11 Thermal Properties of Matter

Facts that Matter

- Heat is the form of energy transferred between two (or more) systems or a system and its surroundings by virtue of temperature difference. The SI unit of heat energy transferred is expressed in joule (*J*).

In CGS system, unit of heat is calorie and kilocalorie (kcal).

1 cal = 4.186 J and 1 kcal = 1000 cal = 4186 J.

- Temperature of a substance is a physical quantity which measures the degree of hotness or coldness of the substance. The SI unit of temperature is kelvin (K) and °C is a commonly used unit of temperature.
- A branch of science which deals with the measurement of temperature of a substance is known as thermometry. A device used to measure the temperature of a body is called thermometer.
- A thermometer calibrated for a temperature scale is used to measure the value of given temperature on that scale. For the measurement of temperature, two fixed reference points are selected. The two convenient fixed reference points are the ice point and the steam point of water at standard pressure, which are known as freezing point and boiling point of water at standard pressure.
- The two familiar temperature scales are the Fahrenheit temperature scale and the celsius temperature scale. The ice and steam point have values 32°F and 212°F respectively, on the Fahrenheit scale and 0°C and 100°C on the celsius scale. On the Fahrenheit scale, there are 180 equal intervals between two reference points, and on the celsius scale, there are 100.
- If t_C and t_F are temperature values of a body on Celsius temperature scale and Fahrenheit temperature scale respectively, then the relationship between Fahrenheit and Celsius temperature is given by

$$\frac{t_c - 0}{100} = \frac{t_F - 32}{180}$$

- An ideal gas obeys the following law. That is $PV = \mu RT$, where P, V and T are the pressure, volume and temperature of the gas respectively. μ is the number of moles in an ideal gas and $R = 8.31 \text{ J mol}^{-1} K^{-1}$ is known as universal gas constant. The equation, $PV = \mu RT$ is known as ideal gas equation.
- The absolute minimum temperature for an ideal gas, inferred by extrapolating the straight line $P - T$ graph is found to be – 273.15 °C and is designated as absolute zero. Absolute temperature scale (T) and Celsius scale are related by

$$t° C = T - 273.15$$

• Thermal Expansion

The increase of size of a body due to the increase in the temperature is called thermal expansion. Three types of expansions can take place in solids viz. linear, superficial and volume expansion.

(*i*) **Linear Expansion:** The increase in the length of a solid on heating is called linear expansion.

If the temperature of a rod of original length l is raised by a small amount Δt, its length increases by Δl. Then the linear expansion is given by

$$\boxed{\Delta l = l \propto \Delta t}$$

where α is the coefficient of linear expansion of the given solid. The unit of α is per degree celsius ($°C^{-1}$) in the CGS and per kelvin (K^{-1}) in the SI system.

(*ii*) **Superficial or Area Expansion:** The increase in surface area of the solid on heating is called superficial expansion.

If A_0 is the area of a solid at 0°C and A_t its area at t°C then

$$A_t = A_0 (1 + \beta t)$$

where β is known as the coefficient of superficial expansion. Unit of β is $°C^{-1}$ or K^{-1}.

(*iii*) **Volume Expansion:** The increase in volume of the solid on heating is called volume expansion.

The change in the volume of a solid with a change in temperature Δt is given by

$$\Delta V = V\gamma \Delta t$$

where γ is the coefficient of volume expansion.

• The relation among coefficients of linear expansion (α), superficial expansion (β) and volume expansion (γ) is given as

$$\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$$

• For a given solid, the three coefficients of expansion α, β, γ are not constant. Their values depend on the temperature range.

• Liquids have volume expansion only. If we do not take into account the expansion of solid container, then the expansion of liquid is called apparent expansion. On the other hand, if we take into account the expansion of solid too, it is referred as the real expansion of liquid. It is found that $\gamma_r = \gamma_a + \gamma_g$, where γ_r = real expansion coefficient of liquid, γ_a = apparent expansion coefficient of liquid and γ_g = volume expansion coefficient of container vessel (glass).

• Water exhibits an anomalous behaviour. It contracts on heating between 0 °C and 4 °C but expands on heating beyond 4 °C. Thus, specific volume of water is minimum at 4 °C or density of water is maximum at 4 °C. This property of water has an important environmental effect.

• Thermal Stress

When a rod is held between two fixed supports and its temperature is increased, the fixed supports do not allow the rod to expand, which results in a stress which is called thermal stress.

Thermal stress in the rod is given by

$$\frac{F}{A} = \text{Thermal stress} = Y \alpha \Delta t$$

where Y is the Young's modulus for the material of the rod, A is the area cross-section of the rod, α is the coefficient of linear expansion and F is the developed force in the rod.

$$F = \frac{YA\Delta l}{l}.$$

- **Thermal Capacity**

The thermal capacity of a body is the quantity of heat required to raise the temperature of the whole of the body through a unit degree. It is measured in calorie per °C or joule per *K*.

If *Q* be the amount of heat needed to produce a change in temperature (Δt) of the substance, then thermal capacity of the substance is given by

$$S = \frac{Q}{\Delta t}$$

Dimensional formula of heat capacity is [$ML^2T^{-2}K^{-1}$].

- **Specific Heat Capacity**

The specific heat capacity (also referred to as specific heat) of a substance is the amount of heat required to raise the temperature of a unit mass of substance through 1 °C. It is measured in cal g^{-1}(°C)$^{-1}$ or J kg^{-1} K^{-1}.

The specific heat capacity of a substance is given by

$$S = \frac{1}{m}\frac{Q}{\Delta t}$$

where *m* is mass of substance and *Q* is the heat required to change its temperature Δt.

- Molar specific heat capacity of a substance is defined as the amount of heat required to raise the temperature of 1 mole of the substance by 1°C.

It is given by

$$C = \frac{1}{\mu}\frac{Q}{\Delta t}$$ where μ is number of moles of a substance.

The unit of molar specific heat capacity is J $mole^{-1}$ K^{-1} in SI system and Cal mol^{-1} °C^{-1} in CGS system.

The dimensional formula of molar specific heat capacity is [ML^2 T^{-2} K^{-1} $mole^{-1}$].

- **Calorimetry**

Calorimetry is concerned with the measurement of heat, the basic apparatus for this purpose being called the *calorimeter.*

When two bodies at different temperatures are 'mixed', heat 'flows' from the body at a higher temperature to the one at a lower temperature, until a common 'equilibrium' temperature is reached. Assuming this 'heat exchange' to be confined to the two bodies alone (*i.e,* neglecting any heat loss to the surroundings) we have, from the law of energy conservation:

Heat gained by one body = heat lost by the other.

- Transition of matter from one state (solid, liquid and gas) to another is called a change of state.
- The change of state from solid to liquid is called melting and from liquid to solid is called fusion.

It is observed that the temperature remains constant until the entire amount of the solid substance melts *i.e.,* both the solid and liquid states of the substance co-exist in thermal equilibrium during the change of state from solid to liquid.

- The temperature at which a solid melts is called its melting point. The value of melting point of a solid is characteristic of the substance and depends on pressure also.
- Melting of ice under increased pressure and refreezing on reducing the pressure is called regelation.
- The change of state from liquid to vapour (or gas) is called vaporisation. The temperature at which the liquid and vapour states of a substance co-exist is called its boiling point.

• The change from solid state to vapour state without passing through the liquid state is called sublimation.

• The Basic Heat Formula

The heat Q required to raise the temperature of a mass m of a substance of specific heat capacity s through t degrees is given by

$$Q = m \times S \times t$$

i.e., Heat required = mass × specific heat × change in temperature

• Latent Heat

Latent heat of a substance is the amount of heat energy required to change the state of unit mass of the substance from solid to liquid or from liquid to gas/vapour without any change in temperature.

• The latent heat of fusion (L_f) is the heat per unit mass required to change a substance from solid into liquid at the same temperature and pressure. The latent heat of vaporisation (L_v) is the heat per unit mass required to change a substance from liquid to vapour state without change in temperature and pressure.

• Heat Transfer

Heat can be transferred from one place to another by three different methods, namely, conduction, convection and radiation. Conduction usually takes place in solids, convection in liquids and gases, and no medium is required for radiation.

(*i*) **Conduction:** According to Maxwell, conduction is the flow of heat through an unequally heated body from places of higher temperature to those of lower temperature. Rate of heat transfer is given by

$$H = \frac{Q}{t} = \frac{KA(T_1 - T_2)}{l}$$

where K is called Thermal Conductivity and A is area of cross-section.

(*ii*) **Convection:** Maxwell defines convection as the flow of heat by the motion of the hot body itself carrying its heat with it.

(*iii*) **Radiation:** Radiation is the mode of heat transfer in which heat travels directly from one place to another without the agency of any intervening medium.

• Thermal conductivity is defined as heat energy transferred in unit time from unit area having a unit difference in temperature over unit length. It is expressed in $\text{Js}^{-1}\ \text{m}^{-1}\ °\text{C}^{-1}$ or $W\ \text{m}^{-1}\ K^{-1}$.

• Thermal Resistance

The thermal resistance of a body is a measure of its opposition to the flow of heat through it. It is defined as

$$\text{Thermal resistance} = \frac{\text{temperature difference at the two ends}}{\text{rate of flow of heat through it}}$$

or

$$\text{Thermal resistance} = \frac{\text{length or thickness of the material}}{\text{thermal conductivity} \times \text{area}} = \frac{l}{KA}.$$

• Newton's Law of Cooling

Newton's law of cooling states that the rate of loss of heat of a body is directly proportional to the difference in temperature of the body and the surroundings, provided the difference in temperature is small, not more than 40 °C.

i.e., $$\frac{dT}{dt} = -K(T - T_s).$$

– ve sign implies that as time passes, temperature T decreases.

• When an object at temperature T_1 is placed in a surrounding of temperature T_2 the net energy radiated per second is, $P = eA\sigma\left(T_1^4 - T_2^4\right)$.

• Black Body Radiation

(*i*) **Emissive Power :** The amount of heat energy rediated per unit area of the surface of a body, per unit time and per unit wavelength range is constant which is called as the 'emissive power' (e_λ) of the given surface, given temperature and wavelength. Its S.I. unit is $Js^{-1}\ m^{-2}$.

(*ii*) **Absorptive Power :** When any radiation is incident over a surface of a body, a part of it gets reflected, a part of it gets refracted and the rest of it is absorbed by that surface. Therefore, the 'absorptive power' of a surface at a given temperature and for a given wavelength is the ratio of the heat energy absorbed by a surface to the total energy incident on it at a certain time. It is represented by (a_λ). It has no unit as it is a ratio.

$$\therefore \quad a_\lambda = \frac{\text{Amount of heat energy absorbed}}{\text{Total heat energy incident}}$$

(*iii*) **Perfect Black Body :** A body is said to be a perfect black body is its absorptivity is 1. It neither reflects nor transmits but absorbs all the thermal radiations incident on it irrespetive of their wavelengths.

(*iv*) **Wein's Displacement Law :** This law states that as the temperature increases, the maximum value of the radiant energy emitted by the black body, move towards shorter wavelengths. Wein found that "The product of the peak wavelength (λ_m) and the Kelvin temperature (T) of the black body should remain constant."

$$\lambda_m \times T = b$$

Where b is constant known as Wein's constant. Its value is 2.898×10^{-3} mk.

(*v*) **Stefan's Law :** This law states that the thermal radiations energy emitted per second from the surface of a black body is directly proportional to its surface area A and to the fourth power of its absolute temperature T.

If H be the thermal radiation energy emitted per unit time, then

$$H \propto AT^4 = \sigma \times AT^4$$

Where σ is a universal constant known as Stephan-Boltzman constant. Its value in S.I. unit is 5.67×10^{-8} W $m^{-2}k^{-4}$.

Emission coefficient or degree of blackness of a body is represented by a dimensionless quantity ε, $0 \le \varepsilon \le 1$. If $\varepsilon = 1$ then the body is perfectly black body. Hence

$$H = A\varepsilon\sigma T^4$$

Let us consider an object at absolute temperature T and To be the temperature of the surroundings.

$\therefore H_1$ = Rate of energy emitted by the body $= A\sigma T^4$

H_2 = Rate of energy absorbed by the body from the surroundings $= A\sigma T_0^4$

H = Net rate of loss of energy $= H_1 - H_2$

$$= A\sigma T^4 - A\sigma T_0^4 = A\sigma\left(T^4 - T_0^4\right)$$

(*vi*) **The Solar Constant :** The average energy emitted from the surface of the sun, absorbed per unit area, per minute by the earth is constant which is called as solar constant which is represented by S whose value is 8.135 $Jm^{-2}\ min^{-1}$.

