11 Dual Nature of Radiation and Matter

Facts that Matter

- Many discoveries and observations of Roentgen, J.J. Thomson, R.A. Millikan, etc., established that electrons are fundamental, universal constituent of matter. Electrons are negatively charged particles having charge to mass ratio

$$e/m = 1.76 \times 10^{11} \text{ C kg}^{-1}.$$

- Metals have free electrons which are responsible for their conductivity. However, the free electrons cannot normally escape out of the metal surface. If an electron attempts to come out of the metal, the metal surface acquires a positive charge and pulls the electron back to the metal. The free electron is thus held inside the metal surface by the attractive force of the ions. Consequently, the electron can come out of the metal surface only if it has got sufficient energy to overcome the attractive pull of the surface called *surface barrier.* A certain minimum amount of energy is required to be given to an electron to pull it out from the surface of the metal. *This minimum energy required by an electron to escape from the metal surface is called the work function* (ϕ).

• Electron Emission

The phenomenon of emission of electrons from the surface of a metal is called electron emission.

(*i*) *Thermionic Emission.* The emission of electrons from the surface of metal with the help of thermal energy is called thermionic emission.

(*ii*) *Field or Cold Cathode Emission.* It is the phenomenon of emission of electrons from the surface of metal under the application of a strong electric field.

(*iii*) *Photoelectric Emission.* Electrons emitted from a metal surface with the help of suitable electromagnetic radiations.

(*iv*) *Secondary Emission.* It is the phenomenon of emission of electrons from the surface of metal in large number when fast moving electrons (called primary electrons) strike the metal surface.

• Effect of Electric and Magnetic Field on the Motion of an Electron

(*a*) *Electric field.* The force F_E experienced by an electron of charge e in the electric field of intensity E is given by

$$F_E = eE$$

(*b*) *Magnetic field.* The force experienced by an electron 'e' in a magnetic field of strength B weber/m^2 is given by

$$F_B = Bev$$

where v is the velocity of electron moving in the field and the magnetic field is perpendicular to the direction of motion.

If the magnetic field is parallel to the direction of motion of electron, then, $F_B = 0$.

• Photoelectric Effect

Photoelectric effect is the phenomenon of emission of electrons from the surface of metals, when radiations of suitable frequency fall on them.

• Hertz's Observations

During his experiments on electromagnetic waves Hertz noticed that sparking across the detector enhanced when emitter plate was illuminated by ultraviolet light.

This was due to the free charged particles (*i.e.,* electrons) which escaped into the surrounding space from the metal surface on absorbing enough energy to overcome their force of attraction with positive ions in the material.

• Hallwachs' and Lenard's Observations

When *UV* (Ultraviolet) radiations were allowed to fall on the emitter plate of an evacuated glass tube enclosing two electrodes, current flows in the circuit. As soon as the *UV* radiations were stopped, the current flow also stopped. These observations indicate that when ultraviolet radiations fall on the emitter plate *C*, electrons are ejected from it, which are attracted towards the positive, collector plate *A* by the electric field. The electrons flow through the evacuated glass tube, resulting in the current flow. Thus, light falling on the surface of the emitter causes current in the external circuit. They studied the variation of photocurrent with collector plate potential, and with frequency and intensity of incident light.

Hallwachs connected a negatively charged zinc plate to an electroscope. He observed that the zinc plate lost its charge when it was illuminated by *UV* light. Positive charge on a positively charged zinc plate was found to be increased when it was illuminated by *UV* light. From these observations he concluded that negatively charged particles were emitted from the zinc plate under the action of ultraviolet light.

When *UV* light was allowed to fall on the emitter plate, no electrons were emitted at all when the frequency of the incident light was smaller than a certain minimum value, called the threshold frequency. This minimum frequency depends on the nature of the material of the emitter plate. It was found that certain metals like zinc, cadmium, magnesium, etc., responded only to ultraviolet light, having short wavelength, to cause electron emission from the surface. However, some alkali metals such as lithium, sodium, potassium, caesium and rubidium were sensitive even to visible light. All these photosensitive substances emit electrons when they are illuminated by light. After the discovery of electrons, these electrons were termed as photoelectrons. The phenomenon is called photoelectric effect.

• Experimental Study of Photoelectric Effect

A schematic view of the arrangement used for the experimental study of photoelectric effect is shown in Fig. 11.1.

It consists of an evacuated glass/quartz tube having a photosensitive plate *C* and another metal plate *A*. Monochromatic light from the source *S* of sufficiently short wavelength passes through the window *W* and falls on the plates *C*. A transparent quartz window is sealed on to the glass tube, which permits ultraviolet radiations to pass thought it and irradiate the photosensitive plate *C*. The electrons are emitted by the plate *C* and are collected by the plate *A*, by the electric field created by the battery. The battery maintains the potential difference between the plates *C* and *A*, that can be varied. The polarity of the plates *C* and *A* can be reversed by a commutator. Thus the plate *A* can be maintained at a desired positive or negative potential with respect to the plate *C*. When the plate *A* is positive with respect to plate *C,* electrons are attracted to it. The emission of electrons causes flow of electric current in the circuit called photo current.

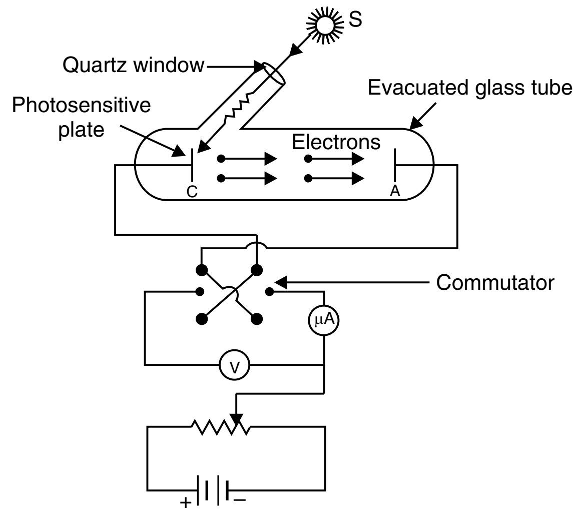

Fig. 11.1

In this experiment the following observations are made:

(*i*) Photo current increases linearly with increase in intensity of incident radiations keeping **frequency** of radiation and **potential difference** between plates *A* and *C* **constant** as shown in Fig. 11.2.

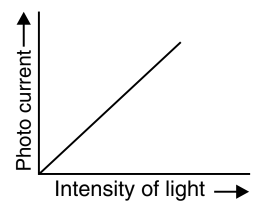

Fig. 11.2

(*ii*) When potential difference between plates *A* and *C* increases photo current increases up to the certain value and beyond this value becomes constant (saturated) for a **particular frequency** of incident radiation as shown in Fig. 11.3.

The minimum negative, (retarding) potential V_0 given to the plate *A* for which the photo current stops or becomes zero is called the *cut-off or stopping potential.*

For a given frequency of the incident radiation, the stopping potential is independent of its intensity.

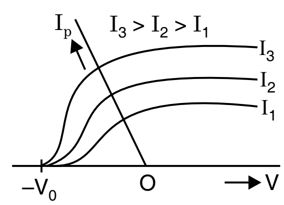

Fig. 11.3

(*iii*) For different frequencies of incident radiations if stopping potential increases, the photo current increases and gets saturated for the same value of photo current (I_p) as shown in Fig. 11.4.

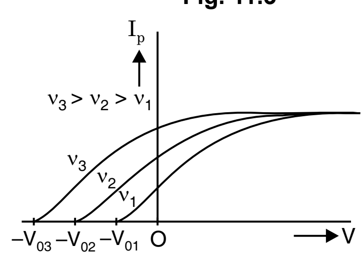

Fig. 11.4

(*iv*) Stopping potential increases with increase in the frequency of the incident radiation. This variation for different metals is shown in Fig. 11.5.

The graph shown in Fig. 11.5 shows that:

(*a*) The stopping potential V_0 varies linearly with the frequency of incident radiation for a given photosensitive material.

(*b*) There exists a certain minimum cut-off frequency ν_0 for which the stopping potential is zero.

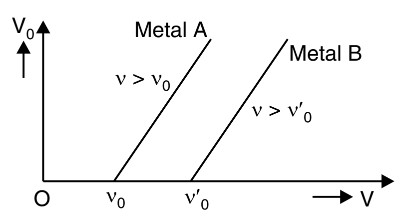

Fig. 11.5

These observations have the implications:

(*i*) The maximum kinetic energy of photoelectron varies linearly with the frequency of incident radiation, but is independent of its intensity.

(*ii*) For a frequency ν of incident radiation, lower than the cut-off frequency ν_0, no photoelectric emission is possible even if the intensity is large.

This minimum, cut-off frequency ν_0 is called *threshold frequency*. It is different for different materials. On the basis of these experimental observations the following facts can be concluded as the *laws of photoelectric effect*.

- For a given photosensitive material and frequency of incident radiations (above the threshold frequency), the photoelectric current is directly proportional to the intensity of incident light.
- For given photosensitive material and frequency of incident radiation, saturation current is found to be proportional to the intensity of incident radiation whereas the stopping potential is independent of its intensity.
- For a given photosensitive material, there exists a certain minimum cut-off frequency of the incident radiation, called the threshold frequency, below which no emission of photo electrons takes place, no matter how intensity of the incident light is. Above the threshold frequency, the stopping potential or equivalently the maximum kinetic energy of the emitted photoelectrons increases linearly with the frequency of the incident radiation, but is independent of its intensity.
- The photoelectric emission is an instantaneous process without any apparent time lag (~ 10^{-9} s or less), even when the incident radiation is made exceedingly dim.

• Wave Theory of Light

According to the wave picture of light, the free electrons over the surface of the metal (over which the beam of radiation falls) absorb the radiant energy continuously. The greater the intensity of radiation, the greater are the amplitude of electric and magnetic fields. Consequently, the greater the intensity, the greater should be the energy by each electron. In this picture, the maximum kinetic energy of the photoelectrons on the surface is then expected to increase with increase is intensity. Also, no matter what the frequency of radiation is, a sufficient intense beam of radiation (over sufficient time) should be able to impart enough energy to the electrons, so that they exceed the minimum energy needed to escape from the metal surface. A threshold frequency, therefore, should not exist. These expectations of the wave theory directly contradict the observations of photoelectric effect. The picture is unable to explain the most basic features of photoelectric emission.

• Einstein's Equation of Photoelectric Effect

Photoelectric effect does not takes place by continuous absorption of energy from radiation. Radiation energy is built up of discrete units—the so called *quanta of energy radiation*. Each quanta of radiant energy has energy $h\nu$ where h is Planck's constant and ν is the frequency of light.

If a quanta of radiant energy ($h\nu$) called photon falls on a metallic surface, this energy in totally imported to a single electron and some part of this energy is used by electron just to overcome the surface barrier (ϕ) and rest part remains in the form of kinetic energy with the electron. If energy of photon is less than the whole function of the electron, no electron will emitted out, thus for photoelectric emission a minimum amount of energy *i.e., threshold energy* equal at least equal to the work function in needed ($\phi = h\nu_0$, where ν_0 is threshold energy). Applying the conservation law of energy,

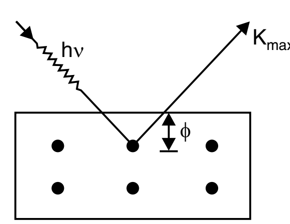

Fig. 11.6

$$h\nu = \phi + K_{max}$$

$\because$ ϕ minimum $\therefore$ kinetic energy of electron emitted will be maximum

$$K_{max} = h\nu - \phi$$

or

$$K_{max} = h\nu - h\nu_o \quad ...(i)$$

or

$$\frac{1}{2}m v_{max}^2 = h(\nu - \nu_0)$$

Also,

$$K_{max} = eV_0$$

$$\therefore \quad eV_0 = h(\nu - \nu_0) \quad ...(ii)$$

Using Eqns. (*i*) and (*ii*) the value of Planck's constant *h* and work function can be determined by the graph drawn between K_{max} or V_0 versus frequency as shown in Fig. 11.7.

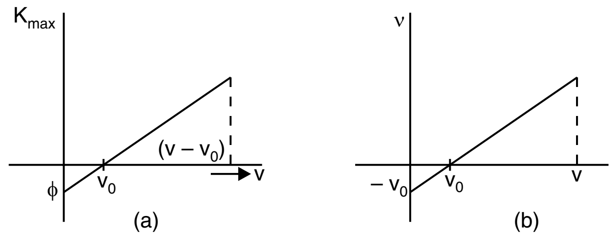

Fig. 11.7

$$\because \quad K_{max} = h(\nu - \nu_0)$$

$$\therefore \quad h = \frac{K_{max}}{(\nu - \nu_0)}$$ is the slope of graph line shown in Fig. 11.7(*a*).

Also the intercept of this graph line is the work function.

Similarly,

$$eV = h(\nu - \nu_0)$$

or

$$\frac{V}{(\nu - \nu_0)} = \frac{h}{e}$$ is the slope of graph line shown in Fig. 11.7(*b*).

The intercept of this line gives the value of slopping potential.

• Particle Nature of Light

(*i*) In interaction of radiation with matter, radiation as if it is made up of particles called photons.

(*ii*) Each photon has energy E (= $h\nu$) and momentum $P \left(= \frac{h\nu}{c} = \frac{h}{\lambda}\right)$

Whatever the intensity of radiation may be. By increasing the intensity of light of a given wavelength, there is only an increase in the number of photons per second crossing a given area, with each photon having the same energy. Thus, photon energy is independent of intensity of radiation.

(*iii*) Photons are electrically neutral and are not deflected by electric and magnetic field.

(*iv*) In a photon-particle collision (such as photon-electron collision), the total energy and total momentum are conserved. However, the number of photons may not conserved in a collision. The photons may be absorbed and a new photon may be created.

• Wave Nature of Matter

de-Broglie hypothesis. According to de-Broglie a moving material particle sometimes acts as a wave and sometimes as a particle or a wave is associated with moving material particle which controls the particle in every respect. The wave associated with moving particle is called *matter wave* or *de-Broglie wave* whose wavelength called de-Broglie wavelength, is given by

$$\lambda = \frac{h}{mv}$$

where m and v are the mass and velocity of the particle and h is a Planck's constant.

Derivation of de-Broglie wavelength.

According to Planck's quantum theory, the energy of a photon of a radiation of frequency ν and wavelength λ is

$$E = h\nu \qquad ...(i)$$

where h is a Planck's constant. If photon is considered to be a particle of mass m, the energy associated with it, according to Einstein mass energy relation, is given by

$$E = mc^2 \qquad ...(ii)$$

From (*i*) and (*ii*), we get

$$h\nu = mc^2 \quad \text{or} \quad m = \frac{h\nu}{c^2}$$

Since each photon moves with the same velocity c, therefore, momentum of photon,

$$p = \text{mass} \times \text{velocity}$$

i.e.,
$$p = \frac{h\nu}{c^2} \times c$$

$$= \frac{h\nu}{c} = \frac{h}{c/\nu} = \frac{h}{\lambda} \qquad \left(\because \lambda = \frac{c}{\lambda}\right)$$

or,
$$\lambda = \frac{h}{p} \qquad ...(iii)$$

de-Broglie assumed that the equation (*iii*) be equally applicable to both the photons of radiation and other material particles.

If a material particle of mass m, is moving with velocity v, then momentum of the particle $p = mv$. According to de-Broglie hypothesis, the wavelength of wave associated with the moving material particle is

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

This is de-Broglie wave equation for material particle.

