12 Atoms

Facts that Matter

• Thomson's Model of Atom

Every atom consists of a positively charged sphere of radius of the order of 10^{-10} m in which the negatively charged electrons are uniformly embedded like plums in a pudding.

This model could not explain scattering of α-particles through thin foils and hence discarded.

• Rutherford's α-Particle Scattering Experiment

In 1911, Rutherford suggested an experiment for α-particle scattering. For the experiment, *H*-Guiger and *E*. Marsden considered $^{214}_{83}Bi$ as a source of α-particles. A collimated beam of α-particles of energy 5.5 MeV was allowed to fall on a 2.1×10^{-7} m thick gold foil.

When it was detected with the help of an α-particle detector (made up of ZnS screen and a microscope), it was found that α-particle got scattered. The scattered α-particles produced scintillations on the zinc sulphide screen. These scintillations were counted at different angles from the direction of incident beam.

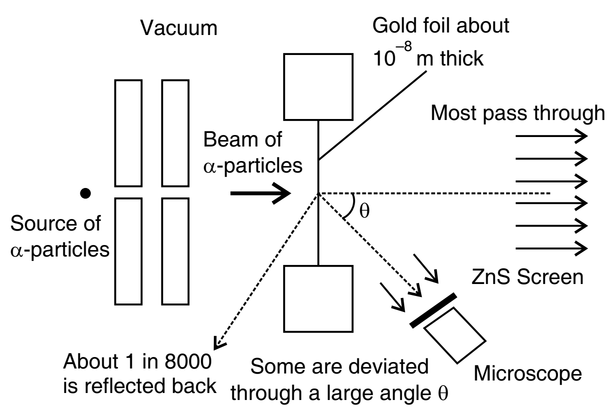

Fig. 12.1

Observations:

- Most α-particles passed through the foil undeviated and reached the screen.
- A large number of α-particles suffered large deflections.
- A small number of α-particles retraced its own path (deflected at 180°).

Explanation:

- Since most of α-particles passed without deflection, a large part of the atom should consist of empty space.
- For α-particles to be deflected at 180°, most of the atomic mass must be concentrated in a small space and should be positively charged. This was called nucleus.
- α-particles scatter due to intense electric field near a nucleus.

Limitations:

This model could not explain instability of the atom because according to classical electromagnetic theory the electron revolving around the nucleus must continuously radiate energy in the form of electromagnetic radiations and hence it should fall into the nucleus.

- **Distance of Closest Approach**

When a α-particle of mass m and velocity v moves directly towards a nucleus of atomic number 'z', its distance of closest approach is given by

$$r_0 = \frac{2k\,Z_e^2}{E} = \frac{4k\,Z_e^2}{mv^2}$$

where, $$E = \frac{1}{2}mv^2$$

and $$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^2$$

- **Impact Parameter**

Impact parameter is defined as the perpendicular distance of the initial velocity vector of the alpha particle from the central line of the nucleus, when the particle is far away from the nuclus of the atom.

Impact parameter, $$b = \frac{ze^2 \cot\frac{\theta}{2}}{4\pi\varepsilon_0 E} = \frac{ze^2 \cot\frac{\theta}{2}}{4\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)}$$

For large values of b, $\cot\frac{\theta}{2}$ is large and scattering angle (θ) is small *i.e.,* if α-particles are travelling away from the nucleus, they suffer small deflection.

For small value of b, $\cot\frac{\theta}{2}$ is small and scattering angle θ is large. *i.e.,* if α-particles are travelling close to the nucleus, they suffer large deflection.

When $b = 0$, $\cot\frac{\theta}{2} = 0$; then $\frac{\theta}{2} = 90°$. Hence $\theta = 180°$. *i.e.,* the α-particles travelling directly towards the nucleus, retraces its own path.

- **Bohr's Postulates of Atomic Theory**

Bohr combined classical and early quantum concepts and gave his theory in the form of three postulates.

(*i*) An electron in an atom could revolve in certain stable orbits without the emission of radiant energy contrary to the prediction of electromagnetic theory. According to this postulates, each atom has certain definite stable states in which it can exist and each possible state has definite total energy. These are called stationary states of the atom or orbit.

(*ii*) The electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of $h/2\pi$ where h in Planck's constant ($= 6.6 \times 10^{-34}$ Js). Thus, the angular momentum L of the orbiting electron is quantised, *i.e.*,

$$L = \frac{nh}{2\pi} = mvr$$

(v = velocity of electron and r = radius of orbit).

(*iii*) An electron might make a transition from one of its specified non-radiating orbits to another of lower energy. When it does so, a photon is emitted having energy equal to the energy difference between the initial and final states. The frequency of the emitted photon in giving by

$$\nu = \frac{1}{h}(E_i - E_f)$$

where E_i and $\boldsymbol{E_f}$ are the energies of the initial and final states and $E_i > \boldsymbol{E_f}$.

• Velocity of Electron in its Orbit

Let an electron is revolving in an atom of atomic number z in its orbit of radius R. The required centrepetal force for this electron is provided by electrostatic force between the electron and nucleus.

Thus, $$\frac{mv^2}{R} = \frac{k(e)(Ze)}{R^2}$$

or $$(mvR)v = kZe^2$$

or $$\frac{nh}{2\pi}.v = kZe^2$$

or $$\boxed{v = \frac{2\pi kZe}{nh}} \quad ...(i)$$

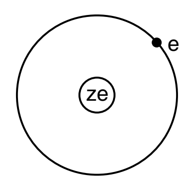

Fig. 12.2

• Radius of the Electron's Orbit

$\because$ $$\frac{mv^2}{R} = \frac{k(Ze)(e)}{R^2}$$

$\therefore$ $$R = \frac{kZe^2}{mv^2}$$

Putting the value of v from Eq. (*i*)

$$R = \frac{kZe^2}{m\left(\frac{2\pi kZe^2}{nh}\right)^2}$$

or $$\boxed{R = \frac{n^2h^2}{4\pi^2 kZe^2 m}} \qquad ...(ii)$$

• Energy of the Electron in its Orbit

(*i*) **Kinetic energy**

$$\because \quad \frac{mv^2}{R} = \frac{k(Ze)(-e)}{R^2}$$

$$\therefore \quad \frac{1}{2}mv^2 = \frac{1}{2}\cdot\frac{kZe^2}{R} \qquad ...(iii)$$

(*ii*) **Potential energy**

Potential energy of two charges (e) and (Ze) separated by R is given by

$$U = \frac{k(Ze)(-e)}{R}$$

$$= -\frac{kZe^2}{R} \qquad ...(iv)$$

Thus, total energy of the electron in its orbit

$$E = \frac{1}{2}\frac{kze^2}{R} - \frac{kze^2}{R}$$

or $$E = -\frac{1}{2}\frac{kze^2}{R} \qquad ...(v)$$

Substituting the value of R from Eq. (*ii*)

$$E = -\frac{1}{2}\frac{kZe^2}{\left(\frac{n^2h^2}{4\pi^2 kZe^2 m}\right)}$$

or $$\boxed{E = -\frac{2\pi^2k^2Z^2e^4m}{n^2h^2}} \qquad ...(vi)$$

or $$E = -13.6\ \frac{Z^2}{n^2}\ \text{eV.}$$

• Spectral Lines in Hydrogen Atom

- Let E_1 and E_2 be the energies of an electron in orbit n_1 and n_2 respectively. When electron jumps from orbit of higher energy (n_2) to orbit of lower energy (n_1) it emits energy of frequency ν in the form of photon.

$$h\nu = E_2 - E_1$$

where, $$E_2 = -\frac{2\pi^2 k^2 Z^2 e^4 m}{n_2^2 h^2}$$

and $$E_1 = -\frac{2\pi^2 k^2 Z^2 e^4 m}{n_1^2 h^2}$$

∴ $$h\nu = \frac{2\pi^2 k^2 Z^2 e^4 m}{h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

or $$\frac{a}{\lambda} = \frac{2\pi^2 k^2 Z^2 e^4 m}{h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

or $$\frac{1}{\lambda} = \frac{2\pi^2 k^2 Z^2 e^4 m}{ch^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

or $$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

where R is Rydberg's constant.

and $\frac{1}{\lambda}$ is wave number.

- When electron jumps from any higher energy level to that lower orbit, the distribution of energy wavelength wise is called spectrum.
- When electron jumps from any higher energy level n_2 = 2, 3, 4, ... to first orbit n_1 = 1, the sequence of spectral lines obtained *Lyman series.*

  For Lymen series,

$$\frac{1}{\lambda} = R\left[1 - \frac{1}{n_2^2}\right]$$

$$\lambda = \frac{n_2^2}{R\,(n_2^2 - 1)} = \frac{4}{3R}, \frac{9}{8R}, \frac{16}{15R}, \ldots$$

- When electron jumps from any higher energy level n_2 = 3, 4, 5... to lower energy level n_1 = 2, the sequence of spectral lines obtained is called *Balmer series*.

$$\frac{1}{\lambda} = R\left[\frac{1}{4} - \frac{1}{n_2^2}\right]$$

or $$\lambda = \frac{4\,n_2^2}{R\,(n_2^2 - 4)} = \frac{36}{5R}, \frac{64}{12R}, \ldots$$

- When electron jumps from any higher energy level n_2 = 4, 5, 6 ... to lower energy n_1 = 3, the sequence of spectral lines obtained is called *Paschen series.*

  For Paschen series,

$$\frac{1}{\lambda} = R\left[\frac{1}{9} - \frac{1}{n_2^2}\right]$$

or $$\lambda = \frac{9n_2^2}{R\,(n_2^2 - 9)} = \frac{144}{7R}, \frac{225}{16R}, \ldots$$

- When electron jumps from any heigher energy level n_2 = 5, 6, 7... to lower energy level n_1 = 4, the sequence of spectral lines obtained is called Breckett series.

