2 Electrostatic Potential and Capacitance

Facts that Matter

• Electric Potential Energy

When an electric charge is brought near to another electric charge, the work is to be done against electrostatic force. This amount of work done is stored in the form of electrostatic energy in the system.

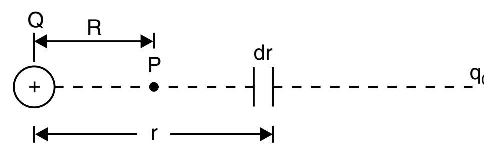


Fig. 2.1

Let a charge q_0 is brought from infinity to point P. The amount of work done for elementary displacement dr,

$$dw = \vec{F} . \vec{dr}$$

$$= -\frac{1}{4\pi\varepsilon_0} . \frac{Qq_0}{r^2} . dr$$

(–ve sign is due to opposite direction of $\vec{F}$ and $\vec{dr}$)

The total amount of work done which stores in the form of potential energy of the system,

$$W = U = -\frac{1}{4\pi\varepsilon_0} Qq_0 \int_{\infty}^{R} \frac{1}{r^2} dr$$

$$= -\frac{1}{4\pi\varepsilon_0} Qq_0 \left[-\frac{1}{R} + \frac{1}{\infty}\right]$$

or $$U = \frac{1}{4\pi\varepsilon_0} . \frac{Qq_0}{R}$$

• Electric Potential

It is defined as the amount of work done in bringing a unit positive charge from infinity to the point of consideration in electric field.

Thus, electric potential,

$$V = \frac{W}{q_0}$$

$$= \frac{1}{4\pi\varepsilon_0} . \frac{Qq_0}{R} / q_0$$

or $$V = \frac{1}{4\pi\varepsilon_0} . \frac{Q}{R}$$ [where Q is the source charge]

• Potential Difference

Difference of potential across two points or the amount of work done in bringing a unit positive charge from one point to another is called the potential difference.

The potential due to Q at point A,

$$V_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R_A}$$

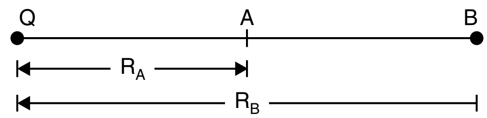


Fig. 2.2

and potential due to Q at point B,

$$V_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R_B}$$

∴ Potential difference

$$V_{BA} = V_A - V_B = \frac{1}{4\pi\varepsilon_0} Q \left[\frac{1}{R_A} - \frac{1}{R_B}\right]$$

or
$$V_{BA} = V_B - V_A = \frac{1}{4\pi\varepsilon_0} Q \left[\frac{1}{R_B} - \frac{1}{R_A}\right]$$

Also,
$$V_{BA} = \frac{W_{BA}}{q_0} = \frac{1}{4\pi\varepsilon_0} Q \int_{R_B}^{R_A} \frac{1}{\pi^2}\, dr$$

$$= \frac{1}{4\pi\varepsilon_0} Q \left[\frac{1}{R_A} - \frac{1}{R_B}\right]$$

or
$$V_{AB} = \frac{W_{AB}}{q_0} = \frac{1}{4\pi\varepsilon_0} Q \int_{R_A}^{R_B} \frac{1}{r^2}\, dr$$

$$= \frac{1}{4\pi\varepsilon_0} Q \left[\frac{1}{R_B} - \frac{1}{R_A}\right]$$

• Potential Energy due to System of N charges

Let $q_1, q_2, q_3, \ldots, q_N$ charges are placed at positions $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots, \vec{r}_N$ respectively. The potential energy of this system of charges,

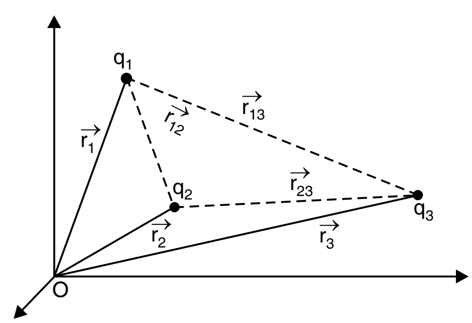


Fig. 2.3

$$U = U_{12} + U_{13} \ldots U_{1N} + U_{23} + U_{24} \ldots U_{2N} + U_{34} + U_{35} \ldots\ldots U_3N \ldots\ldots U_{n-1}U_N$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \ldots, \frac{q_1q_N}{r_{1N}}\right] + \frac{1}{4\pi\varepsilon_0}\left[\frac{q_2q_3}{r_{23}} + \frac{q_2q_4}{r_{24}} + \ldots, \frac{q_2q_N}{r_{2N}}\right]$$

$$+ \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3q_4}{r_{34}} + \frac{q_3q_5}{r_{35}} + \ldots, \frac{q_3q_N}{r_{3N}}\right] \ldots\ldots \frac{q_{N-1}q_N}{r_{N-1}N}\Bigg]$$

or
$$U = \frac{1}{4\pi\varepsilon_0} \sum_{\substack{i,j=1 \\ j>i}}^{N} \frac{q_i\, q_j}{r_{ij}}$$

• Relation between Electric Field and Potential

$\because \quad dV = -\frac{1}{q_0}\int \vec{F}\,.\,\vec{dr}$ and $\int \vec{F} = q_0\,\vec{E}$

$\therefore \quad dV = -\frac{1}{q_0}\int q_0\,\vec{E}\,.\,\vec{dr}$

or $\quad dV = -\int Edr$ or $V = -\int Edr$

Potential difference is –ve line integral of electric field. If one point is taken at infinity, then electric potential at the other point will be

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r}$$ (Q is the source charge)

$\because \quad V = -\int Edr$

differentiating it

$$dV = -Edr$$

or $\quad E = -\frac{dV}{dr}$

Electric field intensity is the –ve gradient of electric potential.

• Potential difference is path independent

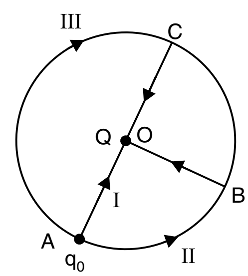


Fig. 2.4

Let q_0 is taken from point A to O through three different paths.
The work done along path $AO(I)$,

$$W_I = -q_0\int_A^0 \vec{E}\,.\,\vec{dr} \qquad ...(i)$$

Work done along path AB_0 (II)

$$W_{II} = -q_0\int_A^B Edr - q_0\int_B^0 Edr \qquad ...(ii)$$

Work done along path ACO (III)

$$W_{III} = -q_0\int_A^C Edr - \int_C^0 Edr \qquad ...(iii)$$

$\because \quad \int_A^B Edr = \int_A^C Edr = 0$ (The force and displacement are perpendicular to each other)

$\therefore$ From Eqns (i), (ii) and (iii)

$$W_I = W_{II} = W_{III} = -q_0\int_A^0 Edr = -q_1\int_B^0 Edr$$

$$= -q_0\int_C^0 Edr.$$

Hence work done or the potential difference is path independent. It depends only on final and initial positions.

• Electric Field is Conservative Field

Let q_0 is taken from A to B then to A. The work done from A to B

$$W_{AB} = -q_0 \int_A^B Edr$$

and work done from B to A

$$W_{BA} = -q_0 \int_B^A Edr$$

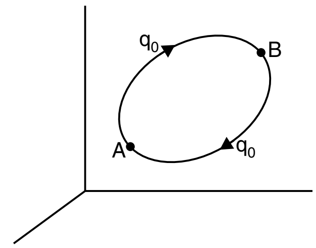


Fig. 2.5

Total work done

$$W = W_{AB} + W_{BA}$$

$$= -q_0 \left[\int_A^B Edr + \int_B^A Edr\right]$$

$$= -q_0 \left[\int_A^B Edr - \int_A^B Edr\right] = 0$$

∵ Work done along a closed path in electric field is zero, hence the electric field is conservative field.

• Electric Potential Due to a Dipole

Let there be an electric dipole of dipole moment $\vec{P} = 2q\vec{l}$. And a point P at the distance of r from the dipole where electric potential is to be determine.

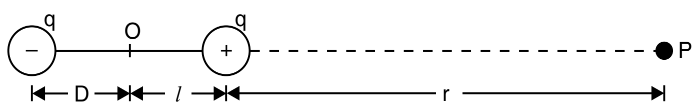


Fig. 2.6

(*i*) **At axial position**

The electric potential due $+q$ at P,

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(r-l)}$$

and the electric potential due $-q$ at P,

$$V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(-q)}{(r+l)}$$

The net potential at P,

$$V = V_1 + V_2$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot q\left[\frac{1}{r-l} + \frac{1}{r+l}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot q\left[\frac{r+l-r+l}{(r^2-l^2)}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot q\left[\frac{2rl}{r^2-l^2}\right] \quad \therefore \quad l << r$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{P}{(r^2-l^2)} \quad \therefore \quad l^2 \text{ can be neglected}$$

or

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{P}{r^2}$$

(*ii*) **At equitorial position**

Electric potential due to $+q$ at P,

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\sqrt{r^2 + l^2}}$$

Electric potential due – q at P,

$$V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(-q)}{\sqrt{r^2 + l^2}}$$

the net potential at P,

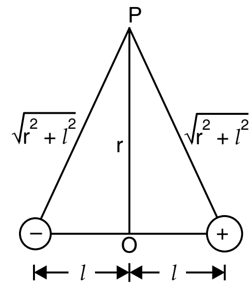


Fig. 2.7

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0} \cdot q \left[\frac{1}{r^2 - l^2} - \frac{1}{\sqrt{r^2 + l^2}}\right]$$

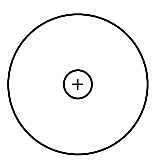
$$= 0$$

The electric potential due to dipole at equatorial position is zero. However, the electric field intensity at equitorial point due to dipole is non-zero. It means at point where electric potential is zero the electric field will not necessarily is zero and vice-versa.

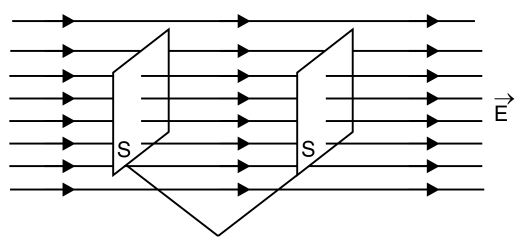
• Equipotential Surface

The surface at which potential at all points is same called equipotential surface. The work done across any two points on equipotential surface is always zero. The electric field is always normal to the equipotential surface.

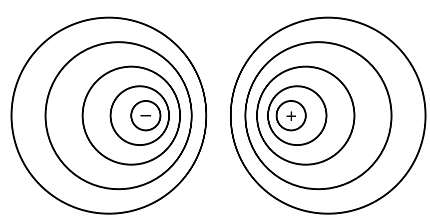
Some equipotential surfaces are given below:



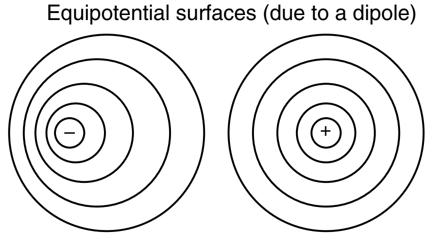
Equipotential surface
[due to point charge]



Equipotential surface (due to uniform electric field)



Equipotential surfaces (due to a dipole)



Equipotential surfaces (due to positive charges)

Fig. 2.8

• Dielectrics and Polarisation

Dielectrics are the substances which do not have free electrons to conduct electricity. These are of two types:

(*i*) **Polar dielectrics:** In the atoms of polar dielectrics the negative charge centre and positive charge centre do not conside and the form a dipole. Hence, all the atoms of polar dielectrics are dipoles. These dipoles are randomly distributed in the substance, hence, the net dipole moment and electric field is zero. When these are placed in electric field, the orientation of atomic dipoles takes place and get oriented in the direction of field. This orientation of atomic dipoles is called polarisation of dielectrics.

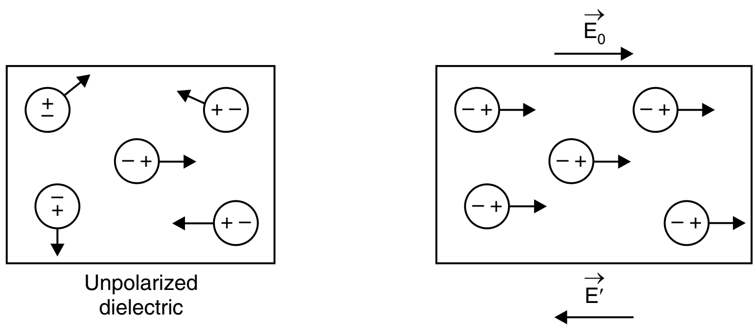


Fig. 2.9

During polarisation of dielectrics an induced field in the opposite direction of applied field is developed due to which intensity of applied field decreases. If $\vec{E}_0$ and $\vec{E}'$ be the applied and induced field respectively, then net field after polarisation

$$\vec{E} = \vec{E}_0 + \vec{E}'$$

or

$$|\vec{E}| = |\vec{E}_0| - |\vec{E}'|$$

The ratio of the applied field to resultant field is called dielectric constant.

$$K = \frac{E_0}{E_0 - E'}$$

(*ii*) **Non-polar dielectrics:** In the atoms of non-polar dielectrics the negative charge centre and positive charge centre of the atoms coincide, hence they do not form dipole. The net field and dipole moment of the substance is zero. When these non-polar dielectrics are placed in electric field, the charge centres stretched gets displaced and becomes dipoles having direction of their dipole moments in the direction of applied field. This is called displacement of dielectrics. In this process an electric field in the opposite direction of applied field is developed due to which net field is decreased.

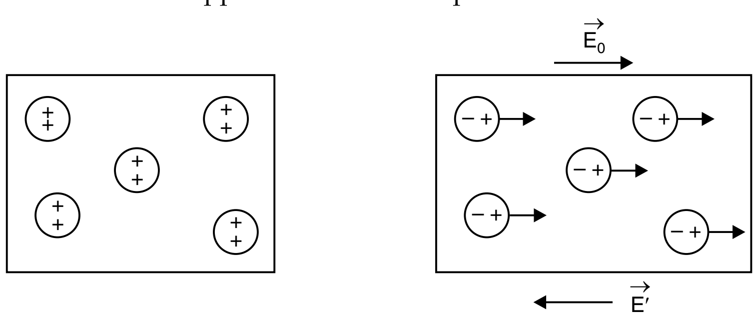


Fig. 2.10

If $\vec{E_0}$ and $\vec{E'}$ be the applied and induced field respectively, then net electric field

$$\vec{E} = \vec{E_0} + \vec{E'}$$

or

$$|\vec{E}| = |\vec{E_0}| - |\vec{E'}|$$

Also, the dielectric constant

$$K = \frac{E_0}{E_0 - E'}$$

• Effect of Temperature on Dielectric Constants

When temperature increases the randomness of atomic dipoles of the dielectrics increases and the intensity of induced field decreases due to which the dielectric constant $K = \frac{E_0}{E_0 - E}$ decreases.

- Dielectric constant or relative permittivity has no unit. Its value for free space, air or vacuum is unity.
- $\because$ $$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$$

$\therefore$ $$\varepsilon_0 = \frac{1}{36\pi \times 10^9} \text{ N}^{-1}\text{m}^{-2} \text{ C}^2$$

(The absolute electrical permittivity of free space, air or vacuum.)

• Polarisation Density

The induced dipole moment developed per unit volume in a dielectric slab on placing it inside the electric field is known as polarisation density. If n be the number of atoms per unit volume, then

$$P = np$$

where, p = induced dipole moment per atom. Using Gauss's Theorem,

$$E = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_p}{\varepsilon_0}$$

$$E = E_0 - \frac{P}{\varepsilon_0}$$

• Electric Susceptability (χ)

The polarisation density of a dielectric substance is directly proportional to the reduced value of the electric field and may be expressed as

$$P = \chi E$$

χ *i.e.,* electrical susceptibility is a dimension less quantity.

• Dielectric Strength

Dielectric strength of a substance is defined as the maximum value of electric field that can be applied to the substance without its electric breakdown. For vacuum, its value is infinity.

- As $\vec{E}$, electric field is related to free charge density σ in vacuum, same role in taken up by $\vec{D}$ (electric displacement vector) inside a dielectric medium.

The quantities $\vec{P}, \vec{E}, \vec{D}$ are parallel to each other. The induced charge density

$$\sigma_p = \vec{P} . n$$

∴ $$\vec{E} . n = \sigma - \frac{\vec{P} n}{\varepsilon_0}$$

or $$(\varepsilon_0 \vec{E} + \vec{P})\, n = \sigma$$

where, $$\varepsilon_0 \vec{E} + \vec{P} = \vec{D}$$

Such that, $$\vec{D} . n = \sigma$$

or $$\frac{D}{E} = \frac{\sigma\varepsilon_0}{\sigma - \sigma_P} = \varepsilon_0 k_P,$$

where k_D is dielectric constant

or $$\vec{D} = \varepsilon_0 k \vec{E}$$

⇒ $$\vec{P} = \vec{D} - \varepsilon_0 \vec{E} = \varepsilon\, (k_D - 1)\, \vec{E}$$

But, $$\frac{P}{E} = \chi, \text{ (Electrical susceptibility)}$$

∴ $$\chi = \varepsilon_0\, (k - 1)$$

Capacitance

When electric charge is given to a body, its electric potential and electric field increase. Beyond the certain value of electric field the surrounding medium of the body gets ionised and the quantity of charge given beyond this limit *h* the body gets neutralised due to presence of opposite nature of charge developed in the surrounding medium due to ionization. Thus, the electric charge cannot be given to a body beyond the certain limit. Every body has its ability to hold the charge called capacity. The measure of the capacity or charge holding capacity is known as capacitance.

The charge given to the body is directly proportional to the increase in potential of the body.

$$q \propto V$$

or $$q = CV$$

where C is the co-efficient of electrical capacity or capacitance.

$$C = \frac{q}{V}, \text{ its SI unit is farad.}$$

If $$V = 1 \text{ volt, then } C = q$$

Thus, capacitance is defined as the amount of charge required to raise the potential of a body by 1 volt.

- **Capacitance of a Spherical Body**

Let q charge is given to a body of radius R to raise its potential by V volt.

$\therefore$
$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$$

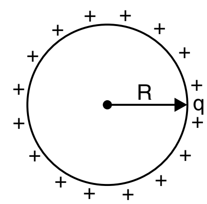


Fig. 2.11

and capacitance,

$$C = \frac{q}{V}$$

$$= \frac{q}{\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}}$$

or
$$C = 4\pi\varepsilon_0 R$$

For a medium of dielectric constant ε_r

$$C = 4\pi\varepsilon_0\varepsilon_r R.$$

Capacitor

When a metallic plate is placed near to a charged plate the electric field in the surrounding of these plates becomes zero and the surrounding medium remains unaffected. This combination can be given now more charge and thus is called a capacitor. A dielectric medium can be put between the plate of the capacitor to decrease the intensity of electric field.

In presence of dielectric, the capacitor can store more charge.

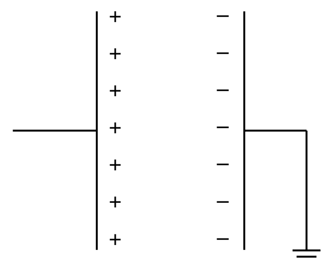


Fig. 2.12

Capacitance of a Parallel Plate Capacitor

Let there be parallel plate capacitor each of area A and separation d. If the + σ and – σ be the electric charge density of respective plates, then the electric field intensity between the plates

$$E = \frac{\sigma}{2E_0} + \frac{\sigma}{2E_0} = \frac{\sigma}{\varepsilon_0}$$

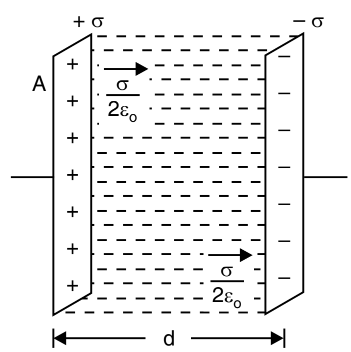


Fig 2.13

And potential difference between the plates

$$V = Ed = \frac{\sigma}{\varepsilon_0} d$$

And the capacitance,

$$C = \frac{q}{V} = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0} d}$$

or
$$C = \frac{\varepsilon_0 A}{d}$$

If the medium between the plates in field with dielectric of dielectric constant $\varepsilon_r = k$, then

$$C = \frac{\varepsilon_0 k A}{d}$$

Combination of Capacitors

(*i*) **Series Combination:** When capacitors are connected one by one and same charge stores in all, the combination is called series combination. Let three capacitors of capacitances C_1, C_2 and C_3 are connected across source of potential difference V in series.

