13 Kinetic Theory

Facts that Matter

- The kinetic theory was developed in the nineteenth century by Maxwell, Boltzman and others. Kinetic theory explains the behaviour of gases based on the idea that the gas consists of rapidly moving atoms or molecules.

- **Ideal Gas**

An ideal gas or a perfect gas is that gas which strictly obeys gas laws such as Boyle's law, Charle's law, Gay Lussac's law etc.
An ideal gas has following characteristics:

(*i*) Molecule of an ideal gas is a point mass with no geometrical dimensions.
(*ii*) There is no force of attraction or repulsion amongst the molecules of the gas.

- **Kinetic Theory and Gas Pressure**

The pressure of a gas is the result of continuous bombardment of the gas molecules against the walls of the container. According to the kinetic theory, the pressure P exerted by an ideal gas is given by

$$P = \frac{1}{3}\rho\bar{c}^2$$

where ρ is the density of the gas and $\bar{c}^2$ is the mean square speed of the gas molecules.
If a container has n molecules each of mass m, then

$$P = \frac{1}{3}\frac{nm}{V}\bar{c}^2$$

where V is the volume of the container.

- **Boyle's Law**

According to this law, the volume (V) of a fixed mass of a gas is inversely proportional to the pressure (P) of the gas, provided temperature of the gas is kept constant.

i.e. $$V \propto \frac{1}{P} \text{ or } PV = \text{constant}$$

- **Charle's Law**

According to this law, the volume (V) of a given mass of a gas is directly proportional to the temperature of the gas, provided pressure of the gas remains constant.

i.e., $$V \propto T \text{ or } \frac{V}{T} = \text{a constant} \Rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2}.$$

- **Gay Lussac's Law (or Pressure Law)**

According to this law, the pressure P of a given mass of a gas is directly proportional to its absolute temperature T, provided the volume V of the gas remains constant.

i.e., $$P \alpha T \text{ or } \frac{P}{T} = \text{a constant} \Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

- **Equation of State of an Ideal Gas**

The relationship between pressure P, volume V and absolute temperature T of a gas is called its equation of state. The equation of state of an ideal gas

$$PV = nRT$$

where n is the number of moles of the enclosed gas and R is the molar gas constant which is the same for all gases and its value is

$$R = 8.315 \text{ JK}^{-1} \text{ mol}^{-1}$$

- **Avagadro's Law**

Equal volumes of all gases under S.T.P. contain the same number of molecules equalling 6.023×10^{23}.

- **Graham's Law of Diffusion of Gases**

It states that rate of diffusion of a gas is inversely proportional to the square root of the density of the gas.

i.e., $$r \propto \sqrt{\frac{1}{\rho}}$$

Hence, denser the gas, the slower is the rate of diffusion.

- **Dalton's Law of Partial Pressures**

According to this law, the resultant pressure exerted by a mixture of non-interacting gases is equal to the sum of their individual pressures.

i.e., $$P = P_1 + P_2 + ...P_n$$

- Mean (or average) speed of molecules of a gas is defined as the arithmetic mean of the speeds of gas molecules.

i.e., mean speed, $$v_{\text{mean}} = \frac{v_1 + v_2 + ... + v_n}{n} = \sqrt{\frac{8k_BT}{\pi m}}$$

where m is mass of 1 molecule of given gas and T = temperature of gas.

- Root mean square speed of gas molecules is defined as the square root of the mean of the squares of the speeds of gas molecules.

i.e., $$v_{\text{rms}} = \sqrt{\frac{v_1^2 + v_2^2 + ... + v_n^2}{n}} = \frac{3P}{\rho} = \sqrt{\frac{3k_BT}{m}}$$

- Most probable speed of gas molecules is defined as the speed which is possessed by maximum number of molecules in a gas

i.e., $$v_{mp} = \sqrt{\frac{2k_BT}{m}}$$

- **Kinetic Interpretation of Temperature**

The total average kinetic energy of all the molecules of a gas is proportional to its absolute temperature (T). Thus, the temperature of a gas is a measure of the average kinetic energy 'U' of the molecules of the gas.

$$U = \frac{3}{2} RT$$

According to this interpretation of temperature, the average kinetic energy U is zero at T = 0, *i.e.,* the motion of molecules ceases altogether at absolute zero.

- **Degrees of Freedom**

The total number of independent co-ordinates required to specify the position of a molecule or the number of independent modes of motion possible with any molecule is called degree of freedom.

Mono-, di-, and polyatomic (N) molecules have, 3, 5 or (3 N–K) number of degrees of freedom where K is the number of constraints [restrictions associated with the structure].

- **Law of Equipartition of Energy**

For a dynamic system in thermal equilibrium, the energy of the system is equally distributed amongst the various degrees of freedom and the energy associated with each degree of freedom per molecule is $\frac{1}{2}$ kT, where k is Boltzman constant.

- **Mean Free Path**

Mean free path of a molecule in a gas is the average distance travelled by the molecule between two successive collisions.

If $\lambda_1, \lambda_2, \lambda_3$ ....λ_N be the free paths travelled by the molecule in N successive collisions. Then, mean free path is given by

$$\lambda = \frac{\lambda_1 + \lambda_2 + \lambda_3 + ...\lambda_N}{N} = \left|\frac{1}{\sqrt{2\pi d^2 n}} = \frac{kT}{\sqrt{2\pi d^2 p}}\right|$$

where, $d \rightarrow$ molecular diameter; $n \rightarrow$ number of molecules per unit volume

$T \rightarrow$ absolute temperature; $P \rightarrow$ pressure

(*i*) Smaller the number of molecules per unit volume of the gas, larger is the mean free path.
(*ii*) Smaller the diameter, larger is the mean free path.
(*iii*) Smaller the density, larger is the mean free path. In the case of vacuum, $\rho = 0$, $\lambda \rightarrow \infty$.
(*iv*) Smaller the pressure of a gas, larger is the mean free path.
(*v*) Higher the temperature of a gas, larger is the mean free path.

- **IMPORTANT TABLES**

TABLE 13.1 Predicted values of specific heat capacities of gases (ignoring vibrational modes).

Nature of Gas	C_V ($J\ mol^{-1}\ K^{-1}$)	C_P ($J\ mol^{-1}\ K^{-1}$)	$C_P - C_V$ ($J\ mol^{-1}\ K^{-1}$)	γ
Monoatomic	12.5	20.8	8.31	1.67
Diatomic	20.8	29.1	8.31	1.40
Triatomic	24.93	33.24	8.31	1.33

TABLE 13.2 Measured values of specific heat capacities of some gases

Nature of Gas	*Gas*	C_V ($J\ mol^{-1}\ K^{-1}$)	C_P ($J\ mol^{-1}\ K^{-1}$)	$C_P - C_V$ ($J\ mol^{-1}\ K^{-1}$)	γ
Monoatomic	He	12.5	20.8	8.30	1.66
Monoatomic	Ne	12.7	20.8	8.12	1.64
Monoatomic	Ar	12.5	20.8	8.30	1.67
Diatomic	H_2	20.4	28.8	8.45	1.41
Diatomic	O_2	21.0	29.3	8.32	1.40
Diatomic	N_2	20.8	29.1	8.32	1.40
Triatomic	H_2O	27.0	35.4	8.35	1.31
Polyatomic	CH_4	27.1	35.4	8.36	1.31

TABLE 13.3 Specific heat capacities of solids

Substance	*Sp. heat* $J\ kg^{-1}\ K^{-1}$	*Molar sp. heat* $J\ mole^{-1}\ K^{-1}$
Aluminium	900.0	24.4
Copper	386.4	24.5
Lead	127.7	26.5
Silver	236.1	25.5
Tungsten	134.4	24.9
Carbon	506.5	6.1

NCERT TEXTBOOK QUESTIONS SOLVED

13.1. *Estimate the fraction of molecular volume to the actual volume occupied by oxygen gas at STP. Take the diameter of an oxygen molecule to be 3 Å.*

Ans. Diameter of an oxygen molecule, $d = 3$ Å $= 3 \times 10^{-10}$ m. Consider one mole of oxygen gas at STP, which contain total $N_A = 6.023 \times 10^{23}$ molecules.

Actual molecular volume of 6.023×10^{23} oxygen molecules

$$V_{\text{actual}} = \frac{4}{3}\pi r^3 \cdot N_A$$

$$= \frac{4}{3} \times 3.14 \times (1.5)^3 \times 10^{-3} \times 6.02 \times 10^{23} \text{ m}^3$$

$$= 8.51 \times 10^{-6} \text{ m}^3$$

$$= 8.51 \times 10^{-3} \text{ litre} \qquad [\because 1 \text{ m}^3 = 10^3 \text{ litre}]$$

$\therefore$ Molecular volume of one mole of oxygen

$$V_{\text{actual}} = 8.51 \times 10^{-3} \text{ litre}$$

At STP, the volume of one mole of oxygen

$$V_{\text{molar}} = 22.4 \text{ litre}$$

$$\frac{V_{\text{actual}}}{V_{\text{molar}}} = \frac{8.51 \times 10^{-3}}{22.4} = 3.8 \times 10^{-4} \simeq 4 \times 10^{-4}.$$

13.2. *Molar volume is the volume occupied by 1 mol of any (ideal) gas at standard temperature and pressure (STP : 1 atmospheric pressure, 0 °C). Show that it is 22.4 litres.*

Ans. For one mole of an ideal gas, we have

$$PV = RT \Rightarrow V = \frac{RT}{P}$$

Putting $R = 8.31$ J mol^{-1} K^{-1}, $T = 273K$ and $P = 1$ atmosphere $= 1.013 \times 10^5$ Nm^{-2}

$$\therefore \quad V = \frac{8.31 \times 273}{1.013 \times 10^5} = 0.0224 \text{ m}^3$$

$$= 0.0224 \times 10^6 \text{ cm}^3 = 22400 \text{ ml} \qquad [1 \text{ cm}^3 = 1\text{ml}]$$

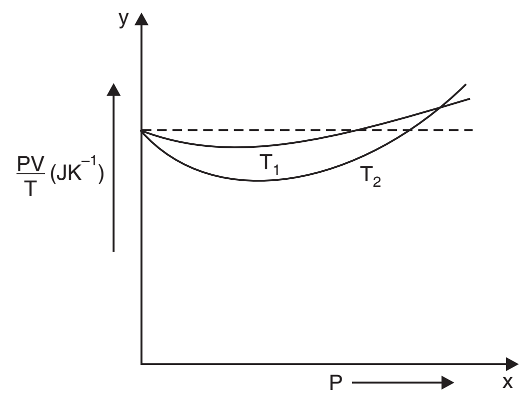
13.3. *Following figure shows plot of PV/T versus P for 1.00×10^{-3} kg of oxygen gas at two different temperatures.*

(a) What does the dotted plot signify?

