10 Wave Optics

Facts that Matter

- **Wavefront**

It is the locus of all points having same phase or it is the surface of constant phase. The wave fronts are of three types.

(*i*) **Spherical wavefront :** It is the wavefront of a point source as shown in Fig. 10.1 (*a*)

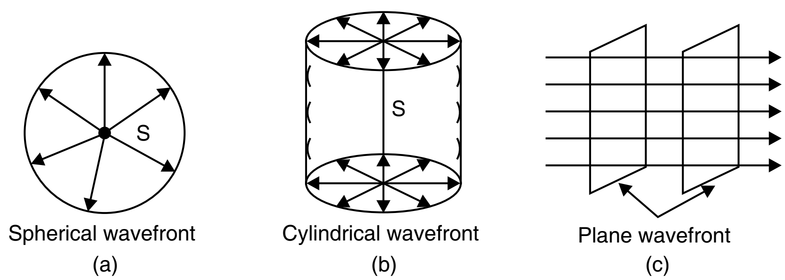


Fig. 10.1

(*ii*) **Cylinderical wavefront :** It is the wavefront of linear source as shown in Fig. 10.1 (*b*).

(*iii*) **Plane wavefront :** The wavefront at distant position irrespective of the nature of source is plane wavefront as shown in Fig. 10.1 (*c*).

- The plane wavefront can be converted into spherical wavefront and vice versa. [Fig. 10.2]

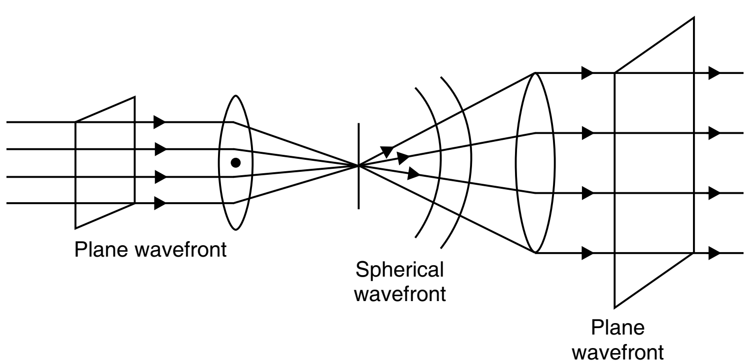


Fig. 10.2

• Huygen's Principle

It states that:

(*i*) Every point on a wavefront acts as the source of secondary wavelets. The secondary wavelet sends out waves in all directions just as primary source of light. The light coming from secondary source or wavelets is identical in all respects as coming from primary source.

(*ii*) The position of new wavefront can be obtained by drawing an envelope of radius c.t. where *t* is time and *c* is the velocity of light. The positions of two wavefronts at time *t* and *t′* are shown in the Fig. 10.3 (*a*) and (*b*)

(*iii*) The direction of wave propagation is always normal to the wavefront.

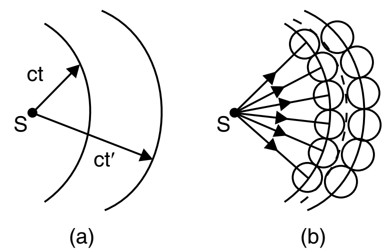


Fig. 10.3

Note. Huygen's construction is based on the principle that every point of a wavefront is a source of secondary wavefront. The envelope of these wavefronts *i.e.,* the surface tangent to all the secondary wavefront gives the new wavefront.

• Reflection on the Basis of Wave Theory

As we know from the previous chapter, the two laws of reflection are:

(*i*) Angle of incidence is equal to angle of reflection.

(*ii*) The incident ray, the reflected ray and the normal at the point of incidence all lie in the same plane.

Using Huygen's Principle to prove above conditions

Let us assume a plane wave AB incident at an angle *i* on a reflecting surface *MN*. If *v* is the speed of the wave in the medium and *t* is the time taken by the wavefront to cover the distance *BC*, then

$$BC = vt$$

To construct the reflected wavefront we draw an arc (representing reflected wavefront) of radius *vt* from the point *A*. Draw a tangent on the arc (*i.e.,* reflected wavefront). We obtain

$$AE = BC = vt$$

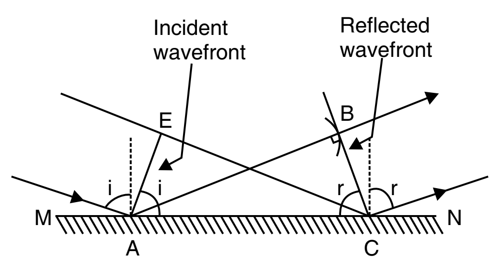


Fig. 10.4

From the triangles *ECA* and *BAC* we will find that they are congruent. This is the law of reflection.

Thus, $\angle i = \angle r$

• Refraction on the Basis of Wave Theory

In the previous chapter we have already discussed the two laws of refraction, *i.e.,*

(*i*) The incident ray, the normal to the refracting surface at the point of incidence and the refracted ray all lie in the same plane.

(*ii*) For a given pair of media and for light of a given wavelength, the ratio of the 'sin *i*' of the angle of incidence to the ′sin *r*′ of angle of refraction is constant.

Using Huygen's Principle to prove above conditions

Let us assume PP' represent the surface separating medium 1 and medium 2. Let v_1 and v_2 represent the speed of light in medium 1 and medium 2 respectively. Take a wavefront AB incident on the interface at an angle i. If t is the time taken by the wavefront to travel the distance BC. Thus,

$$BC = v_1 t$$

We draw an arc (representing refracted wavefront) of radius $v_2 t$ from the point A in the second medium. Draw a tangent on this arc. CE given refracted wavefront. From the triangles ABC and AEC we obtain

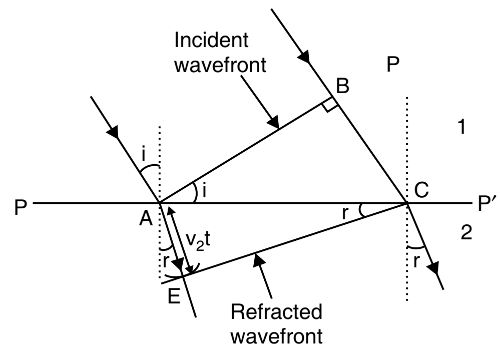


Fig. 10.5

$$\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC} \quad ...(i)$$

and $$\sin r = \frac{AE}{AC} = \frac{v_2 t}{AC} \quad ...(ii)$$

where i and r are the angles of incidence and refraction respectively. Thus

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} \quad ...(iii)$$

Now, if C represents the speed of light in vacuum, then,

$$n_1 = \frac{c}{v_1} \quad ...(iv)$$

and $$n_2 = \frac{c}{v_2} \quad ...(v)$$

In terms of the refractive indices, equation (iii) can be written as

$$n_1 \sin i = n_2 \sin r$$

This is the Snell's law of refraction.

• Interference (Definition)

The term 'interference' in general refers to any situation where two or more waves overlap each other in the same region of space. But usually, interference refers to the superposition of two coherent waves of same frequency moving in the same direction.

• Young's Double Slit Experiment

It was carried out in 1802 by Thomas Young to prove the wave nature of light.

In this experiment, light from a single source is split into two components using two slits. At a distance D, a screen is placed on which the interference pattern is obtained.

Observation:

- The interference pattern disappears, if one of the two slits is closed.
- If two different sources are used, no permanent pattern of interference is obtained.

Condition for Sustained Interference:

- The two sources of light should emit light continuously.
- The light waves should be of same wavelength. (Monochromatic)
- The light waves should be of same or comparable amplitude.
- The two waves must be in same phase or bear a constant phase difference.

- The two sources of light must lie close to each other.
- The two sources must be very narrow.

In sustained interference, the positions of maximum and minimum brightness remain fixed.

Experiment

Two slits S_1 and S_2 are made in an opaque screen, parallel and very close to each other. These two are illuminated by another narrow slit S which in turn is lit by a bright source. Light wave spread out from S and fall on both S_1 and S_2. Any phase difference between S_1 and S_2 is unaffected

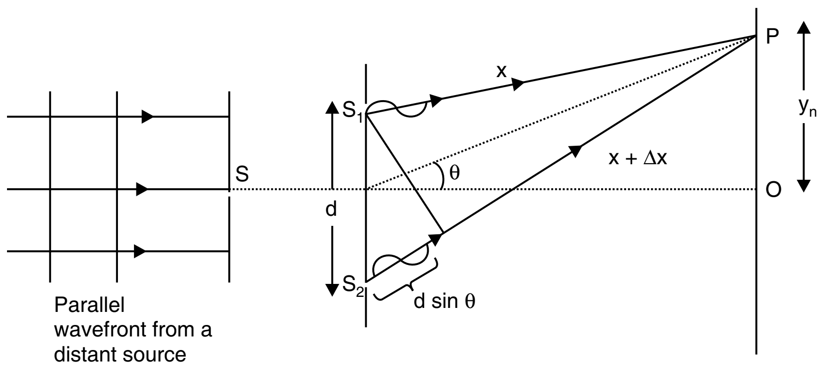


Fig. 10.6

and remain constant. The light waves going from S_1 and S_2 to any point P on the screen interface with each other. At some point the waves superpose in such a way that the resultant intensity is greater than the sum of the intensities due to separate waves **Constructive Interference** while at some other point, it is lesser than the sum of the separate intensities **Destructive interference**. Thus, the overall picture is a pattern of dark and bright bands known as fringe pattern. The dark bands are known as dark fringes and the bright bands are known as bright fringes.

Theory

Let us assume that two waves each of angular frequency ω from sources, S_1 and S_2 reach the point P. If the waves have amplitudes a_1 and a_2 respectively. Then

$$y_1 = a_1 \sin(\omega t - kx) \quad \text{...(i)}$$

and

$$y_2 = a_2 \sin[\omega t - k(x + \Delta x)]$$

$$= a_2 \sin(\omega t - kx - \phi) \quad \text{...(ii)}$$

where,

$$\phi = k\Delta x = \frac{2\pi}{\lambda}(\Delta x) \quad (\Delta x = \text{path difference}) \quad \text{...(iii)}$$

So, by the principle of superposition, the resultant wave at P will be

$$y = y_1 + y_2$$

$$\Rightarrow \quad y = a_1 \sin(\omega t - kx) + a_2 \sin(\omega t - kx - \phi)$$

$$= a_1 \sin(\omega t - kx) + a_2[\sin(\omega t - kx)\cos\phi - \cos(\omega t - kx)\sin\phi)$$

or,

$$y = [(a_1 + a_2\cos\phi)\sin(\omega t - kx)] - [(a_2 \sin\phi).\cos(\omega t - kx)] \quad \text{...(iv)}$$

Let $a_1 + a_2 \cos\phi = a \cos\alpha$

$$a_2 \sin\phi = a \sin\alpha$$

so, $a^2[\cos^2\alpha + \sin^2\alpha] = (a_1 + a_2\cos\phi)^2 + (a_2 \sin\phi)^2$ (squaring and adding above both)

or,

$$\boxed{a^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$$

and $\alpha = \tan^{-1}\left[\frac{a_2 \sin\phi}{a_1 + a_2\cos\phi}\right]$

Hence equation (*iv*) becomes

$$y = a\sin(\omega t - kx)\cos\alpha - a\cos(\omega t - kx)\sin\alpha$$

or, $$y = a\sin(\omega t - kx - \alpha) \quad (v)$$

From above equation, it is clear that in case of superposition of two waves of equal frequencies propagating almost in the same direction, resultant is harmonic wave of same frequency ω and wavelength $\left(\lambda = \frac{2\pi}{k}\right)$ but amplitude

$$a^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi$$

Now, as intensity of wave is given by

$$I = \frac{1}{2}\rho v\omega^2 a^2 = ka^2 \qquad \left(k = \frac{1}{2}\rho v\omega^2\right)$$

So the intensity of resultant wave

$$I = k(a_1^2 + a_2^2 + 2a_1a_2\cos\phi)$$

or, $$\boxed{I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi} \qquad [\text{As } I_1 = ka_1^2;\ I_2 = ka_2^2]$$

- **Condition for Constructive Interference**

For intensity to be maximum at P,

$$\cos\phi = 1$$

∴ Phase difference, $\phi = +2n\pi$

where $n = 0, 1, 2$...........

Now, $\frac{2\pi}{\lambda}(\Delta x) = \Delta\phi = +2\pi n$

or, $$\Delta x = +n\lambda$$

where $n = 0, 1, 2$..........

and $$I_{max} = I_1 + I_2 + 2\sqrt{I_1I_2}$$

or, $$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

or, $$I_{max} \propto (a_1 + a_2)^2$$

- **Condition for Destructive Interference**

Intensity will be minimum when

$$\cos\phi = \text{minimum} = -1$$

i.e., $$\phi = +\pi, +3\pi, +5\pi$$

or, $$\phi = +(2n-1)\pi \text{ with } n = 1, 2, 3...............$$

or, $$\frac{2\pi}{\pi}(\Delta x) = +(2n-1)\pi \qquad \left[\because \Delta\phi = \frac{2\pi}{\lambda}(\Delta x)\right]$$

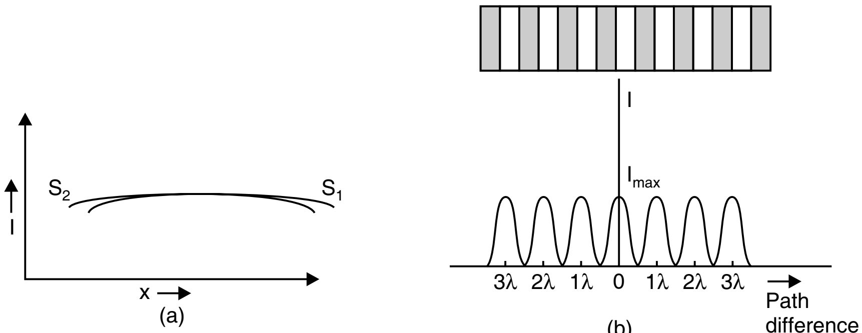
or, $$\Delta x = +(2n-1)\lambda/2 \text{ with } n = 1, 2, 3...........$$

also, $$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

or, $$I_{min} \propto (a_1 - a_2)^2$$

• Ratio of Intensity of Wave at Maxima and Minima

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$



(a) Illumination on the screen due to slits S_1 and S_2 respectively when only one slit is open.

(b) Interference pattern when both S_1 and S_2 are open.

Fig. 10.7

Ratio of intensity of wave due to two sources:

Let I_1 and I_2 be the intensities and a_1 and a_2 be the amplitudes of waves from slits S_1 and S_2 respectively. Then

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$$

Relation between slit width (ω), I and a is given by

$$\frac{\omega_1}{\omega_2} = \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$$

• Position of Bright Fringes

Position of bright fringes is given by

$$Y_n = n\lambda\frac{D}{d}$$

where n = 0, 1, 2..........

• Position of Dark Fringes

Position of dark fringes is given by

$$Y_n = (2n-1)\frac{\lambda D}{2d}$$

where, n = 1, 2, 3............

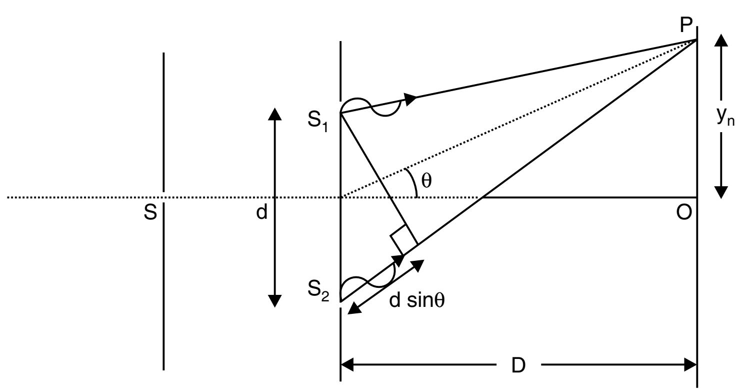


Fig. 10.8

• Fringe Width (ω)

The separation between two consecutive dark (or bright) fringes is known as fringe width (ω). It is also denoted by 'β'.

$$\therefore \quad \omega = Y_{n+1} - Y_n = (n+1)\frac{\lambda D}{d} - \frac{n\lambda D}{d}$$

$$\Rightarrow \quad \boxed{\omega = \frac{\lambda D}{d}}$$

Angular fringe width or angular separation between fringes is

$$\theta = \frac{\omega}{D}$$

$$\boxed{\theta = \frac{\lambda}{d}}$$

• Coherent Waves

Two waves of same frequency are said to be 'coherent' if their phase difference does not change with time, *i.e.,* their phase difference is independent of time.