• Photoelectric Cells

Photoelectric cell is a device that converts light energy into electrical energy. It is of three types:

(*i*) Photoemissive cell
(*ii*) Photovoltaic cell
(*iii*) Photoconductive cell

• Compton Scattering

It is the phenomenon of increase in the wavelength of X-ray photons which occurs when these radiations are scattered on striking an electron. The difference in the wavelength of scattered and incident photons is called compton shift, which is given by

$$\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\phi)$$

where ϕ is the angle of scattering of the X-ray photon and m_0 is the rest mass of the electron.

• Millikan's Oil Drop Method

R.A. Millikan obtained a value for the charge of the electron by observing the effect of an electric field on the motion of a charged oil drop. He first used Stokes' law to determine the radius r of the drop by using the relation

$$\frac{4\pi}{3} r^3 (\rho - \sigma) g = 6\pi\eta r v$$

or,
$$r = \frac{9}{2}\left[\frac{\eta v}{(\rho - \sigma) g}\right]$$

Here ρ and σ are the densities of oil and air respectively, η is the coefficient of viscosity of air and v is the terminal velocity acquired by the freely falling oil drop of radius r.

The electronic charge was determined from a large number of observations by using the relation

$$e = (q' - q)_{\text{min}} = \frac{6\pi\eta r}{E} (v'' - v')_{\text{min}}$$

Here v' and v'' are the terminal velocities of the oil drop when it has charges q and q' respectively and is moving under the action of a vertical electric field E in addition to the force of gravity.

Mass of electron. The mass of the electron was calculated using the value of e/m obtained by Thomson and of 'e' obtained by Millikan.

$$m = \frac{e}{\left(e/m\right)}$$

• Thomson's Method for e/m Measurement of Cathode Rays

J.J. Thomson determined e/m of the cathode rays by subjecting them to crossed electric and magnetic field, *i.e.,* fields that are at right angles to each other.

Force due to electric field, F_E = Force due to magnetic field F_B

or,
$$eE = Bev \Rightarrow v = \frac{E}{B}$$

Also,
$$\frac{e}{m} = \frac{E}{B^2 R} = \frac{V/d}{B^2 R} = \frac{Vx}{B^2 lLd}$$

Where,

R = radius of circular arc in the presence of magnetic field B
x = shift of the electron beam on the screen
V = potential difference between the two electrodes (*i.e.,* P and Q)
d = distance between the two electrodes
l = length of the field
L = distance between the centre of the field and the screen.

• Davisson and Germer's Experiment

It provides first experimental proof towards the concept of wave nature of material particles.

Experimental Arrangement

A fine beam of electrons is allowed to fall on Ni crystal using an electron gun. The electrons are scattered in all directions by the atoms of the crystal. The intensity of the electron beam in a given direction can be found using a detector. A graph is plotted between the scattering angle ϕ and the intensity of scattered electron beam.

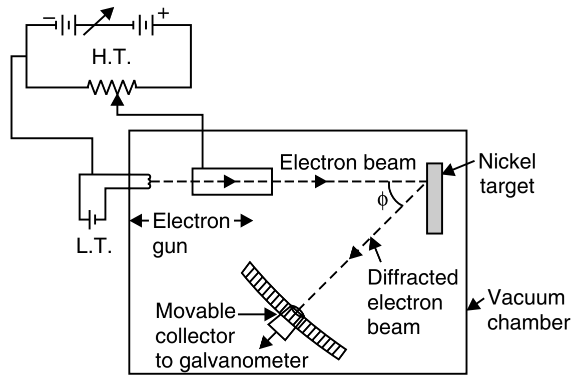

Fig. 11.8

Findings

- The intensity of scattered electrons depends upon the angle of scattering ϕ.
- A spike (a bump) is seen for all voltages. It is found to be maximum at $\phi = 50°$, which is the angle between the incident and the scattered beam.
- The size of the spike is the maximum at 54 volts.
- The size of the bump decreases on further increase of the voltage.

Experiment

It is found that for latitude angle (scattering angle) $\phi = 50°$, the glancing angle (angle between scattered beam of electrons with the plane of atoms of crystal) is given as

$$\theta + \phi + \theta = 180°$$

$$2\theta = 180° - \phi$$

$$\Rightarrow \quad \theta = \frac{1}{2}(180 - \phi) = \frac{1}{2}(180° - 50°)$$

$$\theta = 65°$$

From Bragg's law,

$$2d \sin\theta = n\lambda = \lambda \qquad \text{(As } n = 1 \text{ for first order diffraction)}$$

For nickel crystal $d = 0.91$ Å

$$\lambda = 2 \times 0.91 \times \sin 65° = 1.65 \text{ Å}$$

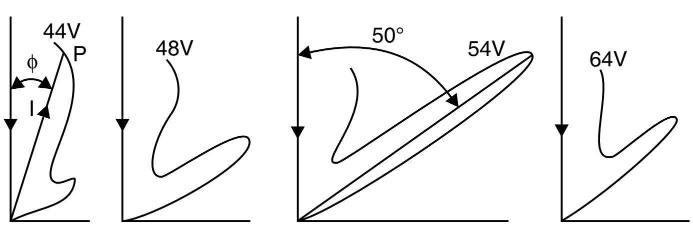

Fig. 11.9

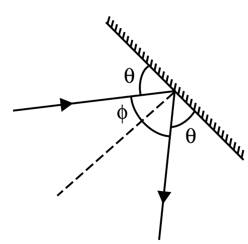

Fig. 11.10

According to de-Broglie hypothesis, the wavelength of electron accelerated through a potential V is

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ Å}$$

For $\quad V = 54$ volt

$$\lambda = 0.166 \text{ nm} = 1.66 \times 10^{-10} \text{ m} = 1.66 \text{ Å}$$

Thus, this experiment confirms the existence of de-Broglie waves associated with electrons.

QUESTIONS FROM TEXTBOOK

11.1. *Find the*

(*a*) *maximum frequency, and*

(*b*) *minimum wavelength of X-rays produced by 30 kV electrons.*

Sol. Here, $V = 30 \text{ kV} = 30 \times 10^3 \text{V} = 3 \times 10^4 \text{V}$

(*a*) $\boxed{h\nu_{max} = eV}$

$$\nu_{max} = \frac{eV}{h} = \frac{1.6 \times 10^{-19} \times 3 \times 10^4}{6.63 \times 10^{-34}} \text{ Hz}$$

$$= 7.24 \times 10^{18} \text{ Hz}$$

(*b*) $$\boxed{\lambda_{max} = \frac{c}{\nu_{max}}} = \frac{3 \times 10^8}{7.24 \times 10^{18}} \text{ m}$$

$\Rightarrow$ $$\lambda_{max} = 0.414 \times 10^{-10} \text{ m} = 0.414 \text{ Å}$$

$$= 0.0414 \text{ nm.}$$

11.2. *The work function of caesium metal is 2.14 eV. When light of frequency 6×10^{14} Hz is incident on the metal surface, photoemission of electrons occurs. What is the*

(*a*) *maximum kinetic energy of the emitted electrons,*

(*b*) *stopping potential, and*

(*c*) *maximum speed of the emitted photoelectrons?*

Sol. (*a*) $$\boxed{\text{K.E.}_{max} = h\nu - \phi_0} = \frac{6.63 \times 10^{-34} \times 6 \times 10^{14}}{1.6 \times 10^{-19}} - 2.14$$

$$E_{max} = 0.346 \text{ eV} = 0.35 \text{ eV}$$

(*b*) $\boxed{eV_0 = \text{K.E.}_{max}}$

$\Rightarrow$ $$V_0 = \frac{\text{K.E.}_{max}}{e} = \frac{0.35 \text{ eV}}{e} = 0.35 \text{ V}$$

(*c*) $$\frac{1}{2} mv^2_{max} = 0.346 \text{ eV} = 0.346 \times 1.6 \times 10^{-19} \text{ J}$$

$\Rightarrow$ $$v_{max} = \sqrt{\frac{0.346 \times 1.6 \times 10^{-19} \times 2}{9.1 \times 10^{-31}}} \text{ ms}^{-1}$$

$$= 3.488 \times 10^5 \text{ ms}^{-1}$$

$$= 348.8 \text{ km s}^{-1}$$

or, $$v_{max} = 349 \text{ km s}^{-1}.$$

11.3. *The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?*

Sol. $\boxed{\text{K.E.}_{max} = \text{eV}_0}$

$$= 1.5 \times 1.6 \times 10^{-19} \text{ J}$$

$$= 2.4 \times 10^{-19} \text{ J.}$$

11.4. *Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42 mW.*

(*a*) *Find the energy and momentum of each photon in the light beam.*

(*b*) *How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross-section which is less than the target area), and*

(*c*) *How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon?*

Sol. Given,

$$\text{Wavelength, } \lambda = 632.8 \text{ nm} = 632.8 \times 10^{-9} \text{ m}$$

$$\text{Frequency, } \nu = \frac{c}{\lambda} = \frac{3\times10^{8}}{632.8\times10^{-9}} \text{ Hz}$$

$$= 4.74 \times 10^{14} \text{ Hz}$$

(*a*)

$$E = h\nu$$

$$= 6.63 \times 10^{-34} \times 4.74 \times 10^{14} \text{ J}$$

$$= 3.14 \times 10^{-19} \text{ J.}$$

$$\boxed{p \text{ (momentum)} = \frac{h}{\lambda}} = \frac{6.63\times10^{-34}}{632.8\times10^{-9}} = 1.05 \times 10^{-27} \text{ kg ms}^{-1}$$

(*b*) Power emitted, $P = 9.42 \text{ mW} = 9.42 \times 10^{-3}$ W

$$\boxed{P = nE}$$

$$n = \frac{P}{E} = \frac{9.42\times10^{-3}\text{ W}}{3.14\times10^{-19}\text{ J}} = 3 \times 10^{16} \text{ photons/sec.}$$

(*c*) Velocity of hydrogen atom

$$= \frac{\text{Momentum '}p\text{' of } H_2 \text{ atom } (mv)}{\text{Mass of } H_2 \text{ atom} (m)}$$

$$\Rightarrow \quad v = \frac{1.05\times10^{-27}}{1.673\times10^{-27}} \text{ ms}^{-1}$$

$$= 0.63 \text{ ms}^{-1}.$$

11.5. *The energy flux of sunlight reaching the surface of the earth is 1.388 × 10^3 W/m^2. How many photons (nearly) per square metre are incident on the earth per second? Assume that the photons in the sunlight have an average wavelength of 550 nm.*

Sol. Energy flux of sunlight

= Total energy per square metre per second

$= 1.388 \times 10^3 \text{ Wm}^{-2}$

Energy of each photon,

$$\boxed{E = \frac{hc}{\lambda}}$$

$$\therefore \quad P = nE = \frac{6.63\times10^{-34}\times3\times10^{8}}{550\times10^{-9}} \text{ J} = 3.62 \times 10^{-19} \text{ J}$$

Number of photons incident on earth's surface per square metre per second

$$= \frac{\text{Total energy per square metre per second}}{\text{Energy of one photon}}$$

$$= \frac{1.388 \times 10^{3}}{3.62 \times 10^{-19}} = 3.8 \times 10^{21}.$$

11.6. *In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be 4.12 × 10⁻¹⁵ Vs. Calculate the value of Planck's constant.*

Sol. The slope of the cut-off voltage versus frequency of incident light

$$\frac{\Delta V}{\Delta \nu} = 4.12 \times 10^{-15} \text{ Vs} = 4.12 \times 10^{-15} \frac{\text{J . s.}}{\text{C}}$$

So, By multiplying it with the charge of an electron, which is the fundamental charge ($e = 1.6 \times 10^{-19}$ C) we get,

$$\because \quad E = h\nu$$

$$h = \frac{E}{\nu} = \text{J.}s.$$

$$h = 4.12 \times 10^{-15} \times 1.6 \times 10^{-19}$$

$$\Rightarrow \quad h = 6.592 \times 10^{-34} \text{ Js.}$$

11.7. *A 100 W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength of the sodium light is 589 nm. (a) What is the energy per photon associated with the sodium light? (b) At what rate are the photons delivered to the sphere?*

Sol. Given,

$$P \text{ (power)} = 100 \text{ W}$$

$$\lambda = 589 \times 10^{-9} \text{ m}$$

(*a*) Energy of each photon

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{589 \times 10^{-9}} \text{ J}$$

$$\Rightarrow \quad E = 3.38 \times 10^{-19} \text{ J}$$

(*b*) Number of photons delivered to sphere per second

$$n = \frac{\text{Energy radiated per second}}{\text{Energy of each photon}} \qquad P = nE$$

$$\text{or,} \quad n = \frac{100}{3.38 \times 10^{-19}} = 3 \times 10^{20} \text{ photons/s.}$$

11.8. *The threshold frequency for a certain metal is 3.3×10¹⁴ Hz. If light of frequency 8.2 × 10¹⁴ Hz is incident on the metal, predict the cut-off voltage for the photoelectric emission.*

Sol. Given,

$$\nu_0 = 3.3 \times 10^{14} \text{ Hz}$$

$$\nu = 8.2 \times 10^{14} \text{ Hz}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

Using Einstein's photoelectric equation,

$$\boxed{\frac{1}{2} mv^2_{\text{ max}} = h(\nu - \nu_0) = eV_0}$$

$$\Rightarrow \quad V_0 = \frac{h(\nu - \nu_0)}{e}$$

$$\Rightarrow \quad V_0 = \frac{6.62 \times 10^{-34}}{1.6 \times 10^{-19}}(8.2 \times 10^{14} - 3.3 \times 10^{14})$$

or, $\quad V_0 = 2.03$ V.

11.9. *The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?*

Sol. $\phi_0 = 4.2 \text{ eV} = 4.2 \times 1.6 \times 10^{-19} \text{ J} = 6.72 \times 10^{-19}$ J

$$\boxed{E = \frac{hc}{\lambda}}$$

$$\Rightarrow \quad E = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{330 \times 10^{-9}} = 6.018 \times 10^{-19} \text{ J}$$

As energy of incident photon $E < \phi_0$, hence no photoelectric emission will take place.