(b) Which is true : $T_1 > T_2$ or $T_1 < T_2$?

(c) What is the value of PV/T where the curves meet on the y-axis?

(d) If we obtained similar plots for 1.00×10^{-3} kg of hydrogen, would we get the same value of PV/T at the point where the curves meet on the y-axis? If not, what mass of hydrogen yields the same value of PV/T (for low pressure high temperature region of the plot)? (Molecular mass of H_2 = 2.02 u, of O_2 = 32.0 u, R = 8.31 J mol^{-1} K^{-1}.)



Ans. (*a*) The dotted plot corresponds to 'ideal' gas behaviour as it is parallel to P-axis and it tells that value of PV/T remains same even when P is changed.

(*b*) The upper position of PV/T shows that its value is lesser for T_1, thus $T_1 > T_2$. This is because the curve at T_1 is more close to dotted plot than the curve at T_2. Since the behaviour of a real gas approaches the perfect gas behaviour, as the temperature is increased.

(*c*) Where the two curves meet, the value of PV/T on *y-axis* is equal to μR. Since ideal gas equation for μ moles is $PV = \mu RT$

where, $$\mu = \frac{1.00 \times 10^{-3} kg}{32 \times 10^{-3} kg} = \frac{1}{32}$$

$$\therefore \quad \text{Value of } \frac{PV}{T} = \mu R = \frac{1}{32} \times 8.31 \text{ JK}^{-1} = 0.26 \text{ JK}^{-1}$$

(*d*) If we obtained similar plots for 1.00×10^{-3} kg of hydrogen, we will not get the same value of $\frac{PV}{T}$ at the point, where the curves meet on the *y-axis*. This is because molecular mass of hydrogen is different from that of oxygen.

For the same value of $\frac{PV}{T}$, mass of hydrogen required is obtained from

$$\frac{PV}{T} = nR = \frac{m}{2.02} \times 8.31 = 0.26$$

$$m = \frac{2.02 \times 0.26}{8.31} \text{ gram} = 6.32 \times 10^{-2} \text{ gram.}$$

13.4. *An oxygen cylinder of volume 30 litre has an initial gauge pressure of 15 atmosphere and a temperature of 27 °C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atmosphere and its temperature drops to 17 °C. Estimate the mass of oxygen taken out of the cylinder. (R = 8.31 J mol^{-1} K^{-1}, molecular mass of O_2 = 32 u.)*

Ans. Initial volume, V_1 = 30 litre = 30×10^3 cm^3 = $30 \times 10^3 \times 10^{-6}$ m^3 = 30×10^{-3} m^3

Initial pressure, P_1 = 15 atm = $15 \times 1.013 \times 10^5$ N m^{-2}

Initial temperature, T_1 = (27 + 273) K = 300 K

Initial number of moles,

$$\mu_1 = \frac{P_1V_1}{RT_1} = \frac{15 \times 1.013 \times 10^5 \times 30 \times 10^{-3}}{8.31 \times 300} = 18.3$$

Final pressure, P_2 = 11 atm = 11 × 1.013 × 10^5 N m^{-2}

Final volume, V_2 = 30 litre = 30 × 10^{-3} cm^3

Final temperature, T_2 = 17 + 273 = 290 K

Final number of moles,

$$\mu_2 = \frac{P_2V_2}{RT_2} = \frac{11 \times 1.013 \times 10^5 \times 30 \times 10^{-3}}{8.31 \times 290} = 13.9$$

Number of moles taken out of cylinder

= 18.3 – 13.9 = 4.4

Mass of gas taken out of cylinder

= 4.4 × 32 g = 140.8 g = 0.141 kg.

13.5. *An air bubble of volume 1.0 cm^3 rises from the bottom of a lake 40 m deep at a temperature of 12°C. To what volume does it grow when it reaches the surface, which is at a temperature of 35 °C.*

Ans. Volume of the bubble inside, V_1 = 1.0 cm^3 = 1 × 10^{-6} m^3

Pressure on the bubble, P_1 = Pressure of water + Atmospheric pressure

= pgh + 1.01 × 10^5 = 1000 × 9.8 × 40 + 1.01 × 10^5

= 3.92 × 10^5 + 1.01 × 10^5 = 4.93 × 10^5 Pa

Temperature, T_1 = 12 °C = 273 + 12 = 285 K

Also, pressure outside the lake, P_2, = 1.01 × 10^5 N m^{-2}

Temperature, T_2 = 35 °C = 273 + 35 = 308 K, volume V_2 =?

Now $$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

∴ $$V_2 = \frac{P_1V_1}{T_1} \cdot \frac{T_2}{P_2} = \frac{4.93 \times 10^5 \times 1 \times 10^{-6} \times 308}{285 \times 1.01 \times 10^5} = 5.3 \times 10^{-6} \text{ m}^3$$

13.6. *Estimate the total number of air molecules (inclusive of oxygen, nitrogen, water vapour and other constituents) in a room of capacity 25.0 m^3 at a temperature of 27 °C and 1 atm pressure.*

Ans. Here, volume of room, V = 25.0 m^3, temperature, T = 27 °C = 300 K and

Pressure, P = 1 atm = 1.01 × 10^5 Pa

According to gas equation,

$$PV = \mu RT = \mu N_A \cdot k_B T$$

Hence, total number of air molecules in the volume of given gas,

$$N = \mu \cdot N_A = \frac{PV}{k_B T}$$

∴ $$N = \frac{1.01 \times 10^5 \times 25.0}{(1.38 \times 10^{-23}) \times 300} = 6.1 \times 10^{26}.$$

13.7. *Estimate the average thermal energy of a helium atom at (i) room temperature (27 °C), (ii) the temperature on the surface of the Sun (6000 K), (iii) the temperature of 10 million kelvin (the typical core temperature in the case of a star).*

Ans. (*i*) Here, T = 27 °C = 27 + 273 = 300 K

Average thermal energy = $\frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = 6.2 \times 10^{-21}$ J.

(*ii*) At $T = 6000$ K,

Average thermal energy $= \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 6000 = 1.24 \times 10^{-19}$ J.

(*iii*) At T = 10 million $K = 10^7$ K

Average thermal energy = $\frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 10^7 = 2.1 \times 10^{-16}$ J

13.8. *Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic), and the third contains uranium hexafluoride (polyatomic). Do the vessels contain equal number of respective molecules? Is the root mean square speed of molecules the same in the three cases? If not, in which case is v_{rms} the largest?*

Ans. Equal volumes of all the gases under similar conditions of pressure and temperature contains equal number of molecules (according to Avogadro's hypothesis). Therefore, the number of molecules in each case is same.

The rms velocity of molecules is given by

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

Clearly $$v_{\text{rms}} \propto \frac{1}{\sqrt{m}}$$

Since neon has minimum atomic mass m, its rms velocity is maximum.

13.9. *At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the rms speed of a helium gas atom at –20 °C? (atomic mass of Ar = 39.9 u, of He = 4.0 u).*

Ans. Let C and C' be the rms velocity of argon and a helium gas atoms at temperature T K and T' K respectively.

Here, $M = 39.9;\quad M' = 4.0;\quad T = ?;\quad T' = -20 + 273 = 253$ K

Now, $$C = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT}{39.9}} \quad \text{and} \quad C' = \sqrt{\frac{3RT'}{M'}} = \sqrt{\frac{3R \times 253}{4}}$$

Since $C = C'$

Therefore, $$\sqrt{\frac{3RT}{39.9}} = \sqrt{\frac{3R \times 253}{4}} \quad \text{or} \quad T = \frac{39.9 \times 253}{4} = 2523.7 \text{ K.}$$

13.10. *Estimate the mean free path and collision frequency of a nitrogen molecule in a cylinder containing nitrogen at 2.0 atm and temperature 17 °C. Take the radius of a nitrogen molecule to be roughly 1.0 Å. Compare the collision time with the time the molecule moves freely between two successive collisions (Molecular mass of N_2 = 28.0 u).*

Ans. Here, $P = 2.0 \text{ atm} = 2 \times 1.013 \times 10^5 \text{ Pa} = 2.026 \times 10^5$ Pa

$T = 17\ ^\circ\text{C} = 17 + 273 = 290$ K

Radius, $R = 1.0 \text{ Å} = 1 \times 10^{-10}$ m, Molecular mass = 28 u

$\therefore$ $m = 28 \times 1.66 \times 10^{-27} = 4.65 \times 10^{-26}$ kg

Also, $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$, $k = 1.38 \times 10^{-23} \text{ JK}^{-1}$

Now for one mole of a gas,

$$PV = RT \Rightarrow V = \frac{RT}{P} = \frac{8.31 \times 290}{2.026 \times 10^5}$$

$$\Rightarrow \qquad V = 1.189 \times 10^{-2}\ m^3$$

$\therefore$ Number of molecules per unit volume, $n = \dfrac{N}{V}$

$$\therefore \qquad n = \frac{6.023 \times 10^{23}}{1.189 \times 10^{-2}} = 5.06 \times 10^{25}\ m^{-3}$$

Now, mean free path,

$$\lambda = \frac{1}{\sqrt{2}\pi n d^2} = \frac{1}{\sqrt{2}\pi n (2r)^2}$$

$$= \frac{1}{1.414 \times 3.14 \times 5.06 \times 10^{25} \times \left(2 \times 1 \times 10^{-10}\right)^2} = 1.1 \times 10^{-7}\ m.$$

Also, $$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 290}{28 \times 10^{-3}}} = 5.08 \times 10^2\ ms^{-1}$$

$\therefore$ Collision frequency,

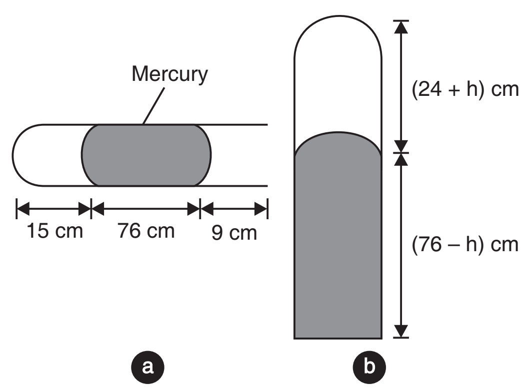
$$v = \frac{v_{rms}}{\lambda} = \frac{5.08 \times 10^2}{1.1 \times 10^{-7}} = 4.62 \times 10^9\ s^{-1}$$

Time between successive collisions $= \dfrac{1}{v} = \dfrac{1}{4.62 \times 10^9} = 2.17 \times 10^{-10}$ s

Also the collision time $= \dfrac{d}{v_{rms}} = \dfrac{2 \times 1 \times 10^{-10}}{5.08 \times 10^2} s = 3.92 \times 10^{-13}$ s.