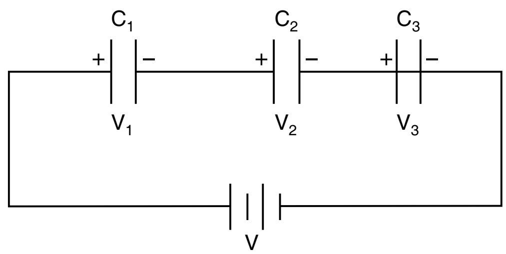


Fig. 2.14

The potential difference across each capacitor V_1, V_2 and V_3 respectively is given as,

$$V_1 = \frac{q}{C_1},\ V_2 = \frac{q}{C_2}$$

and $$V_3 = \frac{q}{C_3} \text{ respectively.}$$

Where q is the charge stored. If C be the equivalent capacitance of the combination,

then $$V = \frac{q}{C} \text{ also, } V = V_1 + V_2 + V_3$$

$\therefore$ $$\frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

or $$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

- In series combination the equivalent capacitance is less than the lowest capacitance present in the combination.
- In series combination the potential difference distributes in inverse ratios of the capacitances.

i.e., $$\frac{V_1}{V_2} = \frac{C_2}{C_1}$$

(*ii*) **Parallel Combination:** When capacitors are connected across two same points and potential difference across all remains same, the combination is called parallel combination. Let three capacitors of capacitances C_1, C_2 and C_3 are connected in parallel. The charge stored in all the capacitors respectively is given as

$$q_1 = C_1V,\ q_2 = C_2V$$

and $$q_3 = C_3V$$

If C be the equivalent capacitance of the combination and q be the net charge stored then

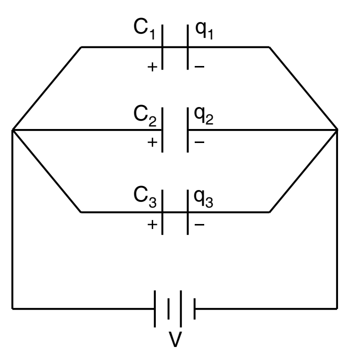


Fig. 2.15

$$q = CV \quad \text{also,} \quad q = q_1 + q_2 + q_3$$

$\Rightarrow$ $$CV = C_1V + C_2V + C_3V$$

or $$C = C_1 + C_2 + C_3$$

- In parallel combination the equivalent capacitance is greater than the largest capacitance present in the combination.
- In parallel combination, the charge distributes in the direct ratio of the capacitances *i.e.,*

$$\frac{q_1}{q_2} = \frac{C_1}{C_2}$$

Capacitance of a Parallel Plate Capacitor Having Dielectric Partially Filled between Its Plate

Let there be parallel plate capacitor of area of each plate A and separation d. A material of dielectric constant k of thickness t ($t < d$) is partially filled between the plates of the capacitor. The capacitor is divided between two parts

(*i*) The capacitor having no dielectric region between the plates.

(*ii*) The capacitor having dielectric region between its plate (t)

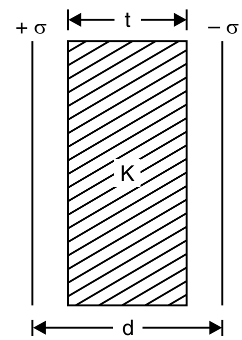


Fig. 2.16

The electric field intensity of the region without dielectric,

$$E = \frac{\sigma}{\varepsilon_0}$$

and the potential difference of this region

$$V_1 = \frac{\sigma}{\varepsilon_0}(d - t)$$

The electric field intensity of the region having dielectric

$$E = \frac{\sigma}{\varepsilon_0 k}$$

and the potential difference of this region,

$$V_2 = \frac{\sigma}{\varepsilon_0 k}(t)$$

The net potential difference across the plates of the capacitor,

$$V = V_1 + V_2$$

$$= \frac{\sigma}{\varepsilon_0}(d - t) + \frac{\sigma}{\varepsilon_0 k}(t)$$

$$= \frac{\sigma}{\varepsilon_0}\left(d - t + \frac{t}{k}\right)$$

The capacitance of the capacitor,

$$C = \frac{q}{V} = \frac{\sigma_A}{\frac{\sigma}{\varepsilon_0}\left(d - t + \frac{t}{k}\right)}$$

or $$C = \frac{\varepsilon_0 A}{\left(d - t + \frac{t}{k}\right)}$$

If $d = t$ (capacitor is completely filled with dielectric)

then $$C = \frac{\varepsilon_0 A k}{d}$$

For metal $k = \infty$

∴ The capacitance of the capacitor having metallic plate between its plates,

$$C = \frac{\varepsilon_0 A}{\left(d - t + \frac{t}{\infty}\right)}$$

or $$C = \frac{\varepsilon_0 A}{(d - t)}$$

Energy Stored in A Capacitor

The amount of work done in raising the potential of a capacitor is stored in the form of energy of the capacitor. Let the potential of a capacitor is increased by dV by giving a charge q.

The amount of work done or energy stored in the capacitor,

$$dU = q dV = CV\, dV$$

And total energy stored in the capacitor

$$U = \int CV\, dV = \frac{CV^2}{2}$$

or $$U = \frac{1}{2} CV^2 \qquad \because \quad C = \frac{q}{V}$$

∴ $$U = \frac{1}{2} qV \qquad \because \quad V = \frac{q}{C}$$

∴ $$U = \frac{1}{2} \frac{q^2}{C}$$

Common Potential

When two capacitors, charged to the different potential, are connected the redistribution of charge takes place and the capacitors acquired same potential called common potential.

Common potential $$V_C = \frac{\text{Net charge stored}}{\text{Net capacitance}}$$

$$= \frac{q_1 + q_2}{C_1 + C_2}$$

or $$V_C = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

Energy Loss

When two capacitors charged to the different potentials are connected, they acquired a common potential,

$$V_C = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$\because$ Initial energy of the system of two capacitors C_1 and C_2 charged to the potentials V_1 and V_2 respectively, (without connecting)

$$U_1 = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

Thus final energy stored in the system (after connection)

$$U_2 = \frac{1}{2}(C_1 + C_2)(V_C)^2$$

$\therefore$ Energy loss,

$$\Delta U = U_1 - U_2$$

$$= \frac{1}{2}\left[C_1V_1^2 + C_2V_2^2 - \frac{(C_1 + C_2)(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)^2}\right]$$

$$= \frac{1}{2}\left[C_1V_1^2 + C_2V_2^2 - \frac{1}{(C_1 + C_2)}(C_1^2V_1^2 + C_2V_2^2 + 2C_1C_2V_1V_2)\right]$$

$$= \frac{1}{2}\left[\frac{C_1^2V_1^2 + C_2^2V_2^2 + C_1C_2V_1^2 + C_1C_2V_2^2 - C_1^2V_1^2 - C_2^2V_2^2 - 2C_1C_2\,V_1V_2}{(C_1 + C_2)}\right]$$

$$= \frac{1}{2}\frac{C_1C_2}{(C_1 + C_2)}(V_1^2 + V_2^2 - 2V_1V_2)$$

$$\Delta U = \frac{1}{2}\frac{C_1C_2\,(V_1 - V_2)^2}{(C_1 + C_2)}$$

$\because$ ΔU is always positive $\therefore$ There is always loss of energy due heat dissipation in connecting wire.

• Van de Graaff Generator

In 1931, R.J. Van de Graaff designed an electrostatic generator to generate very high potential of the order of 5×10^6 V. This high potential can be used to accelerate a charge particle.

- P_1 and P_2 are pullyes
- C_1 is spraying comb
- C_2 is collecting comb
- Van de Graaff generator is based on two electrostatic phenomona:

(*i*) The electric charge takes place in air or gases readily at pointed ends (Point action).

(*ii*) If a metallic sphere is kept inside a conducting shell and is kept in contact with each other than as charges are supplied to internal sphere, the outer conductor starts accepting charges by induction.

- In Van de Graaff generator a high tension battery attached to spraying comb C_1, which ionises the air and positives are is carried by the conveyor belt from pulley P_1 to pulley P_2 from where it is collected by collecting comb C_2 by induction. The charges move from C_2 to the surface S (outer surface of metallic dome). There S is a small leakage of charges from the surface S. However, a large amount of charge continuously gets accumulated over the surface. After certain time large amount of charge gets collected on the surface and hence discharge of charge also become very high. At certain point of time the amount of charges leaking through the surface becomes equal to the amount of charges collecting over surface per unit time. This is the point at which potential of the surface becomes maximum. At this potential, positive alpha particles, etc., are accelerated and are made to hit the target.

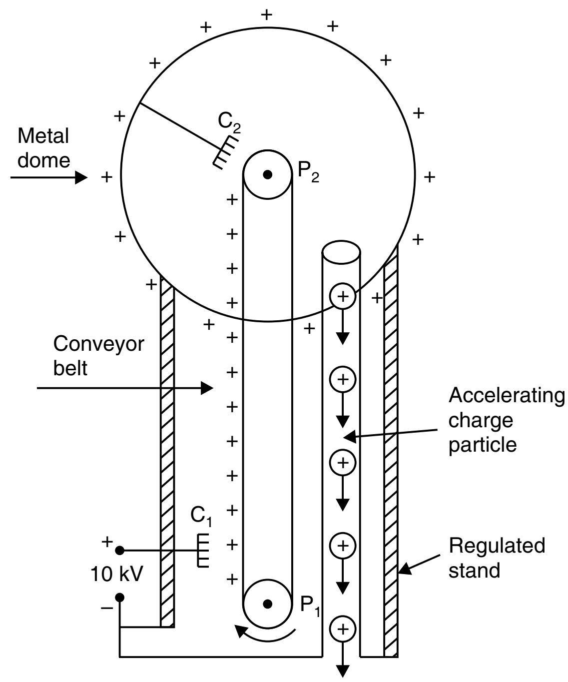


Fig. 2.17

- In Van de Graaff generator the electrostatic potential inside a shell (having radius R, and charge Q on the shell)

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}, \text{ where } r \geq R$$

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}, \text{ where } r \leq R$$

If a small sphere of radius r_0 and charge q is enclosed by the large spherical shell, then the potential due to inner sphere at a distance r from the centre of the shell will be

$$V_i = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Hence, the total potential at $r = R$,

$$V_R = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{R}+\frac{q}{R}\right]$$

and total potential at $r = r_0$,

$$V_{r_0} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{R}+\frac{q}{r_0}\right]$$

Thus, the potential difference, between points $r = r_0$ and $r = R$ is

$$V_{r_0} - V_R = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_0} - \frac{1}{R}\right]$$

For positive charge q, whatever is the magnitude and the sign of the charge Q, the internal small sphere will be at higher potential then the outer shell.

- The Van de Graaff generator is used in

(*i*) x-ray

(*ii*) nuclear research

(*iii*) study of effect due to high electric field.

QUESTIONS FROM TEXTBOOK

2.1. *Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.*

Sol. The other possibility of point where total potential due to q_1 and q_2 are zero may be that point lies out side the segment joining q_1 and q_2

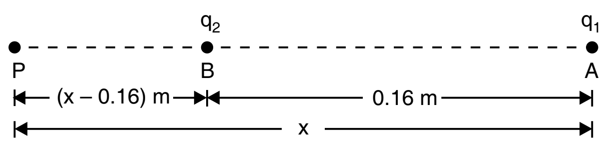


Fig. 2.18

Let required point P lies x m from q_1 then $V_1 + V_2 = 0$

$$\frac{Kq_1}{x} + \frac{Kq_2}{(x-0.16)} = 0$$

$$K\left[\frac{5\times10^{-8}}{x} - \frac{3\times10^{-8}}{(x-0.16)}\right] = 0$$

or

$$\frac{5}{x} - \frac{3}{x-0.16} = 0$$

$$\frac{5x - 0.8 - 3x}{x(x-0.16)} = 0$$

$$2x - 0.8 = 0$$

$$x = \frac{0.8}{2}$$

$$x = 0.4 \text{ m}$$

$$= 40 \text{ cm}$$

required point is 40 cm away from q_1 and (40 – 16) = 24 cm from q_2.

2.2. *A regular hexagon of side 10 cm has a charge 5 μC at each of its vertices. Calculate the potential at the centre of the hexagon.*

Sol. Given, $q = 5 \times 10^{-6}\ C$

$r = 10\ \text{cm} = 0.1\text{m}$

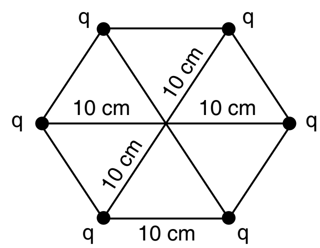


Fig. 2.18

Potential at the centre of the hexagon is given by

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r}+\frac{q}{r}+\frac{q}{r}+\frac{q}{r}+\frac{q}{r}+\frac{q}{r}\right]$$

$$= 6\times\left(\frac{1}{4\pi\varepsilon_0}\right)q/r$$

or, $$V = \frac{6\times 9\times 10^9\times 5\times 10^{-6}}{0.1} = 2.7 \times 10^6\ \text{V}.$$

2.3. *Two charges 2μC and –2μC are placed at points A and B, 6 cm apart.*

(a) Identify an equipotential surface of the system.

(b) What is the direction of electric field at every point on this surface?

Sol. Given, $q_1 = 2\mu\text{C} = 2 \times 10^{-6}\ \text{C}$

$q_2 = -2\mu\text{c} = -2 \times 10^{-6}\ \text{C}$

$r = 0.06\ \text{m}$

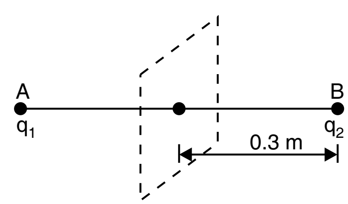


Fig. 2.19

(*a*) Potential will be zero due to both charges at equipotential surface

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{x}+\frac{q_2}{(0.06-x)}\right] = 0$$

or, $$\frac{q_1}{x} = -\frac{q_2}{(0.06-x)}$$

or, $$\frac{2\times 10^{-6}}{x} = -\frac{(-2\times 10^{-6})}{[(0.06)-x]}$$

or, $$x = 0.06 - x$$

$$x = \frac{0.06}{2} = 0.03\ \text{m}$$

i.e., the plane normal to *AB* and passing through its mid-point has zero potential everywhere.

(*b*) The direction of electric field is normal to the plane in the *AB* direction.

2.4. *A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7} C distributed uniformly on its surface. What is the electric fields* (*a*) *inside the sphere,*

(*b*) *just outside the sphere,*

(*c*) *at a point 18 cm from the centre of the sphere?*

Sol. Given, $q = 1.6 \times 10^{-7}$ C

$r = 0.12$ m

(*a*) Electric field is zero inside the sphere, because charge reside on outer surface of conductor.

(*b*) Electric field just outside the sphere is given by

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}} = 9\times10^9 \times \frac{1.6\times10^{-7}}{(0.12)^2}$$

$$= 10^5 \; NC^{-1}$$

(*c*) At a point 0.18 m from the centre of the sphere

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = 9\times10^9 \times \frac{1.6\times10^{-7}}{(0.18)^2}$$

$$= 4.4 \times 10^4 \; NC^{-1}$$

2.5. *A parallel plate capacitor with air between the plates has a capacitance of 8 pF (1pF = 10^{-12}F). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?*

Sol. Given, $C = 8\ pF = 8 \times 10^{-12}$ F

$$\boxed{C_0 = \frac{\varepsilon_0 A}{d}} \quad \text{or,} \quad \frac{\varepsilon_0 A}{d} = 8 \times 10^{-12}$$

If the distance is reduced to half *i.e.*, $d/2$ and space between the plates is filled by a substance of $k = 6$.

$$\boxed{C = kC_0}$$

Then, $$C = \frac{k\varepsilon_0 . A}{d/2}$$

$$= \frac{2k(\varepsilon_0 A)}{d}$$

$$= 2 \times 6 \times 8 \times 10^{-12}$$

$$= 96 \times 10^{-12} \text{ F}$$

or, $C = 96$ pF.

2.6. *Three capacitors each of capacitance 9 pF are connected in series:*

(*a*) *What is the total capacitance of the combination?*

(*b*) *What is the potential difference across each capacitor if the combination is connected to a 120 V supply?*

Sol. Given, three capacitors

$$C_1 = C_2 = C_3 = 9 \text{ pF}$$
$$= 9 \times 10^{-12} \text{ F}$$

(*a*) Since the capacitors are connected in series.

$$\frac{1}{C} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

$$\frac{1}{C} = \frac{1}{9\times10^{-12}}+\frac{1}{9\times10^{-12}}+\frac{1}{9\times10^{-12}}$$

$$= \frac{1}{10^{-12}}\left[\frac{1}{9}+\frac{1}{9}+\frac{1}{9}\right]$$

or, $$\frac{1}{C} = \frac{1}{10^{-12}}\times\frac{3}{9} = \frac{1}{3\times10^{-12}}$$

or, $$C = 3\times10^{-12} = 3 \text{ pF}$$

(*b*) Given, $$V = 120 \text{ volt}$$

$$V = V_1 + V_2 + V_3$$

as capacitances are equal so

$$V_1 = V_2 = V_3 = V' \text{ (let)}$$

$$V = 3V'$$

$$V' = \frac{V}{3} = \frac{120}{3} \qquad (\because C_1 = C_2 = C_3 = C)$$

$$= 40 \text{ volt.}$$

2.7. *Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.*

(a) What is the total capacitance of the combination?

(b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.

Sol. Given,

$$C_1 = 2 \text{ pF} = 2\times10^{-12} \text{ F}$$

$$C_2 = 3 \text{ pF} = 3\times10^{-12} \text{ F}$$

$$C_3 = 4 \text{ pF} = 4\times10^{-12} \text{ F}$$

(*a*) Since the capacitors are connected in parallel then

$$C = C_1 + C_2 + C_3$$

$$= (2 + 3 + 4)\times10^{-12}$$

$$= 9\times10^{-12} = 9 \text{ pF}$$

(*b*) Given, $$V = 100 \text{ volt}$$

$$q_1 = C_1V = 2\times10^{-12}\times100$$

$$= 2\times10^{-10} \text{ C}$$

$$q_2 = C_2V = 3\times10^{-12}\times100$$

$$= 3\times10^{-10} \text{ C}$$

$$q_3 = C_3V = 4\times10^{-12}\times100$$

$$= 4\times10^{-10} \text{ C}$$

2.8. *In a parallel plate capacitor with air between the plates, each plate has an area of 6×10^{-3} m² and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?*

Sol. Given, $d = 3 \times 10^{-3}$ m

$A = 6 \times 10^{-3}\ \text{m}^2$

$$C = \frac{\varepsilon_0 \cdot A}{d} = \frac{9\times10^{-12}\times6\times10^{-3}}{3\times10^{-3}}$$

$= 18 \times 10^{-12}$ F = 18pF

Here, $V = 100\ V$

$q = CV = 18 \times 10^{-12} \times 100$

$= 1.8 \times 10^{-9}\ C$

2.9. *Explain what would happen if in the capacitor given in Question 2.8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates,*

(a) while the voltage supply remained connected.

(b) after the supply was disconnected.

Sol. (*a*) Given, $k = 6$

$t = d = 3$ mm $= 3 \times 10^{-3}$ m

$C = kC_0$

$C = 6 \times 18 \times 10^{-12}$ F

$= 108 \times 10^{-12}$ F

$= 108$ F

As, $Q = CV = 108 \times 10^{-12} \times 100$

$= 108 \times 10^{-10}$

or, $Q = 1.08 \times 10^{-8}$ C

while voltage supply remained connected,

$C = 108$ pF and $Q = 1.08 \times 10^{-8}$ C

(*b*) After the supply was disconnected, the charge remains same

i.e., $q = 1.8 \times 10^{-9}$ C

As, $Q = CV$

$$V = \frac{Q}{C} = \frac{1.8\times10^{-9}}{108\times10^{-12}} = 16.66\ \text{V}$$

The charge remains constant *i.e.,* $Q = 1.08 \times 10^{-8}$ C after the supply was disconnected, the voltage will come down to 16.6 V.

2.10. *A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor?*

Sol. Given, $V = 50\ V$

$C = 12$ pF $= 12 \times 10^{-12}$ F

Electrostatic energy stored in the capacitor is given as

$$\boxed{U = \frac{1}{2}CV^2}$$

$$= \frac{1}{2}\times12\times10^{-12}\times(50)^2$$

or, $U = 1.5 \times 10^{-8}$ J.

2.11. *A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?*

Sol. Given,

$$V = 200 \text{ V}$$
$$C = 600 \text{ pF} = 600 \times 10^{-12} \text{ F}$$
$$\boxed{Q = CV}$$
$$= 600 \times 10^{-12} \times 200$$

or, $$Q = 12 \times 10^{-8} \text{ C}$$

Electrostatic energy stored in capacitor

$$\boxed{U_1 = \frac{1}{2}CV^2} = \frac{1}{2} \times 600 \times 10^{-12} \times (200)^2$$
$$= 12 \times 10^{-6} \text{ J}$$

Now, the supply is disconnected and the capacitor is connected to another similar uncharged capacitor. Therefore, the *charge is divided equally between the two capacitors*.

Hence, $$Q_1 = Q_2 = \frac{12 \times 10^{-8}}{2} = 6 \times 10^{-8} \text{ C}$$

and $$V_1 = V_2 = \frac{Q_1}{C_1} = \frac{6 \times 10^{-8}}{600 \times 10^{-12}} = 100 \text{ V}$$

Total capacitance $$C = C_1 + C_2$$
$$= 600 \times 10^{-12} \text{ F} + 600 \times 10^{-12} \text{ F}$$
$$= 1200 \times 10^{-12} \text{ F}$$

Now, the electrostatic energy stored is given as

$$U_2 = \frac{1}{2}CV^2 = \frac{1}{2} \times 1200 \times 10^{-12} \times (100)^2$$

or, $$U_2 = 6 \times 10^{-6} \text{ J}$$

Electrostatic energy lost in the process

$$= U_1 - U_2$$
$$= 12 \times 10^{-6} - 6 \times 10^{-6}$$
$$= 6 \times 10^{-6} \text{ J.}$$

2.12. *A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of -2×10^{-9} C from a point P(0, 0, 3 cm) to a point Q(0, 4 cm, 0), via a point R (0, 6 cm, 9 cm).*

Sol. Given charge q = 8 mC = 8×10^{-3} C is located at origin and the small charge ($q_0 = -2 \times 10^{-9}$ C) is taken from point P (0, 0, 3 cm) to a point Q (0, 4, cm, 0) through point R (0, 6 cm, 9 cm) which is shown in the figure.

Initial separation between q_0 and q is r_P = 3 cm = 0.03 m

Final separation between q_0 and q is r_Q = 4 cm = 0.4 m

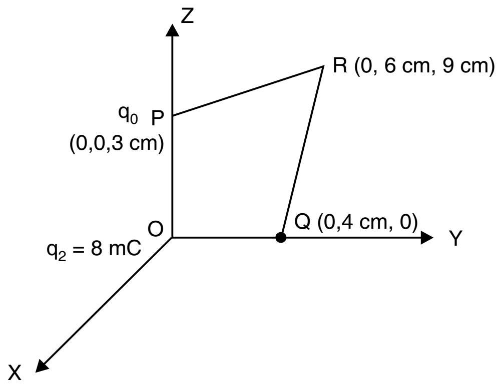


Fig. 2.20

Work done in taking the charge q_0 from point P to Q does not depend on the path followed and depends only upon r_P and r_Q *i.e.,* initial and final positions.

$$W = \frac{1}{4\pi\varepsilon_0} q q_0 \left(\frac{1}{r_Q} - \frac{1}{r_P}\right)$$

or,
$$W = 9 \times 10^9 \times 8 \times 10^{-3} \times (-2 \times 10^{-9}) \times \left(\frac{1}{0.04} - \frac{1}{0.03}\right)$$
$$= 1.2 \text{ J.}$$

2.13. *A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to these charges array at the centre of the cube.*

Sol. Diagonal DF of cube

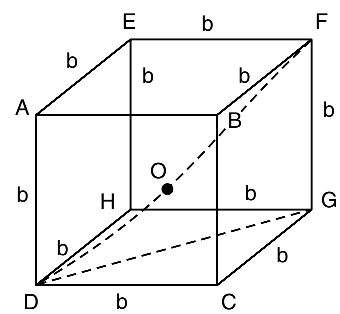


Fig. 2.21

$$DF = \sqrt{b^2 + b^2 + b^2}$$
$$DF = b\sqrt{3}$$

Thus,
$$DO = \frac{DF}{2} = \frac{\sqrt{3}}{2}b$$

Due to one charge q the potential at the centre O is given by

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}} = \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{\frac{\sqrt{3}b}{2}}\right)$$

Due to eight charges the total potential at the centre O is given as

$$V = 8\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{\frac{\sqrt{3}}{2}b}\right)$$
$$= \frac{4q}{\sqrt{3}\pi\varepsilon_0 b}$$

Remark: Due to two opposite corners D and F electric field intensity at the centre 'O' are equal in magnitude and opposite in direction. Therefore, they cancel out each other. Similarly all other intensities cancel out each other and the total electric field at centre is zero.

2.14. *Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential and electric field:*

(a) at the mid-point of the line joining the two charges, and

(b) at a point 10 cm from this mid-point in a plane normal to the line and passing through the mid-point.

Sol. (*a*) Potential at the mid-point of the line joining the two charges is

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1}+\frac{q_2}{r_2}\right]$$

$$= 9\times10^9\left[\frac{15\times10^{-6}}{0.15}+\frac{2.5\times10^{-6}}{0.15}\right]\text{V}$$

$$= 9\times10^9\times10^{-6}\left[10+\frac{50}{3}\right]$$

$$= 9\times10^3\times\frac{80}{3}$$

or, $V = 2.4 \times 10^5$ V

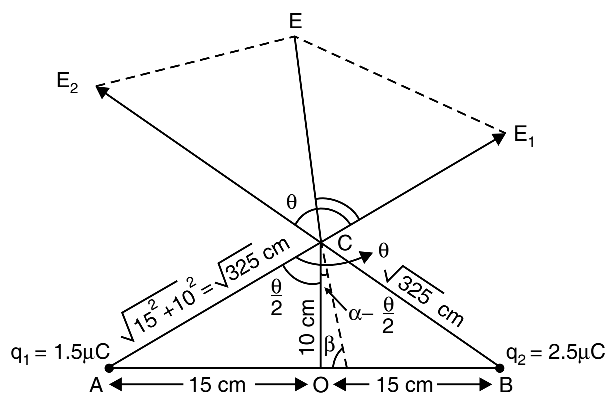


Fig. 2.22

Electric field at the mid-point O due to charge at A

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}$$

$$= 9\times10^9\times\frac{1.5\times10^{-6}}{(0.15)^2}$$

$= 6 \times 10^5$ Vm^{-1} along *OB*

Electric field at the mid-point O due to charge at B

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2} = 9\times10^9\times\frac{2.5\times10^{-6}}{(0.15)^2}$$

$= 10 \times 10^5$ Vm^{-1} along *OA*

Thus, the total electric field at the mid-point O is

$$E = 10 \times 10^5 - 6 \times 10^5$$
$$= 4 \times 10^5 \text{ Vm}^{-1} \text{ (along } BA\text{)}$$

(*b*) Potential at the point C due to the two charges is

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1}+\frac{q_2}{r_2}\right]$$
$$= 9\times10^9\left[\frac{1.5\times10^{-6}}{\sqrt{325}\times10^{-2}}+\frac{2.5\times10^{-6}}{\sqrt{325}\times10^{-2}}\right]V$$
$$= \frac{9\times10^9\times10^{-6}}{10^{-2}}\cdot\frac{4.0}{\sqrt{325}}\text{V}$$
$$= \frac{9\times4}{18.02}\times10^5 V = 2 \times 10^5 \text{ V}$$

Electric field at C due to charge at A

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}$$
$$= 9 \times 10^9 \times \frac{1.5\times10^{-6}}{\left(\sqrt{325}\times10^{-2}\right)^2}\text{Vm}^{-1}$$
$$= \frac{9\times1.5}{325}\times10^7 \text{ Vm}^{-1}$$
$$= 4.15 \times 10^5 \text{ Vm}^{-1}$$

Electric field at C due to charge at B

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2}$$
$$= 9\times10^9\times\frac{2.5\times10^{-6}}{325\times10^{-4}}$$
$$= 6.92 \times 10^5 \text{ Vm}^{-1}$$

If the angle between E_1 and E_2 be θ, then

$$\tan\frac{\theta}{2} = \frac{0.15}{0.10} = 1.5$$
$$\theta/2 = 56.3° \text{ or, } \theta = 112.6°$$

Thus, magnitude of resultant field at C is

$$E = \sqrt{E_1^2+E_2^2+2E_1E_2\cos\theta}$$
$$= \sqrt{\left(4.15\times10^5\right)^2+\left(6.92\times10^5\right)^2+2\times4.15\times10^5\times6.92\times10^5\cos112.6°}$$
$$= 10^5\sqrt{17.2+47.8-2\times4.15\times6.92\cos67.4°}$$

$(\because \cos(180-\theta) = -\cos\theta)$

$$= 10^5\sqrt{43} = 6.56 \times 10^5 \text{ Vm}^{-1}$$

Let the field E makes angle α with the field E_1.

Now, $$\alpha = \frac{E_2 \sin\theta}{E_1 + E_2 \cos\theta}$$

$$= \frac{6.92\times10^5 \times 0.9232}{4.15\times10^5 - 6.92\times10^5\times0.384}$$

$$= \frac{6.39}{4.15-2.66} = \frac{6.39}{1.49} = 4.2876$$

$\therefore$ $$\alpha = \tan^{-1}(4.2876) \approx 76.9°$$

If field E makes angle β with the direction BA, then

$$\beta = 90° - \left(\alpha - \frac{\theta}{2}\right)$$

$$= 90° + \frac{\theta}{2} - \alpha$$

$$= 90° + 56.3° - 76.9° = 69.\ 4°$$

Therefore, angle of 69.4° is made by the electric field with the line joining the two charges. q_2 to q_1.

2.15. *A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q.*

(a) A charge q is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell?

(b) Is the electric field inside a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape? Explain.

Sol. (*a*) As *a* '+ q' charge place at the centre of the shell, it will create *a* '$-q$' charge on the inner surface of the shell.

The charge on the outer surface will increase by $+q$ due to the $-q$ charge on the inner surface by induction. Therefore, there will be total $(Q + q)$ charge on the outer surface of the shell and $-q$ charge on the inner surface of the shell.

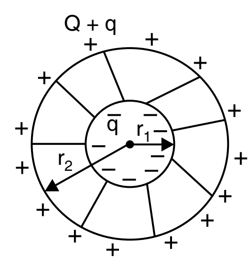


Fig. 2.23

Now surface area of inner surface = $4\pi r_1^2$

and surface area of outer surface = $4\pi r_2^2$

Thus, charge density on the outer surface = $\dfrac{Q+q}{4\pi r_2^2}$

and charge density on the inner surface = $\dfrac{-q}{4\pi r_1^2}$.

(*b*) As charge in shell reside on outer surface so, the net charge on the inner surface of the cavity is zero as per the Gaussian theorem. Although the net charge is zero yet the electric field may not be zero if the cavity is not spherical. The surface may not have equal number of positive and negative charges. We assume a loop for this reason, some portion of which is inside the cavity and rest of its part is inside the conductor. Now, consider that there is some electric field inside the cavity. Since inside the conductor total electric field is zero and net work done by the field in bringing a test charge over this loop will not be zero. But this is not possible for an electrostatic field. Therefore, we must conclude that there *is no electric field inside the cavity irrespective of its shape*.

2.16. (*a*) *Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by*

$$(E_2 - E_1).\hat{n} = \frac{\sigma}{\varepsilon_0}$$

where $\hat{n}$ is a unit vector normal to the surface at a point and σ is the surface charge density at that point. (The direction of $\hat{n}$ is from side 1 to side 2.) Hence, show that just outside a conductor, the electric field is $\sigma\,\hat{n}/\varepsilon_0$.

(*b*) *Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another. [**Hint:** For (a), use Gauss's law. (b) For, use the fact that work done by electrostatic field on a closed loop is zero.]*

Sol. (*a*) Near a plane sheet of charge, electric field is given as

$$E = \frac{\sigma}{2\varepsilon_0}$$

Electric field on side 2 if $\hat{n}$ is a unit vector normal to the sheet from side 1 to side 2.

$$E_2 = \frac{\sigma}{2\varepsilon_0} \quad \text{(In the outward direction normal to the side 2)}$$

Now, electric field on side 1 is given as

$$E_1 = \frac{\sigma}{2\varepsilon_0} \quad \text{(In the outward direction normal to the side 1)}$$

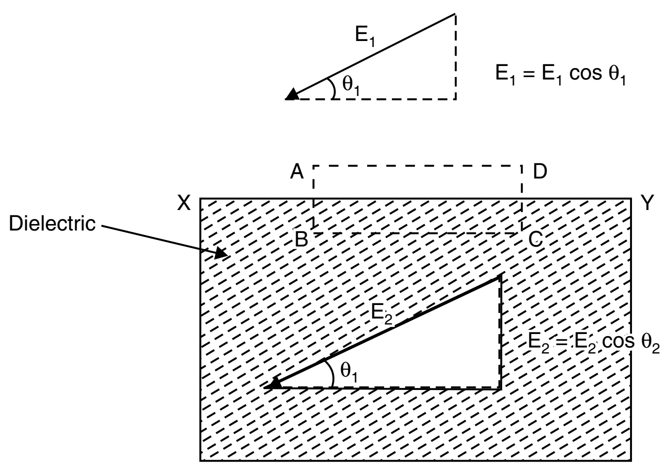


Fig. 2.24

As E_1 and E_2 are in opposite directions so will have opposite sign

$$\therefore \quad (E_2 - E_1)\ \hat{n} = \frac{\sigma}{2\varepsilon_0} - \left(-\frac{\sigma}{2\varepsilon_0}\right) = \frac{\sigma}{\varepsilon_0}\hat{n}$$

There must be discontinuity at the sheet of charge since E_1 and E_2 act in opposite directions.

Now, electric field inside the conductor vanishes.

Hence, $\quad E_1 = 0$

Therefore, electric field outside the conductor

$$E = E_2 = \frac{\sigma}{\varepsilon_0}\hat{n}$$

(*b*) Consider that E_1 and E_2 be the electric field on the two sides of the charged surface and *xy* be the charged surface of dielectric as shown in the figure.

Let a rectangular loop *ABCD* with length *l* and negligible small breadth. Line integral along the closed path *ABCD* will be

$$\oint E.dl = E_1 l - E_2 l = 0$$

or, $$E_1 l \cos\theta_1 - E_2 l \cos\theta_2 = 0$$

$$(E_1 \cos\theta_1 - E_2 \cos\theta_2)l = 0$$

$$(E_1' - E_2') = 0$$

Where E_1' and E_2' are the tangential components of E_1 and E_2 respectively. Hence

$$E'_1 = E_2'$$

Therefore, the tangential component of the electrostatic field is continuous across the surface.

2.17. *A long charged cylinder of linear charged density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders?*

Sol. Two co-axial cylindrical shells *A* and *B* of radii *a* and *b* are possessed by a cylindrical capacitor. Assume *l* be length of the cylindrical shell. Due to the introduction of +*q* charge on the inner cylindrical shell *A*, equal but opposite charge –*q* is induced on the inner surface of the outer cylindrical shell *B*. The induced charge +*q* on its outer surface will flow to earth if the shell *B* is earthed.

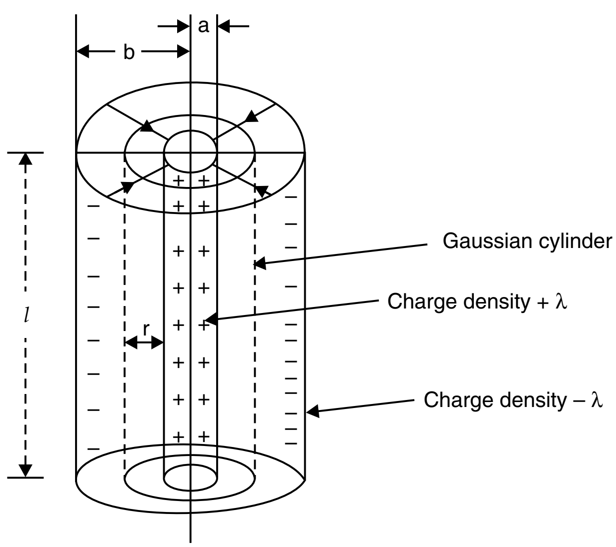


Fig. 2.25

The capacitance of the cylindrical capacitor is given as follows if V is potential difference between the cylindrical shells A and B.

$$C = \frac{q}{V}$$

By applying the Gaussian theorem, we first need to find electric field E in the space between two shells to find out potential difference between the cylindrical shells **A** and B. Let a cylinder of radius r (such that $b > r > a$) and length l as the Gauss surface. Charge enclosed by the Gaussian surface is λl and if λ is charge per unit length on the shell A.

The electric flux will cross through only curved surface of the cylinder (Gauss surface). As the area of curved surface of cylinder is $2\pi rl$, we have by Gaussian theorem

$$\oint E \,.\, ds = \frac{q}{\varepsilon_0}$$

$$E.2\pi rl = \frac{\lambda l}{\varepsilon_0}$$

or, $$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

The field lines are radial and normal to the axis of charged cylinder.

2.18. *In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 Å:*

(a) Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton.

(b) What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (a)?

(c) What are the answers to (a) and (b) above if the zero of potential energy is taken at 1.06 Å separation?

Sol. Given,

$$q_1 = -1.6 \times 10^{-19} \text{ C (electron)}$$

$$q_2 = 1.6 \times 10^{-19} \text{ C (proton)}$$

$$r = 0.53 \text{ Å} = 0.53 \times 10^{-10} \text{ m}$$

(*a*) Potential energy $= \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r}$

$$= \frac{(9\times10^9)(-16\times10^{-19})(16\times10^{-19})}{(0.53\times10^{-10})} \text{ J}$$

$$= -43.47 \times 10^{-19} \text{ J}$$

or, $$= -\frac{43.47\times10^{-19}}{1.6\times10^{-19}} eV = -27.17 \text{ e}V$$

Potential energy is zero at infinite separation. Hence, the potential energy of the system is (–27.17 – 0) or 27.17 *eV* if zero of potential energy is taken at infinite separation.

(*b*) Electron's K.E. $= \dfrac{1}{2}\text{P.E.} = +\dfrac{27.2}{2}$

$$= 13.6 \text{ eV}$$

Total energy of electron = – 27.2 + 13.6

= – 13.6 eV

Amount of work required to free the electron = Increase in energy of electron

= 0 – (–13.6) = 13.6 eV

(*c*) At 1.06 Å (1.06×10^{-10} m) separation, the potential energy of system

$$W = 9\times10^{9}\left[\frac{(1.6\times10^{-19})\times(1.6\times10^{-19})}{1.06\times10^{-10}}\right]$$

$$= 21.74 \times 10^{-19}\text{ J}$$

$$= -\frac{21.74\times10^{-19}}{1.6\times10^{-19}} = -13.585\text{ eV}$$

If it is taken as zero of potential energy, then potential energy of the system

= –27.17 – (–13.585)

= –13.585 eV

2.19. *If one of the two electrons of a H_2 molecule is removed, we get a hydrogen molecular ion H_2^+. In the ground state of a H_2^+, the two protons are separated by roughly 1.5Å, and the electron is roughly 1 Å from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy.*

Sol. Suppose that the distance between two protons p_1 and p_2 be r_{12} and electron (e^-) is placed at r_{13} and r_{23} distance from protons p_1 and p_2 respectively.

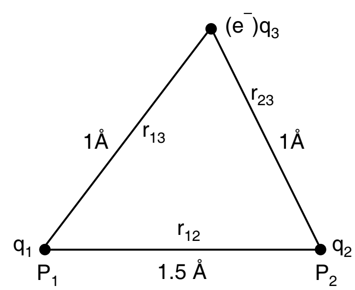


Fig. 2.26

Given,

$$r_{12} = 1.5\text{Å} = 1.5 \times 10^{-10}\text{ m}$$
$$r_{13} = r_{23} = 1\text{Å} = 10^{-10}\text{ m}$$
$$q_1 = q_2 = 1.6 \times 10^{-19}\text{ C (protons)}$$
$$q_3 = -1.6 \times 10^{-19}\text{ C (electrons)}$$

Now, the potential energy of the system

$$W = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}}+\frac{q_1q_3}{r_{13}}+\frac{q_2q_3}{r_{23}}\right)$$

$$= 9\times10^9\left[\frac{1.6\times10^{-19}\times1.6\times10^{-19}}{1.5\times10^{-10}} + \frac{1.6\times10^{-19}\times(-1.6\times10^{-19})}{10^{-10}} + \frac{1.6\times10^{-19}\times(-1.6\times10^{-19})}{10^{-10}}\right]$$

$$W = \frac{1.6\times10^{-19}\times1.6\times10^{-19}\times9\times10^{9}}{10^{-10}}\left[\frac{1}{1.5}-\frac{1}{1}-\frac{1}{1}\right]$$

$$W = 1.6 \times 1.6 \times 9 \times 10^{-19-19+9+10}\left[\frac{2}{3}-2\right]$$

$$W = -2.56 \times 9 \times 10^{-19} \times \frac{4}{3} = -2.56 \times 12 \times 10^{-19}\ \text{J}$$

$$W = \frac{-2.56\times12\times10^{-19}}{1.6\times10^{-19}}\ \text{eV}$$

$$= -1.6 \times 12\ \text{eV}$$

$$= -19.2\ \text{eV}$$

2.20. *Two charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.*

Sol. Suppose that two connected conducting spheres of radii a and b possess charges q_1 and q_2 respectively. On the surface of the two spheres, the potential will be

$$V_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{a}$$

$$V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{b}$$

Till the potentials of two conductors become equal the flow of charges continue.

$$V_1 = V_2$$

or, $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{a} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{b}$$

or, $$\frac{q_1}{q_2} = \frac{a}{b}$$

Now, the electric field on the two spheres is given as

$$E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{a^2}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{b^2}$$

or, $$\frac{E_1}{E_2} = \frac{q_1}{q_2}\cdot\frac{b^2}{a^2} = \frac{a}{b}\cdot\frac{b^2}{a^2} = b/a$$

Therefore, $b : a$ is the ratio of the electric field of the first sphere to that of the second sphere. The surface charge densities of the two spheres are given as

$$\sigma_1 = \frac{q_1}{4\pi a^2}$$ (As the charges are distributed uniformly over the surfaces of conducting spheres)

$$\sigma_2 = \frac{q_2}{4\pi b^2}$$

$$\therefore \quad \frac{\sigma_1}{\sigma_2} = \frac{q_1}{q_2}\cdot\frac{b^2}{a^2} = \frac{a}{b}\cdot\frac{b^2}{a^2} = b/a$$

Therefore, the surface charge densities are *inversely related with the radii* of the sphere. The surface charge density on the sharp and pointed ends of a conductor is higher than on its flatter portion since a flat portion may be taken as a spherical surface of large radius and a pointed portion as that of small radius.

2.21. *Two charges $-q$ and $+q$ are located at points (0, 0, $-a$) and (0, 0, a), respectively.*

(a) What is the electrostatic potential at the points (0, 0, z) and (x, y, 0)?

(b) Obtain the dependence of potential on the distance r of a point from the origin when $r/a >> 1$.

(c) How much work is done in moving a small test charge from the point (5, 0, 0) to (−7, 0, 0) along the x-axis? Does the answer change if the path of the test charge between the same points is not along the x-axis?

Sol. (*a*) When the point $p(0, 0, z)$ is closer to charge $+q$ as shown in figure below. Electrostatic potential at the point p (0, 0, z) is given as follows: (Fig. 2.27)

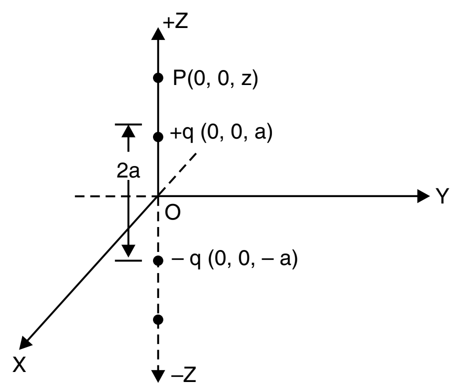


Fig. 2.27

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{z-a} + \frac{-q}{z-(-a)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{z+a-z+a}{z^2-a^2}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\cdot 2a}{z^2-a^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{z^2-a^2}$$

where $p = q.2a$ (dipole moment)

Electrostatic potential at the point $p(0, 0, z)$ is given as follows when the point is closer to charge $-q$ as figure 2.27.

