15 Waves

Facts that Matter

• Waves

Wave is a form of disturbance which travels through a material medium due to the repeated periodic motion of the particles of the medium about their mean positions without any actual transportation of matter.

• Characteristics of wave

The characteristics of waves are as follows:

(*i*) The particles of the medium traversed by a wave execute relatively small vibrations about their mean positions but the particles are not permanently displaced in the direction of propagation of the wave.

(*ii*) Each successive particle of the medium executes a motion quite similar to its predecessors along/perpendicular to the line of travel of the wave.

(*iii*) During wave motion only transfer of energy takes place but not that of a portion of the medium.

Waves are mainly of three types: (*a*) mechanical or elastic waves, (*b*) electromagnetic waves and (*c*) matter waves.

• Mechanical waves

Mechanical waves can be produced or propagated only in a material medium. These waves are governed by Newton's laws of motion. For example, waves on water surface, waves on strings, sound waves etc.

• Electromagnetic Waves

These are the waves which require no material medium for their production and propagation, *i.e.,* they can pass through vacuum and any other material medium. Common examples of electromagnetic waves are visible light; ultra-violet light; radiowaves, microwaves etc.

• Matter waves

These waves are associated with moving particles of matter, like electrons, protons, neutrons etc.

Mechanical waves are of two types:

(*i*) Transverse wave motion, (*ii*) Longitudinal wave motion,

- **Transverse wave motion**

In transverse waves the particles of the medium vibrate at right angles to the direction in which the wave propagates. Waves on strings, surface water waves and electromagnetic waves are transverse waves. In electromagnetic waves (which include light waves) the disturbance that travels is not a result of vibrations of particles but it is the oscillation of electric and magnetic fields which takes place at right angles to the direction in which the wave travels.

- **Longitudinal wave motion**

In these types of waves, particles of the medium vibrate to and fro about their mean position along the direction of propagation of energy. These are also called pressure waves. Sound waves are longitudinal mechanical waves.

- **Wavelength**

The distance travelled by the disturbance during the time of one vibration by a medium particle is called the wavelength (λ). In case of a transverse wave the wavelength may also be defined as the distance between two successive crests or troughs. In case of a longitudinal wave, the wavelength (λ) is equal to distance from centre of one compression (or rarefraction) to another.

- **Wave Velocity**

Wave velocity is the time rate of propagation of wave motion in the given medium. It is different from particle velocity. Wave velocity depends upon the nature of medium.

Wave velocity (v) = frequency (ν) × wavelength (λ)

- **Amplitude**

The amplitude of a wave is the maximum displacement of the particles of the medium from their mean position.

- **Frequency**

The number of vibrations made by a particle in one second is called Frequency. It is represented by ν. Its unit is hertz (Hz) $\nu = \frac{1}{T}$

- **Time Period**

The time taken by a particle to complete one vibration is called time period.

$$T = \frac{1}{\nu}, \text{ it is expressed in seconds.}$$

- The velocity of transverse waves in a stretched string is given by

$$v = \sqrt{\frac{T}{\mu}}$$

where T is the tension in the string and μ is the mass per unit length of the string. μ is also called linear mass density of the string. SI unit of μ is kg m^{-1}.

- The velocity of the longitudinal wave in an elastic medium is given by

$$v = \sqrt{\frac{E}{\rho}}$$

where E is the modulus of elasticity of the medium and ρ is the density of the medium.

In case of solids, E is Young's modulus of elasticity (Y), then

$$v = \sqrt{\frac{Y}{\rho}}$$

In case of fluids, E is replaced by the bulk modulus of elasticity (B), then

$$v = \sqrt{\frac{B}{\rho}}$$

- **Newton's Formula for the velocity of sound in Air**

According to Newton, when sound waves travel in air or in a gaseous media, the change is taking place isothermally and hence, it is found that

$$v = \sqrt{\frac{B_{\text{isothermal}}}{\rho}} = \sqrt{\frac{P}{\rho}}$$

Where p = pressure of air or gas.

Speed of sound in air at STP conditions, calculated on the basis of Newton's formula is 280 ms^{-1}. However, the experimentally determined values is 332 ms^{-1}.

According to Laplace, during propagation of sound waves, the change takes place under adiabatic conditions because gases are thermal insulators and compressions and rarefractions are alternately taking place with a high frequency.

Hence

$$v = \sqrt{\frac{B_{\text{adiabatic}}}{\rho}} = \sqrt{\frac{Yp}{\rho}},$$

where y = ratio of two principal specific heats of the given gas.

Speed of sound in air at STP conditions, as per Laplace's formula comes out to be 332 ms^{-1}.

- **Factors Influencing Velocity of Sound**

The velocity of sound in any gaseous medium is affected by a large number of factors like density, pressure, temperature, humidity, wind velocity etc.

(*i*) The velocity of sound in a gas is inversely proportional to the square root of density of the gas.

(*ii*) The velocity of sound is independent of the change in pressure of the gas, provided temperature remains constant.

(*iii*) The velocity of sound in a gas is directly proportional to the square root of its absolute temperature.

(*iv*) The velocity of sound in moist air is greater than the velocity of sound in dry air.

(*v*) If wind flows at an angle θ to the direction of propagation of sound, the velocity of sound is $v + w \cos \theta$, where w is the velocity of wind.

• General Equation of Progressive Waves

"A progressive wave is one which travels in a given direction with constant amplitude, *i.e.,* without attenuation."

As in wave motion, the displacement is a function of space as well as time, hence displacement relation is expressed as a combined function of position and time as:

$$y\ (x,t) = A \sin (kx - \omega t + \phi)$$

We may also choose a cosine function instead of sine function. Here A, K, ω and ϕ are four constant for a given wave and are known as amplitude, angular wave number, angular frequency and initial phase angle of given wave.

• Relation between phase and path difference

Phase difference. $$\phi = \frac{2\pi}{\lambda} \times \text{path difference}$$

Also, $$\frac{x}{\lambda} = \frac{\phi}{2\pi} = \frac{t}{T},$$ where T is the time period and t is time for a path x or phase ϕ.

• Energy associated with unit volume of a wave is

$$U = \frac{1}{2}\rho V^2.$$

• Intensity is defined as energy per unit area (s) per unit time, *i.e.,* $I = \frac{E}{tS} = \frac{P}{S} = \frac{1}{2}\rho c \omega^2 A^2$, where c is the velocity of wave with amplitude A in a media of density ρ. It is measured in $Js^{-1}\ m^{-2}$ or Wm^{-2}.

• A wave motion can be reflected from a rigid as well as from a free boundary. A travelling wave, at a rigid boundary or a closed end, is reflected with a phase reversal but the reflection at an open boundary takes place without any phase change.

• The Principle of Superposition of Wave

When any number of waves meet simultaneously at a point in a medium, the net displacement at a given time is the algebraic sum of the displacements due to each wave at that time.

If $\vec{y}_1$ and $\vec{y}_2$ represent the displacement of a particle due to the two individual waves. Then at every point in the medium and at each instant of time, the resultant displacement is given by

$$\vec{y} = \vec{y}_1 + \vec{y}_2$$

• Standing waves or Stationary waves

When two sets of progressive wave trains of the same type (*i.e.,* both longitudinal or both transverse) having the same amplitude and time period/frequency/wavelength travelling with same speed along the same straight line in opposite directions superimpose, a new set of waves are formed. These are called stationary waves or standing waves.

The equation of a standing wave is given as:

$$y = 2\ A \sin kx \cos \omega t$$

Special Note:

(*i*) From the above equation it is seen that a particle at any particular point 'x' executes simple harmonic motion and all the particles vibrate with the same frequency.

(*ii*) The amplitude is not the same for different particle but varies with the location 'x' of the particle.

(*iii*) The points having maximum amplitudes are those for which $2A \sin kx$, has a maximum value of $2A$, these are at the positions.

$$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\ldots\ldots\ldots \quad \text{or} \quad x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \ldots\ldots\ldots\ldots$$

i.e., $$x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}; \quad \text{where } n = 0, 1, 2 \ldots\ldots\ldots\ldots$$

These points are called antinodes.

(*iv*) The amplitudes has minimum value of zero at positions

where $$kx = \pi, 2\pi, 3\pi, \ldots\ldots\ldots\ldots \quad \text{or} \quad x = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda \ldots\ldots\ldots\ldots\ldots\ i.e., \ x = \frac{n\lambda}{2}$$

These points are called nodes.

(*v*) It is clear that the separation between consecutive nodes or consecutive antinodes is $\frac{\lambda}{2}$.

(*vi*) As the particles at the nodes do not move at all, energy cannot be transmitted across them.

Progressive Waves	*Stationary Waves*
1. The disturbance progresses onwards; it being handed over from particle to particle. Each particle executes the same type of vibration as the preceding one, though at a different time.	1. The disturbance is stationary, there being no forward or backward movement of the wave. Each particle has its own vibration characteristics.
2. The waves are in the form of crests and troughs, *i.e.,* sine/cosine functions, which move onwards with a definite velocity.	2. The waves have the appearance of a sine/cosine function, which shrink to a straight line, twice in each vibration. It never advances.
3. Every particle has the same amplitude; which it attains in its own time depending upon the progress of the wave.	3. Every particle has a fixed allotted amplitude. Some have zero amplitude (nodes) and some have maximum amplitude (antinodes) always. Each particle attains this at the same given moment.
4. The phase of every particle varies continuously from 0 to 2π.	4. All the particles in one-half of the waves have a fixed phase and all the particles in the other half of the wave have the same phase in the opposite direction simultaneously.
5. No particle remains permanently at rest. Twice during each vibration, the particles are momentarily at rest. Different particles attain this position at different times.	5. There are particles which are permanently at rest (nodes) and all other particles have their own allotted maximum displacement, which they attain simultaneously. These particles are momentarily at rest twice in each vibration, all at the same time.
6. All the particles have the same maximum velocity which they attain one after another, as the wave advances.	6. All the particles attain their individual allotted velocities depending upon their positions, simultaneously. Two particles (nodes) in one waveform have zero velocities all the time.

7. There is a regular flow of energy across every plane along the direction of propagation of the wave. The average energy in a wave is half potential and half kinetic.	7. There is no flow of energy at all, across any plane. Each particle has its own allotted individual energy. They all attain their values of P.E. at one time and all energy becomes K.E. at another given time.

• When a stationary wave is set up in a string of length l fixed at its two ends, in the simplest mode of vibration, nodes are formed at the fixed ends and an antinode is formed at the middle point. The frequency of fundamental mode of vibration (or first harmonic) is given by

$$\nu = \frac{v}{\lambda} = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

• In closed pipes, only odd harmonics are developed, *i.e.*, $\nu = \frac{(2n-1)v}{4l}$, with fundamental frequency of $\left(\frac{v}{4l}\right)$.

• In open pipes, all harmonics with fundamental or first harmonic $\left(\frac{v}{2l}\right)$ are developed.

$$\nu = \frac{nv}{2l}$$, where v is the velocity of sound.

• Frequency of the Stretched String

In general, if the string vibrates in P loops, the frequency of the string under that mode is given by

$$\nu = \frac{P}{2L}\sqrt{\frac{T}{\mu}}$$

Based on this relation three laws of transverse vibrations of stretched strings arise. They are law of length, law of tension and law of mass.

• Law of Length

The fundamental frequency ν is inversely proportional to the length L of the stretched string.

$$\nu \propto \frac{1}{L} \quad \text{or} \quad \nu L = a \text{ constant } (T \text{ being constant})$$

• Law of Tension

The fundamental frequency is directly propotional to the square root of the tension in the string.

$$\nu = \sqrt{T} \quad \text{or} \quad \frac{\nu}{\sqrt{T}} = a \text{ constant } (L, m \text{ being constant})$$

• Law of Mass

The fundamental frequency is inversely proportional to the square root of mass per unit length of the given string when L and T are kept constants.

$$\nu \propto \frac{1}{\sqrt{\mu}}$$

or $$\nu\sqrt{\mu} = a \text{ constant } (L, T \text{ being constant}).$$

• Interference is the interaction with the redistribution of energy when two waves with constant phase difference interact.

If A_1 and A_2 are the two light waves with constant phase difference ϕ interfering, the resultant displacement is, $Y = A \sin(\omega t + \delta)$, where

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi} \quad \text{and} \quad \delta = \frac{A_2 \sin\phi}{A_1 + A_2 \cos\phi}$$

Intensity for equal amplitudes is $4A^2 \cos^2\left(\frac{\phi}{2}\right)$.

• Beats

The phenomenon of regular rise and fall in the intensity of sound, when two waves of nearly equal frequencies travelling along the same line and in the same direction superimpose each other is called beats.

One rise and one fall in the intensity of sound constitutes one beat and the number of beats per second is called beat frequency. It is given as:

$$\nu_b = (\nu_1 - \nu_2)$$

where ν_1 and ν_2 are the frequencies of the two interfering waves; ν_1 being greater than ν_2.

• Doppler Effect

According to Doppler's effect, whenever there is a relative motion between a source of sound and listener, the apparent frequencies of sound heard by the listener is different from the actual frequency of sound emitted by the source.