13.11. *A meter long narrow bore held horizontally (and closed at one end) contains a 76 cm long mercury thread which traps a 15 cm column of air. What happens if the tube is held vertically with the open end at the bottom?*

Ans. When the tube is held horizontally, the mercury thread of length 76 cm traps a length of air = 15 cm. A length of 9 cm of the tube will be left at the open end. The pressure of air enclosed in tube will be atmospheric pressure. Let area of cross-section of the tube be 1 sq. cm.



$\therefore$ $P_1 = 76$ cm and $V_1 = 15$ cm^3

When the tube is held vertically, 15 cm air gets another 9 cm of air (filled in the right handside in the horizontal position) and let h cm of mercury flows out to balance the atmospheric pressure. Then the heights of air column and mercury column are $(24 + h)$ cm and $(76 - h)$ cm respectively.

The pressure of air $= 76 - (76 - h) = h$ cm of mercury.

$$\therefore \qquad V_2 = (24 + h)\ cm^3 \quad \text{and} \quad P_2 = h\ cm$$

If we assume that temperature remains constant, then

$$P_1V_1 = P_2V_2 \quad \text{or} \quad 76 \times 15 = h \times (24 + h) \quad \text{or} \quad h^2 + 24h - 1140 = 0$$

or $$h = \frac{-24 \pm \sqrt{(24)^2 + 4 \times 1140}}{2} = 23.8\ cm \quad \text{or} \quad -47.8\ cm$$

Since h cannot be negative (because more mercury cannot flow into the tube), therefore h = 23.8 cm.

Thus in the vertical position of the tube, 23.8 cm of mercury flows out.

13.12. *From a certain apparatus, the diffusion rate of hydrogen has an average value of 28.7 $cm^3\ s^{-1}$. The diffusion of another gas under the same conditions is measured to have an average rate of 7.2 $cm^3\ s^{-1}$. Identify the gas.*

Ans. According to Graham's law of diffusion of gases, the rate of diffusion of a gas is inversely proportional to the square root of its molecular mass.

If R_1 and R_2 be the rates of diffusion of two gases having molecular masses M_1 and M_2 respectively, then

$$\frac{R_1}{R_2} = \sqrt{\frac{M_2}{M_1}}$$

Now, $R_1 = 28.7\ cm^3\ s^{-1}$, $R_2 = 7.2\ cm^3\ s^{-1}$, $M_1 = 2$, M_2 =?

$$\therefore \quad \frac{28.7}{7.2} = \sqrt{\frac{M_2}{2}} \quad \text{or} \quad \frac{M_2}{2} = \frac{28.7 \times 28.7}{7.2 \times 7.2}$$

$$\text{or} \quad M_2 = \frac{2 \times 28.7 \times 28.7}{7.2 \times 7.2} = 31.78 \approx 32$$

This is molecular mass of oxygen. Therefore, the second gas is oxygen.

13.13. *A gas in equilibrium has uniform density and pressure throughout its volume. This is strictly true only if there are no external influences. A gas column under gravity, for example, does not have uniform density (and pressure). As you might expect, its density decreases with height. The precise dependence is given by the so-called law of atmospheres*

$$n_2 = n_1 \exp [- mg (h_2 - h_1)/k_B T]$$

where n_2, n_1 refer to number density at heights h_2 and h_1 respectively. Use this relation to derive the equation for sedimentation equilibrium of a suspension in a liquid column :

$$n_2 = n_1 \exp [- mg N_A (\rho - P) (h_2 - h_1)/(\rho RT)]$$

where ρ is the density of the suspended particle, and ρ' that of surrounding medium. [N_A is Avogadro's number, and R the universal gas constant.]

[Hint : Use Archimedes principle to find the apparent weight of the suspended particle.]

Ans. Considering the particles and molecules to be spherical, the weight of the particle is

$$W = mg = \frac{4}{3}\pi r^3 \rho g \quad ...(i)$$

where r = radius of the particle and ρ = density of the particle. Its motion under gravity causes buoyant force to act upward which is equal to

$$B = \text{Volume of particle} \times \text{density of the surrounding medium} \times g$$

$$= \frac{4}{3}\pi r^3 \rho' g \quad ...(ii)$$

If F be the downward force acting on the particle, then

$$F = W - B = \frac{4}{3}\pi r^3 (\rho - \rho') g \quad ...(iii)$$

Also $$n_2 = n_1 \exp \left[\frac{-mg}{k_B T}(h_2 - h_1)\right] \quad ...(iv)$$

where k_B = Boltzman constant

n_1 and n_2 are number densities at heights h_1 and h_2 respectively. Here mg can be replaced by effective force F given by equation (*iii*).

$\therefore$ From (*iii*) and (*iv*), we get

$$n_2 = n_1 \exp\left[-\frac{4\pi}{3}r^3 \frac{(\rho-\rho')}{k_B T} g(h_2-h_1)\right]$$

$$= n_1 \exp\left[-\frac{4\pi}{3}r^3 \frac{\rho g\left(1-\frac{\rho'}{\rho}\right)(h_2-h_1)}{\left(\frac{RT}{N_A}\right)}\right] \qquad \left[\because k_B = \frac{R}{N_A}\right]$$

$$n_2 = n_1 \exp\left[-\frac{mg\, N_A\left(1-\frac{\rho'}{\rho}\right)(h_2-h_1)}{RT}\right]$$

which is required relation

where, $\frac{4}{3}\pi r^3 \rho g$ = mass of the particle × g = mg.

13.14. *Given below are densities of some solids and liquids. Give rough estimates of the size of their atoms :*

Substance	*Atomic Mass (u)*	*Density (10^3 Kg m^3)*
Carbon (diamond)	*12.01*	*2.22*
Gold	*197.00*	*19.32*
Nitrogen (liquid)	*14.01*	*1.00*
Lithium	*6.94*	*0.53*
Fluorine (liquid)	*19.00*	*1.14*

*[**Hint :** Assume the atoms to be 'tightly packed' in a solid or liquid phase, and use the known value of Avogadro's number. You should, however, not take the actual numbers you obtain for various atomic sizes too literally. Because of the crudeness of the tight packing approximation, the results only indicate that atomic sizes are in the range of a few Å].*

Ans. In one mole of a substance, there are 6.023 × 10^{23} atoms

$$\therefore \left(\frac{4}{3}\pi R^3\right)\times 6.023\times 10^{23} = \frac{M}{\rho} \quad \text{or} \quad R = \left[\frac{3M}{4\pi\rho\times 6.023\times 10^{23}}\right]^{1/3}$$

For carbon, $M = 12.01 \times 10^{-3}$ kg and $\rho = 2.22 \times 10^3$ kg m^{-3}

$$\therefore \quad R = \left[\frac{3\times 12.01\times 10^{-3}}{4\times 3.14\times 2.22\times 10^3\times 6.023\times 10^{23}}\right]^{1/3}$$

$= 1.29 \times 10^{-10}$ m = 1.29 Å

For gold, $M = 197 \times 10^{-3}$ kg and $\rho = 19.32 \times 10^3$ kg m^{-3}

$$\therefore \quad R = \left[\frac{3\times 197\times 10^{-3}}{4\times 3.14\times 19.32\times 10^3\times 6.023\times 10^{23}}\right]^{1/3}$$

$= 1.59 \times 10^{-10}$ m = 1.59 Å

For nitrogen (liquid)

$$M = 14.01 \times 10^{-3} \text{ kg} \quad \text{and} \quad \rho = 1.00 \times 10^3 \text{ kg m}^{-3}$$

$$\therefore \quad R = \left[\frac{3 \times 14.01 \times 10^{-3}}{4 \times 3.14 \times 1.00 \times 10^3 \times 6.023 \times 10^{23}}\right]^{1/3}$$

$$= 1.77 \times 10^{-10} \text{ m} = 1.77 \text{ Å}$$

For lithium, $M = 6.94 \times 10^{-3}$ kg, $\rho = 0.53 \times 10^3$ kg m^{-3}

$$\therefore \quad R = \left[\frac{3 \times 6.94 \times 10^{-3}}{4 \times 3.14 \times 0.53 \times 10^3 \times 6.023 \times 10^{23}}\right]^{1/3}$$

$$= 1.73 \times 10^{-10} \text{ m} = 1.73 \text{ Å}$$

For fluorine (liquid)

$$M = 19.00 \times 10^{-3} \text{ kg}, \quad \rho = 1.14 \times 10^3 \text{ kg m}^{-3}$$

$$\therefore \quad R = \left[\frac{3 \times 19.00 \times 10^{-3}}{4 \times 3.14 \times 1.14 \times 10^3 \times 6.023 \times 10^{23}}\right]^{1/3}$$

$$= 1.88 \times 10^{-10} \text{ m} = 1.88 \text{ Å}$$

ADDIATIONAL QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *The ratio of vapour densities of two gases at the same temperature is 6 : 9. Compare the r.m.s. velocities of their molecules.*

Ans. The ratio of r.m.s velocities is given as

$$\frac{C_1}{C_2} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{\rho_2}{\rho_1}}; \qquad \frac{C_1}{C_2} = \sqrt{\frac{9}{6}} = \sqrt{3} : \sqrt{2}$$

Q. 2. *The number of molecules in a container is doubled. What will be the effect on the rms speed of the molecules?*

Ans. No effect.

Q. 3. *In given samples of 1 C.C. of hydrogen and 1 C.C. of oxygen at N.T.P., which sample has larger number of molecules?*

Ans. Both the samples will contain the same number of molecules, in accordance with Avogadro's law.