11.10. *Light of frequency 7.21 × 10¹⁴ Hz is incident on a metal surface. Electrons with a maximum speed of 6.0 × 10⁵ m/s are ejected from the surface. What is the threshold frequency for photoemission of electrons?*

Sol. Here,

$$\nu = 7.21 \times 10^{14} \text{ Hz}$$

$$v_{max} = 6.0 \times 10^5 \text{ ms}^{-1}$$

$$m = 9 \times 10^{-31} \text{ kg}$$

Applying Einstein's photoelectric equation,

$$\text{K.E.}_{max} = \frac{1}{2} m v^2_{max} = h(\nu - \nu_0)$$

$$\Rightarrow \quad \boxed{\frac{1}{2} m v^2_{max} = h(\nu - \nu_0)}$$

$$\Rightarrow \quad \nu_0 = \nu - \frac{m v^2_{max}}{2h}$$

$$\Rightarrow \quad \nu_0 = 7.21 \times 10^{14} - \frac{(9.1 \times 10^{-31}) \times (6 \times 10^5)^2}{2 \times (6.63 \times 10^{-34})}$$

$$= 4.74 \times 10^{14} \text{ Hz.}$$

11.11. *Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the emitter is made.*

Sol. Given,

Wavelength, $\lambda = 488 \text{ nm} = 488 \times 10^{-9}$ m

Stopping potential, $V_0 = 0.38$V

As,

$$\boxed{eV_0 = h\upsilon - \phi_0}$$

$$\Rightarrow \quad eV_0 = h\frac{c}{\lambda} - \phi_0$$

$$\Rightarrow \qquad \phi_0 = \frac{hc}{\lambda} - eV_0$$

$$= \left(\frac{6.63\times10^{-34}\times3\times10^{8}}{488\times10^{-9}\times1.6\times10^{-19}} - \frac{1.6\times10^{-19}\times0.38}{1.6\times10^{-19}}\right)$$

$$= (2.55 - 0.38) \text{ eV} = 2.17 \text{ eV}.$$

11.12. *Calculate the*

(*a*) *momentum, and*

(*b*) *de-Broglie wavelength of the electrons accelerated through a potential difference of 56 V.*

Sol. Energy of electron accelerated through potential difference of 56 V

$$= 56 \text{ eV} = 56 \times 1.6 \times 10^{-19} \text{ J}$$

(*a*) As, $$E = \frac{p^2}{2m} \qquad [p = mv, E = \frac{1}{2}mv^2]$$

$$\therefore \qquad p^2 = 2mE$$

$$\Rightarrow \qquad p = \sqrt{2mE}$$

or, $$p = \sqrt{2\times9\times10^{-31}\times56\times1.6\times10^{-19}}$$

$$p = 4.02 \times 10^{-24} \text{ kg ms}^{-1}$$

(*b*) As, $$p = \frac{h}{\lambda}$$

$$\therefore \qquad \lambda = \frac{h}{p} = \frac{6.62\times10^{-34}}{4.02\times10^{-24}}$$

$$= 1.64 \times 10^{-10} \text{ m} = 0.164 \times 10^{-9} \text{ m}$$

or, $$\lambda = 0.164 \text{ nm.}$$

11.13. *What is the*

(*a*) *momentum,*

(*b*) *speed, and*

(*c*) *de-Broglie wavelength of an electron with kinetic energy of 120 eV?*

Sol. (*a*) $$p = \sqrt{2mE}$$

$$\Rightarrow \qquad p = \sqrt{2\times(9\times10^{-31})\times(120\times1.6\times10^{-19})}$$

$$= 5.88 \times 10^{-24} \text{ kg ms}^{-1}$$

(*b*) $$p = mv$$

$$= \frac{p}{m}$$

$$= \frac{5.88\times10^{-24}}{9.1\times10^{-31}}$$

or, $$v = 6.46 \times 10^{6} \text{ m/s}$$

(*c*) $$\boxed{\lambda = \frac{h}{p}} = \frac{6.63\times10^{-34}}{5.88\times10^{-24}} = 1.13 \times 10^{-10} \text{ m}$$

$$= 1.13 \text{ Å}.$$

11.14. *The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which*

(*a*) *an electron, and*

(*b*) *a neutron, would have the same de-Broglie wavelength.*

Sol.

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\Rightarrow \quad E = \frac{h^2}{2\lambda^2 m}$$

(*a*) For electron, $E = \frac{(6.63\times10^{-34})^2}{2\times(589\times10^{-9})^2\times9\times10^{-31}}$

$= 7.03 \times 10^{-25}$ J

(*b*) For neutron, $E = \frac{(6.63\times10^{-34})^2}{2\times(589\times10^{-9})^2\times1.66\times10^{-27}}$

$= 3.81\times10^{-28}$ J.

11.15. *What is the de-Broglie wavelength of*

(a) a bullet of mass 0.040 kg travelling at the speed of 1.0 km/s.

(b) a ball of mass 0.060 kg moving at a speed of 1.0 m/s and

(c) a dust particle of mass 1.0 × 10^{-9} kg drifting with a speed of 2.2 m/s?

Sol. (*a*)

$m = 0.040$ kg

$v = 1\ \text{kms}^{-1} = 10^3\ \text{ms}^{-1}$

$$p = mv$$

$= 0.040 \times 10^3 = 40\ \text{kg ms}^{-1}$

$$\therefore \quad \lambda = \frac{h}{p} = \frac{6.62\times10^{-34}}{40} = 1.7 \times 10^{-35}\ \text{m}$$

(*b*)

$m = 0.060$ kg

$v = 1.0\ \text{ms}^{-1}$

$p = mv = 0.060\ \text{kg ms}^{-1}$

$$\lambda = \frac{h}{p} = \frac{6.62\times10^{-34}}{0.060} = 1.1 \times 10^{-32}\ \text{m}$$

(*c*)

$m = 1.0 \times 10^{-9}$ kg

$v = 2.2\ \text{ms}^{-1}$

$p = mv = 2.2 \times 10^{-9}$

$$\lambda = \frac{h}{p} = \frac{6.62\times10^{-34}}{2.2\times10^{-9}} = 3 \times 10^{-25}\ \text{m.}$$

11.16. *An electron and a photon each have a wavelength of 1.00 nm. Find*

(*a*) *their momenta,*

(*b*) *the energy of the photon, and*

(*c*) *the kinetic energy of electron.*

(Take h = 6.63 × 10^{-34} Js).

Sol. $\lambda_e = \lambda_p = 1.00 \text{ nm} = 1 \times 10^{-9}$ m

(*a*) $$\boxed{p = \frac{h}{\lambda}} = \frac{6.63\times10^{-34}}{1.00\times10^{-9}} \text{ kg ms}^{-1}$$
$$= 6.63 \times 10^{-25} \text{ kg ms}^{-1}$$

(*b*) $$\boxed{E = h\nu = \frac{hc}{\lambda}}$$
$$= \frac{6.63\times10^{-34}\times3\times10^{8}}{1\times10^{-9}} \text{J}$$
$$= 1.989 \times 10^{-16} \text{ J} = \frac{1.989\times10^{-16}}{1.6\times10^{-19}} \text{ eV}$$
$$= 1.243 \text{ keV}$$

(*c*) $$\boxed{\frac{h}{\sqrt{2mE_k}} = \lambda}$$

$\Rightarrow$ $$\sqrt{2mE_k} = \frac{h}{\lambda}$$

or, $$E_k = \frac{h^2}{2m\lambda^2}$$

$\Rightarrow$ $$E_k = \frac{(6.63\times10^{-34})^2}{2\times9.1\times10^{-31}\times(10^{-9})^2}$$
$$= \frac{43.96\times10^{-68}}{18.2\times10^{-49}} \text{J} = 2.4 \times 10^{-19} \text{ J}$$

or, $$E_k = \frac{2.4\times10^{-19}}{1.6\times10^{-19}} \text{eV} = 1.5 \text{ eV.}$$

11.17. (*a*) *For what kinetic energy of a neutron will the associated de-Broglie wavelength be* 1.40×10^{-10} *m?*

(*b*) *Also find the de-Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of* $\frac{3}{2}kT$ *at 300 K.*

Sol. (*a*) Here, $$\lambda = 1.40 \times 10^{-10} \text{ m}$$
$$m = 1.675 \times 10^{-27} \text{ kg}, \; h = 6.63 \times 10^{-34} \text{ Js}$$

As, $$\lambda = \frac{h}{mv}$$
$$v = \frac{h}{m\lambda}$$

or, $$v = \frac{6.63\times10^{-34}}{1.675\times10^{-27}\times1.40\times10^{-10}}$$
$$= 28.28 \times 10^{2} \text{ m/s}$$
$$\text{K.E.} = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 1.675 \times 10^{-27} \times (28.28 \times 10^2)^2$$
$$= 6.634 \times 10^{-21} \text{ J.}$$

(*b*) $$E = \frac{3}{2}kT = \frac{3}{2} \times (1.38 \times 10^{-23}) \times 300 = 6.21 \times 10^{-21} \text{ J}$$

(Here, k is Boltzmann constant. The value of k is 1.38×10^{-38} JK^{-1})

As, $$\lambda = \frac{h}{\sqrt{2mE}}$$

⇒ $$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 6.21 \times 10^{-21} \times 1.675 \times 10^{-27}}}$$

or, $$\lambda = 1.45 \times 10^{-10} \text{ m.}$$

11.18. *Show that the wavelength of electromagnetic radiation equal to the de-Broglie wavelength of its quantum (photon).*

Sol. de-Broglie wavelength of a photon,

$$\lambda = \frac{h}{p}$$

Momentum of a photon, $p = \frac{h\nu}{c}$

Hence, $$\lambda = \frac{h}{\frac{h\nu}{c}} = \frac{c}{\nu}$$

It is same as wavelength of electromagnetic radiation.

11.19. *What is the de-Broglie wavelength of a nitrogen molecule in air at 300 K? Assume that the molecule is moving with the root-mean square speed of molecules at this temperature. (Atomic mass of nitrogen = 14.0076 u).*

Sol. Temperature, $T = 300$ K

Mass of nitrogen molecule, $m = 2 \times 14.0076$ u

$$= 2 \times 14.0076 \times 1.6606 \times 10^{-27} \text{ kg}$$
$$= 46.52 \times 10^{-27} \text{ kg}$$

de-Broglie wavelength, $\lambda = \frac{h}{mv} = \frac{h}{p}$

⇒ $$\lambda = \frac{h}{\sqrt{2mE_k}} = \frac{h}{\sqrt{2m(\tfrac{3}{2}k_BT)}} = \frac{h}{\sqrt{3mk_BT}} \qquad \left(\because \; E_k = \frac{3}{2}k_B\text{J}\right)$$

(Where k_B is Boltzmann constant)

⇒ $$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 46.52 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}}$$

$$= \frac{6.63}{240.37} \times 10^{-9} \text{ m} = 0.0276 \text{ nm}$$
$$= 0.276 \text{ Å.}$$

11.20. *(a) Estimate the speed with which electrons emitted from a heated emitter of an evacuated tube impinge on the collector maintained at a potential difference of 500 V with respect to the emitter. Ignore the small initial speeds of the electrons. The specific charge of the electron, i.e., its e/m is given to be 1.76 × 10^{11} C kg^{-1}.*

(b) Use the same formula you employ in (a) to obtain electron speed for an collector potential of 10 MV. Do you see what is wrong? In what way is the formula to be modified?

Sol. (*a*) Here, $V = 500$ V

$$\frac{e}{m} = 1.76 \times 10^{11} \text{ C kg}^{-1}$$

The work done on the electron by potential difference between the cathode and the anode appears as its kinetic energy. Thus,

$$\frac{1}{2}mv^2 = eV$$

$$\Rightarrow \quad v = \sqrt{\frac{2eV}{m}} = \sqrt{2 \times 1.76 \times 10^{11} \times 500}$$

or, $v = 1.327 \times 10^7 \text{ ms}^{-1}$

(*b*) Here, $V = 10 \text{ MV} = 10 \times 10^6 = 10^7$ V

$$\frac{e}{m} = 1.76 \times 10^{11} \text{ C kg}^{-1}$$

Now, $$v = \sqrt{\frac{2eV}{m}} = \sqrt{2 \times 1.76 \times 10^{11} \times 10^7}$$

or, $v = 1.876 \times 10^9 \text{ ms}^{-1}$

As this speed is greater than the speed of light, so this speed is not possible, because as v approaches c, mass of electron becomes infinite $\left[\because m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}\right]$ so the formula has to be modified.

11.21. *(a) A monoenergetic electron beam with electron speed of 5.20 × 10^6 ms^{-1} is subject to a magnetic field of 1.30 × 10^{-4} T normal to the beam velocity. What is the radius of the circle traced by the beam, given e/m for electron equals 1.76 × 10^{11}C kg^{-1}.*

(b) Is the formula you employ in (a) valid for calculating radius of the path of a 20 MeV electron beam? If not, in what way is it modified?

Sol. (*a*) Velocity, $v = 5.20 \times 10^6$ m/s

Magnetic field, $B = 1.30 \times 10^{-4}$ T

$e/m = 1.76 \times 10^{11} \text{ C kg}^{-1}$

Centripetal force is provided by the force exerted by magnetic field on electron,

$$\therefore \quad \frac{mv^2}{r} = Bev$$

or, $$r = \frac{mv}{Be} = \frac{v}{B(e/m)}$$

or, $$r = \frac{5.20 \times 10^6}{1.30 \times 10^{-4} \times 1.76 \times 10^{11}} \text{ m} = 0.227 \text{ m} = 22.7 \text{ cm}.$$

(*b*) $$\frac{1}{2}mv^2 = 20 \text{ MeV} = 20 \times 1.6 \times 10^{-13} \text{ J}$$

$$v = \sqrt{\frac{2 \times 20 \times 1.6 \times 10^{-13}}{9.1 \times 10^{-31}}} \text{ m/s}$$

$$= 2.65 \times 10^9 \text{ m/s}$$

The velocity is greater than velocity of light. It appears something is wrong with data. However, the electron is clearly moving at relativisitic speed. So, the non-relativistic formula $r = \frac{m_0 v}{eB}$ is not valid. We should use relativistic formula:

$$r = \frac{mv}{eB}$$

$$r = \frac{m_0 v}{eB\sqrt{1-\frac{v^2}{c^2}}}.$$

11.22. *An electron gun with its collector at a potential of 100 V fires out electrons in a spherical bulb containing hydrogen gas at low pressure (–10^{-2} mm of Hg). A magnetic field of 2.83×10^{-4} T curves the path of the electrons in a circular orbit of radius 12.0 cm. (The path can be viewed because the gas ions in the path focus the beam by attracting electrons, and emitting light by electron capture; this method is known as the 'fine beam tube' method.) Determine e/m from the data.*

Sol. Here, $V = 100$ V;

Magnetic field, $B = 2.83 \times 10^{-4}$ T

$r = 12.0 \text{ cm} = 12.0 \times 10^{-2}$ m

When electrons are accelerated through V volt, the gain in K.E. of the electron is given by

$$\frac{1}{2}mv^2 = eV$$

$$\Rightarrow \quad v^2 = \frac{2eV}{m} \quad ...(i)$$

Since the electron moves in circular orbit under magnetic field, therefore, force on the electron due to magnetic field provides the centripetal force to the electron.

$$\therefore \quad evB = \frac{mv^2}{r}$$

$$\text{or,} \quad eB = \frac{mv}{r}$$

$$\text{or,} \quad v^2 = \frac{e^2B^2r^2}{m^2} \quad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$\frac{2eV}{m} = \frac{e^2B^2r^2}{m^2}$$

$$\Rightarrow \quad \frac{e}{m} = \frac{2V}{r^2B^2}$$

or, $$\frac{e}{m} = \frac{2\times 100}{(12\times 10^{-2})^2 \times (2.83\times 10^{-4})^2}$$
$$= 1.73 \times 10^{11} \text{ C kg}^{-1}.$$

11.23. *(a)* *An X-ray tube produces a continuous spectrum of radiation with its short wavelength end at 0.45 Å. What is the maximum energy of a photon in the radiation?*

(b) *From your answer to (a), guess what order of accelerating voltage (for electrons) is required in such a tube?*

Sol. Here,

$$\lambda = 0.45 \text{ Å} = 0.45 \times 10^{-10} \text{ m}$$
$$h = 6.62 \times 10^{-34} \text{ Js}$$
$$c = 3 \times 10^{8} \text{ ms}^{-1}$$

(*a*) The maximum energy of photon is given by

$$\boxed{E = h\nu = h\frac{c}{\lambda}}$$

or, $$E = \frac{6.62\times 10^{-34} \times 3\times 10^{8}}{0.45\times 10^{-10}} = 44 \times 10^{-16} \text{ J}$$

or, $$E = \frac{44\times 10^{-16}}{1.6\times 10^{-19}} \text{ eV} = 27.5 \times 10^{3} \text{ eV}$$
$$= 27.5 \text{ keV}.$$

(*b*) To produce electrons of 27.5 keV, accelerating potential of 27.5 kV or of the order of 30 kV is required.

