14 Oscillations

Facts that Matter

• Periodic Motion

Motions, processes or phenomena, which repeat themselves at regular intervals, are called periodic.

• Oscillatory Motion

The motion of a body is said to be oscillatory motion if it moves to and fro about a fixed point after regular intervals of time. The fixed point about which the body oscillates is called mean position or equilibrium position.

• Simple Harmonic Motion

Simple harmonic motion is a special type of periodic oscillatory motion in which

(*i*) The particle oscillates on a straight line

(*ii*) The acceleration of the particle is always directed towards a fixed point on the line.

(*iii*) The magnitude of acceleration is proportional to the displacement of the particle from the fixed point, *i.e.,* $a \propto -x$

Acceleration $= -\omega^2 x$

• Characteristics of SHM

The displacement x in SHM at time t is given by

$$x = A \sin (\omega t + \phi)$$

where the three constants A, ω and ϕ characterize the SHM, *i.e.,* they distinguish one SHM from another. A SHM can also be described by a cosine function as follows:

$$x = A \cos (\omega t + \delta)$$

• The displacement of an oscillating particle at any instant is equal to the change in its position vector during that time. The maximum value of displacement in an oscillatory motion on either side of its mean position is called "displacement amplitude" or "simple amplitude".

Thus, amplitude $A = x_{max}$.

• The time taken by an oscillating particle to complete one full oscillation to and fro about its mean (equilibrium) position is called the "time period" of SHM.

It is given by

$$T = 2\pi \sqrt{\frac{m}{k}}$$ where, k is force constant (or spring factor) of spring.

• Frequency

The number of oscillations in one second is called frequency. It is expressed in $\sec^{-1}$ or Hertz. Frequency and time period are independent of amplitude.

$$\text{Frequency } (\nu) = \frac{1}{\text{Time period}}\left(\frac{1}{T}\right).$$

- **Phase**

The quantity ($\omega t + \phi$) is called the phase of SHM at time t; it describes the state of motion at that instant. The quantity ϕ is the phase at time $t = 0$ and is called the phase constant or initial phase or epoch of the SHM. The phase constant is the time–independent term in the cosine or sine function.

If displacement of SHM is given by

$$x = A \sin (\omega t \pm \phi), \quad \text{then}$$

Particle velocity $$v = \frac{dx}{dt} = \omega A \cos (\omega t \pm \phi)$$

$$v = \omega A \sqrt{1-\sin^2(\omega t \pm \phi)} = \omega A \sqrt{1-\left(\frac{x^2}{A^2}\right)} = \omega \sqrt{A^2 - x^2}$$

- Velocity amplitude of SHM is equal to $A\omega$. Moreover, the particle velocity is ahead of displacement of the particle by an angle of $\frac{\pi}{2}$.

Phase difference between displacement and velocity is $\frac{\pi}{2}$.

- Acceleration of the particle executing S.H.M is given by

$$a = -\omega^2 A \sin (\omega t \pm \phi) = -\omega^2 \text{A}$$

The phase difference between displacement and the acceleration is π.

- The force responsible for maintaining the S.H.M. is called restoring force.

If the displacement (x) from the equilibrium position is small, the restoring force (F) acting on the body is given by

$$F = -kx$$

where k is a force constant.

- **Energy in S.H.M.**

When a body executes SHM, its energy changes between kinetic and potential, but the total energy is always constant. At any displacement x from the equilibrium position:

$$\text{Kinetic energy } (KE) = \frac{1}{2}mv^2 = \frac{1}{2}m\,\omega^2 \,(A^2 - x^2)$$

$$\text{Potential energy } (PE) = \frac{1}{2}kx^2 = \frac{1}{2}m\,\omega^2 x^2$$

$$\text{Total energy, } E = KE + PE = \frac{1}{2}m\,\omega^2 A^2$$

- Time period and frequency of a particle executing S.H.M. may be expressed in the following way

$$\text{Total period, } T = 2\pi \sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

$$\text{Total period, } \nu = \frac{1}{2\pi}\sqrt{\frac{\text{acceleration}}{\text{displacement}}}$$

- In general, the time period and frequency can be expressed as

$$T = 2\pi\sqrt{\frac{\text{inertial factor}}{\text{spring factor}}}\ ; \quad \nu = \frac{1}{2\pi}\sqrt{\frac{\text{spring factor}}{\text{inertial factor}}}$$

- **Springs in Series**

If two springs, having spring constant k_1 and k_2, are joined in series, the spring constant of the combination is given by

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

- **Springs in Parallel**

If two springs, having spring constants k_1 and k_2, are joined in parallel, the spring constant of the combination is given by

$$k = k_1 + k_2$$

- When one spring is attached to two masses m_1 and m_2, then

$$M = \frac{m_1 m_2}{(m_1 + m_2)} \quad \therefore \quad T = 2\pi\sqrt{\frac{M}{K}}$$

- **Simple Pendulum**

A simple pendulum is the most common example of bodies executing S.H.M. An ideal simple pendulum consists of a heavy point mass body suspended by a weightless inextensible and perfectly flexible string from a rigid support about which it is free to oscillate.

- The time period of simple pendulum of length 'l' is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

The time period of a simple pendulum depends on

(*i*) length of the pendulum and (*ii*) the acceleration due to gravity (g).

- A second's pendulum is a pendulum whose time period is 2s. At a place where $g = 9.8\ \text{ms}^{-2}$, the length of a second's pendulum is found to be 99.3 cm (≈ 1 m).
- If a liquid of density ρ oscillates in a vertical U-tube of uniform cross sectional area A, then the time period of oscillation is given by

$$T = 2\pi\sqrt{\frac{L}{2g}}$$

and frequency, $\nu = \frac{1}{2\pi}\sqrt{\frac{2g}{L}}$ where L is the total length of liquid column.

- If a cylinder of mass m, length L, density of material ρ and uniform area of cross section A, oscillates vertically in a liquid of density σ, then the time period of oscillation is given by

$$T = 2\pi\sqrt{\frac{L\rho}{\sigma g}}$$

• Undamped and Damped Simple Harmonic Oscillations

Undamped Simple Harmonic oscillations: When a simple harmonic system oscillates with a constant amplitude which does not change with time, its oscillations are called undamped simple harmonic oscillations.

Damped Simple Harmonic oscillations: When a simple harmonic system oscillates with a decreasing amplitude with time, its oscillations are called damped simple harmonic oscillations.

The angular frequency of the damped oscillator is given by

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

where b is damping constant.

Time period of damped oscillations is given by

$$T = \frac{2\pi}{\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}}$$

• A system is said to execute free oscillations, if on being displaced or disturbed from its position of equilibrium, it oscillates itself without outside interference.

When a system is compelled to oscillate with a frequency other than its natural frequency, it is said to execute forced oscillations.

The external force which causes forced oscillation, is of sinusoidal nature. It is given as

$$F = F_m \sin(\omega' t) \quad \text{or} \quad F_m \cos(\omega' t).$$

• Resonance is the phenomenon of setting a body into oscillations with large amplitude under the influence of some external periodic force whose frequency is exactly equal to the natural frequency of the given body. Such oscillations are called the "resonant oscillations".

• The two or more oscillations linked together in such a way that the exchange of energy takes place between them are called **coupled oscillators.** The oscillations produced by coupled oscillators are known as coupled oscillations.

• The speed of a mechanic wave depends upon the properties of the medium in which it is travelling. If E is the elastic constant and ρ is the density of the medium then the speed of the wave is given by

$$v = \sqrt{\frac{E}{\rho}}$$

• In case of electric magnetic waves, we know that they are the combinations of the oscillation of electric and magnetic fields in perpendicular directions. Their speed of propagation depends upon the permitivity and the permeability of the medium. If μ_0 is permeability and ε_0 is the permitivity of the medium in vaccum, then

$$v = \frac{1}{\sqrt{\mu_0 \cdot \varepsilon_0}}$$

- **IMPORTANT TABLES**

TABLE 14.1

Physical quantity	*Symbol*	*Dimensions*	*Unit*	*Remarks*
Period	T	$[T]$	s	The least time for motion to repeat itself
Frequency	ν (or f)	$[T^{-1}]$	s^{-1}	$v = \frac{1}{T}$
Angular frequency	ω	$[T^{-1}]$	s^{-1}	$\omega = 2\pi\nu$
Phase constant	ϕ	Dimensionless	rad	Initial value of phase of displacement in SHM
Force constant	k	$[MT^{-2}]$	Nm^{-1}	Simple harmonic motion $F = -kx$

TABLE 14.2 Sin θ as a function of angle θ

θ *(degrees)*	**θ *(radians)***	**sin θ**
0	0	0
5	0.087	0.0015
10	0.174	0.0030
15	0.262	0.0046
20	0.349	0.006

NCERT TEXTBOOK QUESTIONS SOLVED

14.1. *Which of the following examples represent periodic motion?*

(a) A swimmer completing one (return) trip from one bank of a river to the other and back.

(b) A freely suspended bar magnet displaced from its N-S direction and released.

(c) A hydrogen molecule rotating about its centre of mass.

(d) An arrow released from a bow.

Ans. (*a*) It is not a periodic motion. Though the motion of a swimmer is to and fro but will not have a definite period.

(*b*) Since a freely suspended magnet if once displaced from N-S direction and released, it oscillates about this position, it is a periodic motion

(*c*) The rotating motion of a hydrogen molecule about its centre of mass is periodic.

(*d*) Motion of an arrow released from a bow is non-periodic.

14.2. *Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion?*

(a) the rotations of earth about its axis.

(b) motion of an oscillating mercury column in a U-tube.

(c) motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower most point.

(d) general vibrations of a polyatomic molecule about its equilibrium position.

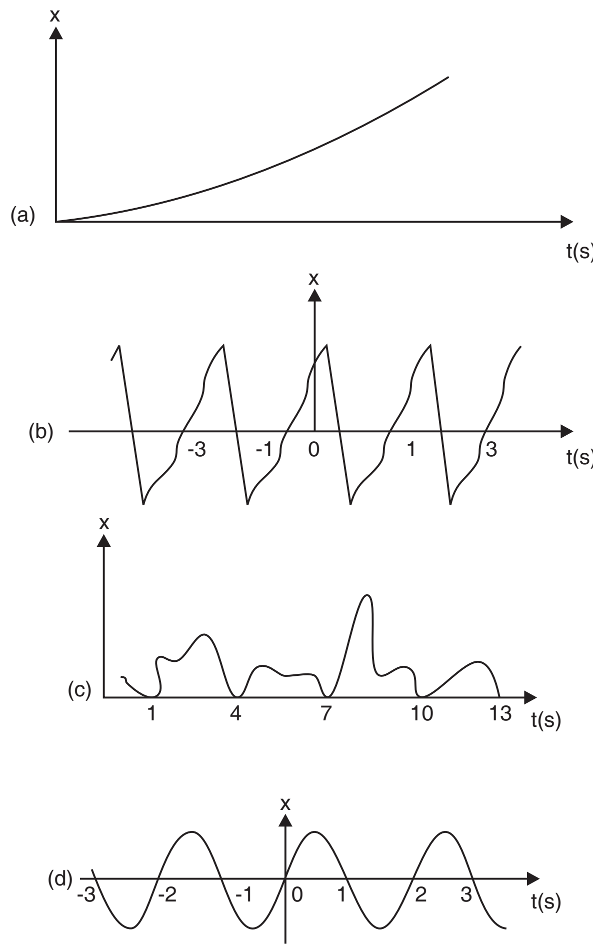
Ans. (*a*) Since the rotation of earth is not to and fro motion about a fixed point, thus it is periodic but not S.H.M.

(*b*) It is S.H.M.

(*c*) It is S.H.M.

(*d*) General vibrations of a polyatomic molecule about its equilibrium position is periodic but non SHM. Infact, it is a result of superposition of SHMs executed by individual vibrations of atoms of the molecule.

14.3. *Fig. depicts four x-t plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion (in case of periodic motion)?*



Ans. Figure (*b*) and (*d*) represent periodic motions and the time period of each of these is 2 seconds. (*a*) and (*c*) are non-periodic motions.

14.4. *Which of the following function of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (ω is any positive constant).*

(a) $\sin \omega t - \cos \omega t$ *(b)* $\sin^2 \omega t$ *(c)* $3 \cos \left(\frac{\pi}{4} - 2\,\omega\, t\right)$ *(d)* $\cos \omega t + \cos 3\,\omega t + \cos 5\,\omega t$

(e) $\exp(-\omega^2 t^2)$ *(f)* $1 + \omega t + \omega^2 t^2$.

Ans. The function will represent a periodic motion, if it is identically repeated after a fixed interval of time and will represent S.H.M if it can be written uniquely in the form of a cos $\left(\frac{2\pi t}{T}+\phi\right)$ or a sin $\left(\frac{2\pi t}{T}+\phi\right)$, where T is the time period.

(*a*) $\sin \omega t - \cos \omega t = \sqrt{2}\left[\frac{1}{\sqrt{2}}\sin \omega t - \frac{1}{\sqrt{2}}\cos \omega t\right]$

$$= \sqrt{2}\left[\sin \omega t \cos \frac{\pi}{4} - \cos \omega t \sin \frac{\pi}{4}\right] = \sqrt{2}\ \sin\left(\omega t - \frac{\pi}{4}\right)$$

It is a S.H.M. and its period is $2\pi/\omega$

(*b*) $\sin^3 \omega t = \frac{1}{3}[3\sin \omega t - \sin 3\omega t]$

Here each term $\sin \omega t$ and $\sin 3\omega t$ individually represents S.H.M. But (*ii*) which is the outcome of the superposition of two SHMs will only be periodic but not **SHMs.** Its time period is $2\pi/\omega$.

(*c*) $3\cos\left(\frac{\pi}{4} - 2\omega t\right) = 3\cos\left(2\omega t - \frac{\pi}{4}\right)$. $[\because \cos(-\theta) = \cos\theta]$

Clearly it represents SHM and its time period is $2\pi/2\omega$.