Let the earth be moving in a circular path of radius r taking sun as its centre.

Taking sun as perfectly black body, the energy radiated per unit time from the surface of the sun is given by

$$H = A\sigma T^4$$

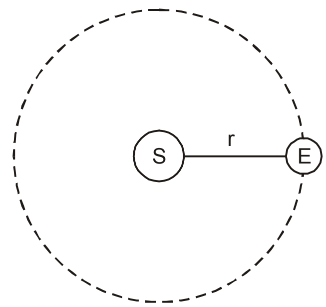
Where A is the surface area of the sun and T its absolute temperature

$$\therefore \quad H = (4\pi R^2) \times \sigma T^4 \quad (R = \text{radius of the sun})$$

Now the energy absorbed by unit area per seco3nd by the earth

$$= \frac{H}{4\pi r^2} = \frac{(4\pi R^2)\sigma T^4}{4\pi r^2}$$

$$= \left(\frac{R}{r}\right)^2 \cdot \sigma T^4$$



Hence the solar constant

$$S = \left(\frac{R}{r}\right)^2 \cdot \sigma T^4 \times 60$$

- **IMPORTANT TABLES**

TABLE 11.1 Coefficient of linear expansion (α) for some materials.

Materials	*α ($10^{-5}\ K^{-1}$)*
Aluminium	2.5
Brass	1.8
Iron	1.2
Copper	1.7
Silver	1.9
Gold	1.4
Pyrex glass	0.32
Lead	0.29

TABLE 11.2 Values of coefficient of volume expansion for some substances

Materials	*α_v(K^{-1})*
Aluminium	7×10^{-5}
Brass	6×10^{-5}
Iron	3.55×10^{-5}
Paraffin	58.8×10^{-5}
Glass (ordinary)	2.5×10^{-5}
Glass (pyrex)	1×10^{-5}
Hard rubber	2.4×10^{-4}
Invar	2×10^{-6}
Mercury	18.2×10^{-5}
Water	20.7×10^{-5}
Alcohol (ethyl)	110×10^{-5}

TABLE 11.3 Specific heat capacity of some substances at room temperature and atmospheric pressure

Substance	*Specific heat capacity ($J\ kg^{-1}\ K^{-1}$)*	*Substance*	*Specific heat capacity ($J\ kg^{-1}\ K^{-1}$)*
Aluminium	900.0	Ice	2060
Carbon	506.5	Glass	840
Copper	386.4	Iron	450
Lead	127.7	Kerosene	2118
Silver	236.1	Edible oil	1965
Tungsten	134.4	Mercury	140
Water	4186.0		

TABLE 11.4 Molar specific heat capacities of some gases

Gas	C_p *($J\ mol^{-1}\ K^{-1}$)*	C_v *($J\ mol^{-1}\ K^{-1}$)*
He	20.8	12.5
H_2	28.8	20.4
N_2	29.1	20.8
O_2	29.4	21.1
CO_2	37.0	28.5

TABLE 11.5 Temperatures of the change of state and latent heats for various substances at 1 atm pressure

Substance	*Melting Point (°C)*	L_f *($10^5\ J\ kg^{-1}$)*	*Boiling Point (°C)*	L_v *($10^5 J\ kg^{-1}$)*
Ethyl alcohol	– 114	1.0	78	8.5
Gold	1063	0.645	2660	15.8
Lead	328	0.25	1744	8.67
Mercury	– 39	0.12	357	2.7
Nitrogen	– 210	0.26	– 196	2.0
Oxygen	– 219	0.14	– 183	2.1
Water	0	3.33	100	22.6

TABLE 11.6 Thermal conductivities of some materials

Materials	*Thermal conductivity ($J\ s^{-1}\ m^{-1}\ K^{-1}$)*
Metals	
Silver	406
Copper	385
Aluminium	205
Brass	109
Steel	50.2
Lead	34.7
Mercury	8.3

Non-metals	
Insulating brick	0.15
Concrete	0.8
Body fat	0.20
Felt	0.04
Glass	0.8
Ice	1.6
Glass wool	0.04
Wood	0.12
Water	0.8
Gases	
Air	0.024
Argon	0.016
Hydrogen	0.14

NCERT TEXTBOOK QUESTIONS SOLVED

11.1. *The triple points of neon and carbon dioxide are 24.57 K and 216.55 K respectively. Express these temperatures on the Celsius and Fahrenheit scales.*

Sol. The relation between kelvin scale and Celsius scale is

$$T_K - 273.15 = T_C \Rightarrow T_C = T_K - 273.15$$

For neon, $T_K = 24.57$ K

$\therefore$ $T_C = 24.57 - 273.15 = -248.58$ °C

For CO_2, $T_K = 216.55$ K

$\therefore$ $T_C = 216.55 - 273.15 = -56.60$ °C

Also, the relation between Kelvin scale and Fahrenheit scale is

$$\frac{T_K - 273.15}{100} = \frac{T_F - 32}{180}$$

$\therefore$ $T_F = \frac{9}{5}(T_K - 273.15) + 32$

Now, for neon, $T_K = 24.57$ K

$\therefore$ $T_F = \frac{9}{5}[24.57 - 273.15] + 32 = -415.44$ °F

For CO_2, $T_K = 216.55$ K

$\therefore$ $T_F = \frac{9}{5}[216.55 - 273.15] + 32 = -69.88$ °F

11.2. *Two absolute scales A and B have triple points of water defined to be 200 A and 350 B. What is the relation between T_A and T_B ?*

Sol. As we know, triple point of water on absolute scale = 273.16 *K*, Size of one degree of kelvin scale on absolute scale A

$$= \frac{273.16}{200}$$

Value of temperature T_A on absolute scale A

$$= \frac{273.16}{200} T_A$$

Value of temperature T_B on absolute scale B

$$= \frac{273.16}{350} T_B$$

Since T_A and T_B represent the same temperature,

$$\therefore \quad \frac{273.16}{200} T_A = \frac{273.16}{350} T_B \quad \text{or} \quad T_A = \frac{200}{350} T_B = \frac{4}{7} T_B.$$

11.3. *The electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law: $R = R_0\, [1 + \alpha\, (T - T_0)]$.*
The resistances is 101.6 Ω at the triple-point of water 273.16 K, and 165.5 Ω at the normal melting point of lead (600.5K). What is the temperature when the resistance is 123.4 Ω ?

Sol. Here, $R_0 = 101.6\ \Omega$; $\quad T_0 = 273.16$ K

Case (*i*) $R_1 = 165.5\ \Omega$; $\quad T_1 = 600.5$ K

Case (*ii*) $R_2 = 123.4\ \Omega$; $\quad T_2 = ?$

Using the relation $\quad R = R_0\, [1 + a\, (T - T_0)]$

Case (*i*) $\quad 165.5 = 101.6\ [1 + \alpha\, (600.5 - 273.16)]$

$$\alpha = \frac{165.5 - 101.6}{101.6 \times (600.5 - 273.16)} = \frac{63.9}{101.6 \times 327 \times 34}$$

Case (*ii*) $\quad 123.4 = 101.6\ [1 + \alpha\, (T_2 - 273.16)]$

or $$123.4 = 101.6\left[1 + \frac{63.9}{101.6 \times 327.34}(T_2 - 273.16)\right]$$

$$= 101.6 + \frac{63.9}{327.34}\ (T_2 - 273.16)$$

or $$T_2 = \frac{(123.4 - 101.6) \times 327.34}{63.9} + 273.16 = 111.67 + 273.16$$

$$= \mathbf{384.83\ K}$$

11.4. *Answer the following:*

(*a*) *The triple-point of water is a standard fixed point in modern thermometry. Why ? What is wrong in taking the melting point of ice and the boiling point of water as standard fixed points (as was originally done in the Celsius scale) ?*

(*b*) *There were two fixed points in the original Celsius scale as mentioned above which were assigned the number 0 °C and 100 °C respectively. On the absolute scale, one of the fixed points is the triple-point of water, which on the Kelvin absolute scale is assigned the number 273.16 K. What is the other fixed point on this (Kelvin) Scale ?*

(*c*) *The absolute temperature (Kelvin scale) T is related to the temperature t_c on the Celsius scale by*

$$t_c = T - 273.15$$

Why do we have 273.15 in this relation, and not 273.16 ?

(*d*) *What is the temperature of the triple-point of water on an absolute scale whose unit interval size is equal to that of the Fahrenheit scale ?*

Sol. (*a*) Triple point of water has a unique value *i.e.,* 273.16 K. The melting point and boiling points of ice and water respectively do not have unique values and change with the change in pressure.

(*b*) On Kelvin's absolute scale, there is only one fixed point, namely, the triple-point of water and there is no other fixed point.

(*c*) On Celsius scale 0 °C corresponds to the melting point of ice at normal pressure and the value of absolute temperature is 273.15 K. The temperature 273.16 K corresponds to the triple point of water.

(*d*) The Fahrenheit scale and Absolute scale are related as

$$\frac{T_F - 32}{180} = \frac{T_K - 273}{100} \qquad ...(i)$$

For another set of temperature T'_F and T'_K,

$$\frac{T'_F - 32}{180} = \frac{T'_K - 273}{100} \qquad ...(ii)$$

Subtracting (*i*) from (*ii*)

$$\frac{T'_F - T_F}{180} = \frac{T'_K - T_K}{100}$$

$$\therefore \quad T'_F - T_F = \frac{180}{100}(T'_K - T_K)$$

For $\quad T'_K - T_K = 1$ K,

$$T'_F - T_F = \frac{180}{100}$$

$\therefore$ For a temperature of triple point *i.e.,* 273.16 K, the temperature on the new scale is

$$= 273.16 \times \frac{180}{100} = 491.688.$$

11.5. *Two ideal gas thermometers A and B use oxygen and hydrogen respectively. The following observations are made:*

Temperature	***Pressure thermometer A***	***Pressure thermometer B***
Triple-point of water	*1.250 × 10⁵ Pa*	*0.200 × 10⁵ Pa*
Normal melting point of sulphur	*1.797 × 10⁵ Pa*	*0.287 × 10⁵ Pa*

(*a*) *What is the absolute temperature of normal melting point of sulphur as read by thermometers A and B ?*

(*b*) *What do you think is the reason behind the slight difference in answers of thermometers A and B ? (The thermometers are not faulty). What further procedure is needed in the experiment to reduce the discrepancy between the two readings ?*

Sol. (*a*) Let T be the melting point of sulphur.

For thermometer A

$$P_{tr} = 1.250 \times 10^5 \text{ Pa}; \quad P = 1.797 \times 10^5 \text{ Pa}$$

Now, $$T_A = T_{tr} \times \frac{P}{P_{tr}}$$

$$T_A = 273.16 \times \frac{1.797 \times 10^5}{1.250 \times 10^5} \text{K} = 392.69 \text{ K}$$

For thermometer B

$$P_{tr} = 0.200 \times 10^5\ Pa; \quad P = 0.287 \times 10^5\ \text{Pa}$$

$$T_B = T_{tr} \times \frac{P}{P_{tr}} = \frac{273.16 \times 0.287 \times 10^5}{0.200 \times 10^5}\text{K} = 391.98\ \text{K}$$

(*b*) The value of the melting point of sulphur found from the two thermometers differ slightly due to the reason that in practice, the gases do not behave strictly as perfect gases *i.e.,* gases are not perfectly ideal.

To reduce the discrepency, readings should be taken for lower and lower pressures and the plot between temperature measured versus absolute pressure of the gas at triple point should be extrapolated to obtain the temperature in the limit pressure tends to zero (if $P \to 0$), when the gases approach ideal gas behaviour.

11.6. *A steel tape 1m long is correctly calibrated for a temperature of 27.0 °C. The length of a steel rod measured by this tape is found to be 63.0 cm on a hot day when the temperature is 45.0 °C. What is the actual length of the steel rod on that day ? What is the length of the same steel rod on a day when the temperature is 27.0 °C ? Coefficient of linear expansion of steel = $1.20 \times 10^{-5} K^{-1}$.*

Sol. On a day when the temperature is 27 °C, the length of 1 cm division on the steel tape is exactly 1 cm, because the tape has been calibrated for 27 °C.

When the temperature rises to 45 °C (that is, $\Delta T = 45 - 27 = 18$ °C), the increase in the length of 1 cm division is

$$\Delta l = \alpha l \Delta T = (1.2 \times 10^{-5}\ °\text{C}^{-1}) \times 1\ \text{cm} \times 18\ °\text{C} = 0.000216\ \text{cm}$$

Therefore, the length of 1 cm division on the tape becomes 1.000216 cm at 45 °C. As the length of the steel rod is read to be 63.0 cm on the steel tape at 45 °C, the actual length of the rod at 45 °C is

$$63.0 \times 1.000216\ \text{cm} = 63.0136\ \text{cm}$$

The length of the same rod at 27 °C is 63.0 cm, because 1 cm mark on the steel tape is exactly 1 cm at 27 °C.

11.7. *A large steel wheel is to be fitted on to a shaft of the same material. At 27 °C, the outer diameter of the shaft is 8.70 cm and the diameter of the central hole in the wheel is 8.69 cm. The shaft is cooled using 'dry ice'. At what temperature of the shaft does the wheel slip on the shaft ? Assume coefficient of linear expansion of the steel to be constant over the required temperature range:*

$$\alpha_{steel} = 1.20 \times 10^{-5}\ K^{-1}.$$

Sol. Here at temperature $T_1 = 27$ °C, diameter of shaft $D_1 = 8.70$ cm

Let at temperature T_2, the diameter of shaft changes to $D_2 = 8.69$ cm and for steel

$$\alpha = 1.20 \times 10^{-5}\ \text{K}^{-1} = 1.20 \times 10^{-5}\ °\text{C}^{-1}$$

$\because$ Change in diameter $\Delta D = D_2 - D_1 = D_1 \times \alpha \times (T_2 - T_1)$

$\therefore \quad 8.69 - 8.70 = 8.70 \times 1.20 \times 10^{-5} \times (T_2 - 27)$

$$\Rightarrow \quad T_2 = 27 - \frac{0.01}{8.70 \times 1.20 \times 10^{-5}} = 27 - 95.8 = -68.8\ °\text{C} \quad \text{or} \quad -69\ °\text{C}.$$

11.8. *A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at 27.0 °C. What is the change in the diameter of the hole when the sheet is heated to 227 °C ? Coefficient of linear expansion of copper = $1.70 \times 10^{-5}\ K^{-1}$.*

Sol. In this problem superficial expansion of copper sheet will be involved on heating.

Here, area of hole at 27° C, $A_1 = \frac{\pi D_1^2}{4} = \frac{\pi}{4} \times (4.24)^2\ \text{cm}^2$

If D_2 cm is the diameter of the hole at 227 °C, then area of the hole at 227 °C,

$$A_2 = \frac{\pi D_2^2}{4} \text{cm}^2$$

Coefficient of superficial expansion of copper is,

$$\beta = 2\alpha = 2 \times 1.70 \times 10^{-5} = 3.4 \times 10^{-5} \text{ °C}^{-1}$$

$$\text{Increase in area} = A_2 - A_1 = \beta A_1 \Delta T \quad \text{or} \quad A_2 = A_1 + \beta A_1 \Delta T = A_1 (1 + \beta \Delta T)$$

$$\frac{\pi D_2^2}{4} = \frac{\pi}{4}(4.24)^2 [1 + 3.4 \times 10^{-5} (227 - 27)]$$

$$\Rightarrow \quad D_2^2 = (4.24)^2 \times 1.0068 \quad \text{or} \quad D_2 = 4.2544 \text{ cm}$$

$$\text{Change in diameter} = D_2 - D_1 = 4.2544 - 4.24 = 0.0144 \text{ cm.}$$

11.9. *A brass wire 1.8 m long at 27 °C is held taut with little tension between two rigid supports. If the wire is cooled to a temperature of – 39 °C, what is the tension developed in the wire, if its diameter is 2.0 mm ? Co-efficient of linear expansion of brass = 2.0 × 10^{-5} K^{-1}; Young's modulus of brass = 0.91 × 10^{11} Pa.*

Sol. Here,

$$l = 1.8 \text{ m,}$$

$$\Delta t = (-39 - 27) \text{ °C} = -66 \text{ °C}$$

$$\alpha = 2.0 \times 10^{-5} \text{ K}^{-1}$$

$$Y = 0.91 \times 10^{11} \text{ Pa}$$

$$A = \frac{\pi D^2}{4} = \frac{22}{7} \times \frac{1}{4} (2 \times 10^{-3})^2 \text{ m}^2$$

Now,

$$Y = \frac{Fl}{A\Delta l} \Rightarrow \Delta l = \frac{Fl}{AY} \quad \text{or} \quad l\alpha\Delta t = \frac{Fl}{AY}$$

or

$$F = -YA\alpha\Delta t$$

or

$$F = -0.91 \times 10^{11} \times \frac{22}{7} \times \frac{1}{4} (2 \times 10^{-3})^2 \times 2.0 \times 10^{-5} \times 66 \text{ N}$$

$$= -3.77 \times 10^2 \text{ N.}$$

11.10. *A brass rod of length 50 cm and diameter 3.0 mm is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at 250 °C, if the original lengths are at 40.0 °C ? Is there a 'thermal stress' developed at the junction ? The ends of the rod are free to expand (Co-efficient of linear expansion of brass = 2.0 × 10^{-5} °C^{-1}, steel = 1.2 × 10^{-5} °C^{-1}.*

Sol. Here, $l_{brass} = l_{steel} = 50$ cm, $d_{brass} = d_{steel} = 3$ mm = 0.3 cm, $\Delta l_{brass} = ?$, $\Delta l_{steel} = ?$

$$\Delta T = 250 - 40 = 210 \text{ °C.}$$

$$\alpha_{brass} = 2 \times 10^{-5} \text{ °C}^{-1} \quad \text{and} \quad \alpha_{steel} = 1.2 \times 10^{-5} \text{ °C}^{-1}$$

Now

$$\Delta l_{brass} = \alpha_{brass} \times l_{brass} \times \Delta T = 2 \times 10^{-5} \times 50 \times 210 = 0.21 \text{ cm}$$

Now

$$\Delta l_{steel} = \alpha_{steel} \times l_{steel} \times \Delta T = 1.2 \times 10^{-5} \times 50 \times 210$$

$$= 0.126 \text{ cm} \approx 0.13 \text{ cm}$$

∴ Total change in length, $\Delta l = \Delta l_{brass} + \Delta l_{steel} = 0.21 + 0.13 = 0.34$ cm

Since the rod is not clamped at its ends, no thermal stress is developed at the junction.

11.11. *The coefficient of volume expansion of glycerine is 49 × 10^{-5} K^{-1}. What is the fractional change in its density for a 30 °C rise in temperature ?*

Sol. Here,

$$\gamma = 49 \times 10^{-5} \text{ °C}^{-1}, \quad \Delta T = 30 \text{ °C}$$

As

$$V = V + \Delta V = V(1 + \gamma \Delta T)$$

∴

$$V' = V(1 + 49 \times 10^{-5} \times 30) = 1.0147 \text{ V}$$

Since $\rho = \frac{m}{V}$, $\rho' = \frac{m}{V'} = \frac{m}{1.0147V} = 0.9855\ \rho$

Fractional change in density $= \frac{\rho - \rho'}{\rho} = \frac{\rho - 0.9855\,\rho}{\rho} = 0.0145.$

11.12. *A 10 kW drilling machine is used to drill a bore in a small aluminium block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 minutes, assuming 50% of power is used up in heating the machine itself or lost to the surroundings? Specific heat of aluminium = 0.91 J g^{-1} K^{-1}.*

Sol. Power = 10 kW = 10^4 W

Mass, m = 8.0 kg = 8×10^3 g

Rise in temperature, ΔT = ?

Time, t = 2.5 min = 2.5 × 60 = 150 s

Specific heat, C = 0.91 Jg^{-1} K^{-1}

Total energy = Power × Time = $10^4 \times 150$ J = 15×10^5 J

As 50% of energy is lost,

∴ Thermal energy available,

$$\Delta Q = \frac{1}{2} \times 15 \times 10^5 = 7.5 \times 10^5 \text{ J}$$

Since $\Delta Q = mc\Delta T$

$$\therefore \quad \Delta T = \frac{\Delta Q}{mc} = \frac{7.5 \times 10^5}{8 \times 10^3 \times 0.91} = 103° \text{ C.}$$

11.13. *A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500°C and then placed on a large ice block. What is the maximum amount of ice that can melt? Specific heat of copper is 0.39 Jg^{-1} $°C^{-1}$. Heat of fusion of water = 335 Jg^{-1}.*

Sol. Here, mass of copper block, m = 2.5 kg = 2500 g

Fall in temperature, ΔT = 500 – 0 = 500°C

Specific heat of copper, c = 0.39 J g^{-1} $°C^{-1}$

Latent heat of fusion, L = 335 J g^{-1}

Let the mass of ice melted be m'

As, Heat gained by ice = Heat lost by copper

$$\therefore \quad m' L = mc\,\Delta T$$

$$m' = \frac{mc\,\Delta T}{L}$$

$$m' = \frac{2500 \times 0.39 \times 500}{335} = 1500 \text{ g} = 1.5 \text{ kg}$$

11.14. *In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150°C is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing 150 cm^3 of water at 27 °C. The final temperature is 40° C. Compute the specific heat of the metal. If heat losses to the surroundings are not negligible, is your answer greater or smaller than the actual value for specific heat of the metal?*

Sol. Mass of metal block, m = 0.20 kg = 200g

Fall in the temperature of metal block,

$$\Delta T = (150 - 40)\ °\text{C} = 110\ °\text{C}$$

If C be the specific heat of metal, then heat lost by the metal block = 200 × C × 110 cal

Volume of water = 150 cm^3

mass of water = 150 g

Increase in temperature of water = (40 – 27) °C = 13°C

Heat gained by water = 150 × 13 cal

Water equivalent of calorimeter, w = 0.025 kg = 25g

Heat gained by calorimeter,

= w × increase in temperature of calorimeter

= 25 × 13 cal

Heat lost by metal block

= Heat gained by water + Heat gained by calorimeter

200 × C × 110 = (150 + 25) 13

$$C = \frac{175 \times 13}{200 \times 110} = 0.1 \text{ Cal g}^{-1} \text{ °C}^{-1} = 0.43 \text{ Jg}^{-1} \text{ K}^{-1}$$

If heat is lost to the surroundings, C will be smaller than the actual value.

11.15. *Given below are observations on molar specific heats at room temperature of some common gases.*

Gas	*Molar specific heat (C_v) (cal mol^{-1} K^{-1})*
Hydrogen	*4.87*
Nitrogen	*4.97*
Oxygen	*5.02*
Nitric oxide	*4.99*
Carbon monoxide	*5.01*
Chlorine	*6.17*

The measured molar specific heats of these gases are markedly different from those for monoatomic gases. Typically, molar specific heat of a monoatomic gas is 2.92 cal/mol K. Explain this difference. What can you infer from the somewhat larger (than the rest) value for chlorine ?

Sol. The gases which are listed in the above table are diatomic gases and not monoatomic gases. For diatomic gases, molar specific heat = $\frac{5}{2}R = \frac{5}{2} \times 1.98 = 4.95$, which agrees fairly well with all observations listed in the table except for chlorine. A monoatomic gas molecule has only the translational motion. A diatomic gas molecule, apart from translational motion, the vibrational as well as rotational motion is also possible. Therefore, to raise the temperature of 1 mole of a diatomic gas through 1°C, heat is to be supplied to increase not only translational energy but also rotational and vibrational energies. Hence, molar specific heat of a diatomic gas is greater than that for monoatomic gas. The higher value of molar specific heat of chlorine as compared to hydrogen, nitrogen, oxygen etc. shows that for chlorine molecule, at room temperature vibrational motion also occurs along with translational and rotational motions, whereas other diatomic molecules at room temperature usually have rotational motion apart from their translational motion. This is the reason that chlorine has somewhat larger value of molar specific heat.

11.16. *(a) At what temperature and pressure can the solid, liquid and vapour phases of CO_2 co-exist in equilibrium ?*

(b) What is the effect of decrease of pressure on the fusion and boiling point of CO_2 ?

(c) *What are the critical temperature and pressure for CO_2 ? What is their significance ?*

(d) *Is CO_2 solid, liquid or gas at (a) – 70 °C under 1 atm (b) – 60 °C under 10 atm (c) 15°C under 56 atm?*

Sol. (*a*) At the triple point, temperature = – 56.6 °C and pressure = 5.11 atm.

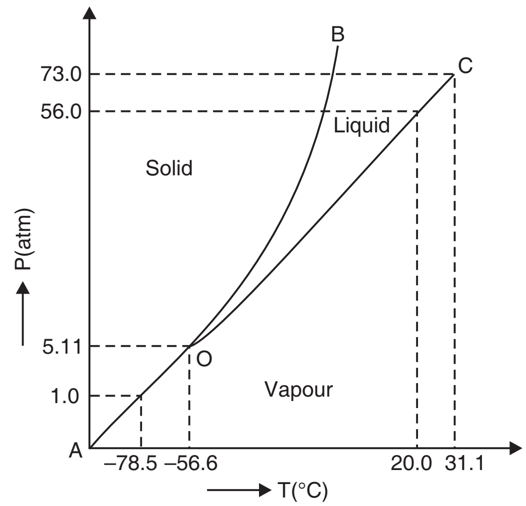
(*b*) Both the boiling point and freezing point of CO_2 decrease if pressure decreases.

(*c*) The critical temperature and pressure of CO_2 are 31.1°C and 73.0 atm respectively. Above this temperature, CO_2 will not liquefy even if compressed to high pressures.

(*d*) (*i*) The point (– 70 °C, 1.0 atm) lies in the vapour region. Hence, CO_2 is vapour at this point.

(*ii*) The point (– 60 °C, 10 atm) lies in the solid region. Hence, CO_2 is solid at this point.

(*iii*) The point (15 °C, 56 atm) lies in the liquid region. Hence, CO_2 is liquid at this point.



P-T phase diagram of CO_2

11.17. *Answer the following questions based on the P – T phase diagram of CO_2 (Fig. of question 17 given above)*

(a) *CO_2 at 1 atm pressure and temperature – 60 °C is compressed isothermally. Does it go through a liquid phase ?*

(b) *What happens when CO_2 at 4 atm pressure is cooled from room temperature at constant pressure ?*

(c) *Describe qualitatively the changes in a given mass of solid CO_2 at 10 atm pressure and temperature – 65 °C as it is heated up at room temperature at constant pressure.*

(d) *CO_2 is heated to a temperature 70 °C and compressed isothermally. What changes in its properties do you expect to observe ?*

Sol. (*a*) No, the CO_2 does not go through the liquid phase. The point (1.00 atm, – 60°C) is to the lift of the triple-point O and below the sublimation curve OA. Therefore, when CO_2 is compressed at this point at constant temperature, the point moves perpendicular to the temperature-axis and enters the solid phase region. Hence, the CO_2 vapour condenses to solid directly without going through the liquid phase.

(*b*) CO_2 at 4.0 atm pressure and room temperature (say, 27 °C) is in vapour phase. This point (4.0 atm, 27°C) lies below the vaporation curve OC and to the right of the triple point O. Therefore, when CO_2 is cooled at this point at constant pressure, the point moves perpendicular to the pressure-axis and enters the solid phase region. Hence, the CO_2 vapour condenses directly to solid phase without going through the liquid phase.

(*c*) When the solid CO_2 at – 65 °C is heated at 10 atm pressure, it is first converted into liquid. A further increase in its temperature brings it into the vapour phase. If a horizontal line at P = 10 atm is drawn parallel to the T-axis, then the points of intersection of line with the fusion and vaporization curve give the fusion and boiling points at 10 atm.

(*d*) Above 31.1°C, the gas cannot be liquefied. Therefore, on being compressed isothermally at 70°C, there will be no transition to the liquid region. However, the gas will depart. more and more from its perfect gas behaviour with the increase in pressure.

11.18. *A child running a temperature of 101°F is given an antipyrin (i.e., a medicine that lowers fever) which causes an increase in the rate of evaporation of sweat from his body. If the fever is brought down to 98° F in 20 minutes, what is the average rate of extra evaporation caused by the drug ? Assume the evaporation mechanism to be the only way by which heat is lost. The mass of the child is 30 kg. The specific heat of human body is approximately the same as that of water, and latent heat of evaporation of water at that temperature is about 580 cal* g^{-1}.

Sol. Decrease of temperature, Δt

$$= 101°F - 98°F = 3°\ F = 3 \times \frac{5}{9}\,°C = 1.67°C$$

specific heat of water $= 1000$ cal kg^{-1} $°C^{-1}$

latent heat of vaporisation, $L = 580 \times 10^3$ cal kg^{-1}

heat lost

$$= 30 \text{ kg} \times 1000 \text{ cal kg}^{-1}\,°C^{-1} \times 1.67\,°C = 50100 \text{ cal}$$

If m' be the mass of water evaporated, then

$$m' = \frac{50100 \text{ cal}}{580 \times 10^3 \text{ cal kg}^{-1}} = 0.086 \text{ kg}$$

This much water has taken 20 minutes to evaporate.

$$\text{So, rate of evaporation} = \frac{0.086 \text{ kg}}{20 \text{ min}} = \frac{86 \text{ g}}{20 \text{ min}} = \mathbf{4.3\ g\ min^{-1}}.$$

11.19. *A 'thermacole' icebox is a cheap and efficient method for storing small quantities of cooked food in summer in particular. A cubical icebox of side 30 cm has a thickness of 5.0 cm. If 4.0 kg of ice is put in the box, estimate the amount of ice remaining after 6 h. The outside temperature is 45°C, and coefficient of thermal conductivity of thermacole is 0.01* $Js^{-1}\ m^{-1}\ °C^{-1}$. *[Heat of fusion of water =* 335×10^3 *J* kg^{-1}*].*

Sol. Each side of the cubical box (having 6 faces) is 30 cm = 0.30 m. Therefore, the total surface area of the icebox exposed to outside air is A = 6 × (0.30 m)^2 = 0.54 m^2. The thickness of the icebox is d = 5.0 cm = 0.05 m, time of exposure t = 6h = 6 × 3600 s and temperature difference $T_1 - T_2 = 45°C - 0°C = 45°C$.

∴ Total heat entering the icebox in 6 h is given by

$$Q = \frac{KA(T_1 - T_2)\,t}{d}$$

$$= \frac{0.01 \text{ Js}^{-1} \text{ m}^{-1} \text{ C}^{-1} \times 0.54 \text{ m}^2 \times 45°C \times (6 \times 3600 \text{ s})}{0.05 \text{ m}}$$

$$= 1.05 \times 10^5 \text{ J}$$

Suppose a mass m of ice melts with this heat. Then $Q = mL$, where L is tatent heat of fusion of water. Thus,

$$1.05 \times 10^5 \text{ J} = m\,(335 \times 10^3) \text{ Jkg}^{-1} \quad \text{or} \quad m = \frac{1.05 \times 10^5 \text{ J}}{335 \times 10^3 \text{ J kg}^{-1}} = 0.313 \text{ kg}$$

The initial mass of ice in the box is 4.0 kg. Therefore,the ice remaining in the box after 6 h is

$$= (4.0 - 0.313) \text{ kg} = 3.687 \text{ kg}.$$

11.20. *A brass boiler has a base area 0.15 m^2 and thickness 1.0 cm. It boils water at the rate of 6.0 kg/min when placed on a gas stove. Estimate the temperature of the part of the flame in contact with the boiler. Thermal conductivity of brass = 109 Js^{-1} m^{-1} K^{-1}.*

(Heat of vaporization of water = 2256 × 10^3 J kg^{-1})

Sol. Here,

$$K = 109 \text{ Js}^{-1} \text{ m}^{-1} \text{ K}^{-1}$$
$$A = 0.15 \text{ m}^2$$
$$d = 1.0 \text{ cm} = 10^{-2} \text{ m}$$
$$T_2 = 100° \text{ C}$$

Let T_1 = temperature of the part of the boiler in contact with the stove.

If Q be the amount of heat flowing per second through the base of the boiler, then

$$Q = \frac{KA\,(T_1 - T_2)}{d}$$

or

$$Q = \frac{109 \times 0.15 \times (T_1 - 100)}{10^{-2}} = 1635\ (T_1 - 100) \text{ Js}^{-1} \quad ...(i)$$

Also heat of vaporisation of water

$$L = 2256 \times 10^3 \text{ J kg}^{-1}$$

Rate of boiling of water in the boiler,

$$M = 6.0 \text{ kg min}^{-1} = \frac{6.0}{60} = 0.1 \text{ kg } s^{-1}.$$

∴ Heat received by water per second, $Q = ML$

$$\Rightarrow \quad Q = 0.1 \times 2256 \times 10^3 \text{ Js}^{-1} \quad ...(ii)$$

∴ From eqn. (*i*) and (*ii*), we get

$$1635\ (T_1 - 100) = 2256 \times 10^2 \quad \text{or} \quad T_1 - 100 = \frac{2256 \times 10^2}{1635} = 138$$

$$T_1 = 138 + 100 = 238°\text{C}.$$

11.21. *Explain why:*

(*a*) a body with large reflectivity is a poor emitter.

(*b*) a brass tumbler feels much colder than a wooden tray on a chilly day.

(*c*) an optical pyrometer (for measuring high temperatures) calibrated for an ideal black body radiation gives too low a value for the temperature of a red hot iron piece in the open, but gives a correct value for the temperature when the same piece is in the furnace.

(*d*) the earth without its atmosphere would be inhospitably cold.

(*e*) heat systems based on circulation of steam are more efficient in warming a building than those based on circulation of hot water.

Sol. (*a*) According to Kirchhoff's law of black body radiations, good emitters are good absorbers and bad emitters are bad absorbers. A body with large reflectivity is a poor absorber of heat and consequently, it is also a poor emitter.

(*b*) Brass is a good conductor of heat, while wood is a bad conductor. When we touch the brass tumbler on a chilly day, heat starts flowing from our body to the tumbler and we feel it cold. However, when the wooden tray is touched, heat does not flow from our hands to the tray and we do not feel cold.

(*c*) An optical pyrometer is based on the principle that the brightness of a glowing surface of a body depends upon its temperature. Therefore, if the temperature of the body is less than 600°C, the image formed by the optical pyrometer is not brilliant and we do not get the reliable result. It is for this reason that the pyrometer gives a very low value for the temperature of red hot iron in the open.

(*d*) The lower layers of earth's atmosphere reflect infrared radiations from earth back to the surface of earth. Thus the heat radiation received by the earth from the sun during the day are kept trapped by the atmosphere. If atmosphere of earth were not there, its surface would become too cold to live.

(*e*) Steam at 100°C possesses more heat than the same mass of water at 100°C. One gram of steam at 100°C possesses 540 calories of heat more than that possessed by 1 gm of water at 100°C. That is why heating systems based on circulation of steam are more efficient than those based on circulation of hot water.

11.22. *A body cools from 80 °C to 50°C in 5 minutes. Calculate the time it takes to cool from 60 °C to 30°C. The temperature of the surroundings is 20 °C.*

Sol. According to Newton's law of cooling, the rate of cooling is proportional to the difference in temperature.

Here average of 80 °C and 50 °C = 65 °C

Temperature of surroundings = 20°C

∴ Difference = 65 – 20 = 45 °C

Under these conditions, the body cools 30°C in time 5 minutes

$$\therefore \quad \frac{\text{Change in temp.}}{\text{Time}} = K \,\Delta T \quad \text{or} \quad \frac{30}{5} = \text{K} \times 45° \qquad ...(i)$$

The average of 60°C and 30°C is 45°C which is 25°C (45 – 20) above the room temperature and the body cools by 30°C (60 – 30) in a time *t* (say)

$$\therefore \quad \frac{30}{t} = K \times 25 \qquad ...(ii)$$

where *K* is same for this situation as for the original.