  For Breckett series

$$\frac{1}{\lambda} = R\left[\frac{1}{16} - \frac{1}{n_2^2}\right]$$

or
$$\lambda = \frac{16n_2^2}{R\,[n_2^2 - 16]} = \frac{400}{9R}, \frac{576}{20R}, \ldots$$

- When electron jumps from any higher energy level n_2 = 6, 7, 8 ... to lower energy level n_1 = 1, the sequence of spectral lines obtained is called *pfund series*.

  For Pfund series,

$$\frac{1}{\lambda} = R\left[\frac{1}{25} - \frac{1}{n_2^2}\right]$$

or
$$\lambda = \frac{25\,n_2^2}{R\,[n_2^2 - 25]} = \frac{900}{11R}, \frac{1225}{24R}, \ldots$$

Above series are shown in Fig. 12.3.

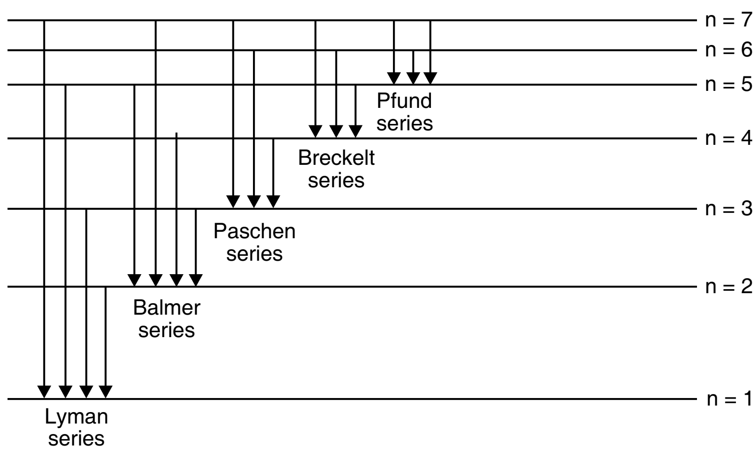

Fig. 12.3

• De Broglie' Explanation of Bohr's Postulate of Quantisation

Louis de Broglie argued that the electron in its orbit, as proposed by Bohr, must be seen as particle wave. In analogy to waves travelling on a string, particle waves too can lead to standing waves under resonant conditions.

When a string is plucked, a vast number of wavelengths are excited. However, only those wavelengths survive which have nodes at the ends and form the standing wave in the string. It means that in a string, standing waves are formed when the total distance travelled by a wave down the string and back is one wavelength, two wavelengths, or any integral number of wavelengths. Waves with other wavelengths interfere with themselves upon reflection and their amplitudes quickly drop to zero. For an electron moving in n^{th} circular orbit of radius r, the total distance is the circumference of the orbit, $2\pi r$.

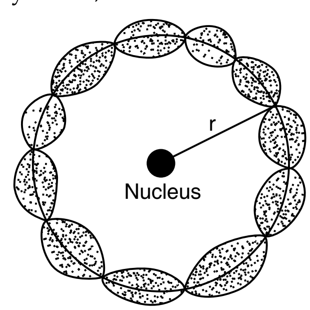

Fig. 12.4

Thus, $2\pi r = n\lambda$ ...(*i*)

where $n = 1, 2, 3...$

Applying de-Broglie wavelength-momentum concept,

$$\lambda = \frac{h}{P} = \frac{h}{mv} \quad ...(ii)$$

From Eqs. (*i*) and (*ii*)

$$2\pi r = n\left(\frac{h}{mv}\right)$$

or

$$mvr = \frac{nh}{2\pi}$$

Thus, the angular momentum of electron (mvr) in its orbit is integral multiple of $h/2\pi$.

- **Limitations of Bohr's Atomic Model**
  - The Bohr model is applicable to hydrogenic atoms. It cannot be extended even to more two electron atoms such as helium. The analysis of atoms with more than one electron was attempted on the lines of Bohr's model for hydrogenic atoms but did not meet with any success. Difficulty lies in the fact that each electron interacts not only with the positively charged nucleus but also with other electrons. The formulation of Bohr model involves electrical force between positively charged nucleus and electron. It does not include the electrical forces between electrons which necessarily appear in multi-electron atoms.
  - While the Bohr's model correctly predicts the frequencies of the light emitted by hydrogenic atoms, the model is unable to explain the relative intensities of the frequencies in the spectrum. In emission spectrum of hydrogen, some of the visible frequencies have weak intensity, others strong. Why? Experimental observations depict that some transitions are more favoured than others. Bohr's model unable to account for the intensity variations.
    Thus, Bohr's model present an elegant picture of the an atom and cannot be generalised to complex atoms.

QUESTIONS FROM TEXTBOOK

12.1. *Choose the correct alternative from the clues given at the end of the each statement:*

(a) The size of the atom in Thomson's model is .... the atomic size in Rutherford's model. (much greater than/no different from/much less than)

(b) In the ground state of .... electrons are in stable equilibrium, while in .... electrons always experience a net force. (Thomson's model/Rutherford's model)

(c) A classical atom based on .... is doomed to collapse. (Thomson's model/Rutherford's model)

(d) An atom has a nearly continuous mass distribution in a .... but has a highly non-uniform mass distribution in .... (Thomson's model/Rutherford's model)

(e) The positively charged part of the atom possesses most of the mass in .... (Rutherford's model/both the models).

Sol. (*a*) No different from
(*b*) Thomson's model, Rutherford's model
(*c*) Rutherford's model
(*d*) Thomson's model, Rutherford's model
(*e*) Both the models.

12.2. *Suppose you are given a chance to repeat the alpha-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below 14 K.) What results do you expect?*

Sol. The nucleus of a hydrogen atom is a proton (mass 1.67×10^{-27} kg) which has only about one-fourth of the mass of an alpha particle (6.64×10^{-27} kg). Because the alpha particle is more massive, it won't bounce back in even a head-on collision with a proton. It is like a bowling ball colliding with a ping-pong ball at rest. Thus, there would be no large angle scattering in this case. In Rutherford's experiment, by contrast, there was large-angle scattering because a gold nucleus is more massive than an alpha-particle. The analogy there is a ping-pong ball hitting a bowling ball at rest.

12.3. *What is the shortest wavelength present in the Paschen series of spectral lines?*

Sol. The shortest wavelength of the spectral line (series limit) of Paschen series is given by

$$\frac{1}{\lambda_{\min}} = R\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right) = \frac{R}{9}$$

$$\Rightarrow \quad \lambda_{\min} = \frac{9}{R} = \frac{9}{1.097 \times 10^7} \text{ m}$$

$$\text{or,} \quad \lambda_{\min} = \frac{9 \times 10^{-7} \times 10^{10}}{1.097} \text{ Å}$$

$$= 8204.2 \text{ Å}.$$

12.4. *A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom make a transition from the upper level to the lower level?*

Sol.

$$E_2 - E_1 = 2.3 \text{ eV} = 2.3 \times 1.6 \times 10^{-19} \text{ J}$$

$$\nu = \frac{E_2 - E_1}{h}$$

$$\Rightarrow \quad \nu = \frac{2.3 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$$\text{or,} \quad \nu = \frac{3.68 \times 10^{15}}{6.6}$$

$$= 0.557 \times 10^{15} \text{ Hz}$$

$$= 5.6 \times 10^{14} \text{ Hz}.$$

12.5. *The ground state energy of hydrogen atom is – 13.6 eV. What are the kinetic and potential energies of the electron in this state?*

Sol. Here, Ground Energy, $E = -13.6$ eV

$$\text{Kinetic Energy, } E_k = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{2r} \qquad [E_k = 2E_p]$$

$$\text{and} \quad \text{Potential Energy, } E_p = -\frac{1}{4\pi\varepsilon_0} \frac{e^2}{r} \qquad \left[\because \; E_p = \frac{-k q_1 q_2}{r}\right]$$

$$\text{Total energy, } E = E_k + E_p$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{2r} - \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r}$$

$$E = -\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r}\right)$$

or, $$-13.6 = -\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r}\right)$$

$\therefore$ $$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} = 27.2$$

$\therefore$ $$E_k = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r} = \frac{27.2}{2}\text{ eV} = 13.6\text{ eV}$$

$$E_p = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r} = -27.2\text{ eV.}$$

12.6. *A hydrogen atom initially in the ground level absorbs a photon, which excites it to the n = 4 level. Determine the wavelength and frequency of photon.*

Sol. Energy of an electron in n^{th} orbit of H atom

$$E_n = \frac{-13.6}{n^2}\text{ eV}$$

$$E_1 = -13.6\text{ eV}$$

Energy is 4^{th} ($n = 4$) level

$$R_4 = \frac{-13.6}{4^2} = -0.85$$

$$\Delta E = E_4 - E_1$$

$$\Delta E = -0.85 - (-13.6)\text{ eV}$$

$$= -0.85 + 13.6$$

$$\Delta E = 12.75\text{ eV}$$

$$h\nu = 12.75\text{ eV}$$

$$h\nu = 12.75 \times 1.6 \times 10^{-19}\text{ J}$$

$$\nu = \frac{12.75 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$$\nu = 3.078 \times 10^{15}\text{ Hz}$$

$$\lambda = \frac{c}{\lambda} = \frac{3 \times 10^8}{3.078 \times 10^{15}}$$

$$\lambda = 974.4\text{ Å.}$$

12.7. *(a) Using the Bohr's model calculate the speed of the electron in a hydrogen atom in the n = 1, 2, and 3 levels. (b) Calculate the orbital period in each of these levels.*