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{+q}{(z+a)} + \frac{-q}{(z-a)}\right]$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{z-a-(z+a)}{z^2-a^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{z-a-z-a}{z^2-a^2}\right]$$

$$= -\frac{1}{4\pi\varepsilon_0}\cdot\frac{2qa}{z^2-a^2}$$

$$V = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{z^2-a^2}$$

So potential at point P $(0, 0, \pm z)$ is

$$V = \pm\frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{(z^2-a^2)}$$

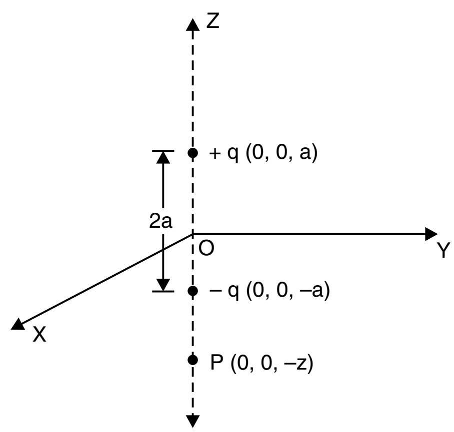


Fig. 2.28

Electric potential at point $(x, y, 0)$ is

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{(x-0)^2 + (y-0)^2 + (0-a)^2}} + \frac{-q}{\sqrt{(x-a)^2 + (y-0)^2 + (0+a)^2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{x^2+y^2+a^2}} - \frac{q}{\sqrt{x^2+y^2+a^2}}\right] = 0$$

(*b*) At a distance $OP = r$ from the origin electric potential

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{PA} - \frac{q}{PB}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{PA} - \frac{1}{PB}\right]$$

Now

$$\frac{1}{PA} = \frac{1}{\sqrt{x^2 + y^2 + (z-a)^2}}$$

$$= [x^2 + y^2 + z^2 - 2az + a^2]^{-1/2}$$

$$= [r^2 - 2az + a^2]^{-1/2} \qquad [\because \; r^2 = x^2 + y^2 + z^2]$$

$$= [r^2 - 2az]^{-1/2} \qquad [\because \; r^2 >>> a^2] \quad (\because a < r)$$

$$\frac{1}{PA} = \frac{1}{r}\left[1 - 2\frac{az}{r^2}\right]^{-1/2} = \frac{1}{r}\left[1 + \frac{az}{r^2}\right] \qquad \text{(Using Binomial theorem)}$$

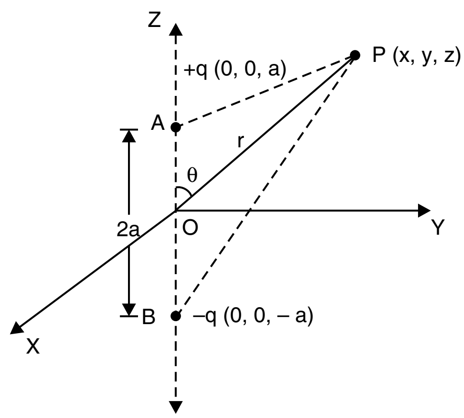


Fig. 2.29

Similarly,

$$\frac{1}{PB} = \frac{1}{\sqrt{x^2 + y^2 + (z+a)^2}} = \frac{1}{\sqrt{x^2 + y^2 + z^2 + a^2 + 2az}}$$

$$= \frac{1}{r}\left[1 - \frac{az}{r^2}\right] \qquad [\because \; x^2 + y^2 + z^2 = r^2 \text{ and } a^2 <<< r^2]$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\left(1 + \frac{az}{r^2}\right) - \frac{1}{r}\left(1 - \frac{az}{r^2}\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0} \cdot \frac{2az}{r^3}$$

$$V = \frac{P\text{r}\cos\theta}{4\pi\varepsilon_0 r^2} \qquad [\because \; p = q.2a,\; z = r\cos\theta]$$

θ = angle between r and $+z$ axis

$\therefore$ The dependence of potential V on r is $\frac{1}{r^2}$ type, like that of a point charge for $a << r$.

(*c*) Due to the dipole, electrostatic potential at point (5, 0, 0) is given as

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{(5-0)^2+(0-0)^2+(0-a)^2}} - \frac{q}{\sqrt{(5-0)^2+(0-0)^2+(0+a)^2}}\right] = 0$$

Due to dipole electrostatic potential at point (–7, 0, 0) is given as

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{(-7-0)^2+(0-0)^2+(0-a)^2}} - \frac{q}{\sqrt{(-7-0)^2+(0-0)^2+(0+a)^2}}\right] = 0$$

In moving small test charge q from the point (5, 0, 0) to (–7, 0, 0) the work done

$$W = q(V_1 - V_2) = q \times 0 = 0$$

As both points lie on X axis so, work done by any charge along X or Y axis or on equitorial line is zero as potential on the equitorial line does not change.

Since the work done by the electrostatic field between two points is not dependent on the path connecting the two points. Therefore the answer will not change if the test charge between the same point is not along X-axis.

2.22. *Figure shows a charge array known as an electric quadrupole. For a point on the axis of the quadrupole, obtain the dependence of potential on r for r/a >> 1, and contrast your results with that due to an electric dipole, and an electric monopole (i.e., a single charge).*

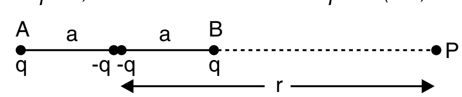


Fig. 2.30

Sol.

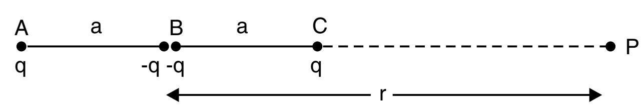


Fig. 2.31

Due to charge $+q$ at A, potential at P

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r+a}$$

Due to charge $-2q$ at B, potential at P

$$= \frac{1}{4\pi\varepsilon_0}\frac{-2q}{r}$$

Due to charge $+q$ at C, potential at P

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r-a}$$

Total electrostatic potential at P

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r+a} - \frac{2q}{r} + \frac{q}{r-a}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r(r-a)-2(r^2-a^2)+r(r+a)}{r(r+a)(r-a)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r^2 - \cancel{ar} - 2r^2 + 2a^2 + r^2 + \cancel{ar}}{r(r^2-a^2)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\,\frac{2a^2}{r(r^2-a^2)}$$

as $\frac{r}{a} >> 1$ *i.e.,* $r^2 >> a^2$, $r^2 - a^2 \to r^2$

Thus $$V = \frac{2a^2 q}{4\pi\varepsilon_0}\cdot\frac{1}{r^3}$$

i.e., $$V \propto \frac{1}{r^3} \text{ for a quadrupole}$$

but $$V \propto \frac{1}{r^2} \text{ for a dipole}$$

and $$V \propto \frac{1}{r} \text{ for a monopole}$$

2.23. *An electrical technician requires a capacitance of 2 μF in a circuit across a potential difference of 1 kV. A large number of 1 μF capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires the minimum number of capacitors.*

Sol. Let possible arrangement requires N capacitors of each 1 μF is n capacitors in series and m series arrangement in parallel

Total capacitors $N = m \times n$

As arrangement works on 1000 V.

P.D. across each capacitor in series arrangement is 400 V given

So $$\frac{1000}{n} = 400$$

$$n = 2.5$$

As number of capacitor cannot be in fraction $\therefore$ $n = 3$ equivalent capacitors in each row of series.

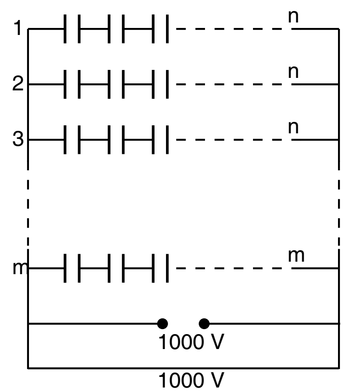


Fig. 2.32

$$\frac{1}{C_s} = \left[\frac{1}{n} + \frac{1}{n} + \ldots n \text{ lines}\right] = \frac{1}{C_s} = m$$

$$C_s = \frac{1}{n}\,\mu\text{F}$$

as the $\frac{1}{n}$ capacitors are in m rows so resultant capacitance of all capacitors equal to

$$\frac{1}{n}+\frac{1}{n}+\frac{1}{n}...+m \text{ lines} = 2$$

$$\frac{m}{n} = 2$$

$$\frac{m}{3} = 2$$

$$m = 6 \text{ rows}$$

$$n = 6 \times 3 = 18$$

2.24. *What is the area of the plates of a 2F parallel plate capacitor, given that the separation between the plates is 0.5 cm? [You will realise from your answer why ordinary capacitors are in the range of μF or less. However, electorlytic capacitors do have a much larger capacitance (0.1 F) because of very minute separation between the conductor.]*

Sol. Given, the separation between plates (d) = 0.5 cm = 5×10^{-3} m the capacitance (c) = 2F

Now,

$$C = \varepsilon_0 \frac{A}{d}$$

$$A = \frac{Cd}{\varepsilon_0} \qquad (\varepsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1})$$

$$= \frac{2\times5\times10^{-3}}{8.85\times10^{-12}}$$

or, $$A = 1.13 \times 10^9 \text{ m}^2$$

$$= 1.13 \times 10^3 \text{ km}^2 = 1130 \text{ km}^2$$

The area of plates should be in kilometres in order to get the capacitance in Farads. Therefore, the ordinary capacitors are in the range of μF.

2.25. *Obtain the equivalent capacitance of the network shown in figure below. For a 300 V supply, determine the charge and voltage across each capacitor.*

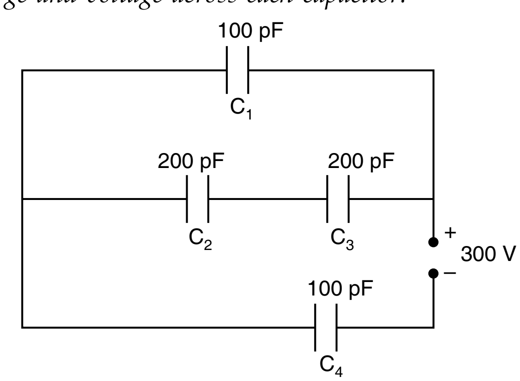


Fig. 2.33

Sol. A similar network is drawn below as given in the problem.

Here, $C_1 = C_4 = 100pF$

$C_2 = C_3 = 200pF$

Assume that the series combination of C_2 and C_3 is C_{23}.

$$\frac{1}{C_{23}} = \frac{1}{C_2}+\frac{1}{C_3} = \frac{1}{200}+\frac{1}{200} = \frac{1}{100}$$

or, $$C_{23} = 100 \text{ pF}$$

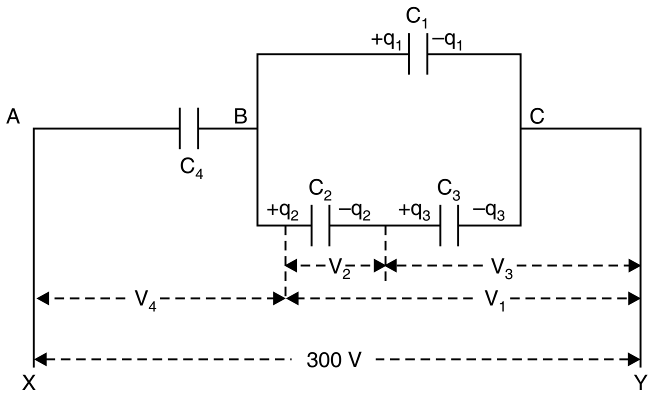


Fig. 2.34

Suppose that the parallel combination of C_1 and C_{23} is C' which is given as

$$C' = C_1 + C_{23} = 100 + 100 = 200 \text{ pF}$$

Let the series combination of C_4 and C' is C which is given as

$$\frac{1}{C} = \frac{1}{C_4}+\frac{1}{C'} = \frac{1}{100}+\frac{1}{200} = \frac{3}{200}$$

or, $$C = \frac{200}{3}\text{pF}$$

Total charge on all capacitors.

$$q = CV$$

$$= \frac{200}{\not{3}}\times 10^{-12}\times \overset{100}{\not{300}}$$

$$q = 2 \times 10^{-8} \text{ C}$$

$$V_4 = \frac{q_4}{C_4} = \frac{2\times 10^{-8}}{100\times 10^{-12}}$$

$$V_4 = 2 \times 100 = 200 \text{ V.}$$

$$V_1 = [300 - 200] = 100 \text{ V}$$

$$q_1 = C_1 V_1$$

$$q_1 = 100 \times 10^{-12} \times 100 = \mathbf{10^{-8}\ C}$$

$$V_2 = V_3$$

$$V_2 + V_3 = 100 \text{ V}$$

$$2V_2 = 100$$

$$V_2 = 50\text{V}$$

$$V_2 = V_S = 50\text{V}$$

$$q_2 = C_2V_2 = 200 \times 10^{-12} \times 50$$

$$q_2 = 10000 \times 10^{-12} = 10^{-8}\ \text{C}$$

$$q_3 = C_3V_3 = 200 \times 10^{12} \times 50 = 10^{-8}\ \text{C}$$

$V_1 = 100$ V $\quad q_1 = 10^{-8}$ C

$V_2 = 500$ V $\quad q_2 = 10^{-8}$ C

$V_3 = 50$ V $\quad q_3 = 10^{-8}$ C

$V_4 = 200$ V $\quad q_4 = 2 \times 10^{-8}$ C.

2.26. *The plates of a parallel plate capacitor have an area of 90 cm² each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply.*

(a) How much electrostatic energy is stored by the capacitor?

(b) View this energy as stored in the electrostatic field between the plates, and obtain the energy per unit volume u. Hence arrive at a relation between u and the magnitude of electric field E between the plates.

Sol. Given, $d = 2.5\ \text{mm} = 2.5 \times 10^{-3}\ \text{m}$

$V = 400$ V

$A = 90\ \text{cm}^2 = 90 \times 10^{-4}\ \text{m}^2$

(*a*) Stored electrostatic energy in the capacitor

$$\boxed{U = \frac{1}{2}CV^2} = \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2$$

$$= \frac{1}{2}\frac{\left(8.85\times10^{-12}\right)90\times10^{-4}\,(400)^2}{2.5\times10^{-3}}$$

$$= \frac{8.85\times90\times16}{2\times2.5}\times10^{-9}$$

$$= 2.55 \times 10^{-6}\ \text{J}$$

(*b*) Volume of the medium between the plates

$$= A \times d$$

$$= 90 \times 10^{-4} \times 2.5 \times 10^{-3}$$

$$= 225 \times 10^{-7}\ \text{m}^3$$

Per unit volume, energy stored $= \dfrac{2.55\times10^{-6}}{255\times10^{-7}}$

$$U = 0.113\ \text{Jm}^{-3}$$

Relation between E and U

$$U = \frac{U}{A\,.\,d} = \frac{\frac{1}{2}CV^2}{A\,.\,d} = \frac{1}{2}\,\frac{V^2}{A\,.\,d}\,\frac{\varepsilon_0 A}{d}$$

$$U = \frac{1}{2}\varepsilon_0\frac{V^2}{d^2}$$

$$\because \qquad E = \frac{V}{d}$$

$$U = \frac{1}{2}\varepsilon_0 E^2$$

2.27. *A 4 µF capacitor is charged by a 200V supply. It is then disconnected from the supply, and is connected to another uncharged 2 µF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation?*

Sol. When 4 µF capacitor is charged by 200V then charge on it is given as

$$Q = CV = 4 \times 10^{-6} \times 200 = 8 \times 10^{-4} \text{ C}$$

Now it is connected to another uncharged capacitor of capacitance 2 µF (2×10^{-6}F).

Thus, $\qquad (4 + 2) = 6$ µF

Until both the capacitor acquire a common potential charge on the first capacitor is shared between them.

After the combination, the common potential = Q/C

$$V' = \frac{8\times10^{-4}C}{6\times10^{-6}F} = 1.33 \times 10^2 \text{ V}$$

$$V' = 133 \text{ V}$$

Before the combination the electrostatic potential energy of the first capacitor

$$U_1 = \frac{1}{2}CV^2 = \frac{1}{2}\left(4\times10^{-6}\right)(200)^2$$

$$U_1 = 8 \times 10^{-2} \text{ J}$$

After the combination electrostatic potential energy of the system

$$U_2 = \frac{1}{2}C'V'^2 = \frac{1}{2}\left(6\times10^{-6}\right)(133)^2$$

$$U_2 = 5.30 \times 10^{-2} \text{ J}$$

Now, lost electrostatic energy by the first capacitor in the form of heat and electromagnetic radiation

$$U = U_1 - U_2 = (8 \times 10^{-2} - 5.3 \times 10^{-2}) = 2.7 \times 10^{-2} \text{ J}$$

2.28. *Show that the force on each plate of a parallel plate capacitor has a magnitude equal to* $\left(\frac{1}{2}\right)QE$, *where Q is the charge on the capacitor, and E is the magnitude of electric field between the plates. Explain the origin of the factor 1/2.*

Sol. Consider surface charge density of the capacitor σ and A as the plate area.

Now, $\qquad Q = \sigma A$

$$E = \sigma/\varepsilon_0 \quad \text{or} \quad \varepsilon_0 = \frac{\sigma}{E} \qquad \ldots(i)$$

If the separation of the capacitor plates is increased by a small distance Δx against the force F. Then, work done by the external agency = $F\cdot\Delta x$

Let u be the energy stored per unit volume or the energy density of capacitor, then *increase in the potential energy* of the capacitor

$$= u \times \text{increase in volume}$$
$$= u.\ A.\ \Delta x$$
$$F.\ \Delta x = uA.\ \Delta x$$
$$F = uA$$
$$= \left(\frac{1}{2}\varepsilon_0 E^2\right) \text{A}$$
$$\boxed{F = \frac{1}{2}\frac{\sigma}{E}E^2 A} \qquad \text{[From } (i)\text{]}$$
$$= \frac{1}{2}\sigma AE$$
$$= \frac{1}{2}QE$$

Therefore, the origin of the factor $\frac{1}{2}$ lies in the fact that field is zero just outside the conductor and it is E inside. Hence, the average value $E/2$ contributes to the force.

2.29. *A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports. Show that the capacitance of a spherical capacitor is given by*

$$C = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$$

where r_1 and r_2 are the radii of outer and inner spheres respectively.

Sol. As a charge – Q is introduced on the inner sphere of radius r_2, it is distributed uniformly on its outer surface. A charge + Q is induced on the outer surface of spherical shell of radius r_1 and Q is induced on its inner surface. The positive charge of the outer surface of shell flows to earth as it is earthed.

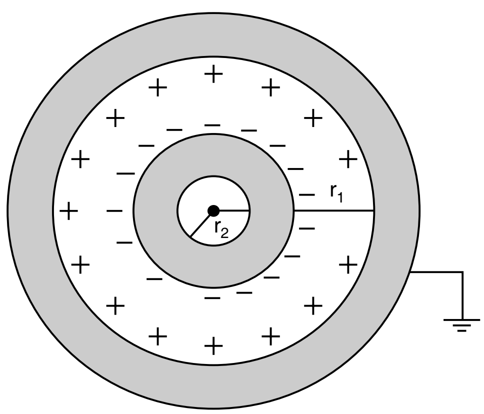


Fig. 2.35

Electric field inside sphere of radius r_2 is zero due to electrostatic shielding.

$$E = 0 \text{ for } r < r_2$$
$$E = 0 \text{ for } r > r_1$$

Electric field exists in between and is directed radially outward.

Electrostatic potential of inner sphere of radius r_2

$$V = \frac{1}{4\pi\varepsilon_0}\left\{\frac{Q}{r_2}-\frac{Q}{r_1}\right\} = \frac{Q}{4\pi\varepsilon_0}\left\{\frac{1}{r_2}-\frac{1}{r_1}\right\}$$

Potential of outer spherical shell = 0

Potential difference = $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_2}-\frac{1}{r_1}\right]$

If C is the capacitance of spherical capacitor

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_2}-\frac{1}{r_1}\right]}$$

$$\boxed{C = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}}$$

2.30. *A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 μC. The space between the concentric spheres is filled with a liquid of dielectric constant 32.*

(a) Determine the capacitance of the capacitor.

(b) What is the potential of the sphere?

(c) Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain why the latter is much smaller.

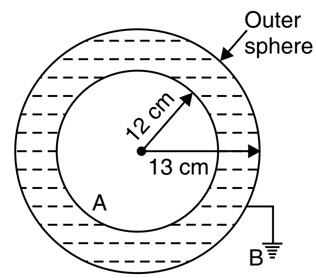


Fig. 2.36

Sol. Given,

r_1 = 12 cm = 12×10^{-2} *m*

r_2 = 13 cm = 13×10^{-2} *m*

q = 2.5 μC = 2.5×10^{-6} C

k = 32

(*a*) From formula,

$$C = kC_0$$

$$\boxed{C = k.4\pi\varepsilon_0 \frac{r_1 r_2}{r_2 - r_1}}$$

$$= \frac{32\times13\times10^{-2}\times12\times10^{-2}}{9\times10^{9}\left(13\times10^{-2}-12\times10^{-2}\right)} \qquad \left[\because 4\pi\varepsilon_0 = \frac{1}{9\times10^9}\right]$$

$$= \frac{32\times13\times12}{9}\times10^{-11} = \frac{1644}{3}\times10^{-11}$$

$$= 5.54 \times 10^{-9} \text{ F}$$

(*b*) Potential of inner sphere,

$$V = \frac{q}{C} = \frac{2.5\times10^{-6}}{5.54\times10^{-9}} = 4.5 \times 10^{2} \text{ V}$$

(*c*) Capacitance of sphere

$$= 4\pi\varepsilon_0$$

$$= \frac{12\times10^{-2}}{9\times10^{9}} = 1.33 \times 10^{-11} \text{ F}$$

Total potential in case of concentric spheres is distributed over two spheres and the potential difference between the two spheres becomes smaller that is why the capacitance of an isolated shpere is much smaller than that of concentric spheres. Since the capacitance is inversely proportional to the potential difference ($C = Q/V$).