For observing interference effects, waves (or sources) must be coherent.

- In case of two coherent sources, the resultant intensity at any point is given by

  $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$, where f is the phase difference between the two coherent waves reaching the point.

  In case of in coherent sources, the resultant intensity at any point is given by

  $I_R = I_1 + I_2 +$ ....

- In case of interference of light (having zero phase difference), at a point on the screen the maximum intensity is obtained for path difference,

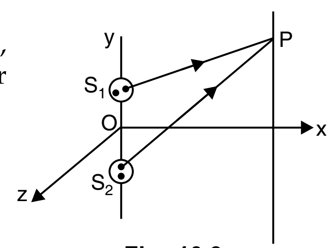


Fig. 10.9

$$\Delta n = S_2 - S_1P$$
$$= \pm n\lambda \text{ with } n = 0, 1, 2, ...$$

  and minimum intersity of the path difference

$$\Delta n = S_2P - S_1P = (2n - 1)\,\lambda/2$$
$$\text{with } n = 1, 2, 3, ...$$

  These intensity maxima and minima are called bright and dark fringes respectively and the array of fringes, interference pattern.

- In case of interference with zero initial difference, maxima corresponding to $n = 0$ (*i.e.,* $\Delta n = 0$) is called **central fringe or zero order maxima** while for $n = 1$, first maxima or bright fringe and so on. For minima, $n = 10$ (*i.e.,* central minima) does not exist and give the first minima.

- As for a given fringe, n is constant each fringe will be the locus of the point P for which $S_2P - S_1P$ = constant.

  Such locus will be a hyperbola with S_1 and S_2 as foci. If this hyperbola is rotated about the line S_1S_2 revolving hyperboloids will be obtained *i.e.,* the interference fringes of two coherent sources are hyperboloids with axis S_1S_2. Fringes seen on the sereen will be the sections of these hyperboloids.

- If screen is placed perpendicular to x-axis, *i.e.,* $y - z$ plane, which is usually the case, the fringes will be hyperbolic with straight-central bright fringe along z-axis as show in Fig. 10.10.

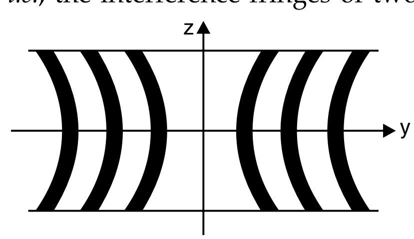


Fig. 10.10

- If the sereen is perpendicular to z-axis, *i.e.*, $x - y$ plane, the fringes will be hyperbolic with straight-central bright fringe along x-axis as shown in Fig. 10.11.
- If the screen is perpendicular to y-axis (line joining the sources) *i.e.*, $x - z$ plane the fringes will be circular (*i.e.*, concentric circles with their centres on the point of intersection of screen with y–axis). In this situation central fringe will be bright if $S_1S_2 = n\lambda$ and dark if $S_1S_2 = (2n - 1)\lambda/2$ as shown in Fig. 10.12

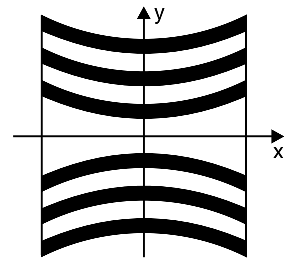


Fig. 10.11

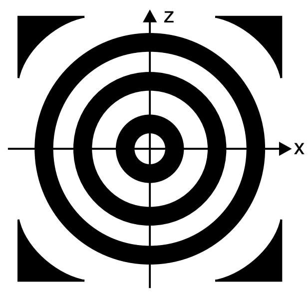


Fig. 10.12

- If instead of pinholes the sources are slits parallel to z-axis, for screen perpendicular to z-axis (*i.e.*, $x - y$ plane) shape of fringe will remain unchanged, *i.e.*, hyperbolic with straight-central bright fringe along x-axis as shown in Fig. 10.11, while for screen perpendicular to n and y-axis fringes will be straight lines parallel to the slits, *i.e.*, z-axis as shown in Fig. 10.13.

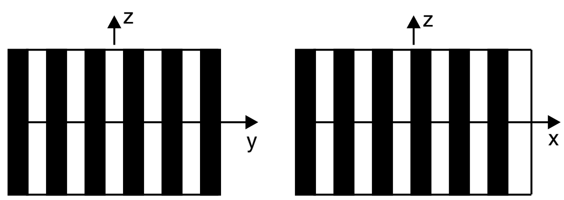


Fig. 10.13

- The shape of fringe depends on nature of source (pinholes or slits) and direction of observation, *i.e.*, position of screen.

• Diffraction of Light

According to Grimaldi, if the size of an obstacle or aperture is comparable with the wavelength of light, light deviates from rectilinear propagation near the edges of the obstacle (or aperture) and entries the geometrical shadow. *This flaring out or encroachment of light in the shadow zone as it passes around obstacles or through small aperture is called diffraction.*

- As result of diffraction, the edges of the shadow (or illuminated region) do not remain well defined and sharp but become blurred and fringed. This is why light filtering through tree leaves does not form sharp illuminated regions on the ground but often casts bright patches with blurred edges on the ground.
- The wavelength of light is very small and the aperture, obstacles, slits, etc., are not commonly available in nature, with comparable of wavelength hence the diffraction of light is not common in day to day activities. However, the wavelength of sound is of the order of the size of doors, windows, etc, therefore, the diffraction of sound in very common in day to day life.

- According to Huygen's wave theory if the size of aperture or slit is much larger than the wavelength of light ($d > \lambda$) the incident plane wavefront will pass through the slit without change in its shape and direction of motion will result rectilinear propagation of light as shown in Fig. 10.14.

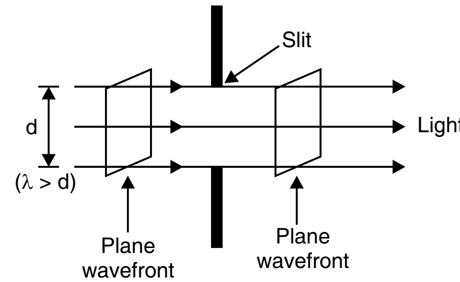


Fig. 10.14

- If width of the aperture or slit is comparable to the wavelength of light ($\lambda<d$), must of the incident wavefront will be obstructed and in accordance with Huygen's wave theory a cylinderical (or spherical) wavefront depending on the aperture (slit or hole) will originate from it. Now, as the direction of wave propagation is normal to the wavefront, after passing through the aperture light will flare out as shown in Fig 10.15. This flaring out or spreading of light is the so called diffraction.

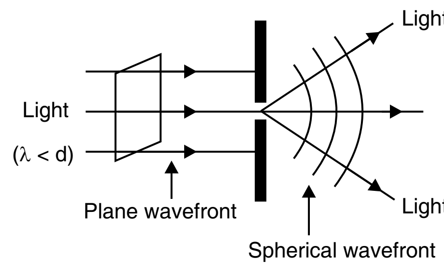


Fig. 10.15

- On the basis of wave theory of diffraction, *Poisson* showed that it leads the '*absured*' prediction that the shadow of an opaque object should have a bright spot at its centre. When experiments were performed to test this 'absured prediction', a bright spot was actually observed at the centre of a shadow. This all in turn, instead of disproving wave-theory established that diffraction is a convincing proof of wave theory.
- In case of diffraction of light at aperture and obstacles (for $\lambda < d$), instead of sharp shadow (or region of uniform illumination) fringe pattern called diffraction pattern is obtained. This diffraction pattern depends on the nature of diffracting device and wavelength of light used. For example in case of *diffraction at single slit*, a central bright band with alternate bright (subsidary maxima) and dark bands (minima) of intensity decreasing are obtained as shown in Fig. 10.16.

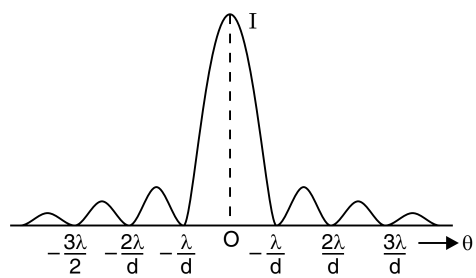


Fig. 10.16

- **Diffraction at Single Slit**

Let a plane wavefront incidents on a slit of width d and diffracted rays converged on screen with help of a convex lens at the distance of D from the plane of the slit as shown in Fig. 10.17.

The slit can be assumed to be divided into two equal halves, the wavelength from the corresponding points of the two halves of the slit will have a path difference of $\lambda/2$, *i.e.*, the wavelets from each half will reach point P in opposite phase. Hence, for second secondary minima

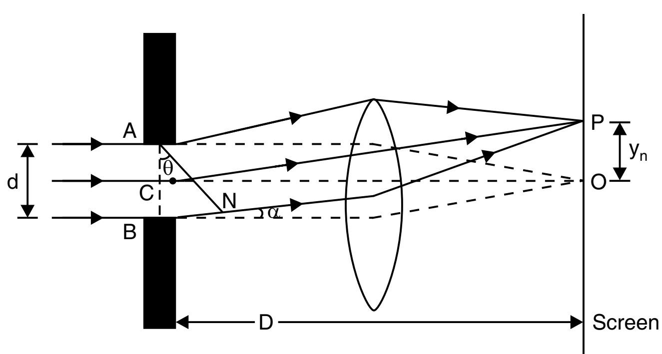


Fig. 10.17

$$2\lambda = d \sin \theta \qquad \left[\because \begin{array}{r} BN = \lambda \\ \frac{d}{2}\sin v = \lambda \end{array}\right]$$

$$\sin \theta = \frac{2\lambda}{d}$$

or $$\sin \theta_n = \frac{n\lambda}{d} \qquad \text{(for } n\text{th secondary minima)}$$

The angular width

$$\theta_n = \frac{n\lambda}{d}$$

If y_n is the distance of nth secondary minimum from the centre of the screen,

then $$\tan \theta_n = \frac{OP}{CO} = \frac{y_n}{D}$$

For small value θ_n

$$\tan \theta_n = \sin \theta_n$$

$$\frac{y_n}{D} = \frac{n\lambda}{d} \quad \text{(from fig. and eq. } I\text{)}$$

or $$y_n = \frac{nD\lambda}{d}$$

The fringe width

$$\beta = y_n - y_{n-1} = \frac{\lambda D}{d}$$

for first secondary maxima

$$\sin \theta_1 = \frac{3\lambda}{2d}$$

($\because$ The wavelets from each half will reach point P such that out of three equal parts two will canceal out leaving one part of wavelets to produce the bright fringe.)

- All secondary maxima and minima are of same width.
- Central maxima is of double width as other maxima.
- If $\beta = d$, then the light can go up to distance D (= DF) without much spreading. This distance is called fresenel's distance

  $\therefore$ for fresenel distance $D = Df$

  $$d = \frac{\lambda D_F}{d}$$

  or $$\boxed{D_F = \frac{d^2}{\lambda}}$$

- In case of diffraction at single slit, position of nth minima is given by $d \sin \theta = n\lambda$, and as nth secondary maxima is approximately half-way between nth and $(n + 1)^{th}$ minima, for it

  $$d \sin \theta = \frac{n\lambda + (n+1)\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

  so $$\alpha = \frac{\phi}{2} = \frac{\pi}{\lambda}(2n+1)\frac{\lambda}{2} = \frac{(2n+1)}{2}\pi$$

  i.e., $$\alpha = 1.5\,\pi, 2.5\,\pi, 3.5\,\pi, \ldots\ldots$$

- Precise theory of diffraction shows that secondary maxima are not exactly midway between minima but are displaced towards the centre of the pattern and the displacement towards the pattern decreases as the order of maxima increases. Exactly,

$$\alpha = 1.43\ \pi, \qquad 2.46\ \pi,\ 3.47\ \pi\ ......$$

$$\therefore \qquad I = I_m \left[\frac{\sin\alpha}{\alpha}\right]^2 = I_m \left[\frac{\sin (2n+1)\pi}{(2n+1)\pi}\right]^2$$

$$= \frac{I_m}{[(2n+1)\pi]^2}$$

$$\Rightarrow \qquad I_1 : I_2 : I_3 = \frac{1}{(1.5\ \pi)^2} : \frac{1}{(2.5\ \pi)^2} : \frac{1}{(3.5\ \pi)^2}$$

Thus, it is evident that the intensity in secondary maxima decreases with increase in order and in first secondary maxima is 4.5% of central maxima.

• Polarisation of Light

Light is an electromagnetic wave having electric and magnetic field vibrations perpendicular to each other as well as perpendicular to the direction of propagation. *The phenomenon due to which the electric vectors of light are restricted in a particular direction is called polarisation.* Polarised light consists of individual photons whose electric field vectors are all aligned in the same direction. Ordinary light is unpolarised because the photons are emitted in a random manner. When light passes through a polariser, the electric field vectors outreacts more strongly with molecules having certain orientation. This causes the incident beam to separate into two beams, whose electric vectors are perpendicular to each other. A horizontal filter absorbs photons whose electric vectors are vertical.

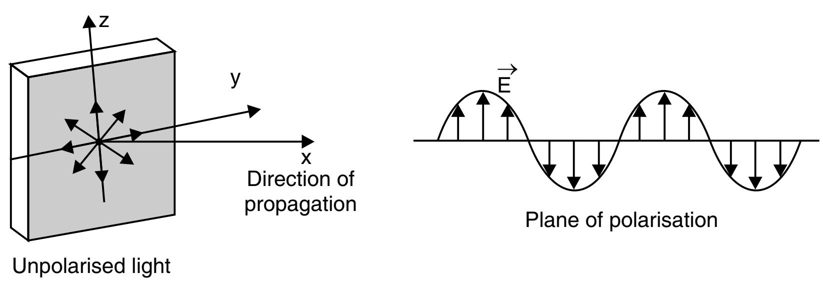


Fig. 10.18

- The tourmalines crystal is used to produce plane polarised light. When light passes through the tourmaline crystal it allows the light to pass thought it whose electric field vectors are parallel to optic axis of tourmaline crystal, hence light gets polarised. The tourmalines crystal is called polarizer or polaroid. If tourmaline crystal is used to study the polarized light it is called analyser. If an analyser is placed perpendicular to the polariser, no light will be seen as shown in Fig. 10.19.

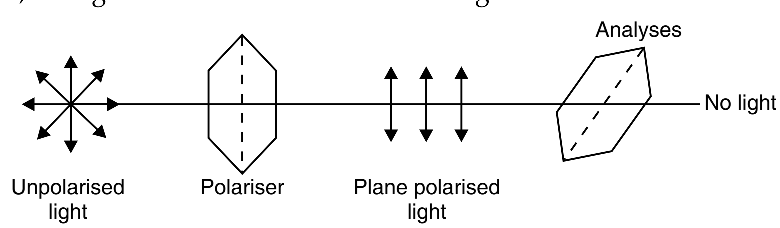
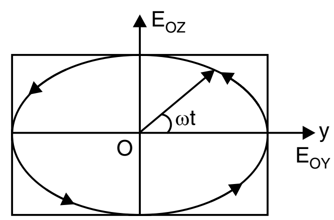


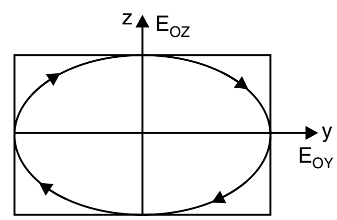
Fig. 10.19

- Polarisation is a convincing proof of transverse nature of wave as an transverse wave vibrations are in a plane perpendicular to the direction of wave-motion and so the wave can be polarised.
- In case of interference of polarised light the interfering waves must have same plane of polarisation, otherwise unpolarised (or partially polarised) light will result.
- Apart from partially polarised and plane (*i.e.*, linearly) polarised light, can also be circularly or elliptically polarised that too left handed or right handed. Elliptically and circularly polarised lights result due to superposition of two mutually perpendicular plane polarised lights differing in phase by ($\pi/2$) with unequal or equal amplitudes of vibrations respectively.