Sol. (*a*) From $$v = \frac{c}{n}\alpha, \quad \text{where } \alpha = \frac{2\pi Ke^2}{ch} = 0.0073$$

$$v_1 = \frac{3 \times 10^8}{1} \times 0.0073 = 2.19 \times 10^6 \text{ m/s}$$

$$v_2 = \frac{3 \times 10^8}{2} \times 0.0073 = 1.095 \times 10^6 \text{ m/s}$$

$$v_3 = \frac{3 \times 10^8}{3} \times 0.0073 = 7.3 \times 10^5 \text{ m/s.}$$

(*b*) Orbital period, $T = \frac{2\pi r}{v}$ As $r_1 = 0.53 \times 10^{-10}$ m

$$T_1 = \frac{2\pi \times 0.53 \times 10^{-10}}{2.19 \times 10^6} = 1.52 \times 10^{-16} \text{ s}$$

As $r_2 = 4\, r_1$ and $v_2 = \frac{1}{2} v_1$

$$T_2 = 8\, T_1 = 8 \times 1.52 \times 10^{-16} \text{ s} = 1.216 \times 10^{-15} \text{ s}$$

As $r_3 = 9\, r_1$ and $v_3 = \frac{1}{3} v_1$

$\therefore$ $T_3 = 27\, T_1 = 27 \times 1.52 \times 10^{-16} \text{ s} = 4.1 \times 10^{-15}$ s.

12.8. *The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. What are the radii of the $n = 2$ and $n = 3$ orbits?*

Sol. $r_0 = 5.3 \times 10^{-11}$ m, $r = r_0 \,.\, n^2$

(*i*) when $n = 2$, $r = 5.3 \times 10^{-11} \times (2)^2$

or, $r = 21.2 \times 10^{-11} \text{ m} = 2.12 \times 10^{-10}$ m [$\because$ $r_n = 0.53 \times n^2$ Å]

(*ii*) when $n = 3$, $r = 5.3 \times 10^{-11} \text{ m} \times (3)^2$

$= 47.7 \times 10^{-11} \text{ m} = 4.77 \times 10^{-10}$ m.

12.9. *A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?*

Sol. In ground state, energy of gaseous hydrogen at room temperature = – 13.6 eV. When it is bombarded with 12.5 eV electro beam, the energy becomes –13.6 + 12.6 = – 1.1 eV.

$\because$ $E_n = \frac{-13.6}{n^2}$ So $n^2 = \frac{-13.6}{-1.1} = 12.3 \Rightarrow n = 3$

The electron would jump from $n = 1$ to $n = 3$, where $E_3 = -\frac{13.6}{3^2} = -1.5$ eV. On de-excitation the electron may jump from $n = 3$ to $n = 2$ giving rise to Balmer series. It may also jump from $n = 3$ to $n = 1$, giving rise to Lyman series.

So, number of spectral line = $\frac{n(n-1)}{2} = \frac{3(3-1)}{2}$ = 3 spectral lines appear.

12.10. *In accordance with the Bohr's model, find the quantum number that characterises the earth's revolution around the sun in an orbit of radius 1.5×10^{11} m with orbital speed 3×10^4 m/s. (Mass of earth = 6.0×10^{24} kg.)*

Sol. According to Bohr's theory

$$mvr = \frac{nh}{2\pi}$$

or, $$n = \frac{2\pi\, mvr}{h}$$

or, $$n = \frac{2 \times 3.14 \times 6.0 \times 10^{24} \times 3 \times 10^{4} \times 1.5 \times 10^{11}}{6.63 \times 10^{-34}}$$

$$= 2.56 \times 10^{74}.$$

12.11. *Answer the following questions, which help you understand the difference between Thomson's model and Rutherford's model better.*

(a) Is the average angle of deflection of α-particles by a thin gold foil predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

(b) Is the probability of backward scattering (i.e., scattering of α-particles at angles greater than 90°) predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

(c) Keeping other factors fixed, it is found experimentally that for small thickness t, the number of α-particles scattered at moderate angles is proportional to t. What clue does this linear dependence on t provide?

(d) In which model is it completely wrong to ignore multiple scattering for the calculation of average angle of scattering of α-particles by a thin foil?

Sol. (*a*) About the same.

(*b*) Much less.

(*c*) It suggests that scattering is predominantly due to a single collision, because the chance of a single collision increases linearly with the number of target atoms, and hence linearly with the thickness of the foil.

(*d*) In Thomson's model, a single collision causes very little deflection. The observed average scattering angle can be explained only by considering multiple scattering. So it is wrong to ignore multiple scattering in Thomson's model. In Rutherford's model, most of the scattering comes through a single collision and multiple scattering effects can be ignored as a first approximation.

12.12. *The gravitational attraction between electron and proton in a hydrogen atom is weaker than the coulomb attraction by a factor of about 10^{-40}. An alternative way of looking at this fact is to estimate the radius of the first Bohr orbit of a hydrogen atom if the electron and proton were bound by gravitational attraction. You will find the answer interesting.*

Sol. The radius of the first orbit of hydrogen atom in Bohr's model is given by

$$r = \frac{n^2 h^2}{4\pi^2 mkZe^2}$$

$$r = \frac{\varepsilon_0\, h^2}{\pi\, me^2} = \frac{4\pi\, \varepsilon_0}{e^2}\left(\frac{h^2}{4\pi^2 m}\right) \qquad \left[\begin{aligned} &\text{here } k = \frac{1}{4\pi\varepsilon_0} \\ &Z = 1, n = 1 \end{aligned}\right]$$

If electrostatic force $\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2}$ is replaced by gravitational force $\frac{GMm}{r^2}$, we put GMm in place of $\frac{e^2}{4\pi\varepsilon_0}$ in above expression.

Hence radius of first orbit under gravitational force

$$r_G = \frac{1}{GMm} \cdot \frac{h^2}{4\pi^2 m}$$

$$= \frac{h^2}{4\pi^2 GMm^2} \qquad \begin{bmatrix} M = \text{mass of proton} \\ m = \text{mass of electron} \end{bmatrix}$$

or, $$r_G = \frac{(6.26 \times 10^{-34})^2}{4 \times (3.14)^2 \; (6.67 \times 10^{-11}) \times (1.672 \times 10^{-27}) \times (9.1 \times 10^{-31})^2}$$

$$= \frac{6626 \times 6626 \times 10^{-74}}{4 \times 3.14 \times 3.14 \times 667 \times 16724 \times 19 \times 19 \times 10^{-112}}$$

$$= 1.21 \times 10^{29} \text{ m.}$$

It is larger than the size of the universe.

12.13. *Obtain an expression for the frequency of radiations emitted when a hydrogen atom de-excites from level n to level (n – 1). For large n, show that the frequency equals the classical frequency of revolution of the electron in the orbit.*

Sol. The frequency ν of the emitted radiation when a hydrogen atom de-excites from level n to level $(n - 1)$ is

$$E = hv = E_2 - E_1$$

$$\nu = \frac{1}{2}\frac{mc^2\alpha^2}{h}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \quad \text{where} \quad \alpha = \frac{2\pi\, Ke^2}{ch} = \text{fine structure constant}$$

$$\nu = \frac{1}{2}\frac{mc^2\alpha^2}{h}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] = \frac{mc^2\alpha^2}{2h}\left[\frac{n^2 - (n-1)^2}{n^2\,(n-1)^2}\right]$$

$$= \frac{mc^2\alpha^2\left[(n+n-1)\,(n-n+1)\right]}{2hn^2\,(n-1)^2}$$

$$\nu = \frac{mc^2\alpha^2\,(2n-1)}{2h\,n^2\,(n-1)^2}.$$

For large n, $(2n - 1) \approx 2n$, and $(n - 1) \approx n$

$$\nu = \frac{mc^2\alpha^2 . 2n}{2h\,n^2 . n^2} = \frac{mc^2\alpha^2}{hn^3}$$

Putting $\alpha = \frac{2\pi\, Ke^2}{ch}$, we get $\nu = \frac{mc^2}{hn^3} \cdot \frac{4\pi^2 K^2 e^4}{c^2 h^2}$

$$\nu = \frac{4\pi^2 mK^2 e^4}{n^3 h^3}$$

In Bohr's atom model, velocity of electron in nth orbit is $v = \frac{nh}{2\pi\, mr}$ and radius of nth orbit is $r = \frac{n^2 h^2}{4\pi^2 mKe^2}$ $(\because \quad Z = 1)$

∴ frequency of revolution of electron $$\nu = \frac{v}{2\pi r} = \frac{nh}{2\pi\, mr}\left(\frac{4\pi^2 mKe^2}{2\pi \cdot n^2 h^2}\right)$$

$$\nu = \frac{Ke^2}{nh \cdot r} = \frac{Ke^2}{nh}\left(\frac{4\pi^2 mKe^2}{n^2 h^2}\right)$$

$$\nu = \frac{4\pi^2 mK^2 e^4}{n^3 h^3} \text{ which is the same as } (i).$$

Hence for large values of n, classical frequency of revolution of electron in nth orbit is the same as the frequency of radiation emitted when hydrogen atom de-excites from level (n) to level ($n - 1$).

