2 Units and Measurements

Facts that Matter

- **Measurement**

The process of measurement is basically a comparison process. To measure a physical quantity, we have to find out how many times a standard amount of that physical quantity is present in the quantity being measured. The number thus obtained is known as the magnitude and the standard chosen is called the unit of the physical quantity.

- **Unit**

The unit of a physical quantity is an arbitrarily chosen standard which is widely accepted by the society and in terms of which other quantities of similar nature may be measured.

- **Standard**

The actual physical embodiment of the unit of a physical quantity is known as a standard of that physical quantity.

- To express any measurement made we need the numerical value (n) and the unit (u).

Measurement of physical quantity = Numerical value × Unit

For example: Length of a rod = 8 m

where 8 is numerical value and m (metre) is unit of length.

- **Fundamental Physical Quantity/Units**

It is an elementary physical quantity, which does not require any other physical quantity to express it. It means it cannot be resolved further in terms of any other physical quantity. It is also known as basic physical quantity.

The units of fundamental physical quantities are called fundamental units.

For example, in M. K. S. system, Mass, Length and Time expressed in kilogram, metre and second respectively are fundamental units.

- **Derived Physical Quantity/Units**

All those physical quantities, which can be derived from the combination of two or more fundamental quantities or can be expressed in terms of basic physical quantities, are called derived physical quantities.

The units of all other physical quantities, which can be obtained from fundamental units, are called derived units. For example, units of velocity, density and force are m/s, kg/m^3, kg m/s^2 respectively and they are examples of derived units.

- **Systems of Units**

Earlier three different units systems were used in different countries. These were CGS, FPS and MKS systems. Now-a-days internationally SI system of units is followed. In SI unit system, seven quantities are taken as the base quantities.

(*i*) **CGS System.** Centimetre, Gram and Second are used to express length, mass and time respectively.

(*ii*) **FPS System.** Foot, pound and second are used to express length, mass and time respectively.

(*iii*) **MKS System.** Length is expressed in metre, mass is expressed in kilogram and time is expressed in second. Metre, kilogram and second are used to express length, mass and time respectively.

(*iv*) **SI Units.** Length, mass, time, electric current, thermodynamic temperature, Amount of substance and luminous intensity are expressed in metre, kilogram, second, ampere, kelvin, mole and candela respectively.

• Definitions of Fundamental Units

TABLE 2.1 SI Base Quantities and Units

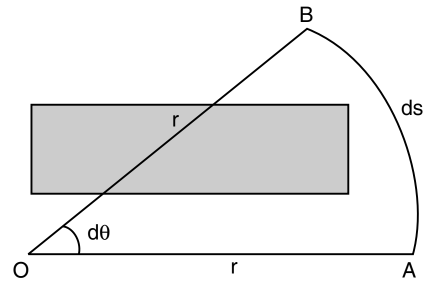
Base quantity	*SI Units*		
	Name	*Symbol*	*Definition*
Length	metre	m	The metre is the length of the path travelled by light in vacuum during a time interval of 1/299, 792, 458 of a second. (1983)
Mass	kilogram	kg	The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at international Bureau of Weights and Measures, at Sevres, near Paris, France. (1889)
Time	second	s	The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom. (1967)
Electric current	ampere	A	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length. (1948)
Thermodynamic Temperature	kelvin	K	The kelvin is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (1967)
Amount of substance	mole	mol	The mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12. (1971)
Luminous intensity	candela	cd	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian. (1979)

• Supplementary Units

Besides the above mentioned seven units, there are two supplementary base units. These are (*i*) radian (rad) for angle, and (*ii*) steradian (sr) for solid angle.

(*i*) **Radian (rad).** It is the unit of plane angle. One radian is an angle subtended at the centre of a circle by an arc of length equal to the radius of the circle.

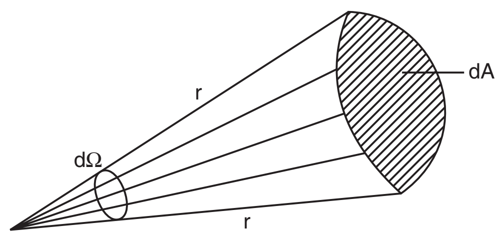
$$d\theta = \left(\frac{ds}{r}\right) \text{ radian}$$



(*ii*) **Steradian (sr).** It is the unit of solid angle. One steradian is the solid angle subtended at the centre of a sphere by its surface whose area is equal to the square of the radius of the sphere. Solid angle in steradian,

$$d\Omega = \frac{\text{area cut out from the surface of sphere}}{(\text{radius})^2}$$

$$d\Omega = \left(\frac{dA}{r^2}\right) \text{ steradian}$$



• Advantages of SI Unit System

SI unit system has following advantages over the other systems of units:

(*i*) It is internationally accepted,
(*ii*) It is a rational unit system,
(*iii*) It is a coherent unit system,
(*iv*) It is a metric system,
(*v*) It is closely related to CGS and MKS systems of units,
(*vi*) Uses decimal system, hence is more user friendly.

• Other Important Units of Length

For measuring large distances *e.g.,* distances of planets and stars etc., some bigger units of length such as 'astronomical unit', 'light year', 'parsec' etc. are used.

- The average separation between the Earth and the sun is called one astronomical unit.

$$1 \text{ AU} = 1.496 \times 10^{11} \text{ m.}$$

- The distance travelled by light in vacuum in one year is called light year.

$$1 \text{ light year} = 9.46 \times 10^{15} \text{ m.}$$

- The distance at which an arc of length of one astronomical unit subtends an angle of one second at a point is called parsec.

$$1 \text{ parsec} = 3.08 \times 10^{16} \text{ m}$$

- Size of a tiny nucleus = 1 fermi = $1f = 10^{-15}$ m
- Size of a tiny atom = 1 angstrom = $1\text{Å} = 10^{-10}$ m

• Parallax Method

This method is used to measure the distance of planets and stars from earth.

Parallax. Hold a pen in front of your eyes and look at the pen by closing the right eye and then the left eye. What do you observe? The position of the pen changes with respect to the background. This relative shift in the position of the pen (object) w.r.t. background is called parallax.

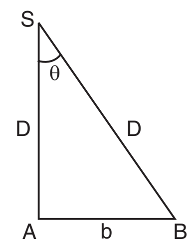
If a distant object *e.g.*, a planet or a star subtends parallax angle θ on an arc of radius b (known as basis) on Earth, then distance of that distant object from the basis is given by

$$s = \frac{b}{\theta}$$

- To estimate size of atoms we can use electron microscope and tunneling microscopy technique. Rutherford's α-particle scattering experiment enables us to estimate size of nuclei of different elements.
- Pendulum clocks, mechanical watches (in which vibrations of a balance wheel are used) and quartz watches are commonly used to measure time. Cesium atomic clocks can be used to measure time with an accuracy of 1 part in 10^{13} (or to a maximum discrepancy of 3 μs in a year).
- The SI unit of mass is kilogram. While dealing with atoms/molecules and subatomic particles we define a unit known as ''unified atomic mass unit'' (1 u), where 1 u = 1.66×10^{-27} kg.

Measuring the distance of a far away planet:

Let us assume S is a planet a distance D from earth. A and B are two observatories on earth.



Distance $AB = b$

Parallax Angle ∠ASB = θ

As the planet is very far away.

So, $\frac{b}{D} << 1$

and hence, θ is very small.

AB is an arc of circle with centre S and radius D

$$D = AS = BS$$

$$AB = b = D\theta, \text{ where } \theta \text{ is in radians}$$

$$\Rightarrow \quad D = \frac{b}{\theta} \quad ...(i)$$

After determining D, size or angular diameter at the planet can be determined using same method. If d is the diameter and α is the angular size of planet, then

$$\alpha = \frac{d}{D} \quad ...(ii)$$

Using these two equations, diameter of planet can be calculated.

Example 2.1. *Calculate the angle of:*

(*a*) *1° (degree)* (*b*) *1′ (minute of arc or arc min)*

(*c*) *1″ (second of ar or arc second)*

in radians. Use 360° = 2π rad, 1° = 60′ and 1′ = 60″.

Solution. (*a*) We have 360° = 2π rad

So, $1° = \frac{\pi}{180}$ rad $= 1.745 \times 10^{-2}$ rad

(*b*) $1° = 60' = 1.745 \times 10^{-2}$ rad

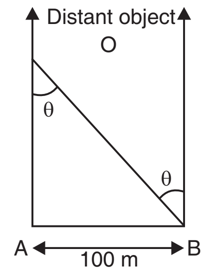
⇒ $1' = 2.908 \times 10^{-4}$ rad $= 2.91 \times 10^{-4}$ rad.

(*c*) $1' = 60'' = 2.908 \times 10^{-4}$ rad

$\Rightarrow$ $1'' = 4.847 \times 10^{-4}$ rad $= 4.85 \times 10^{-6}$ rad.

Example 2.2. *A man wishes to estimate the distance of a nearby tower from him. He stands at a point A in front of the tower C and spots a very distant object O in line with AC. He then walks perpendicular to AC upto B, a distance of 100 m, and sees O again. Since O is very distant, the direction BO is practically the same as AO; but he finds the line of sight of C shifted from the original line of sight by an angle θ = 40°.*

Estimate the distance of the tower C from his original position A.



Solution. We have, parallax angle $\theta = 40°$

In ΔABC, $AB = AC \tan\theta$

$$\Rightarrow \quad AC = \frac{AB}{\tan\theta} = \frac{100 \text{ m}}{\tan 40°} = \frac{100 \text{ m}}{0.8391} = 119 \text{ m}.$$

Example 2.3. *The moon is observed from two diameterically opposite points A and B on Earth. The angle subtended at the moon by the two directions of observation is 1°54°. Given the diameter of earth to be about 1.276×10^2 m, compute the distance of the moon from the earth.*

Solution. We have $\theta = 1°54' = 114'$

$= (114 \times 60)'' \times (4.85 \times 10^{-6})$ rad $\quad (\because \; 1'' = 4.85 \times 10^{-6}$ rad$)$

$= 3.32 \times 10^{-2}$ rad.

Also, $b = AB = 1.276 \times 10^7$ m

Distance between earth and moon can be given as follows:

$$D = \frac{b}{\theta} = \frac{1.276 \times 10^7}{3.32 \times 10^{-2}} = 3.84 \times 10^8 \text{ m}$$

Example 2.4. *The sun's angular diameter is measured to be 1920°. The distance D of the sun from the earth is 1.496×10^{11} m. What is the diameter of the sun*

Solution. Sun's angular diameter $\alpha = 1920°$

$= 1920 \times 4.85 \times 10^{-6}$ rad $= 9.31 \times 10^{-3}$ rad

Sun's diameter $d = \alpha D = (9.31 \times 10^{-3}) \times (1.496 \times 10^{11})$ m $= 1.39 \times 10^9$ m.

Estimation of Molecular Size of Oleic Acid

For this 1 cm^3 of oleic acid is dissolved in alcohol to make a solution of 20 cm^3. Then 1 cm^3 of this solution is taken and diluted to 20 cm^3, using alcohol. So, the concentration of the solution is as follows:

$$\left(\frac{1}{20 \times 20}\right) \text{cm}^3$$

After that some lycopodium powder is lightly sprinkled on the surface of water in a large trough and one drop of this solution is put in water. The oleic acid drop spreads into a thin, large and roughly circular film of molecular thickness on water surface. Then, the diameter of the thin film is quickly measured to get its area *A*. Suppose *n* drops were put in the water. Initially, the approximate volume of each drop is determined (V cm^3).

Volume of n drops of solution = nV cm^3

Amount of oleic acid in this solution

$$= nV\left(\frac{1}{20 \times 20}\right) \text{cm}^3$$

The solution of oleic acid spreads very fast on the surface of water and forms a very thin layer of thickness t. If this spreads to form a film of area A cm^2, then thickness of the film

$$t = \frac{\text{Volume of the film}}{\text{Area of the film}} \quad \text{or} \quad t = \frac{nV}{20 \times 20} \text{ cm}$$

If we assume that the film has mono-molecular thickness, this becomes the size or diameter of a molecule of oleic acid. The value of this thickness comes out to be of the order of 10^{-9} m.

• Dimensions

The dimensions of a physical quantity are the powers to which the fundamental units of mass, length and time must be raised to represent the given physical quantity.

• Dimensional Formula

The dimensional formula of a physical quantity is an expression telling us how and which of the fundamental quantities enter into the unit of that quantity.

It is customary to express the fundamental quantities by a capital letter, *e.g.*, length (L), mass (M), time (T), electric current (I), temperature (K) and luminous intensity (C). We write appropriate powers of these capital letters within square brackets to get the dimensional formula of any given physical quantity.

• Applications of Dimensions

The concept of dimensions and dimensional formulae are put to the following uses:

(*i*) Checking the results obtained

(*ii*) Conversion from one system of units to another

(*iii*) Deriving relationships between physical quantities

(*iv*) Scaling and studying of models.

The underlying principle for these uses is the principle of homogeneity of dimensions. According to this principle, the 'net' dimensions of the various physical quantities on both sides of a permissible physical relation must be the same; also only dimensionally similar quantities can be added to or subtracted from each other.

• If a given physical quantity has a dimensional formula $M^a L^b T^c$, then

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

Where M_1, L_1, T_1 and M_2, L_2, T_2 are the units of mass, length and time in two systems and n_1 and n_2, the numerical values of the physical quantity in these unit systems.

Example 2.5. *Let us consider an equation*

$$\frac{1}{2} mv^2 = mgh$$

where m is the mass of the body, v its velocity g is the acceleration due to gravity and h is the height. Check whether this equation is dimensionally correct.

Solution. This dimensions of LHS are

$$[M]\ [L\ T^{-1}]^2 = [M]\ [L^2\ T^{-2}] = [M\ L^2\ T^{-2}]$$

The dimensions of RHS are

$$[M]\ [L\ T^{-2}]\ [L] = [M]\ [L^2\ T^{-2}] = [M\ L^2\ T^{-2}]$$

The dimensions of LHS and RHS are the same and hence the equation is dimensionally correct.

Example 2.6. *The SI unit of energy is J = kg m² s⁻²; that of speed v is m s⁻¹ and of acceleration a is m s⁻². Which of the formulae for kinetic energy (K) given below can you rule out on the basis of dimensional arguments [m stands for the mass of the body):*

(a) $K = m^2 v^2$ (b) $K = [1/2]\ mv^2$ (c) $K = ma$

(d) $K = [3/16]\ mv^2$ (e) $K = [1/2]\ mv^2 + ma$.

Solution. Every correct formula or equation must have the same dimensions on both sides of the equation. Also, only quantities with the same physical dimensions can be added or subtracted. The dimensions of the quantity on the right side are $[M^2\ L^3\ T^{-3}]$ for (*a*); $[M\ L^2\ T^{-2}]$ for (*b*) and (*d*); $[M\ L\ T^{-2}]$ for (*c*). The quantity on the right side of (*e*) has no proper dimensions since two quantities of different dimensions have been added. Since the kinetic energy (K) has the dimensions of $[M\ L^2\ T^{-2}]$, formulas (*a*), (*c*) and (*e*) are ruled out. Note that dimensional arguments cannot tell which of the two, (*b*) or (*d*), is the correct formula. For this, one must turn to the actual formula. For this, one must turn to the actual definition of kinetic energy (see Chapter 6). The corect formula for kinetic energy is given by (*b*).

Example 2.7. *Consider a simple pendulum, having a bob attached to a string, that oscillates under the action of the force of gravity. Suppose that the period of oscillation of the simple pendulum depends on its length (l), mass of the bob (m) and acceleration due to gravity (g). Derive the expression for its time period using method of dimensions.*

Solution. The dependence of time period T on the quantities l, g and m as a product may be written as:

$$T = k\ l^x\ g^y\ m^z$$

where k is dimensionless constant and x, y and z are the exponents.

By considering dimensions on both sides, we have

$$[L^0\ M^0\ T^{-1}] = [L^1]^x\ [L^1\ T^{-2}]^y\ [M^1]^z = L^{x+y}\ T^{-2y}\ M^z$$

On equating the dimensions on both sides, we have

$$x + y = 0;\quad -2y = 1;\ \text{and}\ z = 0$$

So that

Then, $$T = k\ l^{1/2}\ g^{-1/2}\quad \text{or,}\quad T = k\sqrt{\frac{l}{g}}$$

Note that value of constant k can not be obtained by the method of dimensions. Here it does not matter if some number multiplies the right side of this formula, because that does not affect its dimensions.

Actually, $k = 2\pi$ so that $T = 2\pi\sqrt{\frac{l}{g}}$

• Limitations of Dimensional Analysis

The method of dimensions has the following limitations:

(*i*) by this method the value of dimensionless constant cannot be calculated.

(*ii*) by this method the equation containing trigonometric, exponential and logarithmic terms cannot be analyzed.

(*iii*) if a physical quantity in mechanics depends on more than three factors, then relation among them cannot be established because we can have only three equations by equalizing the powers of *M, L* and *T*.

(*iv*) it doesn't tell whether the quantity is vector or scalar.

• Significant Figures

The significant figures are a measure of accuracy of a particular measurement of a physical quantity.

Significant figures in a measurement are those digits in a physical quantity that are known reliably plus the first digit which is uncertain.

• The Rules for Determining the Number of Significant Figures

(*i*) All non-zero digits are significant.

(*ii*) All zeroes between non-zero digits are significant.

(*iii*) All zeroes to the right of the last non-zero digit are not significant in numbers without decimal point.

(*iv*) All zeroes to the right of a decimal point and to the left of a non-zero digit are not significant.

(*v*) All zeroes to the right of a decimal point and to the right of a non-zero digit are significant.

(*vi*) In addition and subtraction, we should retain the least decimal place among the values operated, in the result.

(*vii*) In multiplication and division, we should express the result with the least number of significant figures as associated with the least precise number in operation.

(*viii*) If scientific notation is not used:

(*a*) For a number greater than 1, without any decimal, the trailing zeroes are not significant.

(*b*) For a number with a decimal, the trailing zeros are significant.

• Error

The measured value of the physical quantity is usually different from its true value. The result of every measurement by any measuring instrument is an approximate number, which contains some uncertainty. This uncertainty is called error. Every calculated quantity, which is based on measured values, also has an error.

• Causes of Errors in Measurement

Following are the causes of errors in measurement:

Least Count Error. The least count error is the error associated with the resolution of the instrument. Least count may not be sufficiently small. The maximum possible error is equal to the least count.

Instrumental Error. This is due to faulty calibration or change in conditions (*e.g.,* thermal expansion of a measuring scale). An instrument may also have a zero error. A correction has to be applied.

Random Error. This is also called chance error. It makes to give different results for same measurements taken repeatedly. These errors are assumed to follow the Gaussian law of normal distribution.

Accidental Error. This error gives too high or too low results. Measurements involving this error are not included in calculations.

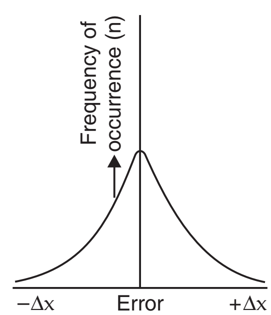


Illustration of Gaussian Law of Normal distribution.

Systematic Error. The systematic errors are those errors that tend to be in one direction, either positive or negative. Errors due to air buoyancy in weighing and radiation loss in calorimetry are systematic errors. They can be eliminated by manipulation. Some of the sources of systematic errors are:

(i) intrumental error

(*ii*) imperfection in experimental technique or procedure

(*iii*) personal errors

• Absolute Error, Relative Error and Percentage Error

– If $a_1, a_2, a_3, \ldots, a_n$ be the measured values of a quantity in several measurements, then their mean is considered to be the true value of that quantity i.e.,

$$\text{true value } a_0 = a_{\text{mean}} = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}$$

– The magnitude of the difference between the true value of the quantity and the individual measurement value is the absolute error of that measurement. Hence, absolute errors in measured values are:

$$\Delta a_1 = a_0 - a_1,\ \Delta a_2 = a_0 - a_2,\ \Delta a_3 = a_0 - a_3,\ \ldots\ldots\ldots,\ \Delta a_n = a_0 - a_n$$

– The arithmetic mean (*i.e.,* the mean of the magnitudes) of all the absolute errors is known as the mean absolute error.

$$\therefore \qquad \Delta a_{\text{mean}} = \frac{\left[|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \ldots + |\Delta a_n|\right]}{n}$$

– The ratio between mean absolute error and the mean value is called relative error.

$$\text{Relative error} = \frac{\text{Mean absolute error}}{\text{Mean value}} = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} = \frac{\Delta a_{\text{mean}}}{a_0}$$

– Percentage error is the expression of the relative error in percentage.

$$\text{Percentage error} = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100\%$$

• Combination of Errors

– If a quantity Z be expressed as the sum or difference of two quantities A and B (*i.e.,* if $Z = A + B$ or $Z = A - B$), then maximum value of error $\Delta Z = \Delta A + \Delta B$.

Hence, when two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.

– If a quantity Z be expressed as product or a quotient of quantities A and B, then the maximum fractional error in Z is given by

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Hence, when two quantities are multiplied or divided, the relative error in the result is the sum of the relative errors in the multipliers.

– If $Z = A^m B^n C^l$ etc., then maximum fractional error in Z is given by

$$\frac{\Delta Z}{Z} = m\frac{\Delta A}{A} + n\frac{\Delta B}{B} + l\frac{\Delta C}{C}$$

- **IMPORTANT TABLES**

TABLE 2.2 Some units retained for general use (Though outside SI)

Name	*Symbol*	*Value in SI Unit*
minute	min	60 s
hour	h	60 min = 3600 s
day	d	24 h = 86400 s
year	y	365.25 d = 3.156×10^{7} s
degree	°	1° = (π/180) rad
litre	L	I $dm^3 = 10^3$ m^3
tonne	t	10^3 kg
carat	c	200 mg
bar	bar	0.1 MPa = 10^5 Pa
curie	Ci	3.7×10^{10} s^{-1}
roentgen	R	2.58×10^{-4} C/kg
quintal	q	100 kg
barn	b	100 $fm^2 = 10^{-28}$ m^2
are	a	1 $dam^2 = 10^2$ m^2
hectare	ha	1 $hm^2 = 10^4$ m^2
standard atmospheric pressure	atm	101325 Pa = 1.013×10^5 Pa

TABLE 2.3 Range and order of lengths

Size of object or distance	*Length (m)*
Size of a proton	10^{-15}
Size of atomic nucleus	10^{-14}
Size of hydrogen atom	10^{-10}
Length of typical virus	10^{-8}
Wavelength of light	10^{-7}
Size of red blood corpuscle	10^{-5}
Thickness of a paper	10^{-4}
Height of the Mount Everest above sea level	10^{4}
Radius of the Earth	10^{7}
Distance of moon from the Earth	10^{8}
Distance of the Sun from the Earth	10^{11}
Distance of Pluto from the Sun	10^{13}
Size of our galaxy	10^{21}
Distance to Andromeda galaxy	10^{22}
Distance to the boundary of observable universe	10^{26}

TABLE 2.4 Range and order of masses

Object	*Mass (kg)*
Electron	10^{-30}
Proton	10^{-27}
Uranium atom	10^{-25}
Red blood cell	10^{-13}
Dust particle	10^{-9}
Rain drop	10^{-6}
Mosquito	10^{-5}
Grape	10^{-3}
Human	10^{2}
Automobile	10^{3}
Boeing 747 aircraft	10^{8}
Moon	10^{23}
Earth	10^{25}
Sun	10^{30}
Milky way galaxy	10^{41}
Observable Universe	10^{55}

TABLE 2.5 Range and order of time intervals

Event	*Time interval (s)*
Life-span of most unstable particle	10^{-24}
Time required for light to cross a nuclear distance	10^{-22}
Period of x-rays	10^{-19}
Period of atomic vibrations	10^{-15}
Period of light wave	10^{-15}
Life time of an excited state of an atom	10^{-8}
Period of a radio wave	10^{-6}
Period of a sound wave	10^{-3}
Wink of eye	10^{-1}
Time between successive human heart beats	10^{0}
Travel time for light from the Moon to the Earth	10^{0}
Travel time for light from the Sun to the Earth	10^{2}
Time period of a satellite	10^{4}
Rotation period of the Earth	10^{5}

Rotation and revolution periods of the moon	10^6
Revolution period of the Earth	10^7
Travel time for light from nearest star	10^8
Average human life-span	10^9
Age of Egyptian pyramids	10^{11}
Time since dinosaurs became extinct	10^{15}
Age of the universe	10^{17}

TABLE 2.6 Dimensional Formulae of Some Physical Quantities

Physical Quantity	*Dimensional Formula*	*Physical Quantity*		*Dimensional Formula*
Area	$[L^2]$	Capacitance		$[M^{-1} L^{-2} T^2 Q^2]$
Volume	$[L^3]$	Electric current		[I or Q T^{-1}]
Density	$[M L^{-3}]$	Electric potential		$[M L^2 T^{-2} Q^{-1}]$
Velocity	$[L T^{-1}]$		or	$[M L^2 T^{-3} I^{-1}]$
Acceleration	$[L T^{-2}]$	Electric field		$[M L^{-2} Q^{-1}]$
Momentum	$[M L T^{-1}]$		or	$[M L T^{-3} I^{-1}]$
Force	$[M L T^{-2}]$	Inductance		$[ML^2 Q^{-2}]$
Energy, work	$[M L^2 T^{-2}]$		or	$[M L^2 T^{-2} I^{-2}]$
Power	$[M L^2 T^{-3}]$	Resistance		$[M L^2 T^{-1} Q^{-2}]$
Frequency	$[T^{-1}]$		or	$[M L^2 T^{-3} I^{-2}]$
Pressure	$[M L^{-1} T^{-2}]$	Magnetic flux		$[M L^2 T^{-1} Q^{-1}]$
Torque, couple	$[M L^2 T^{-2}]$		or	$[M L^2 T^{-2} I^{-1}]$
Moment of inertia	$[M L^2]$	Magnetic field vector H		$[L^{-1} T^{-1} Q]$
Temperature	[K]		or	$[L^{-1} I]$
Heat energy	$[M L^2 T^{-2}]$	Magnetic feild intensity, B		$[M T^{-1} Q^{-1}]$
Entropy	$[M L^2 T^{-2} K^{-1}]$		or	$[M T^{-2} I^{-1}]$
Specific heat capacity	$[L^2 T^{-2} K^{-1}]$	Permeability		$[M L Q^{-2}]$
Specific latent heat	$[L^2 T^{-2}]$		or	$[M L T^{-2} I^{-2}]$
Thermal conductivity	$[M L T^{-3} K^{-1}]$	Permittivity		$[M^{-1} L^{-3} T^2 Q^2]$
Electric charge	[Q or IT]		or	$[M^{-1} L^{-3} T^4 I^2]$

NCERT TEXTBOOK QUESTIONS SOLVED

2.1. *Fill in the blanks*

(a) The volume of a cube of side 1 cm is equal to ..... m^3.

(b) The surface area of a solid cylinder of radius 2.0 cm and height 10.0 cm is equal to ... $(mm)^2$.

(c) A vehicle moving with a speed of 18 km h^{-1} covers .... m in 1 s.

(d) The relative density of lead is 11.3. Its density is .... g cm^{-3} or .... kg m^{-3}.

Sol. (*a*) Volume of cube, $V = (1 \text{ cm})^3 = (10^{-2} \text{ m})^3 = 10^{-6} \text{ m}^3$.

Hence, answer is 10^{-6}.

(*b*) Surface area $= 2\pi rh + 2\pi r^2 = 2\pi r\,(h + r)$

$$= 2 \times \frac{22}{7} \times 2 \times 10\ (10 \times 10 + 2 \times 10) \text{ mm}^2 = 1.5 \times 10^4 \text{ mm}^2$$

Hence, answer is 1.5×10^4.

(*c*) Speed of vehicle $= 18 \text{ km/h} = \frac{18 \times 1000}{3600} \text{ m/s}$

$= 5$ m/s ; so the vehicle covers 5 m in 1 s.

(*d*) Density $= 11.3 \text{ g cm}^{-3} = 11.3 \times \frac{10^{-3}}{10^{-6}} \text{ kg m}^{-3}$

$= 11.3 \times 10^3 \text{ kg m}^{-3}$ $[\because 1 \text{ kg} = 10^3 \text{ g}, 1\text{m} = 10^2 \text{ cm}]$

$= 1.13 \times 10^4 \text{ kg m}^{-3}$

2.2. *Fill in the blanks by suitable conversion of units*

(a) $1 \text{ kg m}^2 \text{ s}^{-2} = \ldots\text{ g cm}^2 \text{ s}^{-2}$ *(b)* 1 m = ... 1y

(c) $3.0 \text{ m s}^{-2} = \ldots\text{ km h}^{-2}$ *(d)* $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ (kg)}^{-2} = \ldots \text{(cm)}^3 \text{ s}^{-2} \text{ g}^{-1}$.

Sol. (*a*) $1 \text{ kg m}^2 \text{ s}^{-2} = \frac{1 \text{ kg m}^2}{s^2} = \frac{1 \times 1000 \times \left(10^2\right)^2}{s^2} \text{ g cm}^2 = 10^7 \text{ g cm}^2 \text{ s}^{-2}$

(*b*) $1 \text{ m} = \frac{1}{9.46 \times 10^5} \text{ ly} \approx \frac{1}{10^{16}} \text{ ly} = 10^{-16} \text{ly}$

(*c*) $3 \text{ ms}^{-2} = \frac{3 \times 10^{-3} \text{km}}{\left(\frac{1}{3600}\right)^2 \text{h}^2} = 3 \times 3600 \times 3600 \times 10^{-3} \text{ km h}^{-2} = 3.888 \times 10^4 \text{ km h}^{-2}$

(*d*) $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2} = 6.67 \times 10^{-11} \frac{\text{kg m}}{s^2} \text{ m}^2 \text{ kg}^{-2}$

$= 6.67 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}$

$= 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg s}^2} = \frac{6.67 \times 10^{-11} \times \left(10^2\right)^3}{\left(10^3\right)^2}$

$= 6.67 \times 10^{-8} \text{ cm}^{-3} \text{ s}^{-2} \text{ g}^{-1}$.

2.3. *A calorie is a unit of heat or energy and it equals about 4.2 J where 1 J = 1 kg m^2 s^{-2}. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β m, the unit of time is γs. Show that a calorie has a magnitude 4.2 α^{-1} β^{-2} γ^2 in terms of the new units.*

Sol. $1 \text{ cal} = 4.2 \text{ kg m}^2 \text{ s}^{-2}$

SI	New system
$n_1 = 4.2$	$n_2 = ?$
$M_1 = 1$ kg	$M_2 = \alpha$ kg
$L_1 = 1$m	$L_2 = \beta\ m$
$T_1 = 1$ s	$T_2 = y$ second

Dimensional formula of energy is $[M^1 L^2 T^{-2}]$

Comparing with $[M^a L^b T^c]$, we get

$$a = 1, \quad b = 2, \quad c = -2$$

Now, $$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c = 4.2 \left[\frac{1\text{ kg}}{\alpha\text{ kg}}\right]^1 \left[\frac{1\text{ m}}{\beta\, m}\right]^2 \left[\frac{1\text{ s}}{\gamma\, s}\right]^{-2}$$

or, $$n_2 = 4.2\, \alpha^{-1}\, \beta^{-2}\, \gamma^2.$$

2.4. *Explain this statement clearly:*

''To call a dimensional quantity 'large' or 'small' is meaningless without specifying a standard for comparison''. In view of this, reframe the following statements wherever necessary:

(a) atoms are very small objects

(b) a jet plane moves with great speed

(c) the mass of Jupiter is very large

(d) the air inside this room contains a large number of molecules

(e) a proton is much more massive than an electron

(f) the speed of sound is much smaller than the speed of light.

Sol. Physical quantities are called large or small depending on the unit (standard) of measurement. For example, the distance between two cities on earth is measured in kilometres but the distance between stars or inter—galactic distances are measured in parsec. The later standard parsec is equal to 3.08×10^{16} m or 3.08×10^{12} km is certainly larger than metre or kilometre. Therefore, the inter-stellar or intergalactic distances are certainly larger than the distances between two cities on earth.

(*a*) The size of an atom is much smaller than even the sharp tip of a pin.

(*b*) A Jet plane moves with a speed greater than that of a superfast train.

(*c*) The mass of Jupiter is very large compared to that of the earth.

(*d*) The air inside this room contains more number of molecules than in one mole of air.

(*e*) This is a correct statement.

(*f*) This is a correct statement.

2.5. *A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the Sun and the Earth in terms of the new unit if light takes 8 min and 20 s to cover this distance?*

Sol. Distance between Sun and Earth

= Speed of light in vacuum × time taken by light to travel from Sun to Earth

= 3×10^8 m/s × 8 min 20 s = 3×10^8 m/s × 500 s = $500 \times 3 \times 10^8$ m.

In the new system, the speed of light in vacuum is unity. So, the new unit of length is 3×10^8 m.

∴ distance between Sun and Earth = 500 new units.

2.6. *Which of the following is the most precise device for measuring length:*

(a) a vernier callipers with 20 divisions on the sliding scale.

(b) a screw gauge of pitch 1 mm and 100 divisions on the circular scale.

(c) an optical instrument that can measure length to within a wavelength of light?

Sol. (*a*) Least count of vernier callipers = $\frac{1}{20}$ = 0.05 mm = 5×10^{-5} m (*b*) Least count of screw gauge = $\dfrac{\text{Pitch}}{\text{No. of divisions on circular scale}}$

$$= \frac{1 \times 10^{-3}}{100} = 1 \times 10^{-5} \text{ m}$$

(*c*) Least count of optical instrument ≈ 6000 Å

(average wavelength of visible light as 6000 Å)

$$= 6 \times 10^{-7} \text{ m}$$

As the least count of optical instrument is least, it is the most precise device out of three instruments given to us.

2.7. *A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. What is the estimate on the thickness of hair?*

Sol. As magnification, $m = \dfrac{\text{thickness of image of hair}}{\text{real thickness of hair}} = 100$

and average width of the image of hair as seen by microscope = 3.5 mm

$\therefore$ Thickness of hair = $\dfrac{3.5 \text{ mm}}{100} = 0.035$ mm

2.8. *Answer the following:*

(a) You are given a thread and a metre scale. How will you estimate the diameter of the thread ?

(b) A screw gauge has a pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale?

(c) The mean diameter of a thin brass rod is to be measured by vernier callipers. Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only?

Sol. (*a*) Wrap the thread a number of times on a round pencil so as to form a coil having its turns touching each other closely. Measure the length of this coil, mode by the thread, with a metre scale. If n be the number of turns of the coil and l be the length of the coil, then the length occupied by each single turn *i.e.*, the thickness of the thread = $\dfrac{1}{n}$.

This is equal to the diameter of the thread.

(*b*) We know that least count = $\dfrac{\text{Pitch}}{\text{number of divisions on circular scale}}$

When number of divisions on circular scale is increased, least count is decreased. Hence the accuracy is increased. However, this is only a theoretical idea.

Practically speaking, increasing the number of turns would create many difficulties. As an example, the low resolution of the human eye would make observations difficult. The nearest divisions would not clearly be distinguished as separate. Moreover, it would be technically difficult to maintain uniformity of the pitch of the screw throughout its length.

(*c*) Due to random errors, a large number of observation will give a more reliable result than smaller number of observations. This is due to the fact that the probability (chance) of making a positive random error of a given magnitude is equal to that of making a negative random error of the same magnitude. Thus in a large number of observations, positive and negative errors are likely to cancel each other. Hence more reliable result can be obtained.

2.9. *The photograph of a house occupies an area of 1.75 cm^2 on a 35 mm slide. The slide is projected on to a screen, and the area of the house on the screen is 1.55 m^2. What is the linear magnification of the projector-screen arrangement?*

Sol. Here area of the house on slide = 1.75 cm^2 = 1.75×10^{-4} m^2 and area of the house of projector-screen = 1.55 m^2

$$\therefore \text{ Areal magnification } = \frac{\text{Area on screen}}{\text{Area on slide}} = \frac{1.55 \text{ m}^2}{1.75 \times 10^{-4} \text{ m}^2} = 8.857 \times 10^3$$

$$\therefore \text{ Linear magnification } = \sqrt{\text{Areal magnification}} = \sqrt{(8.857) \times 10^3}s = 94.1.$$

2.10. *State the number of significant figures in the following:*

(a) 0.007 m^2 (b) 2.64×10^4 kg (c) 0.2370 g cm^{-3}

(d) 6.320 J (e) 6.032 N m^{-2} (f) 0.0006032 m^2

Sol. (*a*) 1 (*b*) 3 (*c*) 4 (*d*) 4 (*e*) 4 (*f*) 4.

2.11. *The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.*

Sol. As Area = $(4.234 \times 1.005) \times 2 = 8.51034 = 8.5$ m^2

Volume = $(4.234 \times 1.005) \times (2.01 \times 10^{-2}) = 8.55289 \times 10^{-2} = 0.0855$ m^3.

2.12. *The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box (b) the difference in the masses of the pieces to correct significant figures?*

Sol. (*a*) Total mass of the box = (2.3 + 0.0217 + 0.0215) kg = 2.3442 kg

Since the least number of decimal places is 1, therefore, the total mass of the box = 2.3 kg.

(*b*) Difference of mass = 2.17 – 2.15 = 0.02 g

Since the least number of decimal places is 2 so the difference in masses to the correct significant figures is 0.02 g.

2.13. *A physical quantity P is related to four observables a, b, c and d as follows:*

$$P = a^3b^2/\left(\sqrt{c}d\right)$$

The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2%, respectively. What is the percentage error in the quantity P? If the value of P calculated using the above relation turns out to be 3.763, to what value should you round off the result?

Sol. As $P = \dfrac{a^3 b^2}{\left(\sqrt{c}d\right)}s = a^3 b^2 c^{-1/2} d^{-1}$

$\therefore$ Maximum fractional error in the mearurement

$$\frac{\Delta P}{P} = 3\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + \frac{\Delta d}{d}$$

As $\dfrac{\Delta a}{a} = 1\%$, $\dfrac{\Delta b}{b} = 3\%$, $\dfrac{\Delta c}{c} = 4\%$ and $\dfrac{\Delta d}{d} = 2\%$

∴ Maximum fractional error in the measurement

$$\frac{\Delta P}{P} = 3 \times 1\% + 2 \times 3\% + \frac{1}{2} \times 4\% + 2\% = 3\% + 6\% + 2\% + 2\% = 13\%$$

If $P = 3.763$, then $\Delta P = 13\%$ of $P = \frac{13P}{100} = \frac{13 \times 3.763}{100} = 0.489$

As the error lies in first decimal place, the answer should be rounded off to first decimal place. Hence, we shall express the value of P after rounding it off as

$$P = 3.8.$$

2.14. *A book with many printing errors contains four different formulas for the displacement y of a particle undergoing a certain periodic motion:*

(a) $y = a \sin \frac{2\pi t}{T}$ (b) $y = a \sin vt$

(c) $y = \left(\frac{a}{T}\right) \sin \frac{t}{a}$ (d) $y = \left(\frac{a}{\sqrt{2}}\right)\left(\sin \frac{2\pi t}{T} + \cos \frac{2\pi t}{T}\right)$

(a = maximum displacement of the particle,

v = speed of the particle, T = time-period of motion)

Rule out the wrong formulas on dimensional grounds.

Sol. According to dimensional analysis an equation must be dimensionally homogeneous.

(*a*) $y = a \sin \frac{2\pi t}{T}$

Here, [L.H.S.] = $[y] = [L]$ and [R.H.S.] = $\left[a \sin \frac{2\pi t}{T}\right] = \left[L \sin \frac{T}{T}\right] = [L]$

So, it is correct.

(*b*) $y = a \sin vt$

Here, $[y] = [L]$ and $[a \sin vt] = [L \sin (LT^{-1}. T)] = [L \sin L]$

So, the equation is wrong.

(*c*) $y = \left(\frac{a}{T}\right) \sin \frac{t}{a}$

Here, $[y] = [L]$ and $\left[\left(\frac{a}{T}\right) \sin \frac{t}{a}\right] = \left[\frac{L}{T} \sin \frac{T}{L}\right] = [LT^{-1} \sin TL^{-1}]$

So, the equation is wrong.

(*d*) $y = \left(a\sqrt{2}\right)\left(\sin \frac{2\pi t}{T} + \cos \frac{2\pi t}{T}\right)$

Here, $[y] = [L], \left[a\sqrt{2}\right] = [L]$

and $\left[\sin \frac{2\pi t}{T} + \cos \frac{2\pi t}{T}\right] = \left[\sin \frac{T}{T} + \cos \frac{T}{T}\right]$ = dimensionless

So, the equation is correct.

2.15. *A famous relation in physics relates 'moving mass' m to the 'rest mass' m_0 of a particle in terms of its speed v and the speed of light c. (This relation first arose as a consequence of special relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c. He writes:*

$$m = \frac{m_0}{\left(1 - v^2\right)^{1/2}} s$$

Guess where to put the missing c.

Sol. From the given equation, $\frac{m_0}{m} = \sqrt{1 - v^2}$

Left hand side is dimensionless.

Therefore, right hand side should also be dimensionless.

It is possible only when $\sqrt{1 - v^2}$ should be $\sqrt{1 - \frac{v^2}{c^2}}$.

Thus, the correct formula is $m = m_0 \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$.

2.16. *The unit of length convenient on the atomic scale is known as an angstrom and is denoted by Å: 1 Å = 10^{-10} m. The size of a hydrogen atom is about 0.5 Å. What is the total atomic volume in m^3 of a mole of hydrogen atoms?*

Sol. Volume of one hydrogen atom = $\frac{4}{3}\pi r^3$ (volume of sphere)

$$= \frac{4}{3} \times 3.14 \times (0.5 \times 10^{-10})\ \text{m}^3 = 5.23 \times 10^{-31}\ \text{m}^3$$

According to Avagadro's hypothesis, one mole of hydrogen contains 6.023×10^{23} atoms.

∴ Atomic volume of 1 mole of hydrogen atoms

$$= 6.023 \times 10^{23} \times 5.23 \times 10^{-31} = 3.15 \times 10^{-7}\ \text{m}^3.$$

2.17. *One mole of an ideal gas at standard temperature and pressure occupies 22.4 L (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen? (Take the size of hydrogen molecule to be about 1 Å.) Why is this ratio so large?*

Sol. Volume of one mole of ideal gas, V_g

$$= 22.4\ \text{litre} = 22.4 \times 10^{-3}\ \text{m}^3$$

Radius of hydrogen molecule = $\frac{1\ Å}{2}$

$$= 0.5\ Å = 0.5 \times 10^{-10}\ \text{m}$$

Volume of hydrogen molecule = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} (0.5 \times 10^{-10})^3\ \text{m}^3 = 0.5238 \times 10^{-30}\ \text{m}^3$$

One mole contains 6.023×10^{23} molecules.

∴ Volume of one mole of hydrogen, V_H

$$= 0.5238 \times 10^{-30} \times 6.023 \times 10^{23}\ \text{m}^3 = 3.1548 \times 10^{-7}\ \text{m}^3$$

Now, $\dfrac{V_g}{V_H} = \dfrac{22.4 \times 10^{-3}}{3.1548 \times 10^{-7}} = 7.1 \times 10^4$

The ratio is very large. This is because the interatomic separation in the gas is very large compared to the size of a hydrogen molecule.

2.18. *Explain this common observation clearly: If you look out of the window of a fast moving train, the nearby trees, houses etc., seem to move rapidly in a direction opposite to the train's motion, but the distant objects (hill tops, the Moon, the stars etc.) seem to be stationary. (In fact, since you are aware that you are moving, these distant objects seem to move with you).*

Sol. The line joining a given object to our eye is known as the line of sight. When a train moves rapidly, the line of sight of a passenger sitting in the train for nearby trees changes its direction rapidly. As a result, the nearby trees and other objects appear to run in a direction opposite to the train's motion. However, the line of sight of distant and large size objects *e.g.*, hill tops, the Moon, the stars etc., almost remains unchanged (or changes by an extremely small angle). As a result, the distant object seems to be stationary.

2.19. *The principle of 'parallax' is used in the determination of distances of very distant stars. The baseline AB is the line joining the Earth's two locations six months apart in its orbit around the Sun. That is, the baseline is about the diameter of the Earth's orbit ≈ 3 × 10¹¹ m. However, even the nearest stars are so distant that with such a long baseline, they show parallel only of the order of 1″ (second) of arc or so. A parsec is a convenient unit of length on the astronomical scale. It is the distance of an object that will show a parallax of 1″ (second) of arc from opposite ends of a baseline equal to the distance from the Earth to the Sun. How much is a parsec in terms of metres?*

Sol. From parallax method we can say

$$\theta = \frac{b}{D}, \quad \text{where } b = \text{baseline}$$

$$D = \text{distance of distant object or star}$$

Since, $\theta = 1''$ (s) and $b = 3 \times 10^{11}$ m

$$\theta = 4.85 \times 10^{-6} \text{ rad.}$$

$$D = \frac{b}{2\theta} = \frac{3 \times 10^{11}}{2 \times 4.85 \times 10^{-6}} \text{ m}$$

or, $$D = \frac{3 \times 10^{11}}{9.7 \times 10^{-6}} \text{ m} = \frac{30 \times 10^{16}}{9.7} \text{ m} = 3.09 \times 10^{16} \text{ m} \approx 3 \times 10^{16} \text{ m.}$$

2.20. *The nearest star to our solar system is 4.29 light years away. How much is this distance in terms of parsecs? How much parallax would this star (named Alpha Centauri) show when viewed from two locations of the Earth six months apart in its orbit around the Sun?*

Sol. As we know, 1 light year = 9.46×10^{15} m

∴ 4.29 light years = $4.29 \times 9.46 \times 10^{15} = 4.058 \times 10^{16}$ m

Also, 1 parsec = 3.08×10^{16} m

∴ 4.29 light years = $\dfrac{4.058 \times 10^{16}}{3.08 \times 10^{16}}$ = 1.318 parsec = 1.32 parsec.

As a parsec distance subtends a parallax angle of 1″ for a basis of radius of Earth's orbit around the Sun (r).

In present problem base is the distance between two locations of the Earth six months apart in its orbit around the Sun = diameter of Earth's orbit ($b = 2r$).

∴ Parallax angle subtended by 1 parsec distance at this basis = 2 second (by definition of parsec).

∴ Parallax angle subtended by the star Alpha Centauri at the given basis

$$\theta = 1.32 \times 2 = 2.64''.$$

2.21. *Precise measurements of physical quantities are a need of science. For example, to ascertain the speed of an aircraft, one must have an accurate method to find its positions at closely separated instants of time. This was the actual motivation behind the discovery of radar in World War II. Think of different examples in modern science where precise measurements of length, time, mass etc., are needed. Also, wherever you can, give a quantitative idea of the precision needed.*

Sol. Extremely precise measurements are needed in modern science. As an example, while launching a satellite using a space launch rocket system we must measure time to a precision of 1 micro second. Again working with lasers we require length measurements to an angstrom unit (1 Å = 10^{-10} m) or even a fraction of it. For estimating nuclear sizes we require a precision of 10^{-15} m. To measure atomic masses using mass spectrograph we require a precision of 10^{-30} kg and so on.

2.22. *Just as precise measurements are necessary in science, it is equally important to be able to make rough estimates of quantities using rudimentary ideas and common observations. Think of ways by which you can estimate the following (where an estimate is difficult to obtain, try to get an upper bound on the quantity):*

(a) the total mass of rain-bearing clouds over India during the Monsoon
(b) the mass of an elephant
(c) the wind speed during a storm
(d) the number of strands of hair on your head
(e) the number of air molecules in your classroom.

Sol. (*a*) The average rainfall of nearly 100 cm or 1 m is recorded by meteorologists, during Monsoon, in India. If A is the area of the country, then

$$A = 3.3 \text{ million sq. km} = 3.3 \times 10^6 \text{ (km)}^2$$
$$= 3.3 \times 10^6 \times 10^6 \text{ m}^2 = 3.3 \times 10^{12} \text{ m}^2$$

Mass of rain–bearing clouds

$$= \text{area} \times \text{height} \times \text{density} = 3.3 \times 10^{12} \times 1 \times 1000 \text{ kg} = 3.3 \times 10^{15} \text{ kg}.$$

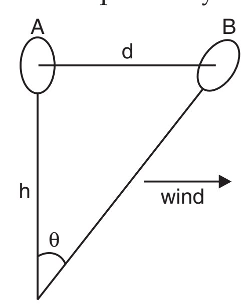
(*b*) Measure the depth of an empty boat in water. Let it be d_1. If A be the base area of the boat, then volume of water displaced by boat, $V_1 = Ad_1$.

Let d_2 be the depth of boat in water when the elephant is moved into the boat.
Volume of water displaced by (boat + elephant), $V_2 = Ad_2$
Volume of water displaced by elephant,

$$V = V_2 - V_1 = A(d_2 - d_1)$$

If ρ be the density of water, then mass of elephant = mass of water displaced by it = $A(d_2 - d_1)\,\rho$.

(*c*) Wind speed can be estimated by floating a gas-filled balloon in air at a known height h. When there is no wind, the balloon is at A. Suppose the wind starts blowing to the right such that the balloon drifts to position B in 1 second.



Now, $AB = d = h\theta$.

The value of d directly gives the wind speed.

(*d*) Let us assume that the man is not partially bald. Let us further assume that the hair on the head are uniformly distributed. We can estimate the area of the head. The thickness of a hair can be measured by using a screw gauge. The number of hair on the head is clearly the ratio of the area of head to the cross-sectional area of a hair.

Assume that the human head is a circle of radius 0.08 m *i.e.*, 8 cm. Let us further assume that the thickness of a human air is 5×10^{-5} m.

Number of hair on the head

$$= \frac{\text{Area of the head}}{\text{Area of cross - section of a hair}}$$

$$= \frac{\pi(0.08)^2}{\pi\left(5\times10^{-5}\right)^2} = \frac{64\times10^{-4}}{25\times10^{-10}} = 2.56\times10^6$$

The number of hair on the human head is of the order of one million.

(*e*) We can determine the volume of the class-room by measuring its length, breadth and height. Consider a class room of size 10 m × 8 m × 4 m. Volume of this room is 320 m^3.

We know that 22.4*l* or 22.4×10^{-3} m^3 of air has 6.02×10^{23} molecules (equal to Avogadro's number).

∴ Number of molecules of air in the class room

$$= \frac{6.02\times10^{23}}{22.4\times10^{-3}} \times 320 = 8.6 \times 10^{27}.$$

2.23. *The Sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding 10^7 K, and its outer surface at a temperature of about 6000 K. At these high temperatures, no substance remains in a solid or liquid phase. In what range do you expect the mass density of the Sun to be, in the range of densities of solids and liquids or gases? Check if your guess is correct from the following data: mass of the Sun = 2.0×10^{30} kg, radius of the Sun = 7.0×10^8 m.*

Sol. Given $M = 2 \times 10^{30}$ kg, $r = 7 \times 10^8$ m

∴ Volume of Sun $= \frac{4}{3}\pi r^3 \times 3.14 \times (7 \times 10^8)^3 = 1.437 \times 10^{27}$ m^3

As $\rho = \frac{M}{V}$, ∴ $\rho = \frac{2\times10^{30}}{1.437\times10^{27}} = 1391.8$ kg m^{-3} $= 1.4 \times 10^3$ kg m^{-3}

Mass density of Sun is in the range of mass densities of solids/liquids and not gases.

2.24. *When the planet Jupiter is at a distance of 824.7 million kilometres from the Earth, its angular diameter is measured to be 35.72'' of arc. Calculate the diameter of Jupiter.*

Sol. Given angular diameter θ = 35.72'' = $35.72 \times 4.85 \times 10^{-6}$ rad

$= 173.242 \times 10^{-6} = 1.73 \times 10^{-4}$ rad

∴ Diameter of Jupiter $= D = \theta \times d = 1.73 \times 10^{-4} \times 824.7 \times 10^9$ m

$= 1426.731 \times 10^5 = 1.43 \times 10^8$ m

2.25. *A man walking briskly in rain with speed v must slant his umbrella forward making an angle θ with the vertical. A student derives the following relation between θ and v: tan θ = v and checks that the relation has a correct limit: as $v \to 0$, $\theta \to 0$, as expected. (We are assuming there is no strong wind and that the rain falls vertically for a stationary man). Do you think this relation can be correct? If not, guess the correct relation.*

Sol. According to principle of homogenity of dimensional equations,

Dimensions of L.H.S. = Dimensions of R.H.S.

Here, $v = \tan\theta$

i.e., $[L^1 T^{-1}]$ = dimensionless, which is incorrect.

Correcting the L.H.S., we get

$$\frac{v}{u} = \tan\theta, \text{ where } u \text{ is velocity of rain.}$$

2.26. *It is claimed that two cesium clocks, if allowed to run for 100 years, free from any disturbance, may differ by only about 0.02 s. What does this imply for the accuracy of the standard cesium clock in measuring a time-interval of 1 s?*

Sol. Total time = 100 years = 100 × 365 × 24 × 60 × 60 s

$$\text{Error in 1 second} = \frac{0.02}{100 \times 365 \times 24 \times 60 \times 60} = 6.34 \times 10^{-12} \text{ s}$$

∴ Accuracy of 1 part in 10^{11} to 10^{12}.

2.27. *Estimate the average mass density of a sodium atom assuming its size to be about 2.5 Å. (Use the known values of Avogadro's number and the atomic mass of sodium). Compare it with the density of sodium in its crystalline phase: 970 kg m^{-3}. Are the two densities of the same order of magnitude? If so, why?*

Sol. It is given that radius of sodium atom, R = 2.5 Å = 2.5×10^{-10} m

∴ Volume of one mole atom of sodium, $V = N_A \cdot \frac{4}{3} \pi R^3$

$$V = 6.023 \times 10^{23} \times \frac{4}{3} \times 3.14 \times (2.5 \times 10^{-10})^3 \text{ m}^3$$

and mass of one mole atom of sodium, M = 23 g = 23×10^{-3} kg

∴ Average mass density of sodium atom, $\rho = \frac{M}{V}$

$$= \frac{23 \times 10^{-3}}{6.023 \times 10^{23} \times \frac{4}{3} \times 3.14 \times \left(2.5 \times 10^{-10}\right)}$$

$$= 6.96 \times 10^2 \text{ kg m}^{-3} = 0.7 \times 10^{-3} \text{ kg m}^{-3}$$

The density of sodium in its crystalline phase = 970 kg m^{-3}

$$= 0.97 \times 10^3 \text{ kg m}^{-3}$$

Obviously the two densities are of the same order of magnitude (= 10^3 kg m^{-3}). It is on account of the fact that in solid phase atoms are tightly packed and so the atomic mass density is close to the mass density of solid.

2.28. *The unit of length convenient on the nuclear scale is a fermi: 1 f = 10^{-15} m. Nuclear sizes obey roughly the following empirical relation:*

$$r = r_0 A^{1/3}$$

where r is the radius of the nucleus, A its mass number, and r_0 is a constant equal to about, 1.2 f. Show that the rule implies that nuclear mass density is nearly constant for different nuclei. Estimate the mass density of sodium nucleus. Compare it with the average mass density of a sodium atom obtained in Exercise 2.27.

Sol. Assume that the nucleus is spherical. Volume of nucleus

$$= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left[r_0 A^{1/3}\right]^3 = \frac{4}{3}\pi r_0^3 A$$

Mass of nucleus = A

$$\therefore \text{ Nuclear mass density } = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} = \frac{A}{\frac{4}{3}\pi r_0^3 A} = \frac{3}{4\pi r_0^3}$$

Since r_0 is a constant therefore the right hand side is a constant. So, the nuclear mass density is independent of mass number. Thus, nuclear mass density is constant for different nuclei.

For sodium, $A = 23$

$\therefore$ radius of sodium nucleus,

$$r = 1.2 \times 10^{-15} (23)^{1/3} \text{ m} = 1.2 \times 2.844 \times 10^{-15} \text{ m} = 3.4128 \times 10^{-15} \text{ m}$$

$$\text{Volume of nucleus} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} (3.4128 \times 10^{-15})^3 \text{ m}^3 = 1.66 \times 10^{-43} \text{ m}^3$$

If we neglect the mass of electrons of a sodium atom, then the mass of its nucleus can be taken to be the mass of its atom.

$\therefore$ Mass of sodium nucleus = 3.82×10^{-26} kg

(Refer to Q. 2.27)

Mass density of sodium nucleus

$$= \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} = \frac{3.82 \times 10^{-26}}{1.66 \times 10^{-43}} \text{ kg m}^{-3} = 2.3 \times 10^{17} \text{ kg m}^{-3}$$

Mass density of sodium atom = 4.67×10^3 kg m^{-3}

(Refer to Q. 2.27)

The ratio of the mass density of sodium nucleus to the average mass density of a sodium atom is

$$\frac{2.3 \times 10^{17}}{4.67 \times 10^3} \text{ i.e., } 4.92 \times 10^{13}.$$

So, the nuclear mass density is nearly 50 million times more than the atomic mass density for a sodium atom.

2.29. *A LASER is source of very intense, monochromatic, and unidirectional beam of light. These properties of a laser light can be exploited to measure long distances. The distance of the Moon from the Earth has been already determined very precisely using a laser as a source of light. A laser light beamed at the Moon takes 2.56 s to return after reflection at the Moon's surface. How much is the radius of the lunar orbit around the Earth?*

Sol. We known that speed of laser light = $c = 3 \times 10^8$ m/s. If d be the distance of Moon from the earth, the time taken by laser signal to return after reflection at the Moon's surface

$$t = 2.56 \text{ s} = \frac{2d}{c} = \frac{2d}{3 \times 10^8 \text{ ms}^{-1}}$$

$$\Rightarrow \quad d = \frac{1}{2} \times 2.56 \times 3 \times 10^8 \text{ m} = 3.84 \times 10^8 \text{ m}.$$

2.30. *A SONAR (sound navigation and ranging) uses ultrasonic waves to detect and locate objects under water. In a submarine equipped with a SONAR the time delay between generation of a probe wave and* *the reception of its echo after reflection from an enemy submarine is found to be 77.0 s. What is the distance of the enemy submarine? (Speed of sound in water = 1450 m s^{-1}).*

Sol. Here speed of sound in water v = 1450 m s^{-1} and time of echo t = 77.0 s.

If distance of enemy submarine be d, then $t = \frac{2d}{v}$

$$\therefore \quad d = \frac{vt}{2} = \frac{1450 \times 77.0}{2} = 55825 \text{ m} \simeq 55.8 \times 10^3 \text{ m or } 55.8 \text{ km.}$$

2.31. *The farthest objects in our Universe discovered by modern astronomers are so distant that light emitted by them takes billions of years to reach the Earth. These objects (known as quasars) have many puzzling features, which have not yet been satisfactorily explained. What is the distance in km of a quasar from which light takes 3.0 billion years to reach us?*

Sol. The time taken by light from the quasar to the observer

$$t = 3.0 \text{ billion years} = 3.0 \times 10^9 \text{ years}$$

As $\quad 1 \text{ ly} = 9.46 \times 10^{15}$ m

∴ Distance of quasar from the observer

$$d = 3.0 \times 10^9 \times 9.46 \times 10^{15} \text{ m} = 28.38 \times 10^{24} \text{ m}$$
$$\simeq 2.8 \times 10^{25} \text{ m or } 2.8 \times 10^{22} \text{ km.}$$

2.32. *It is a well known fact that during a total solar eclipse the disk of the Moon almost completely covers the disk of the Sun. From this fact and from the information you can gather from examples 2.3 and 2.4, determine the approximate diameter of the Moon.*

Sol. From examples 2.3 and 2.4, we get θ = 1920″ and $S = 3.8452 \times 10^8$ m. During the total solar eclipse, the disc of the moon completely covers the disc of the sun, so the angular diameter of both the sun and the moon must be equal.

∴ Angular diameter of the moon, θ = Angular diameter of the sun

$$= 1920'' = 1920 \times 4.85 \times 10^{-6} \text{ rad} \qquad [\because 1'' = 4.85 \times 10^{-6} \text{ rad}]$$

The earth-moon distance, $S = 3.8452 \times 10^8$ m

∴ The diameter of the moon, $D = \theta \times S$

$$= 1920 \times 4.85 \times 10^{-6} \times 3.8452 \times 10^8 \text{ m} = 35806.5024 \times 10^2 \text{ m}$$
$$= 3581 \times 10^3 \text{ m} = 3581 \text{ km.}$$

2.33. *A great physicist of this century (P.A.M. Dirac) loved playing with numerical values of fundamental constants of nature. This led him to an interesting observation. Dirac found that from the basic constants of atomic physics (c, e, mass of electron, mass of proton) and the gravitational constant G, he could arrive at a number with the dimension of time. Further, it was a very large number, its magnitude being close to the present estimate on the age of the universe (≈ 15 billion years). From the table of fundamental constants in this book, try to see if you too can construct this number (or any other interesting number you can think of). If its coincidence with the age of the universe were significant, what would this imply for the constancy of fundamental constants?*

Sol. The values of different fundamental constants are given below:

Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg

Mass of a proton, $m_p = 1.67 \times 10^{-27}$ kg

Speed of light, $c = 3 \times 10^8$ m/s

Gravitational constant, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$$

We have to try to make permutations and combinations of the universal constants and see if there can be any such combination whose dimensions come out to be the dimensions of time. One such combination is:

$$\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \cdot \frac{1}{m_p\, m_e^2\, c^3\, G} s$$

According to Coulomb's law of electrostatics,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{(e)(e)}{r^2} \quad \text{or,} \quad \frac{1}{4\pi\varepsilon_0} = \frac{Fr^2}{e^2} \quad \text{or} \quad \left(\frac{1}{4\pi\varepsilon_0}\right)^2 = \frac{F^2r^4}{e^4}$$

According to Newton's law of gravitation,

$$F = G\frac{m_1\, m_2}{r^2} \quad \text{or} \quad G = \frac{Fr^2}{m_1\, m_2}$$

Now, $$\left[\frac{e^4}{(4\pi\varepsilon_0)^2\, m_p\, m_e^2\, c^3 G}\right] = \left[e^4\left(\frac{F^2r^4}{e^4}\right)\frac{1}{m_p\, m_e^2\, c^3}\frac{m_1\, m_2}{Fr^2}\right]s$$

$$= \left[\frac{Fr^2}{mc^3}\right] = \left[\frac{MLT^{-2}L^2}{ML^3T^{-3}}\right] = [T]$$

Clearly, the quantity under discussion has the dimensions of time.

Substituting values in the quantity under discussion, we get

$$\frac{(1.6\times10^{-19})^4\,(9\times10^9)^2}{(1.69\times10^{-27})(9.1\times10^{-31})^2\,(3\times10^8)^3\,(6.67\times10^{-11})}$$

$$= 2.1 \times 10^{16} \text{ second}$$

$$= \frac{2.1\times10^{16}}{60\times60\times24\times365.25} \text{ years}$$

$$= 6.65 \times 10^8 \text{ years} = 10^9 \text{ years}$$

The estimated time is nearly one billion years.

QUESTIONS BASED ON SUPPLEMENTARY CONTENTS

Q. 1. *The radius of a sphere is measured as (2.1 ± 0.5) cm calculate its surface area with error limits.*

Sol. Radius of the sphere = (2.1 ± 0.5) cm

$\therefore$ $r = 2.1$ and $\Delta r = \pm 0.5$

S.A. = $4\pi r^2 = 4 \times 3.14 \times 2.1 \times 2.1 = 55.4 \text{ cm}^2$

As per the principle of error

$$\frac{\Delta s}{s} = \pm 2\cdot\frac{\Delta r}{r}$$

$$\frac{\Delta s}{55.4} = \pm\frac{2\times 0.5}{2.1}$$

$\therefore$ $\Delta s = \pm$ 26.4 cm

$\therefore$ Error limits are ± 26.4 cm

$\therefore$ Surface area of the sphere = (55.4 ± 26.4) cm^2 **Ans.**

Q. 2. *The voltage across a lamp is (6.0 ± 0.1) volt and the current passing through it is (4.0 ± 0.2) ampere. Find the power consumed by the lamp.*

Sol. Power $P = V \times I$

$P = 6 \times 4 = 24$ watt

Here $\Delta V = \pm$ 0.1 volts

and $\Delta I = \pm 1.6$ A

As per the principle of error;

$$\therefore \quad \frac{\Delta P}{P} = \pm\left(\frac{\Delta V}{V}+\frac{\Delta I}{I}\right); \quad \frac{\Delta P}{24} = \pm\left(\frac{0.1}{6}+\frac{0.2}{4}\right); \quad \frac{\Delta P}{24} = \pm\frac{0.8}{12}$$

$\therefore$ $\Delta P = \pm 1.6$ watt

$\therefore$ Power with error limit is (24 ± 1.6) watt **Ans.**

Q. 3. *The length and breadth of a rectangular block are 25.2 cm and 16.8 cm, which have both been measured to an accuracy of 0.1 cm. Find the area of the rectangular block.*

Sol. Here l = (25.2 ± 0.1) cm

b = (16.8 ± 0.1) cm

Area $= l \times b = 25.2 \times 16.8 = 423.4$ cm^2

As per the principle of error,

$$\frac{\Delta A}{A} = \pm\left(\frac{\Delta P}{P}+\frac{\Delta b}{b}\right)$$

$$\Rightarrow \quad \frac{\Delta A}{423.4} = \pm\left(\frac{0.1}{25.2}+\frac{0.1}{16.8}\right) \quad \Rightarrow \quad \Delta A = \pm\frac{423.4\times 0.1\times 42}{25.2\times 16.8}$$

$\Rightarrow$ $\Delta A = \pm 4.2$ cm^2

Hence the area with error limit = (423.4 ± 4.2) cm^2 **Ans.**

Q. 4. *A force of (2500 ± 5) N is applied over an area of (0.32 ± 0.02) m^2. Calculate the pressure exerted over the area.*

Sol. Here force F = (2500 ± 5) N and area A = (0.32 ± 0.02) m^2

Pressure $P = \dfrac{F}{A}$

$$P = \frac{2500}{0.32}$$

$\therefore \quad P = 7812.5 \text{ Nm}^{-2}$

As per the principle of error,

$$\frac{\Delta P}{P} = \pm\left(\frac{\Delta F}{F} + \frac{\Delta A}{A}\right)$$

$$\frac{\Delta P}{7812.5} = \pm\left(\frac{5}{2500} + \frac{0.02}{0.32}\right)$$

$$\Delta P = \pm 7812.5\left(\frac{1}{500} + \frac{1}{16}\right) = \pm 7812.5 \times \frac{516}{8000} = \pm 503.9 \text{ Nm}^{-2}$$

Hence, the pressure with error limit = (7812.5 ± 503.9) Nm^{-2}

Q. 5. *To find the value of 'g' by using a simple pendulum, the following observations were made :*

Length of thread $\quad l = (100 \pm 0.1)$ *cm*

Time period of oscillation $\quad T = (2 \pm 0.1)$ *sec*

Calculate the maximum permissible error in measurement of 'g'. Which quantity should be measured more accurately and why?

Sol. Here $\quad l = (100 \pm 0.1)$ cm and $T = (2 \pm 0.1)$ sec

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T^2 = 4\pi^2 \times \frac{l}{g}$$

$$\therefore \quad g = \frac{4\pi^2 l}{T^2}$$

As per the principle of error,

$$\frac{\Delta g}{g} = \pm\left(\frac{\Delta l}{l} + 2\frac{\Delta T}{T}\right)$$

$$\frac{\Delta g}{9.8} = \pm\left(\frac{0.1}{100} + 2 \times \frac{0.1}{2}\right)$$

$$\Delta g = \pm 9.8 \times 0.101 = \pm 0.99$$

$\therefore$ Maximum permissible error in the measurement of g = ±0.99. Time period of the pendulum should be measured more accurately as $g \propto \frac{1}{T^2}$.

Q. 6. *For a glass prism of refracting angle 60°, the minimum angle of deviation D_m is found to be 36° with a maximum error of 1.05°. When a beam of parallel light is incident on the prism, find the range of experimental value of refractive index 'μ'. It is known that the refractive index 'μ' of the material of the prism is given by*

$$\mu = \frac{\sin\left(\frac{A + D_m}{2}\right)}{\sin(A/2)}$$

Sol. Error in calculated D_m is ±1.05°

Given : $$\mu = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\frac{A}{2}}$$

$$\mu = \frac{\sin\left(\frac{36^\circ \pm 1.05^\circ}{2}\right)}{\sin\left(\frac{60}{2}\right)^\circ} = \frac{\sin\left(\frac{37.05}{2}\right)}{\sin 30^\circ}$$

$$= \frac{\sin(18.525^\circ)}{\frac{1}{2}} \text{ or } \frac{\sin(17.475^\circ)}{\frac{1}{2}}$$

$\Rightarrow$ 2×0.755 or 2×0.73

$\Rightarrow$ 1.51 or 1.46

Here range of μ is

$$1.46 \leq \mu \leq 1.51$$

Q. 7. The radius of curvature of a concave mirror, measured by a spherometer is given by

$$R = \frac{l^2}{6h} + \frac{h}{2}$$

The value of l and h are 4.0 cm and 0.065 cm respectively where l is measured by a metre scale and h by the spherometer. Find the relative error in the measurement of R.

Sol. Given that $l = 4$ cm and $\Delta l = 0.1$ cm (least count of the metre scale) here l is the distance between the legs of the spherometer.

As $$R = \frac{l^2}{6h} + \frac{h}{2}$$

$\therefore$ $$\frac{\Delta R}{R} = \frac{2\Delta l}{l} + \left(-\frac{\Delta h}{h}\right) + \frac{\Delta h}{h}$$

$\Rightarrow$ $$\frac{\Delta R}{R} = 2\frac{\Delta l}{l} + \frac{\Delta h}{h} + \frac{\Delta h}{h}$$ (Considering the magnitude only)

$$= 2\left(\frac{\Delta l}{l} + \frac{\Delta h}{h}\right) = 2\left(\frac{0.1}{4}\right) + 2 \times \left(\frac{0.001}{0.065}\right)$$

$= 0.05 + 0.03 = 0.08$ **Ans.**

Q. 8. *In Searle's experiment, the diameter of the wire as measured by a screw guage of least count 0.001 cm, is 0.500 cm. The length, measured by a scale of least count 0.1 cm is 110.0 cm. When a weight of 40 N is suspended from the wire, its extension is measured to be 0.125 cm by a micrometer of least count 0.001 cm. Find the Young's modulus of the material of the wire from this data.*

Sol. Young's modulus of the material of the wire is given as

$$Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}} = \frac{F/A}{l/L} = \frac{FL}{A \cdot l}$$

$$= \frac{F \cdot L}{\frac{\pi d^2}{4} \times l} = \frac{4FL}{\pi d^2 l}$$

Here $F = 40$ N, $L = 110$ cm $= 1.1$ m

$l = 0.125$ cm $= 0.00125$ m and $d = 0.500$ cm $= 0.005$ m

$$\therefore \quad Y = \frac{4\times 40\times 1.1}{0.00125\times 3.14\times (0.005)^2} = 2.2 \times 10^{11} \text{ N/m}^2$$

Now
$$\frac{\Delta Y}{Y} = \frac{\Delta L}{L}+\frac{\Delta l}{l}+2\frac{\Delta d}{d}$$

$$\frac{\Delta Y}{2.2\times 10^{11}} = \frac{0.1}{110}\times\frac{0.001}{0.125}+2\frac{0.001}{0.5} = \frac{1}{1100}+\frac{1}{125}+\frac{1}{250}$$

$$\therefore \quad \Delta Y = 2.2\times 10^{11}\times\left[\frac{1}{1100}+\frac{1}{125}+\frac{1}{250}\right] = 0.10758 \times 10^{11}$$

$$= 10.758 \times 10^9 \text{ N/m}^2$$

Hence the Young's modulus of the wire is

$$= (2.2 \times 10^{11} \pm 10.758 \times 10^9) \text{ N/m}^2$$ **Ans.**

Q. 9. *A small error in the measurement of the quantity having the highest power (in a given formula) will contribute maximum percentage error in the value of the physical quantity to whom it is related. Explain why?*

Sol. Let $Z = A^m \times B^n \times C^l$

where $m > n > l$

$\therefore$ Maximum fractional error in Z is given by

$$\frac{\Delta Z}{Z} = m\cdot\frac{\Delta A}{A}+n\cdot\frac{\Delta B}{B}+l\cdot\frac{\Delta C}{C}$$

as $m > n > l$

$\therefore$ $m \times \frac{\Delta A}{A}$ will contribute the maximum percentage error in the value of A.

Q. 10. *The two specific heat capacities of a gas are measured as $C_p = (12.28 \pm 0.2)$ units and $C_v = (3.97 \pm 0.3)$ units. Find the value of the gas constant R.*

Sol. Here $C_p = (12.28 \pm 0.2)$ units

and $C_v = (3.97 \pm 0.3)$ units

We know that

$$C_p - C_v = R$$

$$(12.28 \pm 0.2) \pm (3.97 \pm 0.3) = R$$

$$\Rightarrow (12.28 - 3.97) \pm (0.2 + 0.3) = R$$

$$\Rightarrow (8.31 \pm 0.5) = R$$

Hence $R = (8.31 \pm 0.5)$ units

ADDITIONAL QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *What are derived units?*

Ans. Units of those physical quantities which are derived from the fundamental units are called derived units.

Q. 2. *What do you understand by fundamental physical quantities?*

Ans. Fundamental physical quantities are those quantities which are independent of each other. For example, mass, length, time, temperature, electric current, luminous intensity and amount of substance are seven fundamental physical quantities.

Q. 3. *Define parsec.*

Ans. The distance at which a star would have annual parallax of 1 second of arc.

$$1 \text{ parsec} = 3.08 \times 10^{16} \text{ m}$$

Q. 4. *Define Atomic mass unit (a.m.u.).*

Ans. 1 a.m.u. = $\frac{1}{12}$th mass of carbon-12 atom, *i.e.*, 1.66×10^{-27} kg.

Q. 5. *Which is a bigger unit-light year or parsec?*

Ans. Parsec is bigger unit than light year (1 parsec = 3.26 light year).

Q. 6. *Do Å and A.U. stand for same length?*

Ans. No, $1 \text{ Å} = 10^{-10}$ m

$1 \text{ A.U.} = 1.496 \times 10^{11}$ m

Q. 7. *Name two pairs of physical quantities whose dimensions are same.*

Ans. → Stress and Young's modulus.

→ Work and Energy.

Q. 8. *What is the order of precision of an atomic clock?*

Ans. About 1 in 10^{12} to 10^{13} s.

Q. 9. *What does RADAR stand for?*

Ans. RADAR stands for 'Radio detection and ranging'.

Q. 10. *What does SONAR stand for?*

Ans. SONAR stands for 'sound navigation and ranging'.

Q. 11. *If $f = x^2$, then what is the relative error in f?*

Ans. $\frac{2\Delta x}{x}$.

Q. 12. *Name at least six physical quantities whose dimensions are ML^2T^{-2}.*

Ans. (*i*) Work (*ii*) Torque (*iii*) Moment of force (*iv*) Couple

(*v*) Potential energy (*vi*) Kinetic energy.

Q. 13. *Name four units used in the measurement of extremely short distances.*

Ans. 1 micron (1 μ) = 10^{-6} m

1 nanometre (1 nm) = 10^{-9} m

1 angstrom (1 Å) = 10^{-10} m

1 fermi (1 f) = 10^{-15} m.

Q. 14. *If $x = a + bt + ct^2$ where x is in metre and t in second, then what is the unit of e?*

Ans. According to the principle of homogeneity of dimensions.

$$[ct^2] = [L] \quad \text{or} \quad [c] = [LT^{-2}]$$

So, the unit of c is ms^{-2}.

Q. 15. *Do all physical quantities have dimensions? If no, name four physical quantities which are dimensionless.*

Ans. No, all physical quantities do not possess dimensions. Angle, specific gravity, Poisson's ratio and Strain are four examples of dimensionless quantities.

Q. 16. *Obtain the dimensions of relative density.*

Ans. As relative density is defined as the ratio of the density of given substance and the density of standard distance (water), it is a dimensionless quantity.

Q. 17. *Obtain the dimensional formula for coefficient of viscosity.*

Ans. As $F = \eta A \frac{dv}{dx}$, hence $\eta = \frac{F\,dx}{A\,dv}$

$$\therefore \quad [\eta] = \frac{[F][dx]}{[A][dv]} = \frac{[M\,L\,T^{-2} \cdot L]}{[L^2 \cdot L\,T^{-1}]} = [M^1\,L^{-1}\,T^{-1}]$$

Q. 18. *Do specific heat and latent heat have the same dimensions?*

Ans. No.

Q. 19. *Do mass and weight have the same dimensions?*

Ans. No.

Q. 20. *Given that the value of G in the CGS system as 6.67×10^{-8} dyne cm^2 g^{-2}, find the value in MKS system.*

Ans. 6.67×10^{-8} dyne cm^2 g^{-2} = 6.67×10^{-11} Nm^2/kg^2.

Q. 21. *Is Avogadro's number a dimensionless quantity?*

Ans. No, it has dimensions. In fact its dimensional formula is $[mol^{-1}]$.

Q. 22. *Can a physical quantity have dimensions but still have no units?*

Ans. No, it is not possible.

Q. 23. *Are all constants dimensionless?*

Ans. No, it is not true.

Q. 24. *What is N m^{-1} s^2 equal to?*

Ans. N m^{-1} s^2 is nothing but SI unit of mass *i.e.,* the kilogram.

Q. 25. *Express a joule in terms of fundamental units.*

Ans. [Energy] = $[M\,L^2\,T^{-2}]$, hence 1 joule = 1 kg × 1 m^2 × 1 s^{-2} = 1 kg m^2 s^{-2}.

Q. 26. *What is the dimensional formula for torque?*

Ans. $[M\,L^2\,T^{-2}]$.

Q. 27. *Is nuclear mass density dependent on the mass number? (Given: $r = r_0 A^{1/3}$)*

Ans. No, since density $= \frac{\text{Mass}}{\text{Volume}} = \frac{A}{\frac{4}{3}\pi r^3} = \frac{A}{\frac{4}{3}\pi r_0^3 A}$ is independent of A.

Q. 28. *What does LASER stand for?*

Ans. LASER stands for 'Light Amplification by Stimulated Emission of Radiation'.

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *A body travels uniformly a distance of (13.8 ± 0.2) m in a time (4.0 ± 0.3) s. What is the velocity of the body within error limits?*

Ans. Here, $S = (13.8 \pm 0.2)$ cm; $t = (4.0 \pm 0.3)$ s

$$\therefore \quad V = \frac{13.8}{4.0} = 3.45 \text{ ms}^{-1}$$

Also
$$\frac{\Delta V}{V} = \pm\left(\frac{\Delta S}{S} + \frac{\Delta t}{t}\right) = \pm\left(\frac{0.2}{13.8} + \frac{0.3}{4.0}\right) = \pm 0.0895$$

$$\Delta V = \pm 0.3 \text{ (rounding off to one place of decimal)}$$

$$V = (3.45 \pm 0.3) \text{ ms}^{-1}.$$

Q. 2. *What do you mean by order of magnitude? Explain.*

Ans. The order of magnitude of a numerical quantity (N) is the nearest power of 10 to which its value can be written.

For example. Order of magnitude of nuclear radius 1.5×10^{-14} m is –14.

Q. 3. *A laser signal is beamed towards the planet Venus from Earth and its echo is received 8.2 minutes later. Calculate the distance of Venus from the Earth at that time.*

Ans. We know that speed of laser light, $c = 3 \times 10^8$ m/s

Time of echo, t = 8.2 minutes = 8.2 × 60 seconds

If distance of Venus be d, then $t = \frac{2d}{c}$

$$d = \frac{1}{2}ct = \frac{1}{2} \times 3 \times 10^8 \times 8.2 \times 60 \text{ m}$$

$$= 7.38 \times 10^{10} \text{ m} = 7.4 \times 10^{10} \text{ m}.$$

Q. 4. *The parallax of a heavenly body measured from two points diameterically opposite on earth's equator is 60 second. If the radius of earth is 6.4×10^6 m, determine the distance of the heavenly body from the centre of earth. Convert this distance in A.U. Given 1 A.U. = 1.5×10^{11} m.*

Ans. Given, $R = 6.4 \times 10^6$ m

$$\therefore \quad D = 2R = 2 \times 6.4 \times 10^6 \text{ m} = 12.8 \times 10^6 \text{ m}$$

$$\theta = 60 \text{ second} = \frac{1^\circ}{60} = \frac{\pi}{180} \times \frac{1}{60} \text{ radian}$$

The distance of heavenly body from earth is given by

$$r = \frac{D}{\theta} = \frac{12.8 \times 10^6}{\pi} \times 180 \times 60 = \frac{12.8 \times 180 \times 60 \times 10^6}{3.142}$$

$$\Rightarrow \quad r = 4.399 \times 10^{10} \text{ m}$$

$$\text{or,} \quad r = \frac{4.399 \times 10^{10}}{1.5 \times 10^{11}} \text{ A.U.} = 0.293 \text{ A.U.}$$

Q. 5. *If the length and time period of an oscillating pendulum have errors of 1% and 2% respectively, what is the error in the estimate of g?*

Ans. We know
$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{or} \quad T^2 = 4\pi^2 \frac{l}{g}$$

$$\therefore \qquad g = 4\pi^2 \frac{l}{T^2}$$

$$\therefore \qquad \frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$$

% error in g = 1% + 2 × 2% = 5%.

Q. 6. *If $x = at^2 + bt + c$; where x is displacement as a function of time. Write the dimensions of a, b and c.*

Ans. All the terms should have the same dimension

$$\therefore \qquad [a] = \left[\frac{x}{t^2}\right]s = [LT^{-2}]$$

$$[b] = \left[\frac{x}{t}\right] = [LT^{-1}]$$

$$[c] = [x] = [L]$$

Q. 7. *The number of particles crossing per unit area perpendicular to x-axis in unit time N is given by $N = -D\left(\frac{n_2 - n_1}{x_2 - x_1}\right)s$, where n_1 and n_2 are the number of particles per unit volume at x_1 and x_2 respectively. Deduce the dimensional formula for D.*

Ans.

$$D = -N\left(\frac{x_2 - x_1}{n_2 - n_1}\right)s$$

$$[N] = \frac{N_0}{[L^2T]} = [L^{-2}\ T^{-1}]$$

$$[D] = \frac{[L^{-2}T^{-1}L]}{[L^{-3}]} = [L^2\ T^{-1}]$$

$$[x_2] = [x_1] = [L] \quad \text{and} \quad [n_2] = [n_1] = \frac{N_0}{[L^3]} = [L^{-3}]$$

Q. 8. *An experiment measured quantities a, b, c and then x is calculated by using the relation $x = \frac{ab^2}{c^3}$. If the percentage errors in measurements of a, b and c are ± 1%, ± 2% and ± 1.5% respectively, then calculate the maximum percentage error in value of x obtained.*

Ans. Given $\qquad x = \frac{ab^2}{c^3}$

$$\therefore \qquad \left(\frac{\Delta x}{x}\right)_{max} = \frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c}$$

But $\qquad \frac{\Delta a}{a} = \pm 1\%, \quad \frac{\Delta b}{b} = \pm 2\% \quad$ and $\quad \frac{\Delta c}{c} = \pm 1.5\%$

$$\therefore \qquad \left(\frac{\Delta x}{x}\right)_{max} = 1\% + 2 \times 2\% + 3 \times 1.5\% = (1 + 4 + 4.5)\% = 9.5\%.$$

Q. 9. *If instead of mass, length and time as fundamental quantities, we choose velocity, acceleration and force as fundamental quantities and express their dimensions by V, A and F respectively, show that the dimensions of Young's modulus can be expressed as $[FA^2\ V^{-4}]$.*

Ans. We know that the usual dimensions of Y are

$$\frac{[MLT^{-2}]}{[L^2]},\ i.e.,\ [ML^{-1}\ T^{-2}]$$

To express these in terms of F, A and V, we must express, M, L and T in terms of these new 'fundamental' quantities.

Now, $[V] = [LT^{-1}]$, $[A] = [LT^{-2}]$ and $[F] = [MLT^{-2}]$

It follow that $M = [FA^{-1}]$, $T = [VA^{-1}]$, $L = [V^2\ A^{-1}]$

$$[Y] = [ML^{-1}\ T^{-2}] = [FA^{-1}]\ [V^2\ A^{-1}]^{-1}\ [VA^{-1}]^{-2} = [FA^2\ V^{-4}]$$

Thus the 'new' dimensions of Young's modulus are $[FV^{-4}\ A^2]$

Q. 10. *The density of a cylindrical rod was measured by using the formula*

$$\rho = \frac{4m}{\pi D^2 l}$$

The percentage errors in m, D and l are 1%, 1.5% and 0.5%. Calculate the percentage error in the calculated value of density.

Ans. $\because$ Density $\rho = \dfrac{4m}{\pi D^2 l}$

$$\therefore \quad \left(\frac{\Delta\rho}{\rho}\right)_{max} = \frac{\Delta m}{m} + 2\frac{\Delta D}{D} + \frac{\Delta l}{l}$$

But $\dfrac{\Delta m}{m} = 1\%$, $\dfrac{\Delta D}{D} = 1.5\%$ and $\dfrac{\Delta l}{l} = 0.5\%$

$\therefore$ Maximum percentage error in calculated value of density

$$\left(\frac{\Delta\rho}{\rho}\right)_{max} = 1\% + 2 \times 1.5\% + 0.5\% = (1 + 3 + 0.5)\% = 4.5\%.$$

Q. 11. *The force experienced by a mass moving with a uniform speed v in a circular path of radius r experiences a force which depends on its mass, speed and radius. Prove that the relation is* $f = \dfrac{mv^2}{r}$.

Ans. $f \propto m^a\ v^b\ r^c$

$\therefore \quad [f] = k\ [m]^a\ [v]^b\ [r]^c$

where k is a constant

or $[MLT^{-2}] = [M]^a\ [LT^{-1}]^b\ [L]^c$

Compare the powers of M, L and T, we have

$$a = 1,\quad b + c = 1 \quad \text{and} \quad -b = -2$$

Solving above equations, we get

$$a = 1,\quad b = 2 \quad \text{and} \quad c = -1$$

$\therefore \quad f = k\ M^1\ v^2\ r^{-1}$

or $$f = k\frac{mv^2}{r}$$

Here $$k = 1$$

$$\therefore \quad \boxed{f = \frac{mv^2}{r}}$$

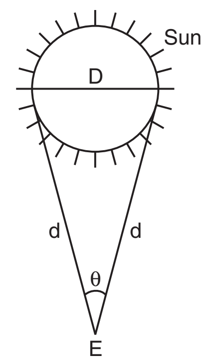
Q. 12. *The distance of the Sun from the Earth is 1.496 × 10¹¹ m (i.e., 1 A.U.). If the angular diameter of the Sun is 2000″, find the diameter of the Sun.*

Ans. Here,

$$\theta = 2000'' = \frac{2000}{3600} \times \frac{\pi}{180} \text{ rad}$$

$$= 9.7 \times 10^{-3} \text{ rad}$$

$$d = 1.496 \times 10^{11} \text{ m}$$



From the figure,

$$\theta = \frac{D}{d}$$

$$\therefore \quad D = \theta\, d$$

$$= 9.7 \times 10^{-3} \times 1.496 \times 10^{11}$$

$$= \mathbf{1.45 \times 10^9 \ m}$$

Q. 13. *Experiments show that the frequency (n) of a tuning fork depends upon the length (l) of the prong, the density (d) and the Young's modulus (Y) of its material. From dimensional considerations, find a possible relation for the frequency of a tuning fork.*

Ans. We are given that

$$n = f(l, d, Y)$$

Assuming that

$$n = k\, l^a\, d^b\, Y^c$$

and substituting dimensions of all the quantities involved, we have

$$[T^{-1}] = [L]^a\, [ML^{-3}]^b\, [ML^{-1}\, T^{-2}]^c$$

Equating powers of M, L and T on both sides, we have

$$b + c = 0$$

$$a - 3b - c = 0$$

$$-2c = -1$$

These give $c = \frac{1}{2}$, $b = -\frac{1}{2}$ and $a = -1$

$$\therefore \quad n = kl^{-1}\, d^{-1/2}\, Y^{1/2} \quad \text{or} \quad n = \frac{k}{l}\sqrt{\frac{Y}{d}}$$

This is the required relation for the frequency of a tuning fork.

Q. 14. *Calculate focal length of a spherical mirror from the following observations: object distance u = (50.1 ± 0.5) cm and image distance v = (20.1 ± 0.2) cm.*

Ans. As
$$\frac{1}{f} = \frac{1}{u}+\frac{1}{v} = \frac{v+u}{uv}$$

$$\therefore \quad f = \frac{uv}{u+v} = \frac{(50.1)(20.1)}{(50.1+20.1)} = 14.3 \text{ cm}$$

Also,
$$\frac{\Delta f}{f} = \pm\left[\frac{\Delta u}{u}+\frac{\Delta v}{v}+\frac{\Delta u+\Delta v}{u+v}\right] = \pm\left[\frac{0.5}{50.1}+\frac{0.2}{20.1}+\frac{0.5+0.2}{50.1+20.1}\right]$$

$$\frac{\Delta f}{f} = \pm\left[\frac{1}{100.2}+\frac{1}{100.5}+\frac{0.7}{70.2}\right] = \pm\,[0.00998 + 0.00995 + 0.00997]$$

$$\Delta f = 0.02990 \times f = 0.0299 \times 14.3 = 0.428 \text{ cm} = 0.4 \text{ cm}$$

$$\therefore \quad f = \mathbf{(14.3 \pm 0.4)\ cm}$$

Q. 15. *The radius of the Earth is 6.37 × 10^6 m and its mass is 5.975 × 10^{24} kg. Find the Earth's average density to appropriate significant figures.*

Ans. Radius of the Earth (R) = 6.37 × 10^6 m

Volume of the Earth (V) = $\frac{4}{3}\pi R^3 m^3 = \frac{4}{3} \times (3.142) \times (6.37 \times 10^6)^3 \text{ m}^3$

Average density (D) = $\frac{\text{Mass}}{\text{Volume}} = \frac{M}{V}$ = 0.005517 × 10^6 kg m^{-3}

The density is accurate only up to three significant figures which is the accuracy of the least accurate factor, namely, the radius of the earth.

Q. 16. *The orbital velocity v of a satellite may depend on its mass m, distance r from the centre of Earth and acceleration due to gravity g. Obtain an expression for orbital velocity.*

Ans. Let orbital velocity of satellite be given by the relation

$v = k\, m^a r^b g^c$ where k is a dimensionless constant and a, b and c are the unknown powers.

Writing dimensions on two sides of equation, we have

$$[M^0 L^1 T^{-1}] = [M]^a [L]^b [L T^{-2}]^c = [M^a L^{b+c} T^{-2c}]$$

Applying principle of homogeneity of dimensional equation, we find that

$$a = 0 \quad \text{...(i)}$$
$$b + c = 1 \quad \text{...(ii)}$$
$$-2c = -1 \quad \text{...(iii)}$$

On solving these equations, we find that

$$a = 0, \quad b = +\frac{1}{2} \quad \text{and} \quad c = +\frac{1}{2}$$

$$\therefore \quad v = k r^{\frac{1}{2}} g^{\frac{1}{2}}$$

or
$$v = k\sqrt{rg}\,.$$

Q. 17. *Check by the method of dimensional analysis whether the following relations are correct.*

(*i*) $$v = \sqrt{\frac{P}{D}} \quad \text{where} \quad v = \text{velocity of sound and}$$

$$P = \text{pressure}, \quad D = \text{density of medium.}$$

(*ii*) $$n = \frac{1}{2l}\sqrt{\frac{F}{m}}, \quad \text{where} \quad n = \text{frequency of vibration}$$

$$l = \text{length of the string}$$

$$F = \text{stretching force}$$

$$m = \text{mass per unit length of the string.}$$

Ans. (*i*)

$$[\text{R.H.S.}] = \sqrt{\frac{[P]}{[D]}} = \sqrt{\frac{[ML^{-1}T^{-2}]}{[ML^{-3}]}} = [LT^{-1}]$$

$$[\text{L.H.S.}] = [v] = [LT^{-1}]$$

$$[\text{R.H.S.}] = [\text{L.H.S.}]$$

Hence, the relation is correct.

(*ii*)

$$[\text{R.H.S.}] = \frac{1}{[l]}\sqrt{\frac{[F]}{[m]}} = \frac{1}{L}\sqrt{\frac{MLT^{-2}}{ML^{-1}}} = \frac{1}{L}[LT^{-1}] = [T^{-1}]$$

$$[\text{L.H.S.}] = \left[\frac{1}{\text{Time}}\right] = \frac{1}{T} = [T^{-1}]$$

Hence, the relation is correct.

Q. 18. *Given that the time period T of oscillation of a gas bubble from an explosion under water depends upon P, d and E where P is the static pressure, d the density of water and E is the total energy of explosion, find dimensionally a relation for T.*

Ans. We are given that

$$T = f(P, d, E)$$

Assuming that $T = k\, P^a\, d^b\, E^c$ and substituting dimensions of all the quantities involved, we have

$$[T] = [M\, L^{-1}\, T^{-2}]^a\, [M\, L^{-3}]^b\, [M\, L^2\, T^{-2}]^c$$

Equating powers of M, L and T on both sides, we have

$$a + b + c = 0$$

$$-a - 3b + 2c = 0$$

$$-2a - 2c = 1$$

Solving these equations, we get

$$a = -5/6 \quad b = 1/2 \quad c = 1/3$$

$$\therefore \quad T = k\, P^{-5/6}\, d^{1/2}\, E^{1/3}$$

or

$$T = k\left(\frac{d^{1/2}\, E^{1/3}}{P^{5/6}}\right)$$

This is the required relation for T.

Q. 19. *The radius of curvature of a concave mirror measured by spherometer is given by* $R = \frac{l^2}{6h} + \frac{h}{2}$. *The values of l and h are 4 cm and 0.065 cm respectively. Compute the error in measurement of radius of curvature.*

Ans. We are given

$$l = 4 \text{ cm}, \quad \Delta l = 0.1 \text{ cm} \qquad \text{(least count of the metre scale)}$$

here l is the distance between the legs of the spherometer.

As $$R = \frac{l^2}{6h} + \frac{h}{2}$$

$$\therefore \quad \frac{\Delta R}{R} = \frac{2\Delta l}{l} + \left(-\frac{\Delta h}{h}\right) + \frac{\Delta h}{h}$$

Considering the magnitudes only, we get

$$\frac{\Delta R}{R} = 2\frac{\Delta l}{l} + \frac{\Delta h}{h} + \frac{\Delta h}{h} = 2\left(\frac{\Delta l}{l} + \frac{\Delta h}{h}\right)$$

$$= 2 \times \frac{0.1}{4} + \frac{2 \times 0.001}{0.065} = 0.05 + 0.03 = 0.08.$$

Q. 20. *The radius of the Earth is* 6.37×10^6*m and its average density is* 5.517×10^3 *kg* m^{-3}*. Calculate the mass of earth to correct significant figures.*

Ans. $$\text{Mass} = \text{Volume} \times \text{density}$$

$$\text{Volume of earth} = \frac{4}{3}\pi R^3 = \frac{4}{3} \times 3.142 \times (6.37 \times 10^6)^3 \text{ m}^3$$

$$\therefore \quad \text{Mass of earth} = \frac{4}{3} \times 3.142 \times (6.37 \times 10^6)^3 \times 5.517 \times 10^3 \text{ kg}$$

$$= 5974.01 \times 10^{21} \text{ kg} = 5.97401 \times 10^{24} \text{ kg}$$

The radius has three significant figures and the density has four. Therefore, the final result should be rounded upto three significant figures. Hence, mass of the earth = 5.97×10^{24} kg.

Q. 21. *Find the dimensions of the following quantities*

(i) Acceleration *(ii) Angle* *(iii) Density*

(iv) Kinetic energy *(v) Gravitational constant* *(vi) Permeability*

Ans. (*i*) $$\text{Acceleration} = \frac{\text{Velocity}}{\text{Time}} \quad \therefore \quad [\text{Acceleration}] = \frac{[\text{Velocity}]}{[\text{Time}]} = \frac{[LT^{-1}]}{[T]} = [LT^{-2}]$$

(*ii*) $$\text{Angle} = \frac{\text{Distance}}{\text{Distance}} \quad \therefore \text{ angle is dimensionless.}$$

(*iii*) $$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \quad \therefore \quad [\text{Density}] = \frac{[\text{Mass}]}{[\text{Volume}]} = \frac{[M]}{[L^3]} = [ML^{-3}]$$

(*iv*) $$\text{Kinetic energy} = \frac{1}{2} \text{ Mass} \times \text{Velocity}^2$$

$$\therefore [\text{Kinetic energy}] = [\text{Mass}] \times [\text{Velocity}]^2 = [ML^2 T^{-2}]$$

(*v*) Constant of gravitation occurs in Newton's law of gravitation

$$F = G\frac{m_1 m_2}{d^2}$$

$$\therefore \qquad [G] = \frac{[F][d^2]}{[m_1][m_2]} = \frac{[MLT^{-2}L^2]}{[M][M]} = [M^{-1}L^3T^{-2}]$$

(*vi*) Permeability occurs in Ampere's law of force

$$\Delta F = \mu\frac{(i_1\Delta l_1)(i_2\Delta l_2)\sin\theta}{r^2}$$

$$\therefore \qquad [\mu] = \frac{[\Delta F][r^2]}{[i_1\Delta l_1][i_2\Delta l_2]} = \frac{[MLT^{-2}]}{[AL].[AL]} = [MLT^{-2}A^{-2}]$$

Q. 22. *The length, breadth and thickness of a block of metal were measured with the help of a Vernier Callipers. The measurements are*

$$l = (5.250 \pm 0.001)\ cm,$$
$$b = (3.450 \pm 0.001)\ cm,$$
$$t = (1.740 \pm 0.001)\ cm.$$

Find the percentage error in volume of the block.

Ans. Volume of the block is given by

$$V = l\ b\ t$$

Relative error in the volume of block

$$\frac{\Delta V}{V} = \frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta t}{t}$$

Here, $\Delta l = 0.001$ cm, $l = 5.250$ cm,
$\Delta b = 0.001$ cm, $b = 3.450$ cm,
$\Delta t = 0.001$ cm, $t = 1.740$ cm.

$$\therefore \qquad \frac{\Delta V}{V} = \frac{0.001}{5.250} + \frac{0.001}{3.450} + \frac{0.001}{1.740}$$
$$= 0.0019 + 0.00289 + 0.00575 = 0.00954$$

$$\therefore \qquad \%\ \text{Error} = \frac{\Delta V}{V}\times 100\% = 0.00954 \times 100\% = 0.9\% \simeq 1\%$$

Q. 23. *Find the value of 60 W on a system having 100 g, 20 cm and 1 minute as the fundamental units.*

Ans. Here n_1 = 60 W. Obviously, the physical quantity is power whose dimensional formula is $[M^1\ L^2\ T^{-3}]$. The first system, in which unit of power is 1 watt, is SI system in which M_1 = 1 kg, L_1 = 1 m and T_1 = 1 s in second system, M_2 = 100 g, L_2 = 20 cm and T_2 = 1 min = 60 s.

$$\therefore \qquad n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^2\left[\frac{T_1}{T_2}\right]^{-3}$$

$$= 60\left[\frac{1\text{ kg}}{100\text{ g}}\right]^1\left[\frac{1\text{ m}}{20\text{ cm}}\right]^2\left[\frac{1\text{ s}}{1\text{ min}}\right]^{-3} = 60\left[\frac{1000\text{ g}}{100\text{ g}}\right]^1\left[\frac{100\text{ cm}}{20\text{ cm}}\right]^2\left[\frac{1\text{ s}}{60\text{ s}}\right]^{-3}$$

$$= 60\times\frac{1000}{100}\times\frac{100}{20}\times\frac{100}{20}\times 60\times 60\times 60 = 3.24 \times 10^9 \text{ units.}$$

Q. 24. *By using the method of dimension, check the accuracy of the following formula:* $T = \frac{rh\rho g}{2\cos\theta}$, *where T is the surface tension, h is the height of the liquid in a capillary tube, ρ is the density of the liquid, g is the acceleration due to gravity, θ is the angle of contact, and r is the radius of the capillary tube.*

Ans. In order to find out the accuracy of the given equation we shall compare the dimensions of T and $\frac{rh\rho g}{2\cos\theta}$.

The dimensions of surface tension, $T = \frac{\text{force}}{\text{length}} = \frac{[MLT^{-2}]}{[L]} = [MT^{-2}]$

The dimensions of $\frac{rh\rho g}{2\cos\theta} = [L]\,[L]\,[ML^{-3}]\,[LT^{-2}] = [MT^{-2}]$ (2 cos θ is dimensionless)

The dimensions of both the sides are the same and hence the equation is correct.

III. LONG ANSWER TYPE QUESTIONS

Q. 1. *P.A.M. Dirac, a great physicist of 20th century found that from the following basic constants, a number having dimensions of time can be constructed:*

(i) charge on electron (e), (ii) permittivity of free space (ε_0), (iii) mass of electron (m_e), (iv) mass of proton (m_p), (v) speed of light (c), (vi) universal gravitational constant (G).

Obtain Dirac's number, given that the desired number is proportional to m_p^{-1} and m_e^{-2}. What is the significance of this number?

Ans. Let X be the desired number, then

$$X = k\, e^u\, \varepsilon_o^x\, m_e^{-2}\, m_p^{-1}\, c^y\, G^z \qquad \text{...(i)}$$

Here k is a dimensionless constant and x, y, z and u are unknowns, whose value is to be obtained from the principle of homogeneity of dimensions. Now

$$[X] = [M^0 L^0 T^1 Q^0]$$
$$[e] = [M^0 L^0 T^0 Q^1]$$
$$[\varepsilon_0] = [M^{-1} L^{-3} T^2 Q]$$
$$[C] = [M^0 L T^{-1}]$$
$$[G] = [M^{-1} L^3 T^{-2}]$$

Substituting dimensions of parameters involved in equation (*i*), we get

$$[M^0 L^0 T^1 Q^0] = Q^u [M^{-1} L^{-3} T^2 Q^2]^x M^{-2} M^{-1} [LT^{-1}]^y [M^{-1} L^3 T^{-2}]^z$$
$$= [M^{-x-3-z} L^{-3x+y+3z} T^{2x-y+2z} Q^{u+2x}]$$

From the principle of homogeneity of dimensions

$$-x - 3 - z = 0 \qquad \text{...(ii)}$$
$$-3x + y + 3z = 0 \qquad \text{...(iii)}$$
$$2x - y - 2z = 1 \qquad \text{...(iv)}$$
$$u + 2x = 0 \qquad \text{...(v)}$$

Solving eqns. (*ii*), (*iii*), (*iv*) and (*v*), we get

$$u = +4, \quad x = -2, \quad y = -3, \quad z = -1$$

$$\therefore \quad x = ke^4\, \varepsilon_0^{-2}\, m_e^{-2}\, m_p^{-1}\, c^{-3}\, G^{-1}$$

or, $$x = \frac{ke^4}{\varepsilon_0^2 m_e^2 m_p c^3 G}$$

Experiments show that

$$k = \frac{1}{16\pi^2}$$

Hence, $$x = \frac{e^4}{16\pi^2 \varepsilon_0^2 m_e^2 m_p c^3 G}$$

Substituting values of all known parameters we find that the value of x is nearly 15 billion years, which is approximately equal to the present estimate of the age of the universe.

Q. 2. *To determine acceleration due to gravity, the time of 20 oscillations of a simple pendulum of length 100 cm was observed to be 40 s. Calculate the value of g and maximum percentage error in the measured value of g.*

Ans. Here $$T = 2\pi\sqrt{\frac{l}{g}}$$

or $$T^2 = 4\pi^2 \frac{l}{g} \quad \text{or} \quad g = 4\pi^2 \frac{l}{T^2}$$

Given $$l = 100 \text{ cm}, \quad T = \frac{40\text{s}}{20} = 2\text{s}$$

$$\therefore \quad g = 4\times(3.14)^2 \times \frac{1000 \text{ cm}}{(2\text{s})^2} = \frac{4\times 9.8596\times 100}{4}\text{cms}^{-2}$$

$$= \mathbf{985.9\ cms^{-2}}$$

Let us now calculate the maximum error

$$g = 4\pi^2 \times \frac{l}{T^2} = 4\pi^2 \frac{1}{\left(\frac{t}{20}\right)^2} \left(\text{taking } T = \frac{t}{20}\right)$$

or $$g = \frac{4\pi^2 l\times(20)^2}{t^2} s$$

Taking log on both sides, we get

$$\log g = \log 4 + 2\log\pi + \log l + 2\log 20 - 2\log t$$

[differentiating both the sides,]

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} - 2\frac{\Delta t}{t}$$

Given l = 100 cm

Δl = 0.1 cm (least count of the metre scale)

t = 40 s, Δt = 0.1 s (least count of a stop watch)

$$\therefore \text{ Maximum error in } g = \frac{0.1}{100} + 2\times\frac{0.1}{40}$$

$$= 0.001 + 0.005 = 0.006 = 0.006 \times 100\% = 0.6\%$$

Here 0.1% is the error in the measurement of length, and 0.5% is the error in the measurement of time. Therefore, time needs more careful measurement.

Q. 3. *It is known that the period T of a magnet of magnetic moment M vibrating in a uniform magnetic field of intensity B depends upon M, B and I where I is the moment of inertia of the magnet about its axis of oscillations. Show that*

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

Ans. We first note that the dimension of I are $[ML^2]$. Also the magnetic moment has the units Am^2 so that its dimensions can be written as $[AL^2]$ where A stands for the dimensions of the electric current. Finally the magnetic field vector B has the units newton (per ampere metre) so that its dimensions can be written as

$$[B] = \frac{[MLT^{-2}]}{[A][L]} = [MT^{-2}A^{-1}]$$

We now assume that

$$T = k\, I^a M^b B^c$$

Substituting dimensions of all the quantities involved, we have

$$[T] = [ML^2]^a [AL^2]^b [MT^{-2}A^{-1}]^c$$
$$= [M^{a+c} L^{2a+2b} T^{-2c} A^{b-c}]$$

Equating powers of M, L, T and A on both sides, we have $a + c = 0$, $2(a + b) = 0$, $-2c = 1$, $b - c = 0$. From the first three equations, we get $c = \frac{-1}{2}$, $a = \frac{1}{2}$ and $b = \frac{-1}{2}$.

These values are consistent with the fourth equation. Thus

$$a = \frac{1}{2},\quad b = \frac{-1}{2} \quad \text{and} \quad c = \frac{-1}{2}$$

$$\therefore \quad T = k\, I^{1/2} M^{-1/2} B^{-1/2} = k\sqrt{\frac{I}{MB}}$$

Experiments show that $k = 2\pi$. Therefore

$$T = 2\pi\sqrt{\frac{I}{MB}}.$$

Q. 4. *Briefly explain how you will estimate the molecular diameter of oleic acid.*

Ans. To determine the molecular diameter of oleic acid, we first of all dissolve 1 mL of oleic acid in 20 mL of alcohol. Then redissolve 1 mL of this solution in 20 mL of alcohol. Hence, the concentration of final solution is

$$\frac{1}{20}\times\frac{1}{20} = \frac{1}{400}\text{th} \text{ part of oleic acid in alcohol.}$$

Now take a large sized trough filled with water. Lightly sprinkle lycopodium powder on water surface. Using a dropper of fine bore gently put few drops (say n) of the solution prepared on to water. The solution drops spread into a thin, large and roughly circular film of molecular thickness on water surface. Quickly measure the diameter of thin circular film and calculate its surface area S.

If volume of each drop of solution be V, then volume of n drops = nV

Volume of oleic acid in this volume of solution = $\frac{nV}{400}$

It t be the thickness of oleic acid film formed over water surface then the volume of oleic acid film = St

$$\therefore \quad St = \frac{nV}{400} \Rightarrow t = \frac{nV}{400\,S}$$

As the film is extremely thin, this thickness t may be considered to be the size of one molecule of oleic acid *i.e.*, t is the molecular diameter of oleic acid.

Experimentally, molecular diameter of oleic acid is found to be of the order of 10^{-9} m.

Q. 5. *Obtain a relation between the distance travelled by a body in time t, if its initial velocity be u and acceleration f.*

Ans. Let the distance covered is S,

Then $S = k\,u^a f^b t^c$; where k is a constant. Writing dimensions on both the sides, we have

$$[L] = [LT^{-1}]^a [LT^{-2}]^b [T]^c \quad \text{or} \quad [L] = [L^{a+b} T^{-a-2b+c}]$$

Comparing powers on both sides, we get

$$1 = a + b \quad \text{and} \quad 0 = -a - 2b + c$$

We have only two equations with three unknowns, therefore, we split the problem into two parts.

(*a*) Let the body have no acceleration,

then $S = k_1 u^a t^b$

or $[L] = [LT^{-1}]^a [T]^b = [L^a T^{-a+b}]$

or $a = 1$

$-a + b = 0$ or $b = 1$

$S = k_1 ut$ ...(*i*)

(*b*) Suppose the body has no initial velocity

then $S = k_2 f^a t^b$

$[L] = k_2 [LT^{-2}]^a [T]^b$

or $[L] = [L^a T^{-2a+b}]$ or $a = 1$

$-2a + b = 0$ or $b = 2a = 2$

$\therefore$ $S = k_2 ft^2$ ...(*ii*)

If the body has both the initial velocity and acceleration comparing (*i*) and (*ii*), we get,

$$S = k_1 ut + k_2 ft^2$$

This is the required equation.

If we put $k_1 = 1,\ k_2 = \frac{1}{2}$, we get

$$\boxed{S = ut + \frac{1}{2} ft^2}$$

IV. MULTIPLE CHOICE QUESTIONS

1. The SI units of magnetic field is

(*a*) weber per metre^2
(*b*) newton per coulomb per (metre per second)
(*c*) newton per ampere per metre
(*d*) all the above

2. The dimensions of energy per unit volume are the same as those of

(*a*) pressure (*b*) force

(*c*) modulus of elasticity (*d*) all the above

3. The SI units of the universal gravitational constant G are

(*a*) kg m^2 s^{-2} (*b*) kg^{-1} m^3 s^{-2} (*c*) Nm^2 kg^{-2} (*d*) N kg^2 m^{-2}

4. The number of particles crossing per unit area perpendicular to X-axis in unit time is

$$N = -D\frac{n_2 - n_1}{x_2 - x_1}$$

where n_1 and n_2 are number of particles per unit volume for the value of x_1 and x_2 respectively. The dimensions of diffusion constant D are

(*a*) $M^0L\,T^2$ (*b*) $M^0L^{\,2}T^{-4}$ (*c*) $M^0L\,T^{-3}$ (*d*) $M^0L^{\,2}T^{-1}$

5. A physical quantity is represented by $X = M^a L^b T^{-c}$. If percentage error in the measurement of M, L and T are α%, β% and γ% respectively, then total percentage error is

(*a*) $(\alpha a - \beta b + \gamma c)\%$ (*b*) $(\alpha a + \beta b + \gamma c)\%$

(*c*) $(\alpha a - \beta b - \gamma c)\%$ (*d*) none of the above

6. 'Parsec' is the unit of:

(*a*) Time (*b*) Distance (*c*) Frequency (*d*) Angular acceleration

7. The density of a cube is measured by measuring its mass and the length of its sides. If the maximum errors in the measurement of mass and length are 3% and 2% respectively, then the maximum error in the measurement of density is

(*a*) 9% (*b*) 7% (*c*) 5% (*d*) 1%

8. A wire has a mass 0.3 ± 0.003 g, radius 0.5 ± 0.005 mm and length 6 ± 0.06 cm. The maximum percentage error in the measurement of its density is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

9. The velocity of a body moving in viscous medium is given by $v = \frac{A}{B}\left[1 - e^{\frac{-t}{B}}\right]$ where t is time, A and B are constants.

Then the dimensions of A are

(*a*) $[M^0\,L^0\,T^0]$ (*b*) $[M^0L^{\,1}T^0]$ (*c*) $[M^0L^{\,1}T^{-2}]$ (*d*) $[M^1L^{\,1}T^{-1}]$

10. The dimensions of entropy are

(*a*) $[M^0\,L^{-1}\,T^0\,K]$ (*b*) $[M^0\,L^{-2}\,T^0\,K^2]$ (*c*) $[ML\,T^{-2}\,K]$ (*d*) $[M\,L^2\,T^{-2}\,K^{-1}]$

Ans. **1.**—(*d*) **2.**—(*d*) **3.**—(*b*) and (*c*) **4.**—(*d*) **5.**—(*b*)
6.—(*b*) **7.**—(*a*) **8.**—(*d*) **9.**—(*b*) **10.**—(*d*)

V. QUESTIONS ON HIGH ORDER THINKING SKILLS (HOTS)

Q. 1. *A laser light beam sent to the moon takes 2.56 s to return after reflection at the Moon's surface. Calculate the radius of the lunar orbit around the earth.*

Ans. Radius of the lunar orbit around the earth = Distance between the moon and the earth Time taken by the laser beam from earth to moon and then back to the earth = 2.56 s.

$\therefore$ Time taken by the laser beam to go from earth to the moon is $t = \frac{2.56}{2} = 1.28$ s

Speed of the laser beam (*i.e.,* light),

$$c = 3 \times 10^8 \text{ ms}^{-1}$$

$\therefore$ Distance between moon and earth,

$$S = c\,t = 3 \times 10^8 \times 1.28$$
$$= 3.84 \times 10^8 \text{ m} = 3.84 \times 10^5 \text{ km.}$$

Q. 2. *The parallactic angle subtended by a distant star is 0.76 on the earth's orbital diameter (1.5 × 10^{11} m). Calculate the distance of the star from the earth.*

Ans. The parallactic angle, $\phi = 0.76 = \frac{0.76 \times \pi}{180 \times 60 \times 60}$ radians

$$= \frac{19\,\pi}{1.62 \times 10^7} \text{ radians}$$

The orbital diameter, say $D = 1.5 \times 10^{11}$ m

$\therefore$ The required distance, $d = \frac{D}{\phi}$

$$= \frac{1.5 \times 10^{11} \times 1.62 \times 10^7}{19\,\pi} \text{ m} = \frac{2.43 \times 10^{18}}{19 \times 3.14} \text{ m}$$

$$= \frac{243 \times 10^{16}}{59.66} \text{ m} = 4.073 \times 10^{16} \text{ m}$$

Since, 1 light year $= 9.5 \times 10^{15}$ m

$$d = \frac{4.073 \times 10^{16}}{9.5 \times 10^{15}} \text{ light year} = 4.29 \text{ light year.}$$

Q. 3. *The heat dissipated in a resistance can be obtained by the measurement of resistance, the current and time. If the maximum error in the measurement of these quantities is 1%, 2% and 1% respectively, what is the maximum error in determination of the dissipated heat?*

Ans. Heat produced H is given by

$$H = \frac{I^2 R t}{J}$$

$$\therefore \quad \frac{\Delta H}{H} = 2\frac{\Delta I}{I} + \frac{\Delta R}{R} + \frac{\Delta t}{t} + \frac{\Delta J}{J}$$

For maximum percentage error,

$$\frac{\Delta H}{H} \times 100 = 2\frac{\Delta I}{I} \times 100 + \frac{\Delta R}{R} \times 100 + \frac{\Delta t}{t} \times 100 + \frac{\Delta J}{J} \times 100$$

$$= 2 \times 2\% + 1\% + 1\% + 0\% = \mathbf{6\%}$$

Q. 4. *E, m, l and G denote energy, mass, angular momentum and gravitational constant respectively. Determine the dimensions of* El^2/m^5G^2.

Ans. Dimensions of E = $[M L^2 T^{-2}]$

Dimensions of l = $[M L^2 T^{-2}]$

Dimensions of m = $[M]$

Dimensions of G = $[M^{-1} L^3 T^{-2}]$

$\therefore$ Dimensions of El^2/m^5G^2

$$= \frac{\left[M L^2 T^{-2}\right]\left[M L^2 T^{-2}\right]^2}{[M]^5 \left[M^{-1} L^3 T^{-2}\right]^2} = 1$$

Thus El^2/m^5G^2 is dimensionless.

Q. 5. *The Reynold's number n_R for a liquid flowing through a pipe depends upon: (i) the density of the liquid ρ, (ii) the coefficient of viscosity η, (iii) the speed of flow of the liquid v, and (iv) the radius of the tube r.*

Obtain dimensionally an expression for n_R. Given, n_R is directly proportional to r.

Ans. Let $$n_R = \rho^x \eta^y v^z r \quad ...(1)$$

Note in Eqn. (1) we have used the information that n_R is directly proportional to r. If this information was *not available* there will be four *unknowns*. By equating powers of M, L and T only *three independent* equations will be obtained and they cannot give values of the four unknowns. Now

$$[n_R] = [M^0 L^0 T^0]$$
$$[\rho] = [M L^{-3}]$$
$$[\eta] = [M L^{-1} T^{-1}]$$
$$[r] = [L]$$

Substituting dimensions of parameters involved in Eqn. (1), we have

$$M^0 L^0 T^0 = (M L^{-3})^x (M L^{-1} T^{-1})^y (L T^{-1})^z [L^{+1}]$$
$$= [M^{x+y} L^{-3x-y+z+1} T^{-y-z}]$$

By the principle of homogeneity of dimensions

$$x + y = 0 \quad ...(2)$$
$$-3x - y + z + 1 = 0 \quad ...(3)$$
$$-y - z = 0 \quad ...(4)$$

Solving these equations, we get

$$x = 1, \quad y = -1, \quad z = 1$$

Hence, $$n_R = k\, r\, \rho^1 \eta^{-1} v^1$$

or $$n_R = \frac{k\, r\, \rho\, v}{\eta}.$$

Q. 6. *The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{4g}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1 s resolution. What is the accuracy in the determination of g?*

Ans. $$g = \frac{4\pi^2 L}{T^2}$$

Here, $T = \frac{t}{n}$ and $\Delta T = \frac{\Delta t}{n}$

Therefore, $\frac{\Delta T}{T} = \frac{\Delta t}{t}$

The errors in both L and t are the least count errors.

$$\left(\frac{\Delta g}{g}\right) = \left(\frac{\Delta L}{L}\right) + 2\left(\frac{\Delta T}{T}\right) = \frac{0.1}{20.0} + 2\left(\frac{1}{90}\right) = 0.032$$

Thus, the percentage error in g is

$$100\left(\frac{\Delta g}{g}\right) = 100\left(\frac{\Delta L}{L}\right) + 2\times 100\left(\frac{\Delta T}{T}\right) = 3\%.$$

Q. 7. *The speed of light in air is 3.00×10^8 ms^{-1}. The distance travelled by light in one year (i.e., 365 days = 3.154×10^7 s) is known as light year. A student calculates one light year = 9.462×10^{15} m. Do you agree with the student? If not, write the correct value of one light year.*

Ans. One light year = speed × time = 9.462×10^{15} m. When two physical quantities are multiplied, the significant figures retained in the final result should not be greater than the least number of significant figures in any of the two quantities. Since, in this case significant figures in one quantity (3.00×10^8 ms^{-1}) are 3 and the significant figures in the other quantity (3.154×10^7 s) are 4, therefore, the final result should have 3 significant figures. Thus, the correct value of one light year = 9.46×10^{15} m.

Q. 8. *A physical quantity $X = \frac{a^2 b^{-3/2}}{c^4}$. A student says that the relative error in $X = 2\frac{\Delta a}{a} - \frac{3}{2}\frac{\Delta b}{b} - 4\frac{\Delta c}{c}$. Do you agree with the student? If not, what is the relative error in X?*

Ans. Errors are always additive. Therefore, the relative error in $X = 2\frac{\Delta a}{a} + \frac{3}{2}\frac{\Delta b}{b} + 4\frac{\Delta c}{c}$.

Q. 9. *If velocity of sound in a gas depends on its elasticity and density, derive the relation for the velocity of sound in a medium by the method of dimensions.*

Ans. If v be the velocity of sound, E the elasticity of the medium and ρ the density of the medium, then

$$v \propto E^a \rho^b \quad \text{or} \quad v = k\, E^a \rho^b \qquad ...(i)$$

where k is a dimensionless constant of proportionality. Writing down the dimensions of both sides of equation (i), we get

$$[M^0 L\, T^{-1}] = [M\, L^{-1}\, T^{-2}]^a\, [M\, L^{-3}]^b$$

$$[M^0 L\, T^{-1}] = [M^{a+b}\, L^{-a-3b}\, T^{-2a}]$$

Comparing powers of M, L and T, we get

$$a + b = 0, \quad -a - 3b = 1, \quad -2a = -1 \quad \text{or} \quad a = \frac{1}{2}$$

∴ $\frac{1}{2} + b = 0$ or $b = \frac{-1}{2}$

From eqn. (i), $v = k\, E^{1/2}\, \rho^{-1/2}$

or $$v = k\sqrt{\frac{E}{\rho}}$$

where the value of k can be determined experimentally.

Q. 10. *Reynold's number N_R (a dimensionless quantity) determines the condition of laminar flow of a viscous liquid through a pipe. N_R is a function of the density of the liquid r, its average speed is v and the coefficient of viscosity of the liquid is h. If N_R is given directly proportional to d (the diameter of the pipe), show from dimensional consideration that $N_R \propto \frac{d\rho v}{\eta}$ the unit of η in S.I. system is kgm^{-1} s^{-1}.*

Ans. As the Reynold's number N_R depends on density ρ, average speed v and coefficient of viscosity η, then let us say

$$N_R \propto \rho^a v^b \eta^c$$

Again N_R is proportional to d, the diameter of the pipe, combining the two quantities we have,

$$N_R \propto \rho^a v^b \eta^c d \quad \text{or} \quad N_R = k\rho^a v^b \eta^c d \qquad ...(i)$$

$$[N_R] = [M^0 L^0 T^0]$$

$$[\rho] = [LT^{-3}]$$

$$[\eta] = [ML^{-1} T^{-1}]$$

$$[d] = [L]$$

Substituting the dimension in (i), we have,

$$[M^0 L^0 T^0] = [ML^{-3}]^a [LT^{-1}]^b [ML^{-1} T^{-1}]^c [L] = [M^{a+c} L^{-3a+b-c+1} T^{-b-c}]$$

Comparing the dimensions of M, L and T, we have,

$$a + c = 0$$

$$-3a + b - c + 1 = 0$$

$$-b - c = 0$$

On simplifying, we get $c = -1$, $b = 1$, $a = 1$

Therefore, the relation (i) becomes

$$N_R = k\rho^{\,1} v^1 \eta^{-1} d$$

or $$N_R = k\cdot\rho\frac{vd}{\eta} \quad \text{or} \quad N_R \propto \rho\frac{vd}{\eta}$$

Q. 11. *It is required to find the volume of a rectangular block. A Vernier Caliper is used to measure the length, width and height of the block. The measured values are found to be 1.37 cm, 4.11 cm and 2.56 cm respectively.*

Sol. The measured (nominal) volume of the block is,

$$V = l \times w \times h = (1.37 \times 4.11 \times 2.56)\ \text{cm}^3 = 14.41\ \text{cm}^3$$

The least count of Vernier Caliper is ± 0.01 cm

∴ Uncertain values can be written as

$$l = (1.37 \pm 0.01)\ \text{cm}$$

$$w = (4.11 \pm 0.01)\ \text{cm}$$

$$h = (2.56 \pm 0.01)\ \text{cm}$$

Lower limit of the volume of the block is,

$$V_{(min)} = (1.37 - 0.01) \times (4.11 - 0.01) \times (2.56 - 0.01)\ \text{cm}^3$$
$$= (1.36 \times 4.10 \times 2.55)\ \text{cm}^3 = 14.22\ \text{cm}^3$$

This is 0.19 cm^3 lower than the nominal measured value.

Similarly the upper limit can also be calculated as follows.

$$V_{(max)} = (1.37 + 0.01) \times (4.11 + 0.01) \times (2.56 + 0.01)\ \text{cm}^3$$
$$= (1.38 \times 4.12 \times 2.57)\ \text{cm}^3 = 14.61\ \text{cm}^3$$

This is 0.20 cm^3 higher than the measured value.

But we choose the higher of these two values as the uncertainty i.e. $(14.41 \pm 0.20)\ \text{cm}^3$

Q. 12. *In an experiment on determining the density of a ectangular block, the dimensions of the block are measured with a Venier Caliper (with a least count of 0.01 cm) and its mass is measured with a beam balance of least count of 0.1 gm. How do we report our result for the density of the block?*

Sol. Let the measured values be :

Mass of the block (m) = 39.3 g

length (l) = 5.12 cm

breadth (b) = 2.56 cm

thickness (t) = 0.37 cm

The density of the block is given by

$$d = \frac{\text{mass}}{\text{volume}} = \frac{m}{l \times b \times h} = \frac{39.3}{5.12 \times 2.56 \times 0.37} = 8.1037\ \text{gram/cm}^3$$

Now the uncertain values are

$$l = \pm 0.01\ \text{cm}$$
$$b = \pm 0.01\ \text{cm}$$
$$t = \pm 0.01\ \text{cm}$$

Maximum relative error, in the density, value is given by

$$\frac{\Delta d}{d} = \frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta t}{t} + \frac{\Delta m}{m} = \frac{0.01}{5.12} + \frac{0.01}{2.56} + \frac{0.01}{0.37} + \frac{0.1}{39.3}$$
$$= 0.0019 + 0.0039 + 0.027 + 0.0024 = 0.0358$$

$$\therefore \quad \Delta d = 0.0358 \times 8.1037 = 0.3\ \text{g/cm}^3\ \text{(approx)}$$

V. VALUE-BASED QUESTIONS

Q. 1. *Suresh went to London to his elder brother Lalit who is a Civil Engineer there. Suresh found there the currency is quite different from his country. He could not understand pound and how it is converted into rupees. He asked there an Englishman how far is the Central London from here.*

He replied that it is 16 miles. Suresh again got confused because he never used these units in India. In the evening Suresh inquired all about it. His brother told him about the unit system used in England. He explained his brother that here F.P.S. system is used. It means, distance is measured in foot, mass in pound and time in seconds whereas in India it is MKS system.

(i) What values are displayed by Suresh?

(ii) How many unit system are there?

Ans. (*i*) Sincerity, Curiosity, dedicated and helping nature

(*ii*) Unit system are :

(*a*) FPS system

(*b*) MKS system

(*c*) CGS system

TEST YOUR SKILLS

1. What do you understand by absolute error?
2. State the ''principle of homogeneity'' of dimensions.
3. Wavelength of sodium light is 5893 Å. Express it in (*i*) nanometre, (*ii*) micron.
4. What is the difference between 4.0 and 4.000?
5. Define relative and percentage error.
6. Discuss three most important causes of error in measurement.
7. The velocity v of a particle depends on the time t according to the equation $v = a + bt + \dfrac{c}{d+t}$.

   Write the dimensions of a, b, c and d.
8. How can you check dimensional accuracy of a physical relation?
9. Give two important limitations of the method of dimensional analysis.
10. The planets move round the sun in nearly circular orbits. Assuming that the period of rotation (T) depends upon the radius (r) of the orbit, the mass (M) of the sun and the gravitational constant (G), show that the planets obey Kepler's third law, *i.e.,* the square of the time period of the planet is proportional to the cube of its orbital radius.

❑❑❑