11.24. *In an accelerator experiment on high-energy collisions of electrons with positrons, a certain event is interpreted as annihilation of an electron-positron pair of total energy 10.2 BeV into two γ-rays of equal energy. What is the wavelength associated with each γ-ray? (1BeV = 10^9eV)*

Sol. Total energy of 2 γ-rays= 10.2 BeV = 10.2×10^9 eV

∴ Energy of each γ-ray

⇒ $$E = \frac{1}{2}(10.2 \times 10^{9} \times 1.6 \times 10^{-19}) \text{ J}$$
$$= 8.16 \times 10^{-10} \text{ J}$$

Using the formula,

$$\boxed{E = h\nu = \frac{hc}{\lambda}}$$

⇒ $$\lambda = \frac{hc}{E} = \frac{6.63\times 10^{-34} \times 3\times 10^{8}}{8.16\times 10^{-10}} = 2.436 \times 10^{-16} \text{ m}.$$

11.25. *Estimating the following two numbers should be interesting. The first number will tell you why radio engineers do not need to worry much about 'photons'. The second number tells you why our eye can never 'count photon' even in barely detectable light.*

(a) *The number of photons emitted per second by a medium wave transmitter of 10 kW power emitting radio waves of length 500 m.*

(b) *The number of photons entering the pupil of our eye per second corresponding to the minimum intensity of white light that we humans can perceive (~10^{-10} Wm^{-2}). Take the area of the pupil to be about 0.4 cm^2, and the average frequency of white light to be about 6 × 10^4 Hz.*

Sol. (*a*) $\lambda = 500$ m

Energy of photon, $E = h\nu$

$$\boxed{E = \frac{hc}{\lambda}} = \frac{6.63\times10^{-34}\times3\times10^{8}}{500}\text{ J}$$

$$E = 3.98 \times 10^{-28}\text{ J}$$

Number of photons emitted/s

$$= \frac{\text{Power of transmitter}}{\text{Energy of one photon}} \quad \boxed{\because \quad P = ne}$$

$$= \frac{10^{4}\text{ Js}^{-1}}{3.98\times10^{-28}\text{ J}}$$

$$= 3 \times 10^{31}\text{ s}^{-1}$$

We see that the energy of a radiophoton is exceedingly small, and the number of photons emitted per second in a radio beam is enormously large. There is, therefore, negligible error involved in ignoring the existence of a minimum quantum of energy (photon) and treating the total energy of a radiowave as continuous.

(*b*) For $\nu = 6 \times 10^{14}$ Hz, $E = h\nu = 6.63 \times 10^{-34} \times 6 \times 10^{14}$ J ; 4×10^{-19} J. Photon flux corresponding to minimum intensity

$$= \frac{10^{-10}\text{ Wm}^{-2}}{4\times10^{-19}\text{ J}} \quad \boxed{P = nE}$$

$$= 2.5 \times 10^{8}\text{ m}^{-2}\text{ s}^{-1}$$

Number of photons entering the pupil per second = $2.5 \times 10^{8} \times 0.4 \times 10^{-4}\text{ s}^{-1} = 10^{4}\text{ s}^{-1}$. Though this number is not as large as in (*a*) above, it is large enough for us never to 'sense' or 'count' individual photons by our eye.

11.26. *Ultraviolet light of wavelength 2271 Å from a 100 W mercury source irradiates a photocell made of molybdenum metal. If the stopping potential is –1.3 volt, estimate the work function of the metal. How would the photocell respond to a high intensity (~ 10^{5} Wm^{-2}) red light of wavelength 6328 Å produced by He-Ne laser?*

Sol. Here, $V_0 = 1.3$ V

$\lambda = 2271\text{ Å} = 2271 \times 10^{-10}$ m

Now, $$\boxed{h\nu = h\nu_0 + \frac{1}{2}mv^2_{max} = \phi_0 + eV_0}$$

or, $$\phi_0 = \frac{hc}{\lambda} - eV_0$$

Taking, $h = 6.62 \times 10^{-34}$ Js

$e = 1.6 \times 10^{-19}$ C

$c = 3 \times 10^{8}\text{ ms}^{-1}$

We have,

$$\phi_0 = \frac{6.62\times10^{-34}\times3\times10^{8}}{2271\times10^{-10}} - 1.6 \times 10^{-19} \times 1.3$$

$$\Rightarrow \quad \phi_0 = 8.745 \times 10^{-19} - 2.08 \times 10^{-19} = 6.665 \times 10^{-19}\text{ J}$$

or, $\phi_0 = \frac{6.665 \times 10^{-19}}{1.6 \times 10^{-19}} = 4.166$ eV.

Threshold wavelength is given by

$$\boxed{\lambda_0 = \frac{hc}{\phi_0}} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{6.665 \times 10^{-19}} = 2.98 \times 10^{-7} \text{m}$$

or, $\lambda_0 = 2980$ Å

Since wavelength 6328 Å is greater than λ_0, the photocell will not respond, when red light of wavelength 6328 Å produced by He-Ne laser is incident on the photocell.

11.27. *Monochromatic radiation of wavelength 640.2 nm (1 nm = 10^{-9} m) from a neon lamp irradiates a photosensitive material made of caesium on tungsten. The stopping voltage is measured to be 0.54 V. The source is replaced by an iron source and its 427.2 nm line irradiates the same photocell. Predict the new stopping voltage.*

Sol. Here, for neon lamp,

$$\lambda = 640.2 \text{ nm} = 640.2 \times 10^{-9} \text{ m}$$

$$V = 0.54 \text{ V}$$

As, $$eV_1 = \frac{hc}{\lambda_1} - \phi_0$$

$$eV_2 = \frac{hc}{\lambda_2} - \phi_0$$

$$eV_2 - eV_1 = hc\left[\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right]$$

$$V_2 - V_1 = \frac{hc}{e}\left[\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right]$$

$$V_2 = V_1 = \frac{hc}{e}\left[\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right]$$

$$V_2 = 0.54 + \frac{6.63 \times 10^{-24} \times 3 \times 10^8}{1.6 \times 10^{-19}}\left[\frac{1}{427.2 \times 10^{-9}} - \frac{1}{640.2 \times 10^{-9}}\right]$$

$$= 0.54 + \frac{6.63 \times 3 \times 10^{-24+8+9}}{1.6}\left[\frac{1}{427.2} - \frac{1}{640.2}\right]$$

$$= 0.54 + \frac{6.63 \times 3}{1.6} \frac{[640.2 - 427.2]}{640.2 \times 427.2} \times 10^{-7}$$

$$= 0.54 + \frac{6.63 \times 3 \times 213 \times 10^{-7}}{1.6 \times 640.2 \times 427.2}$$

$$= 0.54 + 0.97$$

$$V_2 = 1.51 \text{ V}.$$

11.28. *A mercury lamp is convenient source for studying frequency dependence of photoelectric emission, since it gives a number of spectral lines ranging from the UV to the red end of the visible spectrum. In our experiment with rubidium photocell, the following lines from a mercury source were used:*

$$\lambda_1 = 3650 \text{ Å}, \lambda_2 = 4047 \text{ Å}, \lambda_3 = 4358 \text{ Å}, \lambda_4 = 5461 \text{ Å}, \lambda_5 = 6907 \text{ Å}$$

The stopping voltages, respectively were measured to be:

$$V_{01} = 1.28\ V, V_{02} = 0.95\ V, V_{03} = 0.74\ V,\ V_{04} = 0.16\ V, V_{05} = 0V$$

(a) Determine the value of Planck's constant h.

(b) Estimate the threshold frequency and work function for the material.

Sol. (*a*) Using $\nu = \frac{c}{\lambda}$, we first determine frequency in each case and then plot a graph between stopping potential V_0 and frequency ν.

$$\nu_1 = \frac{c}{\lambda_1} = \frac{3\times10^8}{3650\times10^{-10}} = 8.219 \times 10^{14}\text{ Hz}$$

$$\nu_2 = \frac{c}{\lambda_2} = \frac{3\times10^8}{4047\times10^{-10}} = 7.412 \times 10^{14}\text{ Hz}$$

$$\nu_3 = \frac{c}{\lambda_3} = \frac{3\times10^8}{4358\times10^{-10}} = 6.884 \times 10^{14}\text{ Hz}$$

$$\nu_4 = \frac{c}{\lambda_4} = \frac{3\times10^8}{5461\times10^{-10}} = 5.493 \times 10^{14}\text{ Hz}$$

$$\nu_5 = \frac{c}{\lambda_5} = \frac{3\times10^8}{6907\times10^{-10}} = 4.343 \times 10^{14}\text{ Hz}$$

V_0 versus ν plot is shown in figure 11.11.

The first four points lie nearly on a straight line which intercepts the ν axis of threshold frequency $\nu_0 = 5.0 \times 10^{14}$ Hz.

The fifth point ν (= 4.3×10^{14} Hz) corresponds to $\nu < \nu_0$, so there is no photoelectric emission and not stopping voltage is required to stop the current. Slope of V_0 versus ν graph is

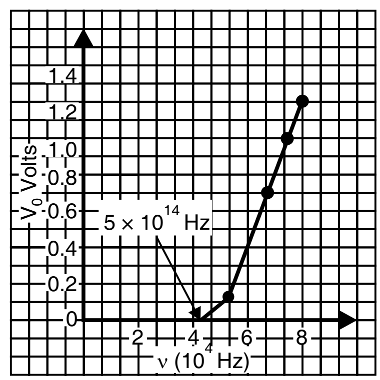

Fig. 11.11

$$\frac{\Delta V}{\Delta \nu} = \frac{(1.28-0)V}{(8.2-5.0)\times10^{14}\text{ s}^{-1}}$$

$$= 4.0 \times 10^{-15}\text{ Vs}$$

From Einstein's photoelectric equation,

$$\boxed{\text{K.E.} = eV = h\nu - W_0}$$

$$\therefore \quad e\Delta V = h\Delta\nu \qquad [W_0 \text{ is a constant}]$$

or, $$\frac{\Delta V}{\Delta \nu} = \frac{h}{e}$$

Hence, $$\frac{h}{e} = 4.0 \times 10^{-15}\text{ Vs}$$

Planck's constant, $h = e \times 4.0 \times 10^{-15}$ Js

or, $h = 1.6 \times 10^{-19} \times 4.0 \times 10^{-15}$ Js

$= 6.4 \times 10^{-34}$ Js

(*b*) Threshold frequency, $\nu_0 = 5.0 \times 10^{14}$ Hz

$\therefore$ Work function, $\phi_0 = h\nu_0 = 6.4 \times 10^{34} \times 5.0 \times 10^{14}$

or, $\phi_0 = \dfrac{6.4 \times 5 \times 10^{-20}}{1.6 \times 10^{-19}}$ eV = 2.00 eV

11.29. *The work function for the following metals is given:*

Na: 2.75 eV; K: 2.30 eV; M_0: 4.17 eV; Ni: 5.15 eV.

Which of these metals will not give photoelectric emission for a radiation of wavelength 3300 Å from a He–Cd laser placed 1m away from the photocell? What happens if the laser is brought nearer and placed 50 cm away?

Sol. (*i*) Work function of Na is

$$\phi_{Na} = 1.92 \text{ eV} = 1.92 \times 1.6 \times 10^{-19} \text{ J}$$

$$\lambda = 3300 \text{ Å} = 3300 \times 10^{-10} \text{ m}$$

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{3300 \times 10^{-10}} \text{ J}$$

$$E = \frac{6.6 \times 3 \times 10^{-34+8+10-2}}{33 \times 10} \text{ J}$$

$$= \frac{6 \times 10^{-18-1}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= \frac{60}{16} \text{ eV}$$

$$E = 3.75 \text{ eV}$$

It is observed that energy of incident radiation is less than Ni and Mo but larger than Na and K. So photoemission current take place from Na and K but not from Mo and Ni. Therefore, Mo and Ni will not give photoelectric emission. If the laser is brought closer the intensity of radiation increases without any change in frequency. This therefore, will not affect the result. However, photoelectric current from Na and K will increase.

11.30. *Light of intensity 10^{-5} W m^{-2} falls on a sodium photocell of surface area 2 cm^2. Assuming that the top 5 layers of sodium absorb the incident energy, estimate time required for photoelectric emission in the wave-picture of radiation. The work function for the metal is given to be about 2 eV. What is the implication of your answer?*

Sol. Number of atoms in 5 layers of sodium

$$= \frac{5 \times \text{area of each layer}}{\text{Effective area of atom}}$$

$$= \frac{5 \times 2 \times 10^{-4}}{10^{-20}} = 10^{17}$$

Assume that there is only one conduction electron per Na atom.

$\therefore$ Number of electrons in 5 layers = 10^{17}

Energy received by an electron per sec

$$= \frac{\text{Power of incident light}}{\text{Number of electrons}}$$

$$= \frac{10^{-5} \times 2 \times 10^{-4}}{10^{17}} = 2 \times 10^{-26} \text{ W}$$

Time required for photoemission

$$= \frac{\text{Energy required per electron}}{\text{Energy absorbed per second per electron}}$$

$$= \frac{2 \times 1.6 \times 10^{-19}}{2 \times 10^{-26}} \text{ s} = 1.6 \times 10^{7} \text{ s.}$$

It is contrary to the observed fact that there is no time lag between the incidence of light and the emission of photoelectrons.

11.31. *Crystal diffraction experiments can be performed using X-rays, or electrons accelerated through appropriate voltage. Which probe has greater energy, an X-ray photon or the electron? (For quantitative comparison, take the wavelength of the probe equal to 1 Å, which is of the order of interatomic spacing in the lattice) ($m_e = 9.11 \times 10^{-31}$ kg).*

Sol. Energy of photon, $\boxed{E = h\nu = \frac{hc}{\lambda}}$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{10^{-10}} \text{ J}$$

$$= 19.89 \times 10^{-16} \text{ J}$$

or, $$E = \frac{19.89 \times 10^{-16}}{1.6 \times 10^{-19}} \text{ eV} = 12.43 \text{ keV}$$

For the case of electron,

$$\boxed{\lambda = \frac{h}{\sqrt{2mE}}}$$

$\Rightarrow$ $$\sqrt{2mE} = \frac{h}{\lambda}$$

or, $$2mE = \frac{h^2}{\lambda^2}$$

or, $$E = \frac{h^2}{2m\lambda^2}$$

$\Rightarrow$ $$E = \frac{\left(6.63 \times 10^{-34}\right)^2}{2 \times 9.11 \times 10^{-31} \times 10^{-20}} \text{ J}$$

$$= \frac{(6.63 \times 10^{-34})^2}{2 \times 9.11 \times 10^{-31} \times 10^{-20} \times 1.6 \times 10^{-19}} \text{ eV}$$

or, $$E = 150.8 \text{ eV}$$

For the same given wavelength, kinetic energy of a photon is much greater than that of electron.

11.32. *(a) Obtain the de-Broglie wavelength of a neutron of kinetic energy 150 eV. As you have seen in Question 11.31, an electron beam of this energy is suitable for crystal diffraction experiments. Would a neutron beam of the same energy be equally suitable? Explain.*
Given $m_n = 1.675 \times 10^{-27}$ kg.