(Left handed elliptically polarised light)

Fig. 10.20



(Right handed elliptically polarised light)

Fig. 10.21

• Polarisation by Reflection

Malus found that ordinary light, when allowed to undergo refraction, the partially reflected light gets partially plane polarised. However, there is an angle of incidence, at which an ordinary light undergoes refraction as well as reflection (partial) and then the partially reflected is richly plane polarised. Such an angle is known as angle of polarisation.

• Brewster' Law

When light is incident at polarising angle at the interface of a refracting medium, the refraction index of the medium in equal to the tangent of polarising angle. In Fig. 10.22.

Mathematically

$$\mu = \tan i_p \qquad \text{...(I)}$$

But from Snell's law

$$\mu = \frac{\sin e_p}{\sin r} \qquad \text{...(II)}$$

$$\Rightarrow \qquad \frac{\sin i_p}{\sin r} = \frac{\sin i_p}{\cos i_p} \qquad \text{from I, II}$$

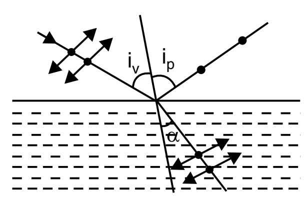


Fig. 10.22

or $\quad \sin r = \cos i_p$

or $\quad \sin r = \sin (90° - i_p)$

or $\quad r = 90° - i_p$

or $\quad r + i_p = 90°$

Thus, the angle of refraction and angle of polarisation is complementary to each other. The reflected and refractive ray are perpendicular to each other.

- **Polarisation by Scattering**

The light form a clear blue portion of the sky shows a rise and fall of intensity when viewed through a polariod which is rotated. This is nothing but sunlight, which has changed its direction (having been scattered) on encountering the molecules of the earth's atmosphere. Shown in Fig. 10.23 the incident sunlight is unpolarised. The dots stands for polarisation perpendicular to plane of the figure. The double arrows is shown in the plane of the figure. Under the influence of the electric field of the incident wave the electrons in the molecules acquire components in both these directions.

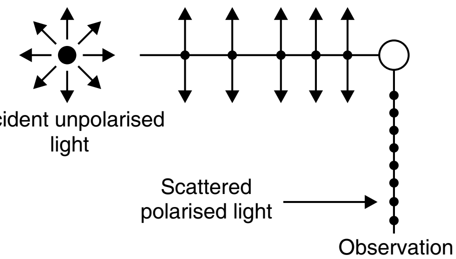


Fig. 10.23

In Fig. 10.23 changes accelerating parallel to the double arrows do not radiate energy towards the observer since their acceleration has no transverse component. The radiation scattered by the molecules is therefore represented by dots. It is polarised perpendicular to the plane of the figure.

- **Intensity of Light Emerging From a Polaroid**

If plane polarised light of intensity I_0 (= KA^2) is incident on a polaroid and its vibrations of amplitude A make an angle θ with transmission axis, then the component of vibrations parallel to transmission axis will be $A \cos \theta$ while perpendicular to it $A \sin \theta$. Now, as polaroid will pass only those vibrations which are parallel to its transmission axis, *i.e.*, $A \cos \theta$, so the intensity of emergent light will be

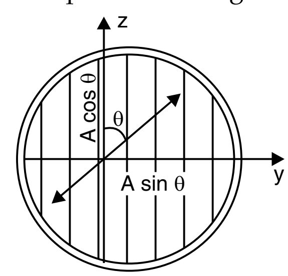


Fig. 10.24

$$I = K(A \cos \theta)^2$$

$$= KA^2 \cos^2 \theta$$

or $$I = I_0 \cos^2 \theta \qquad (\because \quad I_0 = KA^2)$$

This is called *Malus law.*

- If the incident light is unpolarised, then as vibrations are equally probable in all directions (in a plane perpendicular to the direction of wave propagation), θ can have any value from 0 to 2π and hence

$$(\cos^2 \theta) = \frac{1}{2\pi} \int_0^{2\pi} \cos^2 \theta . d\theta$$

$$= \frac{1}{2} \times \frac{1}{2\pi} \int_0^{2\pi} (1 + \cos 2\theta)\, d\theta$$

i.e., $$(\cos^2 \theta)_{av} = \frac{1}{4\pi} \left[\theta + \frac{1}{2} \sin 2\theta \right]_0^{2\pi}$$

$$= \frac{1}{2}$$

$$\therefore \quad I = \frac{I_0}{2}.$$

- If an unpolarised light is connected into plane polarised light its intensity becomes half.

- If light of intensity I, emerging from one polaroid called polariser is incident on a second polaroid called analyser, the intensity of light emerging from the second polaroid in accordance with Malus law will be given by

$$I_2 = I_1 \cos^2 \theta$$

where θ is the angle between the transmission axis of the two polaroids.

• Scattering of Light

When light passes through heterogenous medium, it strikes to the heterogenous particles of the medium and spread out in all the directions. This phenomenon is called scattering of light. In visible spectrum of light, the light of smaller wavelength scatter most and the light of larger wavelength scatter least. The wavelength of light of violet-blue colour in visible spectrum is smaller therefore the scattering of these colours in sun light passing through atmosphere is most and sky appears blue. (Fig. 10.25)

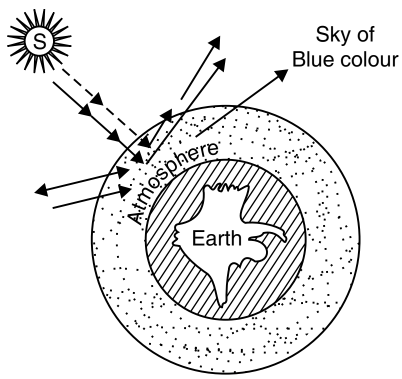


Fig. 10.25

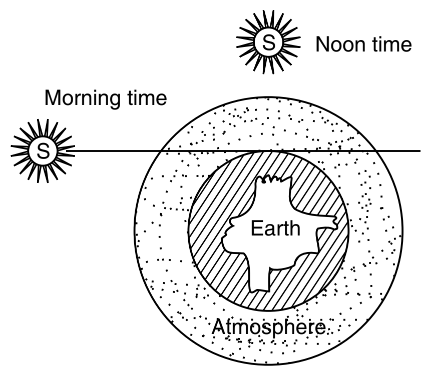


Fig. 10.26

- As shown in Fig. 10.26, at the time of sunrise (morning) sun light has to cover large length of atmosphere to reach the earth, hance there is more scattering. Due to more scattering the light red colour reaches to the surface of earth in large quantity. This is why at the time of sunrise and set, the sun appears red-orange.
- The light from a clear blue portion of the sky shows a rise and fall of intensity when viewed through a polaroid which is rotated. It means scattered light is polarised.

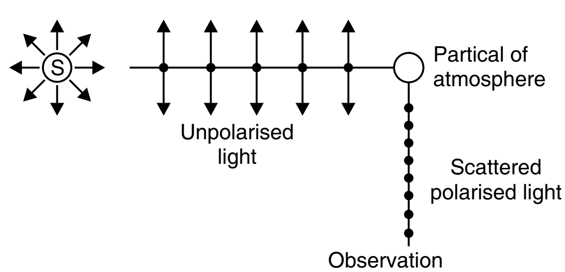


Fig. 10.27

- As shown in Fig. 10.27 the incident sun light is unpolarised. The dots stands for polarisation perpendicular to the plane of the figure. The double arrows show polarisation in plane of the figure. There is no phase relation between these two in unpolarised light. Under the influence of the electric field of the incident wave the electrons in the molecules acquire components of motion in both these directions. In Fig. 10.26 observer is looking at 90° to the direction of sun, therefore charges accelerating parallel to the double arrows do not radiate energy towards the observer, since their acceleration has no transverse component. The radiation scattered by the molecule is therefore, is represented by the dots. It is polarised perpendicular to the plane of the figure.

• Resolving Power of Optical Instruments

The resolution power of an instrument is reciprocal of the resolving limit. The *smallest separation of two point objects at which they appear just separatel is called the limit of resolution.*

Theory of diffraction of light shows that the radius of the central bright region is approximately given by

$$r = \frac{1.22\,\lambda f}{2d} = \frac{0.61\,\lambda f}{d}$$

where f is the focal length of the lens and $2d$ is the diameter of the circular aperture or the diameter of the lens, which is smaller. [Fig. 10.27]

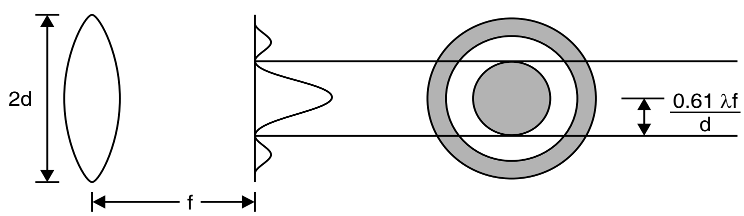


Fig. 10.28

Although the size of the spot is very small, it plays an important role in determine the limit of the optical instrument like a telescope and microscope. For the two stars to be just resolved

$$f\,.\,\theta = r = \frac{0.61\,\lambda f}{d}$$

or

$$\theta = \frac{0.61\,\lambda}{d}$$

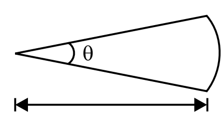


Fig. 10.29

Thus θ will be small of the diameter of the objective is large.

It means the telescope will have better resolving power, if d is large. It is for this reason that for better resolution, a telescope must have a large diameter objective.

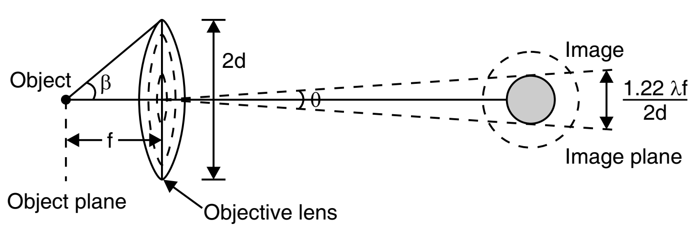


Fig. 10.30

The magnification ratio of image size to object size is given by $\frac{v}{f}$. It can be seen from Fig. 10.30 that

$$\frac{2d}{f} = 2 \tan \beta \qquad ...(i)$$

when the separation between two points in a microscope specimen is comparable to the wavelength λ of light, the diffraction effect becomes important. The image of a point object will again be a diffraction pattern whose size in the image plane will be

$$v\theta = v\left(\frac{1.22\,\lambda}{2d}\right) \qquad ...(ii)$$

Two objects whose images are closer than this distance will not be resolved, they will be seen as one. The corresponding minimum separation, d_{min} in the object plane is given by

$$d_{min} = \frac{1}{m}\left[v\left(\frac{1.22\,\lambda}{2d}\right)\right]$$

$$= \frac{1.22\,\lambda}{2d}\cdot\frac{v}{m}$$

$$d_{min} = \frac{1.22\,\lambda f}{2d} \qquad ...(iii)$$

From Eqs. (*i*) and (*ii*)

$$d_{min} = \frac{1.22\,\lambda}{2\tan\beta} = \frac{1.22\,\lambda}{2\sin\beta} \qquad ...(iv)$$

If the medium between the objective lens is of refractive index n, then

$$d_{min} = \frac{1.22\,\lambda}{2\,n\sin\beta}$$

$n \sin \beta$ is called the numerical aperture

The resolving power of the microscope is reciprocal of the minimum separation of two points seen as distinct.

∴ *R.P.* of microscope

$$R.P. = \frac{2\,n\sin\beta}{1.22\,\lambda}$$

- **Resolving Power of a Telescope**

It is the reciprocal of the smallest angular separation between two distinct objects, so that they appear just separate when seen through telescope.

The least separation,

$$\theta = \frac{1.22\,\lambda}{2d}$$

∴ Resolving power $\frac{1}{\theta} = \frac{2d}{1.22\,\lambda}$

$2d$ is the aperture of the objective lens.

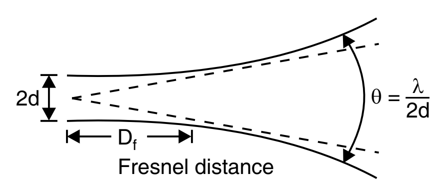


Fig. 10.31

QUESTIONS FROM TEXTBOOK

10.1. *Monochromatic light of wavelength 589 nm is incident from air on a water surface. What are the wavelength, frequency and speed of (a) reflected, and (b) refracted light? Refractive index of water is 1.33.*

Sol. Given, $\lambda = 589$ nm

$c = 3 \times 10^8$ m/s, $\mu = 1.33$

(*a*) For reflected light

wavelength $\lambda = 589 \text{ nm} = 589 \times 10^{-9}$ m

$$\nu = \frac{c}{\lambda} = \frac{3\times10^8}{589\times10^{-9}} = 5.09 \times 10^{14} \text{ Hz.}$$

Speed $v = c = 3 \times 10^8$ m/s

(*b*) For refracted light

$$_a\mu_\omega = \frac{v_a}{v_\omega} \qquad _a\mu_\omega = \frac{v_a\lambda_a}{v_\omega\lambda_\omega}$$

$\therefore$ $v_a = v_\omega$

so $$\lambda_\omega = \frac{\lambda_a}{_a\mu_\omega} = \frac{589\times10^{-9}}{1.33} = 4.42 \times 10^{-7} \text{ m}$$

$$\lambda = \frac{\lambda}{\mu} = \frac{589\times10^{-9}}{1.33} = 4.42 \times 10^{-7}\text{m}$$

As frequency remains unaffected on entering another medium, therefore,

$v_\omega = v_a = 5.09 \times 10^{14}$ Hz

Speed, $$v' = \frac{v_a}{\mu_\omega} = \frac{3\times10^8}{1.33} = 2.25 \times 10^8 \text{ m/s.}$$

10.2. *What is the shape of the wavefront in each of the following cases:*

(a) Light diverging from a point source.

(b) Light emerging out of a convex lens when a point source is placed at its focus.

(c) The portion of the wavefront of light from a distant star intercepted by the Earth.

Sol. (*a*) Spherical

(*b*) Plane

(*c*) Plane (Because a small area on the surface of a large sphere is nearly planar.)

10.3. *(a) The refractive index of glass is 1.5. What is the speed of light in glass? (Speed of light in vacuum is 3.0×10^8 ms^{-1})*

(b) Is the speed of light in glass independent of the colour of light? If not, which of the two colours red and violet travels slower in a glass prism?

Sol. (*a*) Refractive index, $$\mu = \frac{\text{speed of light in vacuum}}{\text{speed of light in the medium}}$$

$\therefore$ Speed of light in glass $$= \frac{\text{speed of light in vacuum}}{\mu_g}$$

$$= \frac{3.0\times10^6}{1.5} = 2.0 \times 10^6 \text{ ms}^{-1}.$$

(*b*) No, the refractive index and the speed of light in a medium depend on wavelength, *i.e.*, colour of light. We know that $\mu_v > \mu_r$. Therefore $v_{\text{violet}} < v_{\text{red}}$. Hence violet component of white light travels slower than the red component.

10.4. *In Young's double-slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.2 m. Determine the wavelength of light used in the experiment.*

Sol. Given, D = 1.4 m, d = 0.28 mm = 0.28×10^{-3}m

Fringe width, $\omega = \dfrac{1.2}{4} = 0.3 \times 10^{-2}$ m

(as the distance 1.2 m is of fourth fringe from central maxima)

Using formula,

Fringe width, $\boxed{\omega = \dfrac{D\lambda}{d}}$

$\therefore \quad \lambda = \dfrac{\omega d}{D} = \dfrac{0.3\times10^{-2}\times0.28\times10^{-3}}{1.4}$

$= 0.06 \times 10^{-5} = 600 \times 10^{-9} = 600$ nm.

10.5. *In Young's double-slit experiment using monochromatic light of wavelength λ, the intensity of light at a point on the screen where path difference is λ, is K units. What is the intensity of light at a point where path difference is λ/3?*

Sol. Phase difference corresponding to λ is 2π and phase difference corresponding to λ/3 is 2π/ 3.

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Let that $I_1 = I_2 = I_0$

In the first case, $K = I_0 + I_0 + 2I_0 \cos 2\pi = 4I_0$

In the second case, $K' = I_0 + I_0 + 2I_0 \cos\dfrac{2\pi}{3}$

$= I_0 + I_0 - 2I_0\left(\dfrac{1}{2}\right) = I_0$

Now, $\dfrac{K'}{K} = \dfrac{I_0}{4I_0} = \dfrac{1}{4}$ or $K' = \dfrac{K}{4}$.