Q. 4. *What are the characteristics of gas molecules?*

Ans. Gas molecules are considered to be tiny, hard, incompressible, perfectly elastic spheres in a state of continuous random motion.

Q. 5. *Which gas possesses higher rms speed - hydrogen or air? Why?*

Ans. At a given temperature rms speed of hydrogen gas molecules is more because density of hydrogen is less than that of air.

Q. 6. *What would be the effect on the rms velocity of gas molecules if the temperature of the gas is increased by a factor of 4?*

Ans. Since, $C \propto \sqrt{T}$

Clearly, C will be doubled.

Q. 7. *Mention two conditions when real gases obey the ideal gas equation PV = RT?*

Ans. (*i*) Low pressure (*ii*) High temperature

Q. 8. *Although the r.m.s. speed of gas molecules is of the order of the speed of sound in that gas, yet on opening a bottle of ammonia in one corner of a room, its smell takes time in reaching the other corner. Explain why?*

Ans. Because the molecules of ammonia move at random and continuously collide with one another. As a result of which they are not able to advance in one particular direction speedily.

Q. 9. *The pressure of a gas at – 173°C is 1 atmosphere. Keeping the volume constant, to what temperature should the gas be heated so that its pressure becomes 2 atmosphere.*

Ans. $P_1/T_1 = P_2/T_2$ or $T_2 = P_2\,T_1/P_1 = \dfrac{2\times(273-173)}{1}$ = 200 K = – 73°C.

Q. 10. *What is the shape of the graph between pressure P and $\frac{1}{V}$ (reciprocal of volume) for a perfect gas at constant temperature?*

Ans. Straight line.

Q. 11. *Does the number of degrees of freedom of a gas molecule change with rise in temperature?*

Ans. Yes, the number of degrees of freedom of a gas molecule may increase with rise in temperature. It is because at higher temperatures vibrational motion may also take place in gas molecules.

Q. 12. *How does the mean free path of a gas depends on its temperature?*

Ans. Mean free path of a gas is directly proportional to its temperature on kelvin scale.

Q. 13. *A vessel is filled with a mixture of two different gases. Will the mean kinetic energy per molecule of both the gases be equal?*

Ans. Yes. This is because the mean kinetic energy per molecule, *i.e.*, $\frac{3}{2}kT$ depends only upon temperature.

Q. 14. *The absolute temperature of the gas is increased 3 times. What will be the increase in root mean square velocity of the gas molecules?*

Ans. Since $C \propto \sqrt{T}$

Therefore, the r.m.s. velocity becomes $\sqrt{3}C$.

Hence, increase in r.m.s. velocity = $\sqrt{3}C - C$ = 0.732 C.

Q. 15. *Four molecules of a gas are having speeds v_1, v_2, v_3 and v_4. (a) What is their average speed? (b) What is the r.m.s. speed?*

Ans. (*a*) $v_{av} = \dfrac{v_1 + v_2 + v_3 + v_4}{4}$ (*b*) $v_{\text{r.m.s.}} = \sqrt{\dfrac{v_1^2 + v_2^2 + v_3^2 + v_4^2}{4}}$

Q. 16. *(a) On the basis of Charle's law, what is the minimum possible temperature?*

(b) What is the volume of gas at absolute zero temperature?

Ans. (*a*) – 273.15°C (*b*) Zero.

Q. 17. *What is the number of degrees of freedom of a molecule of a diatomic gas at room temperature?*

Ans. Generally a molecule of a diatomic gas possesses 5 degrees of freedom at room temperature. 3 due to translational motion and 2 due to rotational motion.

Q. 18. *What is the kinetic energy per molecule of a gas whose pressure is P?*

Ans. $\frac{3\,k_B\,T}{2}$.

Q. 19. *Name the factors on which the degrees of freedom of gas depends.*

Ans. Atomicity and temperature.

Q. 20. *What is the mean translational kinetic energy of a mole of helium gas at 400 K?*

Ans. $\overline{E}_k = \frac{3}{2}RT = \frac{3}{2} \times 8.31 \times 400 = 4986$ J.

Q. 21. *Chlorine and carbon dioxide gases are maintained at 27 °C. Which gas will have higher average molar kinetic energy of translation and why?*

Ans. Both gases have same value of average translational kinetic energy per mole because their temperatures are equal and $\overline{E} = \frac{3}{2}RT$.

Q. 22. *A container has equal number of molecules of hydrogen and carbon dioxide. If a fine hole is made in the container, then which of the two gases shall leak out rapidly?*

Ans. Hydrogen would leak out faster as r.m.s. speed of hydrogen is greater than the r.m.s. speed of CO_2.

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *Why the molecular motion of the molecules ceases at zero kelvin?*

Ans. We know, kinetic energy of a molecule is proportional to the absolute temperature.

i.e., $\frac{1}{2}mv_{\text{rms}}^2 \propto T$

At $T = 0$ (zero kelvin)

$\frac{1}{2}mv_{\text{rms}}^2 = 0$, Since $\frac{1}{2}m \neq 0$, $\therefore$ $v_{\text{rms}} = 0$

Thus molecular motion ceases at zero kelvin.

Q. 2. *Calculate (i) r.m.s. velocity and (ii) mean kinetic energy of one gram molecule of hydrogen at S.T.P. Given density of hydrogen at S.T.P. is 0.09 kg m^{-3}.*

Ans. Here, $\rho = 0.09$ kg m^{-3}

At S.T.P., pressure $P = 1.01 \times 10^5$ Pa.

According to kinetic theory of gases.

$$P = \frac{1}{3}\rho C^2 \quad \text{or} \quad C = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 1.01 \times 10^5}{0.09}} = \mathbf{1837.5\ ms^{-1}}$$

Volume occupied by one mole of hydrogen at S.T.P. = 22.4 litres = 22.4×10^{-3} m^3

$\therefore$ Mass of hydrogen, M = volume × density

$= 22.4 \times 10^{-3} \times 0.09 = 2.016 \times 10^{-3}$ kg

Average K.E./mole = $\frac{1}{2}MC^2$

$= \frac{1}{2} \times (2.016 \times 10^{-3}) \times (1837.5)^2 = \mathbf{3403.4\ J}$

Q. 3. *A vessel A contains hydrogen and another vessel B whose volume is twice of A contains same mass of oxygen at the same temperature. Compare (i) average kinetic energies of hydrogen and oxygen molecules (ii) root mean square speeds of the molecules (iii) pressure of gases in A and B. Molecular weights of hydrogen and oxygen are 2 and 32 respectively.*

Ans. (*i*) For all gases at the same temperature, the kinetic energy per molecule is the same. So, the ratio of the average kinetic energies of hydrogen and oxygen molecules is 1 : 1.

(*ii*) $\frac{C_1}{C_2} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{32}{2}} = 4:1$

(*iii*) We know that $P = \frac{1}{3}\frac{M}{V}C^2$

In the given problem, M is constant

$$\therefore \quad \frac{P_1}{P_2} = \frac{C_1^2}{V_1}\times\frac{V_2}{C_2^2} = \frac{V_2}{V_1}\left[\frac{C_1}{C_2}\right]^2 \quad \text{or} \quad \frac{P_1}{P_2} = \frac{2}{1}\times\frac{16}{1} = 32$$

Q. 4. *At what temperature will the average velocity of oxygen molecules be sufficient so as to escape from the earth? Escape velocity from the earth is 11.0 km/sec and the mass of one molecule of oxygen is 5.34 × 10^{-26} kg (Boltzmann constant k = 1.38 × 10^{-23} Joule/K).*

Ans. We know $\frac{1}{2}mv^2 = \frac{3}{2}kT$ or $T = \frac{mv^2}{3k}$

Here, $m = 5.34 \times 10^{-26}$ kg

$v = 11.0$ km/s $= 11 \times 10^3$ ms^{-1}

$k = 1.38 \times 10^{-23}$ JK^{-1}

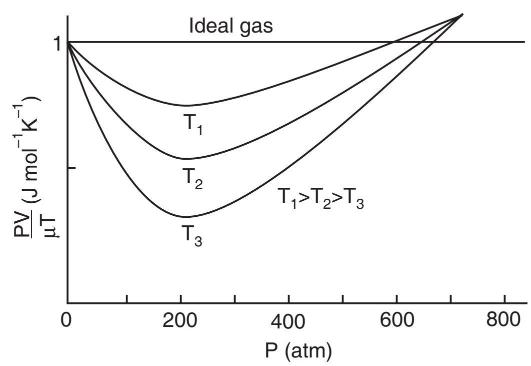
$$\therefore \quad T = \frac{5.34\times10^{-26}\times\left(11\times10^3\right)^2}{3\times1.38\times10^{-23}} = 1.56 \times 10^5 \text{ K}$$

Q. 5. *State ideal gas equation. Draw graph to check whether a real gas obeys this equation. What is the conclusion drawn?*

Ans. According to the ideal gas equation, we have $PV = \mu RT$

Thus, according to this equation $\frac{PV}{\mu T} = R$

i.e., value of $\frac{PV}{\mu T}$ must be a constant having a value 8.31 J mol^{-1} K^{-1}.



Experimentally value of $\frac{PV}{\mu T}$ for real gases was calculated by altering the pressure of gas at different temperatures. The graphs obtained have been shown in the figure. Here, for the purpose of comparison, graph for an ideal gas has also been drawn, which is a straight line parallel to pressure axis. From the graphs it is clear that behaviour of real gases differ from an ideal gas. However, at high temperatures and low pressures behaviour is nearly same as that of an ideal gas.

Q. 6. *On what parameters does the λ (mean free path) depends?*

Ans. We know that

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2\rho} = \frac{m}{\sqrt{2}\pi d^2\rho} = \frac{1}{\sqrt{2}\pi n d^2}$$

∴ λ depends upon:

(*i*) diameter (d) of the molecule, smaller the 'd', larger is the mean free path λ.

(*ii*) $\lambda \propto T$ *i.e.*, higher the temperature larger is the λ.

(*iii*) $\lambda \propto \frac{1}{P}$ *i.e.*, smaller the pressure larger is the λ.

(*iv*) $\lambda \propto \frac{1}{\rho}$ *i.e.*, smaller the density (ρ), larger will be the λ.

(*v*) $\lambda \propto \frac{1}{n}$ *i.e.*, smaller the number of molecules per unit volume of the gas, larger is the λ.