(b) Obtain the de-Broglie wavelength associated with thermal neutrons at room temperature (27°C). Hence explain why a fast neutron beam needs to be thermalised with the environment before it can be used for neutron diffraction experiments.

Sol. (*a*) Here, K.E. of neutron, $E = 150 \text{ eV} = 150 \times 1.6 \times 10^{-19}$ J,

Mass of neutron, $m = 1.675 \times 10^{-27}$ kg.

We know, K.E. of neutron, $E = \frac{1}{2}mv^2$ or $mv = \sqrt{2Em}$

$$\therefore \quad \boxed{\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2Em}}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 150 \times 1.6 \times 10^{-19} \times 1.675 \times 10^{-27}}} = 2.33 \times 10^{-12} \text{ m.}$$

The interatomic spacing ~ 1 Å (= 10^{-10} m) is about a hundred times greater than this wavelength. Therefore, a neutron beam of energy 150 eV is not suitable for diffraction experiment.

(*b*) Here, $T = 27 + 273 = 300$ K,

Boltzmann's constant, $k = 1.38 \times 10^{-23}$ J mol^{-1} K^{-1}

We know, average K.E. of neutron at absolute temperature T is given by $E = \frac{3}{2}kT$. Where k is the Boltzmann's constant.

$$\text{Now,} \quad \boxed{\lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{3mkT}}}$$

$$\therefore \quad \lambda = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 1.675 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}} = 1.45 \times 10^{-10} \text{ m}$$

Since this wavelength is comparable to interatomic spacing (~ 1 Å) in a crystal, therefore, thermal neutrons are suitable probe for diffraction experiments: so a high energy neutron beam should be first thermalised before using it for diffraction.

11.33. *An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de-Broglie wavelength associated with the electrons. If other factors (such as numerical aperture, etc.) are taken to be roughly the same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light?*

Sol. Accelerating voltage,

$V = 50$ kV

∴ Energy of electrons, $E = eV = 50$ *k*eV

or, $E = 50 \times 10^3 \times 1.6 \times 10^{-19}$ J

$$\boxed{\lambda = \frac{h}{\sqrt{2mE}}}$$

$$\Rightarrow \quad \lambda = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 50 \times 10^3 \times 1.6 \times 10^{-19}}}$$

$$\lambda = \frac{6.62 \times 10^{-34}}{1.21 \times 10^{-22}} = 5.47 \times 10^{-12} \text{ m.}$$

Resolving power of microscope $\propto \frac{1}{\lambda}$

$$\therefore \quad \frac{\text{R.P. of electron microscope}}{\text{R.P. of optical microscope}} = \frac{\lambda y}{\lambda} + \frac{5.9 \times 10^{-7}}{5.47 \times 10^{-17}} \cong 10^{+5}$$

λy = wavelength yellow light.

Resolving power of a microscope is inversely proportional to the wavelength of the radiation used. Since the wavelength of yellow light is 5990 Å or 5.99×10^{-7} m, so it follows that electron microscope will have resolving power 10^5 times that of optical microscope.

11.34. *The wavelength of a probe is roughly a measure of the size of a structure that it can probe in some detail. The quark structure of protons and neutrons appears at the minute length-scale of 10^{-15} m or less. This structure was first probed in early 1970's using high energy electron beams produced by a linear accelerator at Stanford, USA. Guess what might have been the order of energy of these electron beams. (Rest mass energy of electron = 0.511 MeV.)*

Sol. Applying formula,

$$\boxed{p = \frac{h}{\lambda}} = \frac{6.63 \times 10^{-34}\,\text{Js}}{10^{-15}\,\text{m}}$$

$$= 6.63 \times 10^{-19}\ \text{Kg ms}^{-1}$$

Use the relativistic formula for energy:

$$\boxed{E^2 = c^2p^2 + m_0^2 c^4}$$

or, $$E^2 = 9 \times (6.63)^2 \times 10^{-22} + (0.511 \times 1.6)^2 \times 10^{-26}$$

$$\simeq 9 \times (6.63)^2 \times 10^{-22}$$

the second term (rest mass energy) being negligible.

Therefore, $$E = 1.989 \times 10^{-10}\ \text{J}$$

or, $$E = \frac{1.989 \times 10^{-10}}{1.6 \times 10^{-11}}\ \text{BeV} = 1.24\ \text{BeV}.$$

Thus, electron energies from the accelerator must have been of the order of a few BeV.

11.35. *Find the typical de-Broglie wavelength associated with a He atom in helium gas at room temperature (27 °C) and 1 atm pressure; and compare it with the mean separation between two atoms under these conditions.*

Sol. Mass of atom,

$$m = \frac{\text{Atomic mass of He}}{\text{Avogadro's number}}$$

$$= \frac{4}{6 \times 10^{23}}\ \text{g}$$

or, $$m = 6.67 \times 10^{-27}\ \text{kg}$$

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{\sqrt{3mkT}}\ \text{m}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 6.67 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}}\ \text{m}$$

or, $$\lambda = 7.3 \times 10^{-11}\ \text{m}$$

Now, $\boxed{PV = RT = kNT}$ (The kinetic gas equation for one mole of a gas.)

or, $\dfrac{V}{N} = \dfrac{KT}{P}$

Mean separation,

$$r_0 = \left(\frac{\text{Molar volume}}{\text{Avogadro's number}}\right)^{1/3} = \left(\frac{V}{N}\right)^{1/3} = \left(\frac{kT}{P}\right)^{1/3}$$

$\Rightarrow$ $$r_0 = \left(\frac{1.38\times10^{-23}\times300}{1.01\times10^{5}}\right)^{1/3} \text{ m}$$

or, $r_0 = 3.4 \times 10^{-9}$ m

The mean separation between two atoms is much larger than the de-Broglie wavelength.

11.36. *Compute the typical de-Broglie wavelength of an electron in a metal at 27°C and compare it with the mean separation between two electrons in a metal which is given to be about 2×10^{-10} m.*

*[**Note:** Questions 11.35 and 11.36 reveal that while the wave-packets associated with gaseous molecules under ordinary conditions are non-overlapping, the electron wave-packets in a metal strongly overlap with one another. This suggests that whereas molecules in an ordinary gas can be distinguished apart, electrons in a metal cannot be distinguished apart from one another. This indistinguishibility has many fundamental implications which you will explore in more advanced Physics courses.]*

Sol. Here, $T = 27°\text{ C} \Rightarrow T = 273 + 27 = 300$ K

$m = 9.1 \times 10^{-31}$ kg

Using formula,

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

$\Rightarrow$ $$\lambda = \frac{6.63\times10^{-34}}{\sqrt{3\times9.1\times10^{-31}\times\left(\dfrac{8.31\times10^{3}}{6\times10^{23}}\right)\times300}}$$

or, $\lambda = 62.15 \times 10^{-10}$ m

Interelectronic separation, $r = 2 \times 10^{-10}$ m

Hence, $r < \lambda$

We find that wave-packets in metals strongly overlap with one another (This is not the case in gas atoms).

11.37. *Answer the following questions:*

(a) *Quarks inside protons and neutrons are thought to carry fractional charges [(+ 2/3)e; (–1/3e). Why do they not show up in Millikan's oil-drop experiment?*

(b) *What is so special about the combination e/m? Why do we not simply talk of e and m separately?*

(c) *Why should gases be insulators at ordinary pressures and start conducting at very low pressures?*

(d) *Every metal has a definite work function. Why do all photoelectrons not come out with the same energy if incident radiation is monochromatic? Why is there an energy distribution of photoelectrons?*

(*e*) *The energy and momentum of an electron are related to the frequency and wavelength of the associated matter wave by the relations*

$$E = h\nu, \ p = \frac{h}{\lambda}$$

But while the value of λ is physically significant, the value of ν (and therefore, the value of the phase speed ν λ) has no physical significance. Why?

Sol. (*a*) Quarks are thought to be confined within a proton or neutron by forces which grow stronger if one tries to pull them apart. Though bound fractional charges may exist in nature, independent charges are still integral multiples of *e*.

(*b*) Basic relations $eV = 1/2\ mv^2$ or $eE = ma$ and $eBv = mv^2/r$, for electric and magnetic fields, respectively, show that the dynamics of electrons is determined not by *e*, and *m* separately but by the combination *e/m*.

(*c*) At low pressures, ions have a chance to reach their respective electrodes and constitute a current. At ordinary pressures, ions have no chance to do so because of collisions with gas molecules and recombination.

(*d*) Work function only indicates the minimum energy required for the electron in the highest level of the conduction band to get out of the metal. Not all electrons in the metal belong to this level. They occupy a continuous band of levels. Consequently, for the same incident radiation, electrons knocked off from different levels come out with different energies.

(*e*) The absolute value of energy *E* (but not momentum *p*) of any particle is arbitrary to within an additive constant. Hence, while λ is physically significant, absolute value of *v* of a matter wave of an electron has no direct physical meaning. The phase speed

$v\lambda$ is likewise not physically significant. The group speed given by $v = \nu\lambda = \frac{\omega}{k}$

$$\frac{dw}{dk} = \frac{dw}{dk}\frac{dv}{d(1/\lambda)} = \frac{dE}{dp} = \frac{d}{dp}\left(\frac{p^2}{2m}\right) = \frac{p}{m}$$

$$\because \quad \boxed{K = \frac{2\pi}{\lambda}, \ \omega = 2\pi\nu} \text{ and } \lambda = \frac{h}{p}$$

is physically meaningful.

MORE QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *An electron and alpha particle have the same de-Broglie wavelength associated with them. How are their kinetic energies related to each other?*

Ans. Kinetic energy, $\boxed{\text{K.E.} = \frac{1}{2}mv^2} = \frac{1}{2}m\frac{p^2}{m^2}$

$$\Rightarrow \quad \text{K.E.} = \frac{p^2}{2m} \qquad (\because p = mv)$$

$$\lambda_e = \lambda_\alpha$$

$$\frac{h}{p_e} = \frac{h}{p_\alpha}$$

$$\Rightarrow \quad p_e^2 = p_\alpha^2 - I$$

$$\text{K.E.} = \frac{1}{2} mv^2$$

$$\text{K.E.} = \frac{p^2}{2m}$$

$$\therefore \quad \frac{\text{K.E.}_e}{\text{K.E.}_\alpha} = \frac{\frac{1}{2}\frac{p_e^2}{2m_e}}{\frac{1}{2}\frac{p_\alpha^2}{2m_\alpha}}$$

$$\therefore \quad \frac{\text{K.E.}_e}{\text{K.E.}_\alpha} = \frac{m_\alpha}{m_e} \times \frac{p_e^2}{p_\alpha} = \frac{m_\alpha}{m_e}.$$

Q. 2. *How does the stopping potential applied to a photocell change, if the distance between the light source and the cathode of the cell is doubled?*

Ans. When the distance between the light source and the cathode is changed, the intensity of light changes accordingly. However, the stopping potential does not depend upon the intensity of incident light.

Q. 3. *de-Broglie wavelength associated with an electron accelerated through a potential difference V is λ. What will be its wavelength when the accelerating potential is increased to 4 V?*

Ans. de-Broglie wavelength,

$$\boxed{\lambda = \frac{1.227}{\sqrt{V}} \text{ nm}}$$

$$\because \quad \lambda \propto \frac{1}{\sqrt{V}}$$

The p.d. is increased 4 times, then the wavelength will be halved.

Q. 4. *Two metals A and B have work function 2 eV and 5 eV respectively. Which metal has lower threshold wavelength?*

Ans. As, $\boxed{\phi_0 = \frac{hc}{\lambda}} \Rightarrow \lambda = \frac{hc}{\phi_0}$

i.e., $\lambda \propto \frac{1}{\phi_0}$

Thus, metal *B* has lower threshold wavelength of radiation since its work function is greater.

Q. 5. *Two lines, A and B in the plot given below show the variation of de-Broglie wavelength, λ versus $\frac{1}{\sqrt{V}}$, where V is the accelerating potential difference, for two particles carrying the same charge. Which one of two represents a particle of smaller mass?*

Ans. $\because \quad \lambda = \frac{h}{p}$

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{1.227}{\sqrt{V}}$$

$$\Rightarrow \quad \text{slope} = \frac{\lambda}{1/\sqrt{V}} = \frac{h/\sqrt{V}}{1/\sqrt{V}}$$

$$\text{slope} = \frac{h}{mv} \times \frac{\sqrt{V}}{1.22}$$

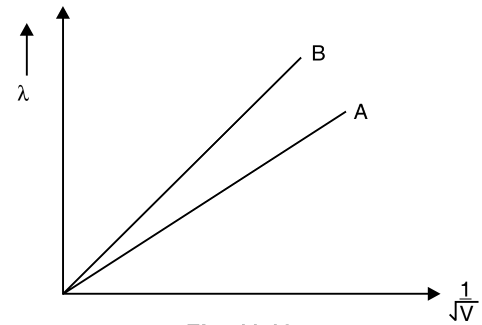

Fig. 11.12

$\because$ Slope of B > slope of A Slope $\propto \frac{1}{m}$

$$\frac{1}{\sqrt{m_B}} > \frac{1}{\sqrt{m_A}} \quad \Rightarrow \quad \sqrt{m_B} < \sqrt{m_A}$$

$$\therefore \quad m_B < m_A$$

Therefore, line B represents a particle of smaller mass.

Q. 6. *The maximum kinetic energy of a photoelectron is 3 eV. What is its stopping potential?*

Ans. Since, $K.E._{max} = eV_0$

$$3 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-19} \; V_0$$

$\therefore$ Stopping potential, $V_0 = 3V$.

Q. 7. *Ultraviolet light is incident on two photosensitive materials having work functions W_1 and W_2 ($W_1 > W_2$). In which case will the kinetic energy of the emitted electrons be greater? Why?*

Ans. K.E. of photoelectron = $h\nu - W$

As given,

$$W_1 > W_2$$

Since, W_2 is lesser than W_1 thus the kinetic energy of the emitted electrons for the photoelectric material having work function W_2 will be greater.

Q. 8. *The frequency (ν) of the incident radiation is greater than threshold frequency (ν_0) in a photocell. How will the stopping potential vary if frequency ν is increased, keeping other factors constant?*

Ans. As,

$$\frac{1}{2} mv_{max}^2 = eV_0 = h(\nu - \nu_0)$$

Therefore, the value of stopping potential (V_0) increases with the increase in frequency (ν) of the incident radiation and K.E. increaseses.

Q. 9. *What is the de-Broglie wavelength associated with an electron accelerated through a potential of 100 volts?*

Ans. de-Broglie wavelength,

$$\boxed{\lambda = \frac{1.227}{\sqrt{V}} \text{ nm}}$$

$$\Rightarrow \quad \lambda = \frac{1.227}{\sqrt{100}} \text{ nm} = 0.123 \text{ nm.}$$

Q. 10. *Show graphically how the stopping potential for a given photosensitive surface varies with the frequency of the incident radiation.*

Ans.

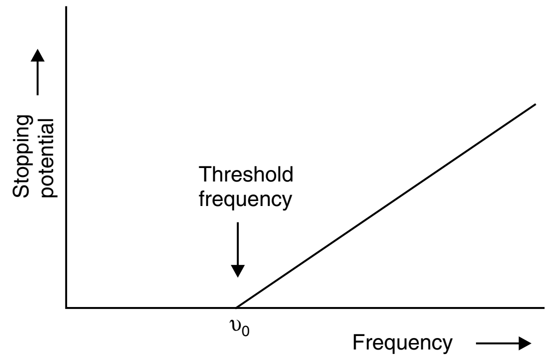

Fig. 11.13

Q. 11. *What is the effect on the velocity of the emitted photoelectrons if the wavelength of the incident light is decreased?*

Ans. There will be an increase in the velocity of the emitted photoelectrons.

