adhesion.

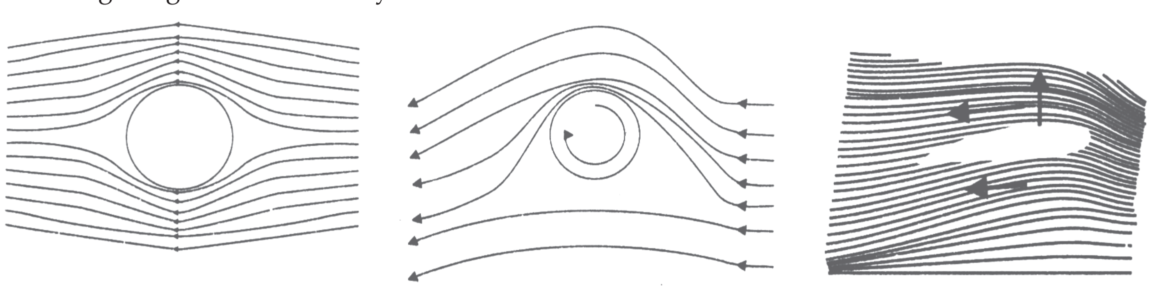
• Torricelli's Theorem

According to this theorem, velocity of efflux *i.e.,* the velocity with which the liquid flows out of on orifice (*i.e.,* a narrow hole) is equal to that which a freely falling body would acquire in falling through a vertical distance equal to the depth of orifice below the free surface of liquid.
The velocity is given by

$$v = \sqrt{2gh}$$

• Magnus Effect

When a ball is given a spin when it is in a streamline of air molecules, it will follow a curved path which is convex towards the greater pressure side. This idea is the basis of the ball from spin bowlers getting a lift and aerodynamics.



- **IMPORTANT TABLES**

TABLE 10.1

Physical Quantity	*Symbol*	*Dimensions*	*Unit*	*Remarks*
Pressure	P	$[M L^{-1}T^{-2}]$	pascal (Pa)	1 atm = 1.013×10^5 Pa, Scalar
Density	ρ	$[ML^{-3}]$	kg m^{-3}	Scalar
Specific Gravity		No	No	$\frac{\rho_{substance}}{\rho_{water}}$, Scalar
Co-efficient of viscosity	η	$[ML^{-1} T^{-1}]$	Pa s or poiseiulles (Pl)	Scalar
Reynold's Number	R_e	No	No	$R_e = \frac{\rho v d}{\eta}$ scalar
Surface Tension	S	$[MT^{-2}]$	Nm^{-1}	Scalar

TABLE 10.2 Densities of some common fluids at STP*

Fluid	*ρ (kg m^{-3})*
Water	1.00×10^3
Sea water	1.03×10^3
Mercury	13.6×10^3
Ethyl alcohol	0.806×10^3
Whole blood	1.06×10^3
Air	1.29
Oxygen	1.43
Hydrogen	9.0×10^{-2}
Interstellar space	$\approx 10^{-20}$

TABLE 10.3 The viscosities of some fluids

Fluid	*ρ (kg m^{-3})*	*Viscosity (mP)*
Water	20	1.0
	10	0.3
Blood	37	2.7
Machine Oil	16	113
	38	34
Glycerine	20	830
Honey		200
Air	0	0.017
	40	0.019

TABLE 10.4 Surface tension of some liquids at the temperatures indicated with the heats of the vaporisation

Liquid	*Temp (°C)*	*Surface Tension (N/m)*	*Heat of vaporisation (kJ/mol)*
Helium	–270	0.000239	0.115
Oxygen	–183	0.0132	7.1
Ethanol	20	0.0227	40.6
Water	20	0.0727	44.16
Mercury	20	0.4355	63.2

NCERT TEXTBOOK QUESTIONS SOLVED

10.1. *Explain why*

(*a*) *The blood pressure in humans is greater at the feet than at the brain.*

(*b*) *Atmospheric pressure at a height of about 6 km decreases to nearly half of its value at the sea level, though the height of the atmosphere is more than 100 km.*

(*c*) *Hydrostatic pressure is a scalar quantity even though pressure is force divided by area.*

Ans. (*a*) The height of the blood column is more for the feet as compared to that for the brain. Consequently, the blood pressure in humans is greater at the feet than at the brain.

(*b*) The variation of air-density with height is not linear. So, pressure also does not reduce linearly with height. The air pressure at a height h is given by $P = P_0 e^{-\alpha h}$ where P_0 represents the pressure of air at sea-level and α is a constant.

(*c*) Due to applied force on liquid, the pressure is transmitted equally in all directions inside the liquid. That is why there is no fixed direction for the pressure due to liquid. Hence hydrostatic pressure is a scalar quantity.

10.2. *Explain why*

(*a*) *The angle of contact of mercury with glass is obtuse, while that of water with glass is acute.*

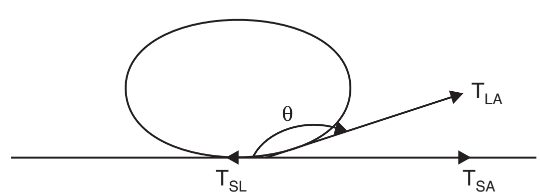
(*b*) *Water on a clean glass surface tends to spread out while mercury on the same surface tends to form drops. (Put differently, water wets glass while mercury does not.)*

(*c*) *Surface tension of a liquid is independent of the area of the surface.*

(*d*) *Water with detergent dissolved in it should have small angles of contact.*

(*e*) *A drop of liquid under no external forces is always spherical in shape.*

Ans. (*a*) Let a drop of a liquid L be poured on a solid surface S placed in air A. If T_{SL}, T_{LA} and T_{SA} be the surface tensions corresponding to solid-liquid layer, liquid-air layer and solid-air layer respectively and θ be the angle of contact between the liquid and solid, then



$$T_{LA}\cos\theta + T_{SL} = T_{SA} \Rightarrow \cos\theta = \frac{T_{SA} - T_{SL}}{T_{LA}}$$

For the mercury-glass interface, $T_{SA} < T_{SL}$. Therefore, cos θ is negative. Thus θ is an obtuse angle. For the water-glass interface, $T_{SA} > T_{SL}$. Therefore cos θ is positive. Thus, θ is an acute angle.

(*b*) Water on a clean glass surface tends to spread out *i.e.,* water wets glass because force of cohesion of water is much less than the force of adhesion due to glass. In case of mercury force of cohesion due to mercury molecules is quite strong as compared to adhesion force due to glass. Consequently, mercury does not wet glass and tends to form drops.

(*c*) Surface tension of liquid is the force acting per unit length on a line drawn tangentially to the liquid surface at rest. Since this force is independent of the area of liquid surface therefore, surface tension is also independent of the area of the liquid surface.

(*d*) We know that the clothes have narrow pores or spaces which act as capillaries. Also, we know that the rise of liquid in a capillary tube is directly proportional to cos θ (Here θ is the angle of contact). As θ is small for detergent, therefore cos θ will be large. Due to this, the detergent will penetrate more in the narrow pores of the clothes.

(*e*) We know that any system tends to remain in a state of minimum energy. In the absence of any external force for a given volume of liquid its surface area and consequently. Surface energy is least for a spherical shape. It is due to this reason that a liquid drop, in the absence of an external force is spherical in shape.

10.3. *Fill in the blanks using the words from the list appended with each statement:*

(a) Surface tension of liquids generally .... with temperature. *(increases/decreases)*

(b) Viscosity of gases .... with temperature, whereas viscosity of liquids .... with temperature. *(increases/decreases)*

(c) For solids with elastic modulus of rigidity, the shearing force is proportional to .... while for fluids it is proportional to .... *(shear strain/rate of shear strain)*

(d) For a fluid in steady flow, the increases in flow speed at a constriction follows from .... while the decrease of pressure there follows from ....

(conservation of mass/Bernoulli's principle)

(e) For the model of a plane in a wind tunnel, turbulence occurs at a .... speed than the critical speed for turbulence for an actual plane. *(greater/smaller)*

Ans. (*a*) decreases (*b*) increases; decreases

(*c*) shear strain; rate of shear strain (*d*) conservation of mass; Bernoulli's principle

(*e*) greater.

10.4. *Explain why*

(a) To keep a piece of paper horizontal, you should blow over, not under, it.

(b) When we try to close a water tap with our fingers, fast jets of water gush through the openings between our fingers.

(c) The size of a needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection.

(d) A fluid flowing out of a small hole in a vessel results in a backward thurst on the vessel.

(e) A spinning cricket ball in air does not follow a parabolic trajectory.

Ans. (*a*) When we blow over the piece of paper, the velocity of air increases. As a result, the pressure on it decreases in accordance with the Bernoulli's theorem whereas the pressure below remains the same (atmospheric pressure). Thus, the paper remains horizontal.

(*b*) By doing so the area of outlet of water jet is reduced, so velocity of water increases according to equation of continuity av = constant.

(*c*) For a constant height, the Bernoulli's theorem is expressed as

$$P + \frac{1}{2}\rho v^2 = \text{constant}$$

In this equation, the pressure P occurs with a single power whereas the velocity occurs with a square power. Therefore, the velocity has more effect compared to the pressure. It is for this reason that needle of the syringe controls flow rate better than the thumb pressure exerted by the doctor.

(*d*) This is because of principle of conservation of momentum. While the flowing fluid carries forward momentum, the vessel gets a backward momentum.

(*e*) A spinning cricket ball would have followed a parabolic trajectory has there been no air. But because of air the Magnus effect takes place. Due to the Magnus effect the spinning cricket ball deviates from its parabolic trajectory.

10.5. *A 50 kg girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter 1.0 cm. What is the pressure exerted by the heel on the horizontal floor?*

Ans. Mass of girl, $m = 50$ kg.

$\therefore$ Force on the heel, $F = mg = 50 \times 9.8 = 490$ N

Diameter, $D = 1.0 \text{ cm} = 1 \times 10^{-2}$ m

$$\therefore \quad \text{Area, } A = \frac{\pi D^2}{4} = \frac{3.14 \times (1 \times 10^{-2})^2}{4} = 7.85 \times 10^{-5} \text{ m}^2$$

$$\therefore \quad \text{Pressure, } P = \frac{F}{A} = \frac{490}{7.85 \times 10^{-5}} = 6.24 \times 10^6 \text{ Pa.}$$

10.6. *Toricelli's barometer used mercury. Pascal duplicated it using French wine of density 984 kg m^{-3}. Determine the height of the wine column for normal atmospheric pressure.*

Ans. We know that atmospheric pressure, $P = 1.01 \times 10^5$ Pa.