10.6. *A beam of light consisting of two wavelength, 650 nm and 520 nm, is used to obtain interference fringes in a Young's double-slit experiment.*

(a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm.

(b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

Sol. Here, λ_1 = 650 nm = 650×10^{-9} m

λ_2 = 520 nm = 520×10^{-9}m

Suppose, d = distance between two slits

D = distance of screen from the slits.

(*a*) For third bright fringe, $n = 3$

$$x = n.\lambda_1.\frac{D}{d}$$

$$= 3 \times 650 \times 10^{-7} \times \frac{120}{0.2} = 0.117 \text{ cm} = 1.17 \text{ mm}$$

(*b*) Let n fringes of wavelength 650 nm coincide with (n + 1) fringes of wavelength 520 nm.

$$x = n\lambda_1 D/d = (n + 1)\ \lambda_2\ D/d \times \lambda_2$$

or, $$x = n \times 650 = (n + 1) \times 520$$

or, $$\frac{n+1}{n} = \frac{650}{520} = \frac{5}{4}$$

or, $$1+\frac{1}{n} = \frac{5}{4} \Rightarrow \frac{1}{n} = \frac{5}{4}-1 = \frac{1}{4}$$

or, $$n = 4$$

Hence, $$x = n\cdot\lambda_1\frac{D}{d}$$

$$= 4 \times 650 \times 10^{-7}\times\frac{120}{0.2}$$

$$= 1.56 \text{ mm.}$$

10.7. *In a double-slit experiment the angular width of a fringe is found to be 0.2° on a screen placed 1 m away. The wavelength of light used as 600 nm. What will be the angular width of the fringe if the entire experiment apparatus is immersed in water? Take refractive index of water to be 4/3.*

Sol. Angular fringe separation,

$$\theta = \frac{\lambda}{d} \text{ or } d = \frac{\lambda}{\theta}$$

In water, $$d = \frac{\lambda'}{\theta'}$$

$$\therefore \quad \frac{\lambda}{\theta} = \frac{\lambda'}{\theta'}$$

or, $$\frac{\theta'}{\theta} = \frac{\lambda'}{\lambda} = \frac{1}{\mu} = 3/4 \qquad \left[\because\ {}_a\mu_\omega = \frac{\lambda_a}{\lambda_\omega}\right]$$

or, $$\theta' = \frac{3}{4}\theta = \frac{3}{4}\times 0.2^\circ = 0.15^\circ.$$

10.8. *What is the Brewster angle for air to glass transition? (Refractive index of glass = 1.5.)*

Sol. Given, $\mu = 1.5$

Using formula,

$$\tan i_p = \mu$$

or, $$i_p = \tan^{-1}(\mu)$$

$$= \tan^{-1}(1.5)$$

Thus, $$i_p = 56.3^\circ.$$

10.9. *Light of wavelength 5000 Å falls on a plane reflecting surface. What are the wavelength and frequency of the reflected light? For what angle of incidence is the reflected ray normal to the incident ray?*

Sol. The wavelength and frequency of the reflected light are the same as that of the incident light.

∴ Wavelength of reflected light = 5000 Å

Frequency of reflected light = c/λ

$$= \frac{3\times10^8}{5000\times10^{-10}} \text{ Hz} = 6 \times 10^{14} \text{ Hz}$$

According to law of reflection, $i = r$

The reflected ray is normal to the incident ray.

$$i + r = 90°$$
$$i + i = 90°$$
$$2i = 90°$$
or, $$i = 45°.$$

10.10. *Estimate the distance for which ray optics is good approximation for an aperture of 4 mm and wavelength 400 nm.*

Sol. Here, $a = 4 \text{ mm} = 4 \times 10^{-3}$ m

$\lambda = 400 \text{ nm} = 400 \times 10^{-9} \text{ m} = 4 \times 10^{-7}$ m

Ray optics is good approximation upto distances equal to Fresnel's distance (Z_F)

$$Z_F = \frac{a^2}{\lambda} = \frac{(4\times10^{-3})^2}{4\times10^{-7}} = 40 \text{ m}.$$

10.11. *The 6563 Å H_α line emitted by hydrogen in a star is found to be red-shifted by 15Å. Estimate the speed with which the star is receding from the Earth.*

Sol. $\lambda = 6563 \text{ Å} = 6563 \times 10^{-10}$ m

$\lambda' - \lambda = 15 \text{ Å} = 15 \times 10^{-10}$ m

$$\boxed{\lambda' - \lambda = \frac{V_s\lambda}{c}}$$

$$V_s = \frac{c}{\lambda}(\lambda' - \lambda)$$
$$= \frac{3.0\times10^8}{6563\times10^{-10}} \times 15 \times 10^{-10}$$
$$= 6.8566 \times 10^5 \text{ ms}^{-1}$$
or, $$V_s = 6.86 \times 10^5 \text{ ms}^{-1}.$$

10.12. *Explain how Newton's Corpuscular theory predicts the speed of light in a medium, say water, to be greater than the speed of light in vacuum. Is the prediction confirmed by the experimental determination of speed of light in water? If not, which alternative picture of light is consistent with experiment?*

Sol. According to Newton's Corpuscular theory of light, when corpuscles of light strike the interface XY, figure separating a denser medium from a rarer medium, the component of their velocity along XY remains the same:

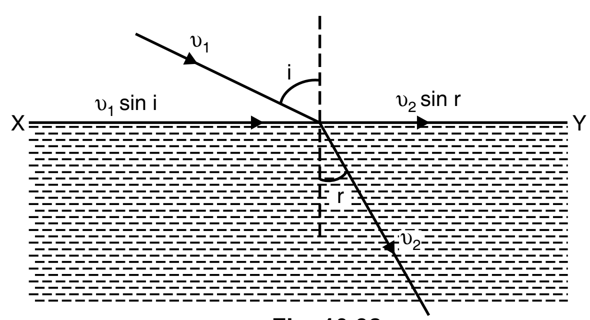


Fig. 10.32

If v_1 is velocity of light in rarer medium (air)

v_2 is velocity of light in denser medium (water)

i is angle of incidence,

r is angle of refraction,

Then component of v_1 along $XY = v_1 \sin i$

component of v_2 along $XY = v_2 \sin r$

As $v_1 \sin i = v_2 \sin r$

$$\therefore \quad \frac{v_2}{v_1} = \frac{\sin i}{\sin r} = \mu$$

As $\mu > 1$ $\therefore$ $v_2 > v_1$

i.e., light should travel faster in water than in air. This prediction of Newton's theory is opposite to the experimental result.

Huygens wave theory predicts that $v_2 < v_1$, which is consistent with experiment.

10.13. *You have learnt in the text how Huygens' principle leads to the laws of reflection and refraction. Use the same principle to deduce directly that a point object placed in front of a plane mirror produces a virtual image whose distance from the mirror is equal to the object distance from the mirror.*

Sol. Let O be a point object in front of plane mirror XY at a normal distance OP from it. Spherical wavefront starts from it. A part RPQ of the wavefront touches the plane mirror at P. Whereas disturbance from R and Q continues moving forward along normals (rays) OR and OQ, that from P reflects back. When disturbances from R and Q reach the mirror at A and C respectively, that from P reaches B'. This gives rise to reflected spherical wavefront $AB'C$. ABC is the virtual position (position in absence of mirror) of the wavefront.

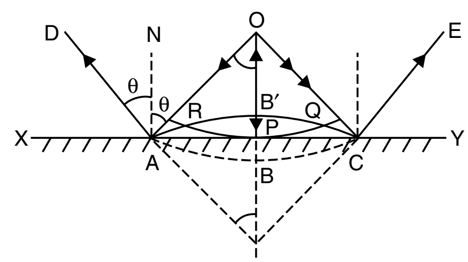


Fig. 10.33

The reflected wavefront $AB'C$, appears to start from I. I becomes virtual image for O as real point object.

Draw AN normal to XY, hence parallel to OP.

Now, OA is incident ray (being normal to incident wavefront ABC) and AD is reflected ray (being normal to reflected wavefront $AB'C$).

Hence, $\angle OAN = \angle DAN = \theta$ $[i = r]$

But $\angle OAN$ = alternate $\angle AOP$

and $\angle DAN$ = corresponding $\angle AIP$

$\angle AOP = \angle AIP$

Now, in ΔAIP and ΔAOP

$\angle AIP = \angle AOP$ (each θ)

$\angle API = \angle APO = 90°$ (each 90°)

AP is common to both

Δ_s become congruent

Hence, $PI = PQ$

i.e., normal distance of image from the mirror = normal distance of object from the mirror.

Thus, virtual image is formed as much behind the mirror as the object in front of it.

10.14. *Let us list some of the factors which could possibly influence the speed of wave propagation: (i) nature of source, (ii) direction of propagation, (iii) motion of source and/or observer, (iv) wave- length, (v) intensity of the wave.*

On which of these factors, if any, does (a) the speed of light in vacuum,

(b) the speed of light in a medium (say, the glass or water) depend?

Sol. (*a*) Speed of light in vacuum is an absolute constant, according to Einstein's theory of relativity. It does not depend upon any of the factors listed above or any other factor.

(*b*) The speed of light in a medium like water or glass

(*i*) does not depend upon the nature of the source.

(*ii*) does not depend upon the direction of propagation, when the medium is isotropic.

(*iii*) does not depend upon motion of the source w.r.t. the medium, but depends on motion of the observer relative to the medium.

(*iv*) depends on wavelength of light, being lesser for shorter wavelength and vice-versa.

(*v*) does not depend upon intensity of light.

10.15. *For sound waves, the Doppler formula for frequency shift differs slightly between the two situations:*

(i) source at rest; observer moving, and

(ii) source moving; observer at rest.

The exact Doppler formulas for the case of light wave in vacuum, are however, strictly identical for these situations. Explain why this should be so. Would you expect the formulas to be strictly identical for the two situations in case of light travelling in a medium?

Sol. Sound waves require a medium for propagation. Thus, even though the situations (*i*) and (*ii*) may correspond to the same relative motion (between the source and the observer), they are not identical physically since the motion of the observer relative to the medium is different in the two situations. Therefore, we can not expect Doppler formulas for sound to be identical for (*i*) and (*ii*). For light waves in vacuum, there is clearly nothing to distinguish between (*i*) and (*ii*). Here only the relative motion between the source and the observer counts. The relativistic Doppler formula is the same for (*i*) and (*ii*). For light propagation in a medium, once again like for sound waves, the two situations are not identical and we should expect the Doppler formulas for this case to be different for the two situations (*i*) and (*ii*).

10.16. *In double-slit experiment using light of wavelength 600 nm, the angular width of a fringe formed on a distant screen is 0.1°. What is the spacing between the two slits?*

Sol. Angular fringe width, $\boxed{\beta_\theta = \frac{\lambda}{d}}$

$$\Rightarrow \qquad d = \frac{\lambda}{\beta_\theta}$$

Now,

$$\lambda = 600 \times 10^{-9}\text{m}, \ \beta_\theta = 0.1° = \frac{0.1 \times \pi}{180} \text{ radian}$$

$$= \frac{0.1 \times 3.14}{180} \text{ radian}$$

$$\therefore \qquad d = \frac{600 \times 10^{-9} \times 180}{0.1 \times 3.14} \text{m} = 3.44 \times 10^{-4} \text{ m.}$$

10.17. *Answer the following questions:*

(a) In a single slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band.

(b) In what way is diffraction from each slit related to interference pattern in a double-slit experiment?

(c) *When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the shadow of the obstacle. Explain why?*

(d) *Two students are separated by a 7 m partition wall in a room 10 m high. If both light and sound waves can bend around obstacles, how is it that the students are unable to see each other even though they can converse easily.*

(e) *Ray optics is based on the assumption that light travels in a straight line. Diffraction effects (observed when light propagates through small apertures/slits or around small obstacles) disprove this assumption. Yet the ray optics assumption is so commonly used in understanding location and several other properties of images in optical instruments. What is the justification?*

Sol. (*a*) Width of central diffraction band = $2D\frac{\lambda}{d}$, so on doubling the width of the slit, the size of the central diffraction band reduces to half value. But the light amplitude becomes double, which increases the intensity four fold.

(*b*) The intensity of interference fringes in Young's double-slit experiment is modified by the diffraction pattern of each slit.

(*c*) Wave diffracted from the edge of the circular obstacle interface constructively at the centre of the shadow producing a bright spot.

(*d*) For diffraction or bending of waves by obstacles or apertures by a large angle, the size of the latter should be comparable to wavelength. If the size of the obstacle/ aperture is much too large compared to wavelength, diffraction is by a small angle. Here the size of partition wall is of the order of a few meters. The wavelength of light is about 5×10^{-7} m, while sound wave of say 1kHz have wavelength of about 0.3 m. Thus, sound waves can bend around the partition while light waves cannot.

(*e*) Typical sizes of the apertures involved in ordinary optical instruments are much large than the wavelength of light. Consequently, the diffraction effects of light are negligibly small in these instruments. Hence, the assumption that light travels in straight lines can be safely used in the optical instruments.

10.18. *Two towers on top of two hills are 40 km apart. The line joining them passes 50 m above a hill halfway between the towers. What is the longest wavelength of radio waves, which can be sent between the towers without appreciable diffraction effects?*

Sol. Size of aperture, $a = 50$ m

Distance of aperture from tower, Z_F

$$= \frac{40}{2} = 20 \text{ km} = 20 \times 10^3 \text{ m.}$$

Fresnel distance, $$\boxed{Z_F = \frac{a^2}{\lambda}}$$

$$\Rightarrow \qquad \lambda = \frac{a^2}{Z_F} = \frac{(50)^2}{20 \times 10^3}$$

or, $$\lambda = 125 \times 10^{-3} \text{ m} = 12.5 \text{ cm.}$$

10.19. *A parallel beam of light of wavelength 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1 m away. It is observed that the first minimum is at a distance of 2.5 mm from the centre of the screen. Find the width of the slit.*

Sol. Given, $D = 1$ m, $n = 1$

$x = 2.5 \text{ mm} = 2.5 \times 10^{-3}$ m

$\lambda = 500 \text{ nm} = 500 \times 10^{-9} \text{ m} = 5 \times 10^{-7}$ m

Using formula,

$$\boxed{x = n\frac{\lambda D}{d}}$$

$\Rightarrow$ $$d = \frac{n\lambda D}{x}$$

or, $$d = \frac{1\times5\times10^{-7}\times1}{2.5\times10^{-3}} = 2 \times 10^{-4} \text{ m} = 0.2 \text{ mm.}$$

10.20. *Answer the following questions:*

(a) When a low flying aircraft passes overhead, we sometimes notice a slight shaking of the picture on our TV screen. Suggest a possible explanation.

(b) As you have learnt in the text, the principle of linear superposition of wave displacement is basic to understanding intensity distributions in diffraction and interference patterns. What is the justification of this principle?

Sol. (*a*) A low flying aircraft reflects the TV signal. The slight shaking on the TV screen may be due to interference between the direct signal and the reflected signal.

(*b*) Superposition principle follows from the linear character of the differential equation governing wavemotion. If y_1 and y_2 are solutions of the wave equation, so is any linear combination of y_1 and y_2. When amplitudes are large (*e.g.*, high intensity laser beams) and non-linear effects are important, the situation is for more complicated.

10.21. *In deriving the single slit diffraction pattern, it was stated that the intensity is zero at angles of nλ/a. Justify this by suitably dividing the slit to bring out the cancellation.*

Sol. Let the slit width a be dividing into n equal parts of width a' so that

$$a' = \frac{a}{n}$$

or, $$a = na'$$

Then angle, $$\boxed{\theta = \frac{n\lambda}{a}} = \frac{n\lambda}{na'}$$

or, $$\theta = \frac{\lambda}{a'}$$

At this angle, each slit part will make first diffraction minimum. Hence, resultant intensity to all slits will be zero in that direction.