(*d*) $\cos \omega t + \cos 3\omega t + \cos 5\omega t$. It represents the periodic but not S.H.M. Its time period is $2\pi/\omega$

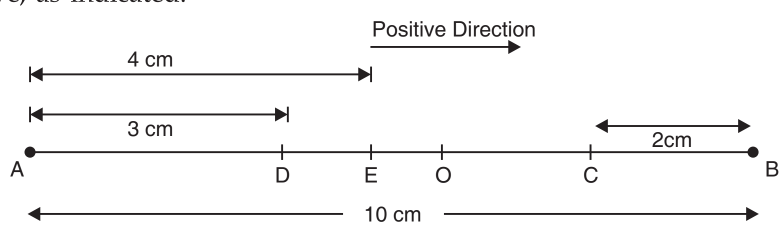
(*e*) $e^{-\omega^2 t^2}$. It is an exponential function which never repeats itself. Therefore it represents non-periodic motion.

(*f*) $1 + \omega t + \omega^2 t^2$ also represents non periodic motion.

14.5. *A particle is in linear simple harmonic motion between two points, A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is*

(a) at the end A,
(b) at the end B,
(c) at the mid-point of AB going towards A,
(d) at 2 cm away from B going towards A,
(e) at 3 cm away from A going towards B, and
(f) at 4 cm away from B going towards A.

Ans. In the fig. (given below), the points A and B, 10 cm apart, are the extreme positions of the particle in SHM, and the point O is the mean position. The direction from A to B is positive, as indicated.



(*a*) At the end A, *i.e.*, extreme position, velocity is zero, acceleration and force are directed towards O and are positive.

(*b*) At the end B, *i.e.*, second extreme position, velocity is zero whereas the acceleration and force are directed towards the point O and are negative.

(*c*) At the mid point O, while going towards A, velocity is negative and maximum. The acceleration and force both are zero.

(*d*) At 2 cm away from B, that is, at C and going towards A: v is negative; acceleration and F, being directed towards O, are also negative.

(*e*) At 3 cm away from A, that is, at D and going towards B: v is positive; acceleration and F, being directed towards O, are also positive.

(*f*) At a distance of 4 cm away from A and going towards A, velocity is directed along BA, therefore, it is positive. Since acceleration and force are directed towards OB, both of them are positive.

14.6. *Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion?*

(a) $a = 0.7x$ (b) $a = -200x^2$

(c) $a = -10x$ (d) $a = 100x^3$

Ans. Only (c) *i.e.*, $a = -10x$ represents SHM. This is because acceleration is proportional and opposite to displacement (x).

14.7. *The motion of a particle executing simple harmonic motion is described by the displacement function.*

$$x(t) = A \cos(\omega t + \phi).$$

If the initial (t = 0) position of the particle is 1 cm and its initial velocity is ω cm/s, what are its amplitude and initial phase angle? The angular frequency of the particle is π s⁻¹. If instead of the cosine function, we choose the sine function to describe the SHM: x = B sin (ωt + α), what are the amplitude and initial phase of the particle with the above initial conditions?

Ans. The given displacement function is

$$x(t) = A \cos(\omega t + \phi) \qquad ...(i)$$

At $t = 0$, $x(0) = 1$ cm. Also, $\omega = \pi\ s^{-1}$

$$\therefore \quad 1 = A \cos(\pi \times 0 + \phi) \Rightarrow A \cos\phi = 1 \qquad ...(ii)$$

Also, differentiating eqn. (*i*) w.r.t. 't'.

$$v = \frac{d}{dt}x(t) = -A\,\omega \sin(\omega t + \phi) \qquad ...(iii)$$

Now at $t = 0$, $v = \omega$

$\therefore$ from eqn. (*iii*), $\omega = -A\,\omega \sin(\pi \times 0 + \phi)$ or $A \sin\phi = -1$ ...(*iv*)

Squaring and adding eqns. (*ii*) and (*iv*).

$$A^2 \cos^2\phi + A^2 \sin^2\phi = 1^2 + 1^2 \quad \text{or} \quad A = \sqrt{2}\ \text{cm}$$

Dividing eqns. (*ii*) and (*iv*),

$$\frac{A \sin\phi}{A \cos\phi} = \frac{-1}{1} \quad \therefore \quad \tan\phi = -1 \Rightarrow \phi = \frac{3\pi}{4}$$

If instead we use the sine function, *i.e.*,

$$x = B \sin(\omega t + \alpha), \quad \text{then } v = \frac{d}{dt} B\omega \cos(\omega t + \alpha)$$

$\therefore$ At $t = 0$, using $x = 1$ and $v = \omega$, we get $1 = B \sin(\omega \times 0 + \alpha)$

or $B \sin\alpha = 1$ ...(*v*)

and $\omega = B\omega \cos(\omega \times 0 + \alpha)$ or $B \cos\alpha = 1$ ...(*vi*)

Dividing (*v*) by (*vi*),

$$\tan\alpha = 1 \quad \text{or} \quad \alpha = \frac{\pi}{4} \quad \text{or} \quad \frac{5\pi}{4}$$

Squaring (v) and (vi), we get

$$B^2 \sin^2 \alpha + B^2 \cos^2 \alpha = 1^2 + 1^2 \Rightarrow B = \sqrt{2} \text{ cm.}$$

14.8. *A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body?*

Ans. $M = 50\text{kg}, \quad y = 20\text{cm} = 0.2 \text{ m}, T = 0.60 \text{ s}$

$$F = ky \quad \text{or} \quad Mg = ky \quad \text{or} \quad k = \frac{Mg}{0.2} = \frac{50 \times 9.8}{0.2} \text{Nm}^{-1}$$

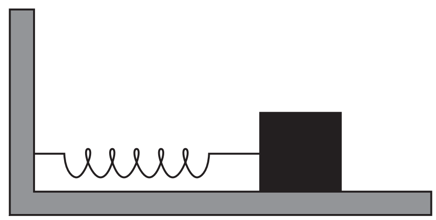
or $\quad K = 2450 \text{ Nm}^{-1}$

Now, $\quad T = 2\pi\sqrt{\frac{m}{k}} \quad \text{or} \quad T^2 = 4\pi^2 \frac{m}{k} \quad \text{or} \quad m = \frac{T^2 k}{4\pi^2}$

or $\quad m = \frac{0.6 \times 0.6 \times 2460 \times 49}{4 \times 484} \text{kg} = 22.3 \text{ kg}$

$\Rightarrow \quad mg = 22.3 \times 9.8 \text{ N} = 218.5 \text{ N} = 22.3 \text{ kgf.}$

14.9. *A spring having with a spring constant 1200 Nm^{-1} is mounted on a horizontal table as shown in Fig. A mass of 3 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released.*



Determine (i) the frequency of oscillations, (ii) maximum acceleration of the mass, and (iii) the maximum speed of the mass.

Ans. Here, $\quad K = 1200 \text{ Nm}^{-1}; \text{ m} = 3.0 \text{ kg}, a = 2.0 \text{ cm} = 0.02 \text{ m}$

(i) Frequency, $\quad \nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2 \times 3.14}\sqrt{\frac{1200}{3}} = 3.2 \text{ s}^{-1}$

(ii) Acceleration, $\quad A = \omega^2 y = \frac{k}{m} y$

Acceleration will be maximum when y is maximum *i.e.*, $y = a$

$\therefore$ max. acceleration, $A_{max} = \frac{ka}{m} = \frac{1200 \times 0.02}{3} = 8 \text{ ms}^{-2}$

(iii) Max. speed of the mass will be when it is passing through mean position

$$V_{max} = a\omega = a\sqrt{\frac{k}{m}} = 0.02 \times \sqrt{\frac{1200}{3}} = 0.4 \text{ ms}^{-1}$$

14.10. *In Exercise 9, let us take the position of mass when the spring is unstreched as x = 0, and the direction from left to right as the positive direction of x - axis. Give x as a function of time t for the oscillating mass if at the moment we start the stopwatch (t = 0), the mass is*

(a) at the mean position,

(b) at the maximum stretched position, and

(c) at the maximum compressed position.

In what way do these functions for SHM differ from each other, in frequency, in amplitude or the initial phase?

Ans. $a = 2 \text{ cm}, \quad \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3}} s^{-1} = 20 s^{-1}$

(*a*) Since time is measured from mean position,

$$x = a \sin \omega t = 2 \sin 20t$$

(*b*) At the maximum stretched position, the body is at the extreme right position. The initial phase is $\frac{\pi}{2}$.

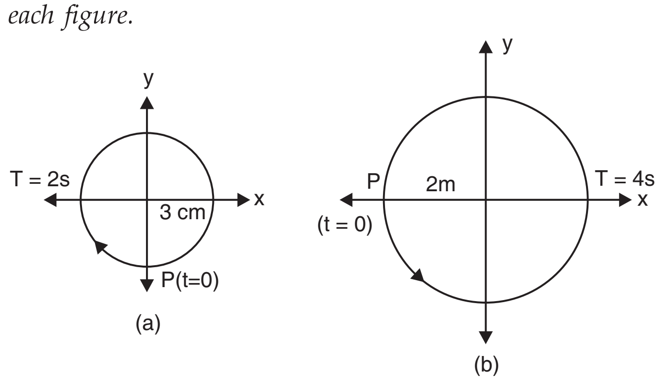
$$\therefore \quad x = a \sin (\omega t + \frac{\pi}{2}) = a \cos \omega t = 2 \cos 20\ t$$

(*c*) At the maximum compressed position, the body is at the extreme left position. The initial phase is $\frac{3\pi}{2}$.

$$\therefore \quad x = a \sin \left(\omega t + \frac{3\pi}{2}\right) = -a \cos \omega t = -2 \cos 20t$$

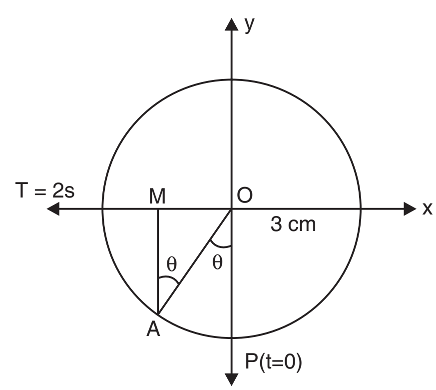
Note: *The functions neither differ in amplitude nor in frequency. They differ only in initial phase.*

14.11. *Following figures correspond to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution (i.e., clockwise or anticlockwise) are indicated on each figure.*



Obtain the corresponding simple harmonic motions of the x-projection of the radius vector of the revolving particle P in each case.

Ans. (1) Let A be any point on the circle of reference of the fig. (a) From A, draw AM perpendicular on x-axis.



If $\angle POA = \theta$, then

$\angle OAM = \theta = \omega t$

$\therefore$ In triangle OAM,

$$\frac{OM}{OA} = \sin \theta$$

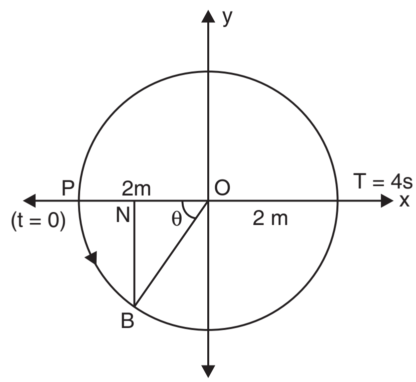
$$\therefore \quad \frac{-x}{3} = \sin \omega t = \sin \frac{2\pi}{T} t$$

[x is –ve in third quadrant]

$$\therefore \quad x = -3 \sin \frac{2\pi}{2} t \quad \text{or} \quad x = -3 \sin \pi t$$

which is the equation of SHM.

(2) Let B be any point on the circle of reference of fig. (b). From B, draw BN perpendicular on x-axis.



$$\text{Then} \quad \angle BON = \theta = \omega t$$

$$\therefore \text{ In } \Delta ONB, \quad \cos\theta = \frac{ON}{OB}$$

$$\text{or} \quad ON = OB\cos\theta$$

$$\therefore \quad -x = 2\cos\omega t$$

$$\Rightarrow \quad x = -2\cos\frac{2\pi}{T}t = -2\cos\frac{2\pi}{4}t$$

$$\therefore \quad x = -2\cos\frac{\pi}{4}t$$ which is equation of SHM

14.12. *Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial (t = 0) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anti-clockwise in every case: (x is in cm and t is in s)*

(a) $x = -2\sin(3t + \pi/3)$ *(b)* $x = \cos(\pi/6 - t)$

(c) $x = 3\sin(2\pi t + \pi/4)$ *(d)* $x = 2\cos\pi t$.

Ans. (a)
$$x = 2\cos\left(3t + \frac{\pi}{3} + \frac{\pi}{2}\right)$$

Radius of the reference circle, r = amplitude of SHM = 2 cm,

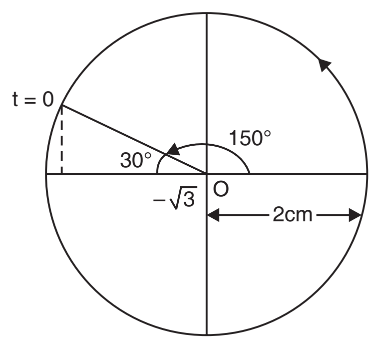
$$\text{At} \quad t = 0, \quad x = -2\sin\frac{\pi}{3}$$

$$= \frac{-2\sqrt{3}}{2} = -\sqrt{3}\text{ cm}$$

$$\text{Also} \quad \omega t = 3t \quad \therefore \omega = 3 \text{ rad/s}$$

$$\cos\phi_0 = -\frac{\sqrt{3}}{2}, \quad \phi_0 = 150°$$

The reference circle is, thus, as plotted below.



