3 Motion in a Straight Line

Facts that Matter

• Introduction

Motion is one of the significant topics in physics. Everything in the universe moves. It might only be a small amount of movement and very-very slow, but movement does happen. Even if you appear to be standing still, the Earth is moving around the sun, and the sun is moving around our galaxy.

"An object is said to be in motion if its position changes with time".

The concept of motion is a relative one and a body that may be in motion relative to one reference system, may be at rest relative to another.

There are two branches in physics that examine the motion of an object.

(*i*) **Kinematics:** It describes the motion of objects, without looking at the cause of the motion.

(*ii*) **Dynamics:** It relates the motion of objects to the forces which cause them.

• Point Object

If the length covered by the objects are very large in comparison to the size of the objects, the objects are considered point objects.

• Reference Systems

The motion of a particle is always described with respect to a reference system. A reference system is made by taking an arbitrary point as origin and imagining a co-ordinate system to be attached to it. This co-ordinate system chosen for a given problem constitutes the reference system for it.

We generally choose a co-ordinate system attached to the earth as the reference system for most of the problems.

• Total Path Length (Distance)

For a particle in motion the total length of the actual path traversed between initial and final positions of the particle is known as the 'total path length' or distance covered by it.

• Types of Motion

In order to completely describe the motion of an object, we need to specify its position. For this, we need to know the position co-ordinates. In some cases, three position co-ordinates are required, while in some cases two or one position co-ordinate is required.

Based on these, motion can be classified as:

(*i*) **One dimensional motion.** A particle moving along a straight-line or a path is said to undergo one dimensional motion. For example, motion of a train along a straight line, freely falling body under gravity etc.

(*ii*) **Two dimensional motion.** A particle moving in a plane is said to undergo two dimensional motion. For example, motion of a shell fired by a gun, carrom board coins etc.

(*iii*) **Three dimensional motion.** A particle moving in space is said to undergo three dimensional motion. For example, motion of a kite in sky, motion of aeroplane etc.

• Displacement

Displacement of a particle in a given time is defined as the change in the position of particle in a particular direction during that time. It is given by a vector drawn from its initial position to its final position.

• Factors Distinguishing Displacement from Distance

→ Displacement has direction. Distance does not have direction.

→ The magnitude of displacement can be both positive and negative.

→ Distance is always positive. It never decreases with time.

→ Distance ≥ | Displacement |

Distance	*Displacement*
(*i*) Length of actual path covered between the initial and final positions/points	(*i*) Length of the shortest path between initial and final points.
(*ii*) Scalar quantity	(*ii*) Vector quantity
(*iii*) Can have only +ve values	(*iii*) Can have –ve, 0, +ve values.

→ Both distance and displacement are measured in metres or kilometre. Their dimension is [L].

Speed	*Velocity*
(*i*) The rate at which distance is covered.	(*i*) The rate at which displacement takes place.
(*ii*) Scalar quantity	(*ii*) Vector quanting.
(*iii*) Can have only +ve values.	(*iii*) Can have –ve, 0, +ve values.

Speed and velocity are expressed in metre per sec, *i.e.*, ms^{-1}. The dimensional formula is $[LT^{-1}]$.

$$\text{Velocity} = \frac{\text{Final position} - \text{Initial position}}{\text{time}} = \frac{x_f - x_i}{t}$$

Velocity can change either by altering magnitude or by changing direction or both.

• Uniform Speed and Uniform Velocity

Uniform Speed. An object is said to move with uniform speed if it covers equal distances in equal intervals of time, howsoever small these intervals of time may be.

Uniform Velocity. An object is said to move with uniform velocity if it covers equal displacements in equal intervals of time, howsoever small these intervals of time may be.

• Variable Speed and Variable Velocity

Variable Speed. An object is said to move with variable speed if it covers unequal distances in equal intervals of time, howsoever small these intervals of time may be.

Variable Velocity. An object is said to move with variable velocity if it covers unequal displacements in equal intervals of time, howsoever small these intervals of time may be.

• Average Speed and Average Velocity

Average Speed. It is the ratio of total path length traversed and the corresponding time interval.

Or

"The distance covered in unit time is called average speed".

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$V_{av} = \frac{\Delta x}{\Delta t}$$

Average Velocity. Average velocity is the displacement divided by the time interval in which the displacement occurs.

Or

"It is that single velocity with which the object can travel the same length in the same time as it generally does with varying velocity".