For sound the observed frequency ν' is given by

$$\nu' = \left(\frac{v + v_0}{v + v_s}\right) . \nu$$

Here ν = true frequency of wave emitted by the source, v = speed of sound through the medium, v_0 the velocity of observer relative to the medium and v_s the velocity of source relative to the medium. In using this formula, velocities in the direction OS (*i.e.*, from observer towards the source) are treated as positive and those opposite to it are taken as negative.

• IMPORTANT TABLES

TABLE 15.1. Speed of Sound in some Media

Medium	Speed (ms^{-1})
Gases	
Air (0°C)	331
Air (20°C)	343
Helium	965
Hydrogen	1284

Liquids	
Water (0°C)	1402
Water (20°C)	1482
Seawater	1522
Solids	
Aluminium	6420
Copper	3560
Steel	5941
Granite	6000
Vulcanised Rubber	54

TABLE 15.2

Physical quantity	*Symbol*	*Dimensions*	*Unit*	*Remarks*
Wavelength	λ	[L]	m	Distance between two consecutive points with the same phase.
Propagation constant	k	$[L^{-1}]$	m^{-1}	$k = \frac{2\pi}{\lambda}$
Wave speed	v	$[LT^{-1}]$	ms^{-1}	$v = V\lambda$
Beat frequency	ν_{beat}	$[T^{-1}]$	s^{-1}	Difference of two close frequencies of superposing waves.

NCERT TEXTBOOK QUESTIONS SOLVED

15.1. *A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?*

Ans. Tension, $T = 200$ N;

Length, $l = 20.0$ m; Mass, $M = 2.50$ kg

Mass per unit length, $\mu = \frac{2.50}{20.0}$ kg m^{-1} = 0.125 kgm^{-1}

Wave velocity, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200\ N}{0.125\ \text{kg m}^{-1}}}$

or $v = \sqrt{1600}\ \text{ms}^{-1} = 40\ \text{ms}^{-1}$

Time, $t = \frac{l}{v} = \frac{20.0}{40}s = \frac{1}{2}s = 0.5\ s.$

15.2. *A stone dropped from the top of a tower of height 300 m high splashes into the water of a pond near the base of the tower. When is the splash heard at the top given that the speed of sound in air is 340 ms^{-1}? (g = 9.8 ms^{-2})*

Ans. Here, h = 300 m, g = 9.8 ms^{-2} and velocity of sound, v = 340 ms^{-1} Let t_1 be the time taken by the stone to reach at the surface of pond.

Then, using $$s = ut + \frac{1}{2}at^2\frac{1}{2}at^2 \Rightarrow h = 0 \times t + \frac{1}{2}gt_1^2$$

$$\therefore \quad t_1 = \sqrt{\frac{2\times 300}{9.8}} = 7.82 \text{ s}$$

Also, if t_2 is the time taken by the sound to reach at a height h, then

$$t_2 = \frac{h}{v} = \frac{300}{340} = 0.88 \text{ s}$$

$\therefore$ Total time after which sound of splash is heard = $t_1 + t_2$

$$= 7.82 + 0.88 = 8.7\text{s}.$$

15.3. *A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that speed of a transverse wave on the wire equals the speed of sound in dry air at 20°C = 340 ms^{1}.*

Ans. Here, l = 12.0 m, M = 2.10 kg

v = 343 ms^{-1}

$$\text{Mass per unit length} = \frac{M}{l} = \frac{2.10}{12.0} = 0.175 \text{ kg m}^{-1}$$

As $$v = \sqrt{\frac{T}{m}}$$

$$\therefore \quad T = v^2.\ \text{m} = (343)^2 \times 0.175 = 2.06 \times 10^4 \text{ N}.$$

15.4. *Use the formula* $v = \sqrt{\frac{\gamma P}{\rho}}$ *to explain why the speed of sound in air*

(a) is independent of pressure. *(b) increases with temperature.*

(c) increases with humidity.

Ans. We are given that $$v = \sqrt{\frac{\gamma P}{\rho}}$$

We know $PV = n\text{R}T$ (for n moles of ideal gas)

$$\Rightarrow \quad PV = \frac{m}{M}RT$$

where m is the total mass and M is the molecular mass of the gas.

$$\therefore \quad P = \frac{m}{M}\cdot\frac{RT}{M} = \frac{\rho RT}{M} \Rightarrow \frac{P}{\rho} = \frac{\text{R}T}{M}$$

(*a*) For a gas at constant temperature, $\frac{P}{\rho}$ = constant

$\therefore$ As P increase, ρ also increases and vice versa. This implies that $v = \sqrt{\frac{\gamma P}{\rho}}$ = constant, *i.e.,* velocity is independent of pressure of the gas.

(*b*) Since $\frac{P}{\rho} = \frac{RT}{M}$, therfore, $v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

Clearly $v \propto \sqrt{T}$ *i.e.,* speed of sound in air increases with increase in temperature.

(*c*) Increase in humidity decreases the effective density of air. Therefore the velocity $\left(v \propto \frac{1}{\sqrt{\rho}}\right)$ increases.

15.5. *You have learnt, that a travelling wave in one dimension is represented by a function y = f(x, t), where x and t must appear in the combination x – v t or x + vt i.e., y = f (x ± vt). Is the converse true? That is, does every function of (x – vt) or (x + vt) represent a travelling wave? Examine, if the following functions for y can possibly represent a travelling wave?*

(a) $(x - vt)^2$ *(b)* $\log\left[\frac{(x+vt)}{x_0}\right]$ *(c)* $\frac{1}{x+vt}$

Ans. No, the converse is not true. The basic requirement for a wave function to represent a travelling wave is that for all values of x and t, wave function must have a finite value. Out of the given functions for y none satisfies this condition. Therefore, none can represent a travelling wave.

15.6. *A bat emits ultrasonic sound of frequency 1000 kHz in air. If this sound meets a water surface, what is the wavelength of (a) the reflected sound, (b) the transmitted sound? Speed of sound in air = 340 ms*$^{-1}$ *and in water = 1486 ms*$^{-1}$*.*

Ans. Here, $\nu = 1000 \times 10^3$ Hz $= 10^6$ Hz, $v_a = 340$ ms^{-1},

$v_w = 1486$ ms^{-1}

Wavelength of reflected sound, λ_a

$$= \frac{v_a}{\nu} = \frac{340}{10^6}\text{ m} = 3.4 \times 10^{-4}\text{ m}$$

Wavelength of transmitted sound, λ_ω

$$= \frac{v_w}{\nu} = \frac{1486}{10^6}\text{ m} = 1.486 \times 10^{-3}\text{m}$$

15.7. *A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in a tissue in which the speed of sound is 1.7 km s*$^{-1}$*? The operating frequency of the scanner is 4.2 MHz.*

Ans. Here speed of sound $\Rightarrow$ $v = 1.7$ km s^{-1} = 1700 ms^{-1}

and frequency v = 4.2 MHz = 4.2×10^6 Hz

$$\therefore \quad \text{Wavelength, } \lambda = \frac{v}{\nu} = \frac{1700}{4.2\times10^6} = 4.1 \times 10^{-4}\text{ m.}$$

15.8. *A transverse harmonic wave on a string is described by*

$$y(x, t) = 3.0 \sin (36t + 0.018 x + \pi/4)$$

where x and y are in cm and t in s. The positive direction of x is from left to right.

(a) Is this a travelling wave or a stationary wave? If it is travelling, what are the speed and direction of its propagation?

(b) What are its amplitude and frequency?

(c) What is the initial phase at the origin?

(d) What is the least distance between two successive crests in the wave?

Ans. The given equation is $y(x, t) = 3.0 \sin (36t + 0.018x + \frac{\pi}{4})$, where x and y are in cm and t in s.

(*a*) The equation is the equation of a travelling wave, travelling from right to left (*i.e.,* along *–ve* direction of x because it is an equation of the type

$$y(x, t) = A \sin (\omega t + kx + \phi)$$

Here, $A = 3.0$ cm, $\omega = 36$ rad s^{-1}, $k = 0.018$ cm and $\phi = \frac{\pi}{4}$.

∴ Speed of wave propagation,

$$v = \frac{\omega}{k} = \frac{36 \text{ rad s}^{-1}}{0.018 \text{ cm}^{-1}} = \frac{36 \text{ rad s}^{-1}}{0.018 \times 10^2 \text{ ms}^{-1}} = 20 \text{ ms}^{-1}$$

(*b*) Amplitude of wave, $A = 3.0$ cm $= 0.03$ m

Frequency of wave $\nu = \frac{\omega}{2\pi} = \frac{36}{2\pi} = 5.7$ Hz

(*c*) Initial phase at the origin, $\phi = \frac{\pi}{4}$

(*d*) Least distance between two successive crests in the wave

$$= \lambda = \frac{2\pi}{k} = \frac{2\pi}{0.018} = 349 \text{ cm} = 3.5 \text{ m}$$

15.9. *For the wave described in Exercise 8, plot the displacement (y) versus (t) graphs for x = 0, 2 and 4 cm. What are the shape of these graphs? In which aspects does the oscillatory motion in travelling wave differ from one point to another : amplitude, frequency or phase?*

Ans. The transverse harmonic wave is

$$y(x, t) = 3.0 \sin \left(36t + 0.018x + \frac{\pi}{4}\right)$$

For $x = 0$,

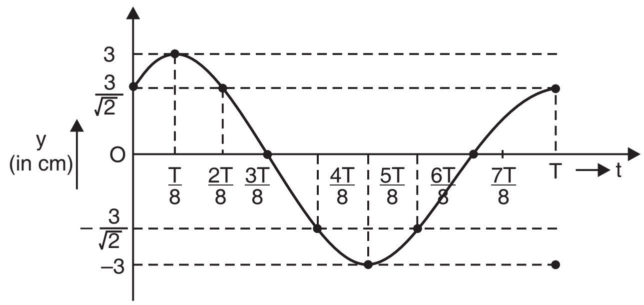
$$y(0, t) = 3 \sin \left(36t + 0 + \frac{\pi}{4}\right) = 3 \sin \left(36t + \frac{\pi}{4}\right) \quad ...(1)$$

Here $\omega = \frac{2\pi}{T} = 36 \Rightarrow T = \frac{2\pi}{36}$

To plot a (y) versus (t) graph, different values of y corresponding to the values of t may be tabulated as under (by making use of eqn. (1)).

t	0	$\frac{T}{8}$	$\frac{2T}{8}$	$\frac{3T}{8}$	$\frac{4T}{8}$	$\frac{5T}{8}$	$\frac{6T}{8}$	$\frac{7T}{8}$	T
y	$\frac{3}{\sqrt{2}}$	3	$\frac{3}{\sqrt{2}}$	0	$\frac{-3}{\sqrt{2}}$	– 3	$\frac{-3}{\sqrt{2}}$	0	$\frac{3}{\sqrt{2}}$

Using the values of t and y (as in the table), a graph is plotted as under



The graph obtained is sinusoidal. Similar graphs are obtained for x = 2 cm and x = 4 cm. The oscillatory motion in the travelling wave only differs in respect of phase. Amplitude and frequency of oscillatory motion remains the same in all the cases.

15.10. *For the travelling harmonic wave*

$$y(x,\ t) = 2.0 \cos 2\pi\ (10t - 0.0080x + 0.35)$$

where x and y are in cm and t in s. Calculate the phase difference between oscillatory motion of two points separated by a distance of

(a) 4 m *(b) 0.5 m*

(a) λ/2 *(b) 3λ/4.*

Ans. The given equation can be rewritten as under:

$$y(x,\ t) = 2.0 \cos [2\pi\ (10t - 0.0080\ x) + 2\pi \times 0.35]$$

or

$$y(x,\ t) = 2.0 \cos \left[2\pi \times 0.0080 \left(\frac{10t}{0.0080} - x\right) + 0.7\pi\right]$$

Comparing this equation with the standard equation of a travelling harmonic wave,

$$\frac{2\pi}{\lambda} = 2\pi \times 0.0080 \quad \text{or} \quad \lambda = \frac{1}{0.0080}\ \text{cm} = 125\ \text{cm}$$

The phase difference between oscillatory motion of two points separated by a distance Δx is given by

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$

(*a*) When Δx = 4 m = 400 cm, then

$$\Delta\phi = \frac{2\pi}{125}\times 400 = 6.4\ \pi\ \text{rad}$$

(*b*) When Δx = 0.5 m = 50 cm, then

$$\Delta\phi = \frac{2\pi}{125}\times 50 = 0.8\ \pi\ \text{rad}$$

(*c*) When $\Delta x = \frac{\lambda}{2} = \frac{125}{2}$ cm, then

$$\Delta\phi = \frac{2\pi}{125} \times \frac{125}{2} = \pi \text{ rad}$$

(*d*) When $$\Delta x = \frac{3\lambda}{4} = \frac{3 \times 125}{4} \text{ cm, then}$$

$$\Delta\phi = \frac{2\pi}{125} \times \frac{3 \times 125}{4} = \frac{3\pi}{2} \text{ rad.}$$

15.11. *The transverse displacement of a string (clamped at its two ends) is given by*

$$y(x, t) = 0.06 \sin \frac{2\pi}{3} x \cos (120\, \pi t)$$

where x, y are in m and t in s. The length of the string is 1.5 m and its mass is 3 × 10^{-2} *kg. Answer the following:*

(i) Does the function represent a travelling or a stationary wave?