If we use French wine of density, $\rho = 984$ kg m^{-3}, then height of wine column should be h_m, such that $P = h\rho g$

$$\Rightarrow \quad h_m = \frac{P}{\rho g} = \frac{1.01 \times 10^5}{984 \times 9.8} = 10.47\text{m} \simeq 10.5 \text{ m}$$

10.7. *A vertical off-shore structure is built to withstand a maximum stress of 10^9 Pa. Is the structure suitable for putting up on top of an oil well in the ocean? Take the depth of the ocean to be roughly 3 km, and ignore ocean currents.*

Ans. Here, Maximum stress = 10^9 Pa, $h = 3 \text{ km} = 3 \times 10^3$ m;

ρ (water) = 10^3 kg/m^3 and $g = 9.8$ m/s^2.

The structure will be suitable for putting upon top of an oil well provided the pressure exerted by sea water is less than the maximum stress it can bear.

Pressure due to sea water, $P = h\rho g = 3 \times 10^3 \times 10^3 \times 9.8$ Pa $= 2.94 \times 10^7$ Pa

Since the pressure of sea water is less than the maximum stress of 10^9 Pa, the structure will be suitable for putting upon top of the oil well.

10.8. *A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 425 cm². What maximum pressure would the smaller piston have to bear?*

Ans. Pressure on the piston due to car

$$= \frac{\text{Weight of car}}{\text{Area of piston}}$$

$$P = \frac{3000 \times 9.8}{425 \times 10^{-4}} \text{ Nm}^{-2} = \mathbf{6.92 \times 10^5 \text{ Pa}}$$

This is also the maximum pressure that the smaller piston would have to bear.

10.9. *A U tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the relative density of spirit?*

Ans. For water column in one arm of *U* tube, $h_1 = 10.0$ cm; ρ_1(density) = 1 g cm^{-3}

For spirit column in other arm of *U* tube, $h_2 = 12.5$ cm; ρ_2 =?

As the mercury columns in the two arms of U tube are in level, therefore pressure exerted by each is equal.

Hence $$h_1\rho_1 g = h_2\rho_2 g \quad \text{or} \quad \rho_2 = \frac{h_1\rho_1}{h_2} = \frac{10 \times 1}{12.5} = 0.8 \text{ } g \text{ cm}^{-3}$$

Therefore, relative density of spirit = $\rho_2/\rho_1 = 0.8/1 = 0.8$

10.10. *In Q.9, if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in the two arms? (Relative density of mercury = 13.6)*

Ans. Let us select two points *A* and *B* lying in the same horizontal plane. Applying Pascal's law (taking into account the force of gravity),

Pressure at *A* = Pressure at *B*

$$\therefore \quad P_0 + h_w\rho_w g = P_0 + h_s\rho_s g + h_m\rho_m g$$

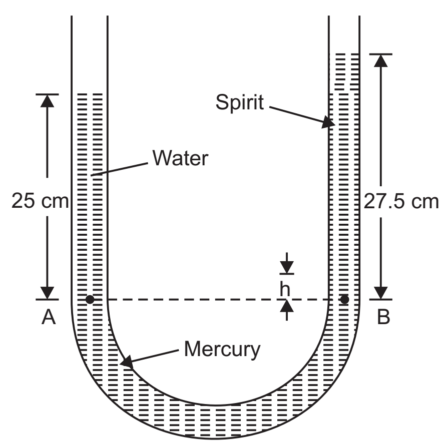
where P_0 is the atmospheric pressure,

Now, $$h_w\rho_w = h_s\rho_s + h_m\rho_m$$

$$\therefore \quad 25 \times 1 = 27.5 \times 0.8 + h \times 13.6$$

or $$h \times 13.6 = 25 - 27.5 \times 0.8$$

or $$h = \frac{25 - 22}{13.6} \text{ cm} = 0.2206 \text{ cm}$$



Mercury will rise in the arm containing spirit; the difference in levels of mercury will be 0.2206 cm.

10.11. *Can Bernoulli's equation be used to describe the flow of water through a rapid motion in a river? Explain.*

Ans. Bernoulli's theorem is applicable only for ther it ideal fluids in streamlined motion. Since the flow of water in a river is rapid, way cannot be treated as streamlined motion, the theorem cannot be used.

10.12. *Does it matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation? Explain.*

Ans. No, it does not matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation, provided the atmospheric pressure at the two points where Bernoulli's equation is applied are significantly different.

10.13. *Glycerine flows steadily through a horizontal tube of length 1.5 m and radius 1.0 cm. If the amount of glycerine collected per second at one end is 4.0×10^{-3} kg s^{-1}, what is the pressure difference between the two ends of the tube? (Density of glycerine = 1.3×10^3 kg m^{-3} and viscosity of glycerine = 0.83 Pa s). [You may also like to check if the assumption of laminar flow in the tube is correct].*

Ans. $l = 1.5$ m, $r = 1 \times 10^{-2}$m,

Volume/s, $$V = \frac{\text{Mass / s}}{\text{Density}} = \frac{4\times10^{-3}}{1.3\times10^{3}}\,\text{m}^3\,\text{s}^{-1} = \frac{4}{1.3}\times10^{-6}\,\text{m}^3\text{s}^{-1}$$

$$\eta = 0.83 \text{ Pa s}$$

Now, $$V = \frac{\pi p r^4}{8\eta l},$$

where p is the pressure difference across the capillary.

or $$p = \frac{8V\eta l}{\pi r^4}$$

Substituting values,

$$p = 8 \times \frac{4}{1.3}\times10^{-6}\times0.83\times1.5\times\frac{7}{22}\times\frac{1}{10^{-8}}\,Pa = \mathbf{9.75 \times 10^2\ Pa}$$

The Reynolds number is 0.3. So, the flow is **laminar.**

10.14. *In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are 70 ms^{-1} and 63 ms^{-1} respectively. What is the lift on the wing if its area is 2.5 m^2? Take the density of air to be 1.3 kg m^{-3}.*

Ans. Let v_1, v_2 be the speeds on the upper and lower surfaces of the wing of aeroplane, and P_1 and P_2 be the pressures on upper and lower surfaces of the wing respectively.

Then $v_1 = 70$ ms^{-1}; $v_2 = 63$ ms^{-1}; $\rho = 1.3$ kg m^{-3}.

From Bernoulli's theorem

$$\frac{P_1}{\rho} + gh + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh + \frac{1}{2}v_2^2$$

$$\therefore \quad \frac{P_1}{\rho} - \frac{P_2}{\rho} = \frac{1}{2}(v_2^2 - v_1^2)$$

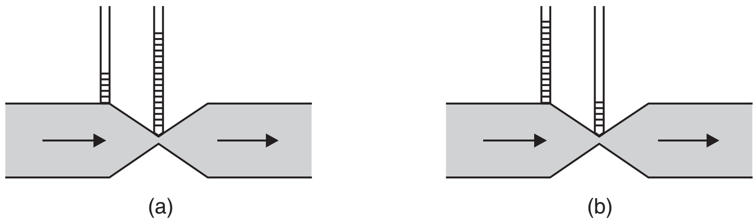
or $$P_1 - P_2 = \frac{1}{2}\rho\,(v_2^2 - v_1^2) = \frac{1}{2}\times1.3\,[(70)^2 - (63)^2]\ \text{Pa} = 605.15\ \text{Pa}.$$

This difference of pressure provides the lift to the aeroplane. So, lift on the aeroplane = pressure difference × area of wings

$$= 605.15 \times 2.5 \text{ N} = 1512.875 \text{ N}$$

$$= 1.51 \times 10^3 \text{ N}.$$

10.15. *Figures (a) and (b) refer to the steady flow of a (non-viscous) liquid. Which of the two figures in incorrect? Why?*



Ans. Figure (*a*) is incorrect. It is because of the fact that at the kink, the velocity of flow of liquid is large and hence using the Bernoulli's theorem the pressure is less. As a result, the water should not rise higher in the tube where there is a kink (*i.e.,* where the area of cross-section is small).

10.16. *The cylindrical tube of a spare pump has a cross-section of 8.0 cm² one end of which has 40 fine holes each of diameter 1.0 mm. If the liquid flow inside the tube is 1.5 m min⁻¹, what is the speed of ejection of the liquid through the holes?*

Ans. Total cross-sectional area of 40 holes, a_2

$$= 40 \times \frac{22}{7} \times \frac{(1\times10^{-3})^2}{4}\text{m}^2 = \frac{22}{7} \times 10^{-5}\text{m}^2$$

Cross-sectional area of tube, $a_1 = 8 \times 10^{-4}$ m²

Speed inside the tube, $v_1 = 1.5$ m min⁻¹ $= \frac{1.5}{60}\text{ms}^{-1}$;

Speed of ejection, $v_2 = ?$

Using $a_2v_2 = a_1v_1$,

we get

$$v_2 = \frac{a_1v_1}{a_2} = \frac{8\times10^{-4}\times\frac{1.5}{60}\times7}{22\times10^{-5}}\text{ms}^{-1} = 0.64 \text{ ms}^{-1}.$$

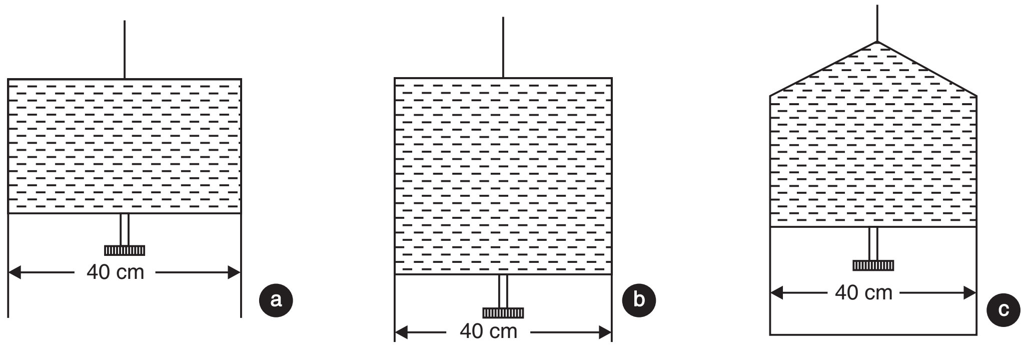
10.17. *A U-shaped wire is dipped in a soap solution, and removed. A thin soap film formed between the wire and a light slider supports a weight of 1.5 × 10⁻² N (which includes the small weight of the slider). The length of the slider is 30 cm. What is the surface tension of the film?*

Ans. In present case force of surface tension is balancing the weight of 1.5×10^{-2} N, hence force of surface tension, $F = 1.5 \times 10^{-2}$ N.

Total length of liquid film, $l = 2 \times 30$ cm $= 60$ cm $= 0.6$ m because the liquid film has two surfaces.

$$\therefore \quad \text{Surface tension, } T = \frac{F}{l} = \frac{1.5\times10^{-2}\text{N}}{0.6\text{ m}} = 2.5 \times 10^{-2} \text{ Nm}^{-1}.$$

10.18. *Figure (a) below shows a thin film supporting a small weight = 4.5 × 10⁻²N. What is the weight supported by a film of the same liquid at the same temperature in Fig. (b) and (c)? Explain your answer physically.*



Ans. (*a*) Here, length of the film supporting the weight = 40 cm = 0.4 m.

Total weight supported (or force) = 4.5×10^{-2} N.

Film has two free surfaces, $\therefore$ Surface tension, $S = \dfrac{4.5 \times 10^{-2}}{2 \times 0.4} = 5.625 \times 10^{-2}\ \text{Nm}^{-1}$.

Since the liquid is same for all the cases (*a*), (*b*) and (*c*), and temperature is also same, therefore surface tension for cases (*b*) and (*c*) will also be the same = 5.625×10^{-2}. In Fig. 7(*b*), 38(*b*) and (*c*), the length of the film supporting the weight is also the same as that of (*a*), hence the total weight supported in each case is 4.5×10^{-2} N.

10.19. *What is the pressure inside a drop of mercury of radius 3.0 mm at room temperature? Surface tension of mercury at that temperature (20°C) is $4.65 \times 10^{-1}\ \text{Nm}^{-1}$. The atmospheric pressure is 1.01×10^5 Pa. Also give the excess pressure inside the drop.*

Ans. Excess pressure $= \dfrac{2\sigma}{R} = \dfrac{2 \times 4.65 \times 10^{-1}}{3 \times 10^{-3}} = 310$ Pa

Total pressure $= 1.01 \times 10^5 + \dfrac{2\sigma}{R} = 1.01 \times 10^5 + 310 = 1.0131 \times 10^5$ Pa

Since data is correct upto three significant figures, we should write total pressure inside the drop as 1.01×10^5 Pa.

10.20. *What is the excess pressure inside a bubble of soap solution of radius 5.00 mm, given that the surface tension of soap solution at the temperature (20 °C) is $2.50 \times 10^{-2}\ \text{Nm}^{-1}$? If an air bubble of the same dimension were formed at depth of 40.0 cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble? (1 atmospheric pressure is 1.01×10^5 Pa).*

Ans. Here surface tension of soap solution at room temperature

$T = 2.50 \times 10^{-2}\ \text{Nm}^{-1}$, radius of soap bubble,

$r = 5.00\ \text{mm} = 5.00 \times 10^{-3}\text{m}$.

$\therefore$ Excess pressure inside soap bubble, $P = P_i - P_0 = \dfrac{4T}{r}$

$$= \frac{4 \times 2.50 \times 10^{-2}}{5.00 \times 10^{-3}} = 20.0 \text{ Pa}$$

When an air bubble of radius $r = 5.00 \times 10^{-3}$ m is formed at a depth h = 40.0 cm = 0.4 m inside a container containing a soap solution of relative density 1.20 or density $\rho = 1.20 \times 10^3\ \text{kg m}^{-3}$, then excess pressure

$$P = P_i - P_0 = \frac{2T}{r}$$

$$\therefore \quad P_i = P_0 + \frac{2T}{r} = (P_a + h\rho g) + \frac{2T}{r}$$

$$= \left[1.01 \times 10^5 + 0.4 \times 1.2 \times 10^3 \times 9.8 + \frac{2 \times 2.50 \times 10^{-2}}{5.00 \times 10^{-3}}\right] \text{ Pa}$$

$$= (1.01 \times 10^5 + 4.7 \times 10^3 + 10.0) \text{ Pa} \simeq 1.06 \times 10^5 \text{ Pa.}$$

10.21. *A tank with a square base of area 1.0 m^2 is divided by a vertical partition in the middle. The bottom of the partition has a small-hinged door of area 20 cm^2. The tank is filled with water in one compartment, and an acid (of relative density 1.7) in the other, both to a height of 4.0 m. Compute the force necessary to keep the door close.*

Ans. Pressure difference across the door

$$= (4 \times 1700 \times 9.8 - 4 \times 1000 \times 9.8) \text{ Pa}$$

$(6.664 \times 10^4 - 3.92 \times 10^4)$ Pa = 2.774×10^4 Pa

Force on the door = Pressure difference × Area of door

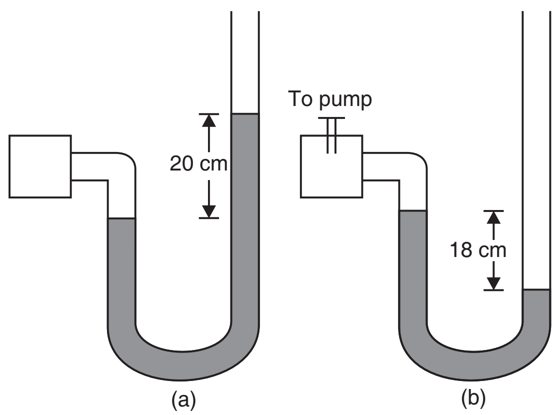
$$= 2.774 \times 10^4 \times 20 \times 10^{-4} \text{ N} = 54.88 \text{ N} = \mathbf{55\ N.}$$

Note. Base area does not affect the answer.

10.22. *A manometer reads the pressure of a gas in an enclosure as shown in Fig. (a). When a pump removes some of the gas, the manometer reads as in Fig. (b). The liquid used in the manometers is mercury and the atmospheric pressure is 76 cm of mercury.*

(a) Give the absolute and gauge pressure of the gas in the enclosure for cases (a) and (b), in units of cm of mercury.

(b) How would the levels change in case (b) if 13.6 cm of water (immiscible with mercury) is poured into the right limb of the manometer? Ignore the small change in the volume of the gas.



Ans. The atmospheric pressure, P = 76 cm of mercury

(*a*) From figure (*a*),

Pressure head, h = 20 cm of mercury

$\therefore$ Absolute pressure = $p + h$ = 76 + 20 = 96 cm of mercury

Also, Gauge pressure = h = 20 cm of mercury

From figure (*b*),

pressure head, h = –18 cm of mercury

$\therefore$ Absolute pressure = $p + h$ = 76 + (–18) = 58 cm of mercury

Also, Gauge pressure = h = –18 cm of mercury (*b*) When 13.6 cm of water is poured into the right limb of the manometer of figure (*b*), then, using the relation:

$$\text{Pressure} = \rho g h = \rho' g' h'$$

We get $h' = \frac{\rho h}{\rho'} = \frac{1 \times 13.6}{13.6} = 1$ cm of mercury [ρ' = density of mercury]

Therefore, pressure at the point B,

$$p_B = P + h' = 76 + 1 = 77 \text{ cm of mercury}$$

If h'' is the difference in the mercury levels in the two limbs, then taking $P_A = P_B$

$\Rightarrow$ $58 + h'' = 77 \Rightarrow h'' = 77 - 58 = 19$ cm of mercury.

10.23. *Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill up to a particular common height. Is the force exerted by the water on the base of the vessel the same in the two cases? If so, why do the vessels filled with water to that same height give different readings on a weighing scale?*

Ans. Pressure (and therefore force) on the two equal base areas are identical. But force is exerted by water on the sides of the vessels also, which has a non-zero vertical component when sides of the vessel are not perfectly normal to the base. This net vertical component of force by water on the sides of the vessel is greater for the first vessel than the second. Hence, the vessels weigh different even when the force on the base is the same in the two cases.

10.24. *During blood transfusion, the needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein? Given: density of whole blood = 1.06×10^3 kg m^{-3}.*

Ans.

$$h = \frac{P}{\rho g} = \frac{2000}{1.06 \times 10^3 \times 9.8} = 0.1925 \text{ m}$$

The blood may just enter the vein if the height at which the blood container be kept must be slightly greater than 0.1925 m *i.e.,* 0.2 m.

10.25. *In deriving Bernoulli's equation, we equated the work done on the fluid in the tube to its change in the potential and kinetic energy. (a) What is the largest average velocity of blood flow in an artery of diameter 2×10^{-3} m if the flow must remain laminar? (b) Do the dissipative forces become more important as the fluid velocity increases? Discuss qualitatively.*

Ans. (*a*) If dissipative forces are present, then some forces in liquid flow due to pressure difference is spent against dissipative forces, due to which the pressure drop becomes large.

(*b*) The dissipative forces become more important with increasing flow velocity, because of turbulence.

10.26. *(a) What is the largest average velocity of blood flow in an artery of radius 2×10^{-3} m if the flow must remain laminar?*

(b) What is the corresponding flow rate? Take viscosity of blood to be 2.084×10^{-3} Pa–s. Density of blood is 1.06×10^3 kg/m³.

Ans. Here, $r = 2 \times 10^{-3}$ m; $D = 2r = 2 \times 2 \times 10^{-3} = 4 \times 10^{-3}$ m;

$\eta = 2.084 \times 10^{-3}$ Pa–s; $\rho = 1.06 \times 10^3$ kg m^{-3}.

For flow to be laminar, $N_R = 2000$

(*a*) Now, $v_c = \frac{N_R \eta}{\rho D} = \frac{2000 \times (2.084 \times 10^{-3})}{(1.06 \times 10^3) \times (4 \times 10^{-3})}$ = 0.98 m/s.

(*b*) Volume flowing per second = $\pi r^2 v_c = \frac{22}{7} \times (2 \times 10^{-3})^2 \times 0.98$ = $\mathbf{1.23 \times 10^{-5}\ m^3 s^{-1}}$.

10.27. *A plane is in level flight at constant speed and each of its wings has an area of 25m². If the speed of the air is 180 km/h over the lower wing and 234 km/h over the upper wing surface, determine the plane's mass. (Take air density to be 1 kg/m³). g = 9.8 m/s².*

Ans. Here speed of air over lower wing, v_1 = 180 km/h = $180 \times \frac{5}{18}$ = 50 ms^{-1}

Speed over the upper wing, v_2 = 234 km/h = $234 \times \frac{5}{18}$ = 65 ms^{-1}

∴ Pressure difference, $P_1 - P_2 = \frac{1}{2}\rho\,(v_2^2 - v_1^2) = \frac{1}{2} \times 1\,(65^2 - 50^2)$ = 862.5 Pa

∴ Net upward force, $F = (P_1 - P_2)A$

This upward force balances the weight of the plane.

∴ $mg = F = (P_1 - P_2)A$ [$A = 25 \times 2 = 50$ m²]

∴ $m = \frac{(P_1 - P_2)\,A}{g} = \frac{862.5 \times 50}{9.8}$ = 4400 N.

10.28. *In Millikan's oil drop experiment, what is the terminal speed of an uncharged drop of radius 2.0×10^{-5} m and density 1.2×10^3 kg m^{-3}. Take the viscosity of air at the temperature of the experiment to be 1.8×10^{-5} Pa–s. How much is the viscous force on the drop at that speed? Neglect buoyancy of the drop due to air.*

Ans. Here radius of drop, $r = 2.0 \times 10^{-5}$ m, density of drop, $\rho = 1.2 \times 10^3$ kg/m³, viscosity of air $\eta = 1.8 \times 10^{-5}$ Pa–s.

Neglecting upward thrust due to air, we find that terminal speed is

$$v_T = \frac{2}{9}\frac{r^2 \rho g}{\eta} = \frac{2 \times (2.0 \times 10^{-5})^2 \times (1.2 \times 10^3) \times 9.8}{9 \times (1.8 \times 10^{-5})}$$

$$= 5.81 \times 10^{-2}\ \text{ms}^{-1} \quad \text{or} \quad 5.81\ \text{cm s}^{-1}$$

Viscous force at this speed,

$$F = 6\pi\eta r v = 6 \times 3.14 \times (1.8 \times 10^{-5}) \times (2.0 \times 10^{-5}) \times (5.81 \times 10^{-2})$$

$$= 3.94 \times 10^{-10}\ \text{N}.$$

10.29. *Mercury has an angle of contact equal to 140° with soda-lime glass. A narrow tube of radius 1.00 mm made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside? Surface tension of mercury at the temperature of the experiment is 0.465 Nm^{-1}. Density of mercury = 13.6×10^3 kgm^{-3}.*

Ans. Radius of tube, r = 1.00 mm = 10^{-3} m

Surface tension of mercury, σ = 0.465 Nm^{-1}

Density of mercury, $\sigma = 13.6 \times 10^3$ kg m^{-3}

Angle of contact, $\theta = 140°$

$$\therefore \quad h = \frac{2\sigma\cos\theta}{r\rho g} = \frac{2 \times 0.465 \times \cos 140°}{10^{-3} \times 13.6 \times 10^3 \times 9.8} = \frac{2 \times 0.465 \times (-0.7660)}{10^{-3} \times 13.6 \times 10^3 \times 9.8}$$

$$= -5.34 \times 10^{-3} \text{ m} = \mathbf{-5.34 \text{ mm}}$$

Negative sign shows that the mercury level is depressed in the tube.

10.30. *Two narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U-tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is 7.3 × 10^{-2} Nm^{-1}. Take the angle of contact to be zero and density of water to be 1.0 × 10^3 kg m^{-3} (g = 9.8 ms^{-2}).*

Ans. Let r_1 be the radius of one bore and r_2 be the radius of second bore of the U-tube. The, if h_1 and h_2 are the heights of water on two sides, then

$$h_1 = \frac{2S\cos\theta}{r_1\rho g} \quad \text{and} \quad h_2 = \frac{2S\cos\theta}{r_2\rho g}$$

On subtraction, we get

$$h_1 - h_2 = \frac{2S\cos\theta}{r_1\rho g} - \frac{2S\cos\theta}{r_2\rho g} = \frac{2S\cos\theta}{\rho g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Here, $S = 7.3 \times 10^{-2}$ Nm^{-1}, $\theta = 0$, $\rho = 1.0 \times 10^3$ kg m^{-3},

$g = 9.8$ ms^{-2}, $r_1 = \frac{3}{2}$ mm $= 1.5 \times 10^{-3}$ m and $r_2 = \frac{6}{2}$ mm

$= 3 \times 10^{-3}$ m

$$\therefore \quad h_1 - h_2 = \frac{2 \times 7.3 \times 10^{-2} \times \cos\theta}{1 \times 10^3 \times 9.8}\left[\frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-3}}\right]$$

$$= 1.49 \times 10^{-5} \times \frac{1}{3 \times 10^{-3}} \approx 4.97 \times 10^{-3} \text{ m} = 4.97 \text{ mm}$$

10.31. *(a) It is known that density ρ of air decreases with height y as*

$$\rho = \rho_0 e^{-y/y_0}$$

where ρ_0 = 1.25 kg m^{-3} is the density at sea level, and y_0 is a constant. This density variation is called the law of atmospheres. Obtain this law assuming that the temperature of atmosphere remains a constant (isothermal conditions). Also assume that the value of g remains constant.

(b) A large He balloon of volume 1425 m^3 is used to lift a payload of 400 kg. Assume that the balloon maintains constant radius as it rises. How high does it rise?

[Take y_0 = 8000 m and ρ_{He} = 0.18 kg m^{-3}].

Ans. (*a*) We know that rate of decrease of density ρ of air is directly proportional to the height *y*. It is given as

$$\frac{d\rho}{dy} = -\frac{\rho}{y_0},$$

where *y* is a constant of proportionality and *–ve* sign signifies that density is decreasing with increase in height. On integration, we get

$$\int_{\rho_0}^{\rho} \frac{d\rho}{\rho} = -\int_0^y \frac{1}{y_0} dy$$

$\Rightarrow$ $$[\log \rho]_{\rho_0}^{\rho} = -\left[\frac{y}{y_0}\right]_0^y,$$ where, ρ_0 = density of air at sea level *i.e.*, $y = 0$

or $$\log_e \frac{\rho}{\rho_0} = -\frac{y}{y_0} \quad \text{or} \quad \rho = \rho_0 e^{-\frac{y}{y_0}}.$$

Here dimensions and units of constant y_0 are same as of y.

(*b*) Here volume of He balloon, $V = 1425\ \text{m}^3$, mass of payload, $m = 400$ kg

$y_0 = 8000$ m, density of He $\rho_{He} = 0.18\ \text{kgm}^{-3}$

$$\text{Mean density of balloon, } \rho = \frac{\text{Total mass of balloon}}{\text{Volume}} = \frac{m + V \cdot \rho_{\text{He}}}{\text{V}} \text{ Pa}$$

$$= \frac{400 + 1425 \times 0.18}{1425} = 0.4608 = 0.46\ \text{kgm}^{-3}$$

As density of air at sea level $\rho_0 = 1.25\ \text{kg m}^{-3}$. The balloon will rise up to a height y where density of air = density of balloon $\rho = 0.46\ \text{kgm}^{-3}$

As $$\rho = \rho_0 e^{-\frac{y}{y_0}} \quad \text{or} \quad \frac{\rho_0}{\rho} = e^{\frac{y_0}{y}}$$

$\therefore$ $$\log_e\left(\frac{\rho_0}{\rho}\right) = \frac{y_0}{y} \quad \text{or} \quad y = \frac{y_0}{\log_e\left(\frac{\rho_0}{\rho}\right)} = \frac{8000}{\log_e\left(\frac{1.25}{0.46}\right)}$$

$$= 8002 \text{ m} \quad \text{or} \quad 8.0 \text{ km}.$$

QUESTIONS BASED ON SUPPLEMENTARY CONTENTS

Q. 1. *What should be the maximum average velocity of water in a tube of diameter 0.5 cm. So that the flow is laminar? The viscosity of water is 0.00125 Nm^{-2} s*

Sol. Here

$$D = 0.5 \text{ cm} = 0.005 \text{ m}$$

$$\rho = 10^3 \text{ kg m}^{-3}$$

$$\eta = 0.00125 \text{ Nm}^{-2} \text{ s}$$

For laminar flow, the Reynold number for water

$$N_R = 2000$$

Let v be the maximum average velocity

$\therefore$ $$N_R = \frac{\rho v D}{\eta} \quad \text{or} \quad v = \frac{N_R \cdot \eta}{\rho \cdot D} = \frac{2000 \times 0.00125}{1000 \times 0.005} = 0.5 \text{ m/s}$$

Q. 2. *What should be the average velocity of water in a tube of radius 0.005 m so that the flow is just turbulent? The viscosity of water is 0.001 Pas.*

Sol. Here $D = 2 \times 0.005 = 0.01$ m; $\rho = 10^3$ kg m^{-3}; $\eta = 0.001$ Pas

For laminar flow of water,

$$N_R = 2000$$

Let v be the average velocity of the water

$$\therefore \quad N_R = \frac{\rho v D}{\eta} \quad \text{or} \quad v = \frac{N_R \cdot \eta}{\rho \cdot D} = \frac{2000 \times 0.001}{1000 \times 0.01} = 0.2 \text{ m/s}$$

Q. 3. *Water flows at a speed of 6 cm/s through a tube of radius 1 cm. Coefficient of viscosity of water at room temperature is 0.001 Poise. What is the nature of the flow?*

Sol. Here $V = 6$ cm/s or 0.06 m/s

$D = 1 \times 2 = 2$ cm or 0.02 m

$\eta = 0.001$ Poise and $\rho = 10^3$ kg/m^3 for water

$$\therefore \quad N_R = \frac{\rho \cdot V \cdot D}{\eta} = \frac{1000 \times 0.06 \times 0.02}{0.001} = 1200 < 2000$$

Hence, the flow of water is Laminar.

ADDITIONAL QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *A steel ball is floating in a trough of mercury. If we fill the empty part of the trough with water, what will happen to the steel ball?*

Ans. It will move up.

Q. 2. *Why machines are sometimes jammed in winter?*

Ans. Due to change in viscosity with temperature.

Q. 3. *Why a hot liquid moves faster than a cold liquid?*

Ans. Due to decrease in viscosity with increase of temperature.

Q. 4. *Under which condition (i) the centre of buoyancy coincides with the centre of gravity (ii) the centre of buoyancy does not coincide with the centre of gravity.*

Ans. (*i*) For a solid body of uniform density, the centre of gravity coincides with the centre of buoyancy. (*ii*) For a solid body having different densities over different parts, its centre of gravity does not coincide with the centre of buoyancy.

Q. 5. *How excess pressure varies with the radius of bubble of drop?*

Ans. $P \propto \frac{1}{r}$.

Q. 6. *What is (i) velocity head (ii) pressure head?*

Ans. (*i*) $\frac{v^2}{2g}$ (*ii*) $\frac{P}{\rho g}$.

Q. 7. *A block of wood is floating in a lake. What is apparent weight of the floating block?*

Ans. The apparent weight of the floating block is equal to zero because the weight of the block acting vertically downwards is balanced by the buoyant force acting on the block upwards.

Q. 8. *Obtain the dimensional formula for coefficient of viscosity.*

Ans. According to Stoke's law, viscous force $F = 6\,\pi\eta rv$

$$\therefore \text{ Dimensions of } \eta \text{ are } \frac{[F]}{[r][v]} = \frac{[\text{M L T}^{-2}]}{[\text{L}] \times [\text{LT}^{-1}]} \; [\text{M}^1 \text{ L}^{-1} \text{ T}^{-1}].$$

Q. 9. *How does the viscous force between two layers of a liquid depend upon the relative velocity between two layers?*

Ans. The viscous force between two layers increases with the increase in relative velocity.

Q. 10. *Why the aeroplanes and cars are given a streamline shape?*

Ans. This is done to reduce the backward drag of the atmosphere.

Q. 11. *What are the values of Reynolds number (N_R) for different types of flows?*

Ans. For streamline or laminar flow $0 < N_R < 2000$,
for turbulent flow, $N_R > 3000$
If N_R lies between 2000 to 3000, the flow is unstable *i.e.,* change from streamline flow to turbulent flow.

Q. 12. *Why is the nib of a pen split?*

Ans. For capillary action.

Q. 13. *Why is the dam of water reservoir thick at the bottom?*

Ans. It is so because the pressure of water in reservoir increases with depth.

Q. 14. *A block of ice is floating in a liquid of specific gravity 1.2 contained in the beaker. What will be the effect on the level of liquid in the beaker when the whole ice melts?*

Ans. **The level of water in a beaker will rise.** It is so because the density of water formed by melting of ice is less than the density of liquid in a beaker. Due to which the volume of the water formed by melting the ice will be more than the volume of the portion of the ice in liquid while floating in liquid.

Q. 15. *What happens to surface tension, when impurity is mixed in liquid?*

Ans. Surface tension of the liquid decreases.

Q. 16. *Why does a spinning cricket ball in air not follow a parabolic path?*

Ans. This is due to Magnus effect.

Q. 17. *How does the rate of flow of a liquid through a tube depend on the radius of its bore?*

Ans. Rate of flow of a liquid through a tube is directly proportional to the fourth power of the radius of its bore *i.e.,* $V \propto r^4$.

Q. 18. *Why does air bubble in a liquid rise up?*

Ans. Because terminal velocity of air bubble is negative.

Q. 19. *Why does the clouds float in the sky?*

Ans. Because they have zero terminal velocity.

Q. 20. *Why do fire fighters have a jet attached to the head of their water pipes?*

Ans. This is done to increase the velocity of water flowing out of the pipe.

Q. 21. *What is the effect:*

(a) of highly soluble impurities on the surface tension of a liquid?
(b) of less soluble impurities on the surface tension of a liquid?
(c) on the angle of contact when the temperature of a liquid is increased?

Ans. (*a*) The surface tension is increased. (*b*) The surface tension decreases.
(c) The angle of contact decreases.

Q. 22. *Why is it that a liquid set in rotation comes to rest after some time?*

Ans. The liquid comes to rest due to the viscous force *i.e.,* due to internal fluid friction between its different layers.

Q. 23. *Surface tension of all lubricating oils and paints is kept low. Why?*

Ans. In order to have low value of surface tension so that it can spread over large area.

Q. 24. *(a) How do trees draw water from the ground?*

(b) The radius of a capillary tube is doubled. What change will take place in the height of the capillary rise?

(c) In a capillary tube water descends and not rises. Guess the material of the capillary tube.

Ans. (*a*) Capillary action. (*b*) Capillary rise will be halved as $h \propto \frac{1}{r}$.

(*c*) Paraffin wax.

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *A cubical block of steel of density 7.8 g cm^{-3} floats on mercury (density 13.6 g cm^{-3}) with its sides vertical. Assume the side of the cube to be 10 cm.*

(a) What length of the block is above the mercury surface?

(b) If water is poured on the mercury surface, what will be the height of the water column, when the water surface just covers the top of the steel block?

Ans. (*a*) Volume of the steel block = 10 × 10 × 10 = 1000 cm^3

Weight of the steel block = 1000 × 7.8 g

Volume of the block below the surface is $(10 - l_1) \times 100$ where l_1 is the length of the block above the surface of mercury.

The weight of mercury displaced by the block

$$= (10 - l_1) \times 100 \times 13.6 \text{ g}$$

According to Archimedes' principle, this must be equal to the weight of the steel block. Therefore,

$$(10 - l_1) \times 100 \times 13.6 = 7800 \quad \text{or} \quad l_1 = 4.26 \text{ cm}$$

(*b*) Let l_2 be the height of the water column

Weight of the block = weight of water displaced + weight of mercury displaced

$$7800 = l_2 \times 100 \times 1 + (10 - l_2) \times 100 \times 13.6$$

which gives $l_2 = 4.6$ cm.

Q. 2. *A large force is needed to normally separate two glass plates having a thin layer of water between them. Why?*

Ans. The thin layer of water between the glass plates forms a concave surface all around. This decreases the pressure on the inner side of the liquid film. Thus, a large amount of force is required to pull them apart against the atmospheric pressure.

Q. 3. *The excess pressure inside a soap bubble is thrice the excess pressure inside a second soap bubble. What is the ratio between the volume of the first and the second bubble?*

Ans. Given $$\frac{4T}{r_1} = \frac{3 \times 4T}{r_2} \quad \text{or} \quad r_2 = 3r_1$$

$$\frac{V_1}{V_2} = \frac{\left(\frac{4}{3}\right)\pi r_1^3}{\left(\frac{4}{3}\right)\pi r_2^3} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

Q. 4. *A piece of an alloy of mass 96 gm is composed of two metals whose specific gravities are 11.4 and 7.4. If the weight of the alloy is 86 gm in water, find the mass of each metal in the alloy.*

Ans. Suppose the mass of the metal of specific gravity 11.4 be m. Now the mass of the second metal of specific gravity 7.4 will be (96 – m).

$$\text{Volume of first metal} = \frac{m}{11.4}\text{cm}^3$$

$$\text{Volume of second metal} = \frac{96 - m}{7.4}\text{ cm}^3$$

$$\text{Total volume} = \frac{m}{11.4} + \frac{96 - m}{7.4}$$

$$\text{Apparent loss of wt. in water} = \left(\frac{m}{11.4} + \frac{96 - m}{7.4}\right)\text{gm wt.}$$

$$\text{Apparent wt. in water} = 96 - \left[\left(\frac{m}{11.4}\right) + \frac{(96 - m)}{7.4}\right]$$

According to the given problem,

$$96 - \left[\left(\frac{m}{11.4}\right) + \frac{(96 - m)}{7.4}\right] = 86 \quad \text{or} \quad \frac{m}{11.4} + \frac{(96 - m)}{7.4} = 10$$

Solving we get, m = **62.7 gm.**

∴ Mass of second metal = 96 – 62.7 = **33.3 gm**

Q. 5. *Why are the wings of an aeroplane rounded outwards while flattened inwards?*

Ans. The special design of the wings increases velocity at the upper surface and decreases velocity at the lower surface. So, according to Bernoulli's theorem, the pressure on the upper side is less than the pressure on the lower side. This difference of pressure provides lift.

Q. 6. *On what factors does the critical velocity of the liquid depend?*

Ans. Critical velocity (v_c) of a liquid is:

(*i*) directly proportional to the coefficient of viscosity of the liquid.

(*ii*) inversely proportional to the density of the liquid *i.e.,* $V_c \propto \frac{1}{\rho}$.

(*iii*) inversely proportional to the diameter of the tube through which it flows *i.e.,* $V_c \propto \frac{1}{D}$.

Q. 7. *The density of atmosphere is 1.29 kg m^{-3} at sea level, where atmospheric pressure is 1.013 × 10^5 Pa. If we assume that atmospheric density does not change with altitude, then what should be the height of the atmosphere?*

Ans. Here $P_a = 1.013 \times 10^5$ Pa and $\rho = 1.29$ kg m^{-3}

If height of air column, assuming its density to be constant, be h, then

$$P_a = h\rho g \Rightarrow h = \frac{P_a}{\rho g} = \frac{1.013 \times 10^5}{1.29 \times 9.8} = 8013 \text{ m} \simeq 8 \text{ km.}$$

Q. 8. *If work required to blow a soap bubble of radius r is W, then what additional work is required to be done to blow it to a radius 3r?*

Ans. Increase in surface area = $2[4\pi(3r)^2 - 4\pi r^2]$

Increase in surface energy = $\sigma \times 2 \times 4\pi \times 8r^2 = 8W$

Additional work done = $8W$

Q. 9. *Two soap bubbles in vacuum having radii 3 cm and 4 cm respectively coalesce under isothermal conditions to form a single bubble. What is the radius of the new bubble?*

Ans. Surface energy of first bubble

$$= \text{Surface area} \times \text{surface tension} = 2 \times 4\pi r_1^2 T = 8\pi r_1^2 T$$

Surface energy of second bubble = $8\pi r_2^2 T$

Let r be the radius of the coalseced bubble.

$\therefore$ Surface energy of new bubble = $8\pi r^2 T$

According to the law of conservation of energy,

$$8\pi r^2 T = 8\pi r_1^2 T + 8\pi r_2^2 T = 8\pi(r_1^2 + r_2^2)T$$

$$\therefore \quad r^2 = r_1^2 + r_2^2 = 3^2 + 4^2 = 9 + 16 = 25$$

$$\therefore \quad r = 5 \text{ cm.}$$

Q. 10. *Derive the condition of floatation of a body.*

Ans. When a body floats in a liquid with a part submerged in the liquid, the weight of the liquid displaced by the submerged part is always equal to the weight of the body.

Let V = volume of the body

σ = density of its material

ρ = density of the liquid in which the body floats such that its volume V' is outside the liquid.

Then volume of the body inside the liquid = $V - V'$

Weight of the displaced liquid = $(V - V')\rho g$

Also weight of the body = $V \sigma g$

For the body to float,

weight of the liquid displaced by the sumberged part = weight of the body

i.e., $\quad (V - V')\rho g = V \sigma g \quad$ or $\quad V' = \dfrac{(\rho - \sigma)V}{\rho}$.

Q. 11. *A small drop of water of surface tension T is squeezed between two clean glass plates so that a thin layer of thickness d and area A is formed between them. If the angle of contact is zero, what is the force required to pull the plates apart.*

Ans. An extremely thin layer of liquid can be considered as the collection of large number of hemispherical drops. In case of a spherical drop, the excess of pressure = $\dfrac{2T}{r}$. But in case of thin layer of liquid, which is a combination of hemispherical drops, the excess pressure is $P = \dfrac{T}{r}$, where $r = \dfrac{d}{2}$.

Therefore, $$P = \frac{T}{\frac{d}{2}} = \frac{2T}{d}$$

Force due to surface tension pushing the two plates together is

$$F = P \times A = \frac{2TA}{d}$$

Q. 12. *Two syringes of different cross-sections (without needles) filled with water are connected with a tightly fitted rubber tube filled with water. Diameters of the smaller piston and larger piston are 1.0 cm and 3.0 cm respectively.*

(a) Find the force exerted on the larger piston when a force of 10 N is applied to the smaller piston.

(b) If the smaller piston is pushed in through 6.0 cm, how much does the larger piston move out?

Ans. Here d_1 = 1.0 cm, d_2 = 3.0 cm, Force on smaller piston F_1 = 10 N.

(*a*) According to Pascal's law of transmission of pressure $P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$.

$$\therefore \quad \frac{F_2}{F_1} = \frac{A_2}{A_1} = \frac{r_2^2}{r_1^2} = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{3.00 \text{ cm}}{1.0 \text{ cm}}\right)^2 = 9$$

$$\Rightarrow \quad F_2 = F_1 \times 9 = 10 \times 9 = 90 \text{ N.}$$

(*b*) Water is considered to be completely incompressible. Therefore, volume covered by the movement of smaller piston inwards is exactly equal to volume moved outwards due to movement of larger piston, distance through which smaller piston is pushed L_1 = 6.0 cm. Let larger piston is pushed by a distance L_2 then

$$V = L_1A_1 = L_2A_2$$

$$\therefore \quad L_2 = L_1\left(\frac{A_1}{A_2}\right) = L_2\left(\frac{d_1}{d_2}\right)^2 = 6.0 \text{ cm} \times \left(\frac{1.0 \text{ cm}}{3.0 \text{ cm}}\right)^2$$

$$= \frac{6}{9} \text{ cm} = 0.67 \text{ cm.}$$

Q. 13. *Find the height to which water at 4°C will rise in a capillary tube of* 10^{-3} *m diameter. Take* $g = 9.8$ *ms*$^{-2}$. *Angle of contact,* $\theta = 0$ *and* $T = 0.072$ *Nm*$^{-1}$.

Ans. Here $D = 10^{-3}$ m; $r = \frac{D}{2} = 0.5 \times 10^{-3}$ m

$g = 9.8$ ms^{-2}, $\theta = 0$, $T = 72 \times 10^{-3}$ Nm^{-1}

$\cos\theta = 1$

Density of water at 4°C = 10^3 kg m^{-3}

Using the relation, $h = \frac{2T\cos\theta}{r\rho g} = \frac{2 \times 72 \times 10^{-3} \times 1}{5 \times 10^{-4} \times 10^3 \times 9.8} = 2.939 \times 10^{-2}$ m.

Q. 14. *A vertical barometer tube 100 cm in length and dipping into mercury contains a small quantity of air. When the open end is 15 cm below the surface of mercury the meniscus is 30 cm from the upper closed end. When the tube is pushed into the mercury so that the open end is 35 cms below the surface, the meniscus is 20 cm from the closed end. Calculate the atmospheric pressure.*

Ans. Let the atmospheric pressure be P and area of cross section of the tube be a.

Case (*i*): Length of air column = 30 cm

∴ Volume of air, V_1 = (20 × a) c.c,

Length of mercury column = 100 – (30 + 15) = 55 cm.

Pressure P_1 of air in tube = (P – 55) cm

Case (*ii*): Length of air column = 20 cm.

∴ Volume of air, V_2 = (20 × a) c.c.

Length of mercury column = 100 – (20 + 35) = 45 cm.

Pressure P_2 of air in tube = (P – 45) cm.

Applying Boyle's law,

$$P_1V_1 = P_2V_2$$

$$(P - 55) \times 30\, a = (P - 45) \times 20\, a$$

Solving, we get, $\mathbf{P = 75\ cm.}$

Q. 15. *Water is escaping from a vessel through a horizontal capillary tube 20 cm long 0.2 mm radius at a point 100 cm below the free surface of water in the vessel. Calculate the rate of flow if coefficient of viscosity of water be 1×10^{-3} Pa–s.*

Ans. Here $l = 20$ cm = 0.20 m; $r = 0.2$ mm $= 2 \times 10^{-4}$ m

$h = 100$ cm = 1 m; $\rho = 10^3$ kg m^{-3}

$\eta = 1 \times 10^{-3}$ Pa–s; $P = h\rho g = 1 \times 10^3 \times 9.8$ Nm^{-2}

Let V be rate of flow (volume of water flowing per second)

$\therefore$ Using the relation, $V = \dfrac{\pi p r^4}{8\eta l}$, we get

$$V = \frac{\pi \times 9.8 \times 10^3 (2 \times 10^{-4})^4}{8 \times 1 \times 10^{-3} \times 0.2} = 3.08 \times 10^{-8}\ \text{m}^3\ \text{s}^{-1}.$$

Q. 16. *A water pipe entering a hose has a dimeter of 2 cm and the speed of water is 0.1 ms^{-1}. Eventually, the pipe tapers to a diameter of 1 cm. Calculate the speed of water in the tapered portion.*

Ans. Here, $v_1 = 0.1$ ms^{-1} ; $r_1 = \dfrac{2}{2}$cm $= 1$ cm $= 10^{-2}$m

$\therefore$ $a_1 = \pi r_1^2 = \pi \times 10^{-4}$ m^2.

At the tapered portion,

$r_2 = \dfrac{1}{2}$cm $= 0.5$ cm $= 0.5 \times 10^{-2}$ m

$\therefore$ $a_2 = \pi r_2^2 = \pi \times 0.25 \times 10^{-4}$ m^2.

Using $a_1 v_1 = a_2 v_2$, we get

$$v_2 = \frac{a_1 v_1}{a_2} = \frac{\pi \times 10^{-4} \times 0.1}{\pi \times 0.25 \times 10^{-4}} = 0.4\ \text{ms}^{-1}.$$

Q. 17. *A hole of area 4 cm^2 is formed in the side of a ship 2.4 m below the water level. What minimum force is required to hold on a patch covering the hole from the inside of the ship? Given that density of sea water = 1.03×10^3 kg m^{-3}.*

Ans. Here, depth of hole below the water level $h = 2.4$ m, density of sea water $\rho = 1.03 \times 10^3$ kg m^{-3} and surface area of hole $A = 4$ cm^2 $= 4 \times 10^{-4}$ m^2.

$\therefore$ Minimum force required to hold on a patch covering the hole from inside the ship

F = Pressure at height h of sea water column × Surface area of hole

$= h\rho g\, A = 2.4 \times 1.03 \times 10^3 \times 9.8 \times 4 \times 10^{-4} = 9.69$ N.

Q. 18. *It is easier to spray water in which some soap is dissolved. Explain why?*

Ans. When the liquid is sprayed, it is broken into small drops. The surface area increases and hence the surface energy is also increased. Therefore, work has to be done to supply the additional energy. Since surface energy is numerically equal to the surface tension, so when soap is dissolved in water, the surface tension of the solution decreases and hence less energy is spent to spray it.

Q. 19. *In a horizontal pipe line of uniform area of cross section, the pressure falls by 8 N-m^{-2} between two points separated by a distance of 1 km. What is the change in kinetic energy per kg of the oil flowing at these points? Density of oil is 806 kg-m^{-3}.*

Ans. According to Bernoulli's theorem,

$$P_1 + \frac{1}{2}\rho\, v_1^2 = P_2 + \frac{1}{2}\rho\, v_2^2 \text{ (pipe is horizontal)}$$

or $$P_1 - P_2 = \frac{1}{2}\rho\,(v_2^2 - v_1^2)$$

or $$\frac{P_1 - P_2}{\rho} = \frac{1}{2}(v_2^2 - v_1^2) = \text{change in K.E. per kg mass.}$$

$\therefore$ Change in K.E. per kg. mass of oil = $\dfrac{P_1 - P_2}{\rho}$.

Substituting the given values, we have

Change in K.E. per kg mass = $\dfrac{8}{800}$ = **10^{-2} J/kg.**

Q. 20. *What should be the maximum average velocity of water in a tube of diameter 2 cm so that flow is laminar? The viscosity of water is 0.001 $Nm^{-2}s$.*

Ans.
$$D = 2 \text{ cm} = 0.02 \text{ m}$$
$$\rho = 10^3 \text{ kg m}^{-3}$$
$$\eta = 0.001 \text{ Nm}^{-2}\text{ s} = 10^{-3} \text{ Nm}^{-2}\text{s}$$

Flow of water will be laminar if

N_R = 1000 where N_R is Reynold number

Let v = maximum average velocity

$\therefore$ Using the relation

$$N_R = \frac{\rho v D}{\eta} \quad \text{or} \quad v = \frac{N_R \eta}{\rho D} = \frac{1000 \times 0.001}{1000 \times 0.02} = 0.05 \text{ ms}^{-1}.$$

III. LONG ANSWER TYPE QUESTIONS

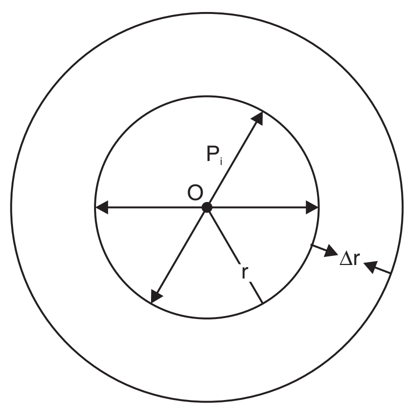
Q. 1. *Derive the expression for excess pressure inside:*

(a) a liquid drop *(b) a liquid bubble* *(c) an air bubble.*

Ans. (*a*) Let r = radius of a spherical liquid drop of centre O. T = surface tension of the liquid. Let P_i and P_0 be the values of pressure inside and outside the drop.

$\therefore$ Excess pressure inside the liquid drop = $P_i - P_0$

Let Δr be the increase in its radius due to excess pressure. It has one free surface outside.



$\therefore$ increase in surface area of the liquid drop

$$= 4\pi(r + \Delta r)^2 - 4\pi r^2$$
$$= 4\pi[r^2 + (\Delta r)^2 + 2r\,\Delta r - r^2]$$
$$= 8\pi r\,\Delta r \qquad ...(i)$$

($\because$ Δr is small $\therefore$ Δr^2 is neglected)

$\therefore$ increase in surface energy of the drop is

$$W = \text{Surface tension} \times \text{increase in area}$$
$$= T \times 8\pi r \, \Delta r \qquad \text{...}(ii)$$

Also W = Force due to excess of pressure × displacement

= Excess pressure × Area of drop × increase in radius

$$= (P_i - P_0)\, 4\pi r^2 \, \Delta r \qquad \text{...}(iii)$$

∴ From eqns (*ii*) and (*iii*), we get

$$(P_i - P_0) \times 4\pi r^2 \, \Delta r = T \times 8\pi r \, \Delta r \quad \Rightarrow \quad P_i - P_0 = \frac{2T}{r}.$$

(*b*) **Inside a liquid bubble:** A liquid bubble has air both inside and outside it and therefore it has two free surfaces.

Thus increase in its surfaces area

$$= 2[4\pi(r + \Delta r)^2 - 4\pi r^2] = 2 \times 8\pi r \, \Delta r = 16\pi r \, \Delta r$$

∴ $W = T \times 16 \, \pi r \, \Delta r$ ...(*i*)

Also $W = (P_i - P_o)\, 4 \, \pi r^2 \times \Delta r$ ...(*ii*)

From eqns (*i*) and (*ii*), we get

$$(P_i - P_0) \times 4\pi r^2 \times \Delta r = T \cdot 16\pi r \, \Delta r \quad \text{or} \quad P_i - P_0 = \frac{4T}{r}$$

(*c*) **Inside an air bubble:** Air bubble is formed inside liquid, thus air bubble has one free surface inside it and liquid is outside.

If r = radius of air bubble

Δr = increase in its radius due to excess of pressure $(P_i - P_0)$ inside it.

T = surface tension of the liquid in which bubble is formed.

∴ increase in surface area = $8\pi r \, \Delta r$.

∴ $W = T \times 8\pi r \Delta r$

Also $W = (P_i - P_0) \times 4\pi r^2 \, \Delta r$

$$\therefore (P_i - P_0) \times 4\pi r^2 \, \Delta r = T \times 8\pi r \, \Delta r \quad \text{or} \quad P_i - P_0 = \frac{2T}{r}.$$

Q. 2. *State law of floatation.*

Compute the volume in m^3 of a life preserver of SG 0.20, which, when worn by a boy weighing 60 kg and having SG equal to 0.9, will just support him, if 3/4 of his body is submerged in fresh water of density 1000 kg m^{-3}. Assume that the life preserver is completely submerged.

Ans. For law of floation, see the chapter in the NCERT Textbook.

The weight of the boy W_b and the weight of the preserver W_p acting downward are just balanced by the upward buoyant force of the preserver B_p and the buoyant force of the boy B_b. Therefore,

$$W_b + W_p = B_b + B_p \qquad \text{...}(1)$$

But $B_b = V_b \, \rho_w \, g$, $W_b = V_b \, \rho_b \, g = 60 \times g$

and $B_p = V_p \, \rho_w \, g$, $W_p = V_p \, \rho_p \, g$

where g is the acceleration due to gravity. V and ρ denote volume and density respectively.

$$\rho_b = 0.9 \times 1000 = 900 \text{ kg m}^{-3}$$

$$\rho_p = 0.20 \times 1000 = 200 \text{ kg m}^{-3} \quad \text{and} \quad V_b = \frac{W_b}{g \, \rho_b}$$

From Eq. (1), we have

$$\frac{3}{4} V_b \rho_w g + V_p \rho_w g = W_b + V_p \rho_p g$$

or $$\frac{3}{4} \frac{W_b}{g\rho_b} \rho_w g + V_p \rho_w g = 60 \times g + V_p \rho_p g$$

or $$\frac{3}{4} \times \frac{60 \times g}{\rho_b} \rho_w + V_p \rho_w g = 60g + V_p \rho_p g$$

or $$45 \frac{\rho_w}{\rho_b} + V_p \rho_w g = 60 + V_p \rho_p$$

or $$V_p(\rho_w - \rho_p) = 60 - 45 \frac{\rho_w}{\rho_b} = 60 - \frac{45 \times 1000}{900} = 10$$

$$V_p = \frac{10}{\rho_w - \rho_p} = \frac{10}{800} = 1.25 \times 10^{-2} \text{ m}^3$$

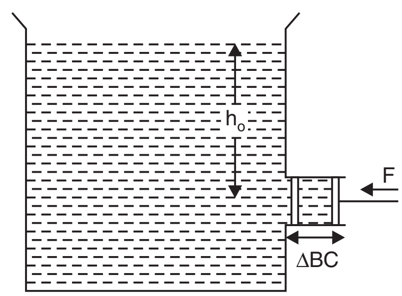
Volume of the life preserver = 0.0125 m^3.

Q. 3. *What are the three forms of energy possessed by a flowing fluid? Find their expressions.*

Ans. Three forms of energy possessed by a flowing fluid are as follows:

1. **Pressure energy:** Let an ideal fluid of density ρ be contained in a rectangular vessel, provided with a small side tube at a depth h_0 below the free surface of fluid in the vessel. At the level of side tube, pressure of fluid along the axis of side tube

$$p = h_0 \rho g$$



If we want to introduce more fluid into the vessel at this very pressure, we can force it through the side tube by doing work on the piston, If 'A' be the cross-section area of the piston, then force acting on the piston $F = PA$.

∴ Work done in moving the piston through a small distance Δr will be

$$\Delta W = F\Delta x = PA\Delta x = P\Delta V$$

As a result of motion of piston, the mass of the fluid forced in the vessel

$$\Delta m = \rho A \Delta x = \rho.\Delta V$$

The work done is stored up in the liquid in the form of its pressure energy.

∴ Pressure energy of liquid per unit mass = Work done per unit mass

$$= \frac{\Delta W}{\Delta m} = \frac{P\Delta V}{\rho \Delta V} = \frac{P}{\rho}$$

and pressure energy per unit volume = p.

2. **Gravitational potential energy:** Let at any stage of its flow a fluid element of mass 'm' be situated at a height 'h' from the reference line (generally taken to be earth's surface), then its gravitational potential energy in that position is mgh.

∴ Gravitational potential energy per unit mass = $\frac{mgh}{m} = gh$

and gravitational potential energy per unit volume = ρgh.

3. Kinetic energy: Let at any stage of its flow, a fluid element of mass '*m*' be moving with a speed '*v*', then the kinetic energy of this fluid element is $\frac{1}{2}mv^2$.

$\therefore$ Kinetic energy per unit mass = $\dfrac{\frac{1}{2}mv^2}{m} = \frac{1}{2}v^2$

and kinetic energy per unit volume = $\frac{1}{2}\rho v^2$

These three forms of energy possessed by a flowing fluid are mutually convertible from one form to another.

IV. MULTIPLE CHOICE QUESTIONS

1. The Bernauli's Theorem is based on the conservation of :

(*a*) mass (*b*) energy (*c*) momentum (*d*) all

2. The mass of water rises in capillary tube of radius R is M. The mass of water that rises in tube of radius 2*R* is

(*a*) M (*b*) M/2 (*c*) 2 M (*d*) 4 M

3. Two small drops of mercury, each of radius R, coalesce to form a single large drop. The ratio of the total surface energies before and after the change is :

(*a*) $1 : 2^{1/3}$ (*b*) $2^{1/3} : 1$ (*c*) 2 : 1 (*d*) 1 : 2

4. For a ball falling in a liquid with constant velocity, ratio of resistance force due to the liquid to that due to gravity is

(*a*) 1 (*b*) $\dfrac{2a^2\rho g}{9\eta^2}$ (*c*) $\dfrac{2a^2(\rho-\sigma)g}{9\eta}$ (*d*) none

5. The work done in increasing the size of a soap film from 10 cm × 6 cm to 10 cm × 11 cm is 3×10^{-4} J. The surface tension of the film is

(*a*) 1.5×10^{-2} N/m (*b*) 3.0×10^{-2} N/m

(*c*) 6.0×10^{-2} N/m (*d*) 11.0×10^{-2} N/m

6. A cylindrical vessel is filled with water upto height *H*. A hole is bored in the wall at a depth *h* from the free surface of water. For maximum angle *h* is equal to

(*a*) H/4 (*b*) H/2 (*c*) 3 H/4 (*d*) H

7. Let W be the work done, when a bubble of volume V is formed from a give solution. How much work is required to be done to form a bubble of volume 2V?

(*a*) W (*b*) 2W (*c*) $2^{1/3}$ W (*d*) $4^{1/3}$ W

8. What is the shape when a non-wetting liquid in displaced in a capillary tube?

(*a*) Concave upwards (*b*) Convex upwards

(*c*) Concave downwards (*d*) Convex downwards

9. Application of Bernaull's Theorem can be seen in

(*a*) dynamic lift of aeroplane (*b*) hydraullic press

(*c*) helicopter (*d*) none of the above

10. At which of the following temperature, the value of surface tension of water is minimum?

(*a*) 4°C (*b*) 25°C (*c*) 50°C (*d*) 75°C

Answers

1. (*b*)	2. (*b*)	3. (*b*)	4. (*a*)	5. (*b*)
6. (*b*)	7. (*d*)	8. (*c*)	9. (*a*)	10. (*d*)

V. QUESTIONS ON HIGH ORDER THINKING SKILLS (HOTS)

Q. 1. *A piece of copper having an internal cavity weight 264 g in air and 221 g in water. Find the volume of the cavity. The density of copper = 8.8 g cm^{-3}.*

Ans. Mass of copper piece in air = 264 g

Mass of copper piece in water = 221 g

Apparent loss of mass = 264 – 221 = 43 g

This is the mass of water displaced by the copper piece when immersed in water.

Volume of copper piece with cavity = 43 cm^3

$$\text{Volume of copper only} = \frac{m}{\rho} = \frac{264}{8.8} = 30 \text{ cm}^3$$

Volume of the cavity = 43 – 30 = 13 cm^3.

Q. 2. *If the excess pressure inside a spherical soap bubble of radius 1 cm is balanced by that due to a column of oil of specific gravity 0.9, 1.36 mm high. Calculate the surface tension.*

Ans. Radius, $r = 1$ cm; $\rho = 0.9$ g cm^{-3}

$h = 1.36$ mm $= 0.136$ cm

Pressure, $P = h\rho g = 0.136 \times 0.9 \times 980 = 119.95$ dyne cm^{-2}

Let T be surface tension of soap solution

$$\therefore \text{ Excess pressure, } P = \frac{4T}{r} \quad \text{or} \quad T = \frac{Pr}{4} = \frac{119.95 \times 1}{4} = 29.988 \text{ dyne cm}^{-1}.$$

Q. 3. *In rising from the bottom of a lake to the top, the temperature of an air bubble remains unchanged, but its diameter gets doubled. What is the depth of the lake? Given h is the barometric height in metres of mercury of relative density ρ at the surface of the lake.*

Ans. At the surface, $P_1 = h\rho g; \quad V_1 = \frac{4}{3}\pi(2r)^3$

At the bottom of depth x, $P_2 = (h\rho g + xg)$ and $V_2 = \frac{4}{3}\pi r^3$

Using Boyle's law, $P_1V_1 = P_2V_2$

$$\therefore \quad h\rho g \times \frac{4}{3}\pi\ (2r)^3 = (h\rho g + xg) \times \frac{4}{3}\pi r^3 \quad \text{or} \quad x = 8h\rho - h\rho = \mathbf{7h\rho \text{ metres.}}$$

Q. 4. *Air is streaming past a horizontal air plane wing such that its speed is 120 ms^{-1} over the upper surface and 90 ms^{-1} at the lower surface. If the density of air is 1.3 kg m^{-3}, find the difference in pressure between the top and bottom of the wing. If wing is 10 m long and has an average width of 2 m, calculate the gross lift of the wing.*

Ans. According to Bernoulli's theorem

$$\frac{P_1}{\rho} + gh_1 + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh_2 + \frac{1}{2}v_2^2$$

For the horizontal flow, $h_1 = h_2$

$$\therefore \quad \frac{P_1}{\rho} + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + \frac{1}{2}v_2^2 \quad \text{...(1)}$$

Here $v_1 = 90 \text{ ms}^{-1}; \quad v_2 = 120 \text{ ms}^{-1};$

$\rho = 1.3 \text{ kg m}^{-3}$

$$\therefore \quad \frac{P_1 - P_2}{\rho} = \frac{1}{2}\left(v_2^2 - v_1^2\right)$$

$$(P_1 - P_2) = \frac{\rho\left(v_2^2 - v_1^2\right)}{2} = \frac{1.3\,(14400 - 8100)}{2} = \frac{1.3 \times 6300}{2}$$

$$P_1 - P_2 = 4.095 \times 10^3 \text{ Nm}^{-2}$$

which is the pressure difference between the top and the bottom of the wing.

Now, Gross lift of the wing (*i.e.* force)

$= (P_1 - P_2) \times$ Area of the wing $= 4.095 \times 10^3 \times 10 \times 2$

$= 8.190 \times 10^4$ N.

Q. 5. *Water stands at a depth H in a tank, whose side walls are vertical. A hole is made on one of the walls at a depth h below the water surface. Find at what distance from the foot of the wall does the emerging stream of water strike the floor and for what value of h this range is maximum.*

Ans. The situation is shown in fig. Here, we have

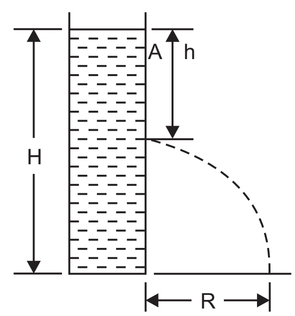
$$v_A = \sqrt{2gh}, \quad \text{...(1)}$$

and $$(H - h) = \frac{1}{2}gt^2 \quad \text{...(2)}$$

The distance R is given by

$$R = v_A \times t. \quad \text{...(3)}$$

From equation (2) $$t = \sqrt{\left(\frac{2(H-h)}{g}\right)} \quad \text{...(4)}$$



Substituting the value of v_A from equation (1) and the value of t from equation (4) in equation (3), we get

$$R = \sqrt{(2gh)} \times \sqrt{\{2(H-h)/g\}}$$

$$= 2\sqrt{\{h(H-h)\}}.$$

The range R will be maximum when

$$dR/dh = 0.$$

$$\therefore \quad 2 \cdot \frac{1}{2}h^{-1/2}\ (H - h)^{1/2} - 2h^{1/2} \cdot \frac{1}{2}\ (H - h)^{-1/2} = 0$$

Solving, we get $h = H/2$.

Q. 6. *A piece of brass (an alloy of copper and zinc) weighs 12.9 g in air. When completely immersed in water it weighs 11.3 g. What is the volume of copper contained in the alloy? RD of copper and zinc are 8.9 and 7.1 respectively.*

Ans. Let m be the mass of copper in the alloy.

Then the mass of zinc in the alloy = $(12.9 - m)$ g.

Volume of copper in the alloy = $\frac{m}{8.9}$ cm^3

Volume of zinc in the alloy = $\frac{(12.9 - m)}{7.1}$ cm^3

Apparent loss of weight of brass = 12.9 – 11.3 = 1.6 g

Volume of the alloy = 1.6 cm^3 (density of water = 1 g cm^{-3})

Hence $\frac{m}{8.9} + \frac{12.9 - m}{7.1} = 1.6$

Solving this equation, we get m = 7.61 g.

Q. 7. *A capillary tube is attached horizontally to a constant head arrangement. If the radius of the capillary tube is increased by 10% then by what percentage the rate of flow of liquid will change.*

Ans. Since,
$$V = \frac{\pi p r^4}{8\eta l}$$
$$r' = r + \frac{10}{100}r = \frac{110}{100}r = 1.1r$$
$$\therefore \quad V' = \frac{\pi p (1.1r)^4}{8\eta l} = \frac{1.464\pi p r^4}{8\eta l} = 1.464\text{ V}$$

$\therefore$ % increase in rate of flow of liquid = $\frac{V' - V}{V} \times 100 = \left(\frac{1.464V - V}{V}\right) \times 100 = 46.4\%$

Q. 8. *What is the excess pressure inside a soap bubble that is 5 cm in diameter, assuming 0.026 Nm^{-1} as the surface tension of the soap solution?*

Ans. Excess pressure inside a soap bubble is
$$(P - P_a) = \frac{4T}{r} \qquad ...(i)$$
Here
$$T = 0.026 \text{ Nm}^{-1}$$
$$r = \frac{5}{2} = 2.5 \text{ cm} = 2.5 \times 10^{-2} \text{ m}$$
$$\therefore \quad P - P_a = \frac{4 \times 0.026 \text{ Nm}^{-1}}{2.5 \times 10^{-2} \text{ m}} = 4.16 \text{ Nm}^{-2}.$$

VI. VALUE-BASED QUESTIONS

Q. 1. *Meera's mother had a history of blood pressure but she did not care of it. One day she feels pain is her heart? Meera took her mother to hospital where the doctor did examination and told that your mother has a trouble in heart. I thought that some artillains are blocked. So she has to undergo angiography immediately. Meera's mother was not prepare mentally. She was afraid of it. Then*

Meera convinced her mother that due to the accumulation of plague on the inner walls of the antillares, the flow of blood has been reduced and giving you trouble. Her mother got convinced and prepare herself for angiography.

(i) What qualities and values were shown by Meera?

(ii) In an artery of radius 'a' blood flows with a uniform speed v. If radius of artery becomes '$\frac{3}{4}a$' due to the accumulation of plague on its inner walls, what will be the flow of the blood through the constriction?

Ans. (*i*) Meera showed an apathy with her mother, conviencing capacity and sharp mind.

(*ii*) According to equation of continuity.

$$AV = A'V'$$

$$\pi a^2 \times V = \pi\left(\frac{3a}{4}\right)^2 \cdot V'$$

$$V' = \frac{a^2 V}{\left(\frac{3a}{4}\right)^2} = \frac{16}{9}V$$

$$\therefore \quad V' = 1.8 \text{ V}$$

Q. 2. *Usha's uncle was suffering from blood pressure but did not consult any doctor. Usha, who was a student of class XII, came to know about her uncle's disease. She requested him to go to hospital with her. She convinced him by telling the effects of blood pressure on heart. Usha's uncle consulted a doctor in the hospital and started taking regular medicine. Within a few days her uncle's blood pressure became normal and he was happy.*

(i) What values were exhibited by Usha?

(ii) State Bernoulli's Theorem.

Ans. (*i*) Usha is very helpful, sensible, cooperative and having good knowledge of diseases and their treatment.

(*ii*) For an incompressible, non-viscous, irrotational liquid having streamlined flow, the sum of the pressure energy, kinetic energy and poential energy per unit mass is constant.

$$\frac{p}{\rho} + \frac{V^2}{2} + gh = \text{constant}$$

But for a steady flow Bernoulli's equation can be simplified as

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

Q. 3. *We know that all the bodies fall downwards with an accelerating velocity due to the acceleration due to gravity 9.8 m/s^2 but rain drops do not fall to earth as the other bodies do. Sohan was a student of science. His father asked Sohan the reason of drops not falling to the earth with the velocity equal to other bodies. He explained this reason that all bodies fall freely irrespective of their mass and size but in case of rain drops, there is a retarding force of viscosity due to the air* *medium. After falling through some distance, the drops attains a constant velocity which is known as terminal velocity. Sohan's father was satisfied with his son's answer.*

(i) What values are displayed by Sohan?

(ii) What do you mean by viscosity and terminal velocity?

Ans. (*i*) Very intelligent, logical explanation, confident and helping nature.

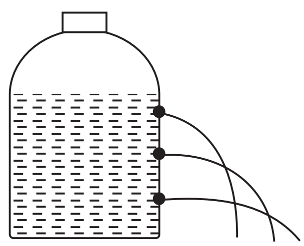
(*ii*) **Viscosity :** It is the property of a fluid by virtue of which an internal frictional force comes into play when the fluid is in motion in the form of layers having relative motion. It opposes the relative motion of the different layers.

Terminal velocity : It is the maximum constant velocity acquired by the body while falling freely in a viscus medium. This is attained when the apparent weight is compensated by the viscous force. It is given by

$$v = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

TEST YOUR SKILLS

1. What is Pascal's law ? Three holes are made in a plastic bottle. When water is filled in the bottle, water jets coming out of holes reach different horizontal distance. How would you explain this?



2. "Pressure is a scalar quantity." Do you agree with this statement? Give reasons for your answer.

3. Golf balls have dimples on their surface. How does this help golf ball, during its flight?

4. Why does a spinning ball from Harbhajan Singh follow a different trajectory, than a pace ball from Ishant Sharma?

5. What do you understand by surface tension? Why does hot food taste better, compared to cold food?

6. When you fouch a soap bubble with dry fingers, it bursts, but when you touch it with fingers, it does not burst. Is it true or false? Give reasons for your answer.

7. Water droplets on lotus leaves are almost spherical in shape, while on plastic plates they are not. Why?

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