MORE QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *How would the angular separation of interference fringes in Young's double slit experiment change when the distance between the slits and screen is halved?*

Ans. As, $$\boxed{\omega = \frac{\lambda D}{d}}$$

$\therefore$ $$\omega_1 = \frac{\lambda.D/2}{d} = \frac{1}{2}\frac{\lambda D}{d}$$

$\Rightarrow$ $$\omega_1 = \frac{1}{2}\beta.$$

Q. 2. *Unpolarised light is incident on a plane surface of glass of refractive index μ at angle i. If the refracted light gets totally polarized, write the relation between the angle i and refractive index μ.*

Ans. $$\boxed{\mu = \tan i_p}$$

Q. 3. *What is the value of refractive index of a medium of polarizing angle 60°?*

Ans. $$\boxed{\mu = \tan i_p}$$

⇒ $$\mu = \tan 60°$$

or, $$\mu = \sqrt{3}.$$

Q. 4. *In a single slit diffraction experiment, the width of the slit is halved. How does it affect the size and intensity of the central maximum?*

Ans. Since, size of central maximum $\propto \frac{\lambda}{a}$

Thus, size becomes double and intensity reduces to $\frac{1}{4}th$.

Q. 5. *If the angle between the pass axis of polarizer and the analyser is 45°, write the ratio of the intensities of original light and the transmitted light after passing through the analyser.*

Ans. Given, $$\theta = 45°$$

Using formula,

$$\boxed{I = I_0 \cos^2\theta}$$

⇒ $$I = I_0 (\cos 45°)^2$$

or, $$I = I_0\left(\frac{1}{\sqrt{2}}\right)^2$$

or, $$\frac{I}{I_0} = \frac{1}{2}$$

⇒ $$I : I_0 = 1 : 2.$$

Q. 6. *How does the angular separation of interference fringes change, in Young's experiment, if the distance between the slits is increased?*

Ans. As, $$\boxed{\omega = \frac{\lambda D}{d}}$$

When separation between slits (d) is increased, then fringe width 'ω' is decreased.

Q. 7. *How does the fringe width of interference fringes change, when whole apparatus of Young's experiment is kept in a liquid of refractive index 1.3?*

Ans. The fringe width in a Young's double-slit experiment is given by

$$\boxed{\omega = \frac{\lambda D}{d}}$$

The fringe width is not affected if the whole apparatus is placed in a liquid of refractive index 1.3.

Q. 8. *A ray of light falls an a transparent slab of μ = 1.732. If reflected and refracted rays are mutually perpendicular, what is the angle of incidence?*

Ans. Using formula,

$$\boxed{\tan i_p = \mu}$$

$\Rightarrow$ $\tan i_p = 1.732$

or, $i_p = \tan^{-1}(1.732) = \tan^{-1}\left(\sqrt{3}\right)$

or $i_p = 60°$.

Q. 9. *How does the resolving power of telescope change when the aperture of the objective is increased?*

Ans. The resolving power of telescope increases on increasing the aperture of objective lens.

Q. 10. *The monochromatic source of light in Young's double-slit experiment is replaced by another monochromatic source of shorter wavelength. What will be the effect?*

Ans. Both the fringe width and the angular separation decrease as $\boxed{\omega = \dfrac{D\lambda}{d}}$.

Q. 11. *State the path difference between two waves for destructive interference.*

Ans. Path difference, $\boxed{\Delta = (2n + 1)\lambda/2}$

where, $n = 0, 1, 2$.................

Q. 12. *Why cannot we obtain interference using two independent sources of light?*

Ans. This is because two independent sources of light cannot be coherent, as their relative phases are changing randomly.

Q. 13. *Why bubbles of colourless soap solution appear coloured in sunlight?*

Ans. This is due to interference of white light from the thin film of soap bubbles.

Q. 14. *What will be the effect on the fringes, if Young's double-slit experiment set up is immersed in water?*

Ans. The fringes become **narrower**.

Q. 15. *Is the speed of light in glass independent of the colour of light?*

Ans. No. The refractive index and hence the speed of light in a medium depends on the wavelength.

Q. 16. *The phase difference between two waves reaching a point is $\pi/2$. What is the resultant amplitude, if the individual amplitudes are 3 mm and 4 mm?*

Ans.
$$\boxed{R = \sqrt{a^2 + b^2 + 2ab\cos\pi/2}}$$

$\Rightarrow$
$$R = \sqrt{3^2 + 4^2 + 2 \times 3 \times 4 \times 0} \qquad (\cos\pi/2 = 0)$$
$$= 5 \text{ mm.}$$

Q. 17. *What is the relation of a wavefront with a ray of light?*

Ans. The rays of light are always normal to the wavefront.

Q. 18. *What is the ratio of slit widths when amplitudes of light waves from them have a ratio $\sqrt{2}:1$.*

Ans.
$$\frac{\omega_1}{\omega_2} = \frac{I_1}{I_2} = \frac{a^2}{b^2}$$

$\Rightarrow$
$$\frac{\omega_1}{\omega_2} = \left(\frac{\sqrt{2}}{1}\right)^2 = \frac{2}{1}$$

$\Rightarrow$
$$\omega_1 : \omega_2 = 2 : 1.$$

Q. 19. *What is a polaroid?*

Ans. It is a thin and large sheet made of crystalline polarising material. It produces plane polarised beam of light.

Q. 20. *Which three phenomena establish the wave nature of light?*

Ans. Interference, diffraction and polarisation of light.

Q. 21. *How does the resolving power of microscope change on (i) decreasing wavelength of light, (ii) decreasing diameter of objective lens.*

Ans. (*i*) increases, (*ii*) decreases.

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *Monochromatic light from a narrow slit illuminates two narrow slits 0.3 mm apart, producing an interference pattern with bright fringes 1.5 mm apart on a screen 75 cm away. Find the wavelength of the light. How will the fringe width be altered if (a) the distance of the screen is doubled and (b) the separation between the slits is doubled?*

Ans. Here,

$$d = 0.3 \text{ mm} = 0.3 \times 10^{-3} \text{ m}$$

$$\beta = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$$

$$D = 7.5 \text{ cm} = 0.75 \text{ m}$$

Wavelength λ is given by

$$\boxed{\beta = \frac{\lambda D}{d}}$$

$$\lambda = \frac{\beta d}{D}$$

$$\Rightarrow \quad \lambda = \frac{1.5\times10^{-3}\times0.3\times10^{-3}}{0.75}$$

$$= 6000 \times 10^{-10} \text{ m} = 6000\text{Å}$$

(*a*) $\beta = \dfrac{\lambda D}{d}$

When D is doubled, fringe width β is also doubled.

$$\beta = 1.5 \times 2 = 3.0 \text{ mm}$$

(*b*) When d is doubled, β is reduced to half *i.e.,*

$$\beta = \frac{1.5}{2} = 0.75 \text{ mm.}$$

Q. 2. *In Young's double-slit experiment, monochromatic light of wavelength 600 m illuminates the pair of slits and produces an interference pattern in which two consecutive bright fringes are separated by 10 mm. Another source of monochromatic light produces the interference pattern in which the two consecutive bright fringes are separated by 8 mm. Find the wavelength of light from the second source.*

What is the effect on the interference fringes if the monochromatic source is replaced by a source of white light?

Ans. Here,

$$\lambda_1 = 600 \text{ nm} = 600 \times 10^{-9} \text{ m}$$

$$\beta_1 = 10 \text{ mm} = 10 \times 10^{-3} \text{ m}$$

$$\beta_2 = 8 \text{ mm} = 8 \times 10^{-3} \text{ m}$$

Let d be the slit width and D be the distance between slit and screen, then

$$\boxed{\beta_1 = \frac{\lambda_1 D}{d}} \text{ and } \beta_2 = \frac{\lambda_2 D}{d}$$

$$\therefore \quad \frac{\beta_1}{\beta_2} = \frac{\lambda_1 D}{d} \Big/ \frac{\lambda_2 D}{d}$$

$$\Rightarrow \quad \frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2}$$

$$\Rightarrow \quad \frac{10\times10^{-3}}{8\times10^{-3}} = \frac{600\times10^{-9}}{\lambda_2}$$

$$\Rightarrow \quad \frac{10}{8} = \frac{600\times10^{-9}}{\lambda_2}$$

$$\text{or,} \quad \lambda_2 = \frac{8\times600\times10^{-9}}{10} = 480 \times 10^{-9} \text{ m}$$

$$= 480 \text{ nm.}$$

If monochromatic source is replaced by white light. We will not fringes on screen as the white light slits will not act as coherent sources of light.

Q. 3. *In a Young's double-slit experiment, the two slits are kept 2 mm apart and the screen is positioned 140 cm away from the plane of the slits. The slits are illuminated with light of wavelength 600 nm. Find the distance of the third bright fringe, from the central maximum, in the interference pattern obtained on the screen.*

If the wavelength of the incident light were changed to 480 nm, find out the shift in the position of third bright fringe from the central maximum.

Ans. Given, $d = 2 \text{ mm} = 2 \times 10^{-3}$ m

$D = 140 \text{ cm} = 1.40$ m

$\lambda = 600 \text{ nm} = 600 \times 10^{-9} = 6 \times 10^{-7}$

Position of bright fringes is given by

$$x_n = n\lambda\frac{D}{d}$$

$\therefore$ Distance of the third bright fringe is

$$x_3 = 3\lambda\frac{D}{d}$$

$$\Rightarrow \quad x_3 = 3 \times 6 \times 10^{-7} \times \frac{1.40}{2\times10^{-3}}$$

$$= 12.6 \times 10^{-4} = 1.26 \times 10^{-3} \text{ m}$$

$$= 1.26 \text{ mm}$$

For $\lambda = 480 \text{ nm} = 480 \times 10^{-9} = 4.8 \times 10^{-7}$m

Distance of the third bright fringe is

$$x_3 = 3\lambda\frac{D}{d}$$

$$= 3 \times 4.8 \times 10^{-7} \times \frac{1.40}{2\times10^{-3}}$$

$$= 10.08 \times 10^{-4} = 1.008 \times 10^{-3} \text{ m}$$

$$= 1.01 \times 10^{-3} \text{ m} = 1.01 \text{ mm.}$$

$\therefore$ Shift in the position of third bright fringe

$$= 1.26 - 1.01 = 0.25 \text{ mm.}$$

Q. 4. *When one of the slits in Young's experiment is covered with a transparent sheet of thickness 3.6 × 10–3 cm the central fringe shifts to a position originally occupied by the 30th bright fringe. If λ = 6000Å, find the refractive index of the sheet.*

Ans. The position of the 30th bright fringe is given by

$$x_n = n\frac{\lambda D}{d}$$

$$x_{30} = 30\frac{\lambda D}{d}$$

Hence the shift of the central fringe is

$$x_0 = 30\frac{\lambda D}{d}$$

But $$x_0 = \frac{D}{d}(\mu-1)t$$

$\therefore$ $$30\frac{\lambda D}{d} = \frac{D}{d}(\mu-1)t$$

$\Rightarrow$ $$(\mu - 1) = \frac{30\lambda}{t} = \frac{30\times(6000\times10^{-10})}{(3.6\times10^{-5})} = 0.5$$

or, $$\mu = 1.5.$$

Q. 5. *What are coherent sources? How does the width of interference fringes in Young's double-slit experiment change when*

(a) the distance between the slits and screen is decreased?

(b) frequency of the source is increased?

Justify your answer in each case.

Ans. Two sources which produce waves of same frequency and the phase difference of produced waves does not change with time then the sources are said to be coherent sources.

$$\beta = \frac{D\lambda}{d} = \frac{D}{d}.\frac{c}{\nu} \qquad [\because c = \nu\lambda]$$

(*a*) The fringe width β decreases if the distance *D* between the slits and screen is decreased. Since $\beta \propto D$.

(*b*) The fringe width β decreased if the frequency ν of the source is increased.

since $$\beta \propto \frac{1}{\nu}.$$

Q. 6. *(a) In a single slit diffraction experiment, a slit of width 'd' is illuminated by red light of wavelength 650 nm. For what value of 'd' will*

(i) the first minimum fall at an angle of diffraction of 30°, and

(ii) the first maximum fall at an angle of diffraction of 30°?

(b) Why does the intensity of the secondary maximum become less as compared to the central maximum?

Ans. (*a*) (*i*) For first minimum fall

$$\lambda = d\sin\theta \qquad n\lambda = d\sin\theta$$

$\Rightarrow$ $$650 = d\sin 30°$$

$$\Rightarrow \quad 650 = d \times \frac{1}{2}$$

$$\text{or,} \quad d = 1300 \text{ nm.}$$

(*ii*) For first maximum fall

$$3\lambda = 2d \sin\theta \qquad \boxed{(2n+1)\frac{\lambda}{2} = d\sin\theta}$$

$$\Rightarrow \quad 3 \times 650 = 2d \sin\theta = 2d \sin 30°$$

$$\text{or,} \quad 1950 = 2d \times \frac{1}{2}$$

$$\text{or,} \quad d = 1950 \text{ nm.}$$

(*b*) As the width of a secondary maximum or the central maximum is directly proportional to λ. Therefore, greater the wavelength of the light used, greater is the width of the maximum.

Q. 7. *Two sources of intensity I and 4I are used in an interference experiment. Find the intensity at a point where the waves from the two sources superimpose with a phase difference of (i) zero (ii) π/2 (iii) π.*

Ans. In case of interference,

$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi}$$

(*i*) As $\phi = 0, \cos\phi = 1$

$$\therefore \quad I = 4I + I + 2\sqrt{4I \times I} \times 1 = 9I$$

(*ii*) As $\phi = \pi/2, \cos\phi = 0$

$$\therefore \quad I = 4I + I + 2\sqrt{4I \times I} \times 0 = 5I$$

(*iii*) As $\phi = \pi, \cos\phi = -1$

$$\therefore \quad I = 4I + I + 2\sqrt{4I \times I} \times (-1) = \text{I.}$$

Q. 8. (*a*) *A ray of light is incident on a glass surface at an angle of 60°. If the reflected and refracted rays are perpendicular to each other, find the refractive index of glass.*

(*b*) *What is the value of the refractive index of a medium of polarizing angle 60°.*

Ans. (a) and (b); we know that if the reflected and refracted rays are perpendicular, the angle of incidence is equal to polarizing angle.

$$i_p = 60°$$

$$\therefore \quad \boxed{\mu = \tan i_p} \Rightarrow \mu = \tan 60°$$

$$\text{or,} \quad \mu = \sqrt{3}$$

$$\text{or,} \quad \mu = 1.732.$$

Q. 9. *A slit of width 3 mm is illuminated by light of λ= 600 nm at normal incidence. If the distance of the screen from the slit is 60 cm, calculate the distance between the first order minimum on both sides of central maximum.*

Ans. Given,

$$d = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$$

$$\lambda = 600 \text{ nm} = 6 \times 10^{-7} \text{ m}$$

$$D = 60 \text{ cm} = 0.60 \text{ m}$$

Distance of first order **minima** from central maximum

$$x_n = n\frac{\lambda D}{d}$$

$$x_1 = 1\times\frac{\lambda D}{d}$$

∴ Distance between first order minimum on both sides of the central maximum

$$2x_1 = \frac{2\lambda D}{d}$$

$$\Rightarrow \qquad 2x_1 = \frac{2\times 6\times 10^{-7}\times 0.6}{3\times 10^{-3}}$$

$$= 0.24\times 10^{-3} \text{ m or } 0.24 \text{ mm.}$$

Q. 10. *How is a wavefront defined? Using Huygen's construction draw a figure showing the propagation of a plane wave reflecting at the interface of the two media. Show that the angle of incidence is equal to the angle of reflection.*

Ans. *Huygen's principle*: (*i*) Every point on a given wavefront acts as a fresh source of secondary wavelets which travel in all directions with the speed of light.

(*ii*) The forward envelope of these secondary wavelets gives the new wavefront at any instant.

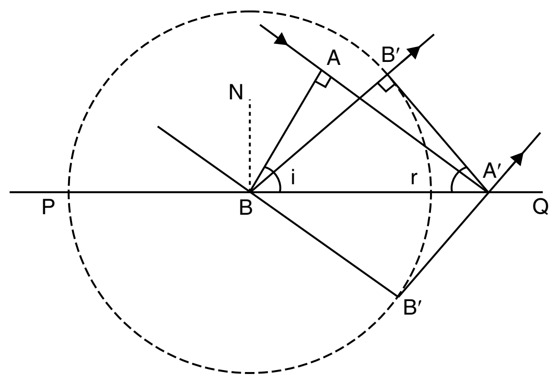


Fig. 10.34

Laws of reflection by Huygen's principle: Let *PQ* be reflecting surface. Let a plane wavefront *AB* moving through the medium (air) towards the surface *PQ* meet at the point *B*. Let *c* be the velocity of light and *t* be the time of *A* to reach *A*' then *AA*' = *ct*.

By the Huygen's principle, secondary wavelets starts from *B* and cover a distance *ct* in time *t* and reaches at *B*′.