2.31. *Answer carefully:*

(a) *Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other. Is the magnitude of electrostatic force between them exactly given by $Q_1 Q_2/4\pi\varepsilon_0 r^2$, where r is the distance between their centres?*

(b) *If Coulomb's law involved $1/r^3$ dependence (instead of $1/r^2$), would Gauss's law be still true?*

(c) *A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the field line passing through that point?*

(d) *What is the work done by the field of a nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?*

(e) *We know that electric field is discontinuous across the surface of a charged conductor. Is electric potential also discontinuous there?*

(f) *What meaning would you give to the capacitance of a single conductor?*

(g) *Guess a possible reason why water has a much greater dielectric constant (= 80) than say, mica (= 6).*

Sol. (*a*) Since the spheres are brought closer, the charge distribution will not remain uniform. Thus the magnitude of electrostatic force between them cannot be given by $Q_1Q_2/4\pi\varepsilon_0 r^2$ as they are not exactly point charges.

(*b*) No, because solid angle

$$d\omega = \frac{ds\cos\theta}{r^2} \text{ and } \neq \frac{ds\cos\theta}{r^3}$$

(*c*) Not definitely. If the field line is a straight line then only the small test charge will move along the line of force. The direction of velocity is not given by line of force, it gives direction of acceleration.

(*d*) Whatsoever be the shape of the orbit work done is always zero because electron will be in the same energy state after it completes an orbit.

(*e*) No, since electric potential is a scalar quantity, it is continuous everywhere.

(*f*) The behaviour of a single conductor is like a capacitor with one plate at infinity. ''The charge required to raise the potential of the conductor by a unit amount is termed as the capacitance of a single conductor''.

(*g*) Water has a much greater dielectric constant than mica because of the high degree of association of water molecules each of which has a permanent dipole moment of about 0.6×10^{-29} cm.

2.32. *A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e., bending of field lines at the ends).*

Sol. Given,

$$q = 3.5\ \mu C = 3.5 \times 10^{-6}\ C$$
$$a = 1.4\ cm = 1.4 \times 10^{-2}\ m$$
$$b = 1.5\ cm = 1.5 \times 10^{-2}\ m$$
$$l = 15\ cm = 15 \times 10^{-2}\ m$$

$$C = \frac{2\pi\varepsilon_0 l}{2.303\log_{10}(b/a)}$$

$$= \frac{2\times\pi\times 8.854\times10^{-12}\times15\times10^{-2}}{2.303\log_{10}\frac{1.5\times10^{-2}}{1.4\times10^{-2}}}$$

$$C = 1.21\times10^{-10}\text{ F}$$

The potential of inner cylinder will be equal to the potential difference between inner and outer cylinder as outer cylinder is earthed.

Hence, potential of inner cylinder

$$V = \frac{q}{C} = \frac{3.5\times10^{-6}}{1.21\times10^{-10}} = 2.89\times10^{4}\text{ V.}$$

2.33. *A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about 10^7 Vm^{-1}. (Dielectric strength is the maximum electric field a material can tolerate without breakdown, i.e., without starting to conduct electricity through partial ionisation.) For safety, we should like the field never to exceed, say 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 pF?*

Sol. Given $V = 1\text{ kV} = 1000\text{ V}$

$K = \varepsilon_r = 3$

Dielectric strength = 10^7 V/m

Due to reasons of safety, electric field at the most should be 10% of dielectric strength.

E = 10% of 10^7 V/m = 10^6 V/m

A = ?

$C = 50\ pF = 50\times10^{-12}$ F

As $E = \frac{V}{d}$

$\therefore$ $d = \frac{V}{E} = \frac{10^3}{10^6} = 10^{-3}$ m

Now, $C = \frac{\varepsilon_0\varepsilon_r A}{d}$

$\therefore$ $A = \frac{Cd}{\varepsilon_0\varepsilon_r} = \frac{50\times10^{-12}\times10^{-3}}{8.85\times10^{-12}\times3}$

or, $A = 1.9\times10^{-3}$ m^2

2.34. *Describe schematically the equipotential surfaces corresponding to*

(a) a constant electric field in the z-direction,

(b) a field that uniformly increases in magnitude but remains in a constant (say, z) direction,

(c) a single positive charge at the origin, and

(d) a uniform grid consisting of long equally spaced parallel charged wires in a plane.

Sol. The surfaces where the potential has a constant value are called equipotential surfaces.

(*a*) When an electric field acting in z-direction is constant, the potential in a direction perpendicular to z-axis remains constant. Therefore, equipotential surface is represented by the planes parallel to x-y plane.

(*b*) The answer is same as (*a*) since the potential in a direction perpendicular to the direction of field remains constant irrespective of the magnitude of the field.

(*c*) The equipotential surfaces are concentric sphere centered at the origin for a single positive charge at the origin. By a constant potential increases with increase in distance from the origin, the separation between the equipotentials differing.

(*d*) Near the grid, the equipotential surfaces are of periodically varying shape which gradually reach the shape of planes parallel to the grid at per distance.

2.35. *In a Van de Graaff type generator a spherical metal shell is to be a 15 × 10⁵ V electrode. The dielectric strength of the gas surrounding the electrode is 5 × 10⁷ Vm⁻¹. What is the minimum radius of the spherical shell required? (You will learn from this exercise why one cannot build an electrostatic generator using a very small shell which requires a small charge to acquire a high potential.)*

Sol. Dielectric strength of gas surrounding the electrodes $E = 5 \times 10^7$ V/m.

Potential of sphere $V = 15 \times 10^5$ V

Suppose radius of the shell = R

For spherical shell $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r}\right)$

As
$$V = \frac{kg}{R} \quad \text{...}(i)$$

$$E = \frac{kg}{R^2}$$

$$E = \frac{V}{R} \quad \text{[From } (i)\text{]}$$

$$R = \frac{V}{E} = 0.3 \text{ m} = 30 \text{ cm}$$

$$R = \frac{15 \times 10^5}{10\% \text{ of } 5 \times 10^7}$$

$$R = \frac{15 \times 10^5}{\frac{10}{100} \times 5 \times 10^5 \times 100}$$

$$= \frac{3}{10} \text{ m}$$

$$R = 30 \text{ cm.}$$

2.36. *A small sphere of radius r_1 and charge q_1 is enclosed by a spherical shell of radius r_2 and charge q_2. Show that if q_1 is positive, charge will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge q_2 on the shell is.*

Sol. Potential of inner sphere due to its one charge

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}$$

Potential of inner sphere due to its presence inside the shell

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

Thus total potential of inner sphere = $V_1 + V_2$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}\right)$$

Potential of shell $V' = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$

Potential difference between inner sphere and shell = $V - V'$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}\right) - \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}$$

Thus, we can conclude that potential difference is independent of the charge q_2 on the shell. Potential of sphere is positive since q_1 is positive. From inner sphere to shell, charge (if positive) will always flow no matter whatsoever charge q_2 on the shell is.

2.37. *Answer the following:*

(a) The top of the atmosphere is at about 400 kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100 V m^{-1}. Why then do we not get an electric shock as we step out of our house into the open? (Assume the house to be a steel cage so there is no field inside!)

(b) A man fixes outside his house one evening a two metre high insulating slab carrying on its top a large aluminium sheet of area 1 m^2. Will he get an electric shock if he touches the metal sheet next morning?

(c) The discharging current in the atmosphere due to the small conductivity of air is known to be 1800 A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral? In other words, what keeps the atmosphere charged?

(d) What are the forms of energy into which the electrical energy of the atmosphere is dissipated during a lightning?

*[**Hint:** The earth has an electric field of about 100 V m^{-1} at its surface in the downward direction, corresponding to a surface charge density = -10^{-9} Cm^{-2}. Due to the slight conductivity of the atmosphere up to about 50 km (beyond which it is good conductor), about + 1800 C is pumped every second into the earth as a whole. The earth, however, does not get discharged since thunderstorms and lightning occurring continually all over the globe pump an equal amount of negative charge on the earth.)*

Sol. (*a*) The surface of earth and the equipotential surface are parallel. Human body is a good conductor. The original equipotential surfaces of open air get modified as we step out into the open but keeping our head and ground at the same potential and we do not get any electric shock.

(*b*) Yes, the aluminium sheet gradually is charged up by the steady discharging current in the atmosphere and raises its voltage to an extent depending on the capacitance of the capacitor (formed by the sheet, slab and the ground).

(*c*) Thunderstorms and ground lightning all over the globe charge the atmosphere continually and discharged through regions of ordinary weather. The two opposing currents are, on an average, in equilibrium.

(*d*) Electrical energy of the atmosphere is dissipated as

(*i*) light energy involved in lightning.

(*ii*) heat and sound energy in accompanying thunder.

MORE QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *Why does the electric field inside a dielectric decrease when it is placed in an external electric field?*

Ans. It is because the dielectric gets polarised in opposite direction.

Q. 2. *No work is done in moving test charge over an equipotential surface. Why?*

Ans. $W = q \times \Delta V$. But $\Delta V = 0$ for an equipotential surface.

$$\therefore \qquad W = 0$$

Q. 3. *What is the work done in moving a test charge q through a distance of 1 cm along the equatorial axis of an electric dipole?*

Ans. As $\qquad V_{eq} = 0$

Work done in moving a positive test charge q through a distance 1 cm is

$$W = qV = q \times 0 = 0$$

Q. 4. *What is the shape of the equipotential surfaces for a uniform electric field?*

Ans. The equipotential surfaces are perpendicular to the direction of electric field.

Q. 5. *Define the term 'potential energy' of charge 'q' at a distance 'r' in an external electric field.*

Ans. Potential energy of a single charge q at a distance 'r' in an external field = $q.V(r)$.

Q. 6. *What is the shape of the equipotential surfaces for an isolated point charge?*

Ans. The equipotential surfaces are concentric spheres whose centres are located at the given point charge.

Q. 7. *A 500 μC charge is at the centre of a square of side 10 cm. Find the work done in moving a charge of 10 μC between two diagonally opposite points on the square.*

Ans. Zero [$\because W = q(V_B - V_A)$ as $V_A = V_C$]

Q. 8. *What is the order of capacitances used in power supplies?*

Ans. It is usually 1μ*F* to 10 μ*F*.

Q. 9. *Define the term 'dielectric constant' of a medium in terms of capacitance of a capacitor.*

Ans. The ratio of the capacitance of the capacitor with the dielectric as the medium to its capacitance with vacuum between its plates is termed as 'dielectric constant'.

Q. 10. *Where does the energy of a capacitor reside?*

Ans. The energy resides in the dielectric medium separating the two plates.

Q. 11. *What is meant by capacitance? Give its SI unit.*

Ans. The ratio of the electric charge on capacitor to its electric potential due to the charge is termed as the capacitance of a conductor. The SI unit of capacitance is Farad.

Q. 12. *Define electric potential at a point in an electric field.*

Ans. Electric potential at a point in an electric field is the amount of work done in moving a unit positive charge from infinity to that point against the electrostatic forces, along any path with acceleration zero.

Q. 13. *What will be the effect on capacity of a parallel plate condenser when area of each plate is doubled and distance between them is also doubled?*

Ans. It will remain unaffected.

Q. 14. *Write the formula for capacity of a parallel plate air capacitor with a metal sheet of thickness t in between the plates.*

Ans. $C = \dfrac{\epsilon_0 A}{d - t}$

Q. 15. *What are the expressions for energy of a charged capacitor?*

Ans. $E = \dfrac{1}{2}QV = \dfrac{1}{2}\dfrac{Q^2}{C} = \dfrac{1}{2}CV^2$

Q. 16. *A capacitor is charged through a potential difference of 200 V, when 0.1 C charge is stored in it. How much energy will it release, when it is discharged?*

Ans. Given, V = 200 volt, q = 0.1 C

Energy released on discharging = energy stored on charging = $\dfrac{1}{2}qV$

$$= \frac{1}{2} \times 0.1 \times 200 = 10\ J.$$

Q. 17. *On inserting a dielectric between the plates of a capacitor, its capacitance is found to increase 5 times. What is the relative permittivity of the dielectric?*

Ans.

$$\boxed{\varepsilon_r = k = \frac{C}{C_0}} = 5$$

Q. 18. *Write an expression for potential $V(\vec{r})$ at a point due to two point charges q_1 and q_2 at position $\vec{r}_1$ and $\vec{r}_2$ respectively.*

Ans.

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{|\vec{r} - \vec{r}_1|} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{|\vec{r} - \vec{r}_2|}$$

Q. 19. *Does the electric potential increase or decrease along the electric line of force?*

Ans. It decreases.

Q. 20. *Is electric potential at any point in space necessarily zero if intensity of electric field at that point is zero?*

Ans. No. At a point mid-way between two equal and similar charges, the electric field strength is zero but the electric potential is not zero.

Q. 21. *Is potential difference a scalar or a vector?*

Ans. It is a scalar.

Q. 22. *An air capacitor is given a charge of 2 μC raising its potential to 200 V. If on inserting a dielectric medium, its potential falls to 50 V, what is the dielectric constant of the medium?*

Ans. When dielectric is introduced, the potential difference between the plates of capacitor decreases by a factor k, the dielectric constant

Thus, $\boxed{k = \dfrac{V}{V'}} = \dfrac{200}{50} = 4.$

Q. 23. *Do electrons tend to go to regions of low potential or high potential?*

Ans. Electrons, being negatively charged, tend to go to regions of high potential. This reduces their potential energy.

Q. 24. *Name the physical quantity which has its unit joule coulomb^{-1}. Is it a scalar or a vector?*

Ans. It is the unit of electric potential. It is a scalar.

Q. 25. *What meaning would you give to capacity of a single conductor?*

Ans. A single conductor can be visualised as a capacitor whose second plate is far away at infinity.

Q. 26. *A parallel plate capacitor has a capacity of 6 μ F in air and 60 μF when dielectric medium is introduced. What is dielectric constant of medium?*

Ans. $K = \frac{C_m}{C_0} = \frac{60}{6} = 10$.

Q. 27. *Can there be a potential difference between two conductors of same volume carrying equal positive charges?*

Ans. Yes, because two conductors of same volume *may have different shapes* and hence different capacitances.

Q. 28. *On which factors does the capacitance of a capacitor depend?*

Ans. It depends on *geometry* of the plates, *distance* between them and nature of *dielectric* medium separating the plates.

Q. 29. *How many picofarads are there in a farad?*

Ans. $1F = 10^{12}\ pF$

Q. 30. *What is the net charge on a charged capacitor?*

Ans. Zero, because one plate has positive charge and the other carries an equal negative charge.

Q. 31. *If the plates of a charged capacitor be suddenly connected to each other by a wire, what will happen?*

Ans. The capacitor will be discharged immediately.

Q. 32. *What are the dimensions of capacitance?*

Ans. $[M^{-1} L^{-2} A^2 T^4]$.

Q. 33. *Sketch a graph to show how charge Q given to a capacitor of capacitance C varies with the potential difference V.*

Ans.

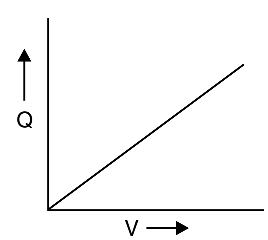


Fig. 2.37

Q. 34. *What is the function of a dielectric in a capacitor?*

Ans. The dielectric increases the capacitance of the capacitor.

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. (*i*) *Can two equipotential surfaces intersect each other? Give reasons.*

(*ii*) *Two charges – q and + q are located at points A (0,0, – a) and B (0,0, + a) respectively. How much work is done in moving a test charge from point P (7, 0, 0) to Q (– 3, 0, 0)?*

Ans. (*i*) No, if two equipotential surfaces intersect then at the point of intersection, there will be two directions of electric field intensity which is not possible.

(*ii*) Potential at P (7, 0, 0) is $V = \frac{kg}{r}$

$$V_1 = \frac{-q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{(7-0)^2+0+(-a-0)^2}} + \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{(7-0)^2+0+(a-0)^2}}$$

or
$$V_1 = \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{49+a^2}}+\frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{49+a^2}} = 0$$

Potential at Q (– 3, 0, 0) is

$$V_2 = \frac{-q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{(-3-0)^2+\left(-a^2\right)}}+\frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{(-3-0)^2+(a)^2}}$$

$$= \frac{-q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{9+a^2}}+\frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{9+a^2}} = 0$$

work done $= q\,(V_2 - V_1) = q\,(0 - 0) = 0$

Thus, $W = 0$.

Q. 2. *Two charges of 5nC and – 2nC, are placed at points (5 cm, 0, 0) and (23 cm, 0, 0) in a region of space. Where there is no other external field. Calculate the electrostatic potential energy of this charge system.*

Ans. Given,

$$q_1 = 5n\text{C} = 5 \times 10^{-9}\ \text{C}$$
$$q_2 = -\,2n\text{C} = -\,2 \times 10^{-9}\ \text{C}$$

and $r = 23 - 5 = 18$ cm $= 0.18$ m

∴ Electrostatic potential energy of the system is

$$\boxed{U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r}}$$

$$= \frac{9\times10^9\times5\times10^{-9}\times(-2)\times10^{-9}}{0.18}$$

$$U = \frac{-9\times5\times2\times10^{-9}\times10^2}{18} = -5\times10^{-7}\ \text{J.}$$

Q. 3. *A parallel plate capacitor, each with plate area A and separation d, is charged to a potential difference V. The battery used to charge it is then disconnected. A dielectric slab of thickness d and dielectric constant k is now placed between the plates. What change, if any, will take place in*

(i) charge on the plates,

(ii) electric field intensity between the plates,

(iii) capacitance of the capacitor.

Justify your answer in each case.

Ans. (*i*) The charge q on the capacitor plates remains same.

(*ii*) Due to the surface charges induced on the dielectric, the electric field intensity between the capacitor plates decreases.

$$E = \frac{E_0}{K}$$

(*iii*) The capacitance increases due to the decrease in potential difference

$$C = k\,C_0$$

Q. 4. *A 5 μF capacitor is charged by a 100 V supply. The supply is then disconnected and the charged capacitor is connected to another uncharged 3 μF capacitor. How much electrostatic energy of the first capacitor is lost in the process of attaining the steady situation?*

Ans. Initial energy stored in 5 μ*F* capacitor,

$$U_i = \frac{1}{2}CV^2 = \frac{1}{2}\times 5\times 10^{-6}\times(100)^2$$
$$= 2.5\times 10^{-2}\text{ J.}$$

Charge on 5 m*F* capacitor $= 5\times 10^{-6}\times 100$ [∵ $q = CV$]

$= 5\times 10^{-4}$ C.

The two capacitors attain a common potential V when they are connected together.

$$V = \frac{\text{total charge}}{\text{total capacitor}} = \frac{5\times 10^{-4}}{(5+3)\times 10^{-6}} = \frac{125}{2}\ V.$$

Final energy of the combination,

$$U_f = \frac{1}{2}\times(5+3)\times 10^{-6}\times\left(\frac{125}{2}\right)^2 = 1.56\times 10^{-2}\text{ J}$$

Electrostatic energy lost in the process of attaining the steady state

$$= U_i - U_f = (2.5 - 1.56)\times 10^{-2}$$
$$= 0.94\times 10^{-2}\text{ J.}$$

Q. 5. *The given graph shows the variation of charge q versus potential difference for two capacitors C_1 and C_2. The two capacitors have same plate separation, but the plate area of C_2 is double that of C_1. Which of the lines in the graph corresponds to C_1 and C_2 and why?*

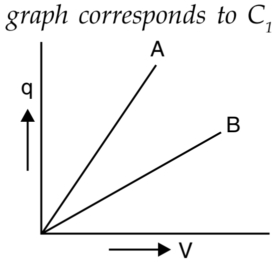


Fig. 2.38

Ans. As $C = \frac{\epsilon_0 A}{d}$, and plate area of C_2 is double than that of C_1, therefore, $C_2 > C_1$. Now

$C = \frac{q}{V}$, which is greater for line A.

$\therefore$ The line A of the graph corresponds to C_2 and line B corresponds to C_1.

Q. 6. *The battery remains connected to a parallel plate capacitor and a dielectric slab is inserted between the plates. What will be the effect on its (i) capacity (ii) charge, (iii) potential difference (iv) electric field, (v) energy stored?*

Ans. When battery remains connected, potential difference V remains constant. Capacity C increases. $Q = CV$ increases as C increase by inserting dielectric slab.

Electric field $E = \frac{E_0}{K} = \frac{V_0}{K\,d}$ will decrease. Energy stored $= \frac{1}{2}CV^2$ increases.

Q. 7. *Given a battery, how would you connect two capacitors, in series or in parallel for them to store the greater (a) total charge (b) total energy?*

Ans. Total charge $q = CV$, and total energy $U = \frac{1}{2}CV^2$. Now, V is constant and $C_p > C_s$,

therefore, parallel combination is required for storing greater charge and greater energy.

Q. 8. *The space between the plates of a parallel plate capacitor is filled consecutively with two dielectric layers of thickness d_1 and d_2 having relative permittivities ϵ_1 and ϵ_2 respectively. If A is area of each plate, what is the capacity of the capacitor?*

Ans. As effective thickness of dielectric is $1/\epsilon$ times the actual thickness, therefore, we may write the capacity of condenser as

$$\boxed{C = \frac{\epsilon_0 A}{\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_1}}}$$

Q. 9. *The separation between the plates of a charged capacitor is to be increased. Explain when work done will be more in case battery is removed after charging the capacitor or battery remains connected.*

Ans. With increase in separation between the two plates, capacity C decreases.

When battery is removed, charge Q and electric field $E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$ = would remain constant. But when battery remains connected, V is constant, Q (= CV) decreases and hence E decreases. Clearly, more work is required to be done in the first case.

Q. 10. *An electric dipole of length 4 cm, when placed with its axis making an angle of 60° with a uniform electric field experiences a torque of $4\sqrt{3}$ Nm. Calculate the*

(i) magnitude of the electric field.

(ii) potential energy of the dipole, if the dipole has charges of ± 8nC.

Ans. (*i*) $\boxed{\tau = pE \sin\theta} = q \times 2a \times E \sin\theta$

$$4\sqrt{3} = 8 \times 10^{-9} \times 4 \times 10^{-2} \times E \times \sin 60°$$

$$E = \frac{4\sqrt{3}\times 2}{32\times 10^{-11}\times\sqrt{3}} = \frac{1}{4}\times 10^{11} = 2.5\times 10^{10} \text{ NC}^{-1}$$

(*ii*)

$$U = -pE\cos\theta$$
$$= -q \times 2a \times E\cos\theta$$
$$= -8\times 10^{-9}\times 2\times 10^{-2}\times 2.5\times 10^{10}\cos 60°$$
$$= -2 \text{ J.}$$

Q. 11. *In which of the following two cases, more work will be done in increasing the separation between the plates of a charged capacitor and why?*

(i) The charging battery remains connected to the capacitor.

(ii) The battery is removed after charging the capacitor.

Ans. In both cases there is a decrease in capacitance. In the first case, V is constant. Since $Q = CV$, thus Q and E decrease. However, in second case charge Q and consequently the electric field $\left(\frac{Q}{\epsilon_0 A}\right)$ would remain constant. Clearly, more work is required to be done in the second case.

Q. 12. *In the above question, if battery is removed after charging the condenser and dielectric slab introduced how are all the five parameters affected?*

Ans. When battery is removed, charge Q remains constant, capacity C increases. Potential difference $V = Q/C$ decreases. Electric field $E = \frac{V}{d}$ decreases. Energy stored $= \frac{1}{2}\frac{Q^2}{C}$ decreases.

Q. 13. *What is an equipotential surface?*

A uniform electric field $\vec{E}$ of 300 NC^{-1} is directed along PQ. A, B and C are three points in the field having x and y coordinates (in metres) as shown in the figure. Calculate potential difference between the points

(i) A and B and (ii) B and C

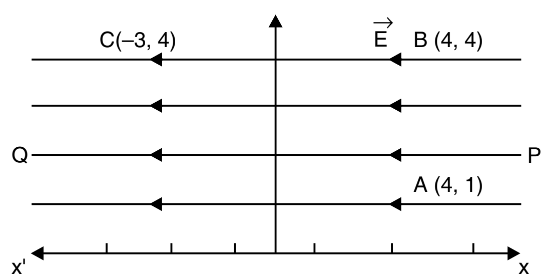


Fig. 2.39

Ans. Equipotential surface is defined as a surface that has same electric potential at its every point.

(*i*) No work is done in taking *a + ve* charge from A to B because the charge moves perpendicular to the electric field.

∴ Potential difference between A and B = 0 (*ii*) Since $E = -\frac{\Delta V}{\Delta x}$

potential difference between B and C

$$\Delta V = - E\, \Delta x$$
$$= - 300 \times 7 = - 2100 \text{ V}$$

Q. 14. *In a parallel plate capacitor, how is the capacity affected, when without changing the charge:*

(i) the distance between the plates is doubled;

(ii) the area of the plate is halved.

Ans. (*i*) $C \propto \frac{1}{d}$. When d is doubled, capacity is halved.

(*ii*) $C \propto A$. When A is halved, capacity is halved.

Q. 15. *n small drops of same size are charged to V volt each. They collapse to form a bigger drop. Calculate the capacity and potential of the bigger drop.*

Ans. If r and R represent the radii of the small drop and the bigger drop respectively, then

$$\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3 \text{ or } R = n^{1/3} r$$

Since *capacity* α *radius*, therefore, the capacity of the bigger drop is $n^{1/3}$ times the capacity of each small drop.

Potential of bigger drop $V = \frac{kg}{R}$

$$= \frac{nq}{4\pi\varepsilon_0 R} = \frac{nq}{4\pi\varepsilon_0 n^{1/3} r} = n^{2/3} \frac{a}{4\pi\varepsilon_0 r}$$

So, the potential of the bigger drop is $n^{2/3}$ times the potential of the smaller drop.

Q. 16. *Two dielectric slabs of dielectric constants k_1 and k_2 are filled in between the two plates, each of area A, of the parallel plate capacitor as shown in the figure. Find the net capacitance of the capacitor.*

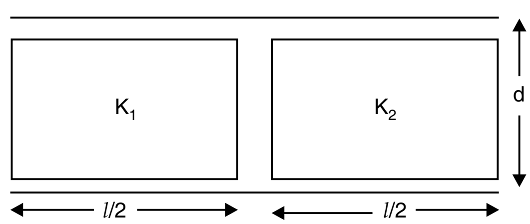


Fig. 2.40

Ans. The arrangement is equivalent to a parallel combination of two capacitors, each with plate area $A/2$ and separation d.

Thus the net capacitance

$$C = C_1 + C_2$$
$$= \frac{\varepsilon_0 (A/2) k_1}{d} + \frac{\varepsilon_0 (A/2) k_2}{d}$$
$$= \frac{\varepsilon_0 A (k_1 + k_2)}{2d}$$

Q. 17. *Two protons A and B are placed between two parallel plates having a potential difference V as shown in figure. Will these protons experience equal or unequal force?*

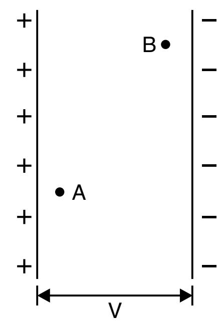


Fig. 2.41

Ans. The electric field betwen the two plates of a parallel plate capacitor is same at all points (except at the edges). So, both the protons *A* and *B* experience equal forces.

Q. 18. *Capacitors P, Q and R have each a capacitance C. A battery can charge the capacitor to a potential difference V. If after charging P, the battery is disconnected from it and the charged capacitor P is connected in following separate instances to Q and R (i) to Q in parallel and (ii) to R in series, then what will be potential differences between the plates of P in the two instances?*

Ans. When capacitor *P* is charged to potential difference *V*, the charge acquired by the capacitor is *CV*.

(*i*) When *P* is connected to *Q* in parallel, then potential difference across each capacitor will be $\frac{CV}{2C}$ or $\frac{V}{2}$.

(*ii*) When *P* is connected to *R* in series, the sharing of charges shall not take place because circuit is not completed. So, potential difference between the plates of *P* shall remain *V*.

Q. 19. *Two point charges 4Q, Q are separated by 1 m in air. At what point on the line joining the charges is the electric field intensity zero?*

Also calculate the electrostatic potential energy of the system of charges, taking the value of charge, $Q = 2 \times 10^{-7}$ *C.*

Ans.

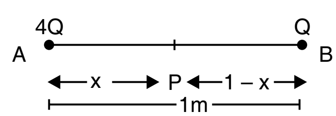


Fig. 2.42

Suppose that the point be at a distance x from 4*Q* charge.

Electric field at *P* due to charge 4*Q* = Electric field at *P* due to *Q* $\boxed{E = \frac{kg}{r}}$

$$\therefore \quad K \times \frac{4Q}{x^2} = K \times \frac{Q}{(1-x)^2}$$

$$\frac{4}{x^2} = \frac{1}{(1-x)^2}$$

$$\Rightarrow \quad \frac{2}{x} = \pm \frac{1}{1-x}$$

$$\frac{2}{x} = \frac{1}{1-x} \quad \text{or,} \quad \frac{2}{x} = -\frac{1}{1-x}$$

$$x = 2 - 2x \quad \text{or,} \quad -x = 2 - 2x$$

$$x + 2x = 2 \quad \text{or,} \quad -x + 2x = 2$$

$$3x = 2 \quad \text{or,} \quad x = 2$$

$$x = \frac{2}{3}$$

$$\because \quad x = 2\text{m (impossible)}$$

Thus, $x = \frac{2}{3}\text{m}$

Electrostatic potential energy of the system is given as

$$\boxed{U = K\frac{q_1 q_2}{r}}$$

or, $U = K.\frac{4Q \cdot Q}{r} = K \cdot \frac{4Q^2}{r}$

$$U = 9\times10^9 \times \frac{4\times\left(2\times10^{-7}\right)^2}{1}$$

$$= 9\times10^9 \times \frac{4\times4\times10^{-14}}{1}$$

$$= 144 \times 10^{-5} = 1.44 \times 10^{-3} \text{ J.}$$

Q. 20. *A parallel plate capacitor with air as dielectric is charged by a d.c. source to a potential V. Without disconnecting the capacitor from the source, air is replaced by another dielectric medium of dielectric constant K. State with reason, how does*

(i) potential difference

(ii) electric field between the plates

(iii) capacity

(iv) charge and

(v) energy stored in the capacitor change.

Ans. (*i*) Since the capacitor remains connected to battery therefore the potential difference would remain unchanged.

(*ii*) Since neither the potential difference nor the separation between the plates is changed therefore the electric field remains unchanged.

(*iii*) Capacity would increase by a factor of K. So, $\boxed{C = KC_0.}$

(*iv*) Since C is increased by a factor of K and V remains unchanged therefore Q is increased by a factor of K. Additional charge flows from the battery to the plates.

(*v*) $U_0 = \frac{1}{2}C_0V_0^2, U = \frac{1}{2}(KC_0)V^2 = KU_0$. So, the energy is increased by a factor of K.

Q. 21. *Electric charge is distributed uniformly on the surface of a spherical rubber balloon. Show how the value of electric intensity and potential vary (i) on the surface, (ii) inside and (iii) outside?*

Ans. On the surface E = constant, V = constant. Inside the surface, $E = 0$, V = constant = potential on surface. Outside the balloon,

$$\boxed{E \propto \frac{1}{r^2}} \quad \text{and} \quad \boxed{V \propto \frac{1}{r}}$$

where r is distance of point from the centre of the balloon.

Q. 22. *Distinguish between electric potential and potential energy.*

Ans. Electric potential at a point is the amount of work done in moving a unit positive charge with zero acceleration from infinity to that point. Potential energy is the energy possessed by the charge by virtue of its particular position. It is equal to amount of work done in carrying the total charge from infinity to that position, against the electrostatic forces. Thus potential energy = potential × charge.

Q. 23. *Suggest an arrangement of three point charges separated by finite distances that has zero electric potential energy.*

Ans. The arrangement of three point charges $-q$, $+q$ and $+q$ shown in figure has total potential energy

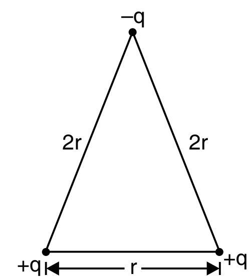


Fig. 2.43

$$U = k\frac{q(q)}{r} + \frac{k(q)(-q)}{(2r)} + \frac{k(-q)(q)}{(2r)}$$

$$U = \text{zero.}$$

Q. 24. *Define the term electric potential due to a point charge. Calculate the electric potential at the centre of a square of side $\sqrt{2}$ m, having charge 100 μC, – 50 μC, 20 μC and – 60 μC at the four corners of the square.*

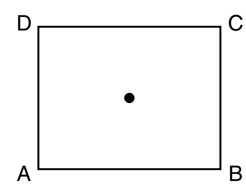


Fig. 2.44

Ans. The amount of work required to carry a unit positive charge with zero acceleration from infinity to a point in electric field against the electrostatic forces is termed as electric potential at that point.

Diagonal of the square $= \sqrt{\left(\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2} = 2$ m

$$r = \text{Half diagonal} = 1 \text{ m}$$

Potential at the centre of the square, $V = \dfrac{kg}{r}$

$$V = 9\times10^9\left[\frac{100\times10^{-6}}{1} - \frac{50\times10^{-6}}{1} + \frac{20\times10^{-6}}{1} - \frac{60\times10^{-6}}{1}\right]$$

$$= 9 \times 10^9 \times 10^{-6} \times 10$$

$$V = 9 \times 10^4 \text{ V.}$$

III. LONG ANSWER TYPE QUESTIONS

Q. 1. *A dielectric slab of thickness 't' is kept in between the plates, each of area 'A', of a parallel plate capacitor separated by a distance 'd', Derive an expression for the capacitance of this capacitor for $t << d$.*

Ans. Let A is the area of the two plates of the parallel plate capacitor and d is the separation between them. A dielectric slab of thickness $t < d$ and area A is kept between the two plates. The total electric field inside the dielectric slab will be,

$$E = \frac{E_0}{K} = E_0 - E',$$

where E' is the opposite field developed inside the slab due to polarisation of slab.

Total potential difference between the plates,

$$V = E_0 (d - t) + Et$$

$$= \frac{\sigma}{\varepsilon_0}(d-t) + \frac{\sigma}{k\varepsilon_0} t$$

$$= \frac{\sigma}{\varepsilon_0}\left[(d-t) + \frac{t}{k}\right]$$

$$V = \frac{q}{A\varepsilon_0}\left[(d-t) + \frac{t}{k}\right]$$

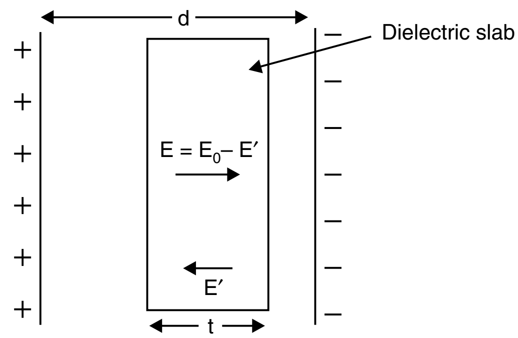


Fig. 2.45

where q is the charge on each plate.

Since, $C = \frac{q}{V}$

or, $C = \frac{q}{\frac{q}{A\varepsilon_0}\left[(d-t) + \frac{t}{k}\right]}$

or, $C = \frac{A\varepsilon_0}{\left[(d-t) + \frac{t}{k}\right]}$.

Q. 2. *Obtain an expression for the capacitance of a parallel plate (air) capacitor. The given figure shows a network of five capacitors connected to a 100 V supply. Calculate the total charge and energy stored in the network.*

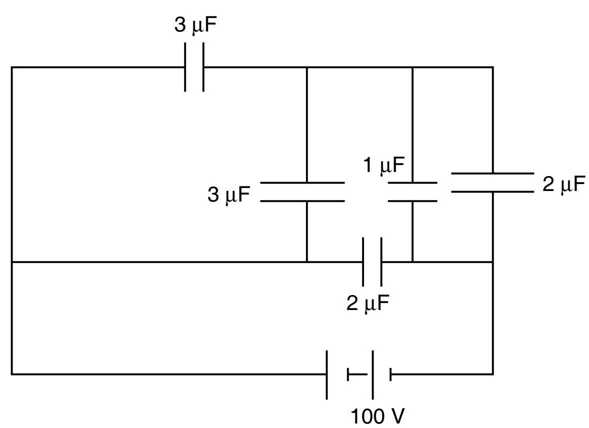


Fig. 2.46

Ans. An expression for capacitance of parallel plate capacitor. See page no. 77–78.

In the given figure the capacitors C_1 and C_2 in parallel combination

$\therefore \quad C_{12} = 1 + 2 = 3\ \mu F$

The circuit reduces to

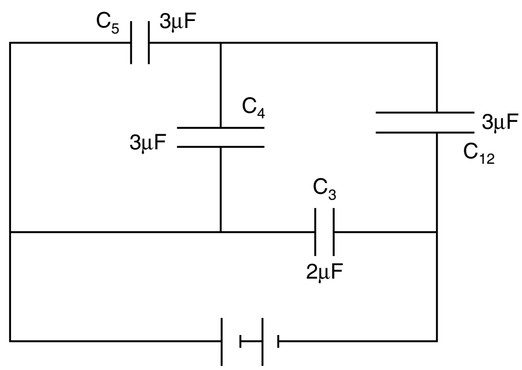


FIg. 2.47

Capacitors C_4 and C_5 are in parallel combinations

$$\therefore \quad C_{45} = 3 + 3 = 6\ \mu F$$

Now, the circuit reduces to

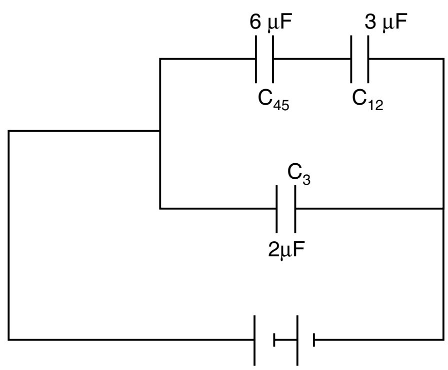


Fig. 2.48

Capacitors C_{12} and C_{45} are in series combinations. $\frac{1}{C_{1245}} = \frac{1}{6} + \frac{1}{3}$

$$\therefore \quad C_{1245} = \frac{3 \times 6}{3+6} = \frac{18}{9} = 2\ \mu F \qquad \left[\because C_5 = \frac{C_1 \cdot C_2}{C_1 + C_2}\right]$$

Now, C_{1245} and C_3 are in parallel combinations, thus equivalent capacitance is

$$\therefore \quad C = C_{1245} + C_3 = 2 + 2 = 4\ \mu F$$

$$\text{Total charge } q = CV$$

$$= 4 \times 10^{-6} \times 100 = 4 \times 10^{-4}\ \text{C}$$

$$\text{Energy stored} = \frac{1}{2} CV^2$$

$$= \frac{1}{2} \times 4 \times 10^{-6} \times (100)^2$$

$$= 2 \times 10^{-6} \times 10^4 = 2 \times 10^{-2}\ \text{J}$$

Q. 3. (*a*) *Explain briefly how a capacitor stores energy on charging. Obtain an expression for the energy thus stored.*

(*b*) *A battery of 10 V is connected to a capacitor of 0.1 F. The battery is now removed and the capacitor is then connected to a second in charged capacitor of same capacitance. Calculate the total energy stored in the system.*

Ans. (*a*) Charges of the capacitor are transferred from one plate to another when it is connected to a battery. The electrostatic potential energy is termed as the work done by the battery in charging the capacitor plates.

Let a capacitor being gradually charged. Its potential at any stage is given as follows:

$$V = \frac{q}{C}$$

In giving an additional charge dq, small amount of work done

$$dW = \frac{q}{C} \cdot dq$$

Total work done in giving a charge Q to the capacitor is

$$W = \int dW = \int \frac{q}{C} dq$$

$$W = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{1}{2}\frac{Q^2}{C}$$

Thus, energy stored in a capacitor is given as follows

$$U = W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 \qquad [\because Q = CV]$$

(*b*) Energy stored in the first capacitor is

$$U_i = \frac{1}{2}CV^2 = \frac{1}{2} \times 0.1 \times (10)^2$$

$$= 5 \text{ J}.$$

Suppose the common potential be V' when the first capacitor is connected across the second capacitor. Then the charge on each capacitor,

$$q' = CV'$$

By the conservation of charge,

$$q' = \frac{q}{2}$$

$\therefore$ Total energy stored in the capacitor is

$$U_f = 2 \times \frac{1}{2} q'V' = q' \times \frac{q'}{C} = \frac{1}{4}\frac{q^2}{C} \qquad [q' = q/2, q = CV]$$

$$= \frac{1}{2} \times \frac{1}{2} CV^2$$

$$= \frac{1}{2} \times 5 = 2.5 \text{ J}.$$

Q. 4. *The area of each plate of parallel plate air capacitor is 150 cm². The distance between its plates is 0.8 mm. It is charged to a potential difference of 1200 volt. What will be its energy? What will be the energy when it is filled with a medium of k = 3 and then charged. If it is charged first as an air capacitor and then filled with this dielectric, what will happen to energy?*

Ans. Given,

$$A = 150 \text{ cm}^2$$
$$= 150 \times 10^{-4} \text{ m}^2$$
$$d = 0.8 = 8 \times 10^{-4} \text{ m}$$
$$V_0 = 1200\ V,\ C_0 = ?\ \text{and}\ E_0 = ?$$
$$\boxed{C_0 = \frac{\varepsilon_0 A}{d}}$$
$$= \frac{8.85 \times 10^{-12} \times 1.50 \times 10^{-4}}{8 \times 10^{-4}}$$
$$= 1.66 \times 10^{-10}\ F$$
$$E_0 = \frac{1}{2} C_0 V_0^2$$
$$= \frac{1}{2} \times 1.66 \times 10^{-10} \times (1200)^2$$
$$= 1.2 \times 10^{-4} \text{ J.}$$

With the dielectric medium, the capacitance becomes

$$C = kC_0 = 3 \times 1.66 \times 10^{-10} \text{ farad}$$

When charged to same potential,

$$V = 1200 \text{ V},$$

Energy
$$\boxed{E = \frac{1}{2} CV^2} = \frac{1}{2}(kC_0)V_0^2$$
$$= k(E_0)$$
$$= 3 \times 1.2 \times 10^{-4}$$
$$= 3.6 \times 10^{-4} \text{ J}$$

When capacitor is charged first as air capacitor, then on filling with the dielectric, its potential becomes

$$\boxed{V = \frac{V_0}{k}} = \frac{1200}{3} = 400 \text{ volt}$$

because capacitance becomes 3 times whereas charge remains the same.

∴ New energy of capacitor

$$U_f = \frac{1}{2} CV^2 = \frac{1}{2}(kC_0)\left(\frac{V_0}{k}\right)^2$$
$$= \frac{1}{2} \frac{C_0 V_0^2}{k} = \frac{1.2 \times 10^{-4}}{3}$$
$$= 4 \times 10^{-5} \text{ J.}$$

Q. 5. *Three charges of + 0.1 C each are placed at the corners of an equilateral triangle as shown in figure. If energy is supplied at the rate of 1 kW, how many days would be required to move the charge at A to a point D which is the mid-point of the line BC?*

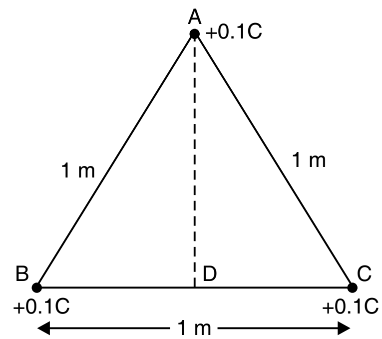


Fig. 2.49

Ans. Potential at A due to charges at B and C is given by

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{0.1}{1} + \frac{1}{4\pi\varepsilon_0}\frac{0.1}{1}$$

$$= 2 \times 9 \times 10^9 \times \frac{1}{10} \text{volt} = 18 \times 10^8 \text{ volt}$$

Potential at D due to charges at B and C is given by

$$V_D = \frac{1}{4\pi\varepsilon_0}\frac{0.1}{0.5} + \frac{1}{4\pi\varepsilon_0}\frac{0.1}{0.5}$$

$$= 2 \times 9 \times 10^9 \times \frac{1}{5} \text{V}$$

$$= 36 \times 10^8 \text{ V}$$

Now, $V_D - V_A = (36 - 18) \times 10^8\ V = 1.8 \times 10^8\ V$

Work done in moving charge 0.1 C from A to D, $W = V.q$

$$W = 0.1\ C \times 18 \times 10^8\ V = 1.8 \times 10^8 \text{ J}$$

We know that

$$\text{Power} = \frac{\text{Work}}{\text{Time}} \text{ or Time} = \frac{\text{Work}}{\text{Power}}$$

Time t taken to move the charge from A to D,

$$= \frac{1.8 \times 10^8 \text{J}}{1kW} = \frac{1.8 \times 10^8 \text{J}}{10^3 Js^{-1}}$$

$$= 1.8 \times 10^5_s \frac{1.8 \times 10^5}{3600} h = 50\ h = \frac{50}{24} = \text{days}$$

$$= 2.08 \text{ days}$$

Q. 6. (*a*) *Determine electrostatic potential energy of a system consisting of two charges 7 μC and − 2μC (and with no external field) placed at (− 9 cm, 0, 0) and (9 cm, 0,0) respectively.*

(*b*) *How much work is required to separate the two charges infinitely away from each other?*

(*c*) *Suppose that the same system of charges is now placed in an external field* $E = A \cdot \frac{1}{r^2}$; $A = 9 \times 10^5\ cm^{-2}$. *What would the electrostatic energy of the configuration be?*

Ans. (*a*) Given,

$$q_1 = 7\ \mu\text{C} = 7 \times 10^{-6} \text{ C}$$

$$q_2 = -2\mu\text{C} = -2 \times 10^{-6} \text{ C}$$

$$r = 9 - (-9) = 18 \text{ cm} = 18 \times 10^{-2} \text{ m}$$

$$\therefore \quad \boxed{U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}} = -0.7 \text{ J}$$

(*b*) Work done $W = U_2 - U_1 = 0 - U = 0 - (-0.7)$

$$= 0.7 \text{ J.}$$

(c) Given, $r = r_2 = 9 \text{ cm} = 0.09 \text{ m}$

The energy of interaction of two charges with the external field E is net electrostatic energy

$$U_f = q_1 V(\vec{r_1}) + q_2 V(\vec{r_2}) + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

$$= q_1 E r_1 + q_2 E r_2 + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \qquad [\because E = V/r]$$

$$= q_1 \frac{A}{r_1^2} r_1 + q_2 \cdot \frac{A}{r_2^2} \cdot r_2 + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

$$= A\frac{q_1}{r_1} + A\frac{q_2}{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

$$= 9\times10^5 \times \frac{7\times10^{-6}}{0.09} + 9\times10^5 \times \frac{-2\times10^{-6}}{0.09} - 0.7$$

$$= 70 - 20 - 0.7 = 49.3 \text{ joule.}$$

Q. 7. *A dielectric slab of thickness 1.0 cm and dielectric constant 5 is placed between the plates of a parallel plate capacitor of plate area 0.01 m² and separation 2.0 cm. Calculate the change in capacity on introduction of dielectric. What would be the change, if the dielectric slab were conducting?*

Ans. Here, $t = 1.0 \text{ cm} = 10^{-2} \text{ m}$

$\in_r = K = 5,\ A = 0.01 \text{ m}^2 = 10^{-2} \text{ m}^2$

$d = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$

Capacity with air in between the plates

$$C_0 = \frac{\in_0 A}{d} = \frac{8.85\times10^{-12}\times10^{-2}}{2\times10^{-2}}$$

$$C_0 = 4.425 \times 10^{-12} \text{ farad}$$

Capacity with dielectric slab in between the plates

$$C = \frac{\in_0 A}{d - t\left(1-\frac{1}{K}\right)}$$

$$= \frac{8.85\times10^{-12}\times10^{-2}}{2\times10^{-2} - 10^{-2}\left(1-\frac{1}{5}\right)}$$

$$C = 7.375 \times 10^{-12} \text{ farad}$$

Capacity with conducting slab in between the plates

$$C' = \frac{\in_0 A}{d-t} = \frac{8.85\times10^{-12}\times10^{-2}}{2\times10^{-2} - 1\times10^{-2}}$$

$$C' = \frac{8.85\times10^{-14}}{10^{-2}} = 8.85\times10^{-12} \text{ farad}$$

Increase in capacity on introduction of dielectric

$$C - C_0 = 7.375 \times 10^{-12} - 4.425 \times 10^{-12}$$
$$= 2.95 \times 10^{-12} \text{ farad}$$

Increase in capacity on introduction of conducting slab

$$C' - C_0 = 8.85 \times 10^{-12} - 4.425 \times 10^{-12}$$
$$= 4.425 \times 10^{-12} \text{ farad.}$$

Q. 8. *Derive an expression for the energy stored in a parallel plate capacitor.*

A parallel plate capacitor with air as dielectric is charged by a d.c. source to a potential 'V'. Without disconnecting the capacitor from the source, air is replaced by another dielectric medium of dielectric constant 10. State with reason, how does (i) electric field between the plates and (ii) energy stored in the capacitor change.

Ans. Work done in charging a capacitor is termed as energy stored in a parallel plate capacitor. Let a capacitor is charged with charge q so that potential difference between its plates is

$$V = \frac{q}{C}$$

Work done to increase the charge around dq is

$$dW = Vdq = \frac{q}{C}dq$$

Total work done to charge the capacitor from 0 to Q is

$$W = \int_0^Q \frac{q}{C}dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{Q^2}{2C}$$

∴ Energy of the capacitor,

$$U = \frac{1}{2}\frac{Q^2}{2C}$$

$$\boxed{U = \frac{1}{2}QV} \text{ or } \boxed{U = \frac{1}{2}CV^2} \qquad \left[\because C = \frac{Q}{V}\right]$$

(*i*) The electric field between the plates remains unchanged ($E = E_0$) because the potential difference across the plates remains unchanged.

(*ii*)
$$U = \frac{1}{2}CV^2 = \frac{1}{2}(10\,C_0)\,V_0^2$$
$$= 10\left(\frac{1}{2}C_0V_0^2\right) = 10\,U_0$$

stored energy increases 10 times.

Q. 9. *A parallel plate capacitor of plate area 2 m² and plate separation 5 mm is charged to 10,000 volt in free space. Calculate the capacitance, charge, charge density, field intensity and the displacement in the space between the plates.*

The charging battery is removed and the space between the plates is filled with a material of relative permittivity 5. Calculate the new capacitance and the potential difference.

Ans. The capacitance, in vacuum, of a parallel plate capacitor is given by $\boxed{C = \varepsilon_0 \frac{A}{d}}$.

$$C_0 = 8.85 \times 10^{-12}\,\text{C}^2\text{N}^{-1}\text{m}^{-2} \times \frac{2\text{m}^2}{5 \times 10^{-3}\,\text{m}}$$

$$C_0 = 3.54 \times 10^{-9} \text{ C}^2\text{N}^{-1} \text{ m}^{-1} = 3.54 \times 10^{-9} \text{ farad}$$

$$\boxed{1 \text{ C}^2\text{N}^{-1}\text{m}^{-1} = 1\frac{C^2}{\text{N m}} = 1\frac{C^2}{\text{J}} = 1\frac{C}{J/C} = 1\frac{C}{V} = 1\,F}$$

Charge on each plate,

$$\boxed{Q = CV} = 3.54 \times 10^{-9} \times 10^4 \text{ C} = 3.54 \times 10^{-5} \text{ C}$$

Free charge density on each plate $\boxed{\sigma = \frac{q}{A}}$

$$\sigma = \frac{3.54\times10^{-5}}{2}\text{C m}^{-2} = 1.77\times10^{-5}\text{C m}^{-2}$$

Electric field intensity, $\boxed{E = \frac{\sigma}{\varepsilon_0}}$

or, $$E = \frac{Q}{\varepsilon_0 A} = \frac{3.54\times10^{-5}C}{8.85\times10^{-12}C^2N^{-1}m^{-2}\times2m^2}$$

$$E = 2 \times 10^6 \text{ NC}^{-1}$$

[Alter. Since the electric intensity equals the potential gradient,

$\therefore$ $$\boxed{E = \frac{V}{d}} = \frac{10^4 \text{ volt}}{5\times10^{-3}\text{m}} = 2\times10^6 V\text{ m}^{-1}]$$

N C^{-1} and V m^{-1} are equivalent units.

Displacement $= 1.77 \times 10^{-5}$ C m^{-2}

New capacitance $\boxed{C = kC_0}$

$$= 5 \times 3.54 \times 10^{-9}\ F = 1.77 \times 10^{-8}\ F$$

New potential difference $\boxed{V = \frac{q}{C}}$

$$V = \frac{3.54\times10^{-5}}{1.77\times10^{-8}}V = 2000\ V$$

Q. 10. *Define the terms (i) Capacitance of capacitor (ii) dielectric strength of a dielectric. When a dielectric is inserted between the plates of a charged parallel plate capacitor, fully, occupying the intervening region, how does the polarisation of the dielectric medium affect the net electric field? For linear dielectrics, show that the introduction of a dielectric increases its capacitance by a factor k, characteristic of the dielectric.*

Ans. (*i*) The amount of charge required to raise the potential of a capacitor by unity is termed as the capacitance of a capacitor.

(*ii*) The maximum electric field that a dielectric medium can with stand without breaking down of its insulating property is called the dielectric strength of a dielectric.

When a dielectric is inserted between the plates of a charged parallel plate capacitor, the field causes a uniform polarisation of the dielectric and the total field in the dielectric is reduced.

The capacitance C_0 is given by

$$C_0 = \frac{\varepsilon_0 A}{d} \quad ...(i)$$

The capacitance C, with dielectric between the plates is

$$C = \frac{\varepsilon_0 kA}{d} \quad ...(ii)$$

The product $\varepsilon_0 k$ is called the permitivity of the medium and is denoted by ε.

$$\therefore \quad \varepsilon = \varepsilon_0 k \Rightarrow k = \frac{\varepsilon}{\varepsilon_0}$$

Where k is called the dielectric constant of the substance.

Dividing (ii) by (i) we get

$$\frac{C}{C_0} = \frac{\varepsilon_0 kA}{d} \times \frac{d}{\varepsilon_0 A}$$

$$\frac{C}{C_0} = k$$

$$\therefore \quad C = kC_0.$$

Therefore, the dielectric constant of a substance is the factor > 1. By which the capacitance increases from its vacuum value, when the dielectric is inserted fully between the plates of a capacitor.

QUESTIONS ON HIGH ORDER THINKING SKILLS (HOTS)

Q. 1. *Obtain the equivalent capacitance of the network in figure. For a 300 V supply, determine the charge and voltage across each capacitor.*

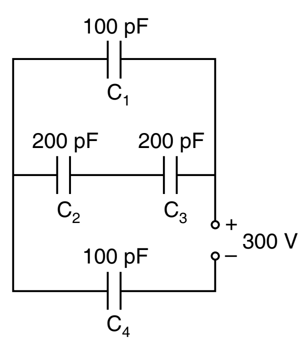


Fig. 2.50

Ans.

$$C_{23} = 100 \text{ pF},$$
$$C_{123} = 200 \text{ pF},$$

C_{123} and C_4 are in series.

$$\therefore \quad C_{1234} = \frac{200 \times 100}{200 + 100} = \frac{200}{3} \text{ pF}$$
$$= 66.67 \text{ pF},$$

The given circuit may be redrawn as shown in following figure

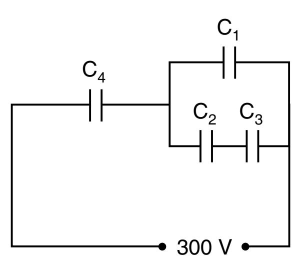


Fig. 2.51

Now, $$\frac{V_{123}}{V_4} = \frac{C_4}{C_{123}} = \frac{100}{200} = \frac{1}{2}$$

So, voltage across C_4 is 200 V. The voltage across the combination of C_1, C_2 and C_3 is 100 V. Since C_1 and C_{23} are in parallel, therefore, the voltage across C_1 as well as across the series combination of C_2 and C_3 is 100 V. Again, C_2 and C_3 are equal therefore, 100 V would be shared equally between C_2 and C_3.

$\therefore \quad V_2 = 50\ V$ and $V_3 = 50\ V$

Again, $\quad Q_1 = 100 \times 10^{-12} \times 100\ C = 10^{-8}\ C$

$Q_2 = 200 \times 10^{-12} \times 50\ C = 10^{-8}\ C$

$Q_3 = 200 \times 10^{-12} \times 50\ C = 10^{-8}\ C$

$Q_4 = 100 \times 10^{-12} \times 200\ C = 2 \times 10^{-8}\ C$

Q. 2. *Two identical plane metallic surfaces A and B are kept parallel to each other in air separated by a distance of 1.0 cm as shown in the figure. Surface A is given a positive potential of 10 V and the outer surface of B is earthed. (i) What is the magnitude and direction of uniform electric field between points Y and Z? (ii) What is work done in moving a charge of 20 μC from point X to Y. Where X is situated on surface A?*

Ans. (*i*) Magnitude of electric field

$$E = \frac{dV}{dr} = \frac{10\ V}{1 \times 10^{-2}\ m}$$

$$= 10^3\ \text{Vm}^{-1}$$

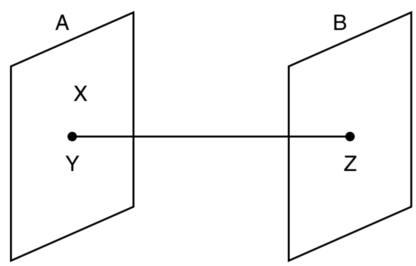


Fig. 2.52

(*ii*) Since surface *A* is an equipotential surface, thus potential difference between *X* and *Y* is given as

$$\Delta V = 0$$

$$\text{Work done} = q.\ \Delta V$$

$$= 0$$

Therefore, no work is done in moving a charge from *X* and *Y*.

Q. 3. *Two charges – q and + q are located at points (0, 0, – a) and (0, 0, a) respectively.*

(a) *What is the electrostatic potential at the points (0, 0, z) and (x, y, 0)?*

(b) *Obtain the dependence of potential on the distance r of a point from the origin when r/a >> 1.*

(c) *How much work is done in moving a small test charge from the point (5, 0, 0) to (– 7, 0, 0) along the x-axis? Does the answer change if the path of the test charge between the same points is not along the x-axis?*

Ans. *See Question 2.21 in section – questions from textbook.*

Q. 4. *Two points charges q_1, q_2, initially at infinity are brought one-by-one to points p_1 and p_2, in external E.F. specified by position vectors $\vec{r_1}$ and $\vec{r_2}$, relative to some origin. What is the potential energy of this charge configuration?*

Ans. Let $\vec{E}$ be the external field.

$\therefore$ Work done on q_2 against the external field

$$= q_2 \cdot V(\vec{r_2})$$

Work done on q_2 against the field due to

$$q_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}} = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

where r_{12} is the distance between q_1 and q_2.

By the superposition principle for fields, we add up the work done on q_2 against the two fields ($\vec{E}$ and that due to q_1). Therefore, work done in carrying

$$q_2 \text{ to } \vec{r}_2 = q_2 \cdot V(\vec{r}_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

Thus, the potential energy of the system = the total work done in assembling the configuration

$$= q_1 \cdot V(\vec{r}_1) + q_2 \cdot V(\vec{r}_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

Q. 5. *An air filled parallel plate capacitor is to be constructed which can store 12 μC of charge when operated at 1200 V. What can be the minimum area of capacitor? The dielectric strength of air is 3 × 10^7 V/m.*

Ans. Here, $Q = 12\ \mu C,\ V = 1200\ V,\ A = ?$

Dielectric strength $= 3 \times 10^7$ V/m

The electric field between the plates should not exceed 10% of the dielectric strength

i.e.,

$$\boxed{E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}} = \frac{10}{100} \times 3 \times 10^7 = 3 \times 10^6 \text{ V/m}$$

$$\therefore \quad A = \frac{Q}{\epsilon_0 \times 3 \times 10^6} = \frac{12 \times 10^{-6}}{8.85 \times 10^{-12} \times 3 \times 10^6} = 0.45 \text{ m}^2$$

Q. 6. *A capacitor of capacitance C_1 = 0.1 microfarad withstands the maximum voltage V_1 = 6.0 kilovolt. While another capacitor of capacitance C_2 = 2.0 microfarad withstands the maximum voltage V_2 = 4.0 kilovolt. What maximum voltage will the system of these two capacitors withstand if they are connected in series?*

Ans. Here, $C_1 = 1.0\ \mu F,\ V_1 = 6.0\ kV = 6 \times 10^3\ V$

∴ Charge on first capacitor

$$q_1 = C_1 V_1 = 1.0 \times 6 \times 10^3\ \mu C = 6000\ \mu C$$

Similarly, charge on 2nd capacitor

$$q_2 = C_2 V_2 = 2.0 \times 4 \times 10^3\ \mu C = 8000\ \mu C$$

In series combination, charge on each capacitor must be the same. As maximum charge on C_1 is 6000 μC; therefore, maximum charge on C_2 must also be 6000 μC:

Hence, maximum voltage for the combination is

$$V' = V_1' + V_2' = \frac{6000}{1.0} + \frac{6000}{2.0} = 9000 \text{ V} = 9 \text{ kV}$$

Q. 7. *If a piece of metal has a charge + 0.1 μC and is placed inside a hollow metal sphere of radius 20 cm (without touching it), what is the potential of the sphere? What will the potential of the sphere become, if*

(a) the sphere is temporarily earthed and then left insulated,

(b) the metal subsequently touched the inside of the sphere?

Ans. On the inner and outer sphere's surface charges of – 0.1 μC and + 0.1 μC are induced respectively. The potential of the sphere relative to earth is determined solely by the outer surface charge q.

$$\therefore \quad \boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}}$$

where r is the radius of the sphere:

$$\text{Now,} \quad V = 9 \times 10^9 \text{ Nm}^2C^{-2} \frac{0.1\times10^{-6}C}{0.2\,m}$$

$$= 4500 \text{ V.}$$

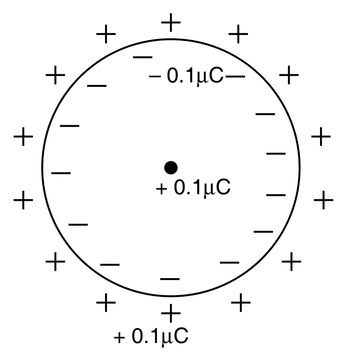


Fig. 2.53

(*a*) The potential of the sphere is momentarily reduced to zero when it is earthed. The positive charge on the outer surface disappears but the induced negative charge inside remains. Therefore, the potential of the sphere is zero.

(*b*) The induced negative charge is neutralized when the metal touches the sphere and no charge remains on the metal or sphere. Both are at the same potential *i.e.*, zero potential.

Q. 8. *A circuit has a section AB as shown in figure. The emf of the source equals E = 10 V, the capacitor capacitances are equal to* C_1 *= 1.0 μF and* C_2 *= 2.0 μF and the potential difference* $V_A - V_B$ *= 5.0 V. Find the voltage across each capacitor.*

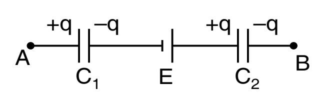


Fig. 2.54

Ans. Let the charge distribution be as shown in figure

$$\therefore \quad V_A - V_B = \frac{q}{C_1} - E + \frac{q}{C_2}$$

$$\text{or} \quad (V_A - V_B) + E = q\left(\frac{1}{C_1}+\frac{1}{C_2}\right)$$

$$= \frac{q(C_2+C_1)}{C_1C_2}$$

$$\therefore \quad q = \frac{\left[(V_A - V_B)+E\right]C_1C_2}{C_1+C_2}$$

Voltage across C_1 is $V_1 = \dfrac{q}{C_1} = \dfrac{\left[(V_A - V_B)+E\right]C_2}{C_1+C_2}$

$$= \frac{(5+10)2.0}{1.0+2.0} = 10 \text{ volt}$$

Voltage across C_2 is V_2

$$\frac{q}{C_2} = \frac{\left[(V_A - V_B)+E\right]C_1}{C_1+C_2}$$

$$= \frac{(5+10)1.0}{1.0\times2.0} = 5 \text{ volt}$$

Q. 9. *Calculate the potential at P due to the charge configuration as shown in the following figure. If $r >> a$, then how will you modify the result?*

Ans. Potential at p due to the given charge configuration is the sum of the potentials due to charges $-q$, $+q$ and $+q$. These charges are at distances $r + a$, r and $r - a$ respectively from the point P.

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\frac{-q}{(r+a)} + \frac{1}{4\pi\varepsilon_0}\frac{q}{r} + \frac{1}{4\pi\varepsilon_0}\frac{q}{r-a}$$

$$\text{or} \quad V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{2qa}{r^2 - a^2}\right)$$

if $r >> a$, then $r^2 - a^2 \approx r^2$

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{2qa}{r^2}\right)$$

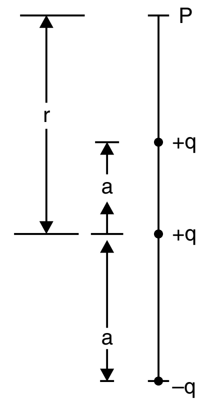


Fig. 2.55

> **Note:** When $r >> a$, the potential at P is simply the potential of a dipole and an isolated charge at distance r.