12.14. *Classically, an electron can be in any orbit around the nucleus of an atom. Then what determines the typical atomic size? Why is an atom not, say, thousand times bigger than its typical size? The question had greatly puzzled Bohr before he arrived at his famous model of the atom that you have learnt in the text. To simulate what he might well have done before his discovery, let us play as follows with the basic constants of nature and see if we can get a quantity with the dimensions of length that is roughly equal to the known size of an atom (~ 10^{-10} m).*

(a) *Construct a quantity with the dimensions of length from the fundamental constants e, m_e, and c. Determine its numerical value.*

(b) *You will find that the length obtained in (a) is many orders of magnitude smaller than the atomic dimensions. Further, it involves c. But energies of atoms are mostly in non-relativistic domain where c is not expected to play any role. This is what may have suggested Bohr to discard c and look for 'something else' to get the right atomic size. Now, the Planck's constant h had already made its appearance elsewhere. Bohr's great insight lay in recognising that h, m_e, and e will yield the right atomic size. Construct a quantity with the dimension of length from h, m_e, and e and confirm that its numerical value has indeed the correct order of magnitude.*

Sol. (*a*) From Coulomb's law for force between hydrogen nucleus and electron.

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e \cdot e}{r^2}$$

$$\Rightarrow \qquad r = \frac{1}{4\pi\varepsilon_0} \frac{e \cdot e}{F \cdot r}$$

But $F.r$ (force × distance) = work or energy = mc^2

$$\therefore \qquad r = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{mc^2} = 2.8 \times 10^{-15} \text{ m.}$$

It is much smaller than typical atomic size.

(*b*) From Bohr's formula for first hydrogen orbit.

$$r = \frac{\varepsilon_0 h^2}{\pi m e^2} = 0.53 \times 10^{-10} \text{ m}$$

It is of the order of atomic size.

12.15. *The total energy of an electron in the first excited state of the hydrogen atom is about – 3.4 eV.*

(a) *What is the kinetic energy of the electron in this state?*

(b) *What is the potential energy of the electron in this state?*

(c) *Which of the answers above would change if the choice of the zero of potential energy is changed?*

Sol. In Bohr's model, $mvr = \frac{nh}{2\pi}$ and $\frac{mv^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$

which gives $E_k = \frac{1}{2} mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r}$; $r = \frac{4\pi\varepsilon_0 h^2}{Ze^2 m} n^2$.

These relations have nothing to do with the choice of the zero of potential energy. Now, choosing the zero of potential energy at infinity, we have

$$E_p = \frac{-Ze^2}{4\pi\varepsilon_0 r} \text{ which gives } E_p = -2\,E_k$$

and $$E = E_k + E_p = -E_k$$

(*a*) The quoted value of $E = -3.4$ eV is based on the customary choice of zero of potential energy at infinity. Using $E = -E_k$, the kinetic energy of electron in this state is + 3.4 eV.

(*b*) Using $E_p = -2\,E_k$, potential energy of the electron is -2×3.4 eV $= -6.8$ eV.

(*c*) If the zero of potential energy is chosen differently, kinetic energy does not change. Its value is + 3.4 eV. This is independent of the choice of the zero of potential energy. The potential energy, and the total energy of the state, however, would alter if a different zero of the potential energy is chosen.

12.16. *If Bohr's quantisation postulate (angular momentum = nh/2π) is a basic law of nature, it should be equally valid for the case of planetary motion also. Why then do we never speak of quantisation of orbits of planets around the sun?*

Sol. Angular momenta associated with planetary motion are incomparably large relative to *h*. For example, angular momentum of the earth in its orbital motion is of the order of 10^{70} h. In terms of the Bohr's quantisation postulate, this corresponds to a very large value of *n* (of the order of 10^{70}). For such large values of *n*, the differences in the successive energies and angular momenta of the quantised levels of the Bohr model are so small compared to the energies and angular momenta respectively for the levels that one can practically consider the levels continuous.

12.17. *Obtain the first Bohr's radius and the ground state energy of a muonic hydrogen atom [i.e., an atom in which a negatively charged muon (μ^-) of mass about 207 m_e orbits around a proton].*

Sol. The first Bohr's radius of *H*-atom is given by

$$r_1 = 4\pi\varepsilon_0 \frac{h^2}{4\pi^2 m_e e^2} = 5.29 \times 10^{-11} \text{ m}$$

If r_1' is the first Bohr's radius of muonic hydrogen atom, then

$$r_1' = 4\pi\varepsilon_0 \frac{h^2}{4\pi^2\,(207\,m_e)\,e^2} = \frac{5.29 \times 10^{-11}}{207}$$

$$= 2.5 \times 10^{-12} \text{ m}$$

The ground state ($n = 1$) energy of *H*-atom is given by

$$E_1 = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \cdot \frac{2\pi^2 m_e e^2}{h^2} = 13.6 \text{ eV}$$

If E_1' is ground state energy of muonic hydrogen atom, then

$$E_1' = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \cdot \frac{2\pi^2 (207\,m_e)\,e^2}{h^2} = -13.6 \times 207$$

$$= -2815.2 \text{ eV.}$$

MORE QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *In a Rutherford's α-scattering experiment with thin gold foil, 8100 scintillations per minute are observed at an angle of 60°. What will be the number of scintillations per minute at an angle of 120°?*

Ans. n_1 (number of scintillations per minute at an angle 60°) = 8100

n_2 (number of scintillations per minute at an angle 120°) =?

The scattering in the Rutherford's experiment is proportional to $\cot^4 \frac{\phi}{2}$.

or $$\frac{n_2}{n_1} = \frac{\cot^4 \phi_2/2}{\cot^4 \phi_1/2}$$

Hence, $$\frac{n_2}{n_1} = \frac{\cot^4\left(\frac{120^\circ}{2}\right)}{\cot^4\left(\frac{60^\circ}{2}\right)} = \frac{\cot^4 60^\circ}{\cot^4 30^\circ} = \left(\frac{\frac{1}{\sqrt{3}}}{\sqrt{3}}\right)^4 = \frac{1}{81}$$

$\Rightarrow$ $$n_2 = \frac{1}{81} \times n_1 = \frac{1}{81} \times 8100 = 100.$$

Q. 2. *Name the series of hydrogen spectrum lying in the infrared region.*

Ans. Paschen series, Brackett series and pfund series.

Q. 3. *Can a hydrogen atom absorb a photon having energy more than 13.6 eV?*

Ans. Yes, it can absorb. But the atom would be ionised.

Q. 4. *Name the series of hydrogen spectrum which does not lie in the visible region.*

Ans. Lyman series.

Q. 5. *Which of the following given transitions in a hydrogen atom emits the photon of lowest frequency?*

(i) $n = 2$ to $n = 1$

(ii) $n = 4$ to $n = 3$.

Ans. *(ii)* $n = 4$ to $n = 3$

Reason: The energy levels got progressively closer as n increases. From the given sets, the closest to each other (hence, minimum energy difference levels) are $n = 4$ to $n = 3$.

Q. 6. *The shortest wavelength in the Lyman Series is 911.6 Å. Then the longest wavelength in the Lyman's series is:*

Ans. For the Lyman Series, we have

$$\frac{\lambda_L}{\lambda_S} = \frac{4}{3}$$

Hence, $$\lambda_L = \frac{4}{3} \times \lambda_S$$

$\Rightarrow$ $$\lambda_L = \frac{4}{3} \times 911.6 \text{ Å} = 1215 \text{ Å}.$$

Q. 7. *Name the series of hydrogen atom spectrum which lies in the visible region.*

Ans. Balmer Series.

Q. 8. *What are the values of first and second excitation potential of hydrogen atom?*

Ans. 10.2 V; 12.09 V.

Q. 9. *What is the ratio of volume of atom to the volume of nucleus?*

Ans. 10^{15}.

Q. 10. *If electron-orbits with principal quantum number n > 3 were not allowed, what would be the number of possible elements?*

Ans. The maximum number of electrons that can be accommodated in orbits with $n = 3$ is

$$2 \times 1^2 + 2 \times 2^2 + 2 \times 3^2 = 28.$$

Q. 11. *What is the impact parameter for scattering of α-particle by 180°?*

Ans. Zero.

Since, $$\text{Impact parameter, } b = \frac{Ze^2 \cot \frac{\theta}{2}}{4\pi\varepsilon_0 \left(\frac{1}{2} mv^2\right)}.$$

Q. 12. *How many times does the electron go round the first Bohr orbit in a second?*

Ans. The frequency of electron is given by

$$\nu = \frac{v}{2\pi r} = \frac{mvr}{2\pi\, mr^2} = \frac{\frac{nh}{2\pi}}{2\pi\, mr^2}$$

or, $$\nu = \frac{nh}{4\pi^2\, mr^2}$$

Hence, $$\nu_1 = \frac{h}{4\pi^2\, mr^2}.$$

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. (*a*) *The energy levels of an atom are as shown below. Which of them will result in the transition of a photon of wavelength 275 nm?*

(*b*) *Which transition corresponds to emission of radiation of maximum wavelength?*

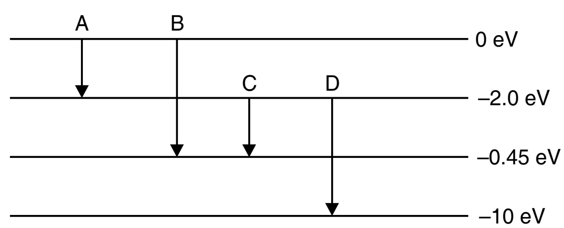

Fig. 12.5

Ans. (*a*) For element *A*

Ground state energy, $E_1 = -2$ eV

Excited state energy, $E_2 = 0$ eV

Energy of photon emitted, $E = E_2 - E_1$

$= 0 - (-2) = 2$ eV

∴ Wavelength of photon emitted,

$$\lambda = \frac{hc}{E}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2 \times 1.6 \times 10^{-19}} = \frac{19.878 \times 10^{-7}}{3.2}$$

$= 6.211 \times 10^{-7}$ m $= 621.1$ nm.