(b)
$$x = \cos\left(t - \frac{\pi}{6}\right)$$

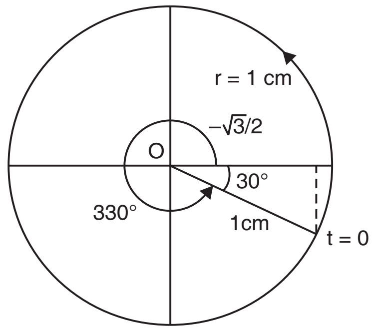
Radius of circle, r = amplitude of SHM = 1 cm.

$$\text{At} \quad t = 0, \quad x = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}\text{cm}$$

$$\text{Also} \quad \omega t = 1t \Rightarrow \omega = 1 \text{ rad/s}$$

$$\cos\phi_0 = \frac{\sqrt{3}}{2}, \quad \phi_0 = -\frac{\pi}{6}$$

The reference circle is, thus as plotted below



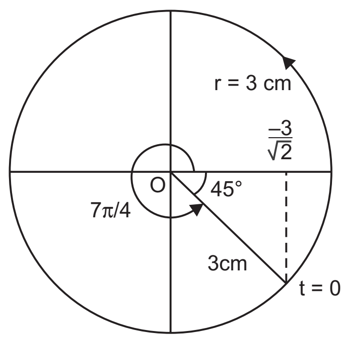
(*c*) $$x = 3\cos\left(2\pi t + \frac{\pi}{4} + \frac{\pi}{2}\right)$$

Here, radius of reference circle, r = 3 cm and at t = 0, $x = 3\sin\frac{\pi}{4} = \frac{3}{\sqrt{2}}$ cm

$$\omega t = 2\pi t \Rightarrow \omega = 2\pi \text{ rad/s}$$

$$\cos\phi_0 = \frac{\frac{3}{\sqrt{2}}}{3} = \frac{1}{\sqrt{2}}$$

Therefore, the reference circle is being shown below.



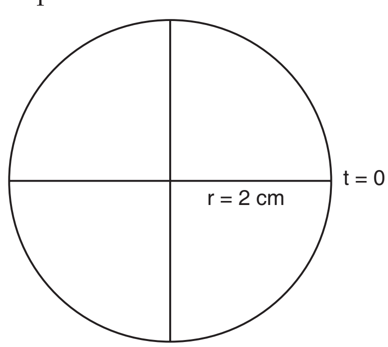
(*d*) $$x = 2\cos\pi t$$

Radius of reference circle, r = 2 cm and at t = 0, x = 2 cm

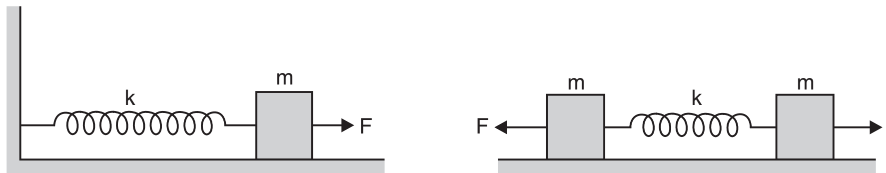
$\therefore$ $$\omega t = \pi t, \quad \text{or} \quad \omega = \pi \text{ rad}/s$$

$$\cos\phi_0 = 1, \quad \phi_0 = 0$$

The reference circle is plotted below.



14.13. *Figure (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F applied at the free end stretches the spring. Figure (b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Figure (b) is stretched by the same force F.*



(a) What is the maximum extension of the spring in the two cases?

(b) If the mass in Fig. (a) and the two masses in Fig. (b) are released free, what is the period of oscillation in each case?

Ans. (*a*) Let y be the maximum extension produced in the spring in Fig. (*a*)

Then $F = ky$ (in magnitude) $\therefore \quad y = \frac{F}{k}$

If fig. (*b*), the force on one mass acts as the force of reaction due to the force on the other mass. Therefore, each mass behaves as if it is fixed with respect to the other.

Therefore, $F = ky \Rightarrow y = \frac{F}{k}$

(*b*) In fig. (*a*), $F = -ky$

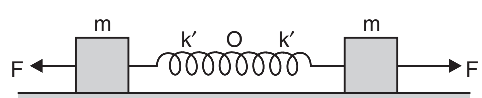
$$\Rightarrow \quad ma = -ky \Rightarrow a = -\frac{k}{m}y \quad \therefore \quad \omega^2 = \frac{k}{m} \quad i.e., \quad \omega = \sqrt{\frac{k}{m}}$$

Therefore, period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

In fig. (*b*), we may consider that the centre of the system is O and there are two springs each of length $\frac{l}{2}$ attached to the two masses, each m, so that k' is the spring factor of each of the springs.

Then, $K' = 2k$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{k'}}$$



$$= 2\pi\sqrt{\frac{m}{2k}}$$

14.14. *The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with an angular frequency of 200 rev/min, what its maximum speed?*

Ans. Stroke of piston = 2 times the amplitude

Let A = amplitude, stroke = 1 m

$\therefore \quad \Rightarrow A = \frac{1}{2}$ m.

Angular frequency, ω = 200 rad/min.

$V_{max} = ?$

We know that the maximum speed of the block when the amplitude is A,

$$V_{max} = \omega A = 200 \times \frac{1}{2} = 100 \text{ m/min.} = \frac{100}{60} = \frac{5}{3}\text{ms}^{-1} = 1.67 \text{ ms}^{-1}.$$

14.15. *The acceleration due to gravity on the surface of moon is 1.7 ms^{-2}. What is the time period of a simple pendulum on the surface of moon if its time-period on the surface of Earth is 3.5 s? (g on the surface of Earth is 9.8 ms^{-2}.)*

Ans. Here, $g_m = 1.7 \text{ ms}^{-2}; \quad g_e = 9.8 \text{ ms}^{-2}; \quad T_m = ?; \quad T_e = 3.5 \text{ s}^{-1}$

Since, $$T_e = 2\pi\sqrt{\frac{1}{g_e}} \quad \text{and} \quad T_m = 2\pi\sqrt{\frac{1}{g_m}}$$

$$\therefore \quad \frac{T_m}{T_e} = \sqrt{\frac{g_e}{g_m}} \Rightarrow T_m = T_e = \sqrt{\frac{g_e}{g_m}} = 3.5\sqrt{\frac{9.8}{1.7}} = 8.4 \text{ s.}$$

14.16. *Answer the following questions:*

(a) *Time period of a particle in SHM depends on the force constant k and mass m of the particle:*

$$T = 2\pi\sqrt{\frac{m}{k}}$$

A simple pendulum executes SHM approximately.

Why then is the time period of a pendulum independent of the mass of the pendulum?

(b) *The motion of a simple pendulum is approximately simple harmonic for small angle oscillations.*

For larger angles of oscillation, a more involved analysis shows that T is greater than $2\pi\sqrt{\frac{l}{g}}$.

Think of a qualitative argument to appreciate this result.

(c) *A man with a wrist watch on his hand falls from the top of a tower. Does the watch give correct time during the free fall?*

(d) *What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity?*

Ans. (*a*) In case of a spring, k does not depend upon m. However, in case of a simple pendulum, k is directly proportional to m and hence the ratio $\frac{m}{k}$ is a constant quantity.

(*b*) The restoring force for the bob of the pendulum is given by

$$F = -mg \sin\theta$$

If θ is small, then $\sin\theta = \theta = \frac{y}{l} \quad \therefore \quad F = -\frac{mg}{l}y$

i.e., the motion is simple harmonic and time period is $T = 2\pi\sqrt{\frac{l}{g}}$.

Cleraly, the above formula is obtained only if we apply the approximation $\sin\theta \approx \theta$.

For large angles, this approximation is not valid and T is greater than $2\pi\sqrt{\frac{l}{g}}$.

(*c*) The wrist watch uses an electronic system or spring system to give the time, which does not change with acceleration due to gravity. Therefore, watch gives the correct time.

(*d*) During free fall of the cabin, the acceleration due to gravity is zero. Therefore, the frequency of oscillations is also zero *i.e.,* the pendulum will not vibrate at all.

14.17. *A simple pendulum of length l and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?*

Ans. In this case, the bob of the pendulum is under the action of two accelerations.

(*i*) Acceleration due to gravity 'g' acting vertically downwards.

(*ii*) Centripetal acceleration $a_c = \frac{v^2}{R}$ acting along the horizontal direction.

$\therefore$ Effective acceleration, $g' = \sqrt{g^2 + a_c^2}$ or $g' = \sqrt{g^2 + \frac{v^4}{R^2}}$

Now time period, $T' = 2\pi\sqrt{\frac{1}{g}} = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \frac{v^4}{R^2}}}}$.

14.18. *A cylindrical piece of cork of density of base area A and height h floats in a liquid of density* ρ_l*. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period*

$$T = 2\pi\sqrt{\frac{h\rho}{\rho_l g}}$$

where ρ *is the density of cork. (Ignore damping due to viscosity of the liquid).*

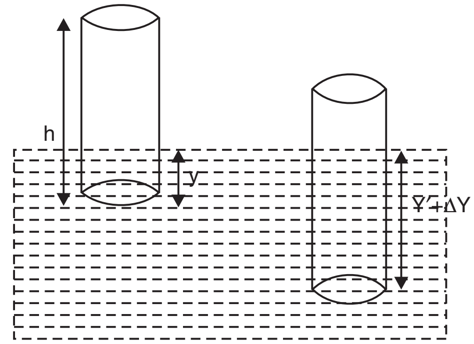
Ans. Say, initially in equilibrium, y height of cylinder is inside the liquid. Then,
Weight of the cylinder = upthrust due to liquid displaced

$\therefore \quad Ah\rho g = Ay\rho_l g$

When the cork cylinder is depressed slightly by Δy and released, a restoring force, equal to additional upthrust, acts on it. The restoring force is

$$F = A(y + \Delta y)\,\rho_l g - Ay\rho_l g = A\rho_l g\Delta y$$

$\therefore \quad$ Acceleration, $a = \frac{F}{m} = \frac{A\rho_l g\Delta y}{Ah\rho} = \frac{\rho_l g}{h\rho} . \Delta y$ and the acceleration is directed in a direction opposite to Δy. Obviously, as $a \propto -\Delta y$, the motion of cork cylinder is SHM, whose time period is given by

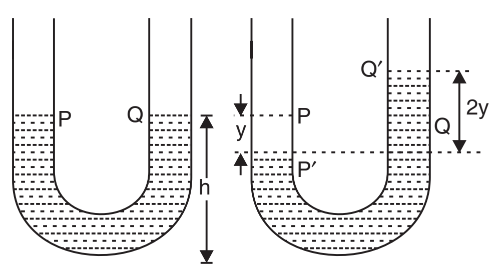


$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

$$= 2\pi\sqrt{\frac{\Delta y}{a}}$$

$$= 2\pi\sqrt{\frac{h\rho}{\rho_l g}}.$$

4.19. *One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the column of mercury in the U-tube executes simple harmonic motion.*



Ans. The suction pump creates the pressure difference, thus mercury rises in one limb of the U-tube. When it is removed, a net force acts on the liquid column due to the difference in levels of mercury in the two limbs and hence the liquid column executes S.H.M. which can be expressed as:

Consider the mercury contained in a vertical U-tube upto the level P and Q in its two limbs.

Let P = density of the mercury.

L = Total length of the mercury column in both the limbs.

A = internal cross-sectional area of U-tube.

m = mass of mercury in U-tube = LAP.

Assume, the mercury be depressed in left limb to P' by a small distance y, then it rises by the same amount in the right limb to position Q'.

$\therefore$ Difference in levels in the two limbs = $P' Q' = 2y$.

$\therefore$ Volume of mercury contained in the column of length

$$2y = A \times 2y$$

$$\therefore \quad m = A \times 2y \times \rho.$$

If W = weight of liquid contained in the column of length $2y$.

Then $W = mg = A \times 2y \times \rho \times g$

This weight produces the restoring force (F) which tends to bring back the mercury to its equilibrium position.

$$\therefore \quad F = -2\,Ay\rho g = -(2A\rho g)y$$

If a = acceleration produced in the liquid column, Then

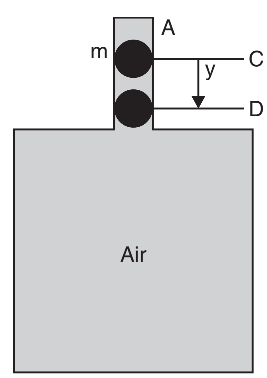
$$a = \frac{F}{m} = -\frac{(2A\rho g)y}{LA\rho} = -\frac{2A\rho g}{LA} = -\frac{2\rho g}{2h\rho}y \quad \text{...}(i) \quad (\because L = 2h)$$

where h = height of mercury in each limb. Now from eqn. (i), it is clear that $a \propto y$ and $-ve$ sign shows that it acts opposite to y, so the motion of mercury in u-tube is simple harmonic in nature having time period (T) given by

$$T = 2\pi\sqrt{\frac{y}{a}} = 2\pi\sqrt{\frac{2h\rho}{2\rho g}} = 2\pi\sqrt{\frac{h\rho}{\rho g}}$$

$$T = 2\pi\sqrt{\frac{h}{g}}$$

4.20. *An air chamber of volume V has a neck area of cross section a into which a ball of mass m just fits and can move up and down without any friction (Fig.). Show that when the ball is pressed down a little and released, it executes SHM. Obtain an expression for the time period of oscillations assuming pressure-volume variations of air to be isothermal.*



Ans. Consider an air chamber of volume V with a long neck of uniform area of cross-section A, and a frictionless ball of mass m fitted smoothly in the neck at position C, Fig. The pressure of air below the ball inside the chamber is equal to the atmospheric pressure. Increase the pressure on the ball by a little amount p, so that the ball is depressed to position D, where $CD = y$.

There will be decrease in volume and hence increase in pressure of air inside the chamber. The decrease in volume of the air inside the chamber, $\Delta V = Ay$

$$\text{Volumetric strain} = \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V} = \frac{Ay}{V}$$

$\therefore$ Bulk Modulus of elasticity E, will be

$$E = \frac{\text{stress (or increase in pressure)}}{\text{volumetric strain}} = \frac{-p}{Ay/V} = \frac{-pV}{Ay}$$

Here, negative sign shows that the increase in pressure will decrease the volume of air in the chamber.

Now, $$p = \frac{-E\,Ay}{V}$$

Due to this excess pressure, the restoring force acting on the ball is

$$F = p \times A = \frac{-E\,Ay}{V} \cdot A = \frac{-E\,A^2}{V}y \quad \text{...}(i)$$

Since $F \propto y$ and negative sign shows that the force is directed towards equilibrium position. If the applied increased pressure is removed from the ball, the ball will start executing linear SHM in the neck of chamber with C as mean position.

In S.H.M., the restoring force,

$$F = -ky \quad \text{...}(ii)$$

Comparing (*i*) and (*ii*), we have

$$\text{Spring factor, } k = EA^2/V$$

Here, $$\text{inertia factor} = \text{mass of ball} = m.$$

$$\text{Period, } T = 2\pi\sqrt{\frac{\text{inertia factor}}{\text{spring factor}}} = 2\pi\sqrt{\frac{m}{EA^2/V}} = \frac{2\pi}{A}\sqrt{\frac{mV}{E}}$$

$\therefore$ Frequency, $$\nu = \frac{1}{T} = \frac{A}{2\pi}\sqrt{\frac{E}{mV}}$$

Note. *If the ball oscillates in the neck of chamber under isothermal conditions, then $E = P$ = pressure of air inside the chamber, when ball is at equilibrium position. If the ball oscillates in the neck of chamber under adiabatic conditions, then $E = gP$, where $g = C_p/C_v$.*

4.21. *You are riding an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation. Estimate the values of (a) the spring constant k and (b) the damping constant b for the spring and shock absorber system of one wheel, assuming that each wheel supports 750 kg. g = 10 m/s^2.*

Ans. (*a*) Here, mass, M = 300 kg, displacement, x = 15 cm = 0.15 m, g = 10 m/s^2. There are four spring systems. If k is the spring constant of each spring, then total spring constant of all the four springs in parallel is

$$K_p = 4k \quad \therefore \quad Mg = k_p x = 4kx$$

$$\Rightarrow \quad K = \frac{Mg}{4x} = \frac{3000 \times 10}{4 \times 0.15} = 5 \times 10^4 \text{ N.}$$

(*b*) For each spring system supporting 750 kg of weight,

$$t = 2\pi\sqrt{\frac{m}{k}} = 2 \times 3.14 \times \sqrt{\frac{750}{5\times10^4}} = 0.77 \text{ sec.}$$

∴ Using $x = x_0\, e^{-\frac{bt}{2m}}$, we get

$$\frac{50}{100}x_0 = x_0\, e^{-\frac{b\times0.77}{2\times750}} \quad \text{or} \quad e^{\frac{0.77b}{1500}} = 2$$

Taking logarithm of both sides,

$$\frac{0.77b}{1500} = \ln 2 = 2.303 \log 2$$

$$\therefore \quad b = \frac{1500}{0.77} \times 2.303 \times 0.3010 = 1350.4 \text{ kg s}^{-1}$$

4.22. *Show that for a particle in linear SHM the average kinetic energy over a period of oscillation equals the average potential energy over the same period.*

Ans. Let the particle executing SHM starts oscillating from its mean position. Then displacement equation is

$$x = A \sin \omega t$$

$$\therefore \quad \text{Particle velocity, } v = A\omega \cos \omega t$$

$$\therefore \quad \text{Instantaneous } K.E., K = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2 \cos^2 \omega t$$

∴ Average value of K.E. over one complete cycle

$$K_{av} = \frac{1}{T}\int_0^T \frac{1}{2}mA^2\omega^2 \cos^2 \omega t\, dt = \frac{mA^2\omega^2}{2T}\int_0^T \cos^2 \omega t\, dt$$

$$= \frac{mA^2\omega^2}{2T}\int_0^T \frac{(1+\cos 2\omega t)}{2}dt = \frac{mA^2\omega^2}{4T}\left[t + \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{mA^2\omega^2}{4T}\left[(T-0) + \left(\frac{\sin 2\omega t - \sin 0}{2\omega}\right)\right] = \frac{1}{4}mA^2w^2 \qquad ...(i)$$

$$\text{Again instantaneous P.E., U} = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2A^2 \sin^2 \omega t$$

∴ Average value of P.E. over one complete cycle

$$U_{av} = \frac{1}{T}\int_0^T \frac{1}{2}m\omega^2A^2 \sin^2 \omega t = \frac{m\omega^2A^2}{2T}\int_0^T \sin^2 \omega t\, dt$$

$$= \frac{m\omega^2A^2}{2T}\int_0^T \frac{(1-\cos 2\omega t)}{2}dt = \frac{m\omega^2A^2}{4T}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{m\omega^2A^2}{4T}\left[(T-0) - \frac{(\sin 2\omega t - \sin 0)}{2\omega}\right] = \frac{1}{4}m\omega^2A^2 \qquad ...(ii)$$

Simple comparison of (*i*) and (*ii*), shows that

$$K_{av} = U_{av} = \frac{1}{4}m\omega^2A^2$$

4.23. *A circular disc, of mass 10 kg, is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. The period of torsional oscillations of found to be 1.5 s. The radius of the disc is 15 cm. Determine the torsional spring constant of the wire. (Torsional spring constant) α is defined by the relation J = – $\alpha\theta$, where J is the restoring couple and θ the angle of twist).*

Ans. $$T = 2\pi\sqrt{\frac{I}{\alpha}} \quad \text{or} \quad T^2 = \frac{4\pi^2 I}{\alpha}$$

or $$\alpha = \frac{4\pi^2 I}{T^2} \quad \text{or} \quad \alpha = \frac{4\pi^2}{T^2}\left(\frac{1}{2}MR^2\right) \quad \text{or} \quad \alpha = \frac{2\pi^2 MR^2}{T^2}$$

or $$\alpha = \frac{2(3.14)^2 \times 10 \times (0.15)^2}{(1.5)^2} \text{ Nm rad}^{-1} = \mathbf{1.97\ Nm\ rad^{-1}}.$$

4.24. *A body describes simple harmonic motion with an amplitude of 5 cm and a period of 0.2s. Find the acceleration and velocity of the body when the displacement is (a) 5 cm (b) 3 cm (c) 0 cm.*

Ans. Here, r = 5 cm = 0.05 m; T = 0.2 s; $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.2}$ = 10π rad/s

When displacement is y, then

$$\text{acceleration, } A = -\omega^2 y$$

$$\text{velocity, } V = \omega\sqrt{r^2 - y^2}$$

Case (*a*) When y = 5 cm = 0.05 m; $A = -(10\pi)^2 \times 0.05 = -5\pi^2$ m/s²

$$V = 10\pi\sqrt{(0.05)^2 - (0.05)^2} = 0.$$

Case (*b*) When y = 3 cm = 0.03 m

$$A = -(10\pi)^2 \times 0.03 = -3\pi^2 \text{ m/s}^2$$

$$V = 10\pi\sqrt{(0.05)^2 - (0.03)^2} = 10\pi \times 0.04 = 0.4\pi \text{ m/s}$$

Case (c) When $y = 0$, $A = -(10\pi)^2 \times 0 = 0$

$$V = 10\pi\sqrt{(0.05)^2 - 0^2} = 10\pi \times 0.05 = 0.5\pi \text{ m/s}.$$

4.25. *A mass attached to a spring is free to oscillate, with angular velocity ω, in a horizontal plane without friction or damping. It is pulled to a distance x_0 and pushed towards the centre with a velocity v_0 at time t = 0. Determine the amplitude of the resulting oscillations in terms of the parameters ω, x_0 and v_0.*

Ans. $$x = \alpha \cos(\omega t + \theta)$$

$$v = \frac{dx}{dt} = -a\omega \sin(\omega t + \theta)$$

When $t = 0$, $x = x_0$ and $\frac{dx}{dt} = -v_0$

∴ $$x_0 = a \cos\theta \quad ...(i)$$

and $$-v_0 = -a\,\omega \sin\theta \quad \text{or} \quad a \sin\theta = \frac{v_0}{\omega} \quad ...(ii)$$

Squaring and adding (*i*) and (*ii*), we get

$$a^2(\cos^2\theta + \sin^2\theta) = x_0^2 + \frac{v_0^2}{\omega^2} \quad \text{or} \quad a = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}.$$

ADDIATIONAL QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *What happens to the time period of a simple pendulum if its length is doubled?*

Ans. The time period is increased by a factor of $\sqrt{2}$.

Q. 2. *What is the phase difference between particle velocity and particle acceleration in SHM?*

Ans. Particle acceleration in SHM is ahead in phase by $\frac{\pi}{2}$ as compared to the particle velocity.

Q. 3. *At what points is the energy entirely kinetic and potential in SHM?*

Ans. At mean position, the energy is entirely K.E. At extreme positions, the energy is entirely P.E.

Q. 4. *How would the period of spring mass system change, when it is made to oscillate horizontally and then vertically?*

Ans. The time period remains same in both the cases.

Q. 5. *Can a simple pendulum vibrate at the centre of Earth?*

Ans. No. This is because of zero value of g at the centre of Earth.

Q. 6. *Is oscillation of a mass suspended by a spring simple harmonic in nature?*

Ans. Yes, it is if the spring is perfectly elastic.

Q. 7. *Why does the time period of a swing not change when two persons sit on it instead of one?*

Ans. $T = 2\pi\sqrt{\frac{l}{g}}$, so it does not depend upon the mass.

Q. 8. *Can a motion be periodic but not oscillatory? If your answer is yes, give an example and if not explain why?*

Ans. Yes, uniform circular motion is the example of it.

Q. 9. *The amplitude of a harmonic oscillator is doubled. How does its energy change?*

Ans. As $E \propto A^2$, the energy of harmonic oscillator will became 4 times, its original value when its amplitude is doubled.

Q. 10. *Two exactly similar simple pendula are vibrating with amplitudes 1 cm and 3 cm. What is the ratio of their energies of vibration?*

Ans. $$\frac{E_1}{E_2} = \frac{a_1^2}{a_2^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}.$$

Q. 11. *At what point the velocity and acceleration are zero in S.H.M?*

Ans. The velocity is zero at the extreme point of motion and acceleration is zero at the mean position of motion.

Q. 12. *Can the motion of an artificial satellite around earth be taken as S.H.M?*

Ans. No, it is a circular and periodic motion but not to and fro about a mean position which is essential for SHM.

Q. 13. *What provides restoring force in the following cases?*

(i) a spring compressed and then left free to vibrate.

(ii) Water disturbed in U-tube.

(iii) Pendulum disturbed from its mean position.

Ans. *(i) Elasticity of the material of the spring* *(ii) Weight of water*

(iii) Weight of pendulum.

Q. 14. *Sometimes, when an automobile picks up speed, its body begins to rattle. Why?*

Ans. This is because of resonant vibrations.

Q. 15. *On what factors does the energy of a harmonic oscillator depend?*

Ans. Energy of a harmonic oscillator depends on the mass, frequency and amplitude of oscillation.

Q. 16. *What determines the natural frequency of a body?*

Ans. Natural frequency of a body depends upon (*i*) elastic properties of the material of the body and (*ii*) dimensions of the body.

Q. 17. *What is a second's pendulum?*

Ans. A pendulum, whose time period is 2 seconds is called a second's pendulum.

Q. 18. *What fraction of the total energy is kinetic energy when the displacement is one-half of amplitude?*

Ans. $$\frac{\text{K.E.}}{\text{Total energy}} = \frac{\frac{1}{2}m\omega^2\left(a^2 - \frac{a^2}{4}\right)}{\frac{1}{2}m\omega^2 a^2} = \frac{3}{4}.$$

Q. 19. *What will be the change in the time period of a loaded spring when taken to moon?*

Ans. No change, since $T = 2\pi\sqrt{\frac{m}{k}}$.

Q. 20. *Is simple harmonic motion always linear?*

Ans. No, it is not essential. Simple harmonic motion may be either a linear simple harmonic motion or an angular SHM.

Q. 21. *Define force constant.*

Ans. Force constant is defined as the restoring force developed in a body per unit displacement.

Q. 22. *How many times in one vibration, K.E. and P.E. become maximum?*

Ans. Two times.

Q. 23. *Is the damping force constant on a system executing SHM?*

Ans. No, because damping force depends upon velocity and is more when the system moves fast and is less when the system moves slow.

Q. 24. *Two springs of force constants k_1 and k_2 are joined in series. What is the force constant of the combination?*

Ans. The force constant k of series combination is given by

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \quad \text{or} \quad k = \frac{k_1 k_2}{k_1 + k_2}.$$

Q. 25. *What will be the time period of oscillation, if the length of a second pendulum is one third?*

Ans. $$\frac{T_2^2}{T_1^2} = \frac{l_2}{l_1} = \frac{\left(\frac{l}{3}\right)}{l} = \frac{1}{3} \quad \text{or} \quad \frac{T_2^2}{T_1^2} = \frac{(2)^2}{3} \quad \text{or} \quad T_2 = \frac{2}{\sqrt{3}} \text{ s.}$$

Q. 26. *A driver wearing an electronic digital watch goes down into sea water with terminal velocity v. How will the time in the water proof watch be affected?*

Ans. It will not be affected as its action is independent of gravity and buoyant force.

Q. 27. *Two springs of force constant k_1 and k_2 are joined in parallel. What is the force constant of the combination?*

Ans. Force constant k of parallel combination is given by $k = k_1 + k_2$. Thus, force constant of the parallel combination is equal to the sum of individual force constants of two springs.

Q. 28. *Why should the amplitude of the vibrating pendulum be small?*

Ans. When amplitude of the vibrating pendulum is small, then angular displacement of the bob used in simple pendulum is small. Here the restoring force $F = mg \sin\theta = mg\,\theta = mgx/l$, where x is the displacement of the bob and l is the length of pendulum. Hence $F \propto x$. Since F is directed towards mean position, therefore the motion of the bob of simple pendulum will be S.H.M. if θ is small.

Q. 29. *What is the total energy of a simple harmonic oscillator?*

Ans. $\frac{1}{2}m\omega^2 r^2$, where r = amplitude, ω = angular frequency, m = mass of the oscillator.

Q. 30. *When a pendulum clock gains time, what adjustment should be made?*

Ans. When a pendulum clock gains time, it means it has gone fast *i.e.,* it makes more vibrations per day than required. This shows that the time period of oscillation has decreased. Therefore, to correct it, the length of pendulum should be properly increased.

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *A particle is subjected to two simple harmonic motions*

$$x_1 = A_1 \sin \omega t \quad \text{and} \quad x_2 = A_2 \sin\left(\omega t + \frac{\pi}{3}\right)$$

Find (a) the displacement at $t = 0$

(b) the maximum speed of the particle and

(c) the maximum acceleration of the particle

Ans. (*a*) At $t = 0, \quad x_1 = A_1 \sin \omega t = 0$

And $$x_2 = A_2 \sin\left(\omega t + \frac{\pi}{3}\right) = \frac{A_2\sqrt{3}}{2}$$

Thus the resultant displacement at $t = 0$ is

$$x = x_1 + x_2 = \frac{A_2\sqrt{3}}{2}$$

(*b*) $$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\frac{\pi}{3}}$$

$$A = \sqrt{A_1^2 + A_2^2 + A_1A_2}$$

The maximum speed is

$$V_{max} = \omega A = \omega\sqrt{A_1^2 + A_2^2 + A_1A_2}$$

(*c*) The maximum acceleration is

$$a_{max} = \omega^2 A = \omega^2\sqrt{A_1^2 + A_2^2 + A_1A_2}$$

Q. 2. *A spring of force constant k has a mass M suspended from it. If the spring is cut into two halves, and the same mass is attached to one of the pieces, what will be the frequencies of oscillation of the mass?*

Ans. When the spring is cut into two equal halves, the force constant of each part will be doubled.

Therefore, the original frequency,

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$$

will becomes $\nu' = \frac{1}{2\pi}\sqrt{\frac{2k}{M}} = \sqrt{2}\nu$

Q. 3. *A particle is executing SHM according to the equation*

$$x = 5 \sin \pi t$$

where x is in cm. How long will the particle take to move from the position of equilibrium to the position of maximum displacement?

Ans. The displacement of the particle varies with time according to the equation.

$$x = 5 \sin \pi t$$

Maximum displacement = amplitude = 5cm

At time $t = 0$, $x = 0$ (equilibrium position). Hence time t taken by the particle to move from $x = 0$ to $x = 5$ cm is given by

$$5 = 5 \sin \pi t \quad \text{or} \quad 1 = \sin \pi t \quad \text{or} \quad \pi t = \frac{\pi}{2} \quad \Rightarrow \quad t = 0.5 \text{ s.}$$

Q. 4. *A girl is swinging in the sitting position. How will the period of the swing be changed if she stands up?*

Ans. This can be explained using the concept of a simple pendulum. We know that the time period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \quad i.e., \quad T \propto \sqrt{l}\,.$$

When the girl stands up, the distance between the point of suspension and the centre of mass of the swinging body decreases *i.e.*, l decreases, so T will also decrease.

Q. 5. *A particle of mass 0.8 kg is executing simple harmonic motion with an amplitude of 1.0 metre and periodic time* $\frac{11}{7}$ *sec. Calculate the velocity and the kinetic energy of the particle at the moment when its displacement is 0.6 metre.*

Ans. We know that, $v = \omega\sqrt{(a^2 - y^2)}$

Further $\omega = \frac{2\pi}{T}$

$$\therefore \quad v = \frac{2\pi}{T}\sqrt{(a^2 - y^2)} = \frac{2 \times 3.14}{\left(\frac{11}{7}\right)}\sqrt{[(1.0)^2 - (0.6)^2]} = 3.2 \text{ m/sec.}$$

Kinetic energy at this displacement is given by

$$K = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.8 \times (3.2)^2 = 4.1 \text{ joule.}$$

Q. 6. *The displacement x (in cm) of an oscillating particle varies with time t (in seconds) according to the equation.*

$$x = 2 \cos (0.5\, \pi t + \frac{\pi}{3})$$

Find

(a) amplitude of oscillation
(b) the time period of oscillation
(c) the maximum velocity of the particle
(d) the maximum acceleration of the particle.

Ans. The displacement of the particle is given by

$$x = 2 \cos (0.5\, \pi t + \frac{\pi}{3}) \text{ cm}$$

To find the amplitude and time period of the oscillation, we compare this equation with

$$x = A \cos(\omega t + \delta)$$

(*a*) Amplitude $A = 2$ cm

(*b*) Angular frequency $\omega = 0.5\,\pi$ rad s^{-1}

or $$T = \frac{2\pi}{\omega} = \frac{2\pi}{0.5\pi} = 4s.$$

(*c*) Maximum velocity $V_{max} = |A\omega|$

$$= 2 \times 0.5\,\pi = \pi \text{ cm s}^{-1} = 3.142 \text{ cm s}^{-1}$$

(*d*) Maximum acceleration $a_{max} = |-\omega^2 A|$

$$= \omega^2 A = (0.5\,\pi)^2 \times 2 = \frac{\pi^2}{2}\text{cm s}^{-2} = 4.935 \text{ cm s}^{-2}.$$

Q. 7. *A block with a mass of 3.0 kg is suspended from an ideal spring having negligible mass and stretches the spring by 0.2 m.*

(a) What is the force constant of the spring?

(b) What is the period of oscillation of the block if it is pulled down and released?

Ans. (*a*) Force constant $k = \frac{F}{l} = \frac{mg}{l}$.

Here $m = 3.0$ kg and elongation in length of spring $l = 0.2$ m

$\therefore$ Force constant $k = \frac{3.0 \times 9.8}{0.2} = 174 \text{ N m}^{-1}$

(*b*) Period of oscillation $T = 2\pi\sqrt{\frac{m}{k}} = 2 \times 3.14 \times \sqrt{\frac{3}{147}} = 0.9$ s.

Q. 8. *The periodic time of a mass suspended by a spring (force constant K) is T. If the spring is cut in three equal pieces, what will be the force constant of each part? If the same mass be suspended from one piece, what will be the periodic time?*

Ans. Consider the spring be made of a combination of three springs in series each of spring constant k. The effective spring constant K is given by

$$\frac{1}{K} = \frac{1}{k}+\frac{1}{k}+\frac{1}{k}=\frac{3}{k} \quad \text{or} \quad K = \frac{k}{3} \quad \text{or} \quad k = 3K$$

$\therefore$ Time period of vibration of a body attached to the end of this spring,

$$T = 2k\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\left(\frac{k}{3}\right)}} = 2\pi\sqrt{\frac{3m}{k}} \quad ...(i)$$

When the spring is cut into three pieces, the spring constant = k,
time period of vibration of a body attached to the end of this spring,

$$T_1 = 2\pi\sqrt{\frac{m}{k}} \quad ...(ii)$$

From eqns. (*i*) and (*ii*),

$$\frac{T_1}{T} = \frac{1}{\sqrt{3}} \quad \text{or} \quad T_1 = \frac{T}{\sqrt{3}}.$$

Q. 9. *A SHM is expressed by the equation $x = A \cos(\omega t + \phi)$ and the phase angle $\phi = 0$. Draw graphs to show variation of displacement, velocity and acceleration for one complete cycle in SHM.*

Ans. Let $x = A \cos(\omega t + \phi)$ and if phase angle ϕ is zero, then

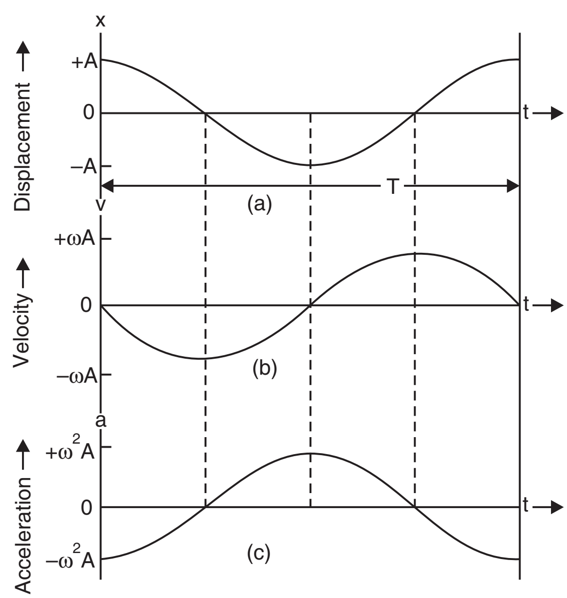
$$x = A \cos \omega t$$

$$\therefore \qquad v = \frac{dx}{dt} = -A\omega \sin \omega t \quad \text{and} \quad a = \frac{dv}{dt} = -A\omega^2 \cos \omega t = -\omega^2 x$$

Thus, values of x, v and a at different times, over one complete oscillation cycle, are

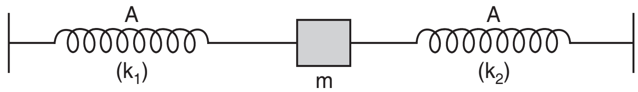
Time t	0	$\frac{T}{4}$	$\frac{T}{2}$	$\frac{3T}{4}$	T
ωt	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	A	0	$-A$	0	A
v	0	$-A\omega$	0	$+A\omega$	0
a	$-A\omega^2$	0	$A\omega^2$	0	$-A\omega^2$

With the given data we plot x–t, v–t and a–t graphs. The graphs have been shown in Fig. (*a*), (*b*) and (*c*).



Q. 10. *A particle of mass 0.1 kg is held between two rigid supports by two springs of force constants 8 N/m and 2 N/m. If the particle is displaced along the direction of the length of the springs, calculate its frequency of vibration.*

Ans. The situation is shown in the fig. When the mass is displaced along the direction of the length of the spring, one spring is compressed while the other is extended but the force due to both the springs is in the same direction. Hence the effective force constant

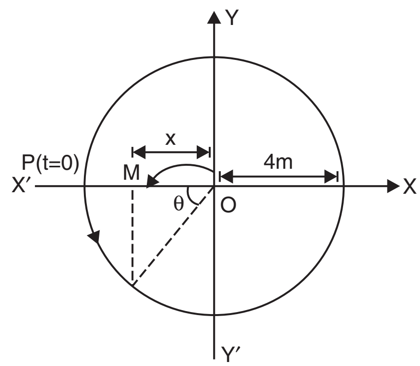


$$k = k_1 + k_2 = 8 \text{ N/m} + 2 \text{ N/m} = 10 \text{ N/m}$$

The frequency of vibration is given by

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{10}{0.1}} \quad \text{or} \quad \nu = \frac{10}{2\pi} = \frac{5}{\pi} \text{ s}^{-1}$$

Q. 11. *A particle P starts its motion at t = 0 and moves along a circle as shown in figure with period of 4 s. Write down the equation representing S.H.M. of the particle P.*



Ans. At $t = 0$, the angle made by OP with x-axis is $+\pi$ radian, so $\phi = \pi$

Let P (t) be the position of the particle at time t. The projection of P (t) on XOX′ diameter is M.

$$\therefore \quad x = OP(t)\cos(\omega t + \phi)$$

$$= 4\cos\left(\frac{2\pi}{T}t + \phi\right)$$

Since $\quad T = 4\text{s}$ and $\phi = +\pi$

$$\therefore \quad n = 4\cos\left(\frac{\pi}{2}t + \pi\right) = -4\cos\left(\frac{\pi}{2}t\right) \qquad (\because \cos(\pi + \theta) = -\cos\theta)$$

Q. 12. *A body executing linear SHM has a velocity of 3 cm s^{-1} when its displacement is 4 cm and a velocity of 4 cm s^{-1} when its displacement is 3 cm.*

(a) Find the amplitude and period of the oscillation.

(b) If the mass of the body is 50g, calculate the total energy of oscillation.

Ans. (*a*) In SHM, the velocity V at a displacement x is given by

$$V = \omega(A^2 - x^2)^{1/2} \quad \text{or} \quad V^2 = \omega^2(A^2 - x^2)$$

Now $\quad V = 3$ cm s^{-1} when $x = 4$ cm. Therefore

$$9 = \omega^2(A^2 - 16) \qquad ...(i)$$

Also $\quad V = 4$ cm s^{-1} when $x = 3$ cm. Therefore

$$16 = \omega^2(A^2 - 9) \qquad ...(ii)$$

Simultaneous solution of eqns (*i*) and (*ii*) gives

Amplitude, $\quad A = 5$ cm

and Angular frequency, $\omega = 1$ rad s^{-1}

Hence time period, $T = \dfrac{2\pi}{\omega} = 2\pi$ seconds ≈ 6.28 s

(*b*) $\quad m = 50 \text{ g} = 50 \times 10^{-3}$ kg

$A = 5 \text{ cm} = 5 \times 10^{-2}$ m

$\omega = 1$ rad s^{-1}

$$\text{Total energy} = \frac{1}{2}mA^2\omega^2 = \frac{1}{2}\times(50\times10^{-3})\times(5\times10^{-2})^2(1)^2 = 6.25 \times 10^{-5} \text{ J.}$$

Q. 13. *A simple pendulum in a stationary lift has time period T. What would be the effect on the time period when the left (i) moves up with uniform velocity v (ii) moves down with uniform velocity v (iii) moves up with uniform acceleration a (iv) moves down with uniform acceleration a (v) beings to fall freely under gravity?*

Ans. (*i*) and (*ii*) since acceleration of the lift is zero therefore there will be no effect on time period.

(*iii*) When the lift moves up with uniform acceleration a, the effective value of acceleration due to gravity is $g + a$.

$$\therefore \quad T' = 2\pi\sqrt{\frac{l}{g+a}} \quad \text{Clearly, } T' < T.$$

(*iv*) When the lift moves down with uniform acceleration a, then the effective value of g is $g - a$.

$$T' = 2\pi\sqrt{\frac{l}{g-a}} \quad \text{Clearly, } T' > T.$$

(*v*) When the lift begins to fall freely under gravity, the effective vlaue of g becomes zero. So, T is infinite *i.e.,* the simple pendulum shall not oscillate.

Q. 14. *A particle is vibrating in SHM when the displacements of the particle from its equilibrium position are x_1 and x_2, it has velocities v_1 and v_2 respectively. Show that its time period is given by*

$$T = 2\pi\sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}.$$

Ans. The particle velocity in SHM is given by : $v = \omega\sqrt{A^2 - x^2}$, where A is the amplitude of oscillation.