Dividing eqn. (*i*) by (*ii*), we get

$$\frac{30/5}{30/t} = \frac{K\times 45}{K\times 25} \quad \text{or} \quad \frac{t}{5} = \frac{9}{5} \quad \Rightarrow \quad t = 9 \text{ min.}$$

QUESTIONS BASED ON SUPPLEMENTARY CONTENTS

Q. 1. *A black body at 2000 K emits maximum energy at a wavelength of 1.56 μm. At what temperature will it emit maximum energy at a wavelength of 1.8 μm?*

Sol. Here $\lambda_{m_1} = 1.56\ \mu\text{m}, \quad T_1 = 2000\ \text{K}$

$\lambda_{m_2} = 1.8\ \mu\text{m}, \quad T_2 = ?$

According to Wien's displacement law,

$$\lambda_m = \frac{b}{T}$$

$\therefore \quad \frac{\lambda_{m_1}}{\lambda_{m_2}} = \frac{T_2}{T_1}$

$\Rightarrow \quad \frac{1.56}{1.8} = \frac{T_2}{2000} \Rightarrow T_2 = \frac{1.56 \times 2000}{1.8}$

$\therefore \quad T_2 = 1733.3 \text{ K}$

Q.2. *A spherical black body with a radius of 12 cm radiates 450 W power at 500K. What would be the power of radiation if radius were to be halved and the temperature doubled.*

Sol. According to Stefan's Law,

Energy emitted per unit time i.e. power

$$H = A \times \sigma T^4$$

Here $H_1 = 450$ W, $T = T_1$ and radius $r = r_1$

$H_2 = ?$, $T = 2T_1$ and $r = \frac{r_1}{2}$

$$\therefore \quad H_1 = 4\pi r_1^2 \cdot \sigma T_1^4 \quad (i)$$

and $$H_2 = 4\pi \left(\frac{r_1}{2}\right)^2 \cdot \sigma (2T_1)^4 \quad (ii)$$

Dividing eq. (*i*) by (*ii*) we get,

$$\frac{H_1}{H_2} = \frac{4\pi r_1^2 \cdot \sigma T_1^4}{4\pi \left(\frac{r_1}{2}\right)^2 \cdot \sigma (2T_1)^4}$$

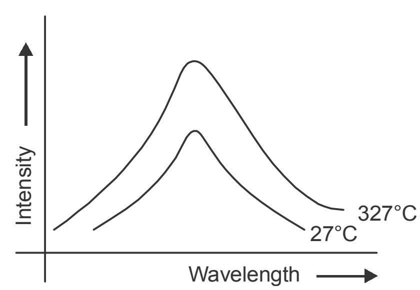
$$\Rightarrow \quad \frac{450}{H_2} = \frac{r_1^2}{\frac{r_1^2}{4}} \times \frac{T_1^4}{16T_1^4} \Rightarrow \frac{450}{H_2} = \frac{1}{4}$$

$\therefore \quad H_2 = 450 \times 4 = 1800$ W

Hence the required power $= 1800$ W

Q. 3. *The spectrum of a black body at two temperatures 27°C and 327°C is shown in the figure. Let A_1 and A_2 be the respective areas under the two curves. Estimate the ratio A_2/A_1.*

Sol. According to Stefan's Law,



$$\text{Area} = \frac{1}{T^4}$$

$$\therefore \quad \frac{A_1}{A_2} = \frac{T_2^4}{T_1^4}$$

$$\Rightarrow \quad \frac{A_2}{A_1} = \frac{T_1^4}{T_2^4}$$

$$\Rightarrow \quad \frac{A_2}{A_1} = \left(\frac{327+273}{27+273}\right)^4 \Rightarrow \frac{A_2}{A_1} = \left(\frac{600}{300}\right)^4$$

$$\Rightarrow \quad \frac{A_2}{A_1} = (2)^4$$

$$\therefore \quad \frac{A_2}{A_1} = \frac{16}{1}$$

$\therefore$ Ratio is 16 : 1

Q. 4. *If each square metre of sun's surface, radiates energy at the rate of 6.3 × 10⁷ J/m²/s and the Stefan's constant is 5.669 × 10⁻⁸ W/m²/k⁴. Calculate the temperature of the sun's energy assuming Stefan's Law applies to the sun's rediation.*

Sol. According to Stefan's law,

$$H = A \times \sigma \times T^4$$

$$6.3 \times 10^7 = 1 \times 5.669 \times 10^{-8} \times T^4$$

$$\Rightarrow \quad T^4 = \frac{6.3 \times 10^7}{5.669 \times 10^{-8}} = \frac{6.3}{5.669} \times 10^{15} = \frac{0.63}{5.669} \times 10^{16}$$

$$\therefore \quad T = 8716.9 \text{ K}$$

Q. 5. *A man, the surface area of whose skin is 2 m², is sitting in a room where area temperature is 20°C. If his skin temperature is 28°C and emissivity of his skin equals 0.97, find the rate at which his body loses heat (Given σ = 5.67 × 10⁻⁸ w/m²/T⁴).*

Sol. According to Stefan's Law,

$$H = \sigma \cdot \varepsilon A\left(T^4 - T_0^4\right)$$

Here $T = (28 + 273)$ K = 310 K

$T_0 = (20 + 273)$ K = 293 K

$\therefore$ $H = (5.67 \times 10^{-8}) \times 0.97 \times 2 \times [(310)^4 - (293)^4] = 92.2$ W

As $(T - T_0)$ is very small. Let us also calculate H by using the approximate form of Stefan-Boltman Law.

$$H = \sigma\varepsilon A\ (T - T_0) = 5.67 \times 10^{-8} \times 0.92 \times 2 \times 17 = 88 \text{ W}$$

$\therefore$ The approximate value is nearly 4.3% lower than its more exact value.

Q. 6. *Two bodies A and B have thermal emisivities of 0.01 and 0.81 respectively. The outer surface area as of both the bodies are same. The two emit the same total radiated power. The wavelength λ_B corresponding to the maximum intensity in radiations from B, is shifted from the wavelength λ_A corresponding to the maximum intensity in radiations from A by 1 μm. If the temperature of body A is 5802 K, find the temperature of body B and the wavelength λ_B.*

Sol. Here $\varepsilon_A = 0.01$ and $\varepsilon_B = 0.81$

$$\left(\frac{dQ}{dt}\right)_A = \left(\frac{dQ}{dt}\right)_B$$

$$\therefore \quad \varepsilon_A \sigma_A T_A^4 = \varepsilon_B \sigma_A T_B^4 \Rightarrow \frac{\varepsilon_A}{\varepsilon_B} = \frac{T_B^4}{T_A^4}$$

But λT = Constant

$$\therefore \quad \lambda_A T_A = \lambda_B T_B$$

$$\therefore \quad \frac{\varepsilon_A}{\varepsilon_B} = \frac{\lambda_A^4}{\lambda_B^4} = \frac{0.1}{0.81} = \left(\frac{1}{3}\right)^4$$

$$\therefore \quad \frac{\lambda_A}{\lambda_B} = \frac{1}{3}$$

$\Rightarrow$ $\lambda_B = 3\lambda_A$ (1)

But $\lambda_B - \lambda_A = 10^{-6}$ (2)

$\therefore$ $3\lambda_A - \lambda_A = 10^{-6}$

$\therefore$ $\lambda_A = 0.5 \times 10^{-6}$ m and $\lambda_B = 3 \times 0.5 \times 10^{-6} = 1.5 \times 10^{-6}$ m

Q. 7. *The tungston filament of an electric lamp has a length of 0.25 m and a diameter of 6×10^{-5} m. The power rating of the lamp is 100 W. If the emissivity of the filament is 0.8, estimate the steady temperature of the filament. Stefan's constant = 5.67×10^{-8} W/m²/k⁴.*

Sol. $A = \pi r^2 = 3.14 \times (3 \times 10^{-5})^2 = 3.14 \times 9 \times 10^{-10}$ m²

According to Stefan's Law

$$H = A\varepsilon\sigma T^4$$

$$T^4 = \frac{H}{A\varepsilon\sigma}$$

$$T^4 = \frac{100}{3.14 \times 9 \times 10^{-10} \times 0.8 \times 5.67 \times 10^{-8}}$$

$\therefore$ $T = 2616$ K

Hence the steady temperature of the filament is 2616 K

Q. 8. *Good reflectors are poor emitters of thermal radiation. Explain.*

Sol. A body with good reflectivity is a poor absorber of heat and the poor absorbers of heat are poor emitters of thermal radiations.

Q. 9. *If the earth did not have an atmosphere it would become intolerable cold. Why?*

Sol. The lower layer of earth's atmosphere reflects infra-red radiations from earth back to the surface of the earth. So the heat radiation received by the earth from the sun during day time are trapped by the atmosphere. Therefore, if the earth did not have atmosphere, its surface would become too cold to tolerate.

Q. 10. *Determine the surface area of the filament of a 100 W incandescent lamp reflecting out its labbled power at 3000 K. Given $\sigma = 5.7 \times 10^{-8}$ W m⁻² k⁻⁴ and emissivity 'ε' of the material of the filament = 0.3.*

Sol. According to Stefan's Law

$$H = A\varepsilon\sigma T^4$$

$$\therefore \quad A = \frac{H}{\varepsilon\sigma T^4} = \frac{100}{0.3 \times 5.7 \times 10^{-8} \times (3000)^4} = 7.25 \times 10^{-5} \text{ m}^2$$

ADDITIONAL QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *What is the relation between the coefficient of linear expansion and coefficient of volume expansion ?*

Ans. Coefficient of volume expansion = 3 × coefficient of linear expansion.

Q. 2. *What is sublimation ?*

Ans. On heating a substance, the change from solid state to vapour state without passing through the liquid state is called sublimation.

Q. 3. *What are the three modes of transmission of heat energy from one point to another point ?*

Ans. Conduction, Convection and Radiation.

Q. 4. *How does density of a solid change as it is heated ?*

Ans. Density of a solid decreases as it is heated (*i.e.,* as its temperature rises) as per following relation:

$\rho' = \rho(1 - \gamma.\Delta T)$, where ρ = density of given solid at temperature T and ρ' = density of given solid at temperature $(T + \Delta T)$.

Q. 5. *Which of two has larger value of specific heat – sand or water ?*

Ans. Water.

Q. 6. *Why two layers of cloth of equal thickness provide warmer covering than a single layer of cloth of double thickness ?*

Ans. This is because the air enclosed between the two layers of cloth acts as a good heat insulator.

Q. 7. *How many fixed points are taken in absolute (kelvin) scale of temperature and what is their value ?*

Ans. On kelvin scale we select only one fixed point, namely, the triple-point of pure water. Its value of T_{tr} = 273.16 K = 0.01°C.

Q. 8. *What is specific heat of a gas in an isothermal process ?*

Ans. Infinite, because $\Delta T = 0$; use $C = \dfrac{\Delta Q}{m\Delta T}$.

Q. 9. *What is the principle of calorimetry ?*

Ans. Heat lost by hot body = Heat gained by cold body.

Q. 10. *What is the value of latent heat of ice ?*

Ans. Latent heat of ice has a value 3.33×10^5 J/kg or 80 kcal/kg.

Q. 11. *Why can water in a metallic pot be boiled quickly if the bottom of the pot is made black and rough than highly polished ?*

Ans. The black and rough surface is a better absorber of heat than a highly polished surface.

Q. 12. *Is the value of temperature coefficient of expansion always positive ?*

Ans. Generally, the value of temperature coefficient of expansion is positive. But in certain cases *e.g.,* water between 0 °C and 4 °C, the expansion coefficient is negative.

Q. 13. *A metal disc has a hole on it. What happens to the size of hole when the disc is heated ?*

Ans. When the disc is heated, alongwith its dimensions the size of hole also increases.

Q. 14. *What happens to the phase of water at its critical point ?*

Ans. At the critical point, water and water vapours are equally dense.

Q. 15. *Express 60 °C in °F.*

Ans. $$\frac{9}{5} \times 60 + 32 = 140°\text{F}$$

Q. 16. *A wooden charcoal and a metal piece of the same dimension are heated in the same oven to the same temperature and then removed in the dark. Which one would shine more and why ?*

Ans. Charcoal will shine more as it is a good absorber and good emitter, so it will emit more energy.

Q. 17. *What is the condition for the difference between the length of a certain brass rod and that of a steel rod to be constant at all temperature ?*

Ans. The condition is that the lengths of the rods are inversely proportional to the coefficients of linear expansion of the materials of the rods.

Q. 18. *What are the S.I. and C.G.S. units of heat ? How are they related ?*

Ans. SI unit of heat is joule and *C.G.S.* unit of heat is calorie. 1 calorie = 4.18 Joule.

Q. 19. *Tea gets cooled when sugar is added to it. Why ?*

Ans. The sugar absorbs heat energy from the tea and hence temperature of the tea decreases.

Q. 20. *What is temperature gradient ?*

Ans. The fall in temperature of a body per unit distance is called the temperature gradient.

Q. 21. *Which substance has maximum value of specific heat ?*

Ans. Out of solids and liquids water has maximum value of specific heat (4186 J kg^{-1} K^{-1}). In gases, hydrogen gas has maximum value of specific heat, which is even more than that of water.

Q. 22. *Does specific heat of water remain constant throughout ?*

Ans. No, the specific heat of water changes with temperature range.

Q. 23. *The temperature gradient in a rod 0.5 m long is 80°C per metre. The temperature of the hotter end is 30°C. What is the temperature of the colder end ?*

Ans. $30 - 0.5 \times 80 = -10\ °C$

Q. 24. *Arrange Cu, Al and Ag in the order of increasing thermal conductivity ?*

Ans. Al, Cu, Ag.

Q. 25. *Solar pond is a device for collecting solar heat. The pond is about one metre deep, filled with saturated salt solution and protected from air current and other disturbances. When exposed to sun, the temperature at the bottom can go as high as 80°C or more. Why is this possible ?*

Ans. It is due to the fact that the thermal conductivity of salt is very high.

Q. 26. *On winter nights, we feel warmer when clouds cover the sky than when the sky is clear. Why ?*

Ans. Clouds are bad conductors. So, heat of the Earth's atmosphere is not conducted out.

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *The design of some physical instrument requires that there be a constant difference in length of 10 cm between an iron rod and copper rod laid side by side at all temperatures. Find their lengths.* ($\alpha_{Fe} = 11 \times 10^{-6}\ °C^{-1}$, $\alpha_{Cu} = 17 \times 10^{-6}\ °C^{-1}$)

Ans. Since the $\alpha_{Cu} > \alpha_{Fe}$ so length of iron rod should be greater than the length of copper rod.

Let the initial lengths of iron and copper rods be l_1 and l_2, then

$$l_1 - l_2 = 10 \text{ cm} \quad ...(i)$$

Also since the difference has to be constant at all the temperatures, so

$$\Delta l = l_1 \alpha_{Fe} \Delta T = l_2 \alpha_{Cu} \Delta T$$

$$\frac{l_1}{l_2} = \frac{\alpha_{Cu}}{\alpha_{Fe}} \quad ...(ii)$$

Solving eqn. (*i*) and (*ii*), we get

$$l_1 = 28.3 \text{ cm} \quad \text{and} \quad l_2 = 18.3 \text{ cm.}$$

Q. 2. *Calculate the temperature whose value is the same on the celsius and Fahrenheit scale ?*

Ans. Let the required temperature in both the scales be x.

i.e., $C = F = x$

Now $$\frac{C}{100} = \frac{F-32}{180}$$

$\therefore$ $$\frac{x}{100} = \frac{x-32}{180}$$

$$180\,x = 100\,x - 3200$$

$$80\,x = -3200 \Rightarrow x = -40$$

$-40°C$ is equivalent to $-40°$ F.

Q. 3. *A cylinder of diameter exactly 1 cm at 30°C is to be slid into a hole in a steel plate. The hole has a diameter of 0.99970 cm at 30°C. To what temperature must the plate be heated ? For steel $\alpha = 1.1 \times 10^{-5}$ (°C^{-1}).*

Ans. The hole will expand in the same way as a circle of steel filling it would expand. The diameter of the hole needs to be changed by

$$\Delta l = 1 - 0.99970 = 0.00030 \text{ cm}$$

But $\Delta l = \alpha\, l\, \Delta T$

$$\therefore \quad \Delta t = \frac{\Delta L}{\alpha l} = \frac{0.00030}{1.1\times10^{-5}\times0.99970} = 27.3°\text{C}$$

The plate must be raised to a temperature of 30 + 27.3 = 57.3 °C

Q. 4. *A steel scale measures the length of a copper rod as 80.00 cm when both are at 20°C, the calibration temperature for the scale. What would the scale read for the length of the rod when both are at 40°C ? α for steel = 11 × 10^{-6} (°C^{-1}) and α for copper = 17 × 10^{-6} (°C)$^{-1}$.*

Ans. The length of 1 cm division of the steel scale at 40 °C is

(1 cm) × (1 + 11 × 10^{-6} × 20) = 1.00022 cm

Length of the copper rod at 40°C will be (80) × (1 + 17 × 10^{-6} × 20) = 80.0272 cm. The number of cm read on the scale will be

$$\frac{80.0272}{1.00022}\text{cm} = 80.0096 \text{ cm}$$

Q. 5. *Two vessels made of two different metals are identical in all respects.They are completely filled with ice at 0°C. The ice in one is melted in 30 minutes and that in another in 10 minutes by heat coming from outside. Compare the thermal conductivities of metals.*

Ans. We know that $Q = \dfrac{KA\,(T_1 - T_2)\,t}{l}$

For given problem, kt = constant or $k \propto \dfrac{1}{t}$.

$$\therefore \quad \frac{k_1}{k_2} = \frac{t_2}{t_1} = \frac{10}{30} = \frac{1}{3}.$$

Q. 6. *Is the rate of cooling the same thing as the rate of loss of heat ? Explain.*

Ans. No. The rate of cooling of a body at a temperature is defined as the fall in temperature per second at that temperature, while the rate of loss of heat is the quantity of heat lost per second from a body at a given temperature.

Q. 7. *Calculate the power developed by a person, while eating 100g of ice per minute. Latent heat of ice = 80 cal g^{-1}.*

Ans. Mass of ice eaten per second,

$$m = \frac{100}{60}\,\text{gs}^{-1} = \frac{5}{3}\,\text{gs}^{-1}$$

$\therefore$ Power developed by a person = $mL = \dfrac{5}{3} \times 80$ cal s^{-1} = $\dfrac{5}{3} \times 80 \times 4.2$ Js^{-1} = 560 W.

Q. 8. *A glass flask of volume 250 cm^3 is just filled with mercury at 20°C. How much mercury overflows when the temperature of the system is raised to 100°C ? The coefficient of volume expansion of glass is 12 × 10^{-6} (°C)$^{-1}$ and that of mercury is 18 × 10^{-5} (°C)$^{-1}$.*

Ans. The increase in the volume of the flask is

$$(\Delta V)_f = V\gamma\,\Delta t = 250 \times 12 \times 10^{-6} \times 80 = 0.24 \text{ cm}^3$$

The increase in the volume of mercury is

$$(\Delta V)_m = 250 \times 18 \times 10^{-5} \times 80 = 3.6 \text{ cm}^3$$

Therefore, the volume of mercury overflowing is

$$3.6 - 0.24 = 3.36 \text{ cm}^3$$

Q. 9. *A mercury barometer has a quartz bulb of volume 0.300 cm³ and a stem of bore 0.0100 cm. How far does the indicating thread of mercury move when the temperature changes from 30°C to 45°C ? Ignore the small expansion of the quartz bulb. The volume expansivity of mercury is 182 × 10⁻⁶ (°C)⁻¹.*

Ans. The increase in volume due to expansion is given by

$$\Delta V = \gamma V \Delta t = 182 \times 10^{-6} \times 0.300 \times 15 = 8.19 \times 10^{-4} \text{ cm}^3$$

Cross-sectional area of the bore = $\pi (0.01/2)^2 = \pi (0.005)^2$ cm^2

Height through which mercury thread rises = $\dfrac{8.19 \times 10^{-4}}{\pi (5 \times 10^{-3})^2}$ = 10.4 cm

Q. 10. *A certain substance has a mass of 50g/ mole. When 300 J of heat is added to 25g of sample of this material, its temperature rises from 25°C to 45°C. Calculate (i) the thermal capacity, (ii) specific heat capacity, and (iii) molar heat capacity of the sample.*

Ans. (*i*) Total heat supplied to sample ΔQ = 300 J and rise in temperature

$$\Delta T = T_2 - T_1 = 45 - 25 = 20 \text{ °C}$$

$\therefore$ Thermal capacity of substance = $\dfrac{\Delta Q}{\Delta T} = \dfrac{300 J}{20°C}$ = 15 J °C^{-1}

(*ii*) As mass of sample m = 25g = 0.025 kg

$\therefore$ Specific heat capacity $C = \dfrac{1}{m} \cdot \dfrac{\Delta Q}{\Delta T} = \dfrac{1}{0.025} \times 15$ = 600 J kg^{-1} °C^{-1}

(*iii*) As the substance has a mass of 50g/mole, hence number of moles in 25g sample

$$\mu = \frac{25}{50} = 0.5 \text{ mole}$$

$\therefore$ Molar heat capacity $C = \dfrac{1}{\mu} \dfrac{\Delta Q}{\Delta t}$

$\Rightarrow$ $C = \dfrac{1}{0.5} \times 15$ J mol^{-1} °C^{-1} or C = 30 J mol^{-1} °C^{-1}.

Q. 11. *Why does a gas not have a unique value of specific heat ?*

Ans. This is because a gas can be heated under different conditions of pressure and volume. The amount of heat required to raise the temperature of unit mass through unit degree is different under different conditions of heating.

Q. 12. *When two bodies having temperatures T_1 and T_2 are brought in contact, then the temperature of this system may not be $\dfrac{(T_1 + T_2)}{2}$. Explain why ?*

Ans. If two bodies made of same material and have the same mass but different temperatures T_1 and T_2 , then their equilibrium temperature T when they are brought in thermal contact will be $\dfrac{(T_1 + T_2)}{2}$. [$\because$ Heat lost = Heat gained]

The equilibrium temperature T will not be necessarily $\dfrac{(T_1 + T_2)}{2}$ if two bodies in thermal equilibrium have different heat capacities.

Q. 13. *Distinguish clearly between the three modes of heat transmission.*

Ans. Three modes of heat transmission are conduction, convection and radiation. Their main points of distinction are as follows:

Conduction	*Convection*	*Radiation*
1. There is no bodily motion of medium particles. Medium particles vibrate to and fro about their mean positions and pass on thermal energy to the neighbouring particles.	1. Heat is transferred from one part of system to another by the actual motion of the particles of the system.	1. Medium has no role as thermal radiation are transmitted without any material medium.
2. No convection currents are formed.	2. Convection currents are formed.	2. Question of formation of convection currents does not arise.
3. Conduction of heat takes place in solids and few liquids like mercury and molten metals.	3. Convection of heat takes place in fluids *i.e.,* liquids as well as gases.	3. Radiant energy directly flows from heat source to the given body at a speed of 3×10^8 m/s as electromagnetic waves.

Q. 14. *What is meant by 'superheated water' and 'super cooled vapour' ? Do these states of water lie on its P-V-T surface ? Give some practical applications of these states of water.*

Ans. 'Superheated water' is water in liquid phase at a temperature above the boiling point of water at the given pressure. 'Super cooled vapour' is water in vapour form at a temperature below its boiling point at the given pressure.

These states are not equilibrium states. They are unstable states and do not lie on the P-V-T surface of water.

Applications: Bubble chamber and cloud chamber for detecting high speed charged nuclear particles.

Q. 15. *What is meant by triple point ? Give the values of triple point pressure and triple point temperature of water.*

Ans. Triple point is a point in the phase diagram, representing a particular pressure and temperature at which the solid, liquid and vapour phases of the substance co-exist.

Triple point pressure of water is 0.46 cm of mercury column or 0.006 atm. and triple point temperature of water is 273.16 K or 0.01°C.

Q. 16. *What are thermal radiations? State their important properties.*

Ans. Thermal radiation refers to radiant energy emitted by a hot body, which travels through vacuum at a speed of 3×10^8 ms^{-1}.

Important properties of thermal radiation are as follows:

(1) Thermal radiations do not need any material medium for their propagation and travel in vacuum with a speed of 3×10^8 ms^{-1} *i.e.,* the speed of light.

(2) Thermal radiations travel in straight line and form shadows.

(3) Thermal radiations can be reflected as well as refracted like light radiation.

(4) Thermal radiations do not affect the medium through which they pass.

(5) Thermal radiations obey inverse square law. Thus, intensity of thermal radiation at a surface is inversely proportional to the square of the distance of the given surface from the heat source.

(6) When a body absorbs thermal radiation, its temperature rises.

(7) Thermal radiations are electromagnetic waves whose wavelengths are more than of visible light.

Q. 17. *A copper calorimeter of mass 100 g contains a lump of ice at 4°C. When 520 calories of heat are given to the calorimeter and its contents, the temperature rises from – 4°C to – 2 °C. The addition of another 41540 calories of heat brings the temperature of the calorimeter and its contents to 2 °C. Determine the specific heat capacity of copper and the mass of ice present in the calorimeter. Given: Latent heat of fusion of ice = 80 cal* g^{-1}

Specific heat capacity of ice = 0.5 cal g^{-1} $(°C)^{-1}$

Ans. Let s be the specific heat capacity of copper and m the mass of ice present in the calorimeter. We then have

$100 \times s \times [-2-(-4)] + m \times 0.5 \times [-2-(-4)] = 520$

or $200s + m = 520$ ...(*i*)

Also $100s \times 2 - (-2) + m \times 0.5 \times (-2) + m \times 80 + m \times 1 \times (2-0) = 41540$ ...(*ii*)

or $400s + 83m = 41540$

Solving eqn. (*i*) and (*ii*), we get

$$m = 500\text{g} \quad \text{and} \quad s = 0.1 \text{ cal g}^{-1} (°\text{C})^{-1}.$$

Q. 18. *On what factors the amount of heat flowing from hot face to the cold face depends ? How ?*

Ans. If Q be the amount of heat flowing from hot to the cold face, then it is found to be:

(*i*) directly proportional to the cross-sectional area (A) of the face.

i.e., $Q \propto A$ ...(1)

(*ii*) directly proportional to the temperature difference between the two faces *i.e.,*

i.e., $Q \propto \Delta\theta$ ...(2)

(*iii*) directly proportional to the time t for which the heat flows.

i.e., $Q \propto t$ ...(3)

(*iv*) inversely proportional to the distance '*d*' between the two faces.

i.e., $Q \propto \frac{1}{\Delta x}$ ...(4)

Combining factors (1) to (4), we get

$$Q \propto \frac{A\Delta\theta}{\Delta x}t \quad \text{or} \quad Q = KA\frac{\Delta\theta}{\Delta x}t$$

where K is the proportionality constant known as the coefficient of thermal conductivity.

Q. 19. *A metallic wire has resistance of 20 ohm at 20°C and a resistance of 21.2 ohm at 40°C. Calculate the temperature coefficient of resistance.*

Ans. Here $R_{20°C} = 20\Omega$, $R_{40°C} = 21.2\ \Omega$.

$$\Delta\theta = 40°C - 20°C = 20°C$$

Using $\alpha = \frac{R_{40°C} - R_{20°C}}{R_{20°C} \times \Delta T}$, we get

$$\alpha = \frac{21.2-20}{20 \times 20} = \frac{1.2}{400} = 3.0 \times 10^{-3}\ °C^{-1}$$

Q. 20. *When 0.2 kg of a body at 100°C is dropped into 0.5 kg of water at 10°C, the resulting temperature is 16°C. Find the specific heat of the body. Specific heat of water is 4.2 ×* 10^3 *J/kg/°C.*

Ans. **For the body,**

$$m_1 = 0.2 \text{ kg}; \Delta T_1 = 100° - 16° = 84°C;$$

$$s_1 = ?$$

For water,

$$m_2 = 0.5 \text{ kg}; \Delta T_2 = 16° - 10° = 6°\text{C};$$
$$s_2 = 4.2 \times 10^3 \text{ J/kg/°C}$$

From law of conservation of energy;

heat lost by body = heat gained by water

i.e., $m_1 s_1 \Delta T_1 = m_2 s_2 \Delta T_2$

$$\text{or} \quad s_1 = \frac{m_2 s_2 \Delta T_2}{m_1 \Delta T_1} = \frac{0.5 \times (4.2 \times 10^3) \times 6}{0.2 \times 84} = \mathbf{0.75 \times 10^3 \text{ J/kg/°C}}$$

Q. 21. *A copper plate has an area of 250 cm² at 0°C. Calculate the area of this plate at 60°C. Given coefficient of linear expansion of copper is 1.7 × 10⁻⁵ °C⁻¹.*

Ans. Here,
$$A_0 = 250 \text{ cm}^2$$
$$\beta = 2\alpha = 2 \times 1.7 \times 10^{-5} = 3.4 \times 10^{-5} \text{ °C}^{-1}.$$
$$\Delta T = (60 - 0) = 60°\text{C}.$$

Using $A_\theta = A_0 (1 + \beta \Delta T)$, we have

$$A_{80°\text{C}} = 250 (1 + 3.4 \times 10^{-5} \times 60) = 250 (1 + .00204) = 250.51 \text{ cm}^2$$

∴ Area of copper plate at 60°C

$$= 250.51 \text{ cm}^2.$$

Q. 22. *What is the temperature which has the same numerical value in kelvin scale and Fahrenheit scale?*

Ans. Let x be the required value in Kelvin and Fahrenheit scales.

Now,
$$\frac{x - 273}{373 - 273} = \frac{x - 32}{212 - 32}$$
$$\frac{x - 273}{100} = \frac{x - 32}{180}$$
$$9(x - 273) = 5(x - 32) \Rightarrow 9x - 2457 = 5x - 160$$
$$4x = 2457 - 160 = 2297$$
$$x = \frac{2297}{4} = 574.25°$$

∴ 574.25 K = 574.25 °F.

Q. 23. *The volume of a metal sphere is 1000 cm³ at 273 K. What would be its volume at 373 K ? Coefficient of linear expansion of the metal is 18 × 10⁻⁶ K⁻¹.*

Ans. Here,
$$V = 1000 \text{ cm}^3 = 10^{-3} \text{ m}^3;$$
$$(T_2 - T_1) = 373 - 273 = 100 \text{ K}$$
$$\alpha = 18 \times 10^{-6} \text{ K}^{-1}; \quad V_{T_2} = ?$$
$$\gamma = 3\mu = 3 \times (18 \times 10^{-6}) = 54 \times 10^{-6} \text{ K}^{-1}$$

As $V_{T_2} = V_{T_1} [1 + \gamma (T_2 - T_1)] = 10^{-3} [1 + 54 \times 10^{-6} \times 100]$

$$= 10^{-3} [1 + 0.0054] = 1.0054 \times 10^{-3} \text{ m}^3$$

Q. 24. *How should one kg of water at 5°C be so divided that one part of it when converted into ice at 0°C, would by this change of state provide a quantity of heat that would be sufficient to vaporise the other part ?*

Ans. Initially 1000g of water is at 5°C.

Let m gram of it be cooled to ice at 0°C.

Heat released due to this = $(m \times 1 \times 5) + (m \times 80)$

$= 5m + 80m = 85m$ cal.

The heat required by $(1000 - m)$ g of water at 5 °C to become steam at 100°C

$= [(1000 - m)(100 - 5) + (1000 - m) 540]$ cal

$= (1000 - m)(95 + 540)$ cal $= (1000 - m)(635)$ cal

Now, $85m = (1000 - m)(635)$ or $720m = 635 \times 1000$

$$\Rightarrow \quad m = \frac{635 \times 1000}{720} = 881.9\text{g}$$

Hence 881.9 g of water by turning into at 0°C will supply heat to evaporate 118.1 g of water.

III. LONG ANSWER TYPE QUESTIONS

Q. 1. *State and explain Newton's law of cooling. Calculate the increase in the temperature of water which falls from a height of 100 m. Assume that 90% of the energy due to fall is converted into heat and is retained by water.* $J = 4.2 \text{ J Cal}^{-1}$.

Ans. Newton's law of cooling states that the rate of loss of heat of a body is directly proportional to the difference in temperature of the body and the surroundings, provided the difference in temperature is small, not more than 40°C.

The rate of loss of heat by a body through radiation also depends upon (*i*) nature of the radiating surface and (*ii*) area of the radiating surface.

Let a body of mass m, specific heat s at temperature T be kept at a place and temperature of surroundings be T_0, where $T > T_0$. There will be loss of heat by the body. Let $d\theta$ be the amount of heat lost by the body in time dt. According to Newton's law of cooling

$$-\frac{d\theta}{dt} \propto (T - T_0) \quad \text{or} \quad -\frac{d\theta}{dt} = K(T - T_0)$$

where K is a constant of proportionality.

Heat in the body, $Q = msT$

$$\therefore \quad -\frac{d}{dt}(msT) = K(T - T_0)$$

$$\text{or} \quad -\frac{dT}{dt} = \frac{k}{ms}(T - T_0) \quad \text{or} \quad -\frac{dT}{dt} = K(T - T_0) \qquad ...(i)$$

where $\frac{k}{ms} = K$ = another constant and

$-\frac{dT}{dt}$ = rate of fall of the temperature of the body.

$$\text{From eqn. } (i) \quad -\frac{dT}{dt} \propto (T - T_0) \qquad ...(ii)$$

Thus, Newton's law of cooling also states that the rate of fall of temperature of a body is directly proportional to the difference of temperature between the body and the surroundings, provided, the temperature difference is not more than 40°C.

Numerical: Here, $h = 100$ m.

m (kg) = mass of water

$\therefore$ Its P.E. at a height $h = mgh$

Energy of fall retained by water *i.e.,* useful work done, W = 90% of mgh

$$= \frac{90}{100} mgh = \frac{90}{100} \times m \times 9.8 \times 100 = 882\ m\ \text{J}.$$

$\therefore$ Heat retained, $Q = \frac{W}{J} = \frac{m \times 882\ \text{J}}{4.2\ \text{J Cal}^{-1}} = m \times 210$ cal ...(*i*)

Specific heat of water, $C = 10^3$ cal Kg^{-1} $(°\text{C})^{-1}$

Let $\Delta\theta$ °C be the rise in the temperature of water.

$\therefore$ Heat gained, $Q = mc\ \Delta\theta = m \times 10^3 \times \Delta\theta$

$= m \times \Delta\theta \times 10^3$ cal ...(*ii*)

From eqn. (*i*) and (*ii*), we get

$$m \times 210 = m \times \Delta\theta \times 10^3 \quad \text{or} \quad \Delta\theta = \frac{210}{10^3} = 0.21°\text{C}.$$

Q. 2. *What is Calorimetry ? State its principle.*

A copper calorimeter of mass 100 g contains 200 g of a mixture of ice and water. Steam at 100°C under normal pressure is passed into the calorimeter and the temperature of the mixture is allowed to rise to 50 °C. If the mass of the calorimeter and its contents is now 330 g, what was the ratio of ice and water in the beginning ? Neglect heat losses. Given

Specific heat capacity of copper = 0.42×10^3 J kg^{-1} K^{-1}

Specific heat capacity of water = 4.2×10^3 J kg^{-1} K^{-1}

Latent heat of fusion of ice = 3.36×10^5 J kg^{-1}

Latent heat of condensation of steam = 22.5×10^5 J kg^{-1}

Ans. For calorimetry and its principle, see text

Heat is lost by steam in getting condensed and gained by water, ice and the calorimeter. Let the calorimeter originally contain x grams of ice and $(200 - x)$ grams of water. We then have

Heat gained by calorimeter $= \frac{100}{1000} \times 0.42 \times 10^3 \times (50 - 0) = 2100$ J

Heat gained by ice $= \frac{x}{1000} \{3.36 \times 10^5 + 4.2 \times 10^3 (50 - 0)\}$

$= x \{(336 + 210)\} = 546\ x$ J

Heat gained by water $= \frac{(200 - x)}{1000} \times 4.2 \times 10^3 \times (50 - 0) = (42000 - 210\ x)$ J

Heat lost by steam $= \frac{(330 - 200 - 100)}{1000} \times [22.5 \times 10^5 + 4.2 \times 10^3 \times (100 - 50)]$

$= 30(2250 + 210) = 73800$ J

Heat gained = Heat lost

$$546x + 2100 + 42000 - 210x = 73800$$

which gives $x = 88.4$ g

Mass of ice in the original mixture = 88.4 g

Mass of water in the original mixture = 200 – 88.4 = 111.6 g

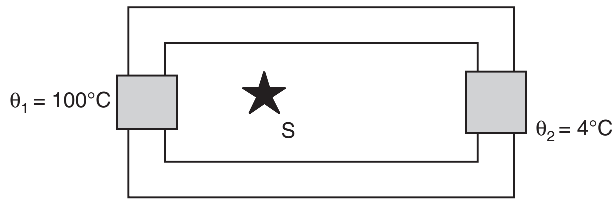
Ratio of ice present to water present = 88.4 : 111.6 = 1 : 1.26

Q. 3. *What do you understand by thermal resistance ?*

A closed cubical box is made of perfectly insulating material and the only way for heat to enter or leave the box is through two solid cylindrical metal plugs, each of cross-sectional area 12 cm^2 and length 8 cm fixed in the opposite walls of the box. The outer surface of one plug is kept at a temperature of 100°C while the outer surface of other plug is maintained at a temperature of 4°C. The thermal conductivity of the material of the plug is 2.0 W/m–°C. A source of energy generating 13 W is enclosed inside the box. Find the equilibrium temperature of the inner surface of the box assuming that it is the same at all points on the inner surface.

Ans. For thermal resistance, see text.

Numerical: The situation is shown in figure. Let the temperature inside the box be θ. The rate at which heat enters the box through the left plug is



$$\frac{\Delta\theta_1}{\Delta t} = \frac{KA(\theta_1 - \theta_2)}{x}$$

The rate of heat generation in the box = 13 W. The rate at which heat flows out of the box through the right plug is

$$\frac{\Delta\theta_1}{\Delta t} + 13\text{ W} = \frac{\Delta\theta_2}{\Delta t} \quad \text{or} \quad \frac{KA}{x}(\theta_1 - \theta) + 13\text{ W} = \frac{KA}{x}(\theta - \theta_2)$$

or $$2\frac{KA}{x}\theta = \frac{KA}{x}(\theta_1 + \theta_2) + 13\text{ W} \Rightarrow \theta = \frac{\theta_1 + \theta_2}{2} + \frac{(13W)\,x}{2KA}$$

or $$\theta = \frac{100°\text{C} + 4°\text{C}}{2} + \frac{(13\text{ W}) \times 0.08\text{m}}{2 \times (2.0W/\text{m}-°\text{C})\,(12 \times 10^{-4}\text{m}^2)}$$

$$= 52°\text{C} + 216.67°\text{C} \approx 269\ °\text{C}.$$

Q. 4. *What is thermal expansion ? Discuss the types of thermal expansions.*

Equal volumes of water and alcohol, when put in similar calorimeters take 100 s and 74 s respectively to cool from 50°C to 40°C. Calculate the specific heat capacity of alcohol given that the thermal capacity of each calorimeter is numerically equal to the volume of either liquid. Take the relative density of alcohol as 0.8 and the specific heat capacity of water as 1 cal per gram per °C.

Ans. For thermal expansion and its classification, see text.

Let V cm^3 be the volume of either liquid. Then the thermal capacity of each calorimeter is also V cal per °C.

Mass of water = $V \times 1 = V$ g

Mass of alcohol = $V \times 0.8 = 0.8\ V$ g

The rate of cooling of the 'water calorimeter'

$$= \frac{1}{100}\{V \times (50-40) + V \times 1 \times (50-40)\} = 5\ V \text{ cal s}^{-1}$$

The rate of cooling of the 'alcohol calorimeter'

$$= \frac{1}{74}\{V \times (50-40) + 0.8\ V \times s \times (50-40)\} = \frac{1}{74}(10\ V + 8\ V\ s)$$

Because identical volumes of the liquids are getting cooled under identical conditions, the rate of cooling is the same in both the cases. Hence

$$5\ V = \frac{1}{74}(10\ V + 8\ V\ s)$$

which gives $s = 0.6$ cal g^{-1} ($°C^{-1}$)

IV. MULTIPLE CHOICE QUESTIONS

1. At about 4°C, a certain amount of water has maximum

(*a*) energy (*b*) specific heat (*c*) density (*d*) volume

2. Dimensional formula of specific heat capacity is

(*a*) $[ML^2T^{-2}L^{-1}]$ (*b*) $[MLT^{-2}K^{-1}]$ (*c*) $[M^0L^2T^{-2}K^{-1}]$ (*d*) $[ML^2T^{-2}K]$

3. If there are no heat losses, the heat released by the condensation of x gram of steam at 100 °C into water at 100 °C can be used to convert y gram of ice at 0 °C into water at 100 °C. Then the ratio $y : x$ is nearly

(*a*) 1 : 1 (*b*) 2 : 1 (*c*) 3 : 1 (*d*) 2.5 : 1

4. The top of a lake gets frozen at a place where the surrounding air is at a temperature of – 20°C. Then

(*a*) the temperature of the layer of water in contact with the lower surface of the ice block will be at 0°C and that at the bottom of the lake will be 4 °C;

(*b*) the temperature of water below the lower surface of ice will be 4 °C right up to the bottom of the lake;

(*c*) the temperature of the water below the lower surface of ice will be 0 °C right up to the bottom of the lake;

(*d*) the temperature of the layer of water immediately in contact with the lower surface of ice will be about – 20°C and that of water at the bottom will be 0 °C.

5. The scale on a steel meter rod is calibrated at 20°C. What will be the error in the reading of 50 cm at 27°C? Take, $\alpha = 1.2 \times 10^{-5}$ °C^{-1}.

(*a*) 0.042 cm (*b*) 0.0042 cm (*c*) 0.021 cm (*d*) 0.0021 cm.

6. The rates of cooling of two different liquids put in exactly similar calorimeters and kept in identical surroundings are the same if

(*a*) the masses of the liquids are equal

(*b*) equal masses of the liquids at the same temperature are taken

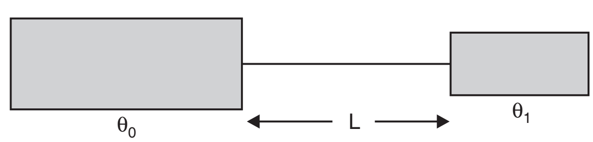
(*c*) different volumes of the liquids at the same temperature are taken

(*d*) equal volumes of the liquids at the same temperature are taken

Ans. **1.**—(*c*) **2.**—(*c*) **3.**—(*c*) **4.**—(*a*) **5.**—(*b*) **6.**—(*d*)

V. QUESTIONS ON HIGH ORDER THINKING SKILLS (HOTS)

Q. 1. *The figure shows a large tank of water at a constant temperature θ_0 and a small vessel containing a mass 'm' of water at an initial temperature θ_1 (< θ_0). A metal rod of length L, area of cross-section A and thermal conductivity K connects the two vessels. Find the time taken for the temperature of the water in the smaller vessel to become θ_2 ($\theta_1 < \theta_2 < \theta_0$). Specific heat capacity of water is 's' and all other heat capacities are negligible.*



Ans. Suppose the temperature of the water in the smaller vessel is θ at time t. In the next time interval dt, a heat, $\Delta\theta$ is transferred to it where

$$\Delta\theta = \frac{KA}{L}(\theta_0 - \theta)\,dt \qquad ...(i)$$

This heat increases the temperature of the water of mass 'm' to $\theta + d\theta$

where $$\Delta\theta = ms\,d\theta \qquad ...(ii)$$

From eqn. (*i*) and (*ii*),

$$\frac{KA}{L}(\theta_0 - \theta)\,dt = ms\,d\theta \quad \text{or} \quad dt = \frac{Lms}{KA}\frac{d\theta}{\theta_0 - \theta} \quad \Rightarrow \quad \int_0^T dt = \frac{Lms}{KA}\int_{\theta_1}^{\theta_2}\frac{d\theta}{\theta_0 - \theta}$$

where T is the time required for the temperature of the water to become θ_2.

Thus, $$T = \left[\frac{Lms}{KA}\ln\frac{\theta_0 - \theta_1}{\theta_0 - \theta_2}\right].$$

Q. 2. *The coefficient of apparent expansion of a liquid when determined using two different vessels A and B are γ_1 and γ_2 respectively. If the coefficient of linear expansion of vessel A is α, find the coefficient of linear expansion of vessel B.*

Ans. We know that coefficient of real expansion of liquid (γ_r) = Coefficient of apparent expansion of the liquid (γ_a) + coefficient of volume expansion (γ_v).

i.e., $$\gamma_r = \gamma_a + \gamma_v = \gamma_a + 3\alpha$$

Since the liquid is same in both the vessel, so value of γ_r is same.

For vessel A, $\gamma_r = \gamma_1 + 3\alpha_1 = \gamma_1 + 3\alpha$ $\qquad [\because \alpha_1 = \alpha, \text{ given}]$

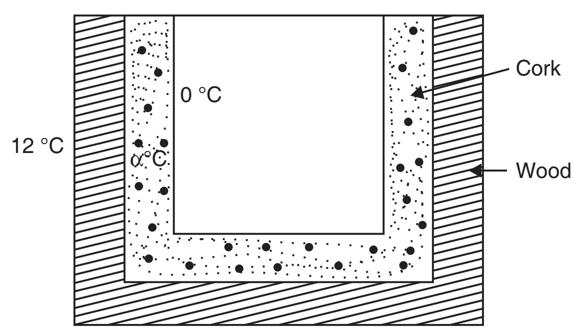
For vessel B, $\gamma_r = \gamma_2 + 3\alpha_2$

$$\therefore \quad \gamma_1 + 3\alpha = \gamma_2 + 3\alpha_2 \quad \text{or} \quad \alpha_2 = \frac{\gamma_1 - \gamma_2}{3} + \alpha.$$

Q. 3. *A block of wood is floating on water at 0°C with a certain volume x above the level of water. The temperature of water is gradually increased from 0°C to 8 °C. How does the volume x change with change in temperature ?*

Ans. As the density of water increases and volume of water decreases from 0 °C to 4°C, so the volume x of the wooden block will increase till the temperature of water becomes 4°C. Now, as the temperature increases from 4°C to 8°C, the density of water decreases and its volume increases above 4°C, therefore the volume x of the block will also decrease.

Q. 4. *An ice box is built of wood 1.75 cm thick, lined inside with cork 2 cm thick. If the temperature of the inner surface of the cork is 0°C and that of the outer surface of the wood is 12°C, what is the temperature of the interface ? The thermal conductivity of wood is 0.0006 and that of cork is 0.00012 CGS units.*



Ans. Let α be the temperature of the interface in the steady state. Since the same amount of heat must flow through wood and cork, we have

$$\frac{k_1 A(12-\alpha)}{l_1} = \frac{k_2 A(\alpha - 0)}{l_2}$$

where l_1 and l_2 are the thicknesses of wood and cork respectively. Their respective conductivities are k_1 and k_2 (Fig.).

Substituting the values we get

$$\frac{6\times10^{-1}}{1.75}\times(12-\alpha) = \frac{12\times10^{-5}}{2}\times\alpha \quad \text{or} \quad 12-\alpha = 1.75\,\alpha\times10^{-1}$$

or $$\alpha = \frac{12}{1.175} = 10.2°C$$

Q. 5. *Find γ for polyatomic gas and hence determine its value for a triatomic gas in which the molecules are linearly arranged.*

Ans. The energy of a polyatomic gas having 'n' degrees of freedom is given by.

$$E = n\times\frac{1}{2}KT\times N = \frac{n}{2}RT$$

$$\therefore \quad C_v = \frac{dE}{dT} = \frac{n}{2}R$$

$$\therefore \quad C_p = C_v + R = \frac{n}{2}R + R = \left(\frac{n}{2}+1\right)R$$

$$\therefore \quad \gamma = \frac{C_p}{C_v} = \frac{n/2+1}{n/2} = 1+\frac{2}{n}.$$

In case of a triatomic gas, $n = 7$

$$\therefore \quad \gamma = 1+\frac{2}{7}=\frac{9}{7}.$$

Q. 6. *A liquid cools from 70°C to 60°C in 5 minutes. Calculate the time taken by the liquid to cool from 60°C to 50°C, if the temperature of the surrounding is constant at 30°C.*

Ans. In the first case

$$T_1 = 70°C,\quad T_2 = 60°C,\ t = 5\text{ min},\ T_0 = 30°C$$

$$t = \frac{2.3026}{K}\log_{10}\frac{T_1-T_0}{T_2-T_0}$$

$$5 = \frac{2.3026}{K}\log_{10}\frac{70-30}{60-30} = \frac{2.3026}{K}\log_{10}\frac{4}{3} \quad \text{...(1)}$$

In the second case

$$T_1 = 60°C,\quad T_2 = 50°C,\quad T_0 = 30°C$$

$$t = \frac{2.3026}{K}\log_{10}\frac{60-30}{50-30} = \frac{2.3026}{K}\log_{10}\frac{3}{2} \quad \text{...(2)}$$

Dividing (2) by (1), we get

$$\frac{t}{5} = \frac{\log_{10}1.5}{\log_{10}1.3333} = \frac{0.1761}{0.1249} = 1.4$$

or $$t = 1.4\times5\text{ min} = \mathbf{7\ min.}$$

Q. 7. *A lead bullet strikes against a steel armour plate with the velocity of 300 ms^{-1}. If the bullet falls dead after the impact, find the rise in temperature of the bullet, assuming that the heat produced is shared equally between the bullet and the target. Specific heat of lead = 0.03 cal g^{-1} °C^{-1}.*

Ans. Let, mass of bullet = m kg

Rise in temperature of the bullet = ΔT°C

Velocity of bullet, v = 300 ms^{-1}

$$\text{K.E. of bullet} = \frac{1}{2}mv^2 = \frac{1}{2}m \times 9 \times 10^4$$

$$= (4.5 \times 10^4 m) \text{ J}$$

Useful K.E. to raise the temperature of the bullet,

$$W = \frac{1}{2} \times 4.5 \times 10^4 \text{ m} = 2.25 \text{ m} \times 10^4 \text{ J}$$

Heat produced in the bullet, $Q = mc\,\Delta T$

$$= 0.03 \times 4200 \text{ m } \Delta T\text{J} \qquad (\because 1 \text{ cal} = 4.2 \text{ J})$$

According to law of conservation of energy

$$Q = W$$

$\therefore$ $126 \text{ m } \Delta T = 2.25 \text{ m} \times 10^4$

or $\Delta T = 178.6°\text{C}$

TEST YOUR SKILLS

1. What is the main cause of transfer of heat from one body to another?
2. On which principle does a thermometer work?
3. What do you understand by specific heat capacity? Water is used both as coolant and as a heater in different situations. What is the reason for this?
4. In coastal areas, it is said that the land warms up more quickly, compared to the ocean surface. Do you agree with this statement? Give reasons for your answer.
5. What do you understand by calorimetry? How would you calculate the specific heat capacity of a given metallic sphere, using calorimetry?
6. When you pop open a hot chapatti the steam coming out of it is apparently hotter than chapatti, why?
7. What is the difference between latent heat of fusion and latent heat of vaporization?

❑❑❑