For element B

$$E_1 = -4.5 \text{ eV}, \quad E_2 = 0 \text{ eV}$$
$$E = 0 - (-4.5) = 4.5 \text{ eV}$$

$$\therefore \qquad \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{4.5 \times 1.6 \times 10^{-19}} = \frac{19.878 \times 10^{-7}}{7.2}$$
$$= 2.76 \times 10^{-7} = 276 \text{ nm.}$$

For element C

$$E_1 = -4.5 \text{ eV}, \quad E_2 = -2 \text{ eV}$$
$$E = -2 - (4.5) = 2.5 \text{ eV}$$

$$\therefore \qquad \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{2.5 \times 1.6 \times 10^{-19}} = \frac{19.878 \times 10^{-7}}{4}$$
$$= 4.969 \times 10^{-7} \text{ m} = 496.9 \text{ nm.}$$

For element D

$$E_1 = -10 \text{ eV}, \quad E_2 = -2 \text{ eV}$$
$$E = -2 - (-10) = 8 \text{ eV}$$

$$\therefore \qquad \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{8 \times 1.6 \times 10^{-19}} = \frac{19.878 \times 10^{-7}}{12.8}$$
$$= 1.552 \times 10^{-7} \text{ m} = 155.2 \text{ nm.}$$

(*b*) Element A has radiation of maximum wavelength 621 nm.

Q. 2. *The Rydberg constant for hydrogen is 10967700 m^{-1}. Calculate the short and long wavelength limits of Lyman series.*

Ans. For Lyman series, the wave number is given by

$$\nu = \frac{1}{\lambda} = R_H \left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

For the short wavelength limit ($\lambda = \lambda_s$), $n = \infty$

$$\text{or} \qquad \bar{\nu}_s = \frac{1}{\lambda_s} = R_H \left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = R_H$$

$$\therefore \qquad \lambda_s = \frac{1}{R_H} = \frac{1}{10967700} \text{ m}$$
$$= 9.116 \times 10^{-8} \text{ m} = 911.6 \text{ Å}$$

For long wavelength limit ($\lambda = \lambda_L$) $n = 2$

$$\therefore \qquad \bar{\nu}_L = \frac{1}{\lambda_L} = R_H \left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4} R_H$$

$$\therefore \qquad \lambda_L = \frac{4}{3 R_H}$$

$$= \frac{4}{3} \times 911.6 \text{ Å} = 1215 \text{ Å}.$$

Q. 3. *The energy of the electron, the hydrogen atom, is known to be expressible in the form*

$$E_n = \frac{-13.6\ eV}{n^2} \quad (n = 1, 2, 3, ...)$$

Use this expression to show that the

(i) electron in the hydrogen atom can not have an energy of – 2 V.

(ii) spacing between the lines (consecutive energy levels) within the given set of the observed hydrogen atom spectrum decreases as n increases.

Ans. As,

$$E_n = -\frac{13.6 \text{ eV}}{n^2}$$

Putting $n = 1, 2, 3 ..... n$, we get

$$E_1 = \frac{-13.6}{12} = -13.6 \text{ eV}$$

$$E_2 = \frac{-13.6}{2^2} = -\frac{13.6}{4} = 3.4 \text{ eV}$$

$$E_3 = \frac{-13.6}{3^2} = -\frac{13.6}{9} = -1.51 \text{ eV}$$

$$E_4 = \frac{-13.6}{4^2} = \frac{-13.6}{16} = -.85 \text{ V}$$

....

....

$$E_n = \frac{-13.6}{\infty^2} = 0 \text{ eV}$$

(*i*) Hence, it can be observed that the electron in the hydrogen atom can not have an energy of – 2V.

(*ii*) As n increases, energies of the excited states come closer and closer together. Therefore, as n increases, E_n becomes less negative until at $n = \infty$, *i.e.*, $E_n = 0$.

Q. 4. *Calculate the nearest distance of approach of an α-particle of energy 2.5 eV being scattered by a gold nucleus (Z = 79).*

Ans. We know that the electrostatic potential at a distance x due to nucleus is given by $Z\ e/4\ \pi\ \varepsilon_0\ x$ where $Z\ e$ is the charge on the nucleus.

The potential energy of an α-particle when it is at a distance x, from the nucleus is given by

$$PE = \left(\frac{Ze}{4\pi\varepsilon_0 x}\right) 2e = \frac{2Ze^2}{(4\pi\varepsilon_0 x)},$$

$2e$ being the charge on α-particle.

Since the α-particle is momentarily stopped at a distance x, its initial kinetic energy is completely changed into potential energy here. Hence

$$\frac{1}{2} m\,v^2 = \frac{2Ze^2}{4\pi\varepsilon_0 x} \qquad \text{(at nearest approach K.E. = P.E.)}$$

or

$$x = \frac{2Ze^2}{4\pi\varepsilon_0} \times \frac{1}{\left(\frac{mv^2}{2}\right)}$$

Now energy of α-particle = $\frac{1}{2} m v^2$ = 2.5 MeV

$$= 2.5 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

$$= 2.5 \times 1.6 \times 10^{-13} \text{ J}$$

Substituting values we get

$$x = \frac{2 \times 79 \times 1.6 \times 1.6 \times 10^{-38} \times 9 \times 10^9}{2.5 \times 1.6 \times 10^{-13}} \text{ m}$$

$$= 9.101 \times 10^{-14} \text{ m.}$$

Q. 5. *In Bohr's theory of hydrogen atom, calculate the energy of the photon emitted during a transition of the electron from the first excitated state to its ground state. Write in which region of the electromagnetic spectrum this transition lies.*

Given Rydberg constant $R = 1.03 \times 10^7 \text{ m}^{-1}$.

Ans. As,

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

Energy of the photon emitted during a transition of the electron from the first excited state to its ground state.

$$E = E_2 - E_1$$

$$= -\frac{13.6}{2^2} - \left(-\frac{13.6}{1^2}\right)$$

$$= -\frac{13.6}{4} + \frac{13.6}{1} = -3.40 + 13.6 = 10.2 \text{ eV}$$

This transition lies in the region of Lyman series.

Q. 6. *The wavelength of the first member of the Balmer series in hydrogen spectrum is 6563 A. What is the wavelength of the first member of Lyman series?*

Ans. Balmer series

$$\frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}$$

Lyman series

$$\frac{1}{\lambda_2} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{4}{3R} \times \frac{5R}{36} = \frac{20}{108} = \frac{5}{27}$$

$$\lambda_2 = \frac{5}{27} \times \lambda_1 = \frac{5}{27} \times 6563 = 1215 \text{ Å}.$$

Q. 7. *The ground state energy of hydrogen atom is – 13.6 eV.*

(i) What is the potential energy of an electron in the 3rd excited state?

(ii) If the electron jumps to the ground state from the 3rd excited state, calculate the wavelength of the photon emitted.

Ans. The energy of an electron in *n*th orbit is given by

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

(i) For 3rd excited state, $n = 4$

$$\therefore \qquad E_4 = -\frac{13.6}{4^2} = -\frac{13.6}{16} = -0.85 \text{ eV}$$

(ii) Required energy to jump electron to the ground state from the 3rd excited state

$$\boxed{E = E_4 - E_1}$$

$$= -\frac{13.6}{4^2} - \left(-\frac{13.6}{1^2}\right)$$

$$= -0.85 + 13.6 = 12.75 \text{ eV}$$

$\therefore$ Wavelength of the photon emitted is

$$\boxed{\lambda = \frac{hc}{E}} \qquad \left(\text{As, } E = \frac{hc}{\lambda}\right)$$

$$\Rightarrow \qquad \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{12.75 \times 1.6 \times 10^{-19}}$$

$$= \frac{19.878 \times 10^{-7}}{20.4} = 0.974 \times 10^{-7}$$

$$= 974 \text{ Å}.$$

Q. 8. *If the average life time of an excited state of hydrogen is of the order of 10^{-8} s, estimate how many rotation an electron makes when it is in the state $n = 2$ and before it suffers a transition to state $n = 1$. Bohr radius $= 5.3 \times 10^{-11}$ m.*

Ans. Velocity of electron in the nth orbit of hydrogen atom

$$\boxed{v_n = \frac{v_1}{n}} = \frac{2.19 \times 10^6}{n} \text{ m/s}$$

If $\qquad n = 2, \quad v_n = \frac{2.19 \times 10^6}{n}$ m/s

Radius of $n = 2$ orbit, $\boxed{r_n = n^2 r_1}$ = 4 × Bohr radius

$$r_n = 4 \times 5.3 \times 10^{-11} \text{ m}$$

Number of revolutions made in 1 sec

$$= \frac{v_n}{2\pi r} = \frac{2.19 \times 10^6}{2 \times 2\pi \times 4 \times 5.3 \times 10^{-11}}$$

Number of revolutions made in 10^{-8} s $= \dfrac{2.19 \times 10^6 \times 10^{-8}}{2 \times 2\pi \times 4 \times 5.3 \times 10^{-11}} = 8.22 \times 10^6$ revolutions.