(ii) Interpret the wave as a superimposition of two waves travelling in opposite directions. What are the wavelength, frequency and speed of propagation of each wave?

(iii) Determine the tension in the string.

Ans. The given equation is

$$y(x, t) = 0.06 \sin \frac{2\pi}{3} x \cos 120\, \pi t \quad \text{...(1)}$$

(*i*) As the equation involves harmonic functions of x and t separately, it represents a stationary wave.

(*ii*) We know that when a wave pulse

$$y_1 = r \sin \frac{2\pi}{\lambda} (vt - x)$$

travelling along + direction of x-axis is superimposed by the reflected wave

$$y_2 = -r \sin \frac{2\pi}{\lambda} (vt + x)$$

travelling in opposite direction, a stationary wave

$$y = y_1 + y_2 = -2r \sin \frac{2\pi}{\lambda} x \cos \frac{2\pi}{\lambda} vt \text{ is formed.} \quad \text{...(2)}$$

Comparing eqns. (1) and (2), we find that

$$\frac{2\pi}{\lambda} = \frac{2\pi}{3} \Rightarrow \lambda = 3\text{m}$$

Also, $$\frac{2\pi}{\lambda} v = 120\, \pi \quad \text{or} \quad v = 60\lambda = 60 \times 3 = 180 \text{ ms}^{-1}$$

Frequency, $$\nu = \frac{v}{\lambda} = \frac{180}{3} = 60 \text{ Hz}$$

Note that both the waves have same wavelength, same frequency and same speed.

(*iii*) Velocity of transverse waves is

$$v = \sqrt{\frac{T}{m}} \quad \text{or} \quad v^2 = \frac{T}{m}$$

$$T = mv^2, \quad \text{where} \quad m = \frac{3 \times 10^{-2}}{1.5} = 2 \times 10^{-2} \text{ kg/m}$$

$$\therefore \quad T = (180)^2 \times 2 \times 10^{-2} = 648 \text{ N.}$$

15.12. *(i) For the wave on a string described in Question 11, do all the points on the string oscillate with the same (a) frequency, (b) phase, (c) amplitude? Explain your answers. (ii) What is the amplitude of a point 0.375 m away from one end?*

Ans. (*i*) For the wave on the string described in questions we have seen that l = 1.5 m and λ = 3 m. So, it is clear that $\lambda = \frac{\lambda}{2}$ and for a string clamped at both ends, it is possible only when both ends behave as nodes and there is only one antinode in between *i.e.,* whole string is vibrating in one segment only.

(*a*) Yes, all the sring particles, except nodes, vibrate with the same frequency ν = 60 Hz.

(*b*) As all string particles lie in one segment, all of them are in same phase.

(*c*) Amplitude varies from particle to particle. At antinode, amplitude = 2A = 0.06 m. It gradually falls on going towards nodes and at nodes, and at nodes, it is zero.

(*ii*) Amplitude at a point x = 0.375 m will be obtained by putting cos (120 πt) as + 1 in the wave equation.

$$\therefore \quad A(x) = 0.06 \sin\left(\frac{2\pi}{3} \times 0.375\right) \times 1 = 0.06 \sin \frac{\pi}{4} = 0.042 \text{ m.}$$

15.13. *Given below are some functions of x and t to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represent (i) a travelling wave, (ii) a stationary wave or (iii) none at all*

(a) $y = 2 \cos (3x) \sin (10t)$ *(b)* $y = 2\sqrt{x - vt}$

(c) $y = 3 \sin (5x - 0.5t) + 4 \cos (5x - 0.5t)$ *(d)* $y = \cos x \sin t + \cos 2x \sin 2t.$

Ans. (*a*) It represents a stationary wave.

(*b*) It does not represent either a travelling wave or a stationary wave.

(*c*) It is a representation for the travelling wave.

(*d*) It is a superposition of two stationary wave.

15.14. *A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5 × 10^{-2} kg and its linear mass density is 4.0 × 10^{-2} kg m^{-1}. What is (a) the speed of a transverse wave on the string, and (b) the tension in the string?*

Ans. Here, n = 45 Hz, $M = 3.5 \times 10^{-2}$ kg

Mass per unit length = $m = 4.0 \times 10^{-2}$ kg m^{-1}

$$\therefore \quad l = \frac{M}{m} = \frac{3.5 \times 10^{-2}}{4.0 \times 10^{-2}} = \frac{7}{8}$$

$$\text{As} \quad \frac{l}{2} = \lambda = \frac{7}{8} \quad \therefore \quad \lambda = \frac{7}{4} \text{ m} = 1.75 \text{ m.}$$

(*a*) The speed of the transverse wave, $v = \nu\lambda = 45 \times 1.75 = 78.75$ m/s

(*b*) As $$v = \sqrt{\frac{T}{m}}$$

$$\therefore \quad T = v^2 \times m = (78.75)^2 \times 4.0 \times 10^{-2} = 248.06 \text{ N.}$$

15.15. *A metre-long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a turning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. The edge effect may be neglected.*

Ans. Frequency of n^{th} mode of vibration of the closed organ pipe of legnth

$$l_1 = (2n-1)\frac{v}{4l_1}$$

Frequency of $(n+1)^{th}$ mode of vibration of closed pipe of length

$$'l_2' = [2(n+1)-1]\frac{v}{4l_2} = (2n+1)\frac{v}{4l_2}$$

Both the modes are given to resonate with a frequency of 340 Hz.

$$\therefore \quad (2n-1)\frac{v}{4l_1} = (2n+1)\frac{v}{4l_2} \quad \text{or} \quad \frac{2n-1}{2n+2} = \frac{l_1}{l_2} = \frac{25.5}{79.3} = \frac{1}{3}$$

[Approximation has been used because edge effect is being ignored. Moreover, we know that in the case of a closed organ pipe, the second resonance length is three times the first resonance length.]

On simplification, $n = 1$

Now, $(2n-1)\frac{v}{4l_1} = 340$. Substituting values

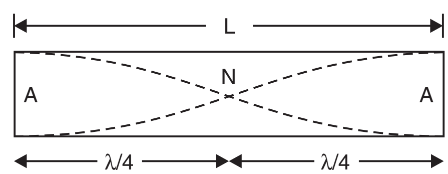
$$(2\times1-1)\frac{v\times100}{4\times25.5} = 340 \quad \text{or} \quad v = 346.8 \text{ ms}^{-1}.$$

15.16. *A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be 2.53 k Hz. What is the speed of sound in steel?*

Ans. Here, $L = 100 \text{ cm} = 1\text{m}, \quad \nu = 2.53\ k \text{ Hz} = 2.53 \times 10^3 \text{ Hz}$

When the rod is clamped at the middle, then in the fundamental mode of vibration of the rod, a node is formed at the middle and antinode is formed at each end.

Therefore, as is clear from Fig.



$$L = \frac{\lambda}{4}+\frac{\lambda}{4}=\frac{\lambda}{2}$$

$$\lambda = 2L = 2 \text{ m}$$

As $v = \nu\lambda$

$\therefore$ $v = 2.53 \times 10^3 \times 2 = \mathbf{5.06 \times 10^3\ ms^{-1}}$

15.17. *A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will the same source be in resonance with the pipe if both ends are open? (speed of sound in air is 340 ms^{-1}).*

Ans. Here length of pipe, $l = 20 \text{ cm} = 0.20 \text{ m}$, frequency $\nu = 430$ Hz and speed of sound in air $v = 340 \text{ ms}^{-1}$

For closed end pipe, $\nu = \frac{(2n-1)\,v}{4l}$, where $n = 1, 2, 3$..........

$$\therefore \quad (2n-1) = \frac{4\nu l}{v} = \frac{4\times430\times0.20}{340} = 1.02$$

$$\Rightarrow \qquad 2n = 1.02 + 1 = 2.02 \quad \Rightarrow \quad n = \frac{0.20}{2} = 1.01$$

Hence, resonance can occur only for first (or fundamental) mode of vibration.

As for an open pipe $\nu = \frac{nv}{2l}$, where $n = 1, 2, 3$..........

$$\therefore \qquad n = \frac{2l\nu}{v} = \frac{2 \times 430 \times 0.20}{340} = 0.51.$$

As $n < 1$, hence, in this case resonance position cannot be obtained.

15.18. *Two sitar strings A and B playing the note 'Ga' are slightly out of tune and produce beats of frequency 6Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3Hz. If the original frequency of A is 324 Hz, what is the frequency of B?*

Ans. Let v_1 and v_2 be the frequencies of strings A and B respectively.

Then, $\nu_1 = 324$ Hz, $\nu_2 = ?$

Number of beats, $b = 6$

$\therefore \qquad \nu_2 = \nu_1 \pm b = 324 \pm 6$ *i.e.,* $\nu_2 = 330$ Hz or 318 Hz

Since the frequency is directly proportional to square root of tension, on decreasing the tension in the string A, its frequency ν_1 will be reduced *i.e.,* number of beats will increase if $\nu_2 = 330$ Hz. This is not so because number of beats become 3.

Therefore, it is concluded that the frequency $\nu_2 = 318$ Hz. because on reducing the tension in the string A, its frequency may be reduced to 321 Hz, thereby giving 3 beats with $\nu_2 = 318$ Hz.

15.19. *Explain why (or how):*

(a) in a sound wave, a displacement node is a pressure antinode and vice versa.

(b) bats can ascertain distances, directions, nature and sizes of the obstacles without any "eyes".

(c) a violin note and sitar note may have the same frequency, yet we can distinguish between the two notes.

(d) solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases, and

(e) the shape of a pulse gets distorted during propagation in a dispersive medium.

Ans. (*a*) In a sound wave, a decrease in displacement *i.e.,* displacement node causes an increase in the pressure there *i.e.,* a pressure antinode is formed. Also, an increase in displacement is due to the decrease in pressure.

(*b*) Bats emit ultrasonic waves of high frequency from their mouths. These waves after being reflected back from the obstacles on their path are observed by the bats. These waves give them an idea of distance, direction, nature and size of the obstacles.

(*c*) The quality of a violin note is different from the quality of sitar. Therefore, they emit different harmonics which can be observed by human ear and used to differentiate between the two notes.

(*d*) This is due to the fact that gases have only the bulk modulus of elasticity whereas solids have both, the shear modulus as well as the bulk modulus of elasticity.

(*e*) A pulse of sound consists of a combination of waves of different wavelength. In a dispersive medium, these waves travel with different velocities giving rise to the distortion in the wave.

15.20. *A train, standing at the outer signal of a railway station blows a whistle of frequency 400 Hz in still air. (i) What is the frequency of the whistle for a platform observer when the train (a) approaches the platform with a speed of 10 ms^{-1}. (b) recedes from the platform with a speed of 10 ms^{-1}?(ii) What is the speed of sound in each case? The speed of sound in still air can be taken as 340 ms^{-1}.*

Ans. Frequency of whistle, ν = 400 Hz; speed of sound, v = 340 ms^{-1} speed of train, v_s= 10 ms ^{1}

(*i*) (*a*) When the train approaches the platform (*i.e.,* the observer at rest),

$$\nu' = \frac{v}{v - v_s} \times \nu = \frac{340}{340 - 10} \times 400 = 412 \text{ Hz.}$$

(*b*) When the train recedes from the platform (*i.e.,* from the observer at rest),

$$\nu' = \frac{v}{v + v_s} \times \nu = \frac{340}{340 + 10} \times 400 = 389 \text{ Hz.}$$

(*ii*) The speed of sound in each case does not change.

$\therefore$ It is 340 ms^{-1} in each case.

15.21. *A train, standing in a station-yard, blows a whistle of frequency 400 Hz in still air. The wind starts blowing in the direction from the yard to the station with a speed of 10ms^{-1}. What are the frequency, wavelength, and speed of sound for an observer standing on the station's platform? Is the situation exactly identical to the case when the air is still and the observer runs towards the yard at a speed of 10 ms^{-1}? The speed of sound in still air can be taken as 340 ms^{-1}?*

Ans. Here actual frequency of whistle of train ν = 400 Hz, speed of sound in still air v = 340 ms^{-1}.

As wind is blowing in the direction from the yard to the station with a speed of v_m = 10 ms^{-1}

$\therefore$ For an observer standing on the platform, the effective speed of sound

$$v' = v + v_m = 340 + 10 = 350 \text{ ms}^{-1}$$

As there is no relative motion between the sound source (rail engine) and the observer, the frequency of sound for the observer, ν = 400 Hz

$\therefore$ Wavelength of sound heard by the observer $\lambda' = \frac{v'}{\nu} = \frac{350}{400}$

$= 0.875$ m.

The situation is not identical to the case when the air is still and observer runs towards the yard at a speed of v_0 = 10 ms^{-1}. In this situation as medium is at rest. Hence $v' = v$ = 340 ms^{-1}.

$$\nu' = \frac{v + v_0}{v}\nu = \frac{340 + 10}{340} \times 400 = 412 \text{ Hz}$$

and $$\lambda' = \lambda = \frac{v}{\nu} = \frac{340}{400} = 0.85 \text{ m}$$

15.22. *A travelling harmonic wave on a string is described by y (x, t) = 7.5 sin (0.0050x + 12t + π/4)*

(a) what are the displacement and velocity of oscillation of a point at x = 1 cm, and t = 1s? Is this velocity equal to the velocity of wave propagation?

(b) Locate the points of the string which have the same transverse displacement and velocity as the x = 1cm point at t = 2s, 5s and 11s.

Ans. The travelling harmonic wave is $y(x, t) = 7.5 \sin(0.0050x + 12t + \pi/4)$

At $x = 1$ cm and $t = 1$ sec,

$$y(1, 1) = 7.5 \sin(0.005 \times 1 + 12 \times 1\ \pi/4) = 7.5 \sin(12.005 + \pi/4) \quad ...(i)$$

Now, $\theta = (12.005 + \pi/4)$ radian

$$= \frac{180}{\pi}(12.005 + \pi/4) \text{ degree} = \frac{12.005 \times 180}{\frac{22}{7}} + 45 = 732.55°.$$

$\therefore$ From (*i*), $y(1, 1) = 7.5 \sin(732.55°) = 7.5 \sin(720 + 12.55°)$

$= 7.5 \sin 12.55° = 7.5 \times 0.2173 = 1.63$ cm

Velocity of oscillation, $v = \frac{dy}{dt}(1, 1) = \frac{d}{dt}\left[7.5 \sin\left(0.005x + 12 + \frac{\pi}{4}\right)\right]$

$$= 7.5 \times 12 \cos\left[0.005x + 12t + \frac{\pi}{4}\right]$$

At $x = 1$cm, $t = 1$sec.

$v = 7.5 \times 12 \cos(0.005 + 12 + \pi/4) = 90 \cos(732.35°)$

$= 90 \cos(720 + 12.55)$

$v = 90 \cos(12.55°) = 90 \times 0.9765 = 87.89$ cm/s.

Comparing the given eqn. with the standard form $y(x, t) = t \sin\left[\frac{\pi}{4}(vt + x) + \phi_0\right]$

We get $r = 7.5$ cm, $\frac{2\pi v}{\lambda} = 12$ or $2\pi\nu = 12$

$$\nu = \frac{6}{\pi}$$

$$\frac{2\pi}{\lambda} = 0.005.$$

$$\therefore \quad \lambda = \frac{2\pi}{0.005} = \frac{2 \times 3.14}{0.005} = 1256 \text{ cm} = 12.56 \text{ m}$$

Velocity of wave propagation, $v = \nu\lambda = \frac{6}{\pi} \times 12.56 =$ **24 m/s.**

We find that velocity at $x = 1$ cm, $t = 1$ sec is not equal to velocity of wave propagation.

(*b*) Now, all points which are at a distance of $\pm\lambda, \pm 2\lambda, \pm 3\lambda$ from $x = 1$ cm will have same transverse displacement and velocity. As $\lambda = 12.56$ m, therefore, all points at distances ± 12.6 m, ± 25.2 m, ± 37.8 m.... from $x = 1$ cm will have same displacement and velocity, as at $x = 1$ point $t = 2$ *s*, 5 *s* and 11 s.

15.23. *A narrow sound pulse (for example, a short pip by a whistle) is sent across a medium. (a) Does the pulse have a definite (i) frequency, (ii) wavelength, (iii) speed of propagation? (b) If the pulse rate is 1 after every 20 s, (that is the whistle is blown for a split of second after every 20 s), is the frequency of the note produced by the whistle equal to 1/20 or 0.05 Hz?*

Ans. (*a*) In a non dispersive medium, the wave propagates with definite speed but its wavelength of frequency is not definite.

(*b*) No, the frequency of the note is not $\frac{1}{20}$ or 0.50 Hz. 0.005 Hz is only the frequency of repetition of the pip of the whistle.

15.24. *One end of a long string of linear mass density 8.0 × 10^{-3} kg m^{-1} is connected to an electrically driven tuning fork of frequency 256 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves at this end have negligible amplitude. At t = 0, the left end (fork end) of the string x = 0 has zero transverse displacement (y = 0) and is moving along positive y-direction. The amplitude of the wave is 5.0 cm. Write down the transverse displacement y as function of x and t that describes the wave on the string.*

Ans. Here, mass per unit length, μ = linear mass density = 8 × 10^{-3} kg m^{-1};

Tension in the string, T = 90 kg = 90 × 9.8 N = 882 N;

Frequency, ν = 256 Hz

and amplitude, A = 5.0 cm = 0.05 m

As the wave propagating along the string is a transverse travelling wave, the velocity of the wave,

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{882}{8\times10^{-3}}}\ \text{ms}^{-1} = 3.32 \times 10^2\ \text{ms}^{-1}$$

Now, $\omega = 2\pi\nu = 2 \times 3.142 \times 256 = 1.61 \times 10^3$ rad s^{-1}

Also, $v = \nu\lambda$ or $\lambda = \dfrac{v}{\nu} = \dfrac{3.32\times10^2}{256}$ m

Propagation constant, $k = \dfrac{2\pi}{\lambda} = \dfrac{2\times3.142\times256}{3.32\times10^2} = 4.84\ \text{m}^{-1}$

∴ The equation of the wave is ,

$$v(x, t) = A \sin(\omega t - kx) = 0.05 \sin(1.61 \times 10^3\, t - 4.84\, x)$$

Here, x, y are in metre and t is in second.

15.25. *A SONAR system fixed in a submarine operates at a frequency 40.0 kHz. An enemy submarine moves towards the SONAR with a speed of 360 km h^{-1}. What is the frequency of sound reflected by the submarine? Take the speed of sound in water to be 1450 ms^{-1}.*

Ans. Here, frequency of SONAR (source) = 40.0 kHz = 40 × 10^3 Hz

Speed of sound waves, v = 1450 ms^{-1}

Speed of observers, v_0 = 360 km/h = 360 × $\dfrac{5}{18}$ = 100 ms^{-1}.

Since the source is at rest and observer moves towards the source (SONAR),

$$\therefore \quad \nu' = \frac{v+v_0}{v}.\nu = \frac{1450+100}{1450}\times40\times10^3 = 4.276 \times 10^4\ \text{Hz}.$$

This frequency (ν') is reflected by the enemy ship and is observed by the SONAR (which now acts as observer). Therefore, in this case, v_s = 360 km/h = 100 ms^{-1}.

$$\therefore \text{ Apparent frequency, } \nu'' = \frac{v}{v-v_s}\nu' = \frac{1450}{1450-100} \times 4.276 \times 10^4$$

$$= 4.59 \times 10^4\ \text{Hz} = 45.9\ \text{kHz}.$$

15.26. *Earthquakes generate sound waves inside the earth. Unlike a gas, the earth can experience both transverse (S) and longitudinal (P) sound waves. Typically the speed of S wave is about 4.0 kms^{-1}. A seismograph records P and S waves from an earthquake. The first P wave arrives 4 min before the first S wave. Assuming the waves travel in straight line, at what distance does the earthquake occur?*

Ans. Here speed of S wave, v_s = 4.0 km s^{-1} and speed of P wave, v_p = 8.0 km s^{-1}. Time gap between P and S waves reaching the resimograph, t = 40 min = 240 s.

Let distance of earthquake centre = sKm

$$\therefore \qquad t = t_s - t_p = \frac{S}{v_s} - \frac{S}{v_p} = \frac{S}{4.0} - \frac{S}{8.0} = \frac{S}{8.0} = 240 \text{ s}$$

or $\qquad s = 240 \times 8.0 = 1920$ km.

15.27. *A bat is flitting about in a cave, navigating via ultrasonic beeps. Assume that the sound emission frequency of the bat is 40 kHz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall?*

Ans. Here, the frequency of sound emitted by the bat, ν = 40 kHz. Velocity of bat, $v_s = 0.03\ v$, where v is velocity of sound.

Apparent frequency of sound striking the wall

$$\nu' = \frac{v}{v - v_s} \times \nu = \frac{v}{v - 0.03v} \times 40 \text{ kHz} = \frac{40}{0.97} \text{ kHz}$$

This frequency is reflected by the wall and is received by the bat moving towards the wall. So $v_s = 0$,

$$v_L = 0.03\ v.$$

$$\nu'' = \frac{(v + v_L)}{v} \times \nu' = \frac{(v + 0.03v)}{v}\left(\frac{40}{0.97}\right) = \frac{1.03}{0.97} \times 40 \text{ kHz} = 42.47 \text{ kHz.}$$

ADDITIONAL QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *Is it possible to have interference between the waves produced by two violins? Why?*

Ans. No. This is because the sounds produced will not have a constant phase difference.

Q. 2. *What is the relation between path difference and phase difference?*

Ans. $\text{Phase difference} = \frac{2\pi}{\lambda} \times \text{path difference}$.

Q. 3. *What determines the type of wave motion in a medium?*

Ans. Type of wave motion is determined by (*i*) nature of the medium, (*ii*) mode of excitation of wave motion.

Q. 4. *What type of graph you expect between speed of sound through a gas and pressure of gas?*

Ans. The graph will be straight line parallel to pressure axis.

Q. 5. *In which type of wave alternate crests and troughs are formed?*

Ans. In a transverse wave.

Q. 6. *What is the phase difference between two successive crests in a transverse wave?*

Ans. Phase difference between two successive crests in a transverse wave is 2π rad.

Q. 7. *What is the nature of light waves?*

Ans. Transverse.

Q. 8. *Which harmonics are absent in a closed organ pipe?*

Ans. All even harmonics are absent.

Q. 9. *In which gas, hydrogen or oxygen, will sound have greater velocity?*

Ans. Since $v \propto \sqrt{\frac{1}{\rho}}$, therefore velocity of sound will be greater in hydrogen gas.

Q. 10. *What is the nature of ultrasonic waves and what is their frequency?*

Ans. Ultrasonic waves are longitudinal waves in nature and have frequency greater than 20 kHz.

Q. 11. *When a source moves at a speed greater than that of sound, will Doppler formula hold? What will happen?*

Ans. No, as it is valid only when $v_s < v$. When $v_s > v$, shock waves are produced.

Q. 12. *What is the audible range of sound frequencies?*

Ans. Between 20 Hz to 20,000 Hz.

Q. 13. *At the same temperature and pressure, the densities of two diatomic gases are d_1 and d_2. What is the ratio of the speeds of sound in these gases?*

Ans.
$$\frac{v_1}{v_2} = \sqrt{\frac{d_2}{d_1}}$$

Q. 14. *In a longitudinal wave, what is the distance between a compression and its nearest rarefraction?*

Ans. $\frac{\lambda}{2}$.

Q. 15. *On what factors does the speed of transverse waves setup in a string depend?*

Ans. Speed of transverse waves setup in a string depends upon the tension (T) in the string and the linear mass density (μ) of the string. In fact, $v = \sqrt{\frac{T}{\mu}}$.

Q. 16. *When a vibrating tunning fork is moved speedily towards a wall, beats are heard. Why?*

Ans. This is due to the difference in the frequency of the incident wave and the apparent frequency of the reflected wave.

Q. 17. *The ratio of amplitude of two waves is 2 : 3. What is the ratio of intensitites of these waves?*

Ans.
$$\frac{I_1}{I_2} = \frac{a^2}{b^2} = \frac{2^2}{3^2} = \frac{4}{9}$$

Q. 18. *If oil of density higher than that of water is used in place of water in a resonance tube, how does the frequency change?*

Ans. The frequency is governed by the air column and does not depend upon the nature of the liquid. So frequency would not change.

Q. 19. *Two medium particles are separated by a distance $\frac{\lambda}{2}$. What is the relationship between phase of these particles at any instant?*

Ans. The phase difference between two given particles = $\frac{2\pi}{\lambda} \times \frac{\lambda}{2}$ = π radian, *i.e.,* the two particles are in mutually opposite phase conditions.

Q. 20. *If radius of a stretched wire is reduced to half, how is the wave speed affected?*

Ans. As $v \propto 1/\sqrt{r}$, therefore, wave speed becomes twice.

Q. 21. *An observer at a sea-coast observes waves reaching the coast. What type of waves does he observe? Why?*

Ans. Elliptical waves while the waves on the surface of water are transverse, the waves just below the surface of water are longitudinal. So the resultant waves are elliptical.

Q. 22. *An observer places his ear at the end of a long steel pipe. He can hear two sounds, when a workman hammers the other end of the pipe. Why?*

Ans. This is because sound is transmitted both through air and medium.

Q. 23. *If tension of a wire is increased to four times, how is the wave speed changed?*

Ans. As $v \propto \sqrt{T}$, therefore, wave speed becomes twice.

Q. 24. *What do you mean by reverberation time?*

Ans. The time during which the intensity of sound decreases to 10^{-6} times its original intensity.

Q. 25. *What is reverberation?*

Ans. The persistence of audible sound after the source has ceased to produce the sound is called reverberation.

Q. 26. *What is the ratio of frequencies of fundamental tone and various overtones formed in a vibrating string?*

Ans. The frequencies of various harmonics (or fundamental tone and overtones) in vibrating string are in the ratio:

$$\nu_1 : \nu_2 : \nu_3 \ldots\ldots = 1 : 2 : 3 : 4 \ldots\ldots\ldots\ldots\ldots$$

Q. 27. *Why bells are made of metal and not wood?*

Ans. This is because wood has high damping.

Q. 28. *How does velocity of sound in air change when temperature rises by 1°C?*

Ans. Velocity of sound in air increases by 0.61 m/s, when temperature rises by 1°C.

Q. 29. *Why do we not hear beats if the frequency of two sounds are widely different?*

Ans. The beats cannot be heard due to persistence of hearing if the difference in frequencies is more than 10.

Q. 30. *Two sound sources produce 20 beats in 5s. By how much do their frequencies differ?*

Ans. Number of beats per second $= \dfrac{20}{5} = 4$

$$\therefore \qquad \nu_1 - \nu_2 = 4.$$

Q. 31. *What sort of waves are formed in a sitar wire when it is once plucked in the middle and then released?*

Ans. Transverse stationary waves are formed in the sitar wire.

Q. 32. *Two astronauts on the surface of moon cannot talk to each other. Why?*

Ans. This is because moon has no atmosphere and sound cannot travel in vacuum.

Q. 33. *Define the terms 'node' and 'antinode'?*

Ans. **Node:** It is a point on stationary wave at which amplitude of vibration of the particle is zero.

Antinode: It is a point on stationary wave at which amplitude of vibration of the particle is maximum

Q. 34. *Does sound travel faster on a wet hot day or a dry cold day? Why?*

Ans. Sound travels faster on a wet hot day due to high temperature and lesser density of wet air.

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *A set of 65 turning forks is so arranged that each gives 3 beats per second with the previous one and the last sounds the octave of first. Find the frequency of first and last forks?*

Ans. According to the given problem the frequency of the last fork is the octave of the first, *i.e.,* if the frequency of first fork is n, then the frequency of last fork is $2n$. This shows that the forks are in increasing frequency order. As each fork gives 3 beats with previous are, hence frequencies are

$$n, (n + 3), (n + 2 \times 3), (n + 3 \times 3), \ldots\ldots 2n$$

In forms an A.P.

$$\therefore \quad a_n = a + (n - 1)\, d \quad \text{or} \quad 2n = n + (65 - 1) \times 3$$

By solving, we get

$$n = 192 \quad \text{and} \quad 2n = 384$$

Frequency of first and last forks are 192 and 384 respectively.

Q. 2. *Transverse waves are generated in two uniform steel wires A and B of diameters 10^{-3} m and 0.5×10^{-3} m respectively, by attaching their free end to a vibrating source of frequency 500 Hz. Find the ratio of the wavelengths if they are stretched with the same tension.*

Ans. The density ρ of a wire of mass M, length L and diameter 'd' is given by

$$\rho = \frac{4M}{\pi d^2 L} = \frac{4m}{\pi d^2}$$

Now $$v_A = \sqrt{\frac{T}{m_A}} \quad \text{and} \quad v_B = \sqrt{\frac{T}{m_B}}$$

$$\therefore \quad \frac{v_A}{v_B} = \sqrt{\frac{m_B}{m_A}} = \frac{d_B}{d_A}$$

but $v_A = \nu\lambda_A$ and $v_A = \nu\lambda_B$, n being the frequency of the source.

Hence $$\frac{\lambda_A}{\lambda_B} = \frac{v_A}{v_B} = \frac{d_B}{d_A} = \frac{0.5 \times 10^{-3}}{10^{-3}} = 0.5$$

Q. 3. *Find the temperature at which the speed of sound in oxygen will be the same as that in nitrogen at 20°C. Given that molar masses of oxygen and nitrogen are 32 and 28 respectively. Both gases are assumed to be ideal.*

Ans. We know that both oxygen and nitrogen are diatomic gases having same value of constant

$$\gamma = 1.40$$

We know that $v = \sqrt{\frac{\gamma RT}{M}}$. As speed of sound in oxygen at T K is same as the speed of sound in nitrogen at $T' = 20$ °C = 293 K, hence

$$v = \sqrt{\frac{\gamma RT}{M_{N_2}}} = \sqrt{\frac{\gamma RT'}{M_{N_2}}}$$

$$\Rightarrow \quad T = T' \cdot \frac{M_{o_2}}{M_{N_2}} = \frac{293 \times 32}{28} = 335 \text{ K} \quad \text{or} \quad 62\ °\text{C}.$$

Q. 4. *What are the uses of ultrasonic waves?*

Ans. Ultrasonic waves are used for the following purposes:

(*i*) They are used in SONAR for finding the range and direction of submarines.

(*ii*) They are used for detecting the presence of cracks and other inhomogeneities in solids.

(*iii*) They are used to kill the bacteria and hence for sterilising milk.

(*iv*) They are used for cleaning the surface of solid.

Q. 5. *A progressive and a stationary wave have frequency 300 Hz and the same wave velocity 360 m/s. Calculate*

(i) the phase difference between two points on the progressive wave which are 0.4 m apart,

(ii) the equation of motion of progressive wave if its amplitude is 0.02m,

(iii) the euqation of the stationary wave if its amplitude is 0.01 m and

(iv) the distance between consecutive nodes in the stationary wave.

Ans. Wave velocity $v = 360 \text{ m/s}$

Frequency $n = 300 \text{ Hz}$

$$\therefore \quad \text{wavelength } \lambda = \frac{V}{f} = \frac{360}{300} = 1.2\text{m}$$

(*i*) The phase difference between two points at a distance one wavelength apart is 2π. Phase difference between points 0.4 m apart is given by

$$\frac{2\pi}{\lambda} \times 0.4 = \frac{2\pi}{1.2} \times 0.4 = \frac{2\pi}{3} \text{ radians.}$$

(*ii*) The equation of motion of a progressive wave is

$$y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

In the case given

$$y = 0.02 \sin 2\pi \left(300\,t - \frac{x}{1.2}\right)$$

(*iii*) The equation of the stationary wave is

$$y = 2A \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi t}{T}$$

Here $2A = 2 \times 0.01 = 0.02$

$$\lambda = 1.2 \text{ m}$$

$$\frac{1}{T} = 300 \text{ Hz}$$

$$\therefore \quad y = 0.02 \cos \frac{2\pi x}{1.2} \sin 600\,\pi t$$

(*iv*) The distance between the two consecutive nodes in the stationary wave is given by

$$\frac{\lambda}{2} = \frac{1.2}{2} m = 0.6 \text{ m}$$

Q. 6. *Write basic conditions for formation of stationary waves.*

Ans. The basic conditions for formation of stationary waves are listed below:

(*i*) The direct and reflected waves must be travelling along the same line.

(*ii*) For stationary wave formation, the superposing waves should either be longitudinal or transverse. A longitudinal and a transverse wave cannot superposition.

(*iii*) For formation of stationary waves, there should not be any relative motion between the medium and oppositely travelling waves.

(*iv*) Amplitude and period of the superposing waves should be same.

Q. 7. *The following equation represents standing wave set up in medium,*

$$y = 4 \cos \frac{\pi x}{5} \sin 40\pi t,$$

where x and y are in cm and t in sec. Find out the amplitude and the velocity of the two component waves and calculate the distance between adjacent nodes. What is the velocity of a medium particle at x = 3 cm at time $\frac{1}{8}$ sec *?*

Ans. The given equation of stationary wave is

$$y = 4 \cos \frac{\pi x}{3} \sin 40\,\pi t \quad \text{or} \quad y = 2 \times 2 \cos \frac{2\pi x}{6} \sin \frac{2x\,(120)\,t}{6} \qquad ...(i)$$

We know that $$y = 2a \cos \frac{2\pi x}{\lambda} \sin \frac{2x\,vt}{\lambda} \qquad ...(ii)$$

By comparing tow equations, we get

$$a = 2 \text{ cm}, \quad \lambda = 6 \text{ cm and } v = 220 \text{ cm/sec.}$$

The component waves are

$$y_1 = a \sin \frac{2\pi}{\lambda}(vt - x) \quad \text{and} \quad y_2 = a \sin \frac{2\pi}{\lambda}(vt + x)$$

Distance between two adjacent nodes = $\frac{\lambda}{2} = \frac{6}{2}$ = 3 cm.

Particle velocity $$\frac{dy}{dt} = 4 \cos \frac{\pi x}{3} \cos (40\pi t).\, 40\pi = 160 \cos \frac{\pi x}{3} \cos 40\pi t$$

$$= 160\,\pi \cos \frac{\pi x}{3} \cos \left(40\pi \times \frac{1}{8}\right) = 160\,\pi \quad [\because \cos \pi = \cos 5\pi = -1]$$

Hence, particle velocity = 160 cm/sec.

Q. 8. *The intensity of sound in a normal conversation at home is about* 3×10^{-6} W m^{-2} *and the frequency of normal human voice is about 1000 Hz. Find the amplitude of waves, assuming that the air is at standard conditions.*

Ans. At standard conditions (STP)

density (ρ) of air $= 1.29$ kg m^{-3}

velocity of sound, $\vartheta = 332.5$ ms^{-1}

Now $I = 2\pi^2 \rho\, n^2 A^2 \vartheta$

where $n = 1000$ Hz, $I = 3 \times 10^{-6}$ Wm^{-2}

$$\therefore \qquad A = \frac{1}{\pi n}\sqrt{\frac{I}{2\rho\vartheta}} = \frac{1}{3.142\times1000}\times\sqrt{\frac{3\times10^{-6}}{2\times1.29\times332.5}}$$

$$= 1.88 \times 10^{-8} \text{ m}$$

Note that the amplitude of sound waves in normal conversation is extremely small.

Q. 9. *A standing wave is represented by $y = 2A \sin kx \cos \omega t$. If one of the component waves is $y_1 = A \sin (\omega t - kx)$, what is the equation of the second component wave?*

Ans. As $\quad 2 \sin A \cos B = \sin (A + B) + \sin (A - B)$

$$y = 2A \sin kx \cos \omega t = A \sin (kx + \omega t) + A \sin (kx - \omega t)$$

According to superposition principle,

$$y = y_1 + y_2;$$

and $\quad y_1 = A \sin (\omega t - kx) = -A \sin (kx - \omega t)$

$$\therefore \quad y_2 = y - y_1 = 2A \sin kx \cos \omega t + A \sin (kx - \omega t)$$

$$= A \sin (kx + \omega t) + 2A \sin (kx - \omega t)$$

$$= A \sin (kx + \omega t) - 2A \sin (\omega t - kx).$$

Q. 10. *Show that a function*

$$y(x, t) = A \sin (kx - \omega t) + B \cos (kx - \omega t)$$

represents a progressive wave. What is the amplitude, wavelength, velocity and initial phase angle of the wave?

Ans. The given function is $y(x, t) = A \sin (kx - \omega t) + B \cos (kx - \omega t)$.

Let us put $A = a \cos \phi$ and $B = a \sin \phi$, where $a = \sqrt{A^2 + B^2}$ and $\tan \phi = \frac{B}{A}$. Then, the above function may be expressed as

$$y(x, t) = a \cos \phi \sin (kx - \omega t) + a \sin \phi \cos (kx - \omega t)$$

$$y(x, t) = a \sin (kx - \omega t + \phi)$$

As it is a single sinusoidal function of space and time, it is representing the equation of a harmonic progressive wave.

Amplitude of wave $= a = \sqrt{A^2 + B^2}$

Wavelength of wave $= \lambda = \frac{2\pi}{k}$.

Wave velocity $= v = \nu\lambda = \frac{\nu}{2\pi} \cdot \frac{2\pi}{k} = \frac{\nu}{k}$.

and initial phase angle of the wave $= \phi$, where $\phi = \tan^{-1}\left(\frac{B}{A}\right)$.

Q. 11. *The intensities due to two sources of sound are I_0 and $4I_0$. What is the intensity at a point where the phase difference between two waves is (i) 0° (ii) $\frac{\pi}{2}$ (iii) π ?*

Ans. If a_1 and a_2 are the amplitudes of two waves, then the resultant amplitude is given by

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos\phi},$$

where ϕ is the phase difference between two waves.

Now, $\quad A^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos\phi$

Expressing this equation in terms of intensity.

$$I = I_1 + 4I_2 + 2\sqrt{I_1}\sqrt{I_2} \cos \phi$$

(*i*) $$I = I_0 + 4I_0 + 2\sqrt{I_0}\sqrt{4I_0} \cos 0° = 9I_0$$

(*ii*) $$I = I_0 + 4I_0 + 2\sqrt{I_0}\sqrt{4I_0} \cos \frac{\pi}{2} = 5I_0$$

(*iii*) $$I = I_0 + 4I_0 + 2\sqrt{I_0}\sqrt{4I_0} \cos \pi = I_0.$$

Q. 12. *Write the applications of beats.*

Ans. Beats are used to:

(*i*) determine an unknown frequency by listening to the beat frequency $\Delta\nu$. Then unknown frequency $\nu' = \nu + \Delta\nu$ where ν is known and it is close to the unknown frequency. Exact values of ν is found by loading and filling the turning fork of unknown frequency from which + or – sign is chosen;

(*ii*) tune musical instruments by sounding them together and reducing beats number to zero;

(*iii*) make a sound rich in musical effect by deliberate introduction of beats between different musical instruments;

(*iv*) to produce very low frequency pulses which otherwise cannot be produced. Beat frequency is the low frequency sound;

(*v*) receive radio programme by superheterodyne method;

(*vi*) detect harmful gases in mines.

Q. 13. *Two strings of the same material and length under the same tension may vibrate with different fundamental frequency. Why?*

Ans. The frequency of vibration of string is

given by $$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$m \text{ (mass per unit length)} = \frac{\text{mass}}{\text{length}} = \frac{\text{volume} \times \text{density}}{l} = \frac{\pi r^2 l \times \rho}{l} = \pi r^2 \rho$$

$$= \pi \left(\frac{D}{2}\right)^2 \rho = \frac{\pi D^2}{4}\rho$$

$$n = \frac{1}{2l}\sqrt{\frac{4T}{\pi D^2 \rho}} = \frac{1}{Dl}\sqrt{\frac{T}{\pi \rho}}$$

$\therefore$ $$n \propto \frac{1}{D}$$ (when l, T, ρ are same)

or nD = constant or $n_1 D_1 = n_2 D_2$

Hence the two strings may vibrate with different frequencies when they have different diameters.

Q. 14. *Compare the velocities of sound in hydrogen (H_2) and carbon dioxide (CO_2). The ratio (γ) of specific heats of H_2 and CO_2 are respectively 1.4 and 1.3.*

Ans. $$\vartheta_1 = \sqrt{\frac{\gamma_1 P}{\rho_1}} \quad \text{and} \quad J_2 = \sqrt{\frac{\gamma_2 P}{\rho_2}}$$

$$\frac{\vartheta_1}{\vartheta_2} = \sqrt{\frac{\gamma_1}{\gamma_2}\frac{\rho_2}{\rho_1}}$$

Since density of a gas is proportional to its molecular weight,

$$\frac{\rho_2}{\rho_1} = \frac{44.01}{2.016} = 21.83$$

$$\frac{\vartheta_1}{\vartheta_2} = \sqrt{\frac{1.4}{1.3}\times 21.83} = 4.85$$

Velocity of sound in hydrogen in 4.85 times that in carbon dioxide.

Q. 15. *Find at what temperature the velocity of sound is air will be* $1\frac{1}{2}$ *times the velocity at 11°C.*

Ans. Suppose velocity of sound in air at t°C is $1\frac{1}{2}$ times the velocity at 11°C.

i.e., $$v_t = \frac{3}{2}v_{11} \quad \text{...}(i)$$

As $$v_t = v_0\sqrt{\frac{273+t}{273}}$$

$\therefore$ from (i), $$v_0 = \sqrt{\frac{273+t}{273}} = \frac{3}{2}v_{11}$$

$$v_0 = \sqrt{\frac{273+t}{273}} = \frac{3}{2}\,v_0\,\frac{284}{273}$$

Squaring both sides, we get

$$\sqrt{\frac{273+t}{273}} = \frac{9}{4}\times\frac{284}{273} \quad \text{or} \quad 1092 + 4t = 2556$$

or $$4t = 2556 - 1092 = 1464 \quad \text{or} \quad t = \frac{1464}{4} = 366°\text{C}.$$

Q. 16. *Wavelength of two notes in air are* $\frac{80}{195}$ *m and* $\frac{80}{193}$ *m. Each note produces five beats per second with a third note of a fixed frequency. Calculate the velocity of sound in air.*

Ans. Here given that, $\lambda_1 = \frac{80}{195}$ m and $l_2 = \frac{80}{193}$ m.

If ν_1 and ν_2 be the corresponding frequencies and v be the velocity of sound in air, then

$$\nu_1 = \frac{v}{\lambda_1} = \frac{195v}{80} \quad \text{and} \quad \nu_2 = \frac{v}{\lambda_2} = \frac{193v}{80}$$

This shows that $\nu_1 > \nu_2$.

Let the frequency of third note be ν, then

$$\nu_1 - \nu = 5 \quad \text{and} \quad \nu - \nu_2 = 5.$$

$$\therefore \quad \nu_1 - \nu_2 = 10 \quad \text{or} \quad \frac{195v}{80} - \frac{193v}{80} = 10$$

$$\Rightarrow \quad 2v = 80 \times 10 \quad \text{or} \quad v = 400 \text{ m/sec.}$$

Q. 17. *An open pipe is suddenly closed at one end with the result that the frequency of the 3rd harmonic of the closed pipe is found to be higher by 100 Hz than the fundamental frequency of the open pipe. What is the fundamental frequency of open pipe?*

Ans.

$$\nu_0 = \frac{\nu}{2L} \quad \text{...(1)}$$

where n_0 = fundamental frequency of open pife. Frequency of third harmonic of closed pipe is

$$\nu_c = 3\left(\frac{v}{4L}\right) \quad \text{...(2)}$$

$\frac{(2)}{(1)}$ gives

$$= \frac{\nu_c}{\nu_0} = \frac{3}{2} \quad \text{or} \quad v_c = \frac{3}{2}\nu_0 \quad \text{...(3)}$$

Also $\nu_c - \nu_0 = 100$ (given)

$$\frac{3}{2}\nu_0 - \nu_0 = 100 \quad \text{or} \quad \frac{\nu_0}{2} = 100 \quad \text{or} \quad \nu_0 = 200 \text{ Hz.}$$

Q. 18. *Two loudspeakers have been installed in an open space to listen to a speech. When both the loudspeakers are in operation, a listener sitting at a particular place receives a very feeble sound. Why? What will happen if one loudspeaker is kept off?*

Ans. When the distance between two loudspeakers from the position of listener is an odd multiple of $\frac{\lambda}{2}$, then due to destructive interference between sound waves from two loudspeakers, a feeble sound is heard by the listener.

When one loudspeaker is kept off, no interference is will take place and the listener will hear the full sound of the operating loudspeaker.

Q. 19. *Two tuning forks A and B when sounded together give 4 beats/sec, A in unison with the note emitted by alength 0.96 m of a sonometer wire under a certain tension while B is in unison with 0.97 m of the same wire under the same tension. Find the frequencies of the forks.*

Ans. Let the frequency of the fork A be n. Since A is in unison with a smaller length of the sonometer wire than B which is in unison with a larger length of the wire, the frequency of fork A should be larger than that of B.

∴ frequency of fork B = (ν – 4) Hz

Now $\quad \nu \times 0.96 = (\nu - 4)(0.97)$

$$\frac{\nu - 4}{\nu} = \frac{96}{97}$$

$$1 - \frac{4}{\nu} = 1 - \frac{1}{97}$$

$$\frac{4}{\nu} = \frac{1}{97}$$

or, $\nu = 4 \times 97 = 388$ Hz

$\therefore$ the frequency of fork $A = 388$ Hz

and that of $B = 384$ Hz.

Q. 20. *The second overtone of an open pipe has the same frequency as the first overtone of a closed pipe 2m long. What is the length of the open pipe?*

Ans. Let L_0 be the length of the open pipe. The fundamental frequency of the pipe is given by

$$\nu_0 = \frac{v}{\lambda_f} = \frac{v}{2L_0}, \quad v = \text{velocity of sound in air}$$

The second overtone of the open pipe has a frequency

$$3\nu_0 = \frac{3v}{2L_0}\text{Hz}$$

The length of the closed pipe $L_C = 2$ m

The frequency of the fundamental omitted by the closed pipe

$$\nu_c = \frac{v}{\lambda} = \frac{v}{4L_c}$$

The first overtone of the closed pipe has a frequency

$$3\nu_c = \frac{3v}{4L_c} = \frac{3v}{4 \times 2} = \frac{3v}{8} \text{ Hz}$$

Now, $3\nu_0 = 3\nu_0$ or $2L_0 = 8$ or $L_0 = 4$ m,

Q. 21. *Consider the wave y(x, t) = 2.2 cos (300t – 0.24x). If the units for y, t and x are mm, s and m respectively, deduce (i) the amplitude, (ii) the frequency, (iii) the wavelength, (iv) the wave velocity, and (v) the amplitude of particle velocity.*

Ans. The given wave equation is $y(x, t) = 2.2 \cos (300t - 0.24x)$

The equation is of the type $y(x, t) = A \cos (\omega t - kx)$.

Comparing the two equations, we obtain

(*i*) Amplitude wave $A = 2.2 \text{ mm} = 2.23 \times 10^{-3}$ m

(*ii*) $\omega = 300 \text{ rad s}^{-1}$

$\therefore$ Frequency $\nu = \frac{\omega}{2\pi} = \frac{300}{2\pi} = 47.7$ Hz

(*iii*) $k = 0.24 \text{ m}^{-1}$

$\therefore$ Wavelength $\lambda = \frac{2\pi}{K} = \frac{2\pi}{0.24} = 26.2$ m

(*iv*) $\therefore$ Wave velocity $v = \nu\lambda = 47.7 \times 26.2 = 1250 \text{ ms}^{-1}$

(*v*) Amplitude of particle velocity $= A\omega = 2.2 \times 10^{-3} \times 300 = 0.66$ mm.

Q. 22. *Calculate the number of beats heard per second is there are three sources of sound of freqeuencies 400, 401 and 402 of equal intensity sounded together.*

Ans. Let us consider the case of three disturbances each of amplitude a and frequencies $(n - 1)$, and $(n + 1)$ respectively. The resultant displacement is given by

$$y = a \sin 2\pi (n-1) t + a \sin 2\pi n t + a \sin 2\pi (n+1)t$$
$$= 2a \sin 2\pi n t \cos 2\pi t + a \sin 2\pi n t$$
$$= a (1 + 2 \cos 2\pi t) \sin 2\pi n t$$

So the resultant amplitude is a $(1 + 2 \cos 2\pi t)$

which is maximum when $\cos 2\pi t = +1$

$\therefore \quad 2\pi t = 2k \quad$ where $k = 0, 1, 2, 3...........$

$t = 0, 1, 2, 3..........$

Thus the time interval between two consecutive maxima is one. This shows that the frequency of maxima is one.

Similarly, the amplitude is minimum when

$$1 + 2 \cos 2\pi t = 0 \quad \text{or} \quad \cos 2\pi t = -\frac{1}{2}$$

or $\quad 2\pi t = 2k\pi + \frac{2\pi}{3} \quad$ (where $k = 0, 1, 2, ...$)

or $\quad t = \left(k + \frac{1}{3}\right) = \frac{1}{3}, \frac{4}{3}, \frac{7}{3}, \frac{10}{3}$

Thus the minima occurs after an interval of one second, *i.e.,* the frequency of minima is also one. Hence, the frequency of beats is also one.

Thus, one beat is heard per second.

Q. 23. *Calculate the speed of sound in a gas in which two waves of wavelengths 1.00 m and 1.01 m produce 10 beats in 3 seconds.*

Ans. Let $\quad v$ = speed of sound in a gas

Frequencies of two waves is

$$\nu_1 = \frac{v}{\lambda_1} = \frac{v}{1.00} \quad \text{and} \quad \nu_2 = \frac{v}{\lambda_2} = \frac{v}{1.01}$$

Given $\quad \nu_1 - \nu_2 = \frac{10}{3}$

or $\quad \frac{v}{1.00} - \frac{v}{1.01} = \frac{10}{3} \quad$ or $\quad \frac{0.01v}{1 \times 1.01} = \frac{10}{3}$

$\therefore \quad v = \frac{10 \times 1 \times 1.01}{3 \times 0.01} = 336.7 \text{ ms}^{-1}$

Q. 24. *Find the velocity of source of sound, when the frequency appears to be (a) double (b) half, the original frequency to a stationary listener.*

Ans. (*a*) $\quad \nu' = 2\nu.$

It is possible is source is approaching the stationary listener *i.e.,* v_s *is* +

As $\quad \nu' = \frac{v - v_L}{v - v_s} \nu$

$\therefore \quad 2\nu = \frac{v}{v - v_s} \nu \quad$ or $\quad 2 = \frac{v}{v - v_s}$

or $2v - 2v_s = v$ or $2v_s = v$ or $v_s = \frac{v}{2}$

Therefore, source should approach the listener with half the velocity of sound propagation.

(*b*) $\nu' = \frac{v}{2}$

It is possible if source is receding away from the stationary listener *i.e.,* v_s is negative and $v_L = 0$

$\therefore$ $\nu' = \frac{v}{v+v_s}\nu$ or $\frac{v}{2} = \frac{v}{v+v_s}\nu$ or $\frac{1}{2} = \frac{v}{v+v_s}$

or $v + v_s = 2v$ or $v_s = v$

Therefore, source should recede away from the listener with the velocity of sound propagation.

III. LONG ANSWER TYPE QUESTIONS

Q. 1. *What do you mean by interference of waves? Distinguish between constructive and destructive interference.*

Standing waves are produced by the superposition of two waves

$$y_1 = 0.05 \sin(3\pi t - 2x) \quad \text{and} \quad y_2 = 0.05 \sin(3\pi t + 2x)$$

where y and x are measured in metres and t in seconds. Find the amplitude of a particle at x = 0.5 m.

Ans. Interference of waves is the phenomenon of redistribution of energy in space on account of superposition of two waves of same nature, same frequency and equal or comparable amplitudes and travelling in the given medium in the same direction

Constructive interference takes place when the two superposing waves are in same phase *i.e.,* crest of one wave (in transverse waves) coincides with crest of another wave and vice-versa. As a result, the resultant amplitude and hence intensity of the resultant wave is maximum. Thus, for constructive interference, the phase difference between the superposing waves $\Delta\phi = 0$ or $2n\pi$, where n is an integer *i.e.,* n =1,2,3.....

Destructive interference takes place when two superposing waves are in mutually opposite phase *i.e.,* in superposing of two transverse waves crest of one wave exactly coincides with trough of another wave. As a result, the resultant amplitude and hence intensity of the resultant wave is minimum. For destructive interference, the phase difference $\Delta\phi = (2n - 1)\pi$, where n = 1, 2, 3.........

Numerical:

The resultant displacement is given by

$$y = y_1 + y_2 = 0.05 \{\sin(3\pi t - 2x) + \sin(3\pi t + 2x)\}$$

Using trigonometric relation

$\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin\alpha \cos\beta$, we have

$$y = 0.1 \cos 2x \sin 3\pi t \quad \text{or} \quad y = A \sin 3\pi t$$

where A, the amplitude of standing waves, is

given by $A = 0.1 \cos 2x$ with

$$x = 0.5 \text{ m}$$

$$\cos 2x = \cos(2 \times 0.5 \text{ rad})$$

$$= \cos (1 \text{ rad}) = \cos \left(\frac{\pi}{3.142}\right) = \cos 57.3° = 0.54$$

Amplitude A at (x = 0.5) = 0.1 × 0.54 = 0.054 m.

Q. 2. *What are the characteristics of stationary waves? Distinguish between stationary waves and progressive waves.*

Ans. Characteristics of Stationary waves

(*i*) Stationary waves are produced in a bounded medium. *A medium whose boundaries are separated from other media by distinct surfaces is called bounded medium.* The boundaries of a bounded medium may be rigid or free. For example, string fixed at both the ends (*i.e.,* string of a guitar), closed and open organ pipes.

(*ii*) There are certain points in the bounded medium (in which stationary waves are formed) which are always in the state of rest. These points are called nodes. If the stationary waves are longitudinal, then the change in pressure and density is maximum at nodes as compared to the other points.

(*iii*) There are points in between the nodes whose displacement is maximum as compared to other points. These points are called *anti-nodes*. In the longitudinal stationary waves, there is no change in pressure and density of the medium at anti-nodes.

(*iv*) The distance between any two successive nodes or antinodes is $\frac{\lambda}{2}$. The distance between a node and the neighbouring anti-node is $\frac{\lambda}{4}$.

(*v*) All particles of the medium lying between two successive nodes vibrate but the amplitude of vibration is different for different particles. The amplitude of vibration is zero at nodes and maximum at anti-nodes.

(*vi*) All particles between two successives nodes vibrate in the same phase. They pass simultaneously through their mean positions and also pass simultaneously through their positions of maximum displacement.

(*vii*) At any instant, the phase of vibration of the particles on one side of a node is opposite from the phase of vibration of the particles on the other side.

(*viii*) All particles of the medium pass through their equilibrium positions (*i.e.,* mean positions) simultaneously twice in each period. That is, the stationary wave takes the form of a straight line twice.

(*ix*) In a stationary wave, the medium splits up into a number of segments. Each segment vibrates up and down as a whole.

(*x*) All the particles except those at nodes, execute simple harmonic motion about their mean positions with the same time period.

(*xi*) In a stationary wave, there is no onward motion of the disturbance from one particle to the other particle.

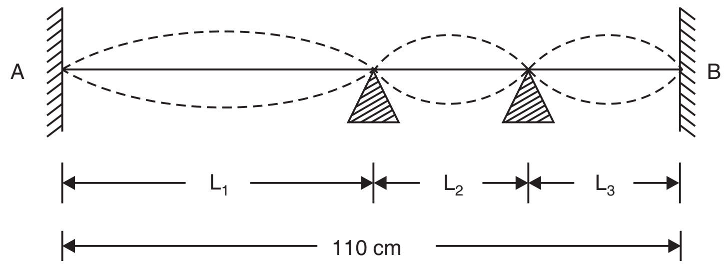
(*xii*) Stationary wave does not advance in the medium, but remains steady at its place. In other words, *stationary wave does not transmit energy in the medium.*

For distinction between progressive and stationary waves, see text.

Q. 3. *Discuss the various factors influencing velocity of sound. A sonometer wire of length 110 cm is stretched with a tension T and fixed at its ends. The wire is divided into three segments by placing two bridges below it. Where should the bridges be placed so that the fundamental frequencies of the segments are in the ratio 1 : 2 : 3?*

Ans. For factors influencing velocity of sound, see text.

Numerical: Let L_1, L_2 and L_3 be the lengths of the segments of wire *AB* (Fig.).



Then

$$L_1 + L_2 + L_3 = 110 \text{ cm} \qquad ...(1)$$

Let n_1 n_2 and n_3 be their respective fundamental frequencies. Thus

$$n_1 = \frac{1}{2L_1}\sqrt{\frac{T}{m}}$$

$$n_2 = \frac{1}{2L_2}\sqrt{\frac{T}{m}} \quad \text{and} \quad n_3 = \frac{1}{2L_3}\sqrt{\frac{T}{m}}$$

Hence $\quad n_1 L_1 = n_2 L_2 = n_3 L_3 \qquad ...(2)$

But $\quad n_1 : n_2 : n_3 = 1 : 2 : 3$

$\therefore \quad n_2 = 2n_1 \quad \text{and} \quad n_3 = 3n_1 \qquad ...(3)$

From (2) and (3) we have

$$L_1 = 2L_2 = 3L_3 \qquad ...(4)$$

Substituting (4) in (1) we get

$$L_1 + \frac{1}{2}L_1 + \frac{1}{3}L_1 = 110 \quad \text{or} \quad L_1 = 60 \text{ cm}$$

Hence $\quad L_2 = 30 \text{ cm} \quad \text{and} \quad L_3 = 20 \text{ cm}$

Thus, the bridges should be placed at distances of 60 cm and 90 cm from end A.

Q. 4. *What do you understand by beat? Explain beats analytically.*

Ans. For definition, see text.

Consider two simple harmonic progressive waves travelling simultaneously in the same direction and in the same medium. Let

(*i*) '*A*' be the amplitude of each wave.

(*ii*) There is no initial phase difference between them.

(*iii*) ν_1 and ν_2 be their frequencies.

If y_1 and y_2 be displacements of the two waves, then

$$y_1 = A \sin 2\pi \nu_1 t \quad \text{and} \quad y_2 = A \sin 2\pi \nu_2 t$$

If y be the result and displacement at any instant, then

$$y = y_1 + y_2 = \text{A} (\sin (2\pi \nu_1 t) + \sin (2\pi \nu_2 t))$$

$$= A\left[2\sin\left(\frac{2\pi(\nu_1+\nu_2)t}{2}\right)\cos\left(\frac{2\pi(\nu_1-\nu_2)t}{2}\right)\right]$$

$$= 2A \cos \pi (\nu_1 - \nu_2) t \sin \pi (\nu_1 + \nu_2) t = R \sin \pi (\nu_1 + \nu_2) t \qquad ... (1)$$

where $\quad R = 2A \cos \pi (\nu_1 - \nu_2) t \qquad ... (2)$

is the amplitude of the resultant displacement and depends upon *t*. The following cases arise.

(*a*) If R is maximum, then

$$\cos \pi (\nu_1 - \nu_2)\, t = \text{max.} = \pm 1 = \cos n\pi$$

$$\therefore \quad \pi (\nu_1 - \nu_2)\, t = n\pi \quad \text{or} \quad t = \frac{n}{\nu_1 - \nu_2} \qquad ...(3)$$

where $n = 0, 1, 2 ....$

∴ Amplitude becomes maximum at times given by

$$t = 0, \frac{1}{\nu_1 - \nu_2}, \frac{2}{\nu_1 - \nu_2}, \frac{3}{\nu_1 - \nu_2}, ...$$

∴ Time interval between two consecutive maxima is

$$= \frac{1}{\nu_1 - \nu_2}$$

$$\therefore \quad \text{Beat period} = \frac{1}{\nu_1 - \nu_2}$$

∴ Beat frequency = $\nu_1 - \nu_2$

∴ no. of beats formed per sec. = $\nu_1 - \nu_2$.

(*b*) If R is minimum, then

$$\cos \pi (\nu_1 - \nu_2)\, t = \min = 0 = \cos (2n + 1) \frac{\pi}{2}$$

$$\therefore \quad \pi (\nu_1 - \nu_2)\, t = (2n + 1) \frac{\pi}{2} \quad \text{or} \quad t = \frac{(2n+1)}{2(\nu_1 - \nu_2)}, \qquad \text{where } n = 0, 1, 2, ...$$

∴ Amplitude becomes minimum at times given by

$$t = \frac{1}{2(\nu_1 - \nu_2)}, \frac{3}{2(\nu_1 - \nu_2)}, \frac{5}{2(\nu_1 - \nu_2)}, ...$$

∴ Time interval between two consecutive minima is = $\frac{1}{\nu_1 - \nu_2}$.

$$\therefore \quad \text{Beat period} = \frac{1}{\nu_1 - \nu_2}.$$

∴ Beat requency = $\nu_1 - \nu_2$

∴ No. of beats formed per sec = $\nu_1 - \nu_2$.

Hence the number of beats formed per second is equal to the difference between the frequencies of two component waves.

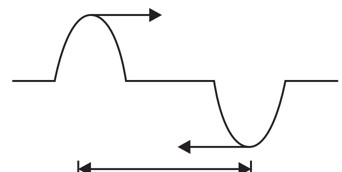
IV. MULTIPLE CHOICE QUESTIONS

1. An empty vessel is partially filled with water. The frequency of vibration of air column in the vessel

(*a*) decreases
(*b*) increases
(*c*) depends on the purity of water
(*d*) remains the same.

2. The time period of mass suspended from a spring is T. If the spring is cut into four equal parts and the same mass is suspended from one of the parts, then the new time period will be
(*a*) T/4 (*b*) T (*c*) T/2 (*d*) 2T

3. Two pulses in a stretched string whose centres are initially 8 cm apart are moving towards each other as shown in figure. The speed of each pulse is 2 cms^{-1}. After 2 second, the total energy of the pulses will be
(*a*) zero
(*b*) purely kinetic
(*c*) purely potential
(*d*) Partly kinetic and partly potential.



4. A transverse wave propagating along X-axis is represented by
$y(x, t) = 8.0 \sin (0.5 \pi x - 4 \pi t - \pi/4)$
where x is in metre and t is in seconds. The speed of the wave is
(*a*) 8 m/s (*b*) 4π m/s (*c*) 0.5π m/s (*d*) $\frac{\pi}{4}$ m/s

5. Two waves represented by $y_1 = a \sin \omega t$ and $y_2 = a \sin (\omega t + \phi)$ with $\phi = \frac{\pi}{2}$ are superposed at any point at a particular instant. The resultant amplitude is:
(*a*) a (*b*) 4 a (*c*) $\sqrt{2}\, a$ (*d*) zero

6. Which of the following statements is true?
(*a*) Both light and sound waves can travel in vacuum
(*b*) Both light and sound waves in air are transverse
(*c*) The sound waves in air are longitudinal, while the light waves are transverse
(*d*) Both light and sound waves in air are longitudinal.

7. A source X of unknown frequency produces 8 beats per second with a source of 250 Hz and 12 beats per second with a source of 270 Hz. The frequency of the source X is
(*a*) 242 Hz (*b*) 258 Hz (*c*) 282 Hz (*d*) 262 Hz

8. Two sound waves with wavelength 5.0 m and 5.5 m respectively, each propagate in a gas with velocity 330 m/s. We expect the following number of beats/sec.
(*a*) 6 (*b*) 12 (*c*) 0 (*d*) 1

9. The whistle of a railway engine is heard in winter at much longer distances. This is due to
(*a*) decrease in velocity of sound in winter.
(*b*) decrease in the density of air w.r.t. height from the surface of the earth.
(*c*) cold air absorbs much smaller energy from sound waves.
(*d*) increase in the density of air w.r.t. height from the surface of the earth.

10. A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then the lowest resonance frequency for this string is
(*a*) 1.05 Hz (*b*) 1050 Hz (*c*) 10.5 Hz (*d*) 105 Hz

Ans. **1.**—(*a*) **2.**—(*d*) **3.**—(*b*) **4.**—(*d*) **5.**—(*c*)
6.—(*d*) **7.**—(*b*) **8.**—(*a*) **9.**—(*a*) **10.**—(*d*)

V. QUESTIONS ON HIGH ORDER THINKING SKILLS (HOTS)

Q. 1. *Two identical steel wires are under tension and are in unison. When the tension in one of the wires is increased by 1 percent, 4 beats per second is heard. Find the original frequency of the wires.*

Ans. By unison of the wires, we mean that they have the same frequency of vibration. Since they are identical in linear density and length, they must be under the same tension to be in unison.

Let the tension in one wire be increased to $T_1 = \frac{101T}{100}$

where T is the original tension. Let ν_1 be the new fundamental frequency.

Now, $$\frac{\nu_1}{\nu} = \sqrt{\frac{T_1}{T}} = \sqrt{\frac{101}{100}}$$

But $\nu_1 > \nu$ and $\nu_1 - \nu = 4$

$$\therefore \quad \frac{\nu+4}{\nu} = \sqrt{\frac{101}{100}} \quad \text{or} \quad 1+\frac{4}{\nu} = \left(1+\frac{1}{100}\right)^{\frac{1}{2}} \approx 1 + \frac{1}{200}$$

$$\therefore \quad \frac{4}{\nu} = \frac{1}{200} \quad \text{or} \quad \nu = 800 \text{ Hz.}$$

Q. 2. (*a*) *If the successive overtones of vibrating string are 280 Hz and 350 Hz, what is the frequency of the fundamental note?*

(*b*) *If the amplitude of a sound wave is tripled, by how many dB will the intensity level increases?*

Ans. (*a*) Here $n\nu = 280$ Hz

and $(n+1)\nu = 350$ Hz

$\therefore \quad (n+1)\,\nu - n\nu = 350 - 280 = 70$

$\nu = 70$ Hz

(*b*) Here $$\frac{a_2}{a_1} = 3$$

$$\therefore \quad \frac{I_2}{I_1} = \left(\frac{a_2}{a_1}\right)^2 = 9$$

Now, $$\log_{10}\left(\frac{I_2}{I_1}\right) = 10\log_{10}\left(\frac{I_2}{I_1}\right) = 10\log_{10}(9) = 10\log_{10}3^2 = 20\log_{10}3 = \log_{10}3^{20}$$

$$\therefore \quad \frac{I_2}{I_1} = 3^{20} \quad \text{or} \quad I_2 = 3^{20} I_1.$$

Q. 3. *Set up a relation between speed of sound in a gas and root mean square velocity of the molecules of that gas.*

Ans. Speed of sound in a gas $v = \sqrt{\frac{\gamma P}{\rho}}$ ...(*i*)

According to kinetic theory of gases, root mean square velocity (C) of molecules of gas is obtained from the relation

$$P = \frac{1}{3}\rho C^2, \quad C = \sqrt{\frac{3P}{\rho}} \qquad ...(ii)$$

Dividing (*i*) by (*ii*), we get

$$\frac{v}{C} = \sqrt{\frac{\gamma}{3}} \quad \text{or} \quad v = \sqrt{\frac{\gamma}{3}} \times C$$

This the required relation.

Q. 4. *A wire having linear density of 0.05 g cm^{-1} is stretched between two rigid supports with a tension of 4.5 × 10^7 dynes. It is observed that me wire resonates at a frequency of 420 Hz. The next higher frequency at which the wire resonates is 490 Hz. Find the length of the wire.*

Ans. Let 420 Hz be the *p*th harmonic, then 490 Hz is the (*p* + 1)th harmonic.
Therefore

$$420 = \frac{p}{2L}\sqrt{\frac{T}{m}} \qquad ...(i)$$

$$\therefore \qquad 490 = \frac{p+1}{2L}\sqrt{\frac{T}{m}} \qquad ...(ii)$$

Dividing (*ii*) by (*i*), we get

$$\frac{490}{420} = \frac{p+1}{p} \quad \Rightarrow \quad p = 6.$$

Substituting this value of *P* in eqn. (*i*), we get

$$420 = \frac{6}{2L} \times \sqrt{\frac{4.5 \times 10^7}{0.05}}$$

which give $L = 214.3$ cm

Q. 5. *A light wave is reflected from the mirror. The incident and the reflected waves superimpose to form stationary wave, but the nodes and anti-nodes are not seen. Why?*

Ans. The distance between successive nodes or anti-nodes is $\frac{\lambda}{2}$. The wavelength (λ) of the light is of the order of 10^{-7}m, so the distance between successive nodes or antinodes is also of the order of 10^{-7} m. Since this distance is very small and cannot be detected by the eye or by an ordinary optical instrument, Hence nodes and antinodes are not seen.

Q. 6. *Two sound waves originating from the same source, travel along different paths in air and then meet at a point. If the source vibrates at a frequency of 1 kHz and one path is 83 cm longer than the other, what will be the nature of interference? The speed of sound in air is 332 m/s.*

Ans. Wavelength of sound wave is

$$\lambda = \frac{v}{\nu} = \frac{332}{1 \times 10^3} = 0.332 \text{ m}$$

Phase difference between the waves arriving at point of observation is

$$\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi \times 0.83}{0.332} = 5\pi$$

Since phase difference is an odd multiple of π, the interference is destructive.

Q. 7. *A metallic rod of length 1m is rigidly clamped at its midpoint. Longitudinal stationary waves are set up in the rod in such a way that there are two nodes on either side of the midpoint. The amplitude of an antinode is 2 × 10^{-6} m. Write the equation of motion at a point 2 cm from the midpoint and those of the constituent waves in the rod. (Young's modulus = 2 × 10^{11} Nm^{-2} and density = 8000 kg m^{-3})*

Ans. The equation of standing wave can be written as

$$y = 2A \sin kx \cos \omega t$$

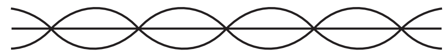
where $k = \frac{2\pi}{\lambda}$ and $\omega = \frac{2\pi V}{\lambda}$.

The standing wave is obtained by adding the equation of two identical progressive waves travelling in opposite directions

$$y_1 = A \sin (kx - \omega t); \quad y_2 = A \text{ Sin } (kx + \omega t)$$

In the present problem the length L of the rod = 1 metre.

i.e., $L = \frac{5\lambda}{2}$ or $\frac{2}{5}$ metre.



Velocity of longitudinal wave is given by

$$V = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2\times10^{11}}{8000}} = 5 \times 10^3 \text{ ms}^{-1}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{2/5} = 5\pi \text{ metre}^{-1}$$

$$\omega = \frac{2\pi V}{\lambda} = \frac{2\pi\times5\times10^3}{2/5} = (25 \times 10^3 \pi)s^{-1}$$

Hence equation of standing wave is

$$y = (2 \times 10^{-6}) \sin 5\pi x \cos 25 \times 10^3 \pi t$$

Equations of component waves are

$$y_1 = (1 \times 10^{-6}) \sin (5\pi x - 25 \times 10^3 \pi t)$$
$$y_2 = (1 \times 10^{-6}) \sin (5\pi x + 25 \times 10^3 \pi t)$$

Q. 8. *An underwater swimmer sends a sound signal to the surface. It produces 5 beats per second when compared with the fundamental note of a pipe 20 cm long closed at one end. What is the wavelength of sound in water? Given velocities of sound in air and water are 360 ms^{-1} and 1500 ms^{-1} respectively.*

Ans. The frequency of the fundamental tone of the pipe is

$$n = \frac{v_a}{4L} = \frac{360}{4\times0.02} = 450 \text{ Hz}$$

$\therefore$ The frequency of sound signal = 450 ± 5 = 445 Hz or 455 Hz

Since the frequency remains unchanged when sound travels from water to air, the frequency of the sound signal in water is either 445 Hz or 455 Hz. Hence the wavelength in water is either

$$\frac{1500}{445} = 3.371 \text{ m} \quad \text{or} \quad \frac{1500}{445} = 3.297 \text{ m}$$

Q. 9. *The amplitude of a wave disturbed propagating in the positive x direction is given by*

$$y = \frac{1}{1+x^2} \quad \text{at } t = 0 \quad \text{and} \quad y = \frac{1}{\left[1+(x-1)^2\right]} \text{ at } t = 2s$$

where x and y in metre. The shape of disturbance does not change during the propagation. What is the velocity of the wave?

Ans. At $t = 0, \quad y = \frac{1}{1+x^2}$

$$\therefore \quad 1 + x^2 = \frac{1}{y}$$

$$x^2 = \frac{1}{y} - 1 = \frac{1-y}{y} \quad x = \left(\frac{1-y}{y}\right)^{\frac{1}{2}}$$

At $\quad t = 2s, \quad y = \frac{1}{[1+(x-1)^2]}$

$$1 + (x-1)^2 = \frac{1}{y}$$

$$(x-1)^2 = \frac{1}{y} - 1 = \frac{1-y}{y}$$

$$(x-1) = \left(\frac{1-y}{y}\right)^{\frac{1}{2}} \Rightarrow x = 1 + \left(\frac{1-y}{y}\right)^{\frac{1}{2}}$$

Since, $\quad v = \frac{x_2 - x_1}{t_2 - t_1}$

$$\therefore \quad v = \frac{1}{2-0} = 0.5 \text{ ms}^{-1}$$

Q. 10. *An open pipe is suddenly closed at one end with the result that the frequency of sound harmonic of the closed pipe is found to be higher by 100 Hz than the fundamental frequency of the open pipe. Calculate the fundamental frequency of the open pipe.*

Ans. Fundamental frequency of open pipe is

$$\nu_0 = \frac{v}{2l} \quad \text{...}(i)$$

Frequency of second harmonic of closed pipe of same length is

$$\nu_c = \frac{3v}{4l} = \frac{3}{2}\left(\frac{v}{2l}\right) = \frac{3}{2}\nu_0$$

Given $\quad \nu_c = \nu_0 + 100 \quad \text{or} \quad \frac{3}{2}\nu_0 = \nu_0 + 100$

$$\Rightarrow \quad \frac{3}{2}\nu_0 - \nu_0 = 100 \quad \text{or} \quad \frac{\nu_0}{2} = 100$$

$$\therefore \quad \nu_0 = 200 \text{ Hz.}$$

VI. VALUE-BASED QUESTIONS

Q. 1. *Rajaram was waiting for train in a waiting room on Delhi Railway station with his father. He felt that when the train was coming towards the station, the intensity of the sound of whisle was gradually increasing and on the platform the sound was maximum and when the train passed away the intensity of the whisle was decreasing. Raja Ram got confused to hear it. He asked his father the reason behind it, but his father could not give the satisfactory answer of it. Then they decided to go to station master to ask this reason. Station master explained them the reason that it is due to the "Doppler's effect".*

(i) What are the values displayed here by Raja Ram?

(ii) Explain the Doppler's effect.

Ans. (*i*) Values are : creativity, sharp observence, intelligence and awareness.

(*ii*) Whenever there is a relative motion between a source of sound and the listener, the apparent frequencies of sound heard by the listner is different from the actual frequency of sound emitted by the source.

For sound the observed frequency ν' is given by

$$\nu' = \left(\frac{v+v_0}{v+v_s}\right)\nu$$

Q. 2. *Ritesh is a very good flute player but he did not know how he gets the different tonned sound by closing and opening the flute holes. He went to his music teacher in the school and asked him to explain the reason of getting the different tonned sound from the flute player. His music teacher explained that when the air is pushed into the flute player pipe, nodes and antinodes are formed and by closing and opening the holes at different heights, we hear the different tonned sound. Ritesh was happy to hear it.*

(i) What values were shown by Ritesh?

(ii) On a certain day, speed of sound in air is 350 m/s. What is the frequency of fundamental note in a closed pipe of length 0.5 m? Also find the frequency of second overtone.

Ans. (*i*) Values displayed are : Creative mind, awareness, scientific interest and keen observer.

(*ii*) Here v = 350 m/s, l = 0.5 m

$\therefore$ For closed pipe fundamental frequency

$$\nu = \frac{v}{4l} = \frac{350}{4 \times 0.5} = 175 \text{ Hz}$$

Frequency of second overtone = 5ν = 5 × 175 = 875 Hz

TEST YOUR SKILLS

1. What is the difference between transverse and longitudinal waves? Explain with suitable examples.

2. Which kind of wave is shown by motion of a kink in a longitudinal spring, produced by displacing one end of the spring sideways?

(*a*) Transverse, (*b*) Longitudinal, (*c*) Both (*a*) and (*b*), (*d*) neither (*a*) nor (*b*).

3. Which kind of wave is shown by ultrasonic waves in air?

(*a*) Transverse, (*b*) Longitudinal, (*c*) Both (*a*) and (*b*), (*d*) neither (*a*) nor (*b*).

4. Which properties of matter govern the speed of wave in a given media?

5. The speed of sound is the highest in which of the following?

Air, Water and Steel

Give reasons for your answer.

6. Out of transverse and longitudinal waves, which wave can propagate in any media? Give reasons for your answer.

❑❑❑