Q. 10. *Find the capacitance of the infinite ladder between points X and Y, figure*

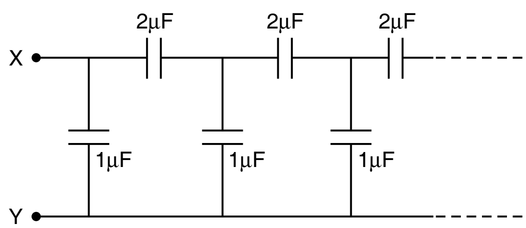


Fig. 2.56

Ans. Let C be the capacitance of the infinite ladder. As the ladder is infinite, addition of one more element of two capacitors (1 μF and 2 μF) across the points X and Y should not change the total capacitance. Therefore, total capacity of the arrangement shown in figure must remain C only.

In figure 2 μF capacitor is in series with capacitance C.

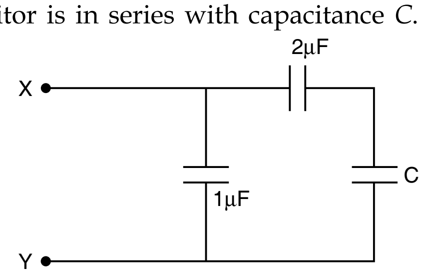


Fig. 2.57

$\therefore$ Their combined capacity = $\frac{2 \times C}{2 + C}$

This combination is in parallel with 1 μF capacitor. The equivalent capacity of the arrangement is

$$1 + \frac{2C}{2+C} = C$$

or, $C^2 + 2C = 2 + 3C$

or, $C^2 - C - 2 = 0$

$\therefore$ $C = 2$ or, -1

As capacitance cannot be negative.

$\therefore$ $C = 2\ \mu F$

Q. 11. *Three identical parallel plate capacitor (air) C_1, C_2, C_3 have capacitance C each. The space between their plates is now filled with dielectrics as shown. If all three capacitors still have equal capacitance, obtain the relation between dielectric constants k, k_1, k_2, k_3 and k_4.*

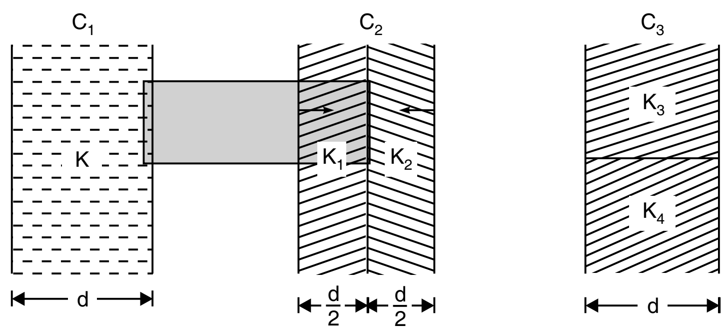


Fig. 2.58

Ans. After introducing the dielectrics the capacitance of capacitors C_1, C_2 and C_3 respectively is given as

$$C = \frac{k\varepsilon_0 A}{d},\ C = \frac{2\varepsilon_0 A}{d}\left[\frac{k_1 k_2}{k_1 + k_2}\right],\ C = \frac{\varepsilon_0 A}{2d}[k_3 + k_4]$$

$$\Rightarrow \quad \frac{k\varepsilon_0 A}{k} = \frac{2\varepsilon_0 A}{d}\left(\frac{k_1 k_2}{k_1 + k_2}\right) = \frac{\varepsilon_0 A}{2d}(k_3 + k_4)$$

$$\Rightarrow \quad k = \frac{2k_1 k_2}{k_1 + k_2} = \frac{k_3 + k_4}{2}$$

Q. 12. *A charge particle of charge 2μC and mass 10 miligram moving with a velocity of 1000 ms^{-1} enters a uniform electric field of strength 10^2 NC^{-1} directed perpendicular to its direction of motion. Find the velocity and acceleration of the particle after 10 s.*

Ans. Acceleration, $a_y = \frac{qE}{m}$

$$= \frac{2 \times 10^{-6} \times 10^2}{10 \times 10^{-6}} = 20\ ms^{-2}$$

And velocity after 10 s perpendicular to the direction of motion, $v_y = u_y + a_y t$

$= 0 + 20 \times 10$

$= 200\ ms^{-1}$

$\because$ There is no force along the direction of motion $\therefore$ velocity along the direction of motion remains same *i.e.*, $v_n = 1000\ ms^{-1}$

$\therefore$ Net velocity after 10 s

$$v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{(1000)^2 + (200)^2}$$

$$= 100\sqrt{104} \text{ ms}^{-1}.$$

Q. 13. *Find the amount of work done in rotating an electric dipole of dipole moment 3 × 10^{-3} cm from its position of stable equilibrium to the position unstable equlibrium in a uniform electric field of intensities 10^4 NC^{-1}.*

Ans. Work done in rotating the dipole from an angle θ_1 to θ_2 of dipole moment with electric field,

$$W = PE (\cos \theta_1 - \cos \theta_2)$$

For stable equilibrium position $\theta_1 = 0°$

and for unstable equilibrium position $\theta_2 = 0°$

$$W = 3 \times 10^{-3} \times 10^4 [1 - (-1)]$$

$$= 60 \text{ J}.$$

Q. 14. *Charges of magnitudes 2q and – q are located at points (a, 0, 0) and (4a, 0, 0). Find the ratio of the flux of electric field due to these charges through concentric spheres of radii 2a and 8a centred at the origin.*

Ans. The locations of the charges are shown in the figure.

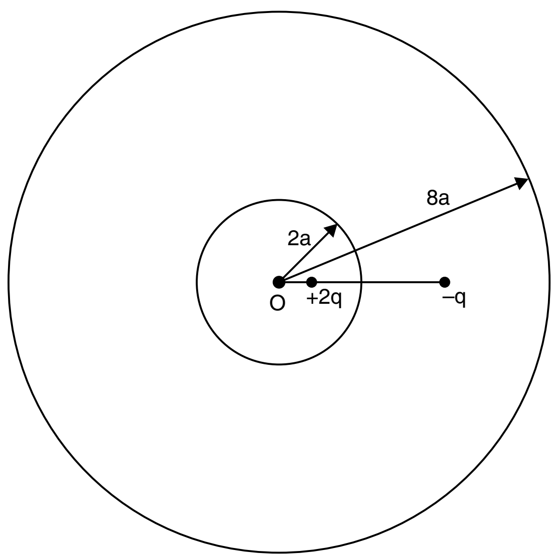


Fig. 2.59

The electric flux through the sphere of radius 2*a*,

$$\phi_1 = \frac{1}{\varepsilon_0}(2q)$$

The electric flux through the sphere of radius 8*a*

$$\phi_2 = \frac{1}{\varepsilon_0}(2q - q) = \frac{1}{\varepsilon_0}(q)$$

$$\therefore \quad \frac{\phi_1}{\phi_2} = \frac{2}{1}.$$

MULTIPLE CHOICE QUESTIONS

1. Equal charges q each are placed at the vertices A and B of an equilateral triangle ABC of side a. The magnitude of electric intensity at the centre is

(*a*) $\dfrac{q}{\Delta\pi\varepsilon_0 a^2}$ (*b*) $\dfrac{\sqrt{2}q}{4\pi\varepsilon_0^2}$

(*c*) $\dfrac{\sqrt{3}q}{4\pi\varepsilon_0 a^2}$ (*d*) $\dfrac{2q}{4\pi\varepsilon_0 a^2}$

2. The capacity of an isolated conducting sphere of radius R is proportional to

(*a*) R^2 (*b*) $\dfrac{1}{R^2}$ (*c*) $\dfrac{1}{R}$ (*d*) R

3. A hollow metal sphere of radius 10 cm is charged such that the potential at its surface is 80 V. The potential at the centre of the sphere is

(*a*) zero (*b*) 80 V

(*c*) 800 V (*d*) 8 V

4. Work done in moving a unit positive charge through a distance of x metre on an equipotential surface is

(*a*) x joule (*b*) $\dfrac{1}{x}$ joule

(*c*) zero (*d*) x^2 joule

5. Two spherical conductors each of capacity C are charged to potential V and $-V$. These are then connected by means of a fine wire. The loss of energy is

(*a*) zero (*b*) $\dfrac{1}{2}CV^2$

(*c*) CV^2 (*d*) $2CV^2$

6. Potential energy of two equal +ve charges 1 μC each held 1 m apart in air is

(*a*) 9×10^{-3} J (*b*) zero (*c*) 9×10^{-3} eV (*d*) 1 J

7. Two infinite plane parallel sheets, separated by a distance d have equal and opposite charge densities σ. Electric field intensity at a point between the sheets is

(*a*) $\dfrac{\sigma}{2\varepsilon_0}$ (*b*) depends upon location of the point

(*c*) $\dfrac{\sigma}{\varepsilon_0}$ (*d*) zero

8. A 2μF capacitor is charged to 100 volt and then its plates are connected by a concluding wire. The heat produced is

(*a*) 0.001 J (*b*) 0.01 J

(*c*) 0.1 J (*d*) 1 J

9. In bringing an electron towards another electron the electrostatic potential energy of the system

(*a*) remains same (*b*) become zero

(*c*) increases (*d*) decreases

10. Equal charges are given to two conducting spheres of different radii. The potential will

(*a*) be more on she smaller sphere

(*b*) be more on the bigger sphere

(*c*) be equal on both the spheres

(*d*) depend on the ratio of the material of the sphere

11. A semi-circular arc of radius '*a*' is charged uniformly and the charge per unit lengths is λ. The electric field at the centre is

(*a*) $\frac{\lambda}{2\pi\varepsilon_0 a^2}$ (*b*) $\frac{\lambda}{4\pi\varepsilon_0 a}$

(*c*) $\frac{\lambda^2}{2\pi\varepsilon_0 a}$ (*d*) $\frac{\lambda}{2\pi\varepsilon_0 a}$

12. Two capacitors *A* and *B* are connected in series with a battery as shown in figure 2.60. When the switch *S* in closed and the two capacitors get charged fully then

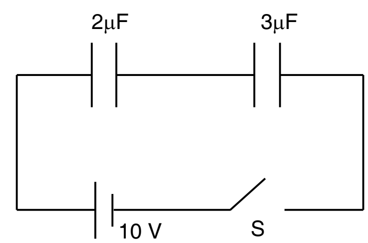


Fig. 2.60

(*a*) the potential difference across the plates of *A* is 4 V and across the plates *B* is 6 V

(*b*) the potential difference across the plates of *A* is 6 V and across the plates of *B* is 4 V

(*c*) the ratio of electrical energies stored in *A* and *B* is 2 : 3

(*d*) the ratio of charges on *A* and *B* is 3 : 2

13. Four capacitors each of 25 μF are connected as shown in figure 2.61. The voltmeter shows a dc of 200 V. The charge on each plate of capacitor is

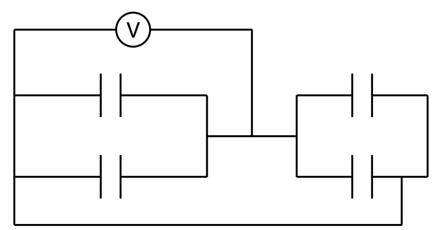


Fig. 2.61

(*a*) $\pm 2 \times 10^{-3}$ C

(*b*) $\pm 5 \times 10^{-3}$ C

(*c*) $\pm 2 \times 10^{-2}$ C

(*d*) $\pm 5 \times 10^{-2}$ C

14. Two charges – 10 C and + 0 C are placed 10 cm apart. Potential at the centre of the line joining the two charges is

(*a*) zero (*b*) 2 V

(*c*) – 2V (*d*) none of those

15. A particle of mass *m* and charge *q* is placed at rest in uniform electric field of intensity ε and then released. The kinetic energy attained by the particle after moving a distance *y* is

(*a*) qEy^2 (*b*) qE^2y

(*c*) qEy (*d*) q^2Ey

Answers

1. (*a*)	**2.** (*a*)	**3.** (*c*)	**4.** (*c*)	**5.** (*d*)
6. (*d*)	**7.** (*a*)	**8.** (*a*)	**9.** (*c*)	**10.** (*d*)
11. (*d*)	**12.** (*b*)	**13.** (*d*)	**14.** (*c*)	**15.** (*c*).

TEST YOUR SKILLS

1. What is an equipotential surface?

2. Two charges 2 μC and – 2 μC are placed at points A and B 6 cm apart.
(*a*) Identify an equipotential surface of the system.
(*b*) What is the direction of the electric field at every point on this surface?

3. What do you understand by electrostatic shielding?

4. Deduce an expression for the electric potential due to an electric dipole at any point on its axis. Mention one contrasting feature of electric potential of a dipole at a point as compared to that due to single charge.

5. An electric charge 10^{-3} μC placed at the origin (0, 0) of $x - y$ coordinate system. The points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2, 0) respectively. Find the potential difference between A and B.

6. Charges are placed on the vertices of a square as shown. Let E be the electric field and V the potential at the centre. If the charges on A and B are interchanged with those on D and C respectively. Then which of the E or V will change or remains unchanged?

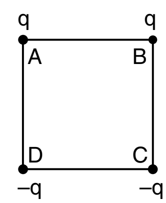


Fig. 2.62

7. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volt. The dielectric slab is slowly removed from between the plates and then reinserted. Find the net work done by the system in this process.

8. A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. What will be the ratio of the energy stored in the capacitor and the work done by the battery?

9. A parallel plate capacitor, each with plate area A and separation d, is charged to a potential difference V. The battery used to charge it is then disconnected. A dielectric slab of thickness d and dielectric constant K is now placed between the plates. What change, if any, will take place in (*i*) charge on the plate (*ii*) electric field intensity between the plates (*iii*) capacitance of the capacitor. Justify your answer in each case.

10. Derive an expression for the capacitance of a parallel plate capacitor having two identical plates, each of area A and separated by a distance d, when the space between the plates is filled with a dielectric medium.

11. Derive an expression for the energy stored in a capacitor.

12. A parallel plate capacitor is charged to a potential difference V by a d.c. source, The capacitor is then disconnected from the source. If the distance between the plates is doubled, state with reason how the following will change.
(*i*) Electric field between the plates,
(*ii*) Capacitance and
(*iii*) Energy stored in the capacitor

13. The electric potential at a point, distance 0.9 m, from a point charge is + 50 V. Find the magnitude and nature of the charge.

14. (*i*) Calculate the electric potential at a point X due to a charge of 0.5 μC located at 10 cm from it.
(*ii*) Also calculate the work done in bringing a charge of 3×10^{-9} C from infinity to the point X.

15. A proton placed in a uniform electric field of magnitude 2000 N/C moves between two points in the direction of electric field. If the distance between the points is 0.2 m, find value of (*i*) p.d. between the points (*ii*) work done.

16. A metal wire is bent into a circle of radius 10 cm. It is given a charge of 200 μC which spread on it uniformly. Calculate the electric potential at its centre.

17. A charge of 12 μC is given to a hollow metallic sphere of radius 0.1 m. Find the potential at the (*i*) surface (*ii*) centre of the sphere.

18. The point charges +4 μC and –6 μC are separated by a distance of 20 cm in air. At what point of the line joining the two charges is the electric potential zero?

19. What is the area of the plates of a parallel plate capacitor of capacitance of 2F with separation between the plates 0.5 cm?

20. Four capacitors are connected as shown in figure given below.

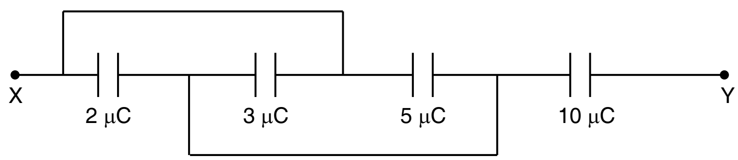


Fig. 2.63

Calculate the equivalent capacitance between *X* and *Y*.

21. The equivalent capacitance of the combination between *A* and *B* in the given figure is 15 μF. Calculate the capacitance of the capacitor *C*.

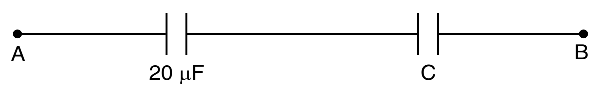


Fig. 2.64

22. Calculate the capacitance of the capacitor *C* in the following figure. The equivalent capacitance of the combination between *P* and *Q* is 30 μF.

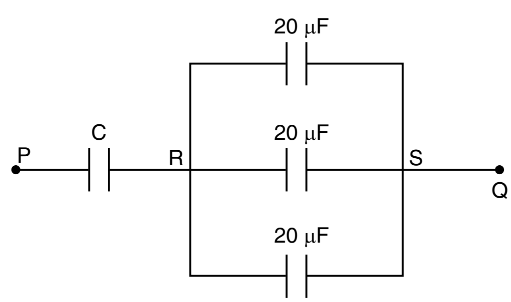


Fig. 2.65

23. Find the equivalent capacitance of the combination of capacitors between the points *A* and *B* as shown in figure. Also calculate the total charge that flows in the circuit when a 100 V battery is connected between the points *A* and *B*.

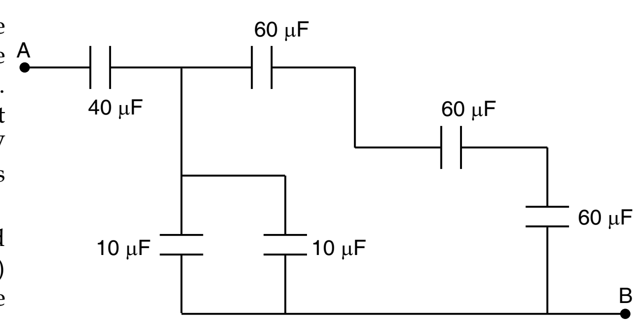


Fig. 2.66

24. Two charges of magnitude 8 *nC* and – 3*nC*, are placed at points (3 cm, 0.0) and (30 cm, 0, 0) in a region of space where there is no other external field. Calculate the electrostatic potential energy of this charge system.

25. Defind the terms (*i*) capacitance of a capacitor (*ii*) strength of dielectric placed between the plates of a charged parallel plate capacitor, fully occupying the intervening region, how does the polarisation of the dielectric medium effect the net electric field? For linear dielectrics show that the introduction of dielectric increases its capacitance by a factor of a characteristic of the dielectric.

26. A uniform electric field $E = E_x i$ N/C for $x > 0$ and $E = E_x i$ N/C for $x < 0$ are given. A right circular cylinder of length l cm and radius r cm has its centre at the origin and its axis along the x-axis. Find out the net outward flux. Using Gauss's law write the expression for the net charge within the cylinder.

27. A 500 μC charge is at the centre of a square of side 10 cm. Find the work done in moving a charge of 10 μC between two diagonally opposite points on the square.

28. Calculate the work done to dissociate the system of the three charges placed on the vertices of a triangle as shown. (Here $q = 1.6 \times 10^{-10}$ C)

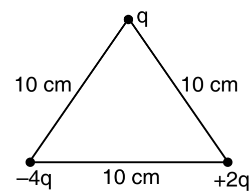


Fig. 2.67

29. Two point charges, $q_1 = 10 \times 10^{-8}$ C and $q_2 = -2 \times 10^{-8}$ C are separated by a distance of 60 cm in air,
    (*i*) Find at what distance from the 1st charge, q_1, would the electric potential be zero
    (*ii*) Also calculate the electrostatic potential energy of the system.

30. Two point charges 4Q and Q are separated by 1 m in air. At what point on the line joining the charge is the electric field intensity zero? Also calculate the electrostatic potential energy of the system of charges, taking the value of charge, $Q = 2 \times 10^{-7}$ C.

31. On charging a parallel plate capacitor to a potential V, the spacing between the plates is halved, and a dielectric medium of $K = 10$ is introduced between the plates, without disconnecting the *d.c.* source. Explain, using suitable expressions, how the (*i*) capacitance, (*ii*) electric field and (*iii*) energy density of the capacitor change?

32. Two charges $-q$ and $+q$ are located at point A (0, 0, – a) and B (0, 0, +a) respectively. How much work is done in moving a test charge from point P (7, 0, 0) to Q (– 3, 0, 0)?

33. Three identical capacitors C_1, C_2 and C_3 of capacitance 6 μF each are connected to a 12V battery as shown in figure 2.68. Find (*i*) charge on each capacitor (*ii*) equivalent capacitance of the network (*iii*) energy stored in the network of capacitors.

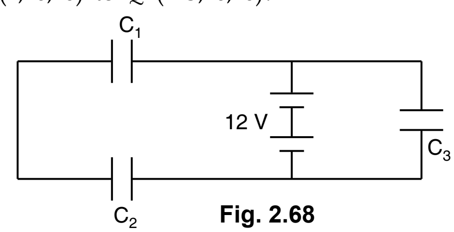


Fig. 2.68

34. The equivalent capacitance of the combination between A and B in the given figure 2.69 is 4 μF.
    (*i*) Calculate the capacitance of the capacitor C.
    (*ii*) Calculate charge on each capacitor if a 12V battery is connected across terminals A and B.
    (*iii*) What will be the potential drop across each capacitor?

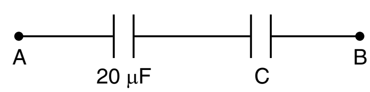


Fig. 2.69

35. Two parallel plate capacitors, X and Y have the same area of plates and same seperation between them. X has air between the plates while Y contained a dielectric medium of $\in_r = 4$.
    (*i*) Calculate capacitance of each capacitor if equivalent capacitance of combination is 4 μF.
    (*ii*) Calculate the potential difference between the plates of X and Y.
    (*iii*) What is the ratio of electrostatic energy stored in X and Y?

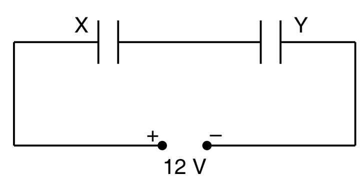


Fig. 2.70

❑❑❑