Q. 9. *A hydrogen atom initially in the ground state absorbs a photon; which excites it to the $n = 4$ level. Determine the wavelength and frequency of photon.*

Ans. As, $$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}$$

$$\Rightarrow \qquad \frac{1}{\lambda} = 1.09 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{4^2}\right) = 1.09 \times 10^7 \times \frac{15}{16}$$

or, $$\lambda = \frac{16}{1.09 \times 10^7 \times 15} = 9.8 \times 10^{-8} \text{ m}$$

$\therefore$ Frequency, $$\boxed{\nu = \frac{c}{\lambda}} = \frac{3 \times 10^8}{9.8 \times 10^{-8}} = 3.06 \times 10^{15} \text{ s}^{-1}.$$

Q. 10. *A doubly ionised lithium atom is hydrogen-like with atomic number 3:*

(*i*) *Find the wavelength of the radiation required to excite the electron in Li^{++} from the first to the third Bohr orbit. (Ionisation energy of the hydrogen atom equals 13.6 eV.)*

(*ii*) *How many spectral lines are observed in the emission spectrum of the above excited system?*

Ans. (*i*) The energy difference of electron in Li^{++} between the first and the third orbit

$$= E_3 - E_1$$

$$\therefore \quad \boxed{E_3 - E_1 = 13.6 \times Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}$$

$$= 13.6 \times (3)^2 \left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$

$$= 13.6 \times 9 \times \frac{8}{9} \times 1.6 \times 10^{-19} \text{ J}$$

Therefore, the equivalent wavelength λ is given by

$$E_3 - E_1 = \frac{hc}{\lambda}$$

or, $$\lambda = \frac{hc}{E_3 - E_1}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{13.6 \times 8 \times 1.6 \times 10^{-19}}$$

$$= 1.137 \times 10^{-8} \text{ m}$$

$$= 113.7 \text{ Å}.$$

(*ii*) The following three spectral lines are observed due to the following transitions:

3rd to 1st orbit

3rd to 2nd orbit

2nd to 1st orbit

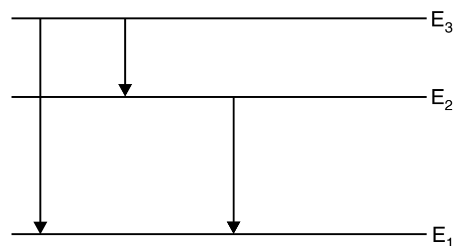

Fig. 12.6

III. LONG ANSWER TYPE QUESTIONS

Q. 1. *A single electron, orbits around a stationary nucleus of charge z_e, where z is a constant and e is the electronic charge. It requires 47.2 eV to excite the electron from the second Bohr orbit to 3rd Bohr orbit. Find,*

(*i*) *The value of z.*

(*ii*) *The energy required to excite the electron from the third to the fourth Bohr orbit.*

(*iii*) *The wavelength of electromagnetic radiation required to remove the electron from the first Bohr orbit to infinity.*

(*iv*) *The kinetic energy, potential energy and angular momentum of the electron in the first Bohr orbit.*

(*v*) *The radius of the first Bohr orbit.*

(Ionisation energy of hydrogen atom = 13.6 eV. Bohr radius = 5.3 × 10^{-11} m, velocity of light = 3 × 10^{8} m/s and Planck's constant = 6.6 × 10^{-34} Js)

Ans. (*i*) For a general hydrogen-like atom

$$E_{n_2} - E_{n_1} = Z^2 E_0 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \text{eV}$$

where E_0 is the ionisation energy of hydrogen atom

$$\Delta E = Z^2 \times 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 47.2$$

or, $$Z^2 \times \frac{13.6}{36} \times 5 = 47.2$$

$$Z^2 = \frac{47.2 \times 36}{13.6 \times 5} = 25$$

$$Z = 5$$

(*ii*) $$E_4 - E_3 = 5^2 \times 13.6\left(\frac{1}{3^2} - \frac{1}{4^2}\right) \text{eV}$$

$$= 25 \times 13.6 \times \frac{7}{144} = 16.53 \text{ eV}$$

Energy required to excite the electron from the third to the fourth Bohr orbit

$$= 16.53 \text{ eV}$$

(*iii*) $$E_\infty - E_1 = Z^2 \times 13.6\left(\frac{1}{1^2} - \frac{1}{\infty}\right) = 13.6 \times 25 \text{ eV}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.6 \times 10^{-34}) \times 3 \times 10^8}{13.6 \times 25 \times 1.6 \times 10^{-19}}$$

$$= 0.03640 \times 10^{-7} = 36.4 \times 10^{-10}$$

$$= 36.4 \text{ Å}$$

The wavelength of electromagnetic radiation required to remove the electron from first Bohr orbit to infinity = 36.4 Å.

(*iv*) Kinetic energy of first Bohr orbit is numerically equal to the energy of the orbit

$$E_1 = -Z^2 E_0 = -25 \times 13.6 \text{ eV}$$

∴ $$\text{K.E.} = 25 \times 13.6 \times 1.6 \times 10^{-19} \text{ J} = 544 \times 10^{-19} \text{ J}$$

Potential energy of electron = – 2 × K.E.

$$= -2 \times 544 \times 10^{-19} \text{ J}$$

$$= -1088 \times 10^{-19} \text{ J}$$

Angular momentum of the electron

$$\boxed{L = mvr = \frac{nh}{2\pi}} = \frac{h}{2\pi}$$

∵ $n = 1$ [For Ist Bohar arbit]

$$L = \frac{6.6 \times 10^{-34}}{2\pi} = 1.05 \times 10^{-34} \text{ Js}$$

(v) Radius r_1 of the first Bohr orbit

$$\boxed{r_n = \frac{n^2 r_0}{Z}} \quad \text{for} \quad n = 1,$$

$$r_1 = \frac{1^2 \times 5.3 \times 10^{-11}}{5} = 1.06 \times 10^{-11} \text{ m.}$$

Q. 2. *Hydrogen atom in its ground state is excited by means of monochromatic radiation of wavelength 975 Å. How many different lines are possible in the resulting spectrum ? Calculate the longest wavelength amongst them. You may assume the ionization energy for hydrogen atom as 13.6 eV.*

Ans. Ionization energy for hydrogen atom = 13.6 eV.

The energy of monochromatic radiation of wavelength 975 Å

$$\boxed{E = \frac{hc}{\lambda}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{975 \times 10^{-10} \times 1.6 \times 10^{-9}} \text{ eV}$$

$$= 12.75 \text{ eV}$$

($\because$ 1 eV = 1.6×10^{-19} J)

$$\therefore \quad 12.75 = 13.6\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

$$\frac{1}{n^2} = 1 - \frac{12.75}{13.6} = \frac{0.85}{13.6} = \frac{1}{16}$$

$$\therefore \quad n = 4$$

$\therefore$ Number of lines possible in the resultant spectrum = 6, as shown in Fig. 12.7 below.

The longest wavelength will be emitted for transition from 4th orbit to 3rd orbit with an energy.

$$\boxed{E_{n_2} - E_{n_1} = E_0 Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]}$$

$$E_{4 \to 3} = 13.6\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 13.6\left(\frac{1}{9} - \frac{1}{16}\right)$$

$$\because \quad Z = 1$$

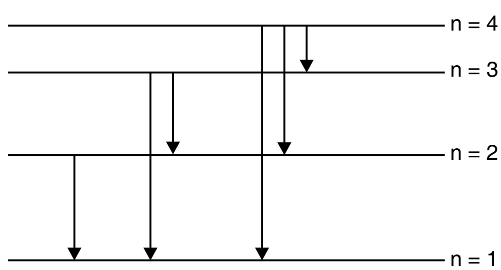

Fig. 12.7

$$E_{4 \to 3} = 13.6 \times \frac{7}{144} \text{ eV} = 13.6 \times \frac{7}{144} \times 1.6 \times 10^{-19} \text{ J}$$

The longest wavelength,

$$\lambda = \frac{hc}{E_{4\to 3}}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8} \times 144}{13.6 \times 7 \times 1.6 \times 10^{-19}} \text{ m}$$

$$\lambda = 1.88 \times 10^{-6} \text{ m} = 18800 \text{ Å}.$$

Q. 3. *In a hydrogen like atom the ionisation energy equals 4 times Rydberg's constant for hydrogen. What is the wavelength of radiations emitted when a jump takes place from the first excited state to the ground state? What is the radius of first Bohr's orbit?*

Ans. The ionization energy E of a hydrogen-like Bohr atom of atomic number z is given by

$$E = -Rz^2 = -\frac{2\pi k^2 z^2 me^4}{n^2h^2}$$

where the Rydberg constant

$$R = \frac{2\pi^2 kme^4}{n^2h^2} = 2.2 \times 10^{-18} \text{ J}$$

As ionization energy

$$E = 4 \times \text{Rydberg constant}$$
$$= 4\,R, \text{ we have}$$

$\therefore$ $$4\,R = RZ^2$$

or, $$Z = 2$$

(*i*) Energy of radiation emitted E when the electron jumps from the first excited state to the ground state is given by

$$E = RZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$
$$= 4R\left(1 - \frac{1}{4}\right)$$
$$= 3\,R$$
$$= 3 \times 2.2 \times 10^{-18} \text{ J}$$

or, $$E = 6.6 \times 10^{-18} \text{ J}.$$

Wavelength of the radiation emitted,

$$\lambda = \frac{hc}{E}$$
$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{6.6 \times 10^{-18}} = 3 \times 10^{-8} \text{ m}$$

(*ii*) Radius of the first Bohr orbit

$$= \frac{\text{Bohr radius of hydrogen atom}}{Z}$$
$$= \frac{5 \times 10^{-11}}{2} = 2.5 \times 10^{-11} \text{ m}.$$

Q. 4. *Draw a labelled diagram for α-particle scattering experiment. Give Rutherford's observation and discuss the significance of this experiment. Obtain the expression which helps us to get an idea of the size of a nucleus, using these observations.*

Ans. See text.

Q. 5. *If the short series limit of the Balmer series for hydrogen is 3646 Å, calculate the atomic number of the element which gives X-ray wavelengths down to 1.0 Å. Identify the element.*

Ans. The short limit of the Balmer series is given by

$$\bar{\nu} = \frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$$

$$\bar{\nu} = \frac{R}{4}$$

$$\therefore \quad R = \frac{4}{\lambda} = \frac{4}{3646} \times 10^{10}\,\text{m}^{-1}$$

Further the wavelengths of the K_α series are given by the relation

$$\bar{\nu} = \frac{1}{\lambda} = R(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

The maximum wave number corresponds to $n = \infty$ and, therefore, we must have

$$\bar{\nu} = \frac{1}{\lambda} = R\,(Z-1)^2$$

$$\text{or} \quad (Z-1)^2 = \frac{1}{R\lambda} = \frac{3646 \times 10^{-10}}{4 \times 1 \times 10^{-10}} = 911.5$$

$$\therefore \quad (Z-1) = \sqrt{911.5}$$

$$\text{or} \quad \cong 30.2$$

$$\text{or} \quad Z = 31.2 \cong 31$$

Thus, the atomic number of the element concerned is 31.

The element having atomic number $Z = 31$ is *Gallium*.

QUESTIONS ON HIGH ORDER THINKING SKILLS (HOTS)

Q. 1. *Using Bohr's formula for energy quantisation, determine:*

(i) the longest wavelength in the Lyman series of hydrogen atom spectrum.

(ii) the excitation energy of the n = 3 level of He^+ atom.

(iii) the ionisation potential of the ground state of Li^{++} atom.

Ans. (*i*) Wavelengths of radiation of the Lyman series are given by

$$\lambda_n = \frac{64\pi^3\varepsilon_0^2 h^3 c}{me^4}\left(\frac{n^2}{n^2 - 1^2}\right)$$

n = 2 corresponds to the longest wavelength

= 1225 Å

(*ii*) The energy required to excite the electron from the ground state ($n = 1$) to the $n = 3$ state is

$$E_3 - E_1 = \frac{mZ^2e^4}{32\pi^2\varepsilon_0^2 h^2}\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$

$$= 48.1 \text{ eV}$$

where $Z = 2$

(*iii*) Ionisation energy is given by

$$E_\infty - E_1 = \frac{mZ^2e^4}{32\pi^2\varepsilon_0^2h^2} \quad \text{(with } Z = 3\text{)}$$

$$= 122 \text{ eV}$$

Thus, ionisation potential is 122 V.

Q. 2. *Which state of the triply ionized Be^{+++} has the same orbital radius as that of the ground state of hydrogen? Compare the energies of two states.*

Ans. Radius of nth orbit is given by

$$r = \frac{n^2h^2}{4\pi^2 mKZe^2} \quad i.e., \quad r \propto \frac{n^2}{Z}$$

For hydrogen, $Z = 1$, $n = 1$ in ground state

$$\therefore \quad \frac{n^2}{Z} = \frac{1^2}{1} = 1$$

For Beryleum, $Z = 4$, As orbital radius is same, $\frac{n^2}{Z} = 1$

$$\therefore \quad n^2 = 1 \times Z = 1 \times 4 = 4$$

$$n = \sqrt{4} = 2$$

Hence $n = 2$ level of *Be* has same radius as $n = 1$ level of hydrogen.

Now, energy of electron in nth orbit is $E = -\frac{2\pi^2 mK^2Z^2e^4}{n^2h^2}$

$$\therefore \quad E \propto \frac{Z^2}{n^2}$$

$$\frac{E_{(Be)}}{E_{(H)}} = \frac{\left[Z^2/n^2\right]_{Be}}{\left[Z^2/n^2\right]_H} = \frac{16/4}{1/1} = 4$$

Q. 3. *Prove that the ionisation energy of hydrogen atom is 13.6 eV.*

Ans. We know that

$$\boxed{E = -\frac{2\pi^2 mK^2Z^2e^4}{n^2h^2}} \quad (n = 1, Z = 1)$$

$$W = k^2\frac{2\pi^2 me^4}{h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Ionisation energy is the energy required to remove an electron from ground state to infinity.

Here, $\quad n_1 = 1, \quad n_2 = \infty$

$$\therefore \quad W = k^2\frac{2\pi^2 me^4}{h^2}\left(\frac{1}{1} - \frac{1}{\infty}\right)$$

$$= k^2\frac{2\pi^2 me^4}{h^2}$$

or, $$W = \frac{(9\times10^{9})^2 \times 2\,(3.142)^2 \times 9\times10^{-31}\times(1.6\times10^{-19})^4}{(6.63\times10^{-34})^2}\text{ J}$$

$$= 21.45 \times 10^{-19}\text{ J} = \frac{21.45\times10^{-19}}{1.6\times10^{-19}}\text{ eV} = 13.4\text{ eV.}$$

Q. 4. *A positronium atom is a bound state of an electron (e^-) and its antiparticle, the positron (e^+) revolving round their centre of mass. In which part of the em spectrum does the system radiate when it de-excites from its first excited state to the ground state?*

Ans. In an ordinary atom, as first approximation, we ignore the motion of the nucleus, being too heavy. In a positronium atom, a positron replaces proton of hydrogen atom. As electron and positron masses are equal, the motion of the positron cannot be ignored.

We consider motion of electron and positron about their centre of mass. A detailed analysis (beyond the scope of this book) shows that formulae of Bohr model apply to positronium atom provided that we replace m_e by what is known as reduced mass of the electron. For positronium, the reduced mass is $m_e/2$. In the transition $n = 2$ to $n = 1$, the wavelength of radiation emitted is double than that of the corresponding radiation emitted for a similar transition in hydrogen atom, which has a wavelength of 1217 Å; and hence is equal to 2 × 1217 = 2434 Å. This radiation lies in the ultra-violet part of the electromagnetic spectrum.

Q. 5. *The wavelength of the first member of Lyman series is 1216 Å. Calculate the wavelength of second member of Balmer series.*

Ans. $$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For first member of Lyman series, $n_1 = 1$ and $n_2 = 2$.

∴ $$\frac{1}{\lambda_1} = R\left(\frac{1}{1^2} - \frac{1}{4}\right)$$

or $$\frac{1}{\lambda_1} = \frac{3R}{4}$$

or $$\lambda_1 = \frac{4}{3R} \quad ...(i)$$

For second member of Balmer series, $n_1 = 2$, $n_2 = 4$

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3R}{16}$$

or, $$\lambda_2 = \frac{16}{3R} \quad ...(ii)$$

Dividing equation (*ii*) by (*i*), we get

$$\frac{\lambda_2}{\lambda_1} = \frac{16}{3R}\times\frac{3R}{4} = 4$$

or $$\lambda_2 = 4\,\lambda_1 = 4 \times 1216\text{ Å} = 4864\text{ Å}.$$

Q. 6. *Determine the speed of electron in n = 3 orbit of He^+. Is the non-relativistic approximation valid?*

Ans. The speed of electron in *n*th orbit is given by

$$v = \frac{2\pi\, KZe^2}{nh}$$

For He, $Z = 2, \quad n = 3$

$$v = \frac{2\pi K2\, e^2}{3h}$$

$$= \frac{4 \times 3.14 \times 9 \times 10^9\ (1.6 \times 10^{-19})^2}{3 \times 6.6 \times 10^{-34}}$$

$$v = 1.46 \times 10^6 \text{ m/s}$$

Now, $$\frac{v}{c} = \frac{1.46 \times 10^6}{3 \times 10^8} = 0.048$$

which is much less than 1. Hence non-relativistic approximation is true.

Q. 7. *Using Rydberg formula, calculate the wavelengths of the first four spectral lines in the Balmer series of hydrogen atom spectrum.*

Ans. The Rydberg formula is

$$E = E_0 Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\frac{hc}{\lambda} = E_0 Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$E_0 = -\ 13.6 \text{ eV}$$

$$\therefore \quad \lambda_{12} = \frac{hc}{21.76 \times 10^{-19} \times \left(\frac{1}{4} - \frac{1}{n_1^2}\right)} \text{ m}$$

$$E_0 = -\ 13.6 \times 1.6 \times 10^{-19} \text{ J}$$

$$E_0 = -\ 21.76 \times 10^{-19} \text{ J}$$

$$\frac{hc}{\lambda_{12}} = -\ 21.76 \times 10^{-19} \left[\frac{1}{2^2} - \frac{1}{n_1^2}\right]$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8 4n_1^2}{21.76 \times 10^{-19} \times (n_1^2 - 4)} \text{ m}$$

$$\lambda_{12} = \frac{3.653\, n_1^2}{(n_1^2 - 4)} \times 10^{-7} \text{ m} = \frac{3653\, n_1^2}{(n_1^2 - 4)} \text{ Å}$$

The wavelengths of the first four lines in the Balmer series correspond to transitions from $n_1 = 3, 4, 5, 6$ to $n_2 = 2$.