Q. 7. *At what temperature would the root-mean square speed of a gas molecule have twice its value at 100°C?*

Ans. We know that

$$C^2 = 3\frac{RT}{Nm} = 3\frac{kT}{m}$$

Thus $$C_1^2 = \frac{3kT_1}{m} \quad \text{and} \quad C_2^2 = \frac{3kT_2}{m}$$

∴ $$\frac{C_1^2}{C_2^2} = \frac{T_1}{T_2}$$

Here $$C_2 = 2C_1, \quad T = 273 + 100 = 373 \text{ K}$$

$$T_2 = T_1 \times \frac{C_2^2}{C_1^2} = 373 \times 4 = 1492 \text{ K} = 1219°\text{C}$$

Q. 8. *A vessel contains two non-reactive gases : neon (monoatomic) and oxygen (diatomic). The ratio of their partial pressures is 3 : 2. Estimate the ratio of (i) number of molecules, (ii) mass density of neon and oxygen in the vessel. Atomic mass of Ne = 20.2 u, molecular mass of O_2 = 32.0 u.*

Ans. (*i*) It is given that $\frac{P_{Ne}}{P_{O_2}} = \frac{3}{2}$

As $$P = \frac{\mu RT}{V}$$

Hence for a given temperature and volume of vessel,

$$\frac{P_{Ne}}{P_{O_2}} = \frac{\mu_{Ne}}{\mu_{O_2}} = \frac{N_{Ne}}{N_{O_2}} = \frac{3}{2}$$

(*ii*) If m_{Ne} and m_{O_2} be the actual masses of the two gases present in the mixture and M_{Ne} and M_{O_2} be their molecular masses, then $\mu_{Ne} = \frac{m_{Ne}}{M_{Ne}}$ and $\mu_{O_2} = \frac{m_{O_2}}{M_{O_2}}$.

If ρ_{Ne} and ρ_{O_2} be the mass densities of two gases, then

$$\frac{\rho_{Ne}}{\rho_{O_2}} = \frac{m_{Ne}/V}{m_{O_2}/V} = \frac{m_{Ne}}{m_{O_2}} = \frac{\mu_{Ne}}{\mu_{O_2}} \times \frac{M_{Ne}}{M_{O_2}} = \frac{3}{2} \times \frac{20.2}{32.0} = \frac{0.947}{1}.$$

Q. 9. *Find the kinetic energy of 1g of nitrogen gas at 77°C. Given, R = 8.31 Jmol^{-1} K^{-1}.*

Ans. For, nitrogen, $M = 28$

$$T = 77 + 273 = 350 \text{ K}$$

$R = 8.31 \text{ Jmol}^{-1} \text{ K}^{-1}$

K.E. of 1g of nitrogen

$$= \frac{3}{2}\frac{RT}{M} = \frac{3 \times 8.31 \times 350}{2 \times 28} = 155.8 \text{ J}$$

Q. 10. *Calculate the temperature at which the root mean square velocity of nitrogen molecules will be equal to 8 km s^{-1}.*

Ans. Given, r.m.s. velocity, C = 8 km s^{-1} = 8 × 10^5 cm s^{-1}

Molar gas constant = R = 8.31 × 10^7 erg mol^{-1} K^{-1}

Molecular weight of nitrogen, M = 28

Let T be the required temperature

Then, $C = \sqrt{\frac{3RT}{M}}$ or $C^2 = \frac{3RT}{M}$ or $T = \frac{MC^2}{3R} = \frac{28 \times (8 \times 10^5)^2}{3 \times 8.31 \times 10^7} \text{K} = 71881 \text{ K}$

Q. 11. *A flask contains argon and chlorine in the ratio of 2 : 1 by mass. The temperature of the mixture is 27 °C. Obtain the ratio of (i) average kinetic energy per molecule, and (ii) root mean square speed v_{rms} of the molecules of the two gases. Atomic mass of argon = 39.9 u; Molecular mass of chlorine = 70.9 u.*

Ans. The important point to remember is that the average kinetic energy (per molecule) of any (ideal) gas (be it monatomic like argon, diatomic like chlorine or polyatomic) is always equal to (3/2) $k_B T$. It depends only on temperature, and is independent of the nature of the gas.

(*i*) Since argon and chlorine both have the same temperature in the flask, the ratio of average kinetic energy (per molecule) of the two gases is 1 : 1.

(*ii*) Now 1/2 $m\, v_{\text{rms}}^2$ = average kinetic energy per molecule = (3/2) $k_B T$ where m is the mass of a molecule of the gas. Therefore,

$$\frac{(V_{rms}^2)_{Ar}}{(V_{rms}^2)_{Cl}} = \frac{(m)_{Cl}}{(m)_{Ar}} = \frac{(M)_{Cl}}{(M)_{Ar}} = \frac{70.9}{39.9} = 1.77$$

where M denotes the molecular mass of the gas. (For argon, a molecule is just an atom of argon.) Taking square root of both sides.

$$\frac{(V_{rms})_{Ar}}{(V_{rms})_{Cl}} = 1.33$$

You should note that the composition of the mixture by mass is quite irrelevant to the above calculation. Any other proportion by mass of argon and chlorine would give the same answers to (*i*) and (*ii*), provided the temperature remains unaltered.

Q. 12. *Two perfect gases at absolute temperatures T_1 and T_2 are mixed. There is no loss of energy. Find the temperature of the mixture if the masses of the molecules are m_1 and m_2 and the number of the molecules in the gases are n_1 and n_2 respectively.*

Ans. According to kinetic theory, the average kinetic energy per molecule of a gas = $\frac{3}{2} k_B T$

Before mixing, the two gases, the average K.E. of all the molecules of two gases

$$= \frac{3}{2} k_B n_1 T_1 + \frac{3}{2} k_B n_2 T_2$$

After mixing, the average K.E. of both the gases = $\frac{3}{2}k_B\ (n_1 + n_2)\ T$

where T is the temperature of mixture. Since there is no loss of energy,

hence, $\frac{3}{2}k_B\,(n_1 + n_2)\,T \;=\; \frac{3}{2}k_B n_1 T_1 + \frac{3}{2}k_B n_2 T_2$ or $T = \frac{n_1 T_1 + n_2 T_2}{(n_1 + n_2)}$.

Q. 13. *At what temperature is the root mean square speed of oxygen atom equal to the r.m.s. speed of helium gas atom at –10°C? Atomic mass of oxygen = 32 and that of helium = 4.0.*

Ans. We know that r.m.s. speed is given by

$$v_{rms} = \left[\frac{3PV}{M}\right]^{1/2} = \left[\frac{3RT}{M}\right]^{1/2} \qquad (\because\ PV = RT)$$

If $(v_{rms})_1$ be the r.m.s. speed of oxygen and $(v_{rms})_2$ be the r.m.s. of helium gas at temperature T_1 and T_2 respectively.

$$(v_{rms})_1 = \left[\frac{3RT_1}{M_1}\right]^{1/2} \quad \text{and} \quad (v_{rms})_2 = \left[\frac{3RT_2}{M_2}\right]^{1/2} \quad \text{or} \quad \frac{(v_{rms})_1}{(v_{rms})_2} = \left[\frac{M_2 T_1}{M_1 T_2}\right]^{1/2}$$

Here $(v_{rms})_1 = (v_{rms})_2$, $M_1 = 32$, $M_2 = 4.0$;

$T_1 = ?$, $T_2 = -10 + 273 = 263$ K.

$$\therefore \qquad 1 = \left[\frac{4 \times T_1}{32 \times 263}\right]^{1/2}$$

Squaring both sides, $\frac{4T_1}{32 \times 263} = 1$

or $T_1 = \frac{32 \times 263}{4} = 2104$ K.

Q. 14. *Three moles of an ideal diatomic gas is taken at a temperature of 300 K. Its volume is doubled keeping its pressure constant. Find the change in internal energy of gas.*

Ans. Here $\mu = 3$, $T_1 = 300$ K and for an ideal monoatomic gas

$$C_v = \frac{5}{2}R.$$

As volume of gas is doubled ($V_2 = 2V_1$) at constant pressure, hence according to Charle's law

$$T_2 = \frac{T_1 V_2}{V_1} = \frac{300 \times 2V_1}{V_1} = 600 \text{ K}$$

$\therefore$ Gain in internal energy $U_2 - U_1 = \mu\ C_v\ (T_2 - T_1) = 3 \times \frac{5}{2}R \times (600 - 300)$

$= 2250\ R = 2250 \times 8.31$ J $= 1.87 \times 10^4$ J

Q. 15. *Calculate the total number of degrees of freedom possessed by the molecules in one cm^3 of H_2 gas at NTP.*

Ans. 22400 cm^3 of every gas contains 6.02×10^{23} molecules.

$\therefore$ Number of molecules in 1 cm^3 of H_2 gas

$$= \frac{6.02 \times 10^{23}}{22400} = 0.26875 \times 10^{20}$$

Number of degrees of freedom of a H_2 gas molecule = 5

$\therefore$ Total number of degrees of freedom of 0.26875×10^{20} molecules

$$= 0.26875 \times 10^{20} \times 5 = 1.34375 \times 10^{20}.$$

Q. 16. *An enclosure of volume four litres contains a mixture of 8 g of oxygen, 14 g of nitrogen and 22 g of carbon dioxide. If the temperature of the mixture is 27 °C, find the pressure of the mixture of gases. Given R = 8.315 J K^{-1} mol^{-1}.*

Ans. Temperature $T = 300$ K

Volume $V = 4$ litre $= 4 \times 10^{-3}$ m^3

The pressure exerted by a gas is given by

$$P = \frac{nRT}{V} = \frac{\text{mass}}{\text{molecular weight}} \times \frac{RT}{V}$$

Pressure exerted by oxygen $P_1 = \frac{8}{32}\frac{RT}{V} = \frac{1}{4}\frac{RT}{V}$

Pressure exerted by nitrogen $P_2 = \frac{14}{28}\frac{RT}{V} = \frac{1}{2}\frac{RT}{V}$

Pressure exerted by carbon dioxide $P_3 = \frac{22}{44}\frac{RT}{V} = \frac{1}{2}\frac{RT}{V}$

From Dalton's law of partial pressures, the total pressure exerted by the mixture is given by

$$P = P_1 + P_2 + P_3 = \frac{1}{4}\frac{RT}{V} + \frac{1}{2}\frac{RT}{V} + \frac{1}{2}\frac{RT}{V}$$

$$= \frac{5}{4}\frac{RT}{V} = \frac{5}{4} \times \frac{8.315 \times 300}{4 \times 10^{-3}} = 7.79 \times 10^5 \text{ N m}^{-2}$$

Q. 17. *The velocities of ten particles in ms^{-1} are 0, 2, 3, 4, 4, 4, 5, 5, 6, 9. Calculate (i) Average speed and (ii) r.m.s. speed.*

Ans. (*i*) **Average speed,**

$$v_{av} = \frac{0+2+3+4+4+4+5+5+6+9}{10} = \frac{42}{10} = 4.2 \text{ ms}^{-1}$$

(*ii*) **R.M.S. speed,**

$$v_{rms} = \left[\frac{(0)^2 + (2)^2 + (3)^2 + (4)^2 + (4)^2 + (4)^2 + (5)^2 + (5)^2 + (6)^2 + (9)^2}{10}\right]^{1/2}$$

$$= \left[\frac{228}{10}\right]^{1/2} = 4.77 \text{ ms}^{-1}$$

Q. 18. *Explain, why it is not possible to increase the temperature of a gas while keeping its volume and pressure constant?*

Ans. According to kinetic theory of gases,

$$P = \frac{1}{3}\rho C^2 = \frac{1}{3}\frac{M}{V}C^2 = \frac{1}{3}\frac{M}{V}KT \quad (\because C^2 = KT. \text{ when } K \text{ is constant})$$

$\therefore$ $T \propto PV$

Now as T is directly proportional to the product of P and V. If P and V are constant, then T is also constant.

Q. 19. *Calculate the root-mean square speed of oxygen molecules at 1092 K. Density of oxygen at STP = 1.424 kg m^{-3}.*

Ans. We first calculate the root-mean square speed of oxygen at *STP*.

$$P_0 = 0.76 \text{ m of Hg} = 1.01 \times 10^5 \text{ N m}^{-2}$$
$$\rho_0 = 1.424 \text{ kg m}^{-3}$$

The root-mean square speed at 0° C is given by

$$c_0 = \sqrt{\frac{3P_0}{\rho_0}} = \sqrt{\frac{3 \times 1.01 \times 10^5}{1.424}} \text{ m s}^{-1} = 4.61 \times 10^2 \text{ m s}^{-1}$$

Now c_{rms} is also given by

$$c_{rms} = \sqrt{\frac{3kT}{m}}$$

$$\therefore \qquad \frac{c_{rms}}{c_0} = \sqrt{\frac{T}{T_0}}$$

Here $T_0 = 273$ K and $T = 1092$ K

$$c_{rms} = c_0 \sqrt{\frac{T}{T_0}} = 4.61 \times 10^2 \times \sqrt{\frac{1092}{273}} = 9.22 \times 10^2 \text{ m s}^{-1}$$

Q. 20. *Why gases at high pressure and low temperature show large deviation from ideal gas behaviour ?*

Ans. When temperature is low and pressure is high, the intermolecular forces become appreciable. Moreover, the volume occupied by the molecules would not be negligibly small as compared to the volume of the gas. Thus, the behaviour of real gases at high pressure and low temperature deviates largely from the behaviour of ideal gases.

Q. 21. *Calculate the diameter of a molecule if $n = 2.79 \times 10^{25}$ molecules per m^3 and mean free path $= 2.2 \times 10^{-8}$.*

Ans. Here $n = 2.79 \times 10^{25}$ molecules m^{-3}; $\lambda = 2.2 \times 10^{-8}$ m.

Using the relation,

$$\lambda = \frac{1}{\sqrt{2}\pi n d^2}$$

We get

$$d^2 = \frac{1}{\sqrt{2}\pi n\lambda} = \frac{1}{\sqrt{2} \times 3.14 \times 2.79 \times 10^{25} \times 2.2 \times 10^{-8}}$$
$$= 0.03666 \times 10^{-17} \text{ m}^2 = 0.367 \times 10^{-18} \text{ m}^2$$

$$\Rightarrow \qquad d = \sqrt{0.367 \times 10^{-18}} \text{ m}^2 = 0.606 \times 10^{-9} \text{ m} = 0.606 \text{ nm.}$$

III. LONG ANSWER TYPE QUESTIONS

Q. 1. *Explain the pressure exerted by an ideal gas and also find the average kinetic energy per molecule of the gas.*

Ans. From kinetic theory of gases, the pressure P exerted by an ideal gas of density ρ and r.m.s. velocity of its gas molecules C is given by

$$P = \frac{1}{3}\rho C^2$$

Mass of unit volume of the gas = 1 × ρ = ρ

Mean kinetic energy of translation per unit volume of the gas is

$$E = \frac{1}{2}\rho C^2, \quad \therefore \quad \frac{P}{E} = \frac{(1/3)\rho C^2}{(1/2)\rho C^2} = \frac{2}{3} \quad \text{or} \quad P = \frac{2}{3}E.$$

"The pressure exerted by an ideal gas is numerically equal to two third of the mean kinetic energy of translation per unit volume of the gas."

Average Kinetic Energy per Molecule of the Gas

Consider one gram mole of an ideal gas occupying a volume V at temperature T. Let m be the mass of each molecule of the gas. Then

$$M = m \times N_A$$

where N_A is Avogadro's number.

If C is the r.m.s. velocity of the gas molecules, then pressure P exerted by ideal gas is

$$P = \frac{1}{3}\rho C^2 = \frac{1}{3}\frac{M}{V}C^2 \quad \text{or} \quad PV = \frac{1}{3}MC^2$$

From perfect gas equation, $PV = RT$, where R is a universal gas constant for one gram mole of the gas.

$$\therefore \qquad \frac{1}{3}MC^2 = RT \quad \text{or} \quad \frac{1}{3}MC^2 = \frac{3}{2}RT$$

$\therefore$ Average kinetic energy of translation of one mole of the gas $= \frac{1}{2}MC^2 = \frac{3}{2}RT$

$$\text{or} \qquad \frac{1}{2}m\,N_A C^2 = \frac{3}{2}RT \qquad (\because M = m\,N_A)$$

$$\text{or} \qquad \frac{1}{2}mC^2 = \frac{3}{2}\left(\frac{R}{N_A}\right)T = \frac{3}{2}k_B T \qquad \left(\because \frac{R}{N} = k_B\right)$$

where k_B is called Boltzmann constant.

$\therefore$ Average K.E. of translation per molecule of gas $= \frac{1}{2}mC^2 = \frac{3}{2}k_B T$.

Q. 2. *What is law of equipartition of energy? Given Avogadro number = 6.02×10^{23} and Boltzmann's constant = 1.38×10^{-23} joule/(molecule-K). Calculate (a) the average kinetic energy of translation of an oxygen molecule at 27 °C, (b) the total kinetic energy of an oxygen molecule at 27 °C, (c) the total kinetic energy in joules of a gram-molecule of oxygen at 27 °C.*

Ans. For 'law of equipartition of energy', see the NCERT Textbook.

Numerical:

Given, Avogadro number $N = 6.02 \times 10^{23}$

Boltzmann's constant $k = 1.38 \times 10^{-23}$ joule/molecule K

Kelvin temperature $T = 27 + 273 = 300$ K.

(*a*) An oxygen molecule has three degrees of freedom with respect of translation. Hence the average kinetic energy of translation of molecule

$$= 3 \times \frac{1}{2}k\,T = \frac{3}{2}k\,T$$

($\because$ Average kinetic energy of a gas molecule per degree of freedom is $\frac{1}{2}k\,T$.)

$$= 3 \times \frac{1}{2}(1.38 \times 10^{-23}) \times 300 = \mathbf{6.21 \times 10^{-21}} \textbf{ joule/molecule}$$

(*b*) The oxygen molecule has five degrees of freedom (three degrees of freedom with respect to translation and two degrees of freedom with respect to rotation as it is a diatomic molecule). Hence the total kinetic energy of the molecule

$$= 5 \times \frac{1}{2} k\,T = \frac{5}{2} k\,T = \frac{5}{2}\ (1.38 \times 10^{-23}) \times 300$$

$$= \mathbf{10.35 \times 10^{-21}\ joule/molecule}$$

(*c*) One gm molecule of oxygen contains N molecules. Hence the total kinetic energy of 1 gm molecule of the gas

$$= N \times \frac{5}{2} k\,T = (6.02 \times 10^{23}) \times 10.35 \times 10^{-21} = \mathbf{6231\ joule/mole.}$$

Q. 3. *What do you understant by mean speed, root mean square speed and most probable speed of gas molecules? On the basis of equipartion law of energy find expressions for two principal molar specific heat of a gas as well as for γ (the ratio of two specific heats) for a gas having P degrees of freedom per molecule.*

Ans. For definitions, see the NCERT Textbook.

We know that in a state of thermal equilibrium, in accordance with the equipartition law of energy, the average energy associated with each degree of freedom is $\frac{1}{2} k_B T$.

If a gas molecule has P degrees of freedom in all, then the average energy per molecule of gas

$$= P \times \frac{1}{2} k_B T = \frac{P}{2} k_B T$$

As one mole of a gas consists of N_A molecules, hence total internal energy of one mole of given gas will be given by

$$U = N_A \times \frac{P}{2} k_B T = \frac{P}{2} RT$$

where $R = N_A \,.\, k_B$ (universal gas constant).

$\therefore$ Molar specific heat of given gas under constant volume condition will be given by

$$C_v = \frac{dU}{dt} = \frac{d}{dT}\left[\frac{P}{2} RT\right] = \frac{P}{2} R \qquad ...(i)$$

For an ideal gas

$$C_p - C_v = R$$

$\therefore$ Molar specific heat of given gas under constant pressure constant

$$C_p = C_v + R = \frac{P}{2} R + R = (P + 2)\frac{R}{2} \qquad ...(ii)$$

$$\gamma = \frac{C_p}{C_v} = \frac{(P+2)\frac{R}{2}}{P\frac{R}{2}} = \frac{P+2}{P} = \left(1 + \frac{2}{P}\right) \qquad ...(iii)$$

Thus, it is clear that values of principal specific heats of a gas as well as their ratio depends on the number of degrees of freedom per molecule of the given gas.