As, $$\frac{1}{2}mv^2_{max} = h\frac{c}{\lambda} - \phi_0.$$

Q. 12. *Plot a graph showing the variation of photoelectric current with anode potential for two light beam of same wavelength but different intensity.*

Ans.

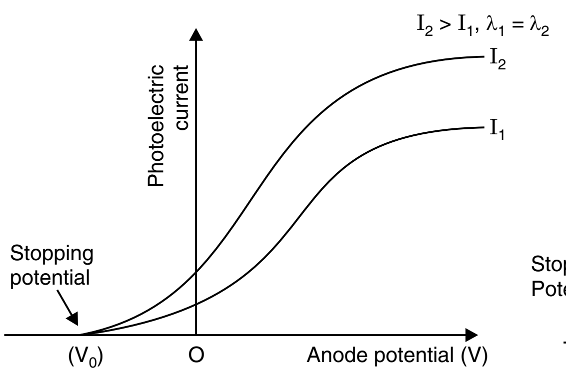

Fig. 11.14

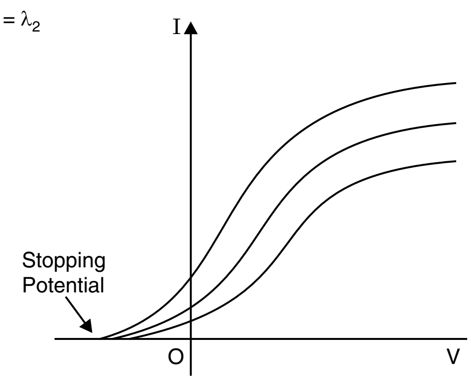

Fig. 11.15

Q. 13. *An electron, an alpha-particle and a proton have the same kinetic energy. Which one of these particles has the largest de-Broglie wavelength?*

Ans. Electron has the largest de-Broglie wavelength as its mass is lesser than the mass of alpha particle and proton.

$$\lambda = \frac{h}{p}$$

Since, $$\lambda = \frac{h}{mv}$$

Thus, $$\lambda \propto \frac{1}{\sqrt{m}}.$$

Q. 14. *Light of frequency 1.5 times the threshold frequency is incident on a photosensitive material. If the frequency is halved and intensity is doubled, what happens to photoelectric current?*

Ans. If the frequency is halved, the frequency of incident light will become $\frac{1.5}{2}$ = 0.75 times the threshold frequency. Hence, photoelectric current will be zero.

Q. 15. *Name the experiment for which the following graph 11.16, showing the variation of intensity of scattered electrons with the angle of scattering, was obtained. Also name the important hypothesis that was confirmed by this experiment.*

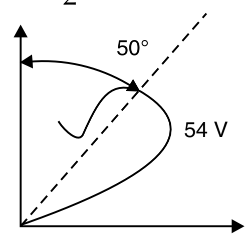

Fig. 11.16

Ans. Davisson and Germer's Experiment.

This experiment confirmed the de-Broglie hypothesis.

Q.16. *Ultraviolet radiations of different frequencies ν_1 and ν_2 are incident on two photosensitive materials having work functions W_1 and W_2 ($W_1 > W_2$) respectively. The kinetic energy of the emitted electrons is same in both the cases. Which one of the two radiations will be of higher frequency?*

Ans. As,

$$\text{K.E.} = h\nu - \phi \quad \text{or} \quad \boxed{\text{K.E.} = h\nu - W}$$

$$\text{K.E.} = h\nu_1 - W_1 \Rightarrow h\nu_1 = \text{K.E.} + W_1$$

and $$\text{K.E.} = h\nu_2 - W_2 \Rightarrow h\nu_2 = \text{K.E.} + W_2$$

Since, $$W_1 > W_2$$

Thus, $$\nu_1 > \nu_2.$$

Q. 17. *How will the photoelectric current change on decreasing the wavelength of incident radiation for a given photosensitive material?*

Ans. Photoelectric current is not affected on decreasing the wavelength of incident radiation.

Q. 18. *Why are alkali metals most suited for photoelectric emission?*

Ans. This is because alkali metals has low work function. It is easy to remove electrons from alkali than from other metals.

Q. 19. *What is the difference between thermionic emission and photoelectric emission?*

Ans. During thermionic emission electrons are emitted from metal surface by providing heat energy, whereas, during photoelectric emission electrons are emitted from metal surface by providing light energy.

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *The emitter in a photoelectric tube has a threshold wavelength of 6000 Å. Determine the wavelength of the light incident on the tube if the stopping potential for this light is 2.5 V.*

Ans. The work function is

$$\boxed{\phi_0 = h\nu_{th}} = \frac{hc}{\lambda th} = \frac{12.4\times10^3 \text{eV}\times\text{Å}}{6000 \text{ Å}}$$

$$= 2.07 \text{ eV}$$

The photoelectric equation then gives

$$eV_0 = h\nu - \phi_0$$

$$\Rightarrow \quad \boxed{eV_0 = \frac{hc}{\lambda} - \phi_0}$$

or, $$2.5 \text{ eV} = \frac{12.4\times10^3 \text{eV.Å}}{\lambda} - 2.07 \text{ eV}$$

or, $$\lambda = 2713 \text{ Å}.$$

Q. 2. *An electron and a proton are accelerated through the same potential. Which one of the two has (i) greater value of de-Broglie wavelength associated with it and (ii) less momentum? Justify answer.*

Ans. (*i*) The de-Broglie wavelength associated with same potential V is given by

$$\boxed{\lambda = \frac{h}{p}} \quad \text{or} \quad \lambda = \frac{h}{mv}$$

$$\Rightarrow \qquad \lambda \propto \frac{1}{\sqrt{m}}$$

As electron's mass is lesser than proton.

Thus, λ electron > λ proton.

(*ii*) As, $$\boxed{\lambda = \frac{h}{p}}$$

$$\Rightarrow \qquad p = \frac{h}{\lambda} \quad \text{or} \quad p \propto \frac{1}{\lambda}$$

Since, $\lambda_e > \lambda_{\text{proton}}$

Hence, momentum of electron will be lesser than proton.

Q. 3. *A proton and an alpha particle are accelerated through the same potential. Which one of the two has (i) greater value of de-Broglie wavelength associated with it, and (ii) less kinetic energy? Justify your answer.*

Ans. (*i*) The de-Broglie wavelength associated with same potential V is

$$\boxed{\lambda = \frac{h}{p}} \quad \text{or} \quad \lambda = \frac{h}{mv}$$

$$\therefore \qquad \lambda \propto \frac{1}{\sqrt{m}}$$

As proton's mass is less than the mass of alpha particle thus,

$$\lambda_{\text{proton}} > \lambda_{\text{alpha.}}$$

$$\boxed{\text{K.E.} = h\nu}$$

(*ii*) As, $$\text{K.E.} = \frac{hc}{\lambda}$$

$$\Rightarrow \qquad \text{K.E.} \propto \frac{1}{\lambda}$$

Since, $\lambda_{\text{proton}} > \lambda_{\text{alpha.}}$

Thus, kinetic energy of proton will be lesser than that of alpha particle.

Q. 4. *Find the de-Broglie wavelengths of (a) a 46-g golf ball with a velocity of 30 m/s, and (b) an electron with a velocity of 10^7 m/s.*

Ans. (*a*) Since $V << c$, we can let $m = m_0$. Hence,

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\,\text{Js}}{(0.046\text{ kg})\,(30\text{m/s})}$$

$$\lambda = 4.8 \times 10^{-34}\text{ m}$$

The wavelength of the golf ball is so small compared with its dimensions that we would not except to find any wave aspects in its behaviour.

(*b*) Again $V << c$, so with $m = m_0 = 9.1 \times 10^{-31}$ kg,

We have,

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\,\text{Js}}{(9.1\times10^{-31}\,\text{kg})(10^{7}\,\text{m/s})}$$

$$= 7.3 \times 10^{-11}\text{ m}$$

The dimensions of atoms are comparable with this figure—the radius of the hydrogen atom, for instance is 5.3×10^{-11} m. It is therefore not surprising that the wave character of moving electrons is the key to understanding atomic structure and behaviour.

Q. 5. *For what kinetic energy of a proton, will the associated de-Broglie wavelength be 16.5 nm?*

Ans. Here, $\lambda = 16.5\text{ nm} = 16.5 \times 10^{-9}\text{ m}$

and mass of proton $m = 1.6 \times 10^{-27}$ kg

Using de-Broglie equation

$$\boxed{\lambda = \frac{h}{mv}}$$

$$\Rightarrow \quad v = \frac{h}{m\lambda}$$

$$\therefore \quad \text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}\,m\,.\,\frac{h^2}{m^2\lambda^2}$$

$$\text{K.E.} = \frac{h^2}{2m\lambda^2}$$

$$= \frac{6.63\times10^{-34}\times6.63\times10^{-34}}{2\times1.6\times10^{-27}\times16.5\times10^{-9}\times16.5\times10^{-9}}$$

$$\text{K.E.} = \frac{6.63\times6.63\times10^{-34-34+27+9+9}}{2\times1.6\times16.5\times16.5}$$

$$\text{K.E.} = 0.05045 \times 10^{-68+45}$$

$$= 5.045 \times 10^{-2} \times 10^{-23}$$

$$\text{K.E.} = 5.045 \times 10^{-25}\text{ J.}$$

Q. 6. *Mention the significance of Davisson and Germer's experiment. An α-particle and a proton are accelerated from rest through the same potential difference V. Find the ratio of de-Broglie wavelengths associated with them.*

Ans. The Davisson and Germer's experiment established famous de-Broglie hypothesis of wave-particle duality by confirming the wave nature of moving particle.

The energy acquired by alpha-particle,

$$E_\alpha = 2\text{eV}$$

The energy acquired by proton,

$$E_p = \text{eV}$$

de-Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mE}}$

Hence, the ratio of wavelengths,

$$\frac{\lambda_\alpha}{\lambda_p} = \frac{\sqrt{m_p E_p}}{\sqrt{m_\alpha E_\alpha}} = \frac{\sqrt{m_p E_p}}{\sqrt{(4m_p)(2E_p)}} \qquad [\because \quad m_\alpha = 4\ m_p \text{ and } E_\alpha = 2E_p]$$

$$\Rightarrow \qquad \frac{\lambda_\alpha}{\lambda_p} = \frac{1}{2\sqrt{2}}$$

or, $\qquad \lambda_\alpha : \lambda_p = 1 : 2\sqrt{2}$.

Q. 7. *Radiations of frequency 10^{15} Hz are incident on two photosensitive surfaces A and B. Following observations are recorded:*

Surface A: No photoemission takes place.

Surface B: Photoemission takes place but photoelectrons have zero energy.

Explain the above observations on the basis of Einstein's photoelectric equation.

How will the observation with surface B change when the wavelength of incident radiations is decreased?

Ans. According to Einstein's photoelectric equation, kinetic energy of a photoelectron is

$$K_{max} = h\nu - \phi_0 \quad \Rightarrow \quad K_{max} = h(\nu - \nu_0)$$

or, $$K_{max} = hc\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right]$$

(*i*) The threshold frequency of surface *A* is greater than 10^{15} Hz, so no photoemission takes place.

(*ii*) For surface *B*, the threshold frequency is equal to 10^{15} Hz. So photoemission takes place but photoelectrons have zero kinetic energy.

When the wavelength λ of the incident radiation is decreased, the kinetic energy of photoelectrons emitted from surface *B* increases.

Q. 8. *Calculate de-Broglie wavelength in nm associated with a ball of mass 66 g moving with a velocity 2.5×10^5 ms^{-1}. Given $h = 6.6 \times 10^{-34}$ Js.*

Ans. Given,

$$m = 66 \text{ g} = 66 \times 10^{-3} \text{ kg}$$
$$v = 2.5 \times 10^5 \text{ ms}^{-1}$$

We know,

$$\lambda = \frac{h}{mv}$$

$$\Rightarrow \qquad \lambda = \frac{6.6 \times 10^{-34}}{66 \times 10^{-3} \times 2.5 \times 10^5}$$

$$= 4 \times 10^{-38} \text{ m} = \frac{4 \times 10^{-38}}{10^{-9}} \text{ nm}$$

$$\Rightarrow \qquad \lambda = 4 \times 10^{-29} \text{ nm.}$$

Q. 9. *Sketch the graphs showing the variation of stopping potential with frequency of incident radiations for two photosensitive materials A and B having threshold frequencies $\nu_0' > \nu_0$ respectively.*

(i) Which of the two metals, A or B has higher work function?

(ii) What information do you get from the slope of the graphs?

(iii) What does the value of the intercept of graph 'A' on the potential axis represent?

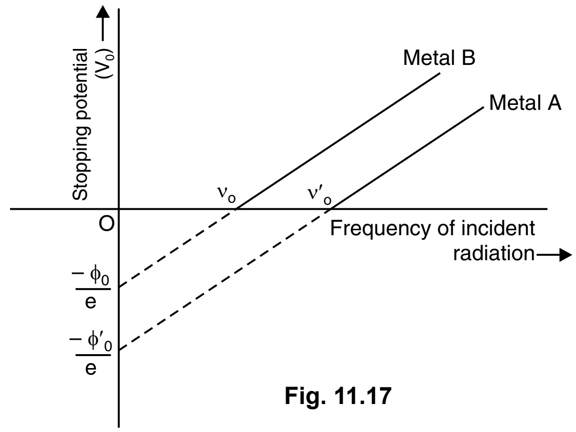

Fig. 11.17

Ans. (*i*) Since work function $\phi = h\nu$

$\therefore \quad \phi_0' = h\nu_0'$

Metal '*A*' has higher work function as $\nu_0' > \nu_0$.

(*ii*) Slope of $\quad V_0 > \nu \quad \therefore$ graph $= \dfrac{h}{e}$

(*iii*) Intercept of graph *A* on the potential axis

$$-\frac{\phi_0'}{e} = -\frac{h\nu_0'}{e}.$$

Q. 10. *A nucleus of mass M initially at rest splits into two fragments of masses m/3 and 2m/3. Find the ratio of de-Broglie wavelength of the two fragments.*

Ans. Let $\quad m_1 = \dfrac{m}{3}$ and $m_2 = \dfrac{2m}{3}$

But $\quad m_1 + m_2 = M \Rightarrow m_1 = M - m_2$

Following the low of conservation of linear momentum,

$$m_1v_1 + m_2v_2 = 0$$

$$\Rightarrow \quad m_1v_1 = -m_2v_2$$

Also, $\quad \lambda = \dfrac{h}{mv}$

Hence, $\quad \left|\dfrac{\lambda_1}{\lambda_2}\right| = \left|\dfrac{m_2v_2}{m_1v_1}\right| = 1.$

Q. 11. *The work function of lithium is 2.3 eV. What does it mean? What is the relation between the work function 'W' and threshold wavelength 'λ' of a metal?*

Ans. The work function of lithium is 2.3 eV. This means that to remove the outermost electron from the ground shell of a lithium atom, an energy of 2.3 eV is required.