To obtain new wavefront, draw circles with point *B* as centre and *ct* (*AA*′ = *BB*′) as radius. Draw a tangent *A*′*B*′ from the point A′.

Then *A*′*B*′ represents the reflected wavelets which travels at right angle. Therefore, incident wavefront *AB* and reflected wavefront *A*′*B*′ and normal lies in the same plane.

In Δ*ABA*′ and *B*′*BA*′

$AA' = BB' = ct$ [∵ $AA' = BB' = BD$ = radii of same circle]

$BA' = BA'$ [common]

$\angle BAA' = \Delta BB'A'$ [each 90°]

∴ $\Delta ABA' \cong \angle DBA'$ [by R.H.S]

$\angle ABA' = \angle B'A'B$ [C.P.C.T]

∴ incident angle *i* = reflected angle *r*

$$\angle i = \angle r$$

Q. 11. *Find the maximum intensity in case of interference of n identical waves each of intensity Io if the interference is*

(i) coherent (ii) incoherent

Ans. (*i*) In case of two coherent sources,

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

I_R will be maximum when

$$\cos\phi = 1 \text{ (maximum)}$$

$$\therefore \quad (I_{max})^{\infty} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$= \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

So for n identical waves each of intensity I_0,

$$(I_{\max})_{\infty} = \left(\sqrt{I_0} + \sqrt{I_0} \ldots\ldots\right)^2$$

$$= \left(n\sqrt{I_0}\right)^2 = n^2 I_0$$

(*ii*) In case of incoherent source,

$$I_R = I_1 + I_2 + \ldots\ldots\ldots$$

$$= I_0 + I_0 + \ldots\ldots\ldots = nI_0.$$

Q. 12. *In Young's double-slit experiment using monochromatic light the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1.6 and thickness 1.964 microns is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the plane of the slits and the screen is doubled. It is found that the distance between the successive maximum now is the same as the observed fringe shift upon the introduction of mica sheet. Calculate the wavelength of the light.*

Ans. Due to introduction of mica sheet, the shift on the screen

$$Y_0 = \frac{D}{d}(\mu - 1)t$$

Now, when the distance between the plane of slits and screen is changed from D to $2D$, fringe width will become,

$$\omega = \frac{2D}{d}(\lambda)$$

According to given problem,

$$\frac{D}{d}(\mu - 1) \quad t = \frac{2D\lambda}{d}$$

$$t = \frac{2\lambda}{(\mu - 1)}$$

or,

$$\lambda = \frac{(\mu - 1)t}{2}$$

$$= \frac{(1.6 - 1) \times 1.964 \times 10^{-6}}{2}$$

$$= 5892 \text{ Å}$$

Q. 13. *The maximum intensity in Young's double-slit experiment is I_0. Distance between the slits is $d = 5\lambda$, where λ is the wavelength of monochromatic light used in the experiment. What will be the intensity of light in front of one of the slits on a screen at a distance $D = 10d$?*

Ans. Path difference, $\boxed{\Delta x = \frac{yd}{D}}$

Here, $y = \frac{d}{2} = \frac{5\lambda}{2}$ (as $d = 5\lambda$) and $D = 10d = 50\lambda$

So, $\Delta x = \left(\frac{5\lambda}{2}\right)\left(\frac{5\lambda}{50\lambda}\right) = \frac{\lambda}{4}$

Corresponding phase difference will be

$$\boxed{\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x)} = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right)$$

$$= \pi/2$$

or, $\frac{\phi}{2} = \frac{\pi}{4}$

$\therefore$ $\boxed{I = I_0 \cos^2\left(\frac{\phi}{2}\right)}$

$$I = I_0 \cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}.$$

Q. 14. *State two conditions to obtain sustained interference of light.*

In Young's double-slit experiment, using light of wavelength 400 nm, interference fringes of width 'X' are obtained. The wavelength of light is increased to 600 nm and the separation between the slits is halved. If one wants the observed fringe width on the screen to be the same in the two cases, find the ratio of the distance between the screen and the plane of the interfering sources in the two arrangements.

Ans. Conditions for sustained interference:

(*i*) The two sources of light should emit light continuously.

(*ii*) The two waves must be in same phase or bear a constant phase difference.

Fringe width in first case,

$$\boxed{\beta = \frac{D\lambda}{d}} = \frac{D \times 400}{d}$$

Fringe width in second case is same.

$\therefore$ $\beta = \frac{D' \times 600}{d/2} = \frac{D' \times 1200}{d}$

$\therefore$ $\frac{D \times 400}{d} = \frac{D' \times 1200}{d}$

or, $\frac{D}{D'} = \frac{3}{1}$

$\Rightarrow$ $D : D' = 3 : 2.$

Q. 15. *Why is interference pattern not detected, when two coherent sources are far apart?*

In Young's experiment, the width of the fringes obtained with light of wavelength 6000 Å is 2.0 mm. Calculate the fringe width if the entire apparatus is immersed in a liquid medium of refractive index 1.33.

Ans. The fringe width will be very small and fringes will not be separately visible if the separation between the two coherent sources is large.

Fringe width in air, $\beta = \dfrac{D\lambda}{d}$

Fringe width in liquid

$$\beta' = \frac{D\lambda'}{d} = \frac{D\lambda}{d\mu}$$

$\Rightarrow \quad \beta' = \dfrac{\beta}{\mu}$

or, $\quad \beta' = \dfrac{2.0}{1.33} = 1.5$ mm.

Q. 16. *In Young's double-slit experiment* $\dfrac{d}{D} = 10^{-4}$ *(d = distance between slits, D = distance of screen from slits). At a point P on the screen resulting intensity is equal to the intensity due to individual slit* I_0*. Find the distance of point P from the central maximum. (λ = 6000Å).*

Ans. $I = 4I_0 \cos^2(\phi/2)$

$I_0 = 4I_0 \cos^2(\phi/2)$

$\therefore \quad \cos(\phi/2) = 1/2$

or, $\quad \phi/2 = \pi/3$

$\Rightarrow \quad \phi = \dfrac{2\pi}{3}$

$$\phi = \left(\frac{2\pi}{\lambda}\right).\Delta x$$

or, $\quad \dfrac{1}{3} = \left(\dfrac{1}{\lambda}\right)Y.\dfrac{d}{D} \quad \dfrac{2\pi}{3} = \dfrac{2\pi}{\lambda}\left(\dfrac{yd}{D}\right) \qquad \left(\because \Delta x = \dfrac{yd}{D}\right)$

$\therefore \quad Y = \dfrac{\lambda}{3\times d/D}$

$\Rightarrow \quad Y = \dfrac{6\times10^{-7}}{3\times10^{-4}} = 2\times10^{-3}$m

or, $\quad Y = 2$ mm.

Q. 17. *Two wavelengths of sodium light 590 nm, 596 nm are used, in turn, to study the diffraction taking place at a single slit of aperture 2 × 10⁻⁶ m. The distance between the slit and the screen is 1.5 m. Calculate the separation between the position of first maximum of the diffraction pattern obtained in the two cases.*

Ans. Separation between the first secondary maximum in the two cases is

$$x_2 - x_1 = 3/2\frac{D\lambda_2}{d} - \frac{3}{2}\frac{D\lambda_1}{d}$$

$\Rightarrow \quad x_2 - x_1 = \dfrac{3D}{2d}(\lambda_2 - \lambda_1)$

$\Rightarrow \quad x_2 - x_1 = \dfrac{3\times1.5}{2\times2\times10^{-6}}(596\times10^{-9} - 590\times10^{-9})$

$$\Rightarrow \qquad x_2 - x_1 = \frac{3\times1.5\times6\times10^{-3}}{4}$$

$$= 6.75 \times 10^{-3} \text{ m} = 6.75 \text{ mm.}$$

Q. 18. *Two coherent light sources of intensity ratio 25 : 4 are employed in an interference experiment what is the ratio of the intensities of the maxima and minima in the interference pattern?*

Ans. Let I_1 and I_2 be the intensities of the two coherent beams and A_1 and A_2 their respective amplitudes.

Now,

Intensity ratio, $\frac{I_1}{I_2} = \frac{25}{4}$, therefore $\boxed{\frac{A_1}{A_2} = \sqrt{\frac{I_1}{I_2}}}$

Amplitude ratio $\frac{A_1}{A_2} = \sqrt{\frac{25}{4}} = \frac{5}{2}$

i.e., $A_1 = 5$ units

and $A_2 = 2$ units

At maxima: $A_{max} = A_1 + A_2 = 7$ units

At minima: $A_{min} = A_1 - A_2 = 3$ units

Hence, $\boxed{\frac{I_{max}}{I_{min}} = \frac{A_{max}^2}{A_{min}^2}}$

$$\Rightarrow \qquad \frac{I_{max}}{I_{min}} = \frac{(7)^2}{(3)^2}$$

$$\Rightarrow \qquad I_{max} : I_{min} = 49 : 9.$$

III. LONG ANSWER TYPE QUESTIONS

Q. 1. *(a) What are polaroids? How are they used to demonstrate that (i) light waves are transverse in nature, (ii) if an unpolarized light wave is incident then light waves will get linearly polarized?*

(b) What is Brewster's angle? When an unpolarized light is incident on a plane glass surface, what should be the angle of incidence so that the reflected and refracted rays are perpendicular to each other?

Ans. (*a*) Polaroid is a thin and large sheet made of crystalline polarizing material. It produces plane polarized beam of light.

(*i*) Light waves are transverse in nature. The electric field associated with a propagation of light waves is always at right angles to the direction of propagation of the wave. If a light wave is incident on the polaroid, the electric vectors along the direction of the aligned molecules get absorbed.

(*ii*) If an unpolarized light wave is incident on a polaroid the the light wave get linearly polarized in a direction perpendicular to the plane of incidence since reflected light contains vibrations of electric vector perpendicular to the plane of incidence.

(*b*) *Brewster's angle:*

Ordinary light, when allowed to undergo refraction, the partially reflected light gets partially plane polarized. However, there is an angle of incidence, at which an ordinary light undergoes refraction as well as reflection (partial) and then the partially reflected ray is richly plane polarized. Such an angle is known as polarizing angle or Brewster's angle. It is denoted by i_p.

Light can be polarized by reflecting it from a transparent medium. The extent of polarization depends on the angle of incident. At a particular angle of incidence, called Brewster's angle, the reflected light is completely polarized as shown in the diagram below:

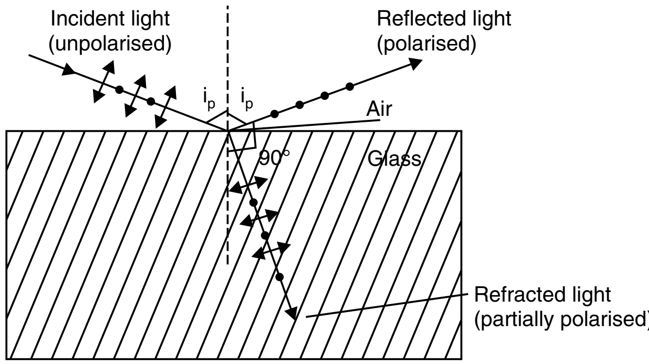


Fig. 10.35

Q. 2. *A narrow monochromatic beam of light of intensity I is incident on a glass plate A as shown in (Fig. a). Another identical glass plate B is kept close to A and parallel to it. Each glass plate reflects 25% of the light intensity incident on it and transmits the remaining. Find the ratio of the minimum and maximum intensities in the interference pattern formed by the two beams obtained after one reflection at each plate.*

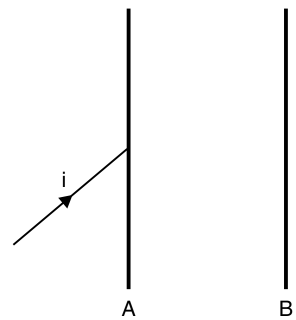


Fig. 10.36

Ans. A beam of light of intensity I is incident on plate A. Since the plate reflects 25% of I, the intensity of the reflected beam I (see Fig. *b*) is

$$I_1 = I \times \frac{25}{100} = \frac{I}{4}$$

The remaining intensity $3I/4$ falls on plate B which reflects 25% of the intensity incident on it. Hence, intensity of beam reflected from B is

$$\frac{3I}{4} \times \frac{25}{100} = \frac{3I}{16}$$

A beam of intensity $3I/16$ falls on plate A which transmits 75% of this intensity. Hence the intensity of beam 2 is

$$I_2 = \frac{3I}{16} \times \frac{75}{100} = \frac{9I}{64}$$

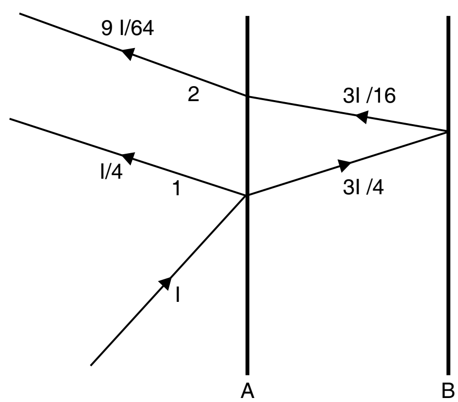


Fig. 10.37

Now, intensity $\propto$(amplitude)^2. If a_1 and a_2 are the amplitudes of beam 1 and 2 respectively, then

$$\boxed{I_1 = ka_1^2} \text{ and } I_2 = ka_2^2$$

or, $$\frac{a_1^2}{a_2^2} = \frac{I_1}{I_2} = \frac{I/4}{9I/64} = \frac{16}{9}$$

or, $$\frac{a_1}{a_2} = \frac{4}{3}$$

or, $$a_1 = 4 \text{ units and } a_2 = 3 \text{ units.}$$

The resultant **amplitude at maxima =** $a_1 + a_2$ and at **minima =** $a_1 - a_2$. Therefore, the ratio of the intensities at maxima and minima of the interference pattern is

$$\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \left(\frac{4+3}{4-3}\right)^2 = 49.$$

Q. 3. (*a*) *What are coherent sources of light? Two slits in Young's double-slit experiment are illuminated by two different sodium lamps emitting light of the same wavelength.*

Why is no interference pattern observed?

(*b*) *Obtain the condition for getting dark and bright fringes in Young's experiment. Hence write the expression for the fringe width.*

(*c*) *If s is the size of the source and its distance from the plane of the two slits, what should be the criteria for the interference fringes to be seen?*

Ans. (*a*) *Coherent sources:*

Sources emitting waves of same frequency or wavelength having either a zero or a constant phase difference are said to be coherent sources of light.

Two independent sources of light do not fulfil the requirement of constant phase difference. As the case of two different sodium lamps is given here. Hence such sources cannot be used for producing interference pattern.

(*b*) For bright fringes (maxima),

Path difference, $\frac{xd}{D} = n\lambda$

$\therefore \quad x = n\frac{\lambda D}{d}$

Where, n = 0, 1, 2, 3...........

For dark fringes (minima),

Path difference, $\frac{xd}{D} = (2n - 1)\lambda/2$

$\therefore \quad x = (2n-1)\frac{\lambda}{2}\frac{D}{d},$

Where n = 0, 1, 2, 3..........

The separation between the centre of two consecutive bright fringes is the width of a dark fringe.

$\therefore$ Fringe width, $\beta = x_n - x_{n-1}$

$$\beta = n\frac{\lambda D}{d} - (n-1)\frac{\lambda D}{d}$$

$$\therefore \quad \beta = \frac{\lambda D}{d}$$

(*c*) The condition for interference fringes is to be:

$$\frac{S}{d} < \lambda/d$$

Q. 4. *A ray of light incident normally on one of the faces of a right-angled isosceles prism is found to be totally reflected as shown.*

(a) What is the minimum value of the refractive index of the material of the prism?

(b) When the prism is immersed in water trace the path of the emergent ray for the same incident ray indicating the values of all the angles. (μ of water = 4/3).

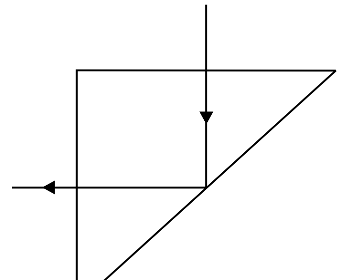


Fig. 10.38

Ans. (*a*) ABC is the section of the prism, B is a right angle. A and C are equal angles *i.e.*, $A = C = 45°$.