For displacement $x = x_1$, $v_1 = \omega\sqrt{A^2 - x_1^2}$ or $v_1^2 = \omega^2(A^2 - x_1^2)$ ...(*i*)

and for displacement $x = x_2$, $v_2 = \omega\sqrt{A^2 - x_2^2}$ or $v_2^2 = \omega^2(A^2 - x_2^2)$ ...(*ii*)

Subtracting (*i*) from (*ii*), we have

$$v_2^2 = v_2^2 = \omega^2\left(x_1^2 - x_2^2\right) \Rightarrow \omega = \sqrt{\frac{(v_2^2 - v_1^2)}{(x_1^2 - x_2^2)}}$$

$$\therefore \text{ Period of oscillation } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{(x_1^2 - x_2^2)}{(v_2^2 - v_1^2)}}.$$

Q. 15. *A horizontal platform with an object placed on it is executing SHM in the vertical direction. The amplitude of oscillation is 2.5 cm. What must be the least period of these oscillations so that the object is not detached from the platform? Take $g = 10\ ms^{-2}$?*

Ans. The object will not detach from the platform, if the angular frequency ω is such that, during the downward motion, the maximum acceleration equals the acceleration due to gravity, *i.e.,*

$$\omega_{max}^2 A = g \quad \text{or} \quad \omega_{max} = \sqrt{\frac{g}{A}}$$

or $$T_{min} = \frac{2\pi}{\omega_{max}} = 2\pi\sqrt{\frac{A}{g}}$$

Now $A = 2.5 \text{ cm} = 2.5 \times 10^{-2} \text{ m}$ and $g = 10 \text{ ms}^{-2}$.

Substituting these values we get

$$T_{min} = \frac{\pi}{10} = 0.314 \text{ s}$$

Q. 16. *A particle executes S.H.M of time period 10 s. The displacement at any instant is given by the relation x = 10 sin ωt. Find (i) velocity of the body 2s after it passes through the mean position and (ii) the acceleration 2s after it passes the mean position (Amplitude is given in cm).*

Ans. (*i*) Velocity at any instant t is given by $v = A\omega \cos \omega t$

Here $A = 10 \text{ cm}, \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{10}$

When $t = 2\text{s}, \; v = 10 \times \frac{2\pi}{10} \cos\left(\frac{2\pi}{10} \times 2\right)$

$= 2\pi \cos (0.4\,\pi) = 1.942 \text{ cm/s}.$

(*ii*) Acceleration at any instant t is given by

$$a = -A\omega^2 \sin \omega t = -10\left(\frac{2\pi}{10}\right)^2 \sin (0.4\,\pi) = -3.755 \text{ cm/s}^2$$

acceleration is numerically equal to 3.754 cm/s^2 and is directed towards the mean position.

Q. 17. *A simple harmonic motion has an amplitude A and time period T. What is the time taken to travel from* $x = A$ *to* $x = \frac{A}{2}$*?*

Ans. Displacement from mean position.

$$= A - \frac{A}{2} = \frac{A}{2}.$$

Now $y = A \cos \omega t$ or $\frac{A}{2} = A \cos \frac{2\pi}{T} t$ or $\cos \frac{2\pi}{T} t = \frac{1}{2}$

$\Rightarrow \quad \cos \frac{2\pi}{T} t = \cos \frac{\pi}{3}$ or $\frac{2\pi}{T} t = \frac{\pi}{3}$

$\therefore \quad t = \frac{T}{6}.$

Q. 18. *The mass 'M' attached to a spring oscillates with a period 2 s. If the mass is increased by 2 kg, the period increases by 1 s. Find the initial mass 'M', assuming that Hooke's law is obeyed.*

Ans. Let the initial mass and time periods be M and T respectively. If Hooke's law is obeyed, then the oscillations of the spring will be simple harmonic having time period T given by

$$T = 2\pi\sqrt{\frac{M}{k}}$$

Given $T = 2 \text{ s}$

$$\therefore \qquad 2 = 2\pi\sqrt{\frac{M}{k}} \quad ; \quad k = \text{spring constant} \qquad ...(i)$$

On increasing the mass by 2 kg

$$3 = 2\pi\sqrt{\frac{M+2}{k}} \qquad ...(ii)$$

Squaring and dividing equation (*ii*) by (*i*) we have

$$\frac{9}{4} = \frac{M+2}{M} = 1 + \frac{2}{M} \quad \text{or} \quad \frac{2}{M} = \frac{9}{4} - 1 = \frac{5}{4}$$

$$\therefore \qquad M = \frac{2 \times 4}{5} = \frac{8}{5} = 1.6 \text{ kg.}$$

Q. 19. *A mass m is dropped in a tunnel along the diameter of earth from a height h (< < R) above the surface of earth. Find the time period of motion, Is the motion simple harmonic?*

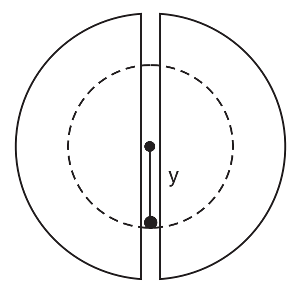
Ans. When a ball is dropped from a height h, it gains velocity due to gravity pull. The body will enter the tunnel of earth with velocity, $v = \sqrt{2gh}$; after a time, $t = \sqrt{2h/g}$. The body will go out of earth on the other side through the same distance before coming back towards the earth. When the body is outside the earth, the restoring force $F \propto (-1/r^2)$ and not $(-r)$ so the motion does not remain SHM but becomes oscillatory. The period of oscillation of the body will be,

$$T = 2\pi\sqrt{\frac{R}{g}} + 4\sqrt{\frac{2h}{g}} \ .$$

Q. 20. *Suppose a tunnel is dug through the earth from one side to the other side along a diameter. Show that the motion of a particle dropped into the tunnel is simple harmonic motion. Find the time period. Neglect all the frictional forces and assume that the earth has a uniform density.*

$G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}; density of earth $= 5.51 \times 10^3$ kg m^{-3}

Ans. Figure shows a tunnel dug along the diameter of the earth. Consider the case of a particle of mass m at a distance y from the centre of the earth. There will be a gravitational attraction of the earth on this particle due to the portion of matter contained in a sphere of radius y. The mass of the sphere of radius y is given by



$$M = \text{Volume} \times \text{density}$$

$$\text{or} \qquad M = \frac{4}{3}\pi y^3 \times d$$

(where d = density of earth).

This mass can be regarded as concentrated at the centre of the earth. The force F between this mass and the particle of mass m is given by

$$F = -\frac{GMm}{y^2}$$

Negative sign shows that the force is of attraction.

$$\therefore \qquad F = -G\left(\frac{4}{3}\pi y^3 d\right)\frac{m}{y^2} = -G \times \left(\frac{4}{3}\pi m d\right)y \quad \text{or} \quad F \propto y.$$

The force is directly proportional to the displacement, hence the motion is simple harmonic motion.

Here, the constant $k = \frac{4}{3}\pi\, m\, dG$.

$$\text{The time period, } T = 2\pi\sqrt{(m/k)} = 2\pi\sqrt{\left(\frac{3m}{4\,\pi\, m\, dG}\right)} = 2\pi\sqrt{\left(\frac{3}{4\,\pi\, dG}\right)}$$

$$= \sqrt{\left(\frac{3\pi}{dG}\right)} = \sqrt{\left(\frac{3\times 3.14}{5.51\times 10^{3}\times 6.67\times 10^{-11}}\right)} = \textbf{42.2 minutes.}$$

Q. 21. *A uniform spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths. l_1 and l_2, where $l_1 = nl_2$ and n is an integer. What are the corresponding force constants k_1 and k_2 in terms of n and k?*

Ans. Here $l = l_1 + l_2$ ...(*i*)

and $l_1 = n\, l_2$ ...(*ii*)

We know, $k = \frac{Mg}{l}$ ...(*iii*)

$\therefore$ $k_1 = \frac{Mg}{l_1}$ ...(*iv*)

and $k_2 = \frac{Mg}{l_2}$ ...(*v*)

Dividing equation (*iv*) by equation (*iii*) we find

$$\frac{k_1}{k} = \frac{l}{l_1} = \frac{l_1 + l_2}{l_1} = 1 + \frac{l_2}{l_1}$$

From equation (*ii*), we find $\frac{l_1}{l_2} = n$

$$\therefore \quad \frac{k_1}{k} = 1 + \frac{1}{n} \quad \text{or} \quad k_1 = \left(\frac{n+1}{n}\right)k$$

From equation (*v*) and (*iii*), we find:

$$\frac{k_2}{k} = \frac{l}{l_2} = \frac{l_1 + l_2}{l_2} = \frac{l_1}{l_2} + 1$$

From equation (*ii*) we have $\frac{l_1}{l_2} = n$.

$$\therefore \quad \frac{k_2}{k} = (n + 1) \quad \therefore \quad k_2 = k(n + 1).$$

Q. 22. *The displacement of two particles executing simple harmonic motion are represented by equations, $y = 4 \sin (10t + \theta)$ and $y_2 = 5 \cos 10t$. What is the phase difference between the velocities of these particles?*

Ans. For 1st particle

$$y_1 = 4 \sin (10t + \theta);$$

$$\text{Velocity} = \frac{dy_1}{dt} = 4 \times 10 \cos (10t + \theta) = 40 \cos (10t + \theta)$$

For second particle

$$y_2 = 5 \cos 10t = 5 \sin (10t + \pi/2)$$

$$\text{Velocity} = \frac{dy_2}{dt} = 5 \times 10 \cos (10t + \pi/2) = 5 \cos (10t + \pi/2)$$

Phase difference between velocities

$$= (10t + \theta) - (10t + \pi/2) = \left(\theta - \frac{\pi}{2}\right).$$

Q. 23. *A block whose mass is 1 kg is fastened to a spring. The spring has a spring constant of 50 Nm^{-1}. The block is pulled to a distance x = 10 cm from the equilibrium position at x = 0 on a frictionless surface from rest at t = 0. Calculate the kinetic, potential and total energies of the block when it is 5 cm away from the mean position.*

Ans. Here mass of block m = 1 kg and spring constant k = 50 Nm^{-1}

$\therefore$ Angular frequency of SHM $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50}{1}} = 7.07$ rad s^{-1}

As at t = 0, the block was x = 0, and then the block was pulled to distance x = 10 cm, it shows that amplitude of oscillation A = 10 – 0 = 10 cm = 0.1 m.

Moreover at any instant the instantaneous displacement $x(t)$ = 5 cm = 0.005 m

$\therefore$ Kinetic energy of the block $k = \frac{1}{2} m\omega^2 (A^2 - x^2)$

$$= \frac{1}{2} \times 1 \times (7.07)^2 \,[(0.1)^2 - (0.05)^2] = 0.19 \text{ J}$$

Potential energy of the block $U = \frac{1}{2} m\omega^2 x^2 = \frac{1}{2} \times 1 \times (7.07)^2 \times (0.05)^2 = 6.25 \times 10^{-2}$ J

and total energy of the block $E = K + U = 0.19 \text{ J} + 6.25 \times 10^{-2} \text{ J}$

$$= 0.25 \text{ J.}$$

Q. 24. *Springs of spring constants k, 2k, 4k, 8k .... are connected in series. A mass m kg is attached to the lower end of the last spring and the system is allowed to vibrate. What is the time period of oscillations?*

Given m = 40 gm; k = 20 N cm^{-1}

Ans. Here, m = 40 g = 0.04 kg;

k = 2.0 N cm^{-1} = 2.0 × 100 Nm^{-1};

effective spring constant K is given by

$$\frac{1}{K} = \frac{1}{k} + \frac{1}{2k} + \frac{1}{4k} + \frac{1}{8k} + \ldots\ldots\ldots = \frac{1}{k}\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots\ldots\ldots\right]$$

$$= \frac{1}{k}\left[\frac{1}{1 - \frac{2}{2}}\right] = \frac{2}{k}$$

or $$K = \frac{k}{2}$$

$\therefore$ Time period, $T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{m}{\frac{k}{2}}} = 2\pi\sqrt{\frac{2m}{k}}$.

$$= 2 \times \frac{22}{7} \times \sqrt{\frac{2 \times 0.04}{2.0 \times 100}} = \mathbf{0.126\ s.}$$

Q. 25. *A cylindrical wooden block of cross-section 15.0 cm² and mass 230 gm is floated over water with an extra weight 50 gm attached to its bottom. The cylinder floats vertically. From the state of equilibrium, it is slightly depressed and released. If the specific gravity of wood is 0.30 and g = 9.8 m per sec², find the frequency of oscillation of the block.*

Ans. Area of cross-section of the block = πr^2 = 15 cm² = 15×10^{-4} m²

Total weight of the block = (230 + 50) = 280 gm = 0.28 kg

Density of wood = 0.30 gm/c.c. = 300 kg/m³

Density of water = 10^3 kg/m³

When the cylinder is depressed in water through a distance y, the

Restoring force = weight of water displaced

$$F = Aydg = (15 \times 10^{-4}) \times 10^3 \times 9.8 \text{ Newton}$$

Restoring force per unit distance $= \frac{F}{y} = k = (15 \times 10^{-4}) \times 10^3 \times 9.8$ newton/metre

$= 1.5 \times 9.8$ N/m

Hence the frequency of oscillation is given by

$$= \frac{1}{2\pi}\sqrt{\left(\frac{k}{m}\right)} = \frac{1}{2\pi}\sqrt{\frac{1.5 \times 9.8}{0.28}} = 1.15 \text{ Hz}$$

Q. 26. *Given the example of the motion in the following cases:*

(i) where magnitude and direction of the acceleration of the particle changes.

(ii) where the magnitude and direction of acceleration of body remains constant.

(iii) where magnitude of acceleration changes but its direction remains constants.

(iv) where the magnitude of acceleration remains constant but its direction changes.

Ans. (*i*) In S.H.M., acceleration is always proportional to displacement but directed opposite to the displacement. So in this case, magnitude as well as direction of acceleration changes.

(*ii*) A body falling under gravity near the surface of the earth.

(*iii*) A body falling under gravity from a height comparable to the radius of the earth.

(*iv*) A body revolving in a circular path with constant speed.

Q. 27. *A particle of mass m is executing simple harmonic oscillations of amplitude A. At* $x = \frac{A}{2}$, *what fraction of its energy is potential? What fraction is kinetic?*

Ans. We know that total energy of a harmonic oscillator,

$$E = \frac{1}{2}m\omega^2 A^2$$

At $x = \frac{A}{2}$, the potential energy of oscillator,

$$U = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 \cdot \left(\frac{A}{2}\right)^2 = \frac{1}{8}m\omega^2 A^2$$

$$\therefore \qquad \frac{U}{E} = \frac{\frac{1}{8}m\omega^2A^2}{\frac{1}{2}m\omega^2A^2} = \frac{1}{4} = \frac{1}{4}\times 100\% = 25\%$$

Again at $\quad x = \frac{A}{2}$, the kinetic energy $K = \frac{1}{2}m\omega^2(A^2 - x^2)$

$$= \frac{1}{2}m\omega^2\left[A^2 - \left(\frac{A}{2}\right)^2\right] = \frac{3}{8}m\omega^2A^2$$

$$\therefore \qquad \frac{K}{E} = \frac{\frac{3}{8}m\omega^2A^2}{\frac{1}{2}m\omega^2A^2} = \frac{3}{4} = \frac{3}{4}\times 100\% = 75\%.$$

Q. 28. *What is the frequency of a second pendulum in an elevator moving up with an acceleration of $\frac{g}{2}$?*

Ans. For second pendulum, frequency $\nu = \frac{1}{2}\,\text{s}^{-1}$.