The relation of work function (W) and wavelength (λ) is given as

$$\boxed{W = h\upsilon}$$

$$W = \frac{hc}{\lambda}.$$

Q. 12. *Why are de-Broglie waves associated with a moving football not visible?*
The wavelength, λ, of a photon and the de-Broglie wavelength of an electron have the same value. Show that the energy of the photon is $\frac{2\lambda mc}{h}$ *times the kinetic energy of the electron, where m, c and h have their usual meanings for electron.*

Ans. Because of large mass of a football, the wavelength associated with a moving football is small. So its wave nature is not visible.

de-Broglie wavelength of electron,

$$\lambda = \frac{h}{p}$$

Momentum of electron,

$$p = \frac{h}{\lambda}$$

K.E. of electron $= \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$ $[\because p = mv]$

Energy of a photon $= \frac{hc}{\lambda}$

$$\therefore \quad \frac{\text{Kinetic energy of photon}}{\text{Kinetic energy of electron}} = \frac{hc}{\lambda}\cdot\frac{2m\lambda^2}{h^2} = \frac{2\lambda mc}{h}$$

$$\therefore \quad \text{Energy of photon} = \frac{2\lambda mc}{\lambda} \times \text{K.E. of electron.}$$

Q. 13. *Red light, however bright it is, cannot produce the emission of electrons from a clean zinc surface. But even weak ultraviolet radiation can do so. Why?*
X-ray of wavelength 'λ' fall on a photosensitive surface, emitting electrons. Assuming that the work function of the surface can be neglected, prove that the de-Broglie wavelength of the electrons emitted will be $\sqrt{h\lambda / 2mc}$.

Ans. The emission of photoelectron depends upon the frequency of the incident light and not on intensity. The threshold frequency required for the zinc surface must be greater than that of red light but less than that of *UV* light. Hence, weak *UV* light is able to produce photoelectron but red light cannot.

The kinetic energy of photoelectron is given by,

$$\boxed{\frac{1}{2}mv^2_{max} = h\nu - \phi_0}$$

$$\Rightarrow \quad \frac{1}{2}mv^2_{max} \approx h\nu = \frac{hc}{\lambda} \quad \text{(Neglecting the term for work function)}$$

or, $$m^2 v^2_{max} = \frac{2mhc}{\lambda}$$

or, $$p = mv_{max} = \sqrt{\frac{2mhc}{\lambda}}$$

But de-Broglie wavelength,

$$\boxed{\lambda = \frac{h}{p}} = \frac{h}{\sqrt{2mhc/\lambda}} = \sqrt{\frac{h\lambda}{2mc}}.$$

Q. 14. *The wavelength of light in the visible region is about 390 nm for violet colour, about 550 nm (average wavelength) for yellow-green colour and about 760 nm for red colour.*

What are the energies of photons in (eV) at the (i) violet end, (ii) average wavelength, yellow-green colour, and (iii) red end of the visible spectrum? (Take h = 6.63 × 10^{-34} Js and 1 eV = 1.6 × 10^{19}J).

Ans. Energy of the incident photon,

$$E = h\nu = \frac{hc}{\lambda}$$

$$\Rightarrow \quad E = (6.63 \times 10^{-34} \text{ Js})\ (3 \times 10^{8} \text{ ms}^{-1})/\lambda$$

$$= \frac{1.989\times10^{-25}\,\text{Jm}}{\lambda}$$

(*i*) For violet light,

$$\lambda_1 = 390 \text{ nm (lower wavelength end)}$$

Incident photon energy

$$E_1 = \frac{1.989\times10^{-25}\,\text{Jm}}{390\times10^{-9}\,\text{m}} = 5.10 \times 10^{-19}\text{ J}$$

$$= \frac{5.10\times10^{-19}\,\text{J}}{1.6\times10^{-19}\,\text{J/eV}} = 3.19 \text{ eV}$$

(*ii*) For yellow-green light, λ_2 = 550 nm (average wavelength)

Incident photon energy,

$$E_2 = \frac{1.989\times10^{-25}\text{ Jm}}{550\times10^{-9}\text{ m}}$$

$$\Rightarrow \quad E_2 = 3.62 \times 10^{-19}\text{ J} = 2.26 \text{ eV}$$

(*iii*) For red light, λ_3 = 760 nm (higher wavelength end)

Incident photon energy, $$E_3 = \frac{1.989\times10^{-25}\,\text{Jm}}{760\times10^{-9}\,\text{m}}$$

$$\Rightarrow \quad E_3 = 2.62 \times 10^{-19}\text{ J} = 1.64 \text{ eV.}$$

Q. 15. *The following graph shown the variation of stopping potential V_0 with the frequency ν of the incident radiation for two photosensitive metals P and Q:*

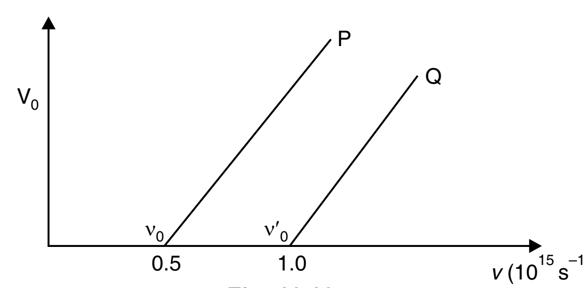

Fig. 11.18

(i) Explain which metal has smaller threshold wavelengths.

(ii) Explain, giving reason, which metal emits photoelectrons having smaller kinetic energy.

(iii) If the distance between the light source and metal P is doubled, how will the stopping potential change?

Ans. (*i*) Suppose the frequency of incident radiations of metal *Q* and *P* be ν_0 and ν_0' respectively.

$$\therefore \quad \nu_0 > \nu_0'$$

$\because \qquad \nu_0 = \dfrac{c}{\lambda_0}$

$\therefore \qquad \dfrac{c}{\lambda_0} > \dfrac{c}{\lambda_0'}$

$\Rightarrow \qquad \lambda_0 < \lambda_0'$

Therefore, metal '*Q*' has smaller wavelength.

(*ii*) As we know,

$$E = h\nu_0$$

$$\Rightarrow \qquad E \propto \nu_0$$

Hence, metal '*P*' has smaller kinetic energy.

(*iii*) Stopping potential remains unaffected because the value of stopping potential for a given metal surface does not depend on the intensity of the incident radiation. It depends on the frequency of incident radiation.

III. LONG ANSWER TYPE QUESTIONS

Q. 1. *Light of wavelength 2000 Å falls on an aluminium surface (work function of aluminium 4.2 eV). Calculate*

(a) the kinetic energy of the fastest and slowest emitted photoelectrons

(b) stopping potential

(c) cut-off wavelength for aluminium.

Ans. (*a*) We know that,

$$\text{Energy of photon} = \frac{hc}{\lambda}$$

$$\Rightarrow \qquad E = \frac{6.62\times10^{-34}\times3\times10^{8}}{2000\times10^{-10}}\ \text{J}$$

$$\text{or,} \qquad E = \frac{6.62\times10^{-34}\times3\times10^{8}}{2\times10^{-7}\times1.6\times10^{-19}}\ \text{eV} = 6.20\ \text{eV}$$

Energy of the fastest emitted photoelectron

$= h(\nu - \nu_0)$ (Where ν_0 is the work function)

$= (6.2 - 4.2)$ eV = 2.0 eV

Since, the emitted electrons from a metal surface have an energy distribution, the minimum energy in this distribution being zero, the energy of slowest photoelectrons is also zero.

(*b*) Since, $eV_0 = \frac{1}{2}mv^2_{max}$ where $\frac{1}{2}mv^2_{max}$ is the maximum energy of the emitted photoelectrons and V_0 is the stopping potential, the stopping potential is 2V.

(*c*) The threshold frequency is related to the work function by the relation

$$\phi_0 = h\nu_0 = \frac{hc}{\lambda_0}$$

$$\therefore \qquad \frac{1}{\lambda_0} = \frac{\phi_0}{hc} = \frac{4.2\times1.6\times10^{-19}\ \text{J}}{6.62\times10^{-34}\ \text{Js}\times3\times10^{8}\ \text{ms}^{-1}}$$

$$\therefore \qquad \lambda_0 = 3\times10^{-7}\ \text{m} = 3000\ \text{Å}.$$

Q. 2. *Define the terms: (i) work function, (ii) threshold frequency and (iii) stopping potential, with reference to photoelectric effect.*

Calculate the maximum kinetic energy of electrons emitted from a photosensitive surface of work function 3.2 eV, for the incident radiation of wavelength 300 nm.

Ans. For definition of terms

– work function

– threshold frequency

– stopping potential (refer text)

Numerical. The maximum kinetic energy of emitted photoelectron is given by

$$\text{K.E.} = \frac{1}{2}mv^2_{\text{max}} = h\nu - \phi_0$$

$$\Rightarrow \quad \boxed{E = \frac{hc}{\lambda} - \phi_0}$$

$$\Rightarrow \quad E = \frac{6.6\times10^{-34}\times3\times10^{8}}{300\times10^{-9}} - 3.2 \times 1.6 \times 10^{-19}$$

$$= 6.6 \times 10^{-19} - 5.12 \times 10^{-19}$$

$$= 1.48 \times 10^{-19}\text{ J.}$$

Q. 3. *Derive the expression for the de-Broglie wavelength of an electron moving under a potential difference of V volt.*

Describe the Davisson and Germer experiment to establish the wave nature of electrons. Draw labelled diagram of the apparatus used.

Ans. See Text for de-Broglie wavelength.

For Davisson and Germer experiment as well as experimental arrangements (see text).

Q. 4. *When a surface is irradiated with light of λ = 4950 Å, a photocurrent appears which vanishes if a retarding potential greater than 0.6 V is applied across the photo tube. When a different source of light is used, it is found that the critical retarding potential is changed to 1.1 V. What is the work function of the surface and the wavelength of the second source? If the photoelectrons (after emission from the source) are subjected to a magnetic field of 10 tesla what changes will be observed in the above two retarding potentials?*

Ans. According to Einstein's equation of photo-electricity

$$\frac{1}{2}mv^2_{\text{max}} = eV_0 = h\nu - \nu_0$$

$$\text{or,} \quad \boxed{eV_0 = \frac{hc}{\lambda} - \phi_0}$$

where ϕ_0 is the work function, λ wavelength of incident light and V_0 is the stopping potential.

For the first source,

$$\lambda_1 = 4950\text{ Å} = 4950 \times 10^{-10}\text{ m}$$

$$V_0 = 0.6\text{ V}$$

$$\therefore \quad 1.6 \times 10^{-19} \times 0.6 = \frac{6.6\times10^{-34}\times3\times10^{8}}{495\times10^{-9}} - \phi_0$$

$$\text{or,} \quad 0.96 \times 10^{-19} = 4 \times 10^{-19} - \phi_0$$

$$\therefore \qquad \phi_0 = 3.04 \times 10^{-19} \text{ J} \qquad \text{...(I)}$$

$$= \frac{3.04 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 1.9 \text{ eV}$$

Let λ_2 be the wavelength of the second source.

Given, $V_0' = 1.1$ V

Therefore,

$$1.6 \times 10^{-19} \times 1.1 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{\lambda_2} - 3.04 \times 10^{-19} \text{ J} \quad \text{(from I)}$$

or,
$$1.76 \times 10^{-19} = \frac{19.8 \times 10^{-26}}{\lambda_2} - 3.04 \times 10^{-19}$$

or,
$$\frac{19.8 \times 10^{-26}}{\lambda_2} = 4.8 \times 10^{-19}$$

$$\therefore \qquad \lambda_2 = \frac{19.8 \times 10^{-26}}{4.8 \times 10^{-19}} \text{ m} = 4.125 \times 10^{-7} \text{ m}$$

$$= 4125 \text{ Å}$$

When the ejected photoelectrons are subjected to the action of a magnetic field no change in retarding potential will be observed. This is because a magnetic field does not alter the kinetic energy of the photoelectrons. The magnetic field only changes the direction of motion.

QUESTIONS ON HIGH ORDER THINKING SKILLS (HOTS)

Q. 1. *In Davisson and Germer experiment if the peak diffraction pattern is obtained at the voltage of 54 V, find the wavelength associated with the electron used for obtaining the diffraction pattern. What should be the angle corresponding to this voltage, which gives maximum diffraction.*

Ans.
$$\boxed{\lambda = \frac{1.227}{\sqrt{V}}} = \frac{1.227}{\sqrt{54}} = 0.165 \text{ nm} = 1.65 \text{ Å}$$

From Bragg's law, $\boxed{2d \sin\theta = n\lambda = \lambda}$ [For first order diffraction, $n = 1$]

For *Ni* crystal, $d = 0.91$ Å

Hence, $2 \times 0.91 \times 10^{-10} \sin\theta = 1.65 \times 10^{-10}$

$$\theta = 65°$$

But
$$\theta = \frac{1}{2}(180° - \phi)$$

Hence, for $\theta = 65°$, the scattering angle, $\phi = 50°$.

Q. 2. *Light of wavelength 5000 Å falls on a sensitive plate with work function 1.90 eV. Calculate (a) the energy of the photon in eV, (b) kinetic energy of the emitted photoelectrons and (c) stopping potential.*

Ans. Given, $h = 6.62 \times 10^{-34}$ Js.

(*a*) $\boxed{E = h\nu} = \dfrac{hc}{\lambda} = \dfrac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{5000 \times 10^{-10}}$ J

or, $E = \dfrac{6.62 \times 3}{5} \times 10^{-19}$ J

$= \dfrac{6.62 \times 3}{5 \times 1.6 \times 10^{-19}} \times 10^{-19}$ eV = 2.48 eV

(*b*) $\boxed{\text{K.E.} = E - \phi_0}$ = (2.48 – 1.90) eV = 0.58 eV

(*c*) eV_0 = K.E. or $V_0 = \dfrac{\text{K.E.}}{e} = \dfrac{0.58 \text{ eV}}{e}$ = 0.58 V.

Q. 3. *A proton and an electron have same de-Broglie wavelength which of them moves fast and which possesses more K.E. Justify your answer.*

Ans. Kinetic energy of particle of mass m having momentum p is

$$K = \frac{1}{2}\frac{p^2}{m} \quad \text{or} \quad p = \sqrt{2mK}$$

de-Broglie wavelength, $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mK}}$

$\therefore$ $p = \dfrac{h}{\lambda}$ ...(*i*)

and $K = \dfrac{h^2}{2m\lambda^2}$ ...(*ii*)

If λ is constant, then from (*i*), $p = a$ constant

i.e., $m_p v_p = m_e v_e$ or $\dfrac{v_p}{v_e} = \dfrac{m_e}{m_p} < 1$ or $v_p < v_e$

If λ is constant, then from (*ii*), $K \propto \dfrac{1}{m}$

$\therefore$ $\dfrac{K_p}{K_e} = \dfrac{m_e}{m_p} < 1$ or $K_p < K_e$

It means the velocity of electron is greater than that of proton. Kinetic energy of electron is greater than that of proton.

Q. 4. *For a photosensitive surface, threshold wavelength is λ_0. Does photoemission occur, if the wavelength (λ) of the incident radiation is (a) more than λ_0 (b) less than λ_0? Justify your answer.*

Ans. From Einstein's photoelectric equation,

$$\boxed{h\nu = h\nu_0 + \frac{1}{2} m v^2_{\text{max}}}$$

$$h\frac{c}{\lambda} = h\frac{c}{\lambda_0} + \frac{1}{2} m v^2_{\text{max}}$$

(*a*) When $\lambda > \lambda_0$, $\frac{1}{2} mv^2_{max}$ is negative.

So, photoemission will not occur.