The ray PQ is normally incident on the face AB. Hence it is normally refracted and the ray QR strikes the face AC at an angle of incidence 45°. It is given that the ray does not undergo refraction but is totally reflected at the face AC. This gives a maximum value for the critical angle as 45°.

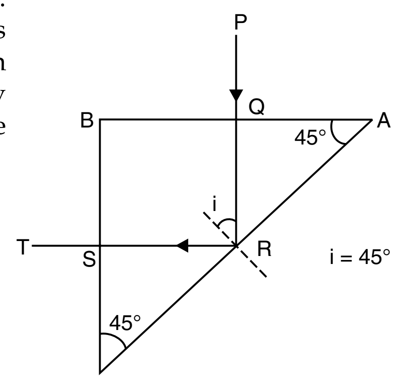


Fig. 10.39

$$\sin C = \sin 45° = \frac{1}{\sqrt{2}} \text{ in the limit}$$

Since, $$\mu = \frac{1}{\sin C} = \frac{1}{\sqrt{2}}$$

or, $$\mu = \frac{1}{\sin 45} \quad \mu_{min} = \sqrt{2}$$

The minimum value of refractive index = $\sqrt{2}$.

(*b*) When the prism is immersed in water the critical angle for the glass-water interface is given by

$$\sin C_1 = \frac{4/3}{\sqrt{2}} = \frac{4}{3\sqrt{2}}$$

$$C_1 = 70.53°$$

The angle of incidence at R continues to be 45° and since 45° < 70.53°.

There is refraction taking place now and the refracted ray is RS. The angle of refraction r is given by $\mu_g \sin i = \mu_w \sin r$

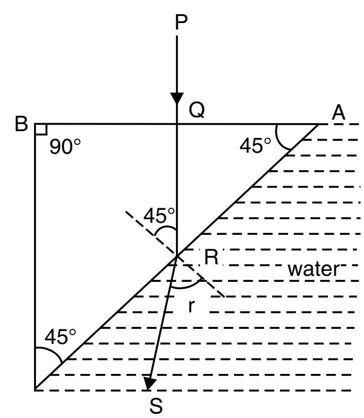


Fig. 10.40

$$_g\mu_\omega = \frac{\sin i}{\sin r}$$

$$\frac{\mu_\omega}{\mu_g} = \frac{\sin i}{\sin r}$$

$$\sqrt{2} \sin 45° = \frac{4}{3} \sin r$$

$$\sin r = \frac{3\sqrt{2}}{4} \sin 45° = \frac{3\sqrt{2}}{4} \times \frac{1}{\sqrt{2}} = \frac{3}{4}$$

$$r = \sin^{-1}\frac{3}{4} = 48°36'$$

∴ The angle of refraction in water = 48°36′.

Q. 5. *What is interference of light? Write two essential conditions for sustained interference pattern to be produced on the screen.*

Draw a graph showing the variation of intensity versus the position on the screen in Young's experiment when (a) both the slits are opened and (b) one of the slit is closed.

What is the effect on the interference pattern in Young's double-slit experiment when:

(i) screen is moved closer to the plane of slits?

(ii) separation between two slits is increased. Explain your answer in each case.

Ans. *Interference of light:* Phenomenon of redistribution of light energy in a medium on account of superposition of light waves from two coherent sources is called interference of light.

Conditions for sustained interference: The two essential conditions of sustained interference are as follows:

(*i*) The two sources of light should emit light continuously.

(*ii*) The light waves should be of same wavelength. (Monochromatic).

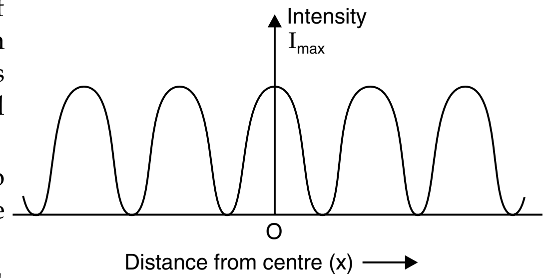


Fig. 10.41

When both the slits are open, we get interference pattern on the screen. Then the following intensity distribution curve is obtained.

When one of the slits is closed, diffraction pattern is obtained on the screen. The following intensity curve is obtained.

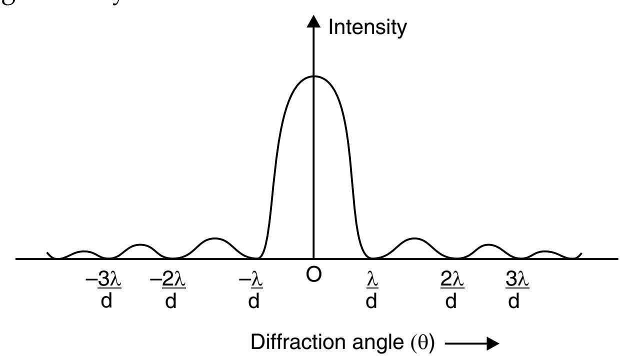


Fig. 10.42

Fringe width, $\boxed{\beta = \frac{D\lambda}{d}}$

(*i*) The distance D decreases, the fringe width β also decreases if screen is moved closer to the plane of the slits.

(*ii*) Fringe width β decreases if separation d between two slits is increased.

Q. 6. (*a*) *What is plane polarised light? Two polaroids are placed at 90° to each other and the transmitted intensity is zero. What happens when one more polaroid is placed between these two, bisecting the angle between them? How will the intensity of transmitted light vary on further rotating the third polaroid?*

(*b*) *If a light beam shows no intensity variation when transmitted through a polaroid which is rotated, does it mean that the light is unpolarized? Explain briefly.*

Ans. (*a*) In plane polarized light electric field E oscillates back and fourth in one plane perpendicular to the propagation of light. The intensity of polarised light through a polarizer at an angle of θ, is given by $\boxed{I = I_0 \cos^2\theta}$. If there are N polaroids placed in between the polarizer and analyzer with equal successive angular separation, then the overall intensity at the analyzer is give by, $\boxed{I = I_0\left(\cos\frac{\pi}{2N}\right)^{2N}}$

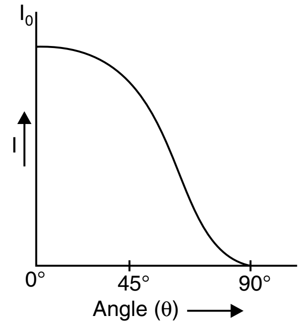


Fig. 10.43

Hence, the intensity of the polarised light coming out from the analyzer is

$$I = I_0\left(\cos\frac{\pi}{4}\right)^{2\times 2} = I_0(\cos 45°)^4 = \frac{I_0}{(\sqrt{2})^4} = \frac{I_0}{4}$$

According to law of Malus, when a beam of completely plane polarized light is incident on an analyzer, the resultant intensity of light (I_0) transmitted from the analyzer varies directly as the square of the cosine of the angle (θ) between planes of transmission of analyzer and polarizer. *i.e.*, $I = I_0\cos^2\theta$.

The required graph is given by figure (*a*).

(*b*) Yes, if a light beam shows no variation when transmitted through a polaroid which is rotated, this means that light is unpolarized. If an unpolarized light from an ordinary source passes through a polaroid sheet P_1, it is observed that its intensity is reduced by half. Rotating P_1 has no effect on the transmitted beam and transmitted intensity remains constant. Now, let an identical piece of polaroid P_2 be placed before P_1. The light from the lamp is reduced in intensity on passing through P_2 alone. In one position, the intensity transmitted by P_2 followed by P_1 is nearly zero. When turned by 90° from this position, P_1 transmits nearly the full intensity emerging from P_2.

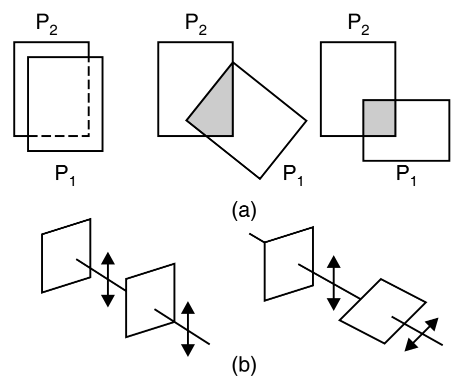


Fig. 10.44

Q. 7. *What is diffraction of light? Draw a graph showing the variation of intensity with angle in a single slit diffraction experiment. Write one feature which distinguishes the observed pattern from the double-slit interference pattern.*

How would the diffraction pattern of a single slit be affected when:

(i) the width of the slit is decreased?

(ii) the monochromatic source of light is replaced by a source of white light?

Ans. *Diffraction of light:* Phenomenon of bending of light around the corners of an obstacle or aperture is called diffraction.

The intensity distribution wave for diffraction is shown in the diagram below.

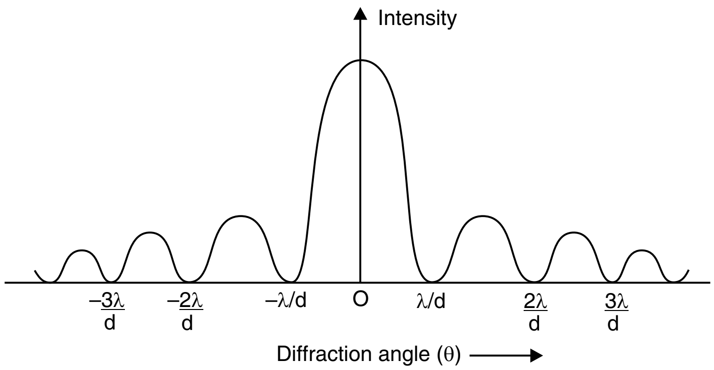


Fig. 10.45

In interference, by 2 slits all bright fringes are of same intensity. In diffraction, the intensity of bright fringes decreases with the increase in distance from the central bright fringe.

(*i*) The diffraction pattern becomes narrower if the width of the slit is decreased.

(*ii*) A coloured diffraction pattern is obtained if monochromatic source is replaced by white light source. The central band is white therefore, the red fringe is wider than the violet fringe etc.

QUESTIONS ON HIGH ORDER THINKING SKILLS (HOTS)

Q. 1. *Two convex lens of same focal length but their aperture and focal lengths 5 cm and 10 cm are used as object lens in two astronomical telescope.*

(a) What will be the ratio of their

(i) resolving power *(ii) magnifying power*

(b) Compare the intensity of images formed in these cases.

Ans. (*a*) (*i*) Ratio of resolving power

$$= \frac{D_1}{1.22\lambda} \bigg/ \frac{D_2}{1.22\lambda}$$

$$= \frac{D_1}{D_2} = \frac{5}{10} = \frac{1}{2}$$

(*ii*) Ratio of magnifying power,

$$= \frac{\left(\frac{f_0}{f_e}\right)_1}{\left(\frac{f_0}{f_e}\right)_2} = \frac{(f_0)_1}{(f_0)_2} = \frac{5}{10} = \frac{1}{2}$$

(*b*) Intensity of image is directly proportional to the size of aperture; hence it will also have a ratio $\frac{1}{2}$.

Q. 2. *The ratio of the intensities at minima to maxima in the interference pattern is 9 : 25. What will be the ratio of the widths of the two slits in the Young's double-slit experiment?*

Ans. Intensity is proportional to width of slit. So, amplitude a_1 and a_2 is proportional to the square root of the width of the slit.

$$\therefore \qquad \frac{a_1}{a_2} = \sqrt{\frac{\omega_1}{\omega_2}}$$

Here ω_1 and ω_2 represent the widths of the two slits.

Now, $$\frac{I_{min}}{I_{max}} = \frac{(a_1 - a_2)^2}{(a_1 + a_2)^2} = \frac{\left(1 - \frac{a_2}{a_1}\right)^2}{\left(1 + \frac{a_2}{a_1}\right)^2}$$

$$\Rightarrow \qquad \frac{9}{25} = \frac{\left(1 - \frac{a_2}{a_1}\right)^2}{\left(1 + \frac{a_2}{a_1}\right)^2} \text{ or } \frac{3}{5} = \frac{1 - \frac{a_2}{a_1}}{1 + \frac{a_2}{a_1}}$$

or, $$8\frac{a_2}{a_1} = 2$$

or, $$\frac{a_1}{a_2} = 4$$

Thus, $$\sqrt{\frac{\omega_1}{\omega_2}} = \frac{4}{1}$$

or, $$\frac{\omega_1}{\omega_2} = \frac{16}{1} \Rightarrow \omega_1 : \omega_2 = 16 : 1.$$

Q. 3. *In a double-slit experiment, two coherent sources have slightly different intensities I and (I + δI), such that δI << I, show that resultant intensity at maxima is near 4I, while that at minima is nearly (δI)²/4I.*

Ans. From $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos 0° = I + (I + \delta I) + 2\sqrt{I(I + \delta I)}$

As $\delta I << I$, therefore, $I_{max} = I + I + 2I = 4I$

Again from $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$$I_{min} = I + (I + \delta I) + 2\sqrt{I(I + \delta I)}\cos 180°$$

$$I_{min} = 2I + \delta I - 2I\left(1 + \frac{\delta I}{I}\right)^{1/2}$$

$$= 2I + \delta I - 2I\left[1 + \frac{1}{2}\frac{\delta I}{I} + \frac{\frac{1}{2}\left(\frac{1}{2} - 1\right)}{2}\left(\frac{\delta I}{I}\right)^2\right]$$

$$= 2I + \delta I - 2I - \delta I + \frac{1}{4}I\left(\frac{\delta I}{I}\right)^2$$

$$I_{min} = \frac{(\delta I)^2}{4I}.$$

Q. 4. *A double-slit is illuminated by light of wavelength 6000Å. The slits are 0.1 cm apart and the screen is placed 1 m away. Calculate (i) angular position of 10 th maximum in radian (ii) separation of two adjacent minima.*

Ans. (*i*) Angular position of *n*th maximum

$$\boxed{\theta_n = \frac{x_n}{D}} = \frac{nD\lambda}{dD} = \frac{n\lambda}{d} \qquad \boxed{\because \quad \chi_n = \frac{nD\lambda}{d}}$$

Angular position of 10th maximum,

$$\theta_{10} = \frac{10 \times 6000 \times 10^{-10}}{0.1 \times 10^{-2}}$$

$$= 6 \times 10^{-3} \text{ radian}$$

(*ii*) Separation between two adjacent minima

i.e., fringe width, $\omega = \frac{D\lambda}{d}$

$$\Rightarrow \qquad \omega = \frac{1 \times 6000 \times 10^{-10}}{10^{-3}}$$

$$= 6 \times 10^{-4} \text{ m} = 0.6 \text{ mm.}$$

Q. 5. *In a Young's experiment, the width of the fringes obtained with light of wavelength 6000Å is 2.0 mm. What will be the fringe width, if the entire apparatus is immersed in a liquid of refractive index 1.33?*

Ans. $\omega = 2 \times 10^{-3}$ m, $\lambda = 6000$ Å $= 6 \times 10^{-7}$ m

$$\boxed{\omega = \frac{D\lambda}{d}}$$

$$\Rightarrow \qquad \frac{D}{d} = \frac{\omega}{\lambda} = \frac{2 \times 10^{-3}}{6 \times 10^{-7}} = \frac{1}{3} \times 10^4$$

When the apparatus is immersed in liquid

Wavelength, $\boxed{\lambda' = \frac{\lambda}{\mu}} = \frac{6 \times 10^{-7}}{1.33}$ m

Fringe width, $\omega' = \frac{D}{d}\lambda'$

$$\Rightarrow \qquad \omega' = \frac{1}{3} \times 10^4 \times \frac{6 \times 10^{-7}}{1.33} \text{ m}$$

$$= 1.5 \times 10^{-3} \text{ m} = 1.5 \text{ mm.}$$

Q. 6. *A beam of light consisting of two wavelengths 6500 Å and 5200 Å, is used to obtain interference fringes in a Young's double-slit experiment.*

(i) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 6500 Å.

(ii) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

The distance between the slits is 2 mm and the distance between the plane of the slits and the screen is 120 cm.