Substituting $n_1 = 3, 4, 5$ and 6, we get

$$\lambda_{32} = 6575 \text{ Å}, \quad \lambda_{42} = 4870 \text{ Å}$$

$$\lambda_{52} = 4348 \text{ Å} \quad \text{and} \quad \lambda_{52} = 4109 \text{ Å}.$$

Q. 8. *Calculate the radius of the first orbit of hydrogen atom. Show that the velocity of electron in the first orbit is $\frac{1}{137}$ times the velocity of light.*

Ans. Since, $$r = \frac{n^2 h^2}{4\pi^2 mKZe^2}$$

Using $n = 1$ for 1st orbit

$$h = 6.6 \times 10^{-34} \text{ Js}$$

$$m = 9 \times 10^{-31} \text{ kg}$$
$$K = 9 \times 10^{9} \text{ Nm}^2 \text{ C}^{-2}$$
$$Z = 1 \text{ for hydrogen, } e = 1.6 \times 10^{-19} \text{ coulomb}$$

we get, $$r = 0.53 \times 10^{-10} \text{ m}$$

Also, $$v = \frac{2\pi Ke^2}{nh} = \frac{c}{n}\left(\frac{2\pi Ke^2}{ch}\right)$$
$$= \frac{c}{1} \times 2 \times \frac{22}{7} \times \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{3 \times 10^8 \times 6.6 \times 10^{-34}}$$
$$v = \frac{1}{137} c .$$

Q. 9. *The energy levels of hydrogen atoms are as shown in Fig. 12.8. Which of the shown transition will result in the emission of a photon of wavelength 275 nm?*

Which of the transition corresponds to emission of radiation of (i) maximum and (ii) minimum wavelength?

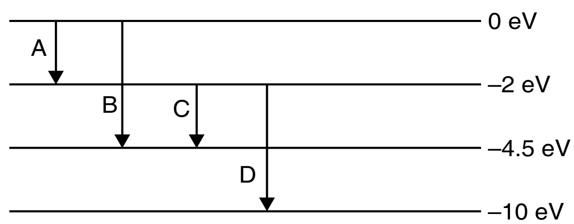

Fig. 12.8

Ans. The energy of photon of wavelength 275 nm

$$\Delta E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{275 \times 10^{-9}} \text{ J}$$
$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{275 \times 10^{-9} \times 1.6 \times 10^{-19}} \text{ eV}$$
$$= \frac{6.6 \times 3}{275 \times 1.6} \times 10^2 \text{ eV}$$
$$= 6.0 \text{ eV}$$

Thus transition is the result.

(*i*) Transition *A*

(*ii*) Transition *D*.

Q. 10. *The short wavelength limits of the Lyman, Paschen and Balmer series, in the hydrogen spectrum, are denoted by λ_L, λ_P and λ_B respectively. Arrange these wavelengths in increasing order.*

Ans. λ_L, λ_B and λ_P.

Q. 11. *The ground state energy of hydrogen atoms is – 13.6 eV.*

(i) Which are the potential and kinetic energy of an electron in the third excited state?

(ii) If the electron jumps to the ground state from the third excited state. Calculate the frequency of photon emitted.

Ans. (*i*) In hydrogen atom in the ground state

$$E = -(\text{K.E.}) = \frac{1}{2}(\text{P.E.})$$

∴ Potential energy of electron in third excited state

$$= \frac{2(E)}{(4)^2} = -\frac{2 \times 13.6}{16} \text{ eV}$$
$$= -1.7 \text{ eV}$$

and kinetic energy of an electron

$$= -\frac{1}{2}(1.7) = 0.85 \text{ eV}$$

(*ii*)
$$\nu = \frac{\Delta E}{h} = \left[\frac{-13.6}{(4)^2} + \frac{13.6}{(1)^2}\right]\frac{1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$$= \frac{11.9 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} \text{ Hz}$$

$$\approx 2.9 \times 10^{15} \text{ Hz.}$$

MULTIPLE CHOICE QUESTIONS

1. If an electron jumps from 1st orbit to 2rd orbit, then it will

(*a*) absorb energy (*b*) release energy

(*c*) no gain of energy (*d*) none of these

2. The wavelength of the first line of Balmer series is 6563 Å. The Rydberg constant for hydrogen is about

(*a*) 1.09×10^7 per m (*b*) 1.09×10^8 per m

(*c*) 1.09×10^9 per m (*d*) 1.09×10^5 per m

3. The Rydberg constant R for hydrogen is

(*a*) $R = -\left[\frac{1}{4\pi\varepsilon_0}\right] \cdot \frac{2\pi^2 me^2}{Ch^2}$ (*b*) $R = \left(\frac{1}{4\pi\varepsilon_0}\right) \cdot \frac{2\pi^2 me^4}{Ch^2}$

(*c*) $R = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \cdot \frac{2\pi^2 me^4}{C^2h^2}$ (*d*) $R = \left[\frac{1}{4\pi\varepsilon_0}\right]^2 \cdot \frac{2\pi^2 me^2}{Ch^3}$

4. The wavelength of the first line of Lyman series of hydrogen is 1216 Å. The wavelength of the second line of the same series will be

(*a*) 912 Å (*b*) 1026 Å (*c*) 3648 Å (*d*) 6566 Å

5. In $_{88}Ra^{226}$ nucleus, there are

(*a*) 138 protons and 88 neutrons (*b*) 138 neutrons and 88 protons

(*c*) 226 protons and 88 electrons (*d*) 226 neutrons and 138 electrons

6. The angular momentum of electron in nth orbit is given by

(*a*) nh (*b*) $n/2\pi n$ (*c*) $nh/2\pi$ (*d*) $n^2h/2\pi$

7. The ionisation energy of 10 times ionised sodium atom is

(*a*) 13.6 eV (*b*) 13.6 × 11 eV (*c*) 13.6/11 eV (*d*) $13.6 \times (11)^2$ eV

8. The ratio of longest wavelength and the shortest wavelength observed in the five spectral series of emission spectrum of hydrogen is

(*a*) 4/3 (*b*) 525/376 (*c*) 25 (*d*) 960/11

9. The absorption transition between the first and the fourth energy states of hydrogen atom are 3. The emission transitions between these states will be

(*a*) 3 (*b*) 4 (*c*) 5 (*d*) 6

10. According to Bohr's theory, the moment of momentum of an electron revolving in second orbit of hydrogen atom will be

(*a*) $2\pi rh$ (*b*) πh (*c*) $\frac{h}{\pi}$ (*d*) $\frac{2h}{\pi}$

11. When an electron jumps from the fourth orbit to the second orbit, one gets the

(*a*) Second line of Paschen series (*b*) Second line of Balmer series

(*c*) First line of Pfund series (*d*) Second line of Lyman series

12. Which of the following transitions in a hydrogen atom emits the photon of highest frequency?

(*a*) $n = 2$ to $n = 6$ (*b*) $n = 6$ to $n = 2$

(*c*) $n = 1$ to $n = 2$ (*d*) $n = 2$ to $n = 1$

13. The ratio of minimum to maximum wavelength in Balmer series is

(*a*) 5 : 9 (*b*) 5 : 36 (*c*) 1 : 4 (*d*) 3 : 4

14. A nucleus represented by the symbol $_ZX^A$ has

(*a*) Z protons and A neutrons (*b*) A protons and $(Z - A)$ neutrons

(*c*) Z neutrons and $(A - Z)$ protons (*d*) Z protons and $(A - Z)$ neutrons

15. An element with atomic number $Z = 11$ emits K_a-X-ray of wavelength λ. The atomic number of element which emits K_a-X-ray of wavelength 4λ is

(*a*) 6 (*b*) 4 (*c*) 11 (*d*) 44

Answers

1. (*a*)	**2.** (*a*)	**3.** (*d*)	**4.** (*b*)	**5.** (*b*)
6. (*c*)	**7.** (*d*)	**8.** (*c*)	**9.** (*d*)	**10.** (*c*)
11. (*b*)	**12.** (*b*)	**13.** (*a*)	**14.** (*d*)	**15.** (*a*)

TEST YOUR SKILLS

1. Write two observations of Rutherford's alpha scattering experiment.
2. Write two conclusions drawn on the basis of alpha scattering experiment.
3. State the limitation of alpha scattering experiment.
4. State the Bohr's postulates of atomic theory.
5. Is Bohr's atomic theory applicable only for hydrogen atom?
6. Explain the Bohr's quantisation condition on the basis of de-Broglie hypothesis.
7. Explain the different spectral lines in hydrogen atom.
8. Does the Bohr's atomic theory explain the variation of intensity of different spectral line on the basis of frequency of photons emitted? Justify your answer.
9. Obtain an expression for the (*i*) velocity and (*ii*) radius of the electron in its orbit.
10. Deduce an expression for the total energy of electron in its orbit.
11. Find the ratio of minimum wavelengths of Lyman series and Paschen series.
12. Find the ratio of minimum frequency of photon obtained in Balmer series and Paschen series.
13. Calculate the angular momentum of an electron in the third orbit of hydrogen atom.
14. Find the frequency of photon emitted due to transition of electron in hydrogen atom from second excited state to the ground state.
15. What will be the kinetic and potential energy of an electron in hydrogen atom in third orbit. (Take total energy of electron in its ground state in – 13.6 eV).
16. What does the negative total energy of electron in its excited state signify?

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