Q. 4. *State the assumptions of the kinetic theory of gases. The density of carbon dioxide gas at 0 °C and at a pressure of 1.0×10^5 newton/metre^2 is 1.98 kg/m^3. Find the root mean square velocity of its molecules at 0 °C and 30 °C. Pressure is constant.*

Ans. The entire structure of the kinetic theory of gases is based on the following assumptions which were first stated by Classius.

1. A gas consists of a very large number of molecules (of the order of Avogadro's number, 10^{23}), which are perfect elastic spheres. They are identical in all respects for a given gas and are different for different gases.
2. The molecules of a gas are in a state of incessant random motion. They move in all directions with different speeds, (of the order of 500 m/s) and obey Newton's laws of motion.
3. The size of the gas molecules is very small as compared to the distance between them. If typical size of a molecule is 2 Å, average distance between the molecules is ≥ 20 Å. Hence volume occupied by the molecules is negligible in comparison to the volume of the gas.
4. The molecules do not exert any force of attraction or repulsion on each other, except during collision.
5. The collisions of the molecules with themselves and with the walls of the vessel are perfectly elastic. As such the momentum and the kinetic energy of the molecules are conserved during collisions, though their velocities change.
6. There is no concentration of the molecules at any point inside the container *i.e.*, molecular density is uniform throughout the gas.
7. A molecule moves along a straight line between two successive collisions and the average straight distance covered between two successive collisions is called the mean free path of the molecules.
8. The collisions are almost instantaneous, *i.e.*, the time of collision of two molecules is negligible as compared to time interval between two successive collisions.

Numerical:

We know that

$$P = \frac{1}{3}\rho v^2$$

$$\therefore \qquad v_{rms} = \sqrt{\left(v^2\right)} = \sqrt{\left(\frac{3P}{\rho}\right)}$$

Given that $P = 1.0 \times 10^5$ newton/metre^2 and

$$\rho = 1.98 \text{ kg/metre}^3$$

$$\therefore \qquad v_{rms} = \sqrt{\left[\left\{\frac{3\times\left(1.0\times10^5\right)}{1.98}\right\}\right]} = \textbf{389 metre/sec.}$$

From kinetic theory of gases, the root mean square speed is directly proportional to the square root of absolute temperature

$$v_{rms} \propto \sqrt{T}$$

$$\therefore \qquad \frac{(v_{rms})_{30}}{(v_{rms})_0} = \sqrt{\left[\left(\frac{273+30}{273+0}\right)\right]} = \sqrt{\left[\left(\frac{303}{273}\right)\right]} = 1.053$$

or $\qquad (v_{rms})_{30} = (v_{rms})_0 \times 1.053 = 389 \times 1.053 = \textbf{410 metre/sec.}$

Q. 5. *What is kinetic interpretation of temperature? Two perfect gases at absolute temperatures T_1 and T_2 are mixed. There is no loss of energy. Find the temperature of the mixture if masses of molecules are m_1 and m_2 and the number of molecules in the gases are μ_1 and μ_2 respectively.*

Ans. For kinetic interpretation of temperature, see text.

Numerical:

We know, K.E. of one molecule of a perfect gas at temperature T is given by

$$E = \frac{3}{2}kT \qquad ...(1)$$

$\therefore$ K.E. of μ_1 molecules of a perfect gas at temperature T_1, $E_1 = \left(\frac{3}{2}k\,T_1\right)\mu_1$

K.E. of μ_2 molecules of a perfect gas at temperature T_2, $E_2 = \left(\frac{3}{2}k\,T_2\right)\mu_2$

When both the gases are mixed, then the total K.E. of the mixture is

$$E = \frac{3}{2}k\ (\mu_1 T_1 + \mu_2 T_2) \qquad ...(2)$$

After mixing, the temperature of the mixture is T, therefore K.E. of the mixture is given by

$$E' = \frac{3}{2}kT\,(\mu_1) + \frac{3}{2}kT\,(\mu_2) = \frac{3}{2}kT\ (\mu_1 + \mu_2) \qquad ...(3)$$

Since there is no loss of energy

$$\therefore \quad E' = E \quad \text{or} \quad \frac{3}{2}kT\ (\mu_1 + \mu_2) = \frac{3}{2}k\ (\mu_1 T_1 + \mu_2 T_2)$$

$$\therefore \quad \boxed{T = \frac{\mu_1 T_1 + \mu_2 T_2}{\mu_1 + \mu_2}}$$

IV. MULTIPLE CHOICE QUESTIONS

1. According to kinetic theory of gases the r.m.s. velocity of the gas molecules is directly proportional to

(*a*) $\sqrt{T}$ (*b*) T^4 (*c*) T (*d*) T^2

2. The speed of sound in a gas is v. The rms speed of molecules of this gas is C. If $\gamma = \frac{C_p}{C_v}$, then the ratio of v and C is

(*a*) $\frac{3}{\gamma}$ (*b*) $0.33\ \gamma$ (*c*) $\sqrt{\frac{3}{\gamma}}$ (*d*) $\sqrt{\frac{\gamma}{3}}$

3. A sealed container with negligible thermal coefficient of expansion contains helium (a monoatomic gas). When it is heated from 300 to 600 K, the average kinetic energy of the helium atom is

(*a*) halved (*b*) left unchanged (*c*) doubled (*d*) becomes $\sqrt{2}$ times

4. During an adiabatic process, the pressure of a gas is proportional to the cube of its absolute temperature. The value of C_p/C_v for that gas is

(*a*) 3/5 (*b*) 4/3 (*c*) 5/3 (*d*) 3/2

5. Two vessels having equal volume contain molecular hydrogen at one atmosphere and helium at two atmosphere pressure respectively. If both samples are at the same temperature the mean velocity of hydrogen molecule is

(*a*) equal to that of helium (*b*) twice that of helium

(*c*) half that of helium (*d*) $\sqrt{2}$ times that of helium

6. The energy density $\frac{u}{V}$ of an ideal gas is related to its pressure P as

(*a*) $\frac{u}{V} = 3P$ (*b*) $\frac{u}{V} = \frac{3}{2}P$ (*c*) $\frac{u}{V} = \frac{1}{3}P$ (*d*) $\frac{u}{V} = \frac{2}{3}P$

7. Oxygen and hydrogen gases are at the same temperature T. The kinetic energy of an oxygen molecule will be equal to

(*a*) 16 times the kinetic energy of a hydrogen molecule

(*b*) 5 times the kinetic energy of a hydrogen molecule

(*c*) the kinetic energy of a hydrogen molecule

(*d*) one-fourth the kinetic energy of a hydrogen molecule

Ans. **1.**—(*c*) **2.**—(*d*) **3.**—(*c*) **4.**—(*b*) **5.**—(*d*)
6.—(*b*) **7.**—(*c*)

V. QUESTIONS ON HIGH ORDER THINKING SKILLS (HOTS)

Q. 1. *An electric bulb of volume 250 cm³ was sealed off during manufacture at a pressure of 10^{-3} mm of Hg at 27°C. Find the number of air molecules in the bulb.*

Ans. Let N be the number of air molecules in the bulb. It is given that

$$P = 10^{-3} \text{ mm of Hg} = 10^{-4} \text{ cm of Hg}$$
$$V = 250 \text{ cm}^3$$
$$T = 273 + 27 = 300 \text{ K}$$

Now $PV = NkT$ ...(*i*)

We know that at STP, one mole of a gas occupies a volume $V_0 = 22400$ cm³ and contains $N_0 = 6.02 \times 10^{23}$ molecules (N_0 is the Avogadro's number) and the pressure $P_0 = 76$ cm of Hg and $T_0 = 273$ K. Also

$$P_0 V_0 = N_0 kT_0 \quad ...(ii)$$

Dividing eqn. (*i*) and (*ii*), we get

$$N = N_0 \times \frac{T_0}{T} \times \frac{P}{P_0} \times \frac{V}{V_0} = \left(6.02 \times 10^{23}\right) \times \left(\frac{273}{300}\right) \times \left(\frac{10^{-4}}{76}\right) \times \left(\frac{250}{22400}\right)$$
$$= 8.045 \times 10^{15} \text{ molecules.}$$

Q. 2. *Calculate the temperature at which r.m.s. velocity of a gas molecule is same as that of a molecule of another gas at 47°C. Molecular weight of first and second gases are 64 and 32 respectively.*

Ans. As $P = \frac{1}{3}\frac{M_m}{V}v_{rms}^2$ or $PV = \frac{1}{3}M_m v_{rms}^2$ or $RT = \frac{1}{3}M_m v_{rms}^2$

or $$\frac{M_m v_{rms}^2}{T} = 3R = \text{Constant}$$

If $(M_m)_1$ and $(M_m)_2$ be the molecular weights of the two gases, $v_{rms} = C_1^2$ and C_2^2, the mean square velocities of two gases and T_1 and T_2 are the absolute temperatures of the two gases, then

$$\frac{(M_m)_1\, C_1^2}{T_1} = \frac{(M_m)_2\, C_2^2}{T_2}$$

According to the given problem

$$\sqrt{C_1^2} = \sqrt{C_2^2} \quad \text{or} \quad C_1^2 = C_2^2$$

$$\therefore \quad \frac{(M_m)_1}{T_1} = \frac{(M_m)_2}{T_2} \quad \text{or} \quad \frac{64}{T_1} = \frac{32}{320} \qquad [\because\ T_2 = 47°\text{C} = 320\ \text{K}]$$

$$T_1 = 640°\text{K} = 267°\text{C}.$$

Q. 3. *A gas is filled in a cylinder fitted with a piston at a definite temperature and pressure. Why the pressure of the gas decreases when the piston is pulled out?*

Ans. When the piston is pulled out, the volume of the gas increases. Now, lesser number of molecules collide against the wall per unit time. Moreover, the collisions take place over a large area. Due to both these reasons, the pressure decreases.

Q. 4. *Two moles of gas A at 27°C are mixed with 3 moles of gas B at 37°C. If both are monoatomic ideal gases, what will be the temperature of the mixture?*

Ans. As there is no loss of energy in the process, therefore, sum of KE of gases A and B = KE of mixture,

$$\mu_1\left(\frac{3}{2}RT_1\right)+\mu_2\left(\frac{3}{2}RT_2\right) = (\mu_1+\mu_2)\frac{3}{2}RT$$

where T is temperature of the mixture.

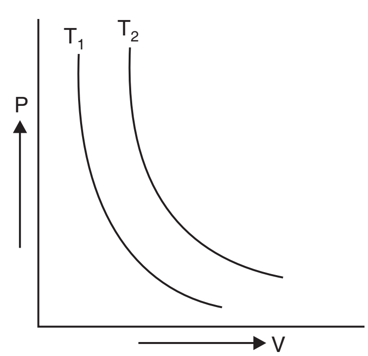
$$\therefore \quad T = \frac{\mu_1 T_1 + \mu_2 T_2}{\mu_1+\mu_2} = \frac{2(27+273)+3(37+273)}{2+3}$$

$$= \frac{600+930}{5} = \frac{1530}{5} = 306\ \text{K} = 306 - 273 = 33°\text{C}.$$

Q. 5. *The ratio of specific heat capacity at constant pressure to the specific heat capacity at constant volume of a diatomic gas decreases with increase in temperature. Explain, why.*

Ans. We know $\gamma = \frac{C_p}{C_v} = 1+\frac{2}{f}$, where f is the degree of freedom of a diatomic gas. The degree of freedom of a diatomic gas increases with the increase in temperature, so γ decreases with increase in temperature.

Q. 6. *Isothermal curves for a given mass of gas are shown at two different temperatures T_1 and T_2. State whether $T_1 > T_2$ or $T_2 > T_1$. Justify your answer.*



Ans. From ideal gas equation

$$PV = \mu RT$$

$$T = \frac{PV}{\mu R}$$

As mass of gas is constant, μ is constant, R is already a constant.

$$\therefore \quad T \propto PV$$

Since PV is greater for the curve at T_2 than for the curve at T_1, therefore, $T_2 > T_1$.

Q. 7. *The volume of air bubble increases 15 times when it rises from bottom to the top of a lake. Calculate the depth of the lake if density of lake water is 1.02 × 10³ kg/m³ and atmospheric pressure is 75 cm of mercury.*

Ans. According to Boyle's law,

$$P_1V_1 = P_2V_2 \quad ...(1)$$

Here P_1 = 75 cm of Hg = 0.75 m of Hg = 0.75 × 13.6 × 10³ × 9.8

= 99.96 × 10³ Nm⁻²

Let volume of bubble at depth $h = x$

i.e., $V_2 = x \quad \therefore \quad V_1 = 16x$

P_2 = 75 cm of Hg + $h\rho_{water}\, g$ = 99.96 × 10³ + h × 10³ × 9.8

Using eqn. (1), we get

$$99.96 \times 10^3 \times 16x = (99.96 \times 10^3 + h \times 10^3 \times 9.8)\, x$$

or $9.8\, h = 99.96 \times 16 - 99.96 = 99.96 \times 15$

$$\therefore \quad h = \frac{99.96 \times 15}{9.8} = 153 \text{ m.}$$

Q. 8. *The root-m]ean square (rms) speed of oxygen molecule (O_2) at a certain temperature T is v. If temperature is doubled and oxygen gas dissociates into atomic oxygen, what is the speed of atomic oxygen?*

Ans. Let C and C' be the root mean square velocity of oxygen gas molecule and atomic oxygen at temperature T and T' respectively. Therefore

$$C = \sqrt{\frac{3RT}{M}} = v \quad \text{and} \quad C' = \sqrt{\frac{3R\,(2T)}{M/2}} = 2\sqrt{\frac{3RT}{M}} = 2v$$

Hence the velocity of atomic oxygen will become double.

Q. 9. *An ideal gas has a specific heat at constant pressure C_p = 5 R/2. The gas is kept in a closed vessel of volume 0.0083 m³ at a temperature of 300 K and a pressure of 1.6 × 10⁶ N m⁻². An amount of 2.49 × 10⁴ J of heat energy is supplied to the gas. Calculate the final temperature and pressure of the gas.*

Ans. $P = 1.6 \times 10^6$ N m⁻², $V = 0.0083$ m³, $T = 300$ K

We know that $P\,V = n\,R\,T$, where $R = 8.3$ J K⁻¹ mol⁻¹

$$\text{Therefore} \quad n = \frac{P\,V}{R\,T} = \frac{1.6 \times 10^6 \times 0.0083}{8.3 \times 300} = \frac{16}{3} \text{mole}$$

$$\text{Now} \quad C_p - C_V = R, \quad \text{therefore} \quad C_V = C_p - R = \frac{5\,R}{2} - R = \frac{3\,R}{2}$$

When heat energy Q is supplied to the gas, the increase ΔT in its temperature is obtained from the relation

$$Q = n\,C_V\,\Delta T \quad \text{or} \quad \Delta T = \frac{Q}{n\,C_V} = \frac{2.49 \times 10^4}{\frac{16}{3} \times \frac{3}{2} \times 8.3} = 375 \text{ K}$$

$\therefore$ Final temperature T' = 300 + 375 = 675 K. Since the gas is kept in a closed vessel, its volume remains constant. Hence the final pressure P' is obtained from the relation.

$$\frac{P'}{T'} = \frac{P}{T} \quad \text{or} \quad P' = P \times \frac{T'}{T} = \frac{1.6 \times 10^6 \times 675}{300} = 3.6 \times 10^6 \text{ N m}^{-2}$$

Q. 10. *Two ideal monoatomic gases A and B at 27°C and 37°C are mixed. The number of moles in gas A are 2 and number of moles in gas B are 3. What will be the temperature of the mixture?*

Ans. Sum of K.E. of gases A and B = K.E. of the mixture

i.e., $$\mu_1 \left(\frac{3}{2} R T_1\right) + \mu_2 \left(\frac{3}{2} R T_2\right) = (\mu_1 + \mu_2)\left(\frac{3}{2} RT\right)$$

$$\therefore \quad T = \frac{\mu_1 T_1 + \mu_2 T_2}{\mu_1 + \mu_2} = \frac{2 \times 300 + 3 \times 310}{2 + 3} = 306 \text{ K}$$

$\therefore$ Temperature of mixture = 33°C.

VI. VALUE-BASED QUESTIONS

Q. 1. *One day, in the morning, Ritu got up at 8 a.m. and saw that rays of sun light coming through the window. The dust particles were seen randomly moving only in a particular region where the sun ray were falling and she could see the particle in the rest of the room. She kept it in her mind and when she reached her school, she first asked her physics teacher the reason of this motion. The teacher explained that the dust molecules or the molecules of the gas always moving in random motion. The particles may also be collide with each other when they come together. Ritu was happy to hear the answer.*

(i) What values, Ritu exhibits?

(ii) What is the relation between C_p and C_v?

(iii) Calculate the value of γ (ratio between C_p and C_v) for diatomic gas.

Ans. (*i*) Awareness, creative, intelligent and sharp mind.

(*ii*) $C_p - C_v = R$.

(*iii*) For diatomic gases, degree of freedom $f = 5$

$$\therefore \quad C_v = \frac{5}{2}R = \frac{5}{2} \times 1.98 = 4.95 \text{ cal/mole/K}$$

$$\therefore \quad C_p = \left(1 + \frac{5}{2}\right) \times 1.98 = 6.93 \text{ cal/mole/K}$$

$$\therefore \quad \gamma = \frac{C_p}{C_v} = \frac{6.93}{4.95} = 1.4$$

Q. 2. *A metre long narrow bore (closed at one end) held horizontally contains a 76 cm long mercury thread, which traps a 15 cm column of air. What happens, if the tube is held vertically with the open end at the bottom?*

(i) Will the mercury thread decrease in length?

(ii) How would you explain it to a student who has very less values?

Ans. (*i*) Yes, when the tube is held vertically, the mercury thread will decrease in length.

(*ii*) Since in a metre long tube, the mercury thread of length 76 cm traps 15 cm of air, a length of 9 cm of tube will be left at the open end. When the tube is held horizontally, the pressure of the trapped air will be equal to the atmospheric pressure 'P' i.e. 76 cm of mercury.

When the tube is held vertically, the length of the air column becomes 15 + 9 = 24 cm. Let h cm of mercury thread flows out of the tube.

$\therefore$ From Boyle's Law

$$P_1V_1 = P_2V_2$$

$$76 \times 15 = h \times (24 + h)$$

$$1140 = 24h + h^2$$

$$\Rightarrow \quad h^2 + 24h - 1140 = 0 \quad \Rightarrow \quad h = 23.8 \text{ or } -47.8 \text{ cm (Rejected)}$$

Hence the mercury thread will decrease in length by 23.8 cm.

Q. 3. *During the lecture of physics, Praveen's teacher Mr. Ashok put a question to the whole class. "Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic) and the third contains uranium hexafluoride (Polyatomic).*

(i) Do the vessels contain equal number of respective molecules?

(ii) Is the root mean square speed of molecules the same in three case?

(iii) According to you what values Praveen showed here?

Ans. The teacher allowed all the student to give the correct answers within 15 minutes. Praveen was able to give the answer correctly as under.

(*i*) Yes, all the three vessels contain equal number of respective molecules of given gases. It is in accordance with Avogadro's Law, which states that under similar condition of temperature and pressure, equal volumes of gases contain equal number of molecules.

(*ii*) Root means square speed $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M_0}} = \sqrt{\dfrac{3K_BT}{m}}$

where M_0 = Molar mass of gas and m = Mass of one molecule

As the three gases have different molar masses, hence V_{rms} will be different.

(*iii*) He is intelligent, courageous, sharp minded, Leadership and creative.

TEST YOUR SKILLS

1. It is said that molecules of gases keep on moving in random directions. Yet it is seen that clouds maintain their shape for considerable time. Explain reasons for this.

2. Is there some difference in densities of air near the floor and near the roof, in a room? Give reasons for your answer.

3. Why is it easier to understand behaviour of gases as compared to behaviour of solids or liquids?

4. What do you understand by ideal gas? Is there a truly ideal gas available in nature?

5. A flask contains argon and chlorine in the ratio of 3 : 2 by mass. Calculate following:

(*i*) Average kinetic energy per molecule

(*ii*) Root mean square speed of the molecules of two gases.

(Atomic mass of Ar = 39.9 u and Molar mass of Cl = 70.9 u).

6. When a gas in a cylinder is compressed by pushing in a piston, what happens to the temperature of gas? Give reasons for your answer.

7. What happens to the energy of air, when it comes out from a tightly filled pneumatic tube?

❑❑❑