When elevator is moving upwards with acceleration a, the effective acceleration due to gravity is $g_1 = g + a = g + \frac{g}{2} = \frac{3g}{2}$.

Since, $\quad \nu = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$

Hence, $\quad \nu^2 \propto g$

$$\therefore \qquad \frac{\nu_1^2}{\nu^2} = \frac{g_1}{g} = \frac{\frac{3g}{2}}{g} = \frac{3}{2} \quad \text{or} \quad \frac{\nu_1}{\nu} = \sqrt{\frac{3}{2}} = 1.225$$

$$\Rightarrow \qquad \nu_1 = 1.225\,\nu = 1.225 \times \frac{1}{2} = 0.612\ \text{s}^{-1}.$$

III. LONG ANSWER TYPE QUESTIONS

Q. 1. *Find the expression for kinetic energy, potential energy and total energy of a particle executing SHM.*

Ans. Let at any instant, the displacement of a particle executing SHM is y, mass of the particle 'm'.

We know, displacement, $y = a \sin \omega t$

velocity, $\quad v = \frac{dy}{dt} = \frac{d}{dt}\, a \sin \omega t \quad$ or $\quad v = a\,\omega \cos \omega t$

kinetic energy $= \frac{1}{2}mv^2$

$$E_k = \frac{1}{2}m\,(a\,\omega \cos \omega t)^2 = \frac{1}{2}ma^2\omega^2 \cos^2 \omega t$$

or $\qquad E_k = \frac{1}{2}ma^2\omega^2(1 - \sin^2 \omega t)$

or, $E_k = \frac{1}{2} ma^2\omega^2(1 - \frac{y^2}{a^2})$ [$\because y = a \sin \omega t$]

or, $E_k = \frac{1}{2} m\omega^2(a^2 - y^2)$

For Potential Energy:

As velocity, $v = a\omega \cos \omega t$

acceleration $= \frac{d}{dt}(v) = (a\omega \cos \omega t) = -a\omega^2 \sin \omega t = -\omega^2(a \sin \omega t) = -\omega^2 y$

Restoring force at any instant, $F = -\omega^2 y. m = -m\omega^2 y$

The negative sign indicates that the restoring force is always directed towards the mean position. In order to maintain the particle at displacement y, a force $m\omega^2 y$ acting away from the mean position has to be applied on the particle. Let dW be the work done by the applied force to displace a given particle through a distance dy away from the mean position.

Then, $dW = m\omega^2 y\, dy$

Let W be the total work done in increasing the displacement from O to y.

Thus $W = \int_0^y m\omega^2 y\, dy = m\omega^2 \int_0^y y dy$

or $W = m\omega^2 \left[\frac{y^2}{2}\right]_0^4 = m\omega^2\left(\frac{y^2}{2} - 0\right) = \frac{1}{2} m\omega^2 y^2$

This work done is stored in the particle as potential

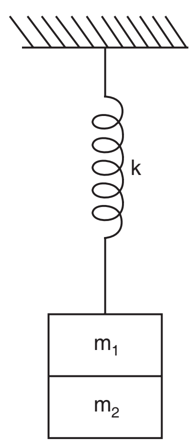
$\therefore$ $E_p = \frac{1}{2} m\omega^2 y^2$.

Total energy, $E = E_k + E_p$

$\Rightarrow$ $E = \frac{1}{2} m\omega^2(a^2 - y^2) + \frac{1}{2} mw^2 y^2$

or $E = \frac{1}{2} m\omega^2 a^2 - \frac{1}{2} m\omega^2 y^2 + \frac{1}{2} mw^2 y^2$ or $E = \frac{1}{2} m\omega^2 a^2$.

Q. 2. *Define simple harmonic motion (SHM). Two masses m_1 and m_2 are suspended together by a massless spring of spring constant k (see Fig.a). When the masses are in equilibrium, m_1 is removed without disturbing the system. Find the angular frequency and amplitude oscillation of m_2*



Ans. For definition, see text. Let x_1 be the extension produced in the spring when it is loaded with mass m_2 alone and x_2 be the further extension when mass m_1 is added to mass m_2 so that $x = x_1 + x_2$ is the total extension produced by $m_1 + m_2$ (see Fig.*b*). Thus we have,

For equilibrium state of m_2.

$m_2 g = k x_1$ ...(1)

For equilibrium state of $(m_1 + m_2)$

$(m_2 + m_2) g = k(x) = k(x_1 + x_2)$ ...(2)

When the mass m_1 is removed, the mass m_2 will move upwards under the unbalanced force $= m_1 g$. Hence

Restoring force (F) on

$$m_2 = -m_1 g$$

Substracting (1) and (2) we have

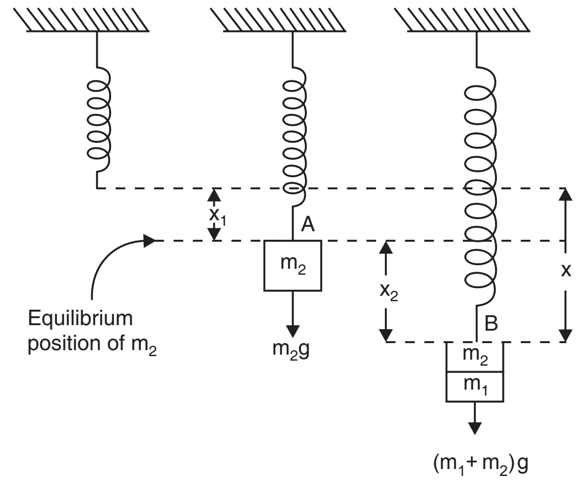
$$m_1 g = k\ x_2 \qquad ...(3)$$

Hence, Restoring force on

$$m_2 = -k\ x_2$$

$\therefore$ acceleration of $m_2 = \dfrac{F}{m_2} = -\dfrac{k}{m_2} x_2$

i.e., accelaration $\propto$ − displacement



Angular frequency is $\omega = \sqrt{\dfrac{k}{m_2}}$

$\therefore$ Frequency of oscillation is $n = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m_2}}$

It is clear that A is the equilibrium postion of m_2 and B its maximum displacement position. Hence $AB = x_2$ is the amplitude of oscillation of m_2 which from Eq. (3) is given by

$$\text{Amplitude} = x_2 = \frac{m_1 g}{k}$$

Q. 3. *Find the expression for time period of motion of a body suspended by two springs connected in parallel and series.*

Ans. Consider a body of mass M suspended by two springs connected in parallel as shown in Fig. (*a*) Let k_1 and k_2 be the spring constants of two springs respectively.

Let the body be pulled down so that each spring is stretched through a distance y. Restoring forces F_1 and F_2 will be developed in the springs S_1 and S_2 respectively.

According to Hooke's law, $F_1 = -k_1 y$

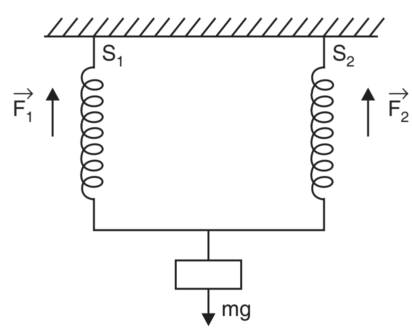
and $F_2 = -k_2 y$

Since both the forces acting in the same direction, therefore, total restoring force acting on the body is given by

$$F = F_1 + F_2 = -k_1\ y - k_2\ y = -(k_1 + k_2)\ y$$

$\therefore$ Acceleration produced in the body is given by

$$a = \frac{F}{M} = -\frac{(k_1 + k_2)\ y}{M} \qquad ...(i)$$



Since $\dfrac{(k_1 + k_2)}{M}$ is constant $\therefore\ a \propto -y$

Hence motion of the body is SHM.

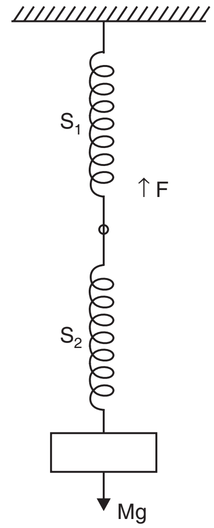
Time period of body is given by

$$T = 2\pi\sqrt{\frac{y}{|a|}} = 2\pi\sqrt{\frac{M}{k_1 + k_2}} \qquad ...(ii)$$

If $k_1 = k_2 = k$

Then $T = 2\pi\sqrt{\dfrac{M}{2k}}$

For series: Consider a body of mass M suspended by two springs S_1 and S_2 which are connected in series as shown in Fig. (*b*). Let k_1 and k_2 be the spring constants of springs S_1 and S_2 respectively.



Suppose at any instant, the displacement of the body from equilibrium position is y in the downward direction. If y_1 and y_2 be the extension produced in the springs S_1 and S_2 respectively, then

$$y = y_1 + y_2 \quad ...(i)$$

Restoring forces developed in S_1 and S_2 are given by

$$F_1 = -k_1 + y_1 \quad ...(ii)$$

$$F_2 = -k_2 + y_2 \quad ...(iii)$$

Multiplying eqns. (*ii*) by k_2 and eqn. (*iii*) by k_1 and adding, we get

$$\therefore \quad k_2 F_1 + k_1 F_2 = -k_1 k_2 (y_1 + y_2) = -k_1 k_2 y$$

[From eqn. (*i*)]

Since both the springs are connected in series, so

$$F_1 = F_2 = F$$

$$\therefore \quad F(k_1 + k_2) = -k_1 k_2 y \quad \text{or} \quad F = -\frac{k_1 k_2}{(k_1 + k_2)\,y}$$

If a be the acceleration produced in the body of mass M, then

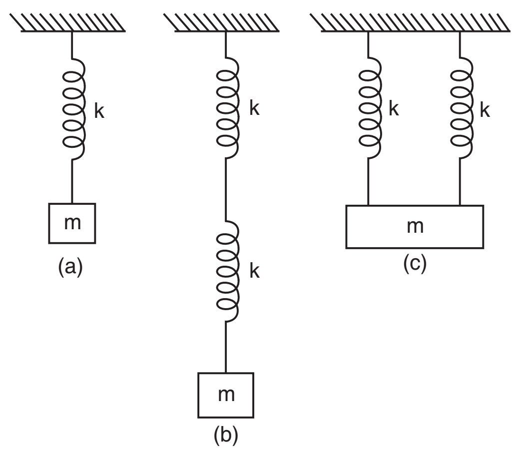
$$a = \frac{F}{M} = -\frac{k_1 k_2 y}{(k_1 + k_2)\,M} \quad ...(iv)$$

Time period of the body is given by

$$T = 2\pi\sqrt{\frac{y}{|a|}} = 2\pi\sqrt{\frac{(k_1 + k_2)M}{k_1 k_2}} \quad \text{[From eqn. } (iv)\text{]}$$

$$T = 2\pi\sqrt{\left(\frac{1}{k_1} + \frac{1}{k_2}\right)M}\,.$$

Q. 4. *What do you understand by undamped and damped simple harmonic oscillations? Show that the time periods for vertical harmonic oscillations of the three systems shown in Figs. (a), (b) and (c) are in the ratio of 1: $\sqrt{2} : \frac{1}{\sqrt{2}}$. All springs are identical, each having a force constant k.*



Ans. For definitions, see the NCERT Textbook.

Let us suppose that an extension x is produced in the spring when a force mg is applied to it. The equilibrium position in case (*a*) is given by

$$F = mg = kx \quad ...(1)$$

The time period in this case is given by

$$T_a = 2\pi\sqrt{\frac{\text{mass}}{\text{spring constant}}} = 2\pi\sqrt{\frac{m}{k}} \quad ...(2)$$

(*b*) In this case, the length of the spring is doubled. Hence a given force mg will double the extension. Let x' be the extension produced and k_{eff} be the force constant of the combination. Thus,

$$F = mg = k_{eff}\, x' = 2\, k_{eff} x \quad (\because\ x' = 2x) \qquad ...(3)$$

Comparing (1) and (3) we get

$$F = k\, x = 2\, k_{eff} x \quad \text{or} \quad k_{eff} = \frac{k}{2}$$

Thus the time period in this case is given by

$$T_b = 2\pi\sqrt{\frac{m}{k_{eff}}} = 2\pi\sqrt{\frac{2m}{k}} = \sqrt{2}\, T_a \qquad ...(4)$$

(*c*) In this case, the extension x'' produced in each spring by a force mg is half that produced in case (*a*), *i.e.*,

$$x'' = \frac{x}{2}$$

If k_{eff} is the force cosntant of the combination in this case, we have

$$F = mg = k_{eff}\, x'' = \frac{k_{eff}}{2} x \qquad ...(5)$$

Comparing (5) with (1) we have

$$k_{eff} = 2k$$

Hence,

$$T_e = 2\pi\sqrt{\frac{m}{k_{eff}}} = 2\pi\sqrt{\frac{m}{2\,k}} = \frac{T_a}{\sqrt{2}} \qquad ...(6)$$

From (2), (4) and (6) we have

$$T_a : T_b : T_c = 1 : \sqrt{2} : \frac{1}{\sqrt{2}}.$$

IV. MULTIPLE CHOICE QUESTIONS

1. A simple pendulum of frequency n is taken upto a certain height above the ground and then dropped along with its support so that it falls freely under gravity. The frequency of oscillations of the falling pendulum will

(*a*) become greater than n (*b*) become zero
(*c*) remain equal to n (*d*) become less than n

2. The length of a simple pendulum is increased by 44%. What is the percentage increase in its time period?

(*a*) 10% (*b*) 20% (*c*) 40% (*d*) 44%

3. A particle executing simple harmonic motion along y-axis has its motion described by the equation $y = A \sin(\omega t) + B$. The amplitude of the simple harmonic motion is

(*a*) A (*b*) B (*c*) $A + B$ (*d*) $\sqrt{A+B}$

4. Masses m and $3m$ are attached to the two ends of a spring of constant k. If the system vibrates freely, the period of oscillation will be

(*a*) $\pi\sqrt{\frac{m}{k}}$ (*b*) $2\pi\sqrt{\frac{3m}{2k}}$ (*c*) $\pi\sqrt{\frac{3m}{k}}$ (*d*) $2\pi\sqrt{\frac{4m}{3k}}$

5. The following are the quantities associated with a body performing SHM.

1. The velocity of the body.

2. The accelerating of the body.

3. The accelerating force acting on the body.

Which of these quantities are exactly in phase with each other?