Ans. (*i*) The distance of the *m*th bright fringe from the central maximum is given by

$$y_m = \frac{m\lambda D}{d}$$

$$\therefore \quad y_3 = \frac{3\lambda D}{d} = \frac{3\times(6500\times10^{-10})\times1.20}{2\times10^{-3}}$$

$$= 1.17 \times 10^{-3} \text{ m} = 1.17 \text{ mm}$$

(*ii*) Let the *n*th bright fringe of wavelength λ_n and the *m*th bright fringe of wavelength λ_m coincide at a distance *y* from the central maximum, then

$$y = \frac{m\lambda_m D}{d} = \frac{n\lambda_n D}{d}$$

or,

$$\frac{m}{n} = \frac{\lambda_n}{\lambda_m} = \frac{6500}{5200} = \frac{5}{4}$$

The least integral value of *m* and *n* which satisfy the above condition are

$$m = 5 \text{ and } n = 4$$

i.e., the 5th bright fringe of wavelength 5200 Å coincides with the 4th bright fringe of wavelength 6500 Å. The smallest value of *y* at which this happens is

$$y_{\min} = \frac{m\lambda_m D}{d} = \frac{5\times(5200\times10^{-10})\times1.20}{2\times10^{-3}}$$

$$= 1.56 \times 10^{-3} \text{ m} = 1.56 \text{ mm.}$$

Q. 7. *How would the angular separation of interference fringes in Young's double-slit experiment change when the distance of separation between slits and screen is doubled?*

Ans. Angular separation, $\theta = \frac{\beta}{D} = \frac{\lambda}{d}$. $\left[\because \text{ fringe width } \beta = \frac{\lambda D}{d}\right]$

It does not depend on *D*, the distance of separation between slits and screen. Therefore, θ remains unaffected.

Q. 8. *How is resolving power of a microscope affected when (i) wavelength of illuminating radiations is decreased (ii) the diameter of objective lens is decreased? Justify.*

Ans. For a microscope,

$$\text{Resolving power} = \frac{2\mu\sin\theta}{\lambda}$$

(*i*) When λ is decreased, resolving power increases.

(*ii*) When diameter of objective lens is decreased, θ decreases, sinθ decreases. Hence resolving power of microscope also decreases.

Q. 9. *In a two-slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from the slits. If the screen is moved by 5×10^{-2} m towards the slits, the change in fringe width is 3×10^{-5} m. If the distance between the slits is 10^{-3} m, calculate the wavelength of light used.*

Ans.

$$\beta = \frac{\lambda}{d}D$$

$$\Delta\beta = \frac{\lambda}{d}\Delta D$$

$\Rightarrow$ $$\lambda = d\frac{\Delta\beta}{\Delta D}$$

$\Rightarrow$ $$\lambda = 10^{-3}\frac{3\times10^{-5}}{5\times10^{-2}}\text{m} = 6 \times 10^{-7} \text{ m}$$

or, $$\lambda = 6 \times 10^{-7} \times 10^{10} \text{ Å} = 6000\text{Å}.$$

Q. 10. *In Young's double-slit experiment how many maximas can be obtained on a screen (including the central maximum) on both sides of the central fringe if λ = 2000 Å and d = 7000Å. Given that perpendicular distance of a screen from the mid-point of two slits is 3.5 cm.*

Ans. For maximum intensity on the screen

$$\boxed{d \sin\theta = n\lambda}$$

or, $$\sin\theta = \frac{n\lambda}{d} = \frac{n(2000)}{(7000)} = \frac{n}{3.5}$$

Since, $\sin\theta < 1 \therefore n = 0, 1, 2, 3$ only............

Thus, only seven maximas can be obtained on both sides of the screen.

Q. 11. *The intensity at the central maxima (0) in a Young's double slit set up is I_0. If the distance OP equals one third of the fringe width of the pattern, show that the intensity, at point P, would equal to $I_0/4$.*

Fig. 10.46

Ans. $$x = \frac{1}{3}\beta \text{ (given)}$$

$\Rightarrow$ $$x = \frac{\lambda D}{3d}$$

$\because$ $$\Delta_p = \frac{xd}{D} \quad \text{(Path difference)}$$

$\therefore$ $$\phi = \frac{2\pi}{\lambda}(\Delta P) \quad \text{(Phase difference)}$$

$$= 2\pi/3$$

But $$I = I_0 \cos^2\frac{\phi}{2}$$

$$= I_0 \cos^2\left(\frac{2\pi}{3}\right) = I_{0/4}$$

Q. 12. (*a*) *A plane wavefront approaches a plane surface separating two media. If medium one is (optically) denser and medium two is optically rarer, construct the refracted wavefront using Huygen's principle.*

(*b*) *Draw the shape of refracted wavefront when a plane wavefront is incident on (i) prism and (ii) convex mirror. Give a brief explanation for the construction.*

Ans. (*a*) Construction of refracted wavefront

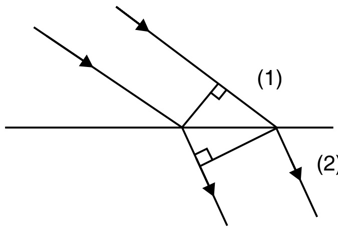


Fig. 10.47

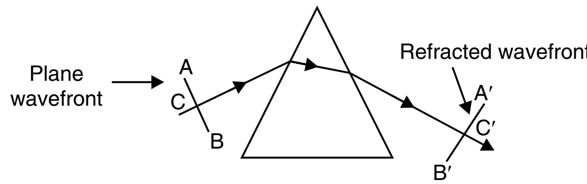


Fig. 10.48

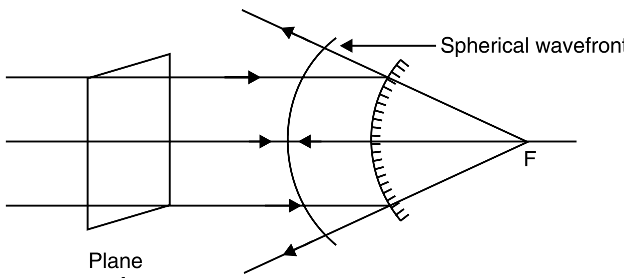


Fig. 10.49

Time taken by any disturbance to travel from incident wavefront to the refracted wavefront is same.

Q. 13. *How will the intensity of maxima and minima, in the Young's double experiment change, if one of the two slits is covered by a transparent paper which transmits only half of light intensity?*

Ans. The intensity of maxima decreases and the intensity of minima increases.

Q. 14. *In the double slit experiment, the pattern on the screen is actually a superposition of single slit diffraction from each slit and the double slit interference pattern in a double slit pattern. In what way is the diffraction from each slit related to the interference pattern in a double slit experiment? Explain.*

Ans. The pattern shows a broader diffraction peak in which there appear several fringes of smaller width due to the double slit interference.

The number of interference fringes depends upon the ratio of the distance between the two slits to the width of a slit.

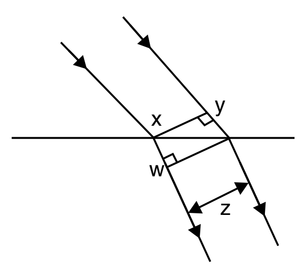


Fig. 10.50

Q. 15. *A plane wavefront of width x is incident on an air-water interface and the corresponding refracted wavefront has a width z as shown. Express the refractive index of air with respect to water in terms of dimension shown in Fig.*

Ans. Defrative index of air with respect to the water from the figure shown can expressed

as $$\mu_{\omega a} = \frac{\sin r}{\sin i} = \frac{\omega}{y}$$

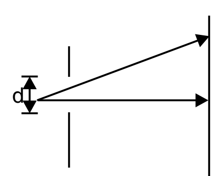


Fig. 10.51

Q. 16. *A slit of width 'd' is illuminated by white light. For what value of 'd' is the first minimum for red light of $\lambda = 650$ nm located at point P. For what value of the wavelength of light will the first diffraction maxima also fall at P?*

Ans. At first minimum, $n = 1$

$\therefore$ $$d \sin 30° = n\lambda$$

or $\frac{d}{2} = 1 \times 650$ nm

or $d = 1300$ nm

for first maxima to be at D

$$d \sin \theta = \frac{3}{2} \lambda'$$

or $$\lambda' = \frac{2d \sin \phi}{3} = \frac{2 \times 1300 \times \sin 30°}{3} \text{ nm}$$

$$= 433.3 \text{ nm.}$$

MULTIPLE CHOICE QUESTIONS

1. The resolving limit of healthy eye is about

(*a*) 1′ (*b*) 1″ (*c*) 1° (*d*) $\frac{1''}{60}$

2. Two waves having intensity in the ratio 25 : 4 produce interference. The ratio of the maximum to minimum intensity is

(*a*) 5 : 2 (*b*) 7 : 3 (*c*) 49 : 9 (*d*) 9 : 49

3. The penetration of light into the region of geometrical shadow is called

(*a*) Polarisation (*b*) Interference

(*c*) Diffraction (*d*) Refraction

4. Two waves $y_1 = A_1 \sin(\omega t - \beta_1)$ and $y_2 = A_2 \sin(\omega t - \beta_2)$ superimpose to form a resultant wave whose amplitude is

(*a*) $\sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos(\beta_1 - \beta_2)}$ (*b*) $\sqrt{A_1^2 + A_2^2 + 2A_1A_2 \sin(\beta_1 - \beta_2)}$

(*c*) $A_1 + A_2$ (*d*) $|A_1 + A_2|$

5. In a wave, the path difference corresponding to a phase difference of ϕ is

(*a*) $\frac{\lambda}{2\lambda}\phi$ (*b*) $\frac{\pi}{\lambda}\phi$

(*c*) $\frac{\lambda}{2\pi}\phi$ (*d*) $\frac{\lambda}{\pi}\phi$

6. Two waves have intensity ratio 25 : 4. What is the ratio of maximum to minimum intensity?

(*a*) $\frac{16}{25}$ (*b*) $\frac{25}{4}$ (*c*) $\frac{9}{49}$ (*d*) $\frac{49}{9}$

7. Newton gave the corpuscular theory on the basis of

(*a*) Newton's rings (*b*) Rectilinear motion

(*c*) Certain corpuscles (*d*) Wavefront

8. The resolving power of a telescope whose lens has a diameter of 1.22 m for a wavelength of 5000 Å is

(*a*) 2×10^5 (*b*) 2×10^6

(*c*) 2×10^2 (*d*) 2×10^4

9. The objective of an astronomical telescope has a large aperture to

(*a*) Reduce spherical aberration (*b*) Increase span of observation

(*c*) Have high resolution (*d*) Wave low dispertation

10. Wavelengths of light used in an optical instrument are λ = 4000 Å and λ = 5000 Å. The ratio of their respective resolving power is
   (*a*) 16 : 25   (*b*) 5 : 4   (*c*) 4 : 5   (*d*) 9 : 1
11. For which property are electrons used in an electron microscope?
   (*a*) Wave nature   (*b*) Negative charge   (*c*) Spin   (*d*) None of these
12. Band spectrum is also called
   (*a*) Molecular spectrum   (*b*) Atomic spectrum
   (*c*) Flash spectrum   (*d*) Line absorption spectrum
13. The angular resolution of a 10 cm diameter telescope at a wave length of 5000 Å is of the order of
   (*a*) 10^{-4} rad   (*b*) 10^{-6} rad   (*c*) 10^{6} rad   (*d*) 10^{-2} rad
14. In Young's double slit experiment, intensity at a point is (1/4) of the maximum intensity angular position of this point is
   (*a*) $\sin^{-1}(\lambda/d)$ (*b*) $\sin^{-1}(\lambda/2d)$ (*c*) $\sin^{-1}(\lambda/3d)$ (*d*) $\sin^{-1}(\lambda/4d)$
15. In case of linearly polarised light, the magnitude of electric field vector
   (*a*) does not change with time   (*b*) varies periodically with time
   (*c*) fringe width decreases   (*d*) fringe width remains same

Answers

1. (*a*)	**2.** (*c*)	**3.** (*c*)	**4.** (*a*)	**5.** (*c*)
6. (*d*)	**7.** (*b*)	**8.** (*b*)	**9.** (*c*)	**10.** (*b*)
11. (*a*)	**12.** (*a*)	**13.** (*b*)	**14.** (*c*)	**15.** (*b*)

TEST YOUR SKILLS

1. What are coherent sources ? Why are coherent sources required to produce interference of light? Give an example of interference of light in everyday life.
2. In young's double slit experiment; the two slits are 0.03 cm apart and screen is placed at a distance of 1.5 m away from the slits. The distance between the central bright fringe and fourth bright fringe is 1 cm. Calculate the wavelength of light used.
3. State the condition under which the phenomenon of diffraction of light takes place. Derive an expression for the width of central maxima due to diffraction of light at a single slit.
4. In Young's double slit experiment, three lights of blue, yellow and red colour are used successively. The fringe width will be maximum for which colour of light and why?
5. In the diffraction at a single slit experiment, how would the width and the intensity of central maximum change if (*i*) slit width is halved (*ii*) visible light of longer wavelength is used?
6. In single slit diffraction experiment, if the width of the slit is doubled, how does the (*i*) intensity of light (*ii*) width of central maximum change? Give reasons for your answer.
7. In Young's double slit experiment, the slits are seperated by 0.24 mm. The screen is 1.2 m away from the slits. The fringe width is 0.3 cm. Calculate the wavelength of light used in the experiment.
8. The light of wavelength 600 nm is incident normally on a slit of width 3 mm. Calculate the linear width of central maximum on a screen kept 3 m away from the slit.
9. (*i*) Give the ratio of velocities of light rays of wavelengths 40 Å and 8000 Å in vacuum
   (*ii*) The polarising, angle of a medium is 60°. What is the refractive index of the medium?
10. A ray of light goes from medium 1 to medium 2. Velocity of light in the two media are c_1 and c_2 respectively. For an angle of incidence θ in medium 1, the corresponding angle of refraction in medium 2 is $\theta/2$.

(*i*) Which of the two media is optically denser and why?
(*ii*) Establish the relationship between θ, c_1 and c_2.

11. If the angle between the axis of polariser and the analyser is 45°, write the ratio of the intensities of original light and transmitted light after passing through the analyser.

12. What type of wavefront will emerge from a (*i*) point source, and (*ii*) distant light source?

13. In a single slit diffraction experiment, when a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the shadow of the obstacle. Explain why?

14. A polarised light is incident on a plane surface of glass of refractive index μ at angle i, if the reflected light gets totally polarised, write the relation between angle i and the refractive index μ.

15. Draw a diagram to show refraction of a plane wavefront incident on a convex lens and hence draw the refracted wavefront.

16. At what angle of incident rays should a light beam strike a glass slab of refractive index $\sqrt{3}$, such that the reflected and the refracted rays are perpendicular to each other.

17. Differentiate between a rays and wavefront.

18. Define the term 'linearly polarised light'. When does the intensity of transmitted light become maximum, when polaroid sheet is rotated between two crossed polariods?

19. In Young's double slit experiment, monochromatic light of wavelength 630 nm illuminates the pair of slits and produces an interference pattern in which two consecutive bright fringes are seperated by 8.1 mm. Another source of monochromatic light produces the interference pattern in which the two consecutive bright fringes are seperated by 7.2 mm. Find the wavelength of light from the second source. What is the effect on interference fringes if the monochromatic source is replaced by a source of white light?

20. (*a*) In a single slit diffration experiment, a slit of width '*d*' is illuminated by red light of wavelength 650 nm. For what value of '*d*' will
(*i*) The first minimum fall at an angle of diffraction of 30°, and
(*ii*) The first maxima fall at an angle of diffraction of 30°.
(*b*) Why does the intensity of the secondray maximum become less as compared to the cenral maximum?

21. Explain both the objective and the eyepiece of a compound microscope must have short focal lengths?

22. Give reason for the following observations:
(*i*) The resultant intensity at any point on the screen varies between zero and four times the intensity due to one slit in Young's double slit experiment.
(*ii*) A few coloured fringes around a central white reason are observed on the screen, when the source of monochromatic light replaced by white light in Young's double slit experiment.
(*iii*) The intensity of transmitted light by a polaroid is half the intensity of light incident on it.

23. State the essential condition for the diffraction to take place.
A parallel beam of monochromatic light falls normally on a narrow slit and light coming out of the slit is obtained on the screen.
Derive an expression for the angular width of the central bright maxima obtained on the screen diffraction describes the limit of rag optics. Give a brief explaination of the statement.

24. How will the intensity of maxima and minima in the Young's double slit experiment change if the one of the two slits is covered by a transparent paper which transmites only half of the light intensity.

25. A light beam is incident on boundary between two transparent media. At a particular angle of incidence the reflected ray is perpendicular to the refracted ray. Obtain an expression for the angle of incidence. Does this angle depend upon the wavelength of light used?

❑❑❑