(*b*) When $\lambda > \lambda_0$, $\frac{1}{2} mv^2_{max}$ is positive.

So, photoemission will occur.

Q. 5. *Find the maximum velocity of photoelectrons emitted by radiation of frequency 3 × 10*^{15} *Hz from a photoelectric surface having a work function 4.0 eV.*

Ans.

$$\frac{1}{2} mv^2_{max} = h\nu - \phi_0$$

$$= 6.63 \times 10^{-34} \times 3 \times 10^{15} - 4 \times 1.6 \times 10^{-19}$$

or,
$$v^2_{max} = \frac{2[19.89 \times 10^{-19} - 6.4 \times 10^{-19}]}{9.1 \times 10^{-31}}$$

$$= \frac{26.98 \times 10^{-19}}{9.1 \times 10^{-31}} = 2.96 \times 10^{12}$$

$$v_{max} = 1.72 \times 10^6 \text{ ms}^{-1}.$$

Q. 6. *Find the de-Broglie wavelength of neutron at 27°C. Given, Boltzmann constant, 1.38 ×* 10^{-23} *J molecule*$^{-1}$ *k*$^{-1}$*, h = 6.63 ×* 10^{-34} *Js; mass of neutron 1.66 ×* 10^{-23} *kg.*

Ans. Here, $T = 27°C = 27 + 273 = 300$ K.

Energy of neutron at temperature *Tk* is

$$E = \frac{3}{2} kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300$$

$$= 6.21 \times 10^{-21} \text{ J}$$

Now,
$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.66 \times 10^{-27} \times 6.21 \times 10^{-21}}}$$

or, $\lambda = 1.46$ Å.

Q. 7. *Ultraviolet light of wavelengths 800 Å and 700 Å, when allowed to fall on hydrogen atom in their ground state, is able to liberate electrons with kinetic energy 1.8 eV and 4.0 eV respectively. Find the value of Planck's constant.*

Ans. The energy E is related to wavelength λ, velocity of light c and Planck's constant is

$$E = \frac{hc}{\lambda} \qquad (h \rightarrow \text{Planck's constant})$$

For radiations of wavelengths λ_1 and λ_2 respectively, energies E_1 and E_2.

$$E_1 = \frac{hc}{\lambda_1} \quad \text{and} \quad E_2 = \frac{hc}{\lambda_2}$$

Subtracting,
$$E_1 - E_2 = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

or, $$h = \frac{(E_1 - E_2)\lambda_1\lambda_2}{c(\lambda_2 - \lambda_1)}$$

∴ $$h = \frac{(4.0 - 1.8) \times 1.6 \times 10^{-19} \times 700 \times 10^{-10} \times 800 \times 10^{-10}}{3 \times 10^8 \times 100 \times 10^{-10}}$$

or, $h = 6.57 \times 10^{-34}$ Js.

Q. 8. *Show, on a graph the nature of variation of the (associated) de-Broglie wavelength (λ) with the accelerating potential (V), for an electron initially at rest.*

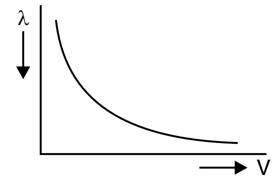

Fig. 11.19

Ans. The variation of λ with V is shown in Fig. 11.19.

Q. 9. *A proton and an alpha particle, both initially at rest, are (suitablly) accelerated so as to have the same kinetic energy. What is the ratio of their de-Broglie wavelengths?*

Ans. ∵ $$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \quad \text{(for same } K\text{)}$$

∴ $$\frac{\lambda p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha}{m_p}} = \sqrt{\frac{4m}{m}} = 2.$$

Q. 10. *What two main observations in photoelectricity led Einstein to suggest the photon theory for the interaction of light with the free electrons in metal? Obtain an expression for threshold frequency for photoelectric emission in terms of the work function of the metal.*

Ans. The two main observations are:

(*i*) The maximum kinetic energy of emitted photoelectron is independent of intensity of light.

(*ii*) For each photoelectron, there must be a threshold frequency of incident light below which no emission takes place.

For the metal of work function ϕ, the kinetic energy of photoelectron emitted due to falling of photon of frequency ν is

$$\frac{1}{2} mv^2_{max} = h\nu - \phi$$
$$= h\nu - h\nu_0$$

where ν_0 is the threshold frequency.

For just emission, $v_{max} = 0$

∴ $$h\nu = h\nu_0 = \phi$$

or $$\nu_0 = \frac{\phi}{h}.$$

Q. 11. *The maximum velocity of electrons, emitted from a metal surface of negligible work function is v, when frequency of light falling on it is f. What will be the maximum velocity when the frequency of incident light made 4f?*

Ans. ∵ $$\frac{1}{2} mv_x^2 = hf - \phi$$

For $$\phi = 0$$

$$v_x^2 = \frac{2hf}{m}, \quad v_x = \sqrt{\frac{2hf}{m}}$$

Thus, for $f' = 4f$

$$v_x' = \sqrt{\frac{2h4f}{m}} = 2v_x$$

So, maximum velocity will be doubled.

Q. 12. *What reasoning led de-Broglie to put forward the concept of matter wave? The wavelength λ, of a photon, and de-Broglie wavelength associated with a particle of mass m has the same value, say λ. Show that the energy of photon is* $\frac{2\lambda mc}{h}$ *times the kinetic energy of the particle.*

Ans. de-Broglie put forward the bold hypothesis that moving particles of matter should display wave-like properties under suitable conditions. He reasoned that nature is symmetrical and that the two basic physical entities, matter and energy must have symmetrical character. If radiation shows a dual nature, so should matter.

$$\because \quad K = \frac{P^2}{2m} \quad \text{or} \quad P = \frac{h}{\lambda}$$

$$\therefore \quad K = \frac{h^2}{2m\lambda^2}$$

Also, $$E\text{ (photon)} = \frac{hc}{\lambda}$$

$$\therefore \quad \frac{K}{E} = \frac{h}{2mc\lambda}$$

or $$E = \frac{2mc\lambda}{h} . K.$$

MULTIPLE CHOICE QUESTIONS

1. For production of characteristic K_β X-rays the electron transmission is

(*a*) $n = 2$ to $n = 1$ (*b*) $n = 3$ to $n = 2$
(*c*) $n = 3$ to $n = 1$ (*d*) $n = 4$ to $n = 2$

2. When ultraviolet rays incident on metal plate then photoelectric effect does not occur, it occurs by incidence of

(*a*) Infrared rays (*b*) Radio waves
(*c*) X-rays (*d*) Light waves

3. The de-Broglie wavelength of an electron in the first Bohr orbit is

(*a*) equal to the circumference of the first orbit.
(*b*) equal to twice the circumference of the first orbit.
(*c*) equal to half the circumference of the first orbit.
(*d*) equal to one-fourth the circumference of two first orbit.

4. The de-Broglie wavelength associated with proton changes by 0.25 percent if its momentum is changed by P_0. The initial momentum was

(*a*) 100 P_0 (*b*) $P_0/400$ (*c*) 401 P_0 (*d*) $P_0/400$

5. Photoelectric effect shows
(*a*) Wave nature of electrons (*b*) Particle nature of light
(*c*) Both (*a*) and (*b*) (*d*) None of these

6. The wavelength associated with a gold ball weighing 200 g and moving at a speet of 5 m/h is of the order of (h = 6.626 × 10^{-34} Js)
(*a*) 10^{-10} m (*b*) 10^{-20} m (*c*) 10^{-30} m (*d*) 10^{-40} m

7. If the kinetic energy of a free electron doubles, its de-Broglie wavelength changes by the factor
(*a*) $\sqrt{2}$ (*b*) $1\sqrt{2}$ (*c*) 2 (*d*) 1/2

8. If the kinetic energy of the particle is increased by 16 times, the percentage change in the de-Broglie wavelength of the particle is
(*a*) 25% (*b*) 75%
(*c*) 60% (*d*) 50%
(*e*) 30%

9. The potential difference applied to an X-ray tube is 5 kV and the current through it is 3.2 mA. Then the number of electrons striking the target per second is
(*a*) 2 × 10^{16} (*b*) 5 × 10^{16}
(*c*) 1 × 10^{17} (*d*) 4 × 10^{15}

10. The lowest frequency of light that will cause the emission of photoelectrons from the surface of a metal (for which work function is 1.65 eV) will be
(*a*) 4 × 10^{10} Hz (*b*) 4 × 10^{11} Hz
(*c*) 4 × 10^{14} Hz (*d*) 4 × 10^{-10} Hz

11. An electron and proton have the same de-Broglie wavelength. Then K.E. of the electron is
(*a*) zero (*b*) infinity
(*c*) equal to K.E. of the proton (*d*) greater than K.E. of proton
(*e*) none of the above

12. Two identical metal plates show photoelectric effect by a light of wavelength λ_A falls on plate *A* and λ_B on plate *B*. ($\lambda_A = 2\lambda_B$). The maximum kinetic energy is
(*a*) $2K_A = K_B$ (*b*) $K_A < K_B/2$
(*c*) $K_A = 2K_B$ (*d*) $K_A = K_B/2$

13. Cathode rays travelling from east to west enter into region of electric field directed towards north to south in the plane of paper. The deflection of cathode rays is towards
(*a*) East (*b*) West (*c*) South (*d*) North

14. The de-Broglie wavelength of a particle moving with a velocity of 2.25 × 10^8 m/s is equal to the wavelength of photon. The ratio of kinetic energy of the particle to the energy of the photon is (velocity of light is 3 × 10^8 m/s)
(*a*) 1/8 (*b*) 3/8 (*c*) 5/8 (*d*) 7/8

15. We may state that the energy E of a photon of frequency V is $E = h\nu$, where h is Plank's constant. The momentum P of a photon is $P = h/\lambda$ where λ is the wavelength of the photon. From the above statement one may conclude that the wave velocity of light is equal to
(*a*) 3 × 10^8 m/s (*b*) E/p
(*c*) Ep (*d*) $(E/p)^2$

Answers

1. (*c*)	**2.** (*c*)	**3.** (*a*)	**4.** (*c*)	**5.** (*b*)
6. (*c*)	**7.** (*b*)	**8.** (*b*)	**9.** (*a*)	**10.** (*c*)
11. (*d*)	**12.** (*b*)	**13.** (*d*)	**14.** (*b*)	**15.** (*b*).

TEST YOUR SKILLS

1. Two metals X and Y have work functions 2 eV and 5 eV respectively. Which metal will emit electrons, when it is irradiated with light of wavelength 400 mm and why?

2. Work functions of three elements A, B and C are as given below

(A) 5.0 eV, (B) 3.8 eV, (C) 2.8 eV

A radiation of wavelength 4125 Å is made to be incident on each of these elements. By appropriate calculations, show in which case photoelectrons will not be emitted.

3. How does the (*i*) photoelectric current and (*ii*) kinetic energy of the photoelectron emitted in a photocell vary if the intensity of the incident radiation is doubled?

4. An electron, an alpha particle and a proton have the same kinetic energy. Which one of these particle has the largest de-Broglie wavelength?

5. In a plot of photoelectric current versus anode potential, how does (*i*) the saturation current vary with anode potential for incident radiation of different frequency but same intensity? (*ii*) the stopping potential vary for incident radiations of different intensity but same frequency? (*iii*) photoelectric current vary for different intensities but same frequency of incident radiations? Justify your answer in each case.

6. Photon of frequency ν has a momentum associated with it. If c is the velocity of light, then what will be momentum of the photon?

7. When light of wavelength 300 nm falls on a photoelectric emitter, photoelectrons are liberated for another emitter, however, light of 600 nm wavelength is sufficient for creating photo emission. What is the ratio of work function of the two emitters?

8. The photosensitive surface is receiving light of wavelength 5000 Å at the rate of 10^{-8} Js^{-1}. Find the number of photons received per second.

9. An electron and alpha particle have the same de-Broglie wavelength associated with them. How are their kinetic energies related to each other?

10. An electromagnetic wave of wavelength λ is incident on a photosensitive surface of negligible work function. If the photo-electrons emitted from this surface have the de-Broglie wavelength λ_1, prove that $\lambda = [2mc/h]\lambda_1^2$.

11. In Davisson and Germer experiment, state the observations which led to (*i*) show the wave nature of electrons and (*ii*) confirm the de-Broglie relation.

12. Write Einstein's photoelectric equation. Explain the terms (*i*) threshold frequency and (*ii*) stopping potential.

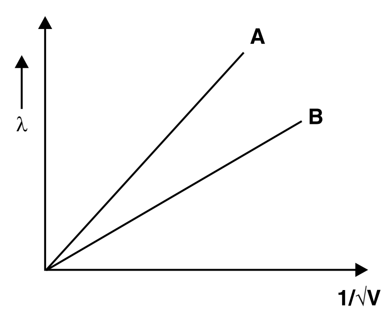

Fig. 11.20

13. Two lines, A and B in the plot given in figure 11.20, show the variation of de Broglie wavelength, λ versus $1/\sqrt{V}$, where V is the accelerating potential difference, for two particles carrying the same charge. Which one of two represents a particle of smaller mass?

14. The following graph shows the variation of stopping potential V_0 with the frequency ν of the incident radiation for two photosensitive metals X and Y:

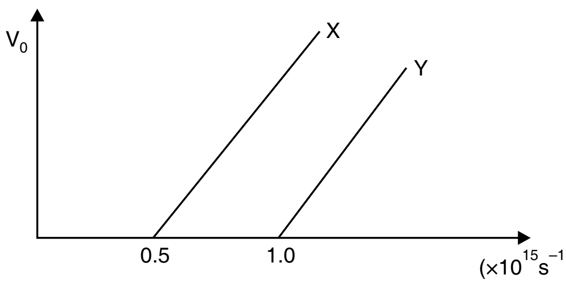

Fig. 11.21

(*i*) Which of the metal has larger threshold wavelength? Give reason.

(*ii*) Explain, giving reason, which metal gives out electrons, having larger kinetic energy, for the same wavelength of the incident radiation.

(*iii*) If the distance between the light source and metal X is halved, how will the kinetic energy of electrons emitted from it change? Give reasons.

15. The figure shows a plot of three curves *a*, *b*, *c* showing the variation of photocurrent Vs. Collector plate potential for three different intensities I_1, I_2 and I_3 having frequencies V_1, V_2 and V_3 respectively incident on a photosensitive surface. Point out the two curve for which the incident radiations have same frequency but different intensities.

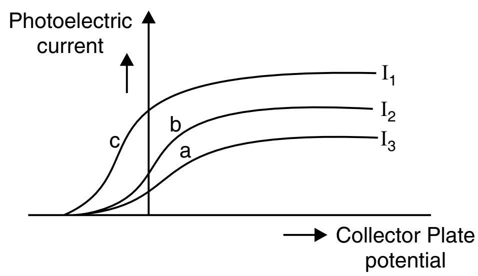

Fig. 11.22

16. A proton and an alpha particle are accelerated through the same potential one of the two has (*i*) greater value of de-Broglie wavelength associated with it, and (*ii*) less kinetic energy? Justify your answer.

17. Light of wavelength 2000 Å falls on a metal surface of work function 4.2 eV. What is the kinetic energy (in eV) of (*i*) the fastest and (*ii*) slowest photoelectrons emitted from the surface?

18. The most probable kinetic energy of a thermal neutron at temperature T may be taken as equal to KT where K is Boltzmann's constant. Taking the mass of a neutron and its associated de-Broglie wavelength as m and λ respectively, state the dependence of λ on m and T.

❑❑❑