(*a*) None of these (*b*) 1 and 2 only

(*c*) 1 and 3 only (*d*) 2 and 3 only

(*e*) 1, 2 and 3 only

6. A heavy brass sphere is hung from a spring and it executes vertical vibrations with period T. The sphere is now immersed in a non-viscous liquid with a density (1/10)th that of brass. When set into vertical vibrations with the sphere remaining inside liquid all the time, the time period will be

(*a*) $\sqrt{\frac{9}{10T}}$ (*b*) $\sqrt{\frac{10}{9T}}$ (*c*) $\sqrt{\left(\frac{9}{10}\right)T}$ (*d*) unchanged

7. A particle executes simple harmonic motion between $x = -A$ and $x = +A$. The time taken for it to go from 0 to $\frac{A}{2}$ is T_1 and to go from $\frac{A}{2}$ to A is T_2. Then

(*a*) $T_1 < T_2$ (*b*) $T_1 > T_2$ (*c*) $T_1 = T_2$ (*d*) $T_1 = 2T_2$

8. Two particles P and Q describe SHM of same amplitude a and frequency v along the same straight line. The maximum distance between two particles is $\sqrt{2}\,a$. The phase difference between the particles is

(*a*) zero (*b*) $\frac{\pi}{2}$ (*c*) $\frac{\pi}{6}$ (*d*) $\frac{\pi}{3}$

Ans. **1.**—(*b*) **2.**—(*b*) **3.**—(*a*) **4.**—(*c*) **5.**—(*d*)
6.—(*b*) **7.**—(*a*) **8.**—(*a*)

V. QUESTIONS ON HIGH ORDER THINKING SKILLS (HOTS)

Q. 1. *A block is resting on a piston which is moving vertically with a simple harmonic motion of period 1 sec. At what amplitude of motion will the block and the piston seperate? What is the maximum velocity of the piston at this amplitude?*

Ans. We know that $y = a \sin \omega t$

$\therefore$ Velocity of the block $= \frac{dy}{dt} = a\omega \cos \omega t$

and acceleration of the block $= \frac{d^2y}{dt^2} = -\omega^2 a \sin \omega t = -\omega^2 y$

For maximum acceleration $y = a$

$$\therefore \quad \left(\frac{d^2y}{dt^2}\right)_{max} = -\omega^2 a$$

The block will be separated form the piston when

$$\omega^2 a = g \quad \text{or} \quad a = \frac{g}{\omega^2} \quad \text{or} \quad a = \frac{gT^2}{4\pi^2} \qquad \left(\because \omega = \frac{2\pi}{T}\right)$$

According to the given problem $T = 1$ sec.

$$\therefore \quad a = \frac{g}{4\pi^2} = \frac{9.8}{4 \times (3.14)^2} = 0.248 \text{ m/sec}^2$$

At this amplitude, the maximum velocity of the block will be

$$\omega a = \frac{2\pi a}{T} = \frac{2 \times 3.14 \times 0.248}{1} = 1.56 \text{ m/sec.}$$

Q. 2. *The potential energy of a particle of mass 1 kg in motion along the x-axis is given by*

$$U = 4\,(1 - \cos 2x)\ J$$

Here x is in metres. Find the period of small oscillations.

Ans.

$$F = -\frac{dU}{dx} = -\frac{d}{dx}[4\,(1 - \cos 2x)] = -8 \sin 2x$$

or $F = -8 \times 2x$ [When x is small, $\sin 2x \approx 2x$]

or $F = -16\,x$

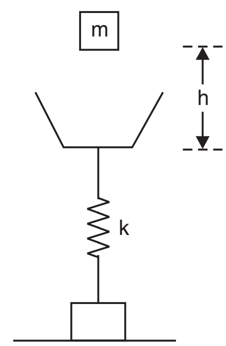
As $F \propto x$ and $-ve$ sign shows that x is directed towards equilibrium position, hence the particle will execute SHM.

Here, spring factor, $k = 16$ N/m

inertia factor, $m = 1$ kg

$\therefore$ time period, $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1}{16}} = \frac{\pi}{2}$ s.

Q. 3. *A body of mass m falls from a height h on to the pan of a spring balance. The masses of the pan and spring are negligible. The spring constant of the spring is k. Having stuck to the pan the body starts performing harmonic oscillations in the vertical direction. Find the amplitude and energy of oscillation.*



Ans. Suppose by falling down through a height h, the mass m compresses the spring balance by a length x.

This P.E. lost by the mass $= mg\,(h + x)$

This is stored up as energy of the spring by compression

$$= \frac{1}{2}kx^2$$

$\therefore$ $mg\,(h + x) = \frac{1}{2}kx^2$ or $\frac{1}{2}kx^2 - mgx - mgh = 0$

or $x^2 - \frac{2mgx}{k} - \frac{2mgh}{k} = 0$

Solving this quadratic equation, we get

$$x = \frac{\frac{2mg}{k} \pm \sqrt{\left(\frac{2mg}{k}\right)^2 + \left(\frac{8mgh}{k}\right)}}{2} = \frac{mg}{k} \pm \frac{mg}{k}\sqrt{1 + \frac{2kh}{mg}}$$

In the equilibrium position, the spring will be compressed through the distance mg/k and hence the amplitude oscillation is

$$A = \frac{mg}{k}\sqrt{1 + \frac{2kh}{mg}}$$

Energy of oscillation $= \frac{1}{2}kA^2 = \frac{1}{2}k\left(\frac{mg}{k}\right)^2\left(1 + \frac{2kh}{mg}\right) = mgh + \frac{(mg)^2}{2k}$

Q. 4. *Two linear simple harmonic motions of equal amplitudes and frequencies ω and 2ω are impressed on a particle along the axes of X and Y respectively. If the initial phase difference between them is $\frac{\pi}{2}$, find the resultant path followed by the particle.*

Ans. Two simple harmonic motions of equal amplitudes (A) and frequencies ω and 2ω and initial phase difference of $\pi/2$ are represented by

$$x = A \sin \omega t \quad ...(i)$$

$$y = A \sin\left(2\omega t + \frac{\pi}{2}\right) = A \cos 2\omega t \quad ...(ii)$$

Since $\cos 2\omega t = (1 - 2\sin^2 \omega t)$

$\therefore \quad y = A[1 - 2\sin^2 \omega t] \quad ...(iii)$

From eqn. (*i*), $\sin^2 \omega t = \dfrac{x^2}{A^2}$

$$\therefore \quad y = A\left[1 - \frac{2x^2}{A^2}\right] = A - \frac{2x^2}{A}$$

$$\Rightarrow \quad \frac{2x^2}{A} + y - A = 0 \quad \text{or} \quad x^2 + \frac{Ay}{2} - \frac{A^2}{2} = 0$$

which is the equation of a parabola. Hence the resultant path followed by the particle is parabolic.

Q. 5. *A simple pendulum with a brass has a time period T. The bob is now immersed in a non-viscous liquid and oscillated. If the density of the liquid is $\frac{1}{9}$ that of brass, find the time period of the same pendulum.*

Ans. Let V be the volume and P be the density of the brass bob. Mass of the bob $m = V\rho$ and weight of bob $= V\rho g$.

Buoyancy force of liquid on bob $= V \dfrac{\rho}{9} g = \dfrac{V\rho g}{9}$

So the effective weight of bob in liquid $= V\rho g - \dfrac{V\rho g}{9} = 8 \dfrac{V\rho g}{9}$

$$\therefore \text{ Acceleration, } g' = \frac{8V\rho g/9}{m} = \frac{8V\rho g/9}{V\rho} = \frac{8g}{9}.$$

$$\text{Time period of the bob} = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{\frac{8g}{9}}} = 2\pi\sqrt{\frac{l}{g}} \times \frac{3}{\sqrt{8}} = \frac{3T}{\sqrt{8}}.$$

Q. 6. *A point particle of mass 0.1 kg is executing S.H.M. of amplitude of 0.1 m. When the particle passes through the mean position, its kinetic energy is 8×10^{-3} joule. Obtain the equation of motion of this particle if the initial phase of oscillation is 45°.*

Ans. The displacement of a particle in S.H.M is given by

$$y = a \sin(\omega t + \phi)$$

$$\text{velocity} = \frac{dy}{dt} = \omega a \cos(\omega t + \phi)$$

The velocity is maximum when the particle passes through the mean position *i.e.,*

$$\left(\frac{dy}{dt}\right)_{max} = \omega a$$

The kinetic energy at this instant is given by

$$\frac{1}{2}m\left(\frac{dy}{dt}\right)^2_{max} = \frac{1}{2}m \times \omega^2 a^2 = 8 \times 10^{-3} \text{ joule} \quad \text{or} \quad \frac{1}{2}\times(0.1)\,\omega^2\times(0.1)^2 = 8 \times 10^{-3}$$

Solving we get $\omega = \pm 4$

Substituting the values of a, ω and ϕ in the equation of SHM., we get

$$y = 0.1 \sin (\pm 4t + \frac{\pi}{4}) \text{ metre.}$$

Q. 7. *Two pendulums of lengths 100 cm and 110.25 cm start oscillating in phase simultaneously. After how many oscillations will they again be in phase together?*

Ans. $T = 2\pi\sqrt{\frac{l}{g}}$, l_1 = 100 cm, l_2 = 110.25 cm

For smaller pendulum, $T_1 = 2\pi\sqrt{\frac{100}{g}}$ ...(*i*)

For larger pendulum, $T_2 = 2\pi\sqrt{\frac{110.25}{g}}$ ...(*ii*)

Let these pendulums oscillate in phase again if larger pendulum completes 'n' oscillations. It means smaller pendulum must complete (n + 1) oscillations.

$$nT_2 = (n + 1)\, T_1$$

or $$\frac{n+1}{n} = \frac{T_2}{T_1} = \sqrt{\frac{110.25}{100}} = 1.05$$

or $$1 + \frac{1}{n} = 1.05 \quad \text{or} \quad \frac{1}{n} = 0.05 = \frac{5}{100} = \frac{1}{20}$$

$\therefore$ $n = 20.$

Hence both pendulums will again oscillate in phase after 20 oscillations of the larger or 21 oscillations of the smaller pendulum.

Q. 8. *The number of harmonic components in the oscillations are represented by,* *$y = 4 \cos^2$ at sin 4t. What are their corresponding angular frequencies?*

Ans.

$$y = 4 \cos^2 2t \sin 4t = 2 (\cos 4t + 1) \sin 4t \quad [\because 2 \cos^2 \theta = \cos 2\theta + 1]$$
$$= 2 \sin 4t \cos 4t + 2 \sin 4t = \sin 8t + 2 \sin 4t$$
$$= 2 \sin 4t + \sin 8t$$

Thus the resulting harmonic oscillation is a combination of two harmonic motions of angular frequencies 4 rad/s and 8 rad/s.

VI. VALUE-BASED QUESTIONS

Q. 1. *Rajeshwer and his friends went to a hill forest for a plant collection tour. One of the friends Rajender Rana went to a different direction and entered a dense forest. He was searching a very useful and promodeal plant. When he searched that plant, he loudly called his friend Rajeshwer. After a few seconds he heard the same sound as he just spoken. He afraid assuming that there was a ghost in nearby forest. He started crying. Somehow Rajeshwar heard his sound and found him.*

Again Rajender Ram made a loud sound, this sound was again copied by somebody. Rajender repeated that same ghost is there. Rajeshwar convinced him that it is nothing but it is reflection of sound. Now Rajender Rana became normal.

(i) What values/qualities were displayed by Rajeshwar?

(ii) What is echo?

(iii) If a reflector is situated at a distance of 860 m from a sound source, what is the time of echo? Speed of sound in air at a room temperature can be taken as 344 m/s.

Ans. (*i*) Rajeshwar displayed the values : having scientific attitude, keen observer, sharp mind, creative and helping nature.

(*ii*) The sound heard by an observer/listner after the reflection from a surface is called echo.

(*iii*) Total distance travelled by sound to come back = 2 × 860 = 1720 m

speed of sound is 344 m/s.

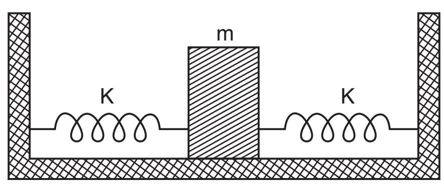
$$\therefore \qquad \text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{1720}{344} = 5 \text{ sec}$$

TEST YOUR SKILLS

1. What is the time period and frequency of the beating of human heart, if it beats 75 times per minute on an average?

2. What is the difference/similarity between simple harmonic motion and uniform circular motion?

3. What is simple harmonic motion? In the following figure, two identical springs are attached to a block and to fixed supports. The spring constant for both springs is *k* and the mass of the block is *M*. Derive the equation for time period in this case.



4. When a spring is showing simple harmonic motion, what are the factors, responsible for storing potential energy and kinetic energy?

5. A block of mass 600 g is attached to a spring. The spring constant is 30 Nm^{-1}. The block is pulled to a distance x = 5 cm from its equilibrium position at x = 0 on a frictionless surface from rest at t = 0. Calculate the kinetic, potential and total energies of the block, when it is 2.5 cm away from the mean position.

6. What do you understand by damping constant?

7. What do you understand by forced oscillation is, important to continue the simple harmonic motion?

8. In case of an earthquake shorter and taller structures are usually not damaged, while medium sized structures are damaged. What is the reason for this?

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