13 Nuclei

Facts that Matter

- In every atom, the positive charge and mass are densely concentrated at the centre of the atom forming its nucleus. The overall dimensions of a nucleus are much smaller than those of an atom. Experiments on scattering of α-particles demonstrated that the radius of a nucleus was smaller than the radius of an atom by a factor of about 10^4. This means the volume of a nucleus is about 10^{-12} times the volume of the atom. In other words, an atom is almost empty. Thus nucleus is the central core of the atom which consists of most mass and positive charge of the atom. The structure of atom and nucleus becomes more clear after the discovery of neutron.

• Discovery of Neutrons

Neutrons were discovered by Chadwick in 1932. When beryllium nuclei are bombarded by α-particles, highly penetrating radiations are emitted, which consists of neutral particles, each having mass nearly that of a proton. These particles were called neutrons.

$$^{9}_{4}\text{Be} + ^{4}_{2}\text{He} \longrightarrow ^{1}_{0}n + ^{12}_{6}\text{C}$$

• Composition of Nucleus

Neutrons and protons are the constituents of a nucleus. Almost the whole mass of the atom is in its nucleus.

Atomic number: The number of protons in the nucleus is called the atomic number. It is denoted by Z.

Mass number: The total number of protons and neutrons present in a nucleus is called the mass number of the element. It is denoted by A.

Number of protons in an atom = Z

Number of electrons in an atom = Z

Number of neutrons in an atom ⇒ $N = A - \mathbf{Z}$

where A is number of nucleons (protons + neutrons).

Nuclear mass: It was observed in Rutherford's α-particle scattering experiment that mass of an atom is concentrated within a very small positively charged region at the centre called nucleus. The total mass of neucleons in the nucleons is called as nuclear mass.

Nuclear mass = mass of protons + mass of neutrons.

Size and shape of the nucleus: The nucleus is nearly spherical. Hence its size is usually given in terms of radius. The radius of nucleus was measured by Rutherford and it was found to have following relation

$$R = R_0 A^{1/3}$$

where $R_0 = 1.1$ fm $= 1.1 \times 10^{-15}$ m and A is mass number of particular element.

Nuclear charge: Nucleus is made of protons and neutrons. Protons have positive charge of magnitude equal to that of electron and neutrons are uncharged. So nuclear charge = Z_e.

Nuclear density: The ratio of the mass of the nucleus to its volume is called nuclear density. As the masses of proton and neutron are roughly equal, the mass of a nucleus is roughly proportional to A.

As volume of a nucleus is]

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A$$

Now, nuclear density $(P) = \dfrac{mA}{\frac{4}{3}\pi R_0^3 A} = \dfrac{3m}{4\pi R_0^3}$

density within a nucleus is independent of A.

- **Isotopes, Isobars, Isotones and Isomers**

Isotopes: Atoms having different mass number (A) but having same atomic number (Z), *e.g.,* $^{235}_{92}U$, $^{238}_{92}U$.

Isobars: The atoms having the same mass number but different atomic number are called isobars. For example: $^{54}_{24}Cr$, $^{54}_{26}Fe$.

Isotones: Nuclei containing same number of neutrons are called isotones. *e.g.,* $^{37}_{17}Cl$ and $^{39}_{17}K$.

Isomers: These are the nuclei with same atomic number and same mass number but in different energy states.

- **Electron Volt**

It is defined as the energy acquired by an electron when it is accelerated through a potential difference of 1 volt and is denoted by eV.

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$
$$1 \text{ MeV} = 10^6 \text{ eV} = 1.602 \times 10^{-13} \text{ J.}$$

- **Atomic Mass Unit**

It is $\frac{1}{12}$th of the actual mass of a carbon atom of isotope $^{12}_{6}C$. It is denoted by 'amu' or just by 'u'.

$$1\ u = 1.66 \times 10^{-27} \text{ kg}$$

1 a.m.u. represents the average mass of a nucleon.

$$1\ u = 931.25 \text{ MeV.}$$

- **Nuclear Forces**

Nuclear force is the strongest force in nature. It acts between the nucleons. The strong forces of attraction, which firmly hold the nucleons in the nucleus, are known as nuclear forces.

The stability of nucleus is due to the presence of these forces. Nuclear forces have following important characteristics:

(*i*) They are attractive, *i.e.,* nucleons exert attractive force on each other hence they are also called cohesive forces.

(*ii*) They are extremely strong. These forces are strongest possible force in nature.

(*iii*) They are charge independent.

(*iv*) They are short-range forces, *i.e.,* they act only over a short range of distances.

(*v*) They are spin dependent, *i.e.,* nuclear forces acting between two nucleons depend on the mutual orientation of the spins of the nucleons.

(*vi*) They are saturated, *i.e.,* their magnitude does not increase with the increase in the number of nucleons, beyond a certain number.

• Mass-Energy Relation

Einstein proved that it is necessary to treat mass as another form of energy. He gave the mass-energy equivalence relation.

$$E = mc^2$$

where m is the mass and c is the velocity of light in vacuum.

$$c \approx 3 \times 10^8 \text{ m/s.}$$

• Mass Defect and Binding Energy

Mass defect: Difference between the sum of the masses of neutrons and protons forming a nucleus and actual mass of the nucleus is called mass defect.

Binding energy: Certain mass disappears in the formation of a nucleus in the form of mass defect. This mass loss reappears in the form of energy (Einstein's theory of mass energy equivalence) called binding energy.

Thus, binding energy is the energy which should be supplied to the nucleus in order to break it up into its constituent particles.

$$\text{B.E.} = [Zm_p + (A - Z)\, m_n - m_N]\, c^2$$

or,

$$\text{B.E.} = [Zm_H + (A - Z)\, m_n - m\,({}_ZX^A)]\, c^2$$

where, $m\,({}_ZX^A)$ is the mass of atom, m_H is mass of the hydrogen atom, m_N is mass of the nucleus, m_p is mass of proton, m_n is mass of neutron.

$$\text{B.E. per nucleon} = \frac{\text{Total B.E.}}{A}.$$

• Binding Energy Curve

A graph between the binding energy per nucleon and the mass number of nuclei is called as the binding energy curve.

The following points may be noted from the binding energy curve:

(*a*) The binding energy per nucleon is maximum ($\approx$ 8.8 MeV) for the nucleus having mass number 56. So, this nucleus is most stable, *i.e.,* iron is the most stable element of periodic table.

(*b*) The light nuclei with $A < 20$ are least stable.

(*c*) The curve has certain peaks indicating that certain nuclei like ${}^4_2\text{He}$, ${}^{12}_6\text{C}$ and ${}^{16}_8\text{O}$ are much more stable than the nuclei in their vicinity.

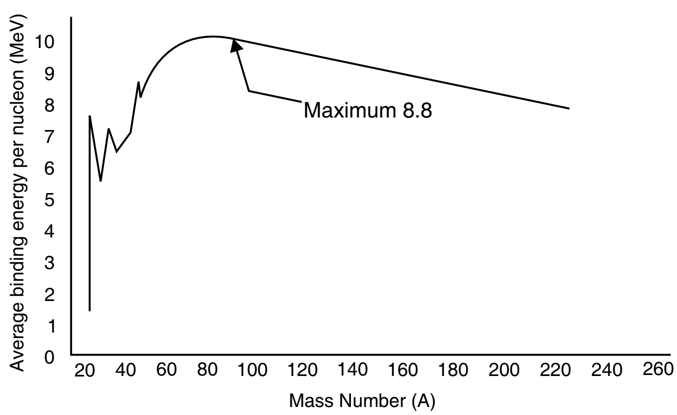

Fig. 13.1

(*d*) For atomic number Z > 56, the curve takes a downside turn indicating lesser stability of these nuclei.

(*e*) Nuclei of intermediate mass are most stable. This means maximum energy is needed to break them into their nucleons.

(*f*) The binding energy per nucleon has a low value for both very light and very heavy nuclei. Hence, if we break a very heavy nucleus (like uranium) into comparatively lighter nuclei then the binding energy per nucleon will increase. Hence, a large quantity of energy will be liberated in this process. This phenomenon is called nuclear fission.

(*g*) Similarly, if we combine two or more very light nuclei (*e.g.,* nucleus of heavy hydrogen ${}_1^2H$) into a relatively heavier nucleus (*e.g.,* ${}_2^4He$), then also the binding energy per nucleon will increase, *i.e.,* again energy will be liberated. This phenomenon is called nuclear fusion.

• Radioactivity

The phenomenon of radioactivity was a chance discovery by Henry Bacquerel who observed that photographic plates wrapped in black paper were affected by radiations from some uranium salts placed nearby.

The phenomenon involves the spontaneous disintegration of the nuclei of heavy elements by the emission of highly penetrating radiations.

Rutherford and Soddy first suggested that radioactivity is due to the spontaneous transformation of one element into another. Elements that are unstable emit radiations to transform themselves into less unstable elements. We call an element as having natural radioactivity, if it is naturally unstable. We also have artificial radioactivity or induced radioactivity in which the instability is induced by an external agency.

• Radioactive Radiations

The radiations emitted by a radioactive element are found to be of three kinds.

(***i***) **Alpha rays:** They consist of alpha particles. Alpha particle is nothing but a helium nucleus (${}_2^4He$) having 2 protons and 2 neutrons. It has a positive charge equal to the charge of 2 protons. Alpha rays are the most ionising but the least penetrative of the three radiations. They have velocities of the order of $10^6 - 10^7$ ms^{-1} and energies from 4 MeV to 9 MeV. They are also characterised by a well-defined range in air.

(***ii***) **Beta rays:** They are negative electrons (or positrons) whose energy as well as ionising power are much less than those of α-rays. Their velocity is much higher than that of α-rays. They are 100 times more penetrating than α-rays. However, they do not have a well-defined range in air.

(***iii***) **Gamma rays:** They are electromagnetic waves of very short wavelengths. The originate in the nucleus, have a very low ionising power but a very high penetrating power.

• Laws of Radioactive Decay

(*i*) Disintegration of radioactive nucleus is random and spontaneous.

(*ii*) The rate of decay of nuclei at any instant is proportional to the number of undecayed radioactive nuclei present at that instant.

$$\frac{dN}{dt} = -\lambda N$$

where λ is called **decay constant** or disintegration constant.

Also, $$N = N_0 e^{-\lambda t}$$

where N_0 is the number of radioactive nuclei present originally.

(*iii*) During radioactive disintegration, two particles are never emitted simultaneously.

(*iv*) An atom does not emit more than one α-particle or more than one β-particle at a time.

• Half-Life and Average Life

The half-life (T) of a radioactive element is the time in which the number of its nuclei is reduced to one-half its initial value.

It can be given as

$$T = \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda}$$

The average life T_{av} of a radioactive element is the reciprocal of the disintegration constant.

$$T_{av} = \frac{1}{\lambda}$$

The activity $\left(= \left| \frac{dN}{dt} \right| \right)$ of a radioactive element at any instant is, of course, just λN:

$$\left| \frac{dN}{dt} \right| = \lambda N.$$

• Units of Radioactivity

The following units have been used for 'measuring' the radioactivity of a radioactive element.

(*i*) **The curie:** The curie is the activity of a radioactive element that is disintegrating at the rate of 3.7×10^{10} disintegration per second. *i.e.,*

1 curie = 3.7×10^{10} disintegration per second.

(*ii*) **The rutherford:** The rutherford is the activity of a radioactive element that is disintegrating at the rate of 1 million disintegrations per second. *i.e.,*

1 rutherford = 10^6 disintegrations per second.

(*iii*) **The becquerel:** This is the unit of radioactivity in the *SI* units.

1 becquerel = 1 Bq = 1 disintegration per second.

- Different radionuclids difference greatly in their rate of decay. A common way to characterise this feature is through the notion of half-life. As the Eq. $N = N_0 e^{-\lambda t}$, for half life period $t = T_{1/2}$, $N = N_0/2$ or $T_{1/2} = \frac{\log_e{}^2}{\lambda}$. Also, $T_{1/2} = \frac{0.693}{\lambda}$ clearly suggests that if N_0 reduces to half of its value in time $T_{1/2}$, the activity (R_0) will also reduce to half its value in the same time according to relation $R = R_0 e^{-\lambda t}$. The variation of N with t is shown in Fig. 13.2.

  It can also shown that

  $$N = N_0 \left(\frac{1}{2}\right)^n$$

  where n is number of half-lines.

  Thus $$\left(\frac{N}{N_0}\right) = \left(\frac{1}{2}\right)^n$$

  The total time in decaying the N_0 into N can be also be calculated by the product of number of half-lives and half life period, *i.e.,* $t = nT_{1/2}$.

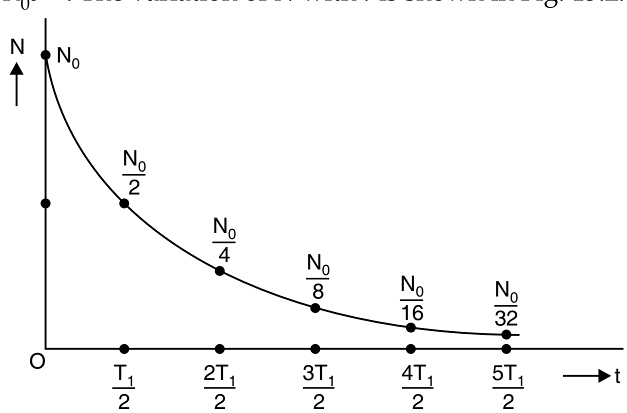

Fig. 13.2

- Another related measure is the

average or mean life τ. This again can be obtained from Eq. $N = N_0 e^{-\lambda t}$. The number of nuclei which decay in the time interval t to $t + \Delta t$ is $R.\Delta t$ (= $lN_0 e^{-\lambda t} \Delta t$). Each of them has lived for time t. Thus the total life of all these nuelci would be $t \lambda N_0 e^{-\lambda t}$. bt. It is clear that some nuclic may live for a short time while others may live longer. Therefore to obtain the mean life, this expression can be integrated over all times from 0 to τ, and divided by the total number N_0 of nuclei at $t = 0$. Thus,

$$t = \frac{\lambda N_0 \int_0^\tau t\, e^{-\lambda t} dt}{N_0}$$

$$= \lambda \int_0^\tau t\, e^{-\lambda t} dt$$

one can show by performing this integral that $\tau = \frac{1}{\lambda}$

$\Rightarrow$ $$T_{1/2} = \frac{0.693}{\lambda}$$

or $$\boxed{T_{1/2} = 0.693\ \tau.}$$

- **Successive Disintegration and Radioactive Equilibrium**

Consrider radioactive decay of ${}^{238}_{92}\text{U}$ to ${}^{234}_{90}\text{Th}$ which further decay to ${}^{234}_{91}\text{Pa}$ such decays are called successive disintegration. Here ${}^{239}_{92}\text{U}$ in called parent nucleus and ${}^{234}_{90}\text{Th}$ daughter nuclei of ${}^{238}_{92}\text{U}$. Any two adjacent nucliei may be considered as parent and daughter nuclei.

Suppose N_1 and N_2 are the numbers of two nuclei at time t, which undergo decay, with decay constants λ_1 and λ_2 respectively, according to the following sequence.

$$A \xrightarrow{\lambda_1} B \xrightarrow{\lambda_2} C$$

Then, rate of disintegration of A,

$$\frac{dN_1}{dt} = -\lambda_1 N_1 \text{ which is equal to the rate of formation of } B$$

Rate disintegration of B,

$$\frac{dN_2}{dt} = -\lambda_2 N_2 \quad ....(i)$$

Net rate of formation of B

$$= \frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \quad ...(ii)$$

The rate increase of the (stable) nuclei C is equal to the rate of decay of nuclei B,

Thus, $$\frac{dN_3}{dt} = -\lambda_2 N_2 \quad ...(iii)$$

Number of nuclei of A at time t,

$$N_1 = N_0 e^{-\lambda t} \quad ...(iv)$$

where N_0 is the initial number of nuclei of A at $t = 0$

Substitutiing N_1 in Eq. (*ii*),

we get $$\frac{dN_2}{dt} + N_2\lambda_2 = \lambda_1 N_0 e^{-\lambda t}$$

or $$\left(\frac{dN_2}{dt} + N_2\lambda_2\right)e^{\lambda_2 t} = (\lambda_1 N_0 e^{-\lambda t})\ e^{\lambda_2 t}$$

$$= \lambda_1 e^{-(\lambda_1 - \lambda_2)t}$$

or $$\frac{dN_2}{dt} e^{\lambda_2 t} + N_2\lambda_2\ e^{\lambda_2 t} = \lambda_1 N_0\ e^{-(\lambda_1 - \lambda_2)t}$$

or $$\frac{d}{dt}(N_2 e^{\lambda_2 t}) = \lambda_1 N_0\ e^{-(\lambda_1 - \lambda_2)t}$$

on integrating, we get

$$N_2 e^{-\lambda_2 t} = \frac{\lambda_1 N_0}{(\lambda_2 - \lambda_1)}\ e^{-(\lambda_1 - \lambda_2)t} + C$$

where C is the constant of integration

at $t = 0$, $N_2 = 0$, hence

$$c = \frac{\lambda_1 N_0}{\lambda_1 - \lambda_2}$$

or $$N_2 = \frac{\lambda_1 N_0}{\lambda_2 - \lambda 1}\ (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \qquad ...(v)$$

$(\lambda_1 < \lambda_2)$

This gives number of daughter nuclei at any instant t, thus N_2 depends only on its own decay constant λ_2 but also on parent's decay constant λ_1.

On substituting the value of N_2 in Eq. (*ii*) and using condition that at $t = 0$, $N_3 = 0$, we get

$$N_3 = N_0\left[1 + \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} - \frac{\lambda_2}{\lambda_2 - \lambda_1} e^{-\lambda_1 t}\right] \qquad ...(vi)$$

After time t, the ratio of activity is

$$\frac{\lambda_2 N_2}{\lambda_1 N_1} = \frac{\lambda_2}{\lambda_1 - \lambda_2}\left(1 - e^{-(\lambda_2 - \lambda_1)t}\right)$$

- The variation of activity with time is shown in Fig. 13.3.

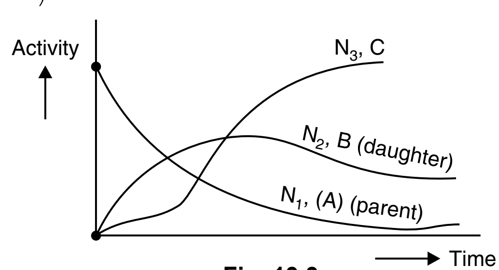

Fig. 13.3

- **Secular and Transient Radioactive Equilibrium**

In a successive disintegration, when the parent has a very-very long half-life, a state is reached when daughter prodcuts are formed at the same rate as they decay. At the stage the proportions of the different radioactive atoms in the mixture is constant, *i.e.*, do not change with time. In this situation the parent is said to be in 'secular radioactive equilibrium' with its daughter product.

Suppose that the parent atom has a much longer half-life than any of the decay product ($T_1 >> T_2$). Then

$$\lambda_1 = 0 \quad \text{and} \quad \lambda_2 << \lambda_3\ ,\ \lambda_1 << \lambda_3$$

Substituting $\lambda_1 << \lambda_2$ in Eq. (*iv*), we get

$$N_2 = \frac{\lambda_1}{\lambda_2} N_0\ (1 - e^{-\lambda_2 t}) \qquad \left[\text{as } \frac{\lambda_1}{\lambda_2 - \lambda_1} = \frac{\lambda_1}{\lambda_2}\right]$$

or $$N_2 = \frac{\lambda_1}{\lambda_2} N_1 (1 - e^{-\lambda_2 t}) \qquad (\because \quad N_1 = N_0)$$

For t tending to infinity, $e^{-\lambda_2 t}$ becomes negligibly small and hence

$$N_2 = \frac{\lambda_1}{\lambda_2} . N_1$$

or $$\boxed{N_1\lambda_1 = N_2\lambda_2}$$

Thus after a sufficient time, the activities of parent and daughter become equal. This condition is known as 'secular equilibrium'.

For a series disintegration $\lambda_1 N_1 = \lambda_2 N_2 = \lambda_3 N_3 = ...$

in term of half lives $\left(T = \frac{\log_e 2}{\lambda}\right)$

$$\frac{N_1}{T_1} = \frac{N_2}{T_2} = \frac{N_3}{T_3} = ...$$

Example of this equilibrium can be seen in the uranium series. Uranium has half-life of 4.5×10^9 years which is very large so that it takes 50×10^6 year for its quatity in a rock to change by 1%.

- The parent is longer lived than the daughter ($\lambda_1 < \lambda_2$), but the half-life of the parent is not very long ($T_1 = T_2$). After t becomes sufficicent large, $e^{-\lambda_2 t}$ becomes negligible compared with $e^{-\lambda_1 t}$ so that the number of the daughter is given by the equation.

$$N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} . N_0 \; e^{-\lambda_1 t} \qquad \text{[using Eq. (}v\text{)]}$$

Comparing it with Eq. (*iv*) and taking that $\lambda_1 < \lambda_2$, it can shown that the daughter eventually decay with the same half life as the parent.

$$\because \qquad N_0 \, e^{-\lambda_1 t} = N_1$$

$$\therefore \qquad N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1$$

or $$\frac{N_1}{N_2} = \frac{\lambda_1}{\lambda_2 - \lambda_1}$$

After a sufficient time the ratio of parent atoms to daughter atoms becomes constant and both eventually decay. This condition is called 'transient equilibrium'.

- When the parent has a shorter half-life time than the daughter ($T_1 < T_2$ or $\lambda > \lambda_2$), no state of equilibrium is attained, if initially we have only the parent atoms, then as the parent atoms decay, the daughter atoms increase in number, pass through a maximum and eventually decay with their own half-life.

• Alpha Decay

In alpha decay the atomic number Z is reduced by 2 unit and atomic mass number A is reduced by 4 units *e.g.*,

$$^{238}_{92}\text{U} \xrightarrow{-1\alpha} {}^{234}_{90}\text{Th}$$

Thus, the number of α-particles emitted can be given as

$$N(\alpha) = \frac{\Delta A}{4} = \frac{\text{change in atomic mass}}{4}$$

As alpha particle consists of two protons and two neutrons. It is the nucleus of ${}^{4}_{2}\mathrm{He}$. The nucelus ${}^{4}_{2}\mathrm{He}$ is particularly stable. Its binding energy is 28.3 MeV. The combination of two neutrons and two protons is particularly strong because of pairing effects. If the last two protons and two neutrons in a nucleus are bound by less than 28.3 MeV, then the emission of an alpha particle is energetically possible.

For alpha decay

$$Q = \left[M\left({}^{A}_{Z}\mathrm{X}\right) - M\left({}^{A-4}_{z-2}\mathrm{D}\right) - M\left({}^{4}_{2}\mathrm{He}\right)\right]c^2$$

If $Q > 0$ then alpha decay is possible. The disintegration energy Q appear in the form of kinetic energy of the daughter nucleus and the alpha particle.

The fraction of the disintegration energy carried off by the alpha particle can be calculated by applying conservation of energy and momentum.

$$Q = K_D + K_\alpha$$

$$P_d \propto P_\alpha$$

Parent nucleus has been assumed to be at rest initially.

Hence the total momentum is zero initially.

If M_D, M_α are masses of daughter and α particle the K.E. of a particle

Also $$K_\alpha = \frac{M_D}{M_D + M_\alpha} \cdot Q \simeq \frac{(A-4)}{A} \cdot Q$$

Kinetic energy of α-particle may be calculated from classical mechanics. After traversing certain distance the α-particles lose their power of ionisation as well as the power of exciting fluorescence or of affecting photographic plate. The range of α-particle means the distance through which the α-particle should travel.

Geiger and Nuttal showed that greater the decay constant of the radioactive substance the greater is the velocity and range (R) of the α-particles emitted. Geiger and Nuttal law states that a graph between $\log_e \lambda$ and $\log_e R$ is straight line.

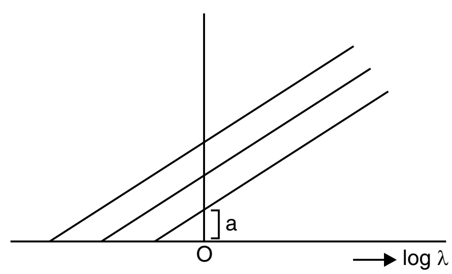

Fig. 13.4

$$\log_e \lambda = a + b \log R$$

Where a and b are cosntants. The constant b is the same for all the three radioactive series, *i.e.*, Uranium Radium Series, Actinium Series and Thorium Series While a differant for each series. The straight lines are, therefore, parallel to each other for three series. (Fig. 13.4)

- **Beta Decay**

When a radioactive nucleus undergoes a beta decay, its atomic number is increased by one unit and there is no change in its atomic mass number.

$${}^{A}_{Z}\mathrm{X} \xrightarrow{-1\beta} {}^{A}_{Z+1}\mathrm{Y}, \qquad e.g.,$$

$${}^{234}_{90}\mathrm{Th} \xrightarrow{-1\beta} {}^{234}_{91}\mathrm{Pa}$$

And the number of β-particles emitted is given as

$$N(\beta) = \left[\frac{\Delta A}{2} - \Delta Z\right]$$

In negative β-decay, a nectron is transformed into a proton and an electron is emitted

$${}^{0}_{1}n \longrightarrow {}^{0}_{1}p + {}^{-1}_{0}e$$

In positive β-decay a proton is transformed into a neutron and a positron is emitted

$${}^{0}_{1}p \longrightarrow {}^{0}_{1}n + e^{+}$$

Experimentally it is found that the β-particles are emitted with a continuous range of kinetic energies up to some maximum value *Q. This* value of a reaction is balanced by kinetic evergy of the particles.

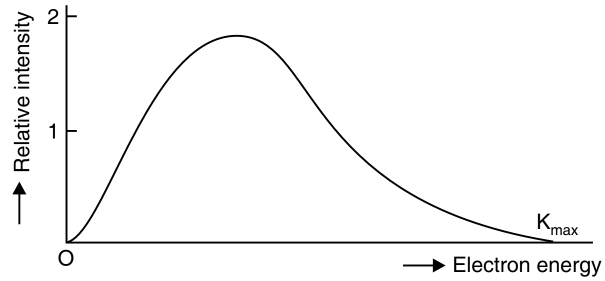

Fig. 13.5

However, because all decaying nuclei have the same initial mass, the *Q* value must be same for each decay. In view of this, why do the emitted particles have different kinetic energies? If there were only two particles have different kinetic energies? If there were only two products in beta decay the electrons must have been monoenergetic with energy K_{max}, which leads to violation of law of conservation of energy. Further analysis shows that law of conservation of both angular momentum and linear momentum are also violated.

Consider beta decay of ${}^{14}_{6}C$ to form ${}^{14}_{7}N$.

$${}^{14}_{6}C \longrightarrow {}^{14}_{7}N + \beta$$

Two strange experimental results puzzled scientists for many years; (1) continuous energy spectrum of β-particles, and (2) Neutron has spin 1/2; it cannot decay to two particles of spin 1/2 which were proton and electron. Secondly ${}^{14}C$ has spin 0, ${}^{14}N$ has spin 1, and the electron has spin 1/2. Resultant of spin 1 of ${}^{14}N$ and spin 1/2 of electron can never be zero. This result seemed to violate the law of conservation of angular momentum.

In order to account for these abnormalities, the great physicist *Pauli* proposed existence of a third particle later named *neutrino*. It has following properties:

(*i*) It has zero electric charge, hence shows no electromagnetic interaction.

(*ii*) Its rest mass is much smaller than that of electron and is possibly zero. Recent experiments suggest that the mass of neutrino is less than $7eV/c^2$.

(*iii*) A spin of 1/2, which satisfies the law of conservation of angular momentum when applied to beta decay.

(*iv*) Very weak interaction with matter, which makes it very difficult to detect.

Following are the general form of beta decay

$$n \longrightarrow p + e^- + \nu^- \quad (\beta^- \text{ decay})$$

$$p \longrightarrow n + e^+ + \nu \quad (\beta^+ \text{ decay})$$

$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + e^- + \nu^- \quad (\beta^- \text{ decay})$$

$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + e^+ + \nu \quad (\beta^+ \text{ decay})$$

where symbol ν represents neutrino and ν^- represents the antineutrino, the antiparticle to the neutrino.

- Electron capture occurs when a parent nucleus captures one of its own ortrital atomic electrons and emits a neutrino. The final product after decay is a nucleus whose charge is $(Z - 1)e$.

$$^{A}_{Z}X + e \longrightarrow {}^{A}_{Z+1}Y + \nu$$

In most cases it is an inner k-shell electron that is captured, and this reffered to as *k-capture*, *e.g.*, $^{7}_{4}Be$ captures an electron to become $^{7}_{3}Li$.

$$^{7}_{4}Be + e \longrightarrow {}^{7}_{3}Li + \nu$$

The Q value for e^- decay and electron capture are

$$Q = (M_x - M_y)\,c^2$$

The Q value for e^+ decay are given by

$$Q = (M_x - M_y - 2\,m_e)\,c^2$$

e^- decay, e^+ decay and electron capture are possible of Q value are positive.

- **Gamma Decay**

During gamma emission from a radioactive substance there is no change in atomic number and atomic mass number. Similar to atoms a nuclcus has excited state. As a result of very strong nuclear interaction, the excitation energies are very high, many MeV. The nucleus may get rid itself of this extra energy by emitting a photon (gamma ray) and underges a transition to some lower energy state. The gamma ray energy $h\nu$ is given by the difference of the higher energy state E_2 and the lower one E_1,

$$h\nu = E_2 - E_1$$

A nuclces may reach an excited state as a result of a violent collision with another particle. Often a nucleus is in excited state after undergoing an alpha to beta decay. The following sequence represents situation in which gamma decay occurs

$$^{12}_{5}B \longrightarrow {}^{12}_{6}C + e^- + \nu^-$$

$$^{12}_{6}C \longrightarrow {}^{12}_{6}C + \gamma$$

Fig. 13.6, shows that the decay scheme for $^{12}_{5}B$, which undergoes beta decay with a half life of 20.4 ms to either of two levels of ^{12}C. It can either (*i*) decay directly to the ground state of ^{12}C by emitting a 13.4 MeV electron or (*ii*) undergo e^- decay to an excited state of $^{12}_{6}C$, followed by gamma decay to the ground state. The later process results in the emission of a 9.0 MeV electron and a 4.4 MeV photon.

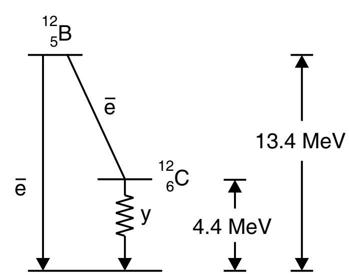

Fig. 13.6

- **Nuclear Fission**

When a heavy nucleus splits into two or more medium intermediate mass nuclei the phenomenon is called nuclear fission. For example when uranium – 235 is bombarded by a thermal neutron, it splits Krypton and Barium with 2 or 3 neutrons and large amount of energy.

$$= {}^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{90}_{36}Kr + {}^{144}_{56}Ba + 2{}^{1}_{0}n + 179.4 \text{ MeV}$$

Fission fragments get very large kinetic energies due to coulomb force of repulsion. As a result of collisions, this energy appears as thermal energy. In Fig. 13.7 a graph between mass difference per nucleon $[M - Zm_p - (A - Z)m_n]/A$ versus A in units of MeV/c^2 is shown in Fig. 13.7.

- This curve is just the negative of the binding energy curve. Figure shows that the **rest energy** per particle of heavy nuclides ($A \leq 200$) is more as compared to nuclides of intermidiate mass.
- Similarly the **rest energy** per particle of light nuclides ($A \leq 20$) is more as compared to nuclides of intermediate mass.
- In fission the total mass decreases and energy is released. For A = 200 the rest energy is about 1 MeV per nucleus greater than for A = 100. Hence in a nuclear fission where approximately equal size fragments are formed about 200 MeV is released.
- Several neutrons are emitted in the fission process which can cause further fission, thereby producing a chain reaction. When ^{235}U captures a thermal neutron the probability resulting ^{236}U nucleus undergoes fission is about 85% and of emitting gamma rays as it de-excites to the ground state is about 15%.

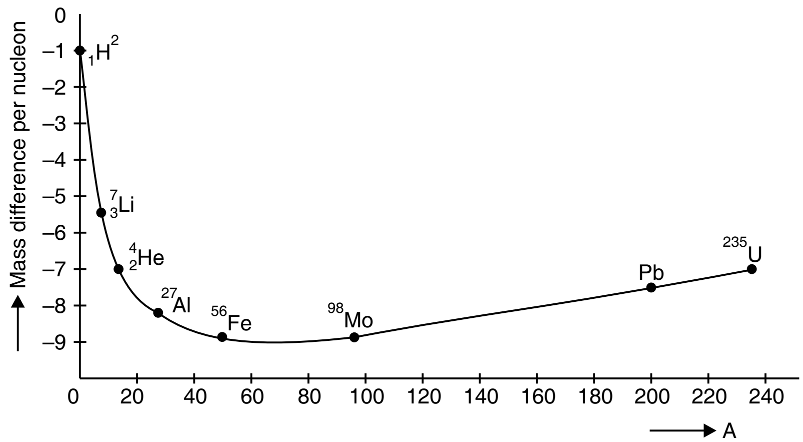

Fig. 13.7

- The critical energy is the energy required by fragments to overcome coulomb forces as shown in Fig. 13.8. For this nucleus, excitation energy produced when ^{239}U captures a neutron is 6.5 MeV. The critical energy is about 6.2 MeV, less than excitation energy. Therefore is the excited state the resulting ^{236}U nucleus has enough evergy to break apart, whereas the critical energy for the fission of the ^{239}U nucleus is 5.9 MeV. After capturing a neutron a ^{238}U nucleus produces an energy of only 5.2 MeV. Therefere, when ^{235}U captures a neutron to form ^{239}U, the excitation energy is not enough for fission to occur unless the neutron is a very fast one. Consequently excited ^{239}U nucleus de-excites by γ or α emission.

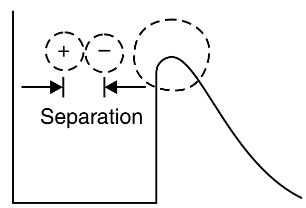

Fig. 13.8

- All nuclei with $Z > 83$ are radioactive. These nuclei may break apart into two nuclei by spontaneous fission, without absorbing a neutron. This spontaneous fission can also be understood by using liquid drop model of positive charges. For relatively small drop, surface tension can overcome the repulsive force of the charges and hold the drop together. Beyond a certain maximum

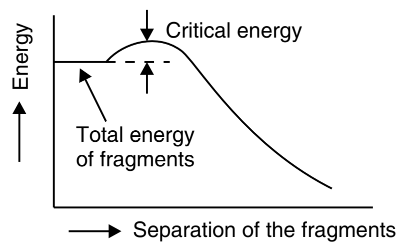

Fig. 13.9

size the drop will be unstable and will spontaneously break apart. Repulsive force is proportional to the number of protons, which is proportional to the number of protons, which is proportional to the volume and hence to R^3. Surface tension is proportional to surface area and hence increases only as R^2. Hence the size of a nucleus and, therefore, the number of elements that are possible are restricted due to spontaneous fission *i.e.,* if beyond a limit the size of a nucleus cannot grow, the nuclei will undergo spontaneous fission, although the probability for spontaneous fission is quite low compared with the other possible decay model. For ^{235}U the half-life for alpha decay is less than that for spontaneous fission is about but alpha decay, inhibited by the coulomb barrier. Even though the process is energetically possible, the large positively charged fission fragments have a very low probability to overcome the coulomb force.

- In nuclear fission many medium mass fragments can be formed. Depending on the perticular reaction, one, two or three neutrons may be emitted. The average number of neutrons emitted in the thermal neutron-induced fission of ^{235}U is about 2.5

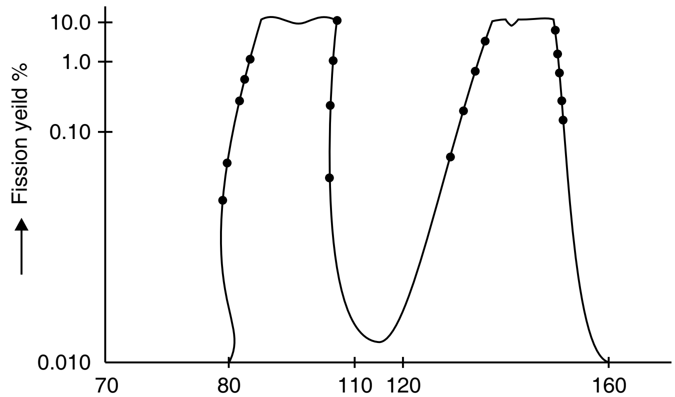

Fig. 13.10

- **Nuclear Reactors**

It is a device which converts nuclear energy into electrical energy (useful energy.) At consists of the following parts:

(*i*) **Fissionable Fuel.** It is a fissionable material such as uranium and plutonium. In nuclear reactor generally enriched uranium ^{235}U is used. The fuel in the form of uranium oxide is placed inside matellic tubes that constitute fuel elements. Cluster of fuel elements constitute fuel core. As the fission reaction proceeds the $^{235}_{92}U$ content decreases and part of $^{238}_{92}U$ is convertid to $^{239}_{94}Pu$ and its isotopes. Hence the fuel elements have to be replaced periodically.

(*ii*) **Moderator.** Moderator is a substance of low nuclear mass which surrounds the full elements. It does not absorb neutrons. It can be water if the uranium is enriched, heavy water or graphite if the uranium is normal. Moderator decreases the energy of the neutrons from above 1 MeV to less than 0.1 eV. In this process neutrons get '*thermalised*', and called thermal neutrons. They remain in thermal equilibrium with the environment.

(*iii*) **Reflector.** The core is surrounded by a substance such as water or graphite. It reduces the loss of neutron from the reactor, otherwise neutrons which are not used in fission would escape from the reactor.

(*iv*) **Control rods.** A series of rods made of a neutron absorbing material (such as *B*, *Hf*, *Cd*) are used to regulate the availability of neutrons for fission. They maintain control over the number of neutrons. They absorb neutrons without becoming radioactive. When these rods start becoming radioactive, they are taken out and dumped into the deep of earth.

(*v*) **Coolant.** A **fuel** is circulated through the core and the moderator which extracts the energy generates in the reactor core and prevents excessives rise in its temperature. The coolent may be water, heavy water, or a gas like He or CO_2. It absorbs very lightly.

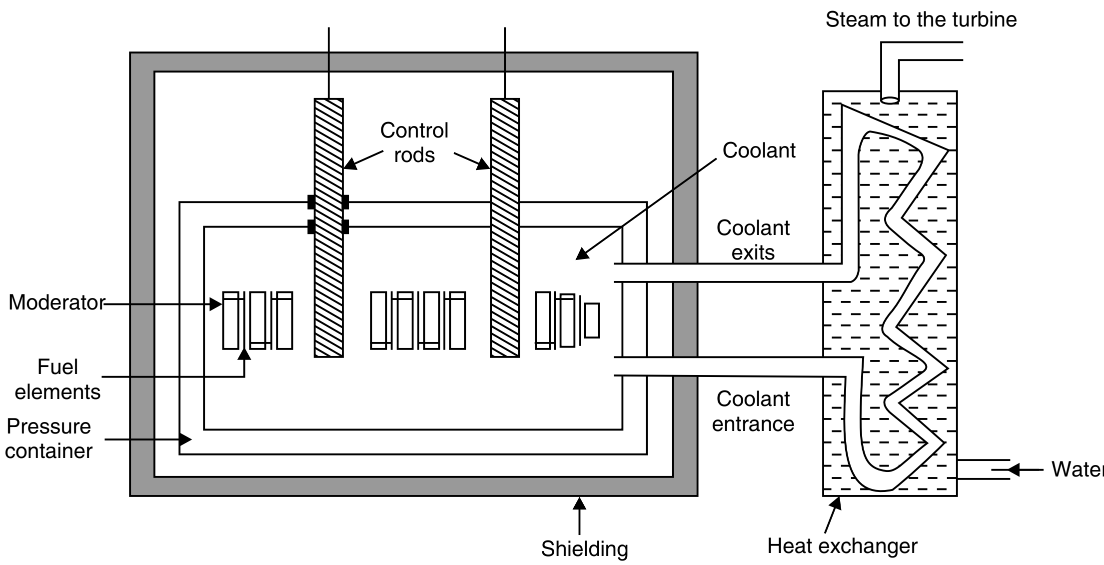

Fig. 13.11

• Nuclear Fusion

When two or more light mass nuclei combine to form a single nucleus the phenomenon is called nuclear fusion. For example when four hydrogen nuclei combine to form a nucleus of helium large amount of energy is released with few positrons.

$$4\ {}^{1}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + 2\,{}^{0}_{1}e\ + 17.6\ \text{MeV}.$$

This can be in different steps as

$${}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{2}_{1}\mathrm{H} + {}_{1}e^{0} + E$$

$${}^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{3}_{1}\mathrm{H} + {}_{1}e^{0} + E$$

$${}^{3}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + E$$

The resultant reaction is same as

$$4\ {}^{1}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + 2\,{}_{1}e^{0} + E$$

The helium nuclei can also combine to form carbon.

$$3\ {}^{4}_{2}\mathrm{He} \rightarrow {}^{12}_{6}\mathrm{C} + \mathrm{E}$$

This carbon can also combine to with the nuclius of hydrogen as

$${}^{12}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{13}_{7}\mathrm{N} + E$$

$${}^{13}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{14}_{7}\mathrm{N} + {}^{0}_{1}e + E$$

$${}^{14}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{15}_{8}\mathrm{O} + E$$

$${}^{15}_{8}\mathrm{O} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He} + {}^{0}_{1}e + E$$

But the resultant reaction is same as

$$4\ {}^{1}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + 2\ {}^{0}_{1}e + E$$

- Very strong coulomb repulsion exists between the positively charged nuclei taking part in nuclear fusion. Thus, very large kinetic energies, of the order of 1 MeV are needed to overcome this repulsion force. These large kinetic energies can be obtained in an accelerator. But in this process scattering of one nucleus by the other is more probable than fusion. Also, the process requires the input of more energy than is recovered.
- To obtain energy from fusion, the particles must be heated to a temperature great enough for the fusion reaction to occur as the result of random thermal collision. Because of thermal energy some particles can overcome the coulomb barrier. A temperature T corresponds to KT = 10 keV is required for a reasonable number of fusion reactions to occur. This temperature is of the order of 10^8 K. Such temperature occur in the core of stars. At this temperatures, a gas consists of positive ions and negative electrons called *plasma*. The main problem lies in confining the plasma long enough for the reactions to take place. The enormous gravitational field of the star confines the plasma in its core.
- The energy required to heat a plasma is proportional to the density of its ions n. The fusion rate is proportional to square of density n^2. The output energy is proportional to n^2t where t is the confinement time. If the output energy is to exceed the input energy, then

$$C_1n^2t > C_2n$$

where C_1 and C_2 are constant.
- *Lawson* derived the following relation between density and confinement time, known as *Lawson's criterion*:

$$nt > 10^{20} \text{ s particle/m}^3.$$

- If Lawson's criterion is met, the energy released by a fusion will must equal the energy input and the thermal energy of the ions is great enough (KT = 10 keV).
- A magnetic field is used to confine the plasma in a device called the Tokmak.

  The plasma is confined in a large toroid. The resultant magnetic field is combination of the magnetic field due to the current in the windings of the toroid and the self-field due to the current of the circulating plasma.

QUESTIONS FROM TEXTBOOK

Data useful in solving the exercises:

e	$= 1.6 \times 10^{-19}$ C	N	$= 6.023 \times 10^{23}$ per mole
$1/(4\pi\varepsilon_0)$	$= 9 \times 10^9$ N m^2/C^2	k	$= 1.381 \times 10^{-23}$ J ^{0}K^{-1}
1 MeV	$= 1.6 \times 10^{-13}$ J	1 u	= 931.5 MeV/c^2
1 year	$= 3.154 \times 10^7$ s		
m_H	= 1.007825 u	m_n	= 1.008665 u
$m\left({}^4_2He\right)$	= 4.002603 u	m_e	= 0.000548 u

13.1. (a) *Two stable isotopes of lithium ^6_3Li and ^7_3Li have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u, respectively. Find the atomic mass of lithium.*

(b) *Boron has two stable isotopes, $^{10}_5B$ and $^{11}_5B$. Their respective masses are 10.01294 u and 11.00931 u, and the atomic mass of boron is 10.81u. Find the abundances of $^{10}_5B$ and $^{11}_5B$.*

Sol. (*a*) Atomic weight = weighted average of the isotopes

$$= \frac{6.01512 \times 7.5 + 7.01600 \times 92.5}{(7.5 + 92.5)}$$

$$= \frac{45.1134 + 648.98}{100} = 6.941 \text{ u}$$

(*b*) Let relative abundance of $^{10}_5B$ be x%

∴ Relative abundance of $^{11}_5B$ = $(100 - x)$%

Proceeding as above,

$$10.811 = \frac{10.01294x + 11.00931 \times (100 - x)}{100}$$

$$x = 19.9\% \quad \text{and} \quad (100 - x) = 80.1\%.$$

13.2. *The three stable isotopes of neon: $^{20}_{10}Ne$, $^{21}_{10}Ne$ and $^{22}_{10}Ne$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of neon.*

Sol. The average atomic mass of neon is

$$m\,(Ne) = [90.51 \times 19.99 + 0.27 \times 20.99 + 9.22 \times 21.99] \times 10^{-2}$$

$$= 20.18 \text{ u.}$$

13.3. *Obtain the binding energy (in MeV) of a nitrogen nucleus $\left(^{14}_{7}N\right)$, given $m\left(^{14}_{7}N\right)$ = 14.00307 u.*

Sol. $^{14}_7N$ nucleus is made up of 7 protons and 7 neutrons. Mass of nucleons forming nucleus

$$= 7\,m_p + 7\,m_n$$

= Mass of 7 protons + Mass of 7 neutrons

$$= 7 \times 1.00783 + 7 \times 1.00867 \text{ u}$$

$$= 7.05431 + 7.06069 \text{ u}$$

$$= 14.11550 \text{ u}$$

Mass of nucleus, m_N = 14.00307 u

Mass defect = 14.11550 − 14.00307 = 0.11243 a.m.u.

Energy equivalent to mass defect = 0.11243 × 931

= 104.67 MeV

∴ Binding energy = 104.67 MeV.

13.4. *Obtain the binding energy of the nuclei $^{56}_{26}Fe$ and $^{209}_{83}Bi$ in units of MeV from the following data: $m\,(^{56}_{26}Fe)$ = 55.934939 u; $m\,(^{209}_{83}Bi)$ = 208.980388 u*

Which nucleus has greater binding energy per nucleon? Take 1 u = 931.5 MeV.

Sol. (*i*) $^{56}_{26}F$ nucleus contains 26 protons and (56 − 26) = 30 neutrons

Mass of 26 protons = 26 × 1.007825 = 26.20345 u

$$\text{Mass of 30 neutrons} = 30 \times 1.008665 = 30.25995 \text{ u}$$
$$\text{Total mass of 56 nucleons} = 56.46340 \text{ u}$$
$$\text{Mass of } {}^{56}_{26}\text{Fe nucleus} = 55.934939 \text{ u}$$
$$\therefore \quad \text{Mass defect, } \Delta m = 56.46340 - 55.934939 = 0.528461 \text{ u}$$
$$\text{Total binding energy} = 0.528461 \times 931.5 \text{ MeV} = 492.26 \text{ MeV}$$
$$\text{Average binding energy per nucleon} = \frac{492.26}{56} = 8.790 \text{ MeV.}$$

(*ii*) ${}^{209}_{83}$Bi nucleus contains 83 protons and (209 – 83) = 126 neutrons.

$$\text{Mass of 83 protons} = 83 \times 1.007825 = 83.649475 \text{ u}$$
$$\text{Mass of 126 neutrons} = 126 \times 1.008665 = 127.091790 \text{ u}$$
$$\text{Total mass of nucleons} = 210.741260 \text{ u}$$
$$\text{Mass of } {}^{209}_{83}\text{Bi nucleus} = 208.980388 \text{ u}$$
$$\text{Mass defect, } \Delta m = 210.741260 - 208.980388 = 1.760872$$
$$\text{Total B.E.} = 1.760872 \times 931.5 \text{ MeV} = 1640.26 \text{ MeV.}$$
$$\text{Average binding energy per nucleon} = \frac{1640.26}{209} = 7.848 \text{ MeV}$$

Hence, ${}^{56}_{26}$Fe has greater B.E. per nucleon than ${}^{209}_{83}$Bi.

13.5. *A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of ${}^{63}_{29}Cu$ atoms (of mass 62.92960 u).*

Sol. Mass of atom = 62.92960 u

Mass of 29 electrons = 29 × 0.000548 u

= 0.015892 u

Mass of nucleus = (62.92960 – 0.015892) u

= 62.913708 u

Mass of 29 protons = 29 × 1.007825 u

= 29.226925 u

Mass of (63 – 29) *i.e.,* 34 neutrons

= 34 × 1.008665 u

= 34.29461 u

Total mass of protons and neutrons

= (29.226925 + 34.29461) u

= 63.521535 u

Binding energy = (63.521535 – 62.913708) × 931.5 MeV

= 0.607827 × 931.5 MeV

$$\text{Required energy} = \frac{6.023 \times 10^{23}}{63} \times 3 \times 0.607827 \times 931.5 \text{ MeV}$$
$$= 1.6 \times 10^{25} \text{ MeV} = 2.6 \times 10^{12} \text{ J.}$$

13.6. *Write nuclear reaction equation for*

(i) *α-decay of* ${}^{226}_{88}Ra$ *(ii)* *α-decay of* ${}^{242}_{94}Pu$

(iii) *β^--decay of* ${}^{32}_{15}P$ *(iv)* *β^--decay of* ${}^{210}_{83}Bi$

(v) β^+*-decay of* $^{11}_{6}C$ *(vi)* β^+*-decay of* $^{97}_{43}Tc$

(vii) Electron capture of $^{120}_{54}Xe$ *.*

Sol. (*i*) $^{226}_{88}Ra \longrightarrow \, ^{222}_{86}Rn + \, ^{4}_{2}He$

(*ii*) $^{242}_{94}Pu \longrightarrow \, ^{238}_{92}U + \, ^{4}_{2}He$

(*iii*) $^{32}_{15}P \longrightarrow \, ^{32}_{16}S + e^{-1} + \bar{\nu}$

(*iv*) $^{210}_{83}Bi \longrightarrow \, ^{210}_{84}Po + e^{-1} + \bar{\nu}$

(*v*) $^{11}_{6}C \longrightarrow \, ^{11}_{5}B + e^{+} + \nu$

(*vi*) $^{97}_{43}Tc \longrightarrow \, ^{97}_{42}Mo + e^{+} + \nu$

(*vii*) $^{120}_{54}Xc + e^{-} \longrightarrow \, ^{120}_{53}I + \nu$

13.7. *A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to (a) 3.125%, (b) 1% of its original value?*

Sol. (*a*) The fraction of the original sample left = $\frac{3.125}{100}$

$$= \frac{1}{32} = \left(\frac{1}{2}\right)^5$$

Hence, there are 5 half lives of T years spent. Thus, the time taken is 5T years.

(*b*) The fraction of the original sample left = $\frac{1}{100} = \left(\frac{1}{2}\right)^n$

or, $2^n = 100 \Leftrightarrow n \log 2 = \log 100$

Hence, $n = \frac{\log 100}{\log 2} = \frac{2}{0.301} = 6.64$

Hence, there are 6.64 half lives of *T* years spent. Thus, the time taken is 6.64 T years.

13.8. *The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive* $^{14}_{6}C$ *present with the stable carbon isotope* $^{12}_{6}C$*. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of* $^{14}_{6}C$*, and the measured activity, the age of the specimen can be approximately estimated. This is the principle of* $^{14}_{6}C$ *dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus Valley Civilisation.*

Sol. Here, normal activity, R_0 = 15 decays/min

Present activity R = 9 decays/min, T = 5730 years, Age t =?

As activity is proportional to the number of radioactive atoms, therefore,

$$\frac{N}{N_0} = \frac{R}{R_0} = \frac{9}{15}$$

But $\frac{N}{N_0} = e^{-\lambda t}$

$\therefore \quad e^{-\lambda t} = \frac{9}{15} = \frac{3}{5}$

$$e^{+\lambda t} = \frac{5}{3}$$

$$\lambda t \log_e e = . \log_e \frac{5}{3} = 2.3026 \log 1.6667$$

$$\lambda t = 2.3026 \times 0.2218 = 0.5109$$

$$t = \frac{0.5109}{\lambda}$$

But $$\lambda = \frac{0.693}{T} = \frac{0.693}{5730} \text{Yr}^{-1}$$

$\therefore$ $$t = \frac{0.5109}{0.693/5730} = \frac{0.5109 \times 5730}{0.693}$$

$$t = 4224.3 \text{ years.}$$

13.9. *Obtain the amount of $^{60}_{27}Co$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of $^{60}_{27}Co$ is 5.3 years.*

Sol. Strength of radioactive source

$$= 8.0 \text{ mCi} = 8.0 \times 10^{-3} \text{ Ci}$$

$$= 8.0 \times 10^{-3} \times 3.7 \times 10^{10} \text{ disintegrations s}^{-1}$$

$$= 29.6 \times 10^{7} \text{ disintegrations s}^{-1}$$

Since the strength of the source decreases with time,

$\therefore$ $$\frac{dN}{dt} = -29.6 \times 10^7$$

But $$\boxed{\frac{dN}{dt} = -\lambda N}$$

$\therefore$ $$-\lambda N = -29.6 \times 10^7$$

or, $$\lambda N = 29.6 \times 10^7$$

or, $$N = \frac{29.6 \times 10^7}{\lambda}$$

or, $$N = \frac{29.6 \times 10^7 \times T}{0.693} \qquad \left(\because \lambda = \frac{0.693}{T}\right)$$

$$= \frac{29.6 \times 10^7 \times 5.3 \times 365 \times 24 \times 60 \times 60}{0.693}$$

$$= 7.139 \times 10^{16}$$

Number of atoms in 60 g of cobalt = 6.023×10^{23}

$$\text{Mass of 1 atom of cobalt} = \frac{60}{6.023 \times 10^{23}} \text{ g}$$

$$\text{Mass of } 7.139 \times 10^{16} \text{ atoms} = \frac{60}{6.023 \times 10^{23}} \times 7.139 \times 10^{16} \text{ g}$$

$$= 7.11 \ \mu\text{g.}$$

13.10. *The half-life of $^{90}_{38}Sr$ is 28 years. What is the disintegration rate of 15 mg of this isotope?*

Sol. Since,

$$\boxed{\lambda = \frac{0.693}{T}}$$

$$\Rightarrow \qquad \lambda = \frac{0.693}{28 \times 365 \times 24 \times 60 \times 60}$$

$$= 7.85 \times 10^{-10} \text{ s}^{-1}$$

90 g of *Sr* contains 6.023×10^{23} atoms

15 mg of Sr contains, $$N_0 = \frac{6.023 \times 10^{23} \times 15 \times 10^{-3}}{90} \text{ atoms}$$

$$N_0 = 1.0038 \times 10^{20} \text{ atoms}$$

Disintegration rate, $$\boxed{\frac{dN}{dt} = -\lambda N_0}$$

$$= -7.85 \times 10^{-10} \times 1.0038 \times 10^{20}$$

$$= 7.88 \times 10^{10} \text{ dps or } Bq$$

$$= \frac{7.88 \times 10^{10}}{3.7 \times 10^{10}} \text{ Ci}$$

$$= 2.13 \text{ Ci.}$$

13.11. *Obtain approximately the ratio of the nuclear radii of the gold isotope $^{197}_{79}Au$ and the silver isotope $^{107}_{47}Ag$.*

Sol. As,

$$\boxed{R \approx A^{1/3}}$$

$$\therefore \qquad \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

$$= \left(\frac{197}{107}\right)^{1/3} = (1.84)^{1/3}$$

$$\Rightarrow \qquad \log_{10}\left(\frac{R_1}{R_2}\right) = \log_{10} (1.84)^{1/3}$$

$$\Rightarrow \qquad \log_{10}\left(\frac{R_1}{R_2}\right) = \frac{1}{3} \log_{10} (1.84)$$

$$= \frac{1}{3} \times 0.2648$$

$$= 0.08827$$

$$\Rightarrow \qquad \frac{R_1}{R_2} = \text{antilog } (0.08827)$$

$$= 1.23.$$

13.12. *Find the Q-value and the kinetic energy of the emitted α-particle in α-decay of (a) $^{226}_{88}Ra$ and (b) $^{220}_{86}Rn$.*

Given $m\left({}^{226}_{88}Ra\right) = 226.02540\ u;\quad m\left({}^{222}_{86}Rn\right) = 222.01750\ u;$

$m\left({}^{220}_{86}Rn\right) = 220.01137\ u;\quad m\left({}^{216}_{84}Po\right) = 216.00189\ u.$

Sol. (*a*) The difference in mass between the original nucleus and the decay products

$$= 226.02540\text{ u} - (222.01750\text{ u} + 4.00260\text{ u})$$
$$= +\ 0.0053\text{ u}$$
$$\text{Energy equivalent} = 0.0053 \times 931.5\text{ MeV}$$
$$= 4.93695\text{ MeV}$$
$$= 4.94\text{ MeV}$$

The decay products would emerge with total kinetic energy 4.94 MeV. Momentum is conserved. If the parent nucleus is at rest, the daughter and the α-particle have momenta of equal magnitude p but opposite direction. Kinetic energy, $K = \frac{p^2}{2m}$. Since p is the same for the two particles therefore the kinetic energy divides inversely as their masses. The α-particle gets $\frac{222}{222+4}$ of the total *i.e.,* $\frac{222}{226} \times 4.94$ MeV or 4.85 MeV.

(*b*) The difference in mass between the original nucleus and the decay products

$$= 220.01137\text{ u} - (216.00189\text{ u} + 4.00260\text{ u})$$
$$= 0.00688\text{ u}$$
$$\text{Energy equivalent} = 0.00688 \times 931.5\text{ MeV}$$
$$= 6.41\text{ MeV}$$
$$E_\alpha = \frac{216}{216+4} \times 6.41\text{ MeV} = 6.29\text{ MeV.}$$

13.13. *The radionuclide* ^{11}C *decays according to* ${}^{11}_{6}C \rightarrow {}^{11}_{5}B + e^+ + \nu$; $T_{1/2} = 20.3$ *min.*

The maximum energy of the emitted positron is 0.960 MeV. Given the mass values:

$m\left({}^{11}_{6}C\right) = 11.011434\ u$ *and* $m\left({}^{11}_{5}B\right) = 11.009305\ u.$

Calculate Q and compare it with the maximum energy of the positron emitted.

Sol. Mass defect

$$= [m\,({}^{11}_{6}C) - 6\,m_e] - [m\,({}^{11}_{5}B) - 5\,m_e + m_e]$$
$$= m\,({}^{11}_{6}C) - m\,({}^{11}_{5}B) - 2\,m_e$$
$$= 11.011434\text{ u} - 11.009305\text{ u} - 2 \times 0.000548\text{ u}$$
$$= 0.001033\text{ u}$$
$$Q = 0.001033 \times 931.5\text{ MeV} = 0.962\text{ MeV}$$
$$Q = E_d + E_e + E_\nu$$

The daughter nucleus is too heavy compared to e^+ and ν. So, it carries negligible energy ($E_d \approx 0$). If the kinetic energy (E_ν) carried by the neutrino is minimum (*i.e.,* zero), the positron carries maximum energy, and this is practically all energy Q. Hence, maximum $E_e \approx Q$.

13.14. *The nucleus $^{23}_{10}Ne$ decays by β^- emission. Write down the β-decay equation and determine the maximum kinetic energy of the electrons emitted from the following data:*

$$m\,(^{23}_{10}Ne) = 22.994466\ u$$
$$m\,(^{23}_{11}Na) = 22.989770\ u$$

Sol. The β-decay of $^{23}_{10}$Ne may be represented as $^{23}_{10}\text{Ne} \longrightarrow {}^{23}_{11}\text{Na} - {}^{0}_{-1}e + \bar{\nu} + Q$

Ignoring the rest mass of antineutrino $(\bar{\nu})$ and electron

Mass defect, $\Delta m = m\,(^{23}_{10}Ne) - m\,(^{23}_{11}Na)$

$= 22.994466 - 22.989770$

$= 0.004696$ u

$\therefore$ $Q = 0.004696 \times 931$ MeV $= 4.372$ MeV.

As $^{23}_{11}$Na is very massive, this energy of 4.3792 MeV, is shared by e^- and $\bar{\nu}$ pair. The maximum K.E. of e^- = 4.372 MeV, when energy carried by $\bar{\nu}$ is zero.

13.15. *The Q-value of a nuclear reaction*
$A + b \longrightarrow C + d$ is defined by $Q = [m_A + m_b - m_c - m_d]\,c^2$
where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

(i) $^1_1H + {}^3_1H \rightarrow {}^2_1H + {}^2_1H$ *(ii)* $^{12}_6C + {}^{12}_6C \rightarrow {}^{20}_{10}Ne + {}^4_2He$
Atomic masses are given to be

$$m\,(^2_1H) = 2.014102\ u; \quad m\,(_1H^3) = 3.016049\ u$$
$$m\,(^{12}_6C) = 12.000000\ u; \quad m\,(_{10}Ne^{20}) = 19.992439\ u.$$

Sol. (*i*) $^1_1\text{H} + {}^3_1\text{H} \rightarrow {}^2_1\text{H} + {}^2_1\text{H}$

$Q = \Delta m \times 931.5$ MeV $= [m\,(^1_1\text{H}) + m\,(^3_1\text{H}) - 2m\,(^2_1\text{H})] \times 931$ MeV
$= [1.007825 + 3.016049 - 2 \times 2.014102] \times 931$ MeV $= -\ 4.03$ MeV

$\therefore$ The reaction is endothermic

(*ii*) $^{12}_6\text{C} + {}^{12}_6\text{C} \rightarrow {}^{20}_{10}\text{Ne} + {}^4_2\text{He}$

$Q = \Delta m \times 931$ MeV $= [2\,m\,(^{12}_6\text{C}) - m\,(^{20}_{10}\text{Ne}) - m\,(^4_2\text{He})] \times 931$ MeV
$= [24.000000 - 19.992439 - 4.002603] \times 931$ MeV $= +\ 4.61$ MeV

$\therefore$ The reaction is exothermic.

13.16. *Suppose, we think of fission of a $^{56}_{26}Fe$ nucleus into two equal fragments $^{28}_{13}Al$. Is the fission energetically possible? Argue by working out Q of the process. Given*
$m\,(^{56}_{26}Fe) = 55.93494\ u$, $m\,(^{28}_{13}Al) = 27.98191\ u$.

Sol. $Q = [m\,(^{56}_{26}\text{Fe}) - 2\,m\,(^{28}_{13}\text{Al})] \times 931.5$ MeV $= [55.93494 - 2 \times 27.98191] \times 931.5$ MeV
$Q = -\ 0.02886 \times 931.5$ MeV $= -\ 26.88$ MeV, which is negative.
The fission is not possible energetically.

13.17. *The fission properties of $^{239}_{94}Pu$ are very similar to those of $^{235}_{92}U$. The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure $^{239}_{94}Pu$ undergo fission?*

Sol. Energy released per fission of

$$^{239}_{94}Pu = 180 \text{ MeV}$$

Quantity of fissionable material = 1 kg

In 239 gm Pu, number of fissionable atom or nuclei

$$= 6.023 \times 10^{23}$$

In 1 g of Pu, number of fissionable atom or nuclei

$$= \frac{6.023 \times 10^{23}}{239}$$

In 1000 gm of Pu, number of fissionable atom or nuclei

$$= \frac{6.023 \times 10^{23}}{239} \times 1000$$

$$= 25.2 \times 10^{23}$$

Energy released in fission of single Pu nucleus

$$= 180 \text{ MeV}$$

Energy released in fission of 25.2×10^{23} Pu nucleus or in fission of 1 kg pure Pu

$$= 180 \times 25.2 \times 10^{23}$$

$$= 4536 \times 10^{23} \text{ MeV}$$

$$= 4.5 \times 10^{26} \text{ MeV.}$$

13.18. *A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much* $^{235}_{92}U$ *did it contain initially? Assume that all the energy generated arises from the fission of* $^{235}_{92}U$ *and that this nuclide is consumed only by the fission process.*

Sol. Power of reactor = 1000 MW = 10^3 MW = 10^9 W = 10^9 Js^{-1}

Energy generated by reactor in 5 years

$$= 5 \times 365 \times 24 \times 60 \times 60 \times 10^9 \text{ J}$$

Energy generated per fission = 200 MeV

$$= 200 \times 1.6 \times 10^{-13} \text{ J}$$

Number of fission taking place or number of U^{235} nuclei required

$$= \frac{5 \times 365 \times 24 \times 60 \times 60 \times 10^9}{200 \times 1.6 \times 10^{-13}}$$

$$= 8.2125 \times 10^{26} \times 6 = 49.275 \times 10^{26}$$

Mass of 6.023×10^{23} nuclei of U

$$= 235 \text{ gm} = 235 \times 10^{-3} \text{ kg}$$

Mass of 8.2125×10^{26} nuclei of U

$$= \frac{235 \times 10^{-3}}{6.023 \times 10^{23}} \times 6 \times 8.2125 \times 10^{26}$$

$$= 1932 \text{ kg}$$

$$\frac{1}{2} \text{ of fuel} = 1932 \text{ kg}$$

$$\text{Total fuel} = 3864 \text{ kg.}$$

13.19. *How long can an electric lamp of 100 W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as:*

$$\cdot {}_1^2H + {}_1^2H \rightarrow {}_2^3He + n + 3.2\ MeV.$$

Sol. When two nuclei of deuterium fuse together,

energy released = 3.2 MeV

Number of deuterium atoms in 2 kg

$$= \frac{6.023 \times 10^{23}}{2} \times 2000 = 6.023 \times 10^{26}$$

When 6.023×10^{26} nuclei of deuterium fuse together, energy released

$$= \frac{3.2}{2} \times 6.023 \times 10^{26}\ \text{MeV}$$

$$= \frac{3.2}{2} \times 6.023 \times 10^{26} \times 1.6 \times 10^{-13}\ \text{J}$$

$$= 1.54 \times 10^{14}\ \text{J or Ws}$$

Power of electric lamp = 100 W

If the lamp glows for time t, then the electrical energy consumed by the lamp is 100 t.

$$\therefore \quad 100\ t = 1.54 \times 10^{14} \quad \text{or} \quad t = 1.54 \times 10^{12}\ \text{s}$$

$$= \frac{1.54 \times 10^{12}}{3.154 \times 10^{7}}\ \text{years} = 4.88 \times 10^{4}\ \text{years.}$$

13.20. *Calculate the height of the potential barrier for a head on collision of two deuterons. Assume that they can be taken as hard spheres of radius 2.0 fm.*

Sol. (**Hint:** *The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other.*) Suppose the two particles are fired at each other with the same kinetic energy K so that they are brought to rest by their mutual Coulomb repulsion when they are just touching each other. We can take this value of K as a representative measure of the height of Coulomb barrier.

$$\text{P.E.} = 2\text{K.E.}$$

$$\frac{e^2}{4\pi\varepsilon_0(2R)} = 2K_e$$

$$2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{(2R)}$$

$$K_e = \frac{e^2}{16\pi\varepsilon_0 R}$$

$$= \frac{(1.6 \times 10^{-19})^2}{16 \times 3.14 \times 8.85 \times 10^{-12} \times 2 \times 10^{-15}}\ \text{J}$$

$$= 2.8788 \times 10^{-14}\ \text{J}$$

$$= \frac{2.8788 \times 10^{-14}}{1.6 \times 10^{-19} \times 10^{3}}\ \text{keV}$$

$$= 179.9\ \text{keV.}$$

13.21. *From the relation $R = R_0 A^{1/3}$, where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e., independent of A).*

Sol. It is found that a nucleus of mass number A has a radius

$$R = R_0 A^{1/3}$$

where, $$R_0 = 1.2 \times 10^{-15} \text{ m}.$$

This implies that the volume of the nucleus, which is proportional to R^3 is proportional to A.

$$\text{Volume of nucleus} = \frac{4}{3}\pi R^3$$

$$= \frac{4}{3}\pi \; (R_0 A^{1/3})^3 = \frac{4}{3}\pi R_0^3 A$$

$$\text{Density of nucleus} = \frac{\text{mass of nucleus}}{\text{volume of nucleus}}$$

$$= \frac{mA}{\frac{4}{3}\pi R_0^3 A}$$

$$= \frac{3m}{4\pi R_0^3}$$

Above derived equation shows that density of nucleus is constant, independent of A, for all nuclei and density of nuclear matter is approximately 2.3×10^7 kg m^{-3} which is very large as compared to ordinary matter, say water which is 10^3 kg m^{-3}.

13.22. *For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K-shell, is captured by the nucleus and a neutrino is emitted).*

$$e^+ + {}^A_ZX \rightarrow {}_{Z-1}^{\;\;A}Y + \nu$$

Show that if β^{++} emission is energetically allowed, electron capture is necessarily allowed but not vice-versa.

Sol. Consider the two competing processes:

$${}^A_ZX \rightarrow {}_{Z-1}^{\;\;A}Y + e^+ + \nu_e + Q_1 \text{ (positron emission)}$$

$$e^- + {}^A_ZX \rightarrow {}_{Z-1}^{\;\;A}Y + \nu_e + Q_2 \text{ (electron capture)}$$

$$Q_1 = \left[m_N\left({}^A_ZX\right) - m_N\left({}_{Z-1}^{\;\;A}Y\right) - m_e\right]c^2$$

$$= \left[m\left({}^A_ZX\right) - Zm_e - m\left({}_{Z-1}^{\;\;A}Y\right) + (Z-1)\,m_e - m_e\right]c^2$$

$$= \left[m\left({}^A_ZX\right) - m\left({}_{Z-1}^{\;\;A}Y\right) - 2m_e\right]c^2$$

$$Q_2 = \left[m_N\left({}^A_ZX\right) + m_e - m_N\left({}_{Z-1}^{\;\;A}Y\right)\right]c^2$$

$$= \left[m\left({}^A_ZX\right) - m\left({}_{Z-1}^{\;\;A}Y\right)\right]c^2$$

This means $Q_1 > 0$ implies $Q_2 > 0$ but $Q_2 > 0$ does not necessarily mean $Q_1 > 0$. Hence the result.

13.23. *In a periodic table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on earth. The three isotopes and their masses are $^{24}_{12}Mg$ (23.98504 u), $^{25}_{12}Mg$ (24.98584 u) and $^{26}_{12}Mg$ (25.98259 u). The natural abundance of $^{24}_{12}Mg$ is 78.99% by mass. Calculate the abundances of other two isotopes.*

Sol.

Isotope	*Abundance Y*	*Atomic mass (Z)*
$_{12}Mg^{24}$	78.99	23.98504
$_{12}Mg^{25}$	x	24.98584
$_{12}Mg^{26}$	$100 - (78.99 + x) = 21.1 - x$	25.98259
	$\Sigma Y = 100$	

Mean atomic mass = 24.312

$$\text{Average atomic mass} = \frac{\Sigma YZ}{\Sigma Y}$$

$$\Rightarrow \quad 24.312 = \frac{78.99 \times 23.98504 + x \times 24.98584 + (21.01 - x)\, 25.98254}{100}$$

or, $2431.2 = 1894.58 + 24.98584\,x + 545.89 - 25.98254\,x$

or, $2431.2 = 2440.47 - .99675\,x$

or, $.99675\,x = 2440.47 - 2431.2 = 9.27$

or, $x = \dfrac{9.27}{.99675} = 9.30$

$\therefore \quad 21.01 - x = 21.01 - 9.30 = 11.71$

Relative abundance of $_{12}Mg^{25}$ = 9.30%

Relative abundance of $_{12}Mg^{26}$ = 11.71%.

13.24. *The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei $^{41}_{20}Ca$ and $^{27}_{13}Al$ from the following data:*

$$m\left(^{40}_{20}Ca\right) = 39.962591 \text{ u}$$

$$m\left(^{41}_{20}Ca\right) = 40.962278 \text{ u}$$

$$m\left(^{26}_{13}Al\right) = 25.986895 \text{ u}$$

$$m\left(^{27}_{13}Al\right) = 26.981541 \text{ u}.$$

Sol. The equation for the neutron separation in first case can be written as,

$$^{41}_{20}Ca + Q_1 \rightarrow {}^{40}_{20}Ca + {}^{1}_{0}n$$

$$\Delta m = m\,(^{40}_{20}Ca) + m\,(^{1}_{0}n) - m\,(^{41}_{20}Ca)$$

$$= 39.962591 + 1.008665 - 40.962278$$

$$= 0.008978 \text{ u}$$

But, $1 \text{ u} \equiv 931.5$ MeV

Hence, $0.008978 \text{ u} \equiv 0.008978 \times 931.5 = 8.363$ MeV

The equation for the neutron separation in second case can be written as,

$$^{27}_{13}Al + Q_2 \rightarrow {}^{26}_{13}Al + {}^{1}_{0}n$$

$$\Delta m = m\,({}^{26}_{13}\text{Al}) + m\,({}^{1}_{0}n) - m\,({}^{27}_{13}\text{Al})$$

But, $1 \text{ u} \equiv 931.5 \text{ MeV}$

Hence, $0.014019 \text{ u} \equiv 0.014019 \times 931.5 = 13.06 \text{ MeV}.$

13.25. *A source contains two phosphorous radio nuclides ${}^{33}_{15}P$ ($T_{1/2}$ = 14.3 d) and ${}^{32}_{15}P$ ($T_{1/2}$ = 25.3 d). Initially, 10% of the decays come from ${}^{33}_{15}P$. How long one must wait until 90% do so?*

Sol. We know that $-\frac{dN}{dt} \propto N$.

So, clearly the initial ratio of the amounts of ${}^{33}_{15}\text{P}$ and ${}^{32}_{15}\text{P}$ is 1:9. We have to find the time after which the ratio is 9 : 1.

Initially, if the amount of ${}^{33}_{15}\text{P}$ is x, the amount of ${}^{32}_{15}\text{P}$ is $9x$. Finally, if the amount of ${}^{33}_{15}\text{P}$ is $9y$, the amount of ${}^{32}_{15}\text{P}$ is y.

Using, $$N = \frac{N_0}{2^{t/T}}$$

$$9y = \frac{x}{2^{t/25.3}}$$

$$y = \frac{9x}{2^{t/14.3}}$$

Dividing, $$9 = \frac{x}{2^{t/25.3}} \times \frac{2^{t/14.3}}{9x}$$

or, $$81 = 2^{\frac{t}{14.3} - \frac{t}{25.3}}$$

or, $$81 = 2^{\frac{11t}{361.79}}$$

or, $$\log_{10} 81 = \frac{11t}{361.79} \log_{10} 2 = \frac{11 \times 0.3010\, t}{361.79}$$

$$= 9.15 \times 10^{-3} \text{ t}$$

$$9.15 \times 10^{-3} \text{ t} = 1.91$$

or, $$t = \frac{1.91 \times 1000}{9.15} d = 208.7 \text{ d.}$$

13.26. *Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α-particle. Consider the following decay processes:*

$${}^{223}_{88}Ra \rightarrow {}^{209}_{82}Pb + {}^{14}_{6}C$$

$${}^{223}_{88}Ra \rightarrow {}^{219}_{86}Rn + {}^{4}_{2}He$$

Calculate the Q-values for these decays and determine that both are energetically allowed.

Sol. (*i*) For decay process ${}^{223}_{88}Ra \rightarrow {}^{209}_{82}Pb + {}^{14}_{6}C + Q$

Mass defect, Δm = mass of Ra^{223} − (mass of Pb^{209} + mass of C^{14})

= 223.01850 − (208.98107 + 14.00324)

= 0.03419 u

∴ $Q = 0.03419 \times 931 \text{ MeV} = 31.83 \text{ MeV}$

(*ii*) For decay process ${}^{223}_{88}\text{Ra} \rightarrow {}^{219}_{86}\text{Rn} + {}^{4}_{2}\text{He} + Q$

$$\begin{aligned}\text{Mass defect, } \Delta m &= \text{mass of Ra}^{223} - (\text{mass of Rn}^{219} + \text{mass of He}^{4}) \\ &= 223.01850 - (219.00948 + 4.00260) \\ &= 0.00642 \text{ u} \\ \therefore \quad Q &= 0.00642 \times 931 \text{ MeV} = 5.98 \text{ MeV}\end{aligned}$$

As Q-values are positive in both the cases, therefore both the decays are energetically possible.

13.27. *Consider the fission of ${}^{238}_{92}\text{U}$ by fast neutrons. In one fission event, no neutrons are emitted and the final end products, after the beta decay of the primary fragments, are ${}^{140}_{58}\text{Ce}$ and ${}^{99}_{44}Ru$. Calculate Q for this fission process. The relevant atomic and particle masses are*

$$\begin{aligned} m\left({}^{238}_{92}\text{U}\right) &= 238.05079 \ u \\ m\left({}^{140}_{58}\text{Ce}\right) &= 139.90543 \ u \\ m\left({}^{99}_{44}Ru\right) &= 98.90594 \ u. \end{aligned}$$

Sol. Fission reaction is

$${}^{238}_{92}\text{U} + {}^{1}_{0}n \rightarrow {}^{140}_{58}\text{Ce} + {}^{99}_{44}\text{Ru} + Q$$

$$\begin{aligned} Q\text{-value} &= (\text{mass of U}^{238} + \text{mass of } {}^{1}_{0}n - \text{mass of Ce}^{140} - \text{mass of Ru}^{99}) \times 931.5 \text{ MeV} \\ &= (238.05079 + 1.00867 - 139.90543 - 98.90594) \times 931.5 \text{ MeV} \\ &= 231.1 \text{ MeV}. \end{aligned}$$

13.28. *Consider the D–T reaction (deuterium-tritium fusion)*

$${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + n$$

(a) Calculate the energy released in MeV in this reaction from the data:

$$\begin{aligned} m\left({}^{2}_{1}H\right) &= 2.014102 \ u \\ m\left({}^{3}_{1}H\right) &= 3.016049 \ u \end{aligned}$$

(b) Consider the radius of both deuterium and tritium to be approximately 2.0 fm. What is the kinetic energy needed to overcome the Coulomb repulsion between the two nuclei? To what temperature must the gas be heated to initiate the reaction?

(**Hint:** *Kinetic energy required for one fusion event = average thermal kinetic energy available with the interacting particles = 2(3kT/2); k = Boltzman's constant, T = absolute temperature.)*

Sol. (*a*) For the process ${}^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + n + Q$

$$\begin{aligned} Q\text{-value} &= [\text{mass of } {}^{2}_{1}\text{H} + \text{mass of } {}^{3}_{1}\text{H} - \text{mass of } {}^{4}_{2}\text{He} - \text{mass of } n] \times 931 \text{ MeV} \\ &= (2.014102 + 3.016049 - 4.002603 - 1.00867) \times 931 \text{ MeV} \\ &= 0.018878 \times 931 = 17.58 \text{ MeV} \end{aligned}$$

(*b*) Repulsive potential energy of two nuclei when they almost touch each other is

$$\begin{aligned} \frac{q^2}{4\pi\varepsilon_0 (2r)} &= \frac{9 \times 10^9 \ (1.6 \times 10^{-19})^2}{2 \times 2 \times 10^{-15}} \text{ joule} \\ &= 5.76 \times 10^{-14} \text{ J} \end{aligned}$$

Classically, *K.E.* at least equal to this amount is required to overcome Coulomb repulsion. Using relation

$$KE = \frac{3}{2}kT$$

$$T = \frac{2\text{K.E.}}{3k} = \frac{2 \times 5.76 \times 10^{-14}}{3 \times 1.38 \times 10^{-23}} = 2.78 \times 10^{9}\,\text{K}$$

In actual practice, the temperature required for triggering the reaction is somewhat less.

13.29. *Obtain the maximum kinetic energy of β-particles, and the radiation frequencies of γ decays in the decay scheme shown in Fig. 13.12. You are given that*

$$m\,(^{198}Au) = 197.968233\ u$$

$$m\,(^{198}Hg) = 197.966760\ u.$$

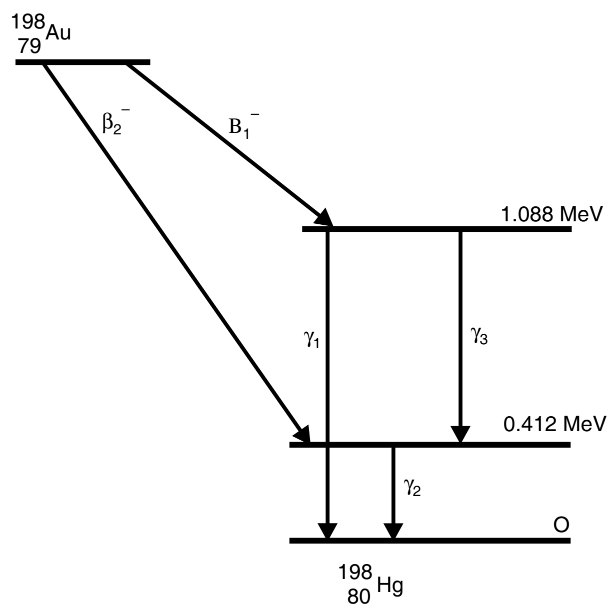

Fig. 13.12

Sol.

$$\nu = \frac{(E_2 - E_1)}{h}$$

$$\nu\,(\gamma_1) = \frac{(1.088 - 0) \times 1.6 \times 10^{-13}}{6.62 \times 10^{-34}} \qquad \left[\begin{array}{l} \because\ 1\,\text{MeV} = 10^{6} \times 1.6 \times 10^{-19}\,\text{J} \\ 1\,\text{MeV} = 1.6 \times 10^{-13}\,\text{J} \end{array}\right]$$

$$= 2.63 \times 10^{20}\ \text{s}^{-1}$$

$$\nu\,(\gamma_2) = \frac{(0.412 - 0) \times 1.6 \times 10^{-13}}{6.62 \times 10^{-34}} = 9.96 \times 10^{20}\,\text{s}^{-1}$$

$$\nu\,(\gamma_3) = \frac{(1.088 - 0.412) \times 1.6 \times 10^{-13}}{6.62 \times 10^{-34}} = 1.63 \times 10^{20}\,\text{s}^{-1}$$

The emission of β_1^- decay may be represented as:

$$^{198}_{79}\text{Au} \rightarrow {}^{198}_{80}\text{Hg} + {}^{0}_{-1}e + E(\beta_1^-) + E(\gamma_1)$$

Where, $E(\gamma_1) = 1.088$ MeV

Now, $E(\beta_1^-) = \left[m\left({}^{198}_{79}\text{Au}\right) - m\left({}^{198}_{80}\text{Hg}\right) - m_e\right] \times 931.5 - E(\gamma_1)$

where $m\left({}^{198}_{79}\text{Au}\right)$ and $m\left({}^{198}_{80}\text{Hg}\right)$ are masses of the ${}^{198}_{79}\text{Au}$ and ${}^{198}_{80}\text{Hg}$ nuclei.

$$\therefore \quad E\left(\beta_1^-\right) = \left[\left\{M\left({}^{198}_{79}\text{Au}\right) - 79\, m_e\right\} - \left\{M\left({}^{198}_{80}\text{Hg}\right) - 80\, m_e\right\} - m_e\right] \times 931.5 - 1.088$$

$$= \left[M\left({}^{198}_{79}\text{Au}\right) - M\left({}^{198}_{80}\text{Hg}\right)\right] \times 931.5 - 1.088$$

$$= (197.968233 - 197.966760) \times 931.5 - 1.088$$

$$= 1.372 - 1.088 = 0.284 \text{ MeV}$$

The emission of β_2^- decay may be represented as:

$${}^{198}_{79}\text{Au} \rightarrow {}^{198}_{80}\text{Hg} + {}^{0}_{-1}e + E(\beta_2^-) + E(\gamma_2)$$

As in case of β_1^- decay, it can be deduced that

$$\therefore \quad E(\beta_2^-) = \left[M\left({}^{198}_{79}\text{Au}\right) - M\left({}^{198}_{80}\text{Hg}\right)\right] \times 931.5 - E(\gamma_2)$$

$$= 1.372 - 0.412 = 0.960 \text{ MeV.}$$

13.30. *Calculate and compare the energy released by (a) fusion of 1.0 kg of hydrogen deep within the sun, and (b) the fission of 1.0 kg of U^{235} in a fission reactor.*

Sol. In sun, four hydrogen nuclei fuse to form a helium nucleus with the release of 26 MeV energy.

$\therefore$ Energy released by fusion of 1 kg of hydrogen

$$= \frac{6 \times 10^{23} \times 26}{4} \times 10^3 \text{ MeV}$$

$$E_1 = 39 \times 10^{26} \text{ MeV}$$

As energy released in fission of one atom of ${}_{92}U^{235}$

$$= 200 \text{ MeV}$$

$\therefore$ Energy released in fission of 1 kg of ${}_{92}U^{235}$

$$= \frac{6 \times 10^{23} \times 1000}{235} \times 200 \text{ MeV}$$

$$E_2 = 5.1 \times 10^{26} \text{ MeV}$$

$$\frac{E_1}{E_2} = \frac{39 \times 10^{26}}{5.1 \times 10^{26}} = 7.65$$

i.e., energy released in fusion is 7.65 times the energy released in fission.

13.31. *Suppose India has a target of producting by 2020 A.D., 2×10^5 MW of electric power, ten percent of which was to be obtained from nuclear power plants. Suppose we are given that on an average, the efficiency of utilization (i.e., conversion to electrical energy) of thermal energy produced in a reactor is 25%. How much amount of fissionable uranium would our country need per year by 2020? Take the heat energy per fission of U^{235} to be about 200 MeV.*

Sol. Target of producing electric power = 100,000 MW. Required electric power from nuclear plants

$$= 100000 \times \frac{10}{100} = 10{,}000 \text{ MW}$$

Therefore, required electric energy from nuclear plants per year

$$= (10{,}000 \times 10^6 \text{ W}) \times 365 \times 24 \times 60 \times 60$$
$$= 3.1536 \times 10^{17} \text{ J}$$

Electrical energy recovered from the fission of one U^{235} nucleus

$$= 200 \times \frac{25}{100} = 50 \text{ MeV}$$
$$= 50 \times 1.6 \times 10^{-13}$$
$$= 8 \times 10^{-12} \text{ J}$$

$\therefore$ Number of fissions of U^{235} nucleus required.

$$= \frac{3.1536 \times 10^7}{8 \times 10^{-12}} = 3.942 \times 10^{28}$$

Number of moles of U^{235} required per year

$$= \frac{3.942 \times 10^{28}}{6.023 \times 10^{23}} = 6.5449 \times 10^4$$

Therefore, mass of U^{235} required per year

$$= 6.5449 \times 10^4 \times 235$$
$$= 1538.054 \text{ g} = 1.538054 \text{ kg.}$$

MORE QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *Two nuclei have mass numbers in the ratio 27:125. What is the ratio of their nuclear radii?*

Ans. Here, $A_1 : A_2 = 27:125$

$\Rightarrow \quad \frac{A_1}{A_2} = \frac{27}{125}$

As, $R = R_0 A^{1/3}$

$\therefore \quad \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$

$$= \left(\frac{27}{125}\right)^{1/3} = \frac{3}{5}$$

$R_1 : R_2$ (ratio of radii) = 3:5.

Q. 2. *Two nuclei have mass numbers in the ratio 1: 3. What is the ratio of their nuclear densities?*

Ans. As the nuclear density is same for all nuclei.

$\therefore \quad \rho_1 : \rho_2 = 1:1$

Q. 3. *Out of the two characteristics – the mass number (A) and the atomic number (Z) of a nucleus, which one does not change during nuclear β-decay?*

Ans. As, $\beta = {}_{-1}e^0$

Thus, the mass number (A) of the element ${}_Z X^A$ does not change during the β-decay.

Q. 4. *Name the absorbing material used to control the reaction rate of neutrons in a nuclear reactor.*

Ans. Heavy water.

Q. 5. *Define the term 'activity' of a radionuclide. Write its SI unit.* (AI CBSE 2007; AI CBSE 2006 C)

Ans. The number of radioactive disintegrations taking place per second in a given sample is called the activity of a sample.

Its *SI* unit is becquerel.

Q. 6. *The radioactive isotope D decays according to the sequence.*

$$D \xrightarrow{\beta^-} D_1 \xrightarrow{\alpha\text{-particle}} D_2$$

If the mass number and atomic number of D_2 are 176 and 71 respectively, what is (i) the mass number, (ii) atomic number of D?

Ans. Decay process can be written as:

$$^{172}_{70}D \xrightarrow{^{0}_{-1}e} {}^{172}_{69}D_1 \xrightarrow{^{4}_{2}He} {}^{176}_{71}D_2$$

(*i*) Mass number of D = 172

(*ii*) Atomic number of D = 70.

Q. 7. *Draw the graph showing the distribution of kinetic energy of electrons emitted during beta decay.*

Ans.

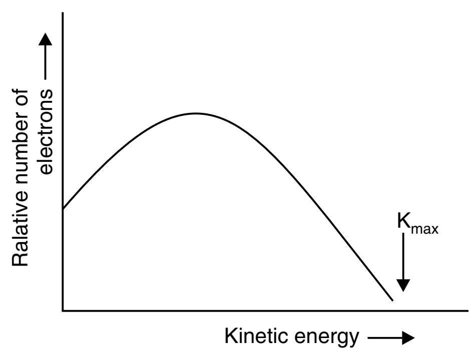

Fig. 13.13

Q. 8. *What is the nuclear radius of Fe^{125}, if that of Al^{27} is 3.6 fermi?*

Ans. Nuclear radius, $R \propto A^{1/3}$

$$\frac{R_{Fe}}{R_{Al}} = \left(\frac{A_{Fe}}{A_{Al}}\right)^{1/3} = \left(\frac{125}{27}\right)^{1/3} = \frac{5}{3} = 1.67$$

$$R_{Fe} = 1.67 \times R_{Al} = \frac{5}{3} \times 3.6 = 6 \text{ fermi.}$$

Q. 9. *Name the reaction which takes place when a slow neutron beam strikes. $^{235}_{92}U$ nuclei. Write the nuclear reaction involved.*

Ans. Nuclear fission of nuclei takes place when a slow neutron strikes $^{235}_{92}U$.

$$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^{1}_{0}n + \text{energy.}$$

Q. 10. *'Heavy water is often used as a moderator in thermal nuclear reactors'? Give reason.*

Ans. Heavy water is used as a moderator because its mass is nearest to that of a neutron and it has negligible chances for neutron absorption.

Q. 11. *A nucleus ${}_nX^m$ emits one alpha particle and one beta particle. Find the mass number and atomic number of the product nucleus.*

Ans.

$$ {}_nX^m \xrightarrow[\alpha\text{-decay}]{{}_2He^4} {}_{n-2}X^{m-4} \xrightarrow[\beta\text{-decay}]{{}_{-1}e^0} {}_{n-1}X^{m-4} $$

$\therefore$ Mass number of product nucleus = $m - 4$

and atomic number of product nucleus = $n - 1$.

Q. 12. *Define decay constant.*

Ans. Decay constant of a radioactive element is the reciprocal of the time during which the number of atoms left in the sample reduces to $\frac{1}{e}$ times the original number of atoms in the sample.

Q. 13. *Which one of ${}^7_3X$ and ${}^4_3Y$ is likely to be more stable? Give reason.*

Ans. ${}^7_3X$ will be more stable than ${}^4_3Y$, because the **odd-even pair is more stable** than odd-odd pair.

Q. 14. *Name two radioactive elements which are not found in observable quantities?*

Ans. Tritium and Plutonium.

Q. 15. *Why is the ionizing power of α-particle greater than that of gamma rays?*

Ans. The ionizing power of α-particle greater than that of gamma rays, because α-particle is positively charged and can interact more strongly with matter than gamma rays.

Q. 16. *The isotope ${}^{16}_8O$ has 8 protons, 8 neutrons and 8 electrons while ${}^8_4Be$ has 4 protons 4 neutrons and 4 electrons. Yet the ratio of their atomic masses is not exactly same. Why?*

Ans. The binding energy does not only depend upon the N/P ratio but, it also varies with the increase in mass number. In the present case, the binding energy per nucleon increases with the increase in the mass number for lighter nuclei.

Q. 17. *Does the ratio of neutrons to proton in a nucleus increase, decrease or remain the same after emission of*

(1) α-particle *(2) β-particle.*

Ans. (1) Increases (This is why, only heavy elements emit α-particle)

(2) Decreases (This is why, only light elements emit β-particle)

Q. 18. *Which one is unstable among neutron, proton, electron and α-particle?*

Ans. Neutron. It decays into proton and electron.

Q. 19. *Name two elementary particles which have almost infinite life time.*

Ans. Electron and proton have almost infinite life time.

Q. 20. *A radioactive material has a half-life of 1 minute. If one of the nuclei decays now, when will the next one decay?*

Ans. The next nucleus can decay any time.

Q. 21. *How are average life and decay constant related?*

Ans. $\tau = \frac{1}{\lambda}$.

Q. 22. *Write the equation of decay of the radioactive nuclei.*

Ans. $N = N_0 e^{-\lambda t}$.

Q. 23. *Why is a neutron most effective as a bullet in nuclear reaction?*

Ans. This is because a neutron carries no charge. It can hit the nucleus directly without being repelled by the nucleus or electrons.

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *Define the activity of a radionuclide. Write its SI unit. Give a plot of the activity of a radioactive species versus time.*

How long will a radioactive isotope, whose half-life is T years, take for its activity to reduce to $\frac{1}{8}$th of its initial value? (AI CBSE 2009)

Ans. For activity of a radionuclide, see Q. 5 (Very Short Answer Type Questions).

The graph between the activity of a radioactive species and time is given in figure 13.14.

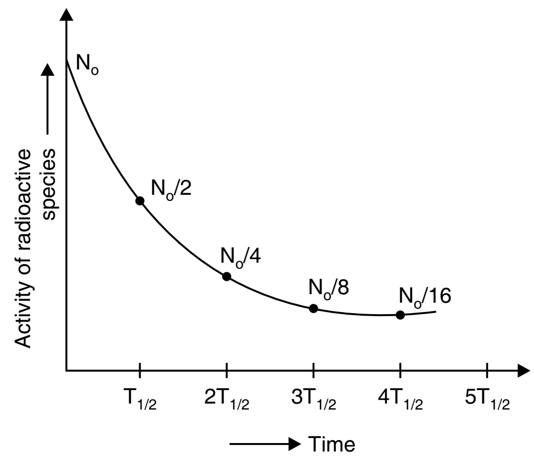

Fig. 13.14

Time taken for activity of a radioactive isotope to reduce to $\frac{1}{8}$th of its initial value

$= 3T_{1/2} = 3T.$ $(\because\ T_{1/2} = T)$

Q. 2. *What is the activity of one gram of $^{226}_{88}Ra$, whose half-life is 1622 years?*

Ans. The number of atoms in 1g of radium is

$$N = \frac{6.023 \times 10^{23}}{226} = 2.666 \times 10^{21} \text{ atoms}$$

The decay constant is related to the half-life by

$$\boxed{\lambda = \frac{0.693}{T_{1/2}}} = \left(\frac{0.693}{1622\,y}\right)\left(\frac{1y}{365\,d}\right)\left(\frac{1d}{8.64 \times 10^4\,s}\right) = 1.355 \times 10^{-11}\,s^{-1}$$

The activity is then found from

$$\boxed{\text{Activity} = \lambda N} = (1.355 \times 10^{-11}\ s^{-1})\ (2.666 \times 10^{21}) = 3.612 \times 10^{10} \text{ disintegrations/s.}$$

The definition of the curie is 1 Ci = 3.7×10^{10} disintegrations/s. This is approximately equal to the value found above.

Q. 3. *Why is it necessary to slow down the neutrons, produced through the fission of $^{235}_{92}U$ nuclei (by neutrons), to sustain a chain reaction? What type of nuclei are (preferably) needed for slowing down fast neutrons?*

Ans. Since slow neutrons have a much higher intrinsic probability of inducing fission in $^{235}_{92}U$ than fast neutrons.

Any substance which is used to slow down fast moving neutrons to thermal energies is called a moderator. Moderators are provided along with the fissionable nuclei for slowing down fast neutrons. The commonly used moderators are water, heavy water (D_2O) and graphite.

Q. 4. *It is found from an experiment that the radioactive substance emits one beta particle for each decay process. Also an average of 8.4 beta particles are emitted each second by 2.5 milligram of substance. The atomic weight of substance is 230. What is the half-life?*

Ans. The activity = 8.4 sec^{-1}

Number of atoms in kilomole (*i.e.,* 230 kg) = 6.02×10^{26}

$$\therefore \qquad N = \frac{6.02 \times 10^{26}}{230} \times 2.5 \times 10^{-6} = 6.54 \times 10^{18}$$

$$8.4 = \lambda N = \lambda \times 6.54 \times 10^{18}$$

$$\lambda = \frac{8.4 \times 10^{-18}}{6.54} = 1.28 \times 10^{-18}/\text{sec}$$

$$\text{Half-life } \boxed{T = \frac{0.693}{\lambda}} = \frac{0.693}{1.28 \times 10^{-18}} = 5.41 \times 10^{17} \text{ sec}$$

$$= \frac{5.41 \times 10^{17}}{3.16 \times 10^{7}} = 1.7 \times 10^{10} \text{ years.}$$

Q. 5. *Would the energy be released or needed for the following D-T reaction $^2_1H + ^3_1H \rightarrow ^4_2He + ^1_0n$ to occur?*

Given: $m(^2_1H) = 2.014102$ u, $m(^3_1H) = 3.016049$ u

$m(^4_2He) = 4.002603$ u, $m(^1_0n) = 1.008665$ u

and $1\ u = 931\ MeV/c^2$

Calculate this energy in MeV.

Ans. Mass (^2_1H) + mass (^3_1H) = 2.014102 + 3.016049

= 5.030151 u

$m(^4_2\text{He}) + m(^1_0n)$ = 4.002603 + 1.008665

= 5.011268 u

Energy is released in this reaction. ($\because$ as mass of reactant is larger than praducts)

Mass defect (Δm) = 5.030151 − 5.011268

= 0.018863 u

Now, energy required = (Δm) × 931 MeV

= 0.018863 × 931 MeV

= 17.561453 MeV

= 17.56 MeV.

Q. 6. *In the deuterium-tritium fusion reaction find the rate at which deuterium and tritium are consumed to produce 1 MW. The Q-value of deuterium-tritium reaction is 17.6 MeV. You can assume that the efficiency is 100%.*

Ans. Energy released per fusion = 17.6 MeV

Number of fusion reactions to produce 1 MW

$$= \frac{10^6}{17.6 \times 1.6 \times 10^{-19} \times 10^6} = 3.55 \times 10^{17}$$

In each reaction one atom of deuterium and one atom of tritium are consumed.

Mass of 3.55×10^{17} atoms of deuterium consumed per second

$$= \frac{2 \times 3.55 \times 10^{17}}{6.023 \times 10^{23}} \text{ g} = 1.1788 \times 10^{-6} \text{ g/s}$$

$$= 1.179 \times 10^{-9} \text{ kg/s.}$$

Mass of tritium consumed per second $= \dfrac{3 \times 3.55 \times 10^{7}}{6.023 \times 10^{23}}$ g/s = 1.768×10^{-6} g/s

$$= 1.768 \times 10^{-9} \text{ kg/s}$$

Q. 7. *If a nucleus $^{56}_{26}Fe$ splits into two nuclei of $^{28}_{13}Al$, would the energy be released or needed for this process to occur?*

Given, mass m ($^{56}_{26}Fe$) = 55.93494 u, m ($^{28}_{13}Al$) = 27.98191 u

and 1 u = 931 MeV/C²

Calculate this energy in MeV.

Ans. Yes, the energy be released as the nuclear reaction is fission which is given as

$$^{56}_{26}\text{Fe} \rightarrow 2\,^{28}_{13}\text{Al} + \text{energy}$$

$$\text{Mass defect } (\Delta m) = 2 \times m\,(^{28}_{13}\text{Al}) - m\,(^{56}_{26}\text{Fe})$$
$$= 2 \times 27.98191 - 55.93494$$
$$= 55.96382 - 55.93494$$
$$= 0.02888 \text{ u}$$

Energy required = 0.02888 × 931 ($\because$ as mass of reactants is smaller than product)

= 26.88728 MeV.

Q. 8. *A radionuclide sample has No nuclei at t = 0. Its number of undecayed nuclei get reduced to No/e at t = τ. What does the term 'τ' stand for? Write, in terms of 'τ', the time interval 'T', in which half of the original number of nuclei of this radionuclide would have got decayed.*

Ans. τ is the average life time of the radioactive element.

and $\tau = 1.44\ T$

i.e., the average life time of a radioactive element is 1.44 times the half-life of the element.

Q. 9. *Draw the graph to show variation of binding energy per nucleon with mass number of different atomic nuclei. Calculate binding energy/nucleon of $^{40}_{20}Ca$ nucleus.*

Given:

mass of $^{40}_{20}Ca$ = 39.962589 u

mass of proton = 1.007825 u

mass of neutron = 1.008665 u

and 1 u = 931 MeV/c².

Ans. The graph showing the variation of binding energy per nucleon with mass number of atomic nuclei is shown on below.

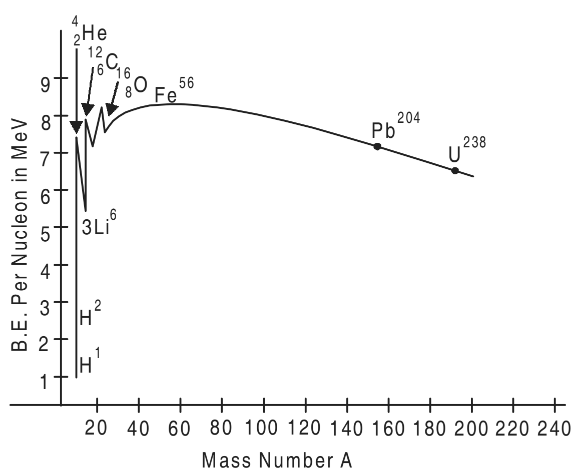

Fig. 13.15

Numerical:

$$Q = [20\, m_n + 20\, m_p - m\,(Ca)]\, c^2$$
$$= (20 \times 1.008665 + 20 \times 1.007825 - 39.962589)\, c^2$$
$$= 0.367211 \text{ u} \times \frac{931.5}{u} \text{ MeV}$$
$$= 342 \text{ MeV}$$

Hence, the B.E. per nucleon of $^{40}_{20}$Ca nucleus

$$= \frac{Q}{A} = \frac{342}{40} = 8.55 \text{ MeV/nucleon.}$$

Q. 10. *Calculate the amount of energy released during the α-decay of* $^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$

Given:

$$\text{atomic mass of } ^{238}_{92}U = 238.05079 \ u$$
$$\text{atomic mass of } ^{234}_{90}Th = 234.04363 \ u$$
$$\text{atomic mass of } ^{4}_{2}He = 4.00260 \ u$$
$$1 \ u = 931.5 \ MeV/c^2$$

Is this decay spontaneous? Give reasons.

Ans. The energy released in the α-decay is

$$Q = [m\,(^{238}_{92}\text{U}) - m\,(^{234}_{90}\text{Th}) - m\,(^{4}_{2}\text{He})]\, c^2$$
$$= [238.05079 - 234.04363 - 4.00260] \times 931.5 \text{ MeV}$$
$$= 0.00456 \times 931.5 = 4.25 \text{ MeV}$$

Since Q-value is positive, the decay process is spontaneous.

Q. 11. *(a) Define the activity of a radioactive nucleus and state its SI unit.*

(b) Two radioactive nuclei X and Y initially contain equal number of atoms. The half life is 1 hour and 2 hours respectively. Calculate the ratio of their rates of disintegration after two hours.

Ans. (*a*) The activity of a radioactive substance may be defined as the rate at which the nuclei of its atoms in the sample disintegrate. Its *SI* unit is becquerel (or *Bq*).

(*b*) After two hours, the ratio of radioactive sample of *X* left,

$$N_1 = \left(\frac{1}{2}\right)^{2/1} N_0 = \frac{1}{4} N_0$$

After two hours, the ratio of radioactive sample of *Y* left,

$$N_2 = \left(\frac{1}{2}\right)^{2/2} N_0 = \frac{1}{2} N_0$$

$$\boxed{R = \lambda N = \frac{0.693\text{ N}}{T_{1/2}}} \qquad \left(R \text{ is rate of decay } \frac{dN}{dt}\right)$$

or $$R \propto \frac{N}{T_{1/2}}$$

So $$\frac{R_1}{R_2} = \frac{(T_{1/2})_2}{(T_{1/2})_1} \times \frac{N_1}{N_2} = \frac{2}{1} \times \frac{\frac{N_0}{4}}{\frac{N_0}{2}} = 1$$

Hence, the activity of the two samples will be equal after 2 hours.

Q. 12. *Define the term decay constant of a radioactive nucleus.*

Two nuclei P, Q have equal number of atoms at t = 0. Their half lives are 3 hours and 9 hours respectively. Compare their rates of disintegration, after 18 hours from the start.

Ans. For definition of term 'decay constant', refer Q. 12 (Very Short Answer Type Questions).

Numerical:

Number of half lives of P in 18 hours $= \frac{18}{3} = 6$

Number of nuclei of P left undecayed after 6 half lives

$$N_1 = N\left(\frac{1}{2}\right)^6$$

Number of half lives of Q in 18 hours $= \frac{18}{9} = 2$

Number of nuclei of Q left undecayed after 2 half lives

$$N_2 = N\left(\frac{1}{2}\right)^2$$

$$\frac{R_1}{R_2} = \frac{\lambda_1 N_1}{\lambda_2 N_2} = \frac{T_2 N_1}{T_1 N_2} \qquad \left[\because R = \lambda N \text{ and } T = \frac{0.693}{\lambda}\right]$$

$$\therefore \qquad \frac{R_1}{R_2} = \frac{9}{3} \times \frac{N\left(\frac{1}{2}\right)^6}{N\left(\frac{1}{2}\right)^2} = \frac{3}{16}$$

Hence, $R_1 : R_2 = 3:16$.

Q. 13. (*a*) *Define the activity of a radioactive nucleus and state its SI unit.*

(*b*) *Tritium has a half-life of 12.5 years against β-decay. What fraction of a sample of pure tritium will remain undecayed after 37.5 years?*

Ans. (*a*) Refer Q. 5 (Very Short Answer Type Questions)

(*b*)
$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

Hence,
$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{37.5}{12.5}} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}.$$

Q. 14. *A radioactive sample contains 2.2 mg of pure* $^{11}_{6}C$, *which has half-life period of 1224 seconds. Calculate*

(*i*) *the number of atoms present initially.*

(*ii*) *the activity when 5 μg of the sample will be left.*

Ans. (*i*) The number of atoms present intially

$$= \frac{2.2 \times 10^{-3} \times 6.023 \times 10^{23}}{11} = 1.2 \times 10^{20} \text{ atoms}$$

(*ii*) The number of atoms present in 5 μg

$$= \frac{5 \times 10^{-6} \times 6.023 \times 10^{23}}{11} = 2.74 \times 10^{17} \text{ atoms}$$

The activity,

$$\boxed{R = \lambda N} = \frac{0.693\,N}{T_{1/2}} = \frac{0.693 \times 2.74 \times 10^{17}}{1224} = 1.55 \times 10^{14}.$$

Q. 15. *A neutron is absorbed by a $^{6}_{3}Li$ nucleus with the subsequent emission of an alpha particle.*

(i) Write the corresponding nuclear reaction.

(ii) Calculate the energy released, in MeV, in this reaction.

[Given: mass $^{6}_{3}Li$ = 6.015126 u; mass (neutron) = 1.0086654 u

mass (alpha particle) = 4.0026044 u

and mass (triton) = 3.0100000 u.

Take 1 u = 931 MeV/c^2].

Ans. Nuclear reaction is given as

$$^{6}_{3}\text{Li} + ^{1}_{0}n \rightarrow ^{4}_{2}\text{He} + ^{3}_{1}\text{H} + Q$$

Mass defect (Δm)

$= [m\,(^{6}_{3}\text{Li}) + m\,(^{1}_{0}n) - m\,(^{4}_{2}\text{He}) - m\,(^{3}_{1}\text{H})]$

$= [6.015126 + 1.0086654 - 4.0026044 - 3.01]$

$= [7.0237914 \text{ u} - 7.0126044 \text{ u}]$

$= 0.0111870 \text{ u}$

Energy released $= 0.0111870 \times 931$

$= 10.415$ MeV.

Q. 16. *Calculate the force between two fission fragments of equal masses and sizes that are produced in the fission of $^{239}_{94}Pu$ (by a thermal neutron) in which 4 neutrons are emitted. Given R_0 = 1.2 fm.*

Ans. Total mass number of $^{239}_{94}\text{Pu}$ and neutron = 240

Mass number of each fragment $= \frac{240 - 4}{2} = 118$

Atomic number of each fragment $= \frac{94}{2} = 47$

Radius of each nucleus formed by the fission of $^{239}_{94}\text{Pu}$ is

$R = R_0\,A^{1/3}$

$= 1.2 \times 10^{-15} \times (118)^{1/3} = 5.886 \times 10^{-15}$ m

Distance between the centre of the two fragments

$= 2 \times 5.886 \times 10^{-15} \text{ m} = 11.77 \times 10^{-15}$ m

Electrostatic force between them

$$= \frac{1}{4\pi\varepsilon_0}\frac{(47 \times 1.6 \times 10^{-19})^2}{(11.77 \times 10^{-15})^2}\text{ N} = 3.763 \times 10^3 \text{ N}.$$

Q. 17. *Explain, with the help of a nuclear reaction in each of the following cases, how the neutron to proton ratio changes during (i) α-decay and (ii) β-decay?*

Ans. (*i*) $$^{238}_{92}\text{U} \rightarrow ^{234}_{90}\text{Th} + ^{4}_{2}\text{He}$$

Neutron to proton ratio before α-decay

$$= \frac{238 - 92}{92} = \frac{146}{92}$$

Neutron to proton ratio after α-decay

$$= \frac{234 - 90}{90} = \frac{144}{90}$$

Since, $\frac{144}{90} > \frac{146}{92}$

Thus, the neutron to proton ratio increases is an α-decay.

(*ii*) $$^{210}_{83}\text{Bi} \rightarrow {}^{210}_{84}\text{Po} + {}^{0}_{-1}e$$

Neutron to proton ratio before β-decay

$$= \frac{210 - 83}{83} = \frac{127}{83}$$

Neutron to proton ratio after β-decay

$$= \frac{210 - 84}{84} = \frac{126}{84}$$

Since, $\frac{126}{84} < \frac{127}{83}$

Thus, the neutron to proton ratio **decreases** in a β-decay.

Q. 18. *Group the following six nuclides into three pairs of (i) isotones, (ii) isotopes and (iii) isobars:*

$$^{12}_{6}C,\ ^{3}_{2}He,\ ^{198}_{80}Hg,\ ^{3}_{1}H,\ ^{197}_{79}Au,\ ^{14}_{6}C$$

How does the size of a nucleus depend on its mass number? Hence explain why the density of nuclear matter should be independent of the size of the nucleus.

Ans. Isotopes: $^{12}_{6}\text{C}$, $^{14}_{6}\text{C}$

Isobars: $^{3}_{2}\text{He}$, $^{3}_{1}\text{H}$

Isotones: $^{198}_{80}\text{Hg}$, $^{197}_{79}\text{Au}$

Radius of nucleus, $\boxed{R = R_0 A^{1/3}}$ ...(*i*)

where, $R_0 \rightarrow$ range of nuclear force (or Nuclear Unit Radius)

The density, $\rho = \frac{M}{\frac{4}{3}\pi R^3}$ ...(*ii*)

But, $M \propto A$ ...(*iii*)

From (*i*) and (*iii*),

$R \propto M^{1/3} \Leftrightarrow R^3 \propto M \Leftrightarrow R^3 = kM$ where $k \rightarrow$ proportionality constant ...(*iv*)

From (*ii*) and (*iv*), we have

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} = \frac{3}{4\pi}\frac{M}{kM} = \text{constant.}$$

Q. 19. *Write the nuclear reactions for the following:*

(*i*) *α-decay of* $^{204}_{84}Po$

(*ii*) *β^--decay of* $^{32}_{15}P$

(*iii*) *β^+-decay of* $^{11}_{6}C$.

Ans. (*i*) $^{204}_{84}Po \rightarrow ^{200}_{82}Pb + ^{4}_{2}He$

(*ii*) $^{32}_{15}P \rightarrow ^{32}_{16}S + ^{0}_{-1}e + \bar{\nu}$

(*iii*) $_{6}C^{11} \rightarrow ^{11}_{5}B + ^{0}_{+1}e + \bar{\nu}$.

Q. 20. *The binding energy per nucleon of $^{16}_{8}O$ is 7.97 MeV and that of O^{17} is 7.75 MeV. Calculate the energy required to remove a neutron from $^{17}_{8}O$.*

Ans. The nuclear reaction is represented as

$$^{17}_{8}O \longrightarrow ^{16}_{8}O + ^{1}_{0}n$$

If m_p represents the mass of a proton and m_n that of a neutron, the energy required to remove a neutron from $^{17}_{8}O$ is given by

$$E = \text{energy equivalent of (mass of } ^{16}_{8}O + m_n - \text{mass of } ^{17}_{8}O)$$

$$= (8\ m_p + 8\ m_n - 7.97 \times 16) + m_n - (8\ m_p + 9\ m_n - 7.75 \times 17)$$

$$= [7.75 \times 17 - 7.97 \times 16] \text{ MeV} = 4.23 \text{ MeV}.$$

Q. 21. *The half-life of $^{238}_{92}U$ against α-decay is 4.5 × 10^9 years. Calculate the activity of 1g sample of $^{238}_{92}U$.*

Ans. $T_{1/2} = 4.5 \times 10^9$ years $= 4.5 \times 10^9 \times 3.156 \times 10^7$ s

Number of atoms in 1g uranium,

$$N = \frac{6.023 \times 10^{23}}{238} \text{ atoms}$$

Activity of the sample = λN

$$= \frac{0.693}{T_{1/2}} \times N$$

$$= \frac{0.693 \times 6.023 \times 10^{23}}{4.5 \times 3.156 \times 10^{16} \times 238}$$

$$= 1.235 \times 10^4 \text{ disintegration/second.}$$

Q. 22. (*a*) *Show that the decay rate 'R' of a sample of a radionuclide is related to the number of radioactive nuclei 'N' at the same instant by the expression R = λ N.*

(*b*) *The half-life of $^{238}_{92}U$ against α-decay is 1.5 × 10^{17} s. What is the activity of a sample of $^{238}_{92}U$ having 25 × 10^{20} atoms?*

Ans. (*a*) Rate of disintegration of a radioactive sample,

$$R = -\frac{dN}{dt} \quad ...(i)$$

According to radioactive decay law,

$$-\frac{dN}{dt} \propto N$$

or, $$-\frac{dN}{dt} = \lambda N \quad ...(ii)$$

∴ $$R = \lambda N$$ [By equation (*i*) and (*ii*)] (*b*) Activity of a sample

$$R = \lambda N = \frac{0.693}{T_{1/2}} \times N \qquad \left(\because \ \lambda = \frac{0.693}{T_{1/2}}\right)$$

$$= \frac{0.693}{1.5 \times 10^{17}} \times 25 \times 10^{20}$$

$$= 11550 \text{ disintegrations/second.}$$

III. LONG ANSWER TYPE QUESTIONS

Q. 1. *Define nuclear force. Give its two most important characteristics.*

What is the energy released if all the deuterium atoms in a lake of cross sectional area 2.56×10^5 (km)^2 and depth 80 m is used in fusion? Given abundance of ${}_1^2H$ = 0.0156% of hydrogen density of water = 10^3 kg m^{-3} energy released due to fusion of one atom of ${}_1^2H$ = 7.17 MeV.

Ans. **For nuclear force, see text.**

Numerical: The volume V of lake is

$$V = 2.56 \times 10^5 \times 10^6 \times 80 \text{ m}^3$$

$$= 2.048 \times 10^{13} \text{ m}^3$$

$\therefore$ Mass of water, M, in lake = $2.048 \times 10^{13} \times 10^3$

$$= 2.048 \times 10^{16} \text{ kg} = 2.048 \times 10^{19} \text{ gm}$$

Now, 18 g of water contains 2 g of hydrogen.

$\therefore$ Mass of hydrogen atom in lake = $\frac{1}{9} \times 2.048 \times 10^{19}$ g

$\therefore$ Number of atoms of hydrogen in lake = $6.023 \times 10^{23} \times \frac{2.048 \times 10^{19}}{9}$

$$= 1.37 \times 10^{42}$$

Since abundance of ${}_1^2$H is only 0.0156% of hydrogen atoms, the number of ${}_1^2$H atoms in lake

$$= 1.37 \times 10^{42} \times 1.56 \times 10^{-4}$$

$$= 2.137 \times 10^{38}$$

Energy released due to fusion of one atom of

$${}_1^2\text{H} = 7.17 \text{ MeV}$$

$\therefore$ Energy released when all ${}_1^2$H atoms present undergo fusion

$$= 2.137 \times 10^{38} \times 7.17 \text{ MeV}$$

$$= 1.532 \times 10^{39} \text{ MeV.}$$

Q. 2. *An unstable element is produced in nuclear reactor at a constant rate R. If its half-life β^--decay is $T_{1/2}$, how much time, in terms of $T_{1/2}$, is required to produce 50% of the equilibrium quantity?*

Ans. We have,

$$\text{Rate of increase of element} = \frac{\text{number of nuclei by reactor}}{1 \text{ second}} - \frac{\text{number of nuclei decaying}}{1 \text{ second}}$$

$$\frac{dN}{dt} = R - \lambda N \quad \text{or} \quad \frac{dN}{dt} + \lambda N = R$$

The solution to this is the sum of the homogeneous solution, $N_h = ce^{-\lambda t}$, where c is a constant, and a particular solution, $N_l = \frac{R}{\lambda}$.

$$N = N_h + N_p = ce^{-\lambda t} + \frac{R}{\lambda}$$

The constant c is obtained from the requirement that the initial number of nuclei be zero,

$$N(0) = 0 = c + \frac{R}{\lambda} \quad \text{or} \quad c = -\frac{R}{\lambda}$$

so that
$$N = \frac{R}{\lambda}(1 - e^{-\lambda t})$$

The equilibrium value is $(t \to \infty) = R/\lambda$. Setting N equal to 1/2 of this value gives

$$\frac{1}{2}\left(\frac{R}{\lambda}\right) = \frac{R}{\lambda}(1 - e^{-\lambda t})$$

$$e^{-\lambda t} = \frac{1}{2} \quad \Rightarrow \quad t = \frac{\ln 2}{\lambda} = T_{1/2}$$

The result is independent of R.

Q. 3. *State the laws of radioactivity.*

A radioactive substance has a half-life period of 30 days. Calculate (i) time taken for $\frac{3}{4}$ of original number of atoms to disintegrate and (ii) time taken for $\frac{1}{8}$ of the original number of atoms to remain unchanged.

Ans. Refer text for laws of radioactivity.

Numerical:

Number of atoms disintegrated = $\left(\frac{3}{4}\right) N_0$

Number of atoms left after time t,

$$N = N_0 - \frac{3}{4}N_0 = \frac{1}{4}N_0$$

Number of half lives in time t days,

T = Half life time

n = no. of half lives

t = time for disintegrates

$$\boxed{n = \frac{t}{T}} = \frac{t}{30}$$

Number of nuclei left after n half lives is given by

$$\boxed{N = N_0\left(\frac{1}{2}\right)^n}$$

$$\therefore \quad \frac{N_0}{4} = N_0\left(\frac{1}{2}\right)^{t/30} \quad \text{or} \quad (2)^{t/30} = 4 = (2)^2$$

$$\text{or,} \quad \frac{t}{30} = 2 \quad \text{or} \quad t = 60 \text{ days}$$

(*ii*) Since, $$\boxed{N = N_0 \left(\frac{1}{2}\right)^n}$$

$$\therefore \quad \frac{N_0}{8} = N_0 \left(\frac{1}{2}\right)^{t/30}$$

or, $$(2)^{t/30} = 8 = (2)^3$$

or, $$\frac{t}{30} = 3$$

or, $$t = 90 \text{ days.}$$

Q. 4. *It is proposed to use the nuclear fusion reaction*

$$^2_1H + ^2_1H \longrightarrow ^4_2He$$

in a nuclear reactor of 200 MW rating. If the energy from the above reaction is used with a 25% efficiency in the reactor, how many grams of deuterium fuel will be needed per day? (The masses of ^2_1H *and* ^4_2He *are 2.0141 atomic mass unit and 4.0026 atomic mass unit (u) respectively.)*

Ans. The given nuclear reaction is

$$^2_1\text{H} + ^2_1\text{H} \longrightarrow ^4_2\text{He}$$

The 'mass defect' for this reaction is

$$\Delta m = (2 \times 2.0141 - 4.0026) \text{ u}$$
$$= (4.0282 - 4.0026) \text{ u} = 0.0256 \text{ u}$$

The energy released in this reaction is, therefore,

$$\Delta E' = 0.0256 \times 931 \text{ MeV} = 23.8336 \text{ MeV}$$
$$= 23.8336 \times 1.6 \times 10^{-13} \text{ J} = 38.134 \times 10^{-13} \text{ J}$$

Since the reaction has a utilization efficiency of 25%, the energy utilized per reaction is

$$\Delta E = 38.134 \times 10^{-13} \times \frac{25}{100} \text{ J} = 9.533 \times 10^{-13} \text{ J}$$

The reactor rating is 200 MW. Hence the total energy required per day is

$$E = 200 \times 10^6 \frac{\text{J}}{\text{s}} \times 24 \times 60 \times 60 \text{ s}$$
$$= 172.8 \times 10^{11} \text{ J}$$

Now, 2 deuterium atoms provide us with an 'available energy' of 9.533×10^{-13} J. Hence the number of deuterium atoms needed per day is

$$n = \frac{E}{\Delta E} = \frac{172.8 \times 10^{11} \times 2}{9.533 \times 10^{-13}} = 36.25 \times 10^{24}$$

Now, 1 mole (= 6.02×10^{23} atoms) of deuterium has a mass of 2.0141 g. Hence, the mass of deuterium needed per day is

$$m = \frac{2.0141 \times 36.25}{6.02 \times 10^{23}} \times 10^{24} \text{ g}$$
$$= 121.3 \text{ g.}$$

QUESTIONS ON HIGH ORDER THINKING SKILLS (HOTS)

Q. 1. *A nucleus with A = 235 splits into two new nuclei whose mass number are in the ratio of 2:1. Find the radii of the new nuclei.*

Ans. We know that the radius R of a nucleus of mass number A is given by the relation.

$$\boxed{R = R_0 A^{1/3}}$$

where experiments show that the value of R_0 is 1.4 fm. Let A_1 and A_2 represent the mass numbers of the new nuclei which are formed when the nucleus with mass number A splits into two halves. Let R_1 and R_2 be the radii of the new nuclei so formed. Then we have

$$A_1 = \frac{1}{3}(235)$$

and $$A_2 = \frac{2}{3}(235)$$

Using, $$R = R_0 (A)^{1/3} \quad \text{we have}$$

$$R_1 = R_0 (A_1)^{1/3}$$

$$= (1.4 \text{ fm}) \left(\frac{235}{3}\right)^{1/3} = 5.99 \text{ fm}$$

and $$R_2 = R_0 (A_2)^{1/3}$$

$$= (1.4 \text{ fm}) \left(\frac{235 \times 2}{3}\right)^{1/3} = 7.55 \text{ fm.}$$

Q. 2. *The radioactive decay rate of a radioactive element is found to be 10^3 disintegrations/sec., at a certain time. If half-life of the element is one second, what would be the decay rate after 1 sec., and after 3 sec.?*

Ans. It is known that radioactive decay rate is directly proportional to the number of nuclei left.

N_0 corresponds to 10^3 (disintegrations/sec.)

As, half-life T = 1 sec., therefore,

(*i*) Number of half lives in 1 sec., n = 1

As, $$\boxed{N = N_0 \left(\frac{1}{2}\right)^n} \quad \therefore \quad N = 1000 \left(\frac{1}{2}\right)^1 = 500$$

$\therefore$ Number of disintegrations/sec. after one sec. = 500

(*ii*) Number of half lives in 3 sec. = $n = \frac{3}{1} = 3$

As, $$\boxed{N = N_0 \left(\frac{1}{2}\right)^n}$$

$\therefore$ $$N = 1000 \left(\frac{1}{2}\right)^3 = \frac{1000}{8} = 125$$

$\therefore$ Number of disintegrations/sec. after three sec. = 125.

Q. 3. *The disintegration rate of a certain radioactive sample at any instant is 4750 disintegrations per minute. 5 minutes after, the rate becomes 2700 disintegrations per minute. Calculate the (i) decay constant and (ii) half-life of sample.*

($\log_{10}$ 1.76 = 0.2455).

Ans. As rate of disintegration is proportional to number of atoms present

$$\therefore \qquad \frac{N_0}{N} = \frac{4750}{2700} = 1.76$$

Now, from $\boxed{N = N_0 e^{-\lambda t}}$

$$\frac{N_0}{N} = e^{\lambda t}; \quad \boxed{\log_e \frac{N_0}{N} = \lambda t}$$

$$\lambda = \frac{\log_e (N_0/N)}{t} = \frac{2.3026 \log_{10} (1.76)}{t}$$

$$= \frac{2.3026 \times 0.2455}{5} = 0.1131 \text{ min}^{-1}.$$

Q. 4. *A radioactive isotope X has a half-life of 3 seconds. At t = 0 second, a given sample of this isotope X contains 8000 atoms. Calculate (a) its decay constant, (b) the time t_1, when 1000 atoms of the isotope X remain in the sample and (c) the number of decays per second in the sample at $t = t_1$ second.*

Ans. (*a*) $\lambda = \frac{0.6931}{T} = \frac{0.6931}{3} = 0.231 \text{ s}^{-1}$

(*b*) $\boxed{t = \frac{1}{\lambda} \log_e \frac{N_0}{N}} = \frac{2.3026}{0.231} \log_{10} 8 = 9\text{s}$

(*c*) $\boxed{-\frac{dN}{dt} = \lambda N} = 0.231 \times 1000 = 231 \text{ s}^{-1}$.

Q. 5. *Calculate the mass of 1 curie of R_aB (Pb^2) from its half-life of 26.8 minutes.*

Ans. Activity of $R_aB = \frac{dN}{dt} = 1$ curie $= 3.7 \times 10^{10}$ disintegrations/s

Half-life of R_aB (T) = 26.8 minutes

If λ is the disintegration constant of R_aB, we have

$$\boxed{\lambda = \frac{0.693}{T}}$$

$$= \frac{0.693}{26.8 \times 60} \text{ s}^{-1}$$

Let N be the number of atoms of R_aB having an activity of 1 curie. Then we have

$$\boxed{\left|\frac{dN}{dt}\right| = \lambda N}$$

or, $$N = \frac{\left|\frac{dN}{dt}\right|}{\lambda}$$

$$\therefore \qquad N = \frac{3.7 \times 10^{10} \times 26.8 \times 60}{0.693}$$

Further we know that 6.02×10^{23} atoms = 1 gram atom = 214 g. Therefore, the mass of R_aB having an activity of 1 curie

$$= \frac{214 \times 3.7 \times 10^{10} \times 26.8 \times 60}{6.02 \times 10^{23} \times 0.693}$$

$$= 30.52 \times 10^{-9} \text{ g}.$$

Q. 6. *Using the present-day abundance of the two main uranium isotopes and assuming that the abundance ratio could never have been greater than unity, estimate the maximum possible age of the Earth's crust. Given that the present-age ratio of main uranium isotopes is 137.8:1.*

Ans. The uranium isotopes involved are ${}^{238}_{92}U$ and ${}^{235}_{92}U$ with a present abundance ratio of 137.8:1.

Now, for ${}^{235}_{92}U$ we have $N_{238} = N_{0238}\, e^{-\lambda_s t}$ where N_{238} and N_{0238} refer to present and original number of ${}^{238}_{92}U$ atoms involved. t is measured from $t = 0$, *i.e.*, is the age of the crust and λ_s is the radioactive decay constant of ${}^{238}_{92}U$ such that

$$\lambda_s = \frac{0.693}{T_s}$$

where T_s is the corresponding half-life,

$$\frac{N_{238}}{N_{0238}} = e^{-\lambda_s t} = e^{\frac{-0.693}{T_{238}}t}$$

Similarly,

$$\frac{N_{235}}{N_{0235}} = e^{-\lambda_s t} = e^{\frac{-0.693}{T_{235}}t}$$

$$\therefore \quad \frac{N_{238}}{N_{235}} \cdot \frac{N_{0235}}{N_{0238}} = \exp\left[0.693\, t\left(\frac{1}{T_{235}} - \frac{1}{T_{238}}\right)\right]$$

Now, $T_{235} = 7.13 \times 10^6$ years

and $T_{238} = 4.5 \times 10^6$ years

Also, $\frac{N_{238}}{N_{235}} = 137.8$ and we assume $\frac{N_{0235}}{N_{0238}} = 1$, the maximum value with $t = t_{max}$.

$$\therefore \quad \log_{10} 137.8 = 0.4343 \times 0.693 \times t_{max} \times (8.2 \times 10^{-8})$$

where t is in year

$$2.1302 = 0.4343 \times 0.693 \times t_{max} \times 8.2 \times 10^{-8}$$

From which $t_{max} = 6 \times 10^9$ years.

Q. 7. *Why heavy stable nucleus must contain more neutrons than protons? What is the effect on neutron to proton ratio in a nucleus when*

(i) an electron is emitted (ii) a positron is emitted?

Ans. Heavy nuclei generally have N/P ratio lower than 1. In order to be stable, the N/P ratio must tend to 1. Hence, heavy stable nucleus must contain more neutrons than protons.

(*i*) N/P ratio increases

(*ii*) N/P ratio decreases.

Q. 8. *We are given the following atomic masses:*

$${}^{238}_{92}U = 238.05079\ u \qquad {}^{4}_{2}He = 4.00260\ u$$

$${}^{234}_{90}Th = 234.04363\ u \qquad {}^{1}_{1}H = 1.00783\ u$$

$${}^{237}_{91}Pa = 237.05121\ u$$

Here the symbol Pa is for the element protactinium (Z = 91).

(a) *Calculate the energy released during the alpha decay of* $^{238}_{92}U$.

(b) *Calculate the kinetic energy of the emitted α-particles.*

(c) *Show that* $^{238}_{92}U$ *cannot spontaneously emit a proton.*

Ans. (*a*) The alpha decay of $^{238}_{92}\text{U}$ is given by

$$^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}$$

The energy released in this process is given by

$$Q = (M_u - M_{\text{Th}} - M_{\text{He}})\, c^2$$

Substituting the atomic masses as given in the data, we find

$$\begin{aligned} Q &= (238.05079 - 234.04363 - 4.00260)\ u \times c^2 \\ &= (0.00456\ u)\ c^2 \\ &= (0.00456\ u)\ (931.5\ \text{MeV/u}) \\ &= 4.25\ \text{MeV} \end{aligned}$$

(*b*) The kinetic energy of the α-particle

$$E_\alpha \approx \left(\frac{A-4}{A}\right) Q = \frac{234}{238} \times 4.25 = 4.18\ \text{MeV}$$

(*c*) If $^{238}_{92}\text{U}$ spontaneously emits a proton, the decay process would be

$$^{238}_{92}\text{U} \rightarrow {}^{237}_{91}\text{Pa} + {}^{1}_{1}\text{H}$$

The Q for this process to happen is

$$\begin{aligned} &= (M_U - M_{\text{Pa}} - M_{\text{H}})\, c^2 \\ &= (238.05079 - 237.05121 - 1.00783)\ u \times c^2 \\ &= (-\ 0.00825\ u)\ c^2 \\ &= -\ (0.00825\ u)\ (931.5\ \text{MeV/u}) \\ &= -\ 7.68\ \text{MeV} \end{aligned}$$

Since the Q-value for this process is negative, it cannot proceed spontaneously. I will require 7.68 MeV energy.

Q. 9. *You are given two nuclei* $^{7}_{3}X$ *and* $^{4}_{3}Y$. *Explain giving reasons, as to which one of the two nuclei is likely to be more stable?*

Ans. In case of $^{7}_{3}X$

$$\frac{\text{neutron number}}{\text{proton number}} = \frac{7-3}{3} = \frac{4}{3} = 1.33$$

In case of $^{4}_{3}Y$

$$\frac{\text{neutron number}}{\text{proton number}} = \frac{4-3}{3} = \frac{1}{3} = 0.33$$

For stability, this ratio has to be close to one.

Obviously, nucleus $^{7}_{3}X$ is more stable than the nucleus $^{4}_{3}Y$.

Q. 10. *10 mg of carbon from living matter produce 200 counts per minute due to a small proportion of the radioactive isotope carbon-14. A piece of ancient wood of mass 10 mg is found to give 50 counts per minute. Find the age of the wood assuming that carbon-14 content of the atmosphere has remained unchanged. The half-life of carbon-14 is 5700 years.*

Ans. Since the amount of ^{14}C in the atmosphere remains constant and a living matter represents the amount of ^{14}C present in the atmoshere, the ancient wood, which has now 50 counts per minute from 10 mg, in the beginning, must have had the same count rate as from living matter. Thus, the age of the ancient wood is the time in which the ^{14}C count rate has decreased from the initial value of 200 counts per minute from 10 mg to the final value of 50 counts per minute from 10 mg.

We know that $T_{1/2}$ for carbon is 5700 years. If n is no of half lives then

$$N = N_0 \cdot \left(\frac{1}{2}\right)^n$$

$$50 = 200 \left(\frac{1}{2}\right)^n$$

$$\frac{50}{200} = \left(\frac{1}{2}\right)^n$$

$$\frac{1}{4} = \left(\frac{1}{2}\right)^n$$

$$\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^n$$

n = 2 half lives required time

= 5700 × 2 = 11400 years = 1.14×10^4 years.

Q. 11. *The mean-life of a radioactive sample is T_m. What is the time in which 50% of this sample would get decayed?*

Ans. The half-life period

$$T_{1/2} = 0.693\ T_m$$

To decay by 50% of the sample one half-life is required $\therefore$ The same will take 0.693 T_m time to decay by 50%.

Q. 12. *Name the physical quantity whose SI unit is becquerel (Bq). How is this quantity related to (i) disintegration constant, (ii) half-life, and (iii) mean-life of the radioactive element?*

Ans. Bq is the SI unit of radioactivity.

(*i*) Radioactivity

$$R = R_0 e^{-\lambda t} \quad \text{or} \quad R = \lambda N$$

where, λ is disintegration constant.

(*ii*) $$R = \left[\frac{0.693}{T_{1/2}}\right] N$$

where $T_{1/2}$ is the half-life period.

(*iii*) $$R = \left[\frac{1}{T_{1/2}}\right] \mathrm{N}$$

where T_m is the mean-life.

Q. 13. *Write the equations for the two types of β-decay. Why is it very difficult to detect the neutrino?*

Ans. For negative β-decay

$$^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}_{1}e^{0} + \overline{\nu}$$

For positive β-decay

$$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}_{1}e^{0} + \nu$$

Neutrino is an uncharged particle which interacts very weakly with matter and hence escapes undetected.

MULTIPLE CHOICE QUESTIONS

1. The half-life period of a radioactive substance is 5 min. The amount of substance decayed in 20 min will be

(*a*) 93.75% (*b*) 75% (*c*) 25% (*d*) 6.25%

2. A nucleus $_nX^m$ emits one alpha and two beta particles (β). The resulting nucleus is

(*a*) $_nX^{m-4}$ (*b*) $_{n-2}X^{m-4}$ (*c*) $_{n-4}X^{m-4}$ (*d*) $_nX^{m-5}$

3. Half-lives of two redioactive substances A and B are respectively 20 minutes and 40 minutes. Initially the samples of A and B have equal number of nuclei. After 80 minutes the ratio of remaining numbers of A and B nuclei is

(*a*) 1 : 16 (*b*) 4 : 1 (*c*) 1 : 4 (*d*) 1 : 1

4. In a nuclear fission, 0.1% mass is converted into energy. The energy released by the fission of 1 kg mass will be

(*a*) 9×10^{16} J (*b*) 9×10^{19} J

(*c*) 9×10^{13} J (*d*) 9×10^{17} J

5. The half-life of I^{131} is 8 days. Given a sample of I^{131} at time $t = 0$, we can assert that

(*a*) no nucleus will decay before $t = 4$ days

(*b*) no nucleus will decay before $t = 8$ days

(*c*) all nuclei will decay before $t = 16$ days

(*d*) a given nucleus may decay after $t = 0$

6. In the nuclear reaction

$_6C^{11} \rightarrow {}_5B^{11} + \beta^+ + X$

What does X stands for?

(*a*) An electron (*b*) A proton (*c*) A neutron (*d*) A neutrino

7. The mass of neutron and proton are 1.0087 and 1.0073 u respectively. If the neutrons and protons combine to form a helium nucleus of mass 4.0015 u, the binding energy of the helium nucleus will be

(*a*) 28.4 MeV (*b*) 20.8 MeV (*c*) 27.3 MeV (*d*) 14.2 MeV

8. The thermal neutrons in a nuclear reactor may be regarded as a gas at a temperature T°K which obeys the laws of kinetic theory. Then the de-Broglie wavelength of such thermal neutrons in terms of temperature T_1 mass of neutron m is given by (*a*) $\lambda = \frac{h}{\sqrt{3mKT}}$ (*b*) $\lambda = \frac{\lambda}{\sqrt{6mKT}}$

(*c*) $\lambda = \frac{\lambda}{\sqrt{5mKT}}$ (*d*) $\lambda = \frac{\lambda}{\sqrt{2mKT}}$

9. Nucleus of an atom of mass no. 24 and charge no. 11 consists of

(*a*) 11 electrons 11 protons and 13 neutrons

(*b*) 11 electrons, 11 protons and 11 neutrons

(*c*) 11 protons and 13 neutrons

(*d*) 11 protons and 13 electrons

10. *Rn* decays into *Po* by emitting an α-particle with half life of 4 days. A sample contains 6.4×10^{10} atoms of *Rn*. After 12 days, the number of atoms of *Rn* left in the sample will be

(*a*) 3.2×10^{10} (*b*) 0.53×10^{10}

(*c*) 2.1×10^{10} (*d*) 0.8×10^{10}

11. It is possible to understand nuclear fission on the basis of the

(*a*) meson theory of the nuclear forces

(*b*) proton-proton cycle

(*c*) independent particle model of the nuclear

(*d*) liquid drop model of the nucleus

12. What per cent of original radioactive substances is left after 5 half-life times?

(*a*) 20% (*b*) 3% (*c*) 5% (*d*) 10%

13. Which of the following is wrongly matched?

(*a*) Barometer-Pressure (*b*) Lactometer-Milk

(*c*) Coulomb's law-charges (*d*) Nuclear reactor-electron

14. Which of the following particles has similar mass to that of the protons?

(*a*) Proton (*b*) Neutron (*c*) Positron (*d*) Neutrino

15. A radioactive substance decays to $\frac{1}{16}$th of its initial activity in 40 days. The half-life of the radioactive substance expressed in days is

(*a*) 20 (*b*) 5 (*c*) 10 (*d*) 2.5

Answers

1. (*a*)	**2.** (*a*)	**3.** (*c*)	**4.** (*c*)	**5.** (*d*)
6. (*d*)	**7.** (*a*)	**8.** (*a*)	**9.** (*c*)	**10.** (*d*)
11. (*d*)	**12.** (*b*)	**13.** (*d*)	**14.** (*c*)	**15.** (*c*).

TEST YOUR SKILLS

1. Calculate the half-life period of a radioactive substance if its activity drops to 1/16th of its initial value in 30 years.
2. Derive an expression for the radius of *n*th orbit of hydrogen atom using Bohr's postulates. Show graphically the (nature of) variation of the radius of orbit with the principal quantum number *n*.
3. Two nuclei have mass numbers in the ratio 1 : 2. What is the ratio of the their nuclei densities?

4. A radioactive nucleus 'A' undergoes a series of decays according to the following scheme:

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

The mass number and atomic number of A are 180 and 72 respectively. What are these numbers for A_4?

5. The radioactive isotopes D decays according to the sequence: $D \xrightarrow{\alpha} D_1 \xrightarrow{\beta} D_2$. If the mass number and atomic number of D_2 are 176 and 71 respectively. What is (*i*) the mass number (*ii*) atomic number of D?

6. Draw a plot of potential energy of a pair of nucleons as a function of their separation. What is the significant of negative potential energy in the graph drawn?

7. Calculate the amount of energy released during the α-decay of ${}^{238}_{92}\text{U} \rightarrow {}^{234}_{92}\text{Th} + {}^{4}_{2}\text{He}$

   Given 1 atomic mass of ${}^{238}_{92}\text{U}$ = 238.0579 u

   2 atomic mass of ${}^{234}_{92}\text{Th}$ = 234.04363 u

   3 atomic mass of ${}^{4}_{2}\text{He}$ = 4.00260 u

   1 u = 931.5 MeV/C^2

   Is the decay spontaneous? Give reason.

8. The activity of a sample of a radioactive material is A_1 at time t_1 and A_2 at time t_2 ($t_2 > t_1$). What is the mean life time?

9. If M_0 is the mass of an oxygen isotope ${}^{17}_{8}\text{O}$, M_P and M_N are the masses of a proton and neutron respectively. Find nuclear binding energy of the isotope.

   The nucleus ${}^{12}_{6}\text{C}$ absorbs an energetic neutron and emits a beta (β) particle. What will be resulting nucleus?

10. The half-life period of a radioactive element X is same as the mean life time of another radio active element Y. Initially if they have the same number of atoms then which will take a faster decay?

11. Draw a diagram to show the variation of binding energy per nucleon with mass number for different nuclei. State with reason why light nuclei usually undergo nuclear fusion.

12. You are given two nuclei ${}^{7}_{3}X$ and ${}^{4}_{3}Y$. Are they isotopes of the same element? State with reasons, which one of the two nuclei is likely to be more stable?

13. Draw a graph showing the variation of binding energy per nucleon with the mass number of different nuclei. State two inferences from the graph.

14. Explain with an example, whether the neutron-proton ratio in a nucleus increases or decreases due to beta (β) decay.

15. Why heavy water is preferred over ordinary water, as a moderator in a nuclear reactor?

16. Compare the radii of two nuclei with mass number 1 and 27 respectively.

17. How many disintegrations per second will occur in one gram of ${}_{92}\text{U}^{238}$, if its half life against alpha decay 8is 1.42×10^{17} s?

18. In the series of radioactive disintegration of ${}^{A}_{Z}X$ first one α-particle and then one β-particle is emitted. What is the atomic number and mass number of the new nucleus formed by these successive disintegrations?

19. Calculate binding energy per nucleon of ${}^{40}_{20}Ca$ nucleus.

   (Given : mass of ${}^{40}_{20}\text{Ca}$ = 39.962589 u; mass of proton = 1.007825 u; mass of neutron = 1.00865 u; 1 atomic mass unit (1u) = 931 MeV)

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