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time taken}}$$

$$\vec{V}_{av} = \frac{\Delta \vec{x}}{\Delta t}$$

The average speed of an object is greater than or equal to the magnitude of the average velocity over a given time interval.

• Instantaneous Speed and Instantaneous Velocity

Instantaneous Speed. The speed of an object at an instant of time is called instantaneous speed.

Or

"Instantaneous speed is the limit of the average speed as the time interval becomes infinitesimally small".

$$v = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{dx}{dt}$$

Instantaneous velocity

The instantaneous velocity of a particle is the velocity at any instant of time or at any point of its path.

or

"Instantaneous velocity or simply velocity is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small."

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

• Acceleration

The rate at which velocity changes is called acceleration.

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

$$a = \frac{v - u}{t}$$

where v and u are final and initial velocity respectively. It is a vector quantity with *S.I.* unit of m/s^2 and has dimensions of $[LT^{-2}]$.

If acceleration is *–ve* (negative), then it is called retardation or deceleration.

• Uniform Acceleration

If an object undergoes equal changes in velocity in equal time intervals it is called uniform acceleration.

• Average and Instantaneous Acceleration

Average Acceleration. It is the change in the velocity divided by the time-interval during which the change occurs.

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

Instantaneous Acceleration. It is defined as the limit of the average acceleration as the time-interval Δt goes to zero.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{dv}{dt}$$

• Kinematical Graphs

The 'displacement-time' and the 'velocity-time' graphs of a particle are often used to provide us with a visual representation of the motion of a particle. The 'shape' of the graphs depends on the initial 'co-ordinates' and the 'nature' of the acceleration of the particle (Fig.)

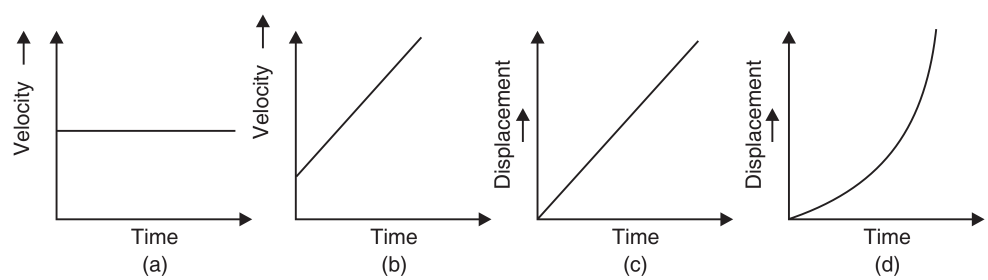


Fig. *Curves (a) and (c) represent motion with a constant speed u. Curves (b) and (d) represent motion with a uniform acceleration* ***a*** *starting with an initial speed* ***u****.*

The following general results are always valid

(*i*) The slope of the displacement-time graph at any instant gives the speed of the particle at that instant.

(*ii*) The slope of the velocity-time graph at any instant gives the magnitude of the acceleration of the particle at that instant.

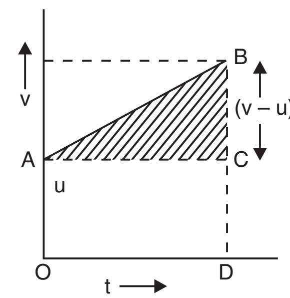
(*iii*) The area enclosed by the velocity-time graph, the time-axis and the two co-ordinates at time instants t_1 to t_2 gives the distance moved by the particle in the time-interval from t_1 to t_2.

• Equations of Motion for Uniformly Accelerated Motion

For uniformly accelerated motion, some simple equations can be derived that relate displacement (x), time taken (t), initial velocity (u), final velocity (v) and acceleration (a). Following equation gives a relation between final and initial velocities v and u of an object moving with uniform acceleration a:

$$v = u + at$$

This relation can be graphically represented by following figure:



The area under this curve is:

Area between instants 0 and t

= Area of triangle ABC + Area of rectangle $OACD$

$$= \frac{1}{2}(v-u)t + ut$$

The area under $v - t$ curve represents the displacement. Therefore, the displacement x of the object is:

$$x = \frac{1}{2}(v-u)t + ut$$

But $$v - u = at$$

So, $$x = \frac{1}{2}at^2 + ut \quad \text{or,} \quad x = ut + \frac{1}{2}at^2$$

The equation for displacement can also be given as follows:

$$x = \frac{v+u}{2}t = vt$$

Earlier we have derived:

$$v = u + at \quad \text{or,} \quad \frac{v+u}{a} = t$$

Substituting the value of t in equation for displacement we get,

$$x = \left(\frac{v+u}{2}\right)\left(\frac{v-u}{2}\right)$$

or, $$x = \frac{v^2 - u^2}{2a} \quad \text{or,} \quad v^2 = u^2 + 2ax$$

So, we have derived following kinematic equation.

$$v = u + at$$

$$x = ut + \frac{1}{2}at^2$$

$$v = u^2 + 2ax$$

If acceleration is uniform or constant, then equations of motion are:

$$v = u + at \qquad ...(i)$$

$$s = ut + \frac{1}{2}at^2 \qquad ...(ii)$$

$$v^2 = u^2 + 2as \quad ...(iii)$$

where u is initial velocity, v is final velocity, a is acceleration and s is the distance covered in time interval t.

For uniformly accelerated motion along a straight line, displacement in a particular instant of time (n^{th} second of the motion) is given by

$$s_{nth} = u + \frac{1}{2} a\,(2n - 1)$$

- Suppose a body is projected vertically upward from a point A with velocity u.

If we take upward direction as positive

(*i*) At time t, its velocity $v = u - gt$

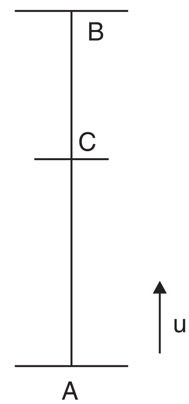
(*ii*) At time t, its displacement from A is given by

$$h = ut - \frac{1}{2} gt^2$$

(*iii*) Its velocity when it has a displacement 'h' is given by

$$v^2 = u^2 - 2gh$$

(*iv*) When it reaches the maximum height from A, its velocity $v = 0$. This happen when $t = \frac{u}{g}$. The body is instantaneously at rest at the highest points.



(*v*) The maximum height reached

$$H = \frac{u^2}{2g}$$

(*vi*) Total time to go up and return to the point of projection = $\frac{2u}{g}$.

(*vii*) At any point C between A and B, where $AC = S$, the velocity v is given by

$$v = \pm\sqrt{u^2 - 2gs}$$

The velocity of body while crossing C upwards = $+\sqrt{u^2 - 2gs}$ and while crossing C downwards is $-\sqrt{u^2 - 2gs}$.

In some problems it is convenient to take the downward direction as positive, in such case all the measurements in downward direction are considered as positive *i.e.*, acceleration will be $+g$. But sometimes we may need to take upward as positive and in such case acceleration will be $-g$.

- **Relative Velocity**

Relative velocity of an object A with respect to another object B is the time rate at which the object A changes its position with respect to the object B.

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$, where $\vec{V}_A$ and $\vec{V}_B$ are the velocities of object A and B. $\left(\vec{V}_A - \vec{V}_B\right)$ indicates the addition of negative of velocity of B to the velocity of A.

→ The relative velocity of two objects moving in the same direction is the difference of the speeds of the objects.

→ The relative velocity of two objects moving in opposite direction is the sum of the speeds of the objects.

- **IMPORTANT TABLES**

TABLE 3.1 Some physical quantities, symbols, dimensions and their units.

S.No.	*Physical quantity*	*Symbol*	*Dimensions*	*Units*
(*i*)	Path length		[L]	m
(*ii*)	Displacement	Δx	[L]	m
(*iii*)	Velocity		$[LT^{-1}]$	ms^{-1}
	(*a*) Average	$\overline{v}$		
	(*b*) Instantaneous	v		
(*iv*)	Speed		$[LT^{-1}]$	ms^{-1}
	(*a*) Average			
	(*b*) Instantaneous			
(*v*)	Acceleration		$[LT^{-2}]$	ms^{-2}
	(*a*) Average	$\overline{a}$		
	(*b*) Instantaneous	a		

NCERT TEXTBOOK QUESTIONS SOLVED

3.1. *In which of the following examples of motion, can the body be considered approximately a point object.*

(a) A railway carriage moving without jerks between two stations.

(b) A monkey sitting on top of a man cycling smoothly on a circular track.

(c) A spinning cricket ball that turns sharply on hitting the ground.

(d) A tumbling beaker that has slipped off the edge of table.

Sol. (*a*) The railway carriage moving without jerks between two stations, so the distance between two stations is considered to be large as compared to the size of the train. Therefore the train is considered as a point object.

(*b*) The monkey may be considered as point object because value of distance covered on a circular track is much greater.

(*c*) As turning of ball is not smooth, thus the distance covered by ball is not large in the reasonable time. Therefore ball cannot be considered as point object.

(*d*) Again a tumbling beaker slipped off the edge of a table cannot be considered as a point object because distance covered is not much larger.

3.2. *The position-time (x - t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in Fig. Choose the correct entries in the brackets below:*

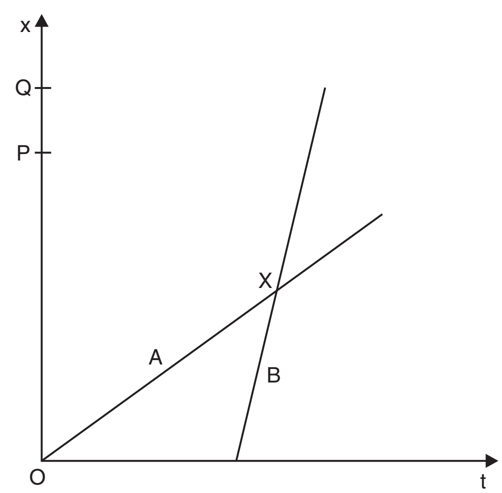
(a) (A/B) lives closer to the school than (B/A).

(b) (A/B) starts from the school earlier than (B/A).

(c) (A/B) walks faster than (B/A).

(d) A and B reach home at the (same/different) time.

(e) (A/B) overtakes (B/A) on the road (once/twice).



Sol. (*a*) *A* lives closer to school than *B*, because *B* has to cover higher distances [*OP* < *OQ*].

(*b*) *A* starts earlier for school than *B*, because $t = 0$ for *A* but for *B*, *t* has some finite time.

(*c*) As slope of *B* is greater than that of *A*, thus *B* walks faster than *A*.

(*d*) *A* and *B* reach home at the same time.

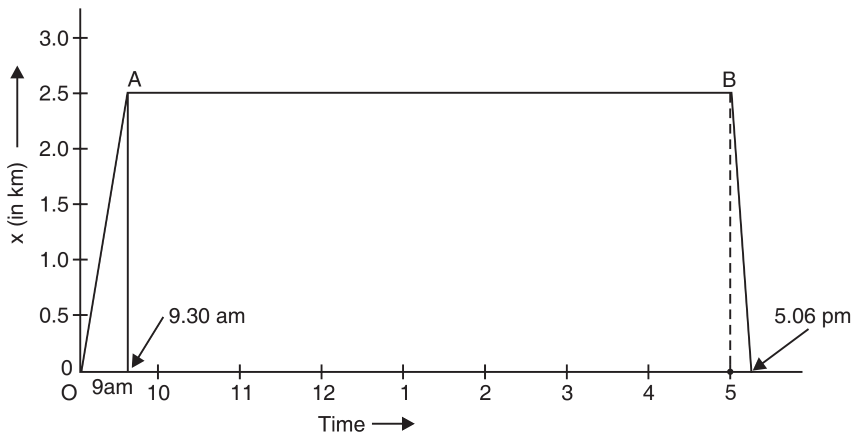
(*e*) At the point of intersection (*i.e.*, *X*), *B* overtakes *A* on the roads once.

3.3. *A woman starts from her home at 9.00 am, walks with a speed of 5 km h*$^{-1}$ *on a straight road up to her office 2.5 km away, stays at the office up to 5.00 pm, and returns home by an auto with a speed of 2.5 km h*$^{-1}$*. Choose suitable scales and plot the x-t graph of her motion.*

Sol. Distance covered while walking = 2.5 km.

Speed while walking = 5 km/h

Time taken to reach office while walking = $\frac{2.5}{5}\text{ h} = \frac{1}{2}\text{ h}$



If O is regarded as the origin for both time and distance, then

at $t = 9.00$ am, $x = 0$

and at $t = 9.30$ am, $x = 2.5$ km

OA is the *x-t* graph of the motion when the woman walks from her home to office. Her stay in the office from 9.30 am to 5.00 pm is represented by the straight line *AB* in the graph.

Now, time taken to return home by an auto

$$= \frac{2.5}{5}\text{ h} = \frac{1}{10}\text{ h} = 6 \text{ minute}$$

So, at t = 5.06 pm, $x = 0$

This motion is represented by the straight line *BC* in the graph. While drawing the *x-t* graph, the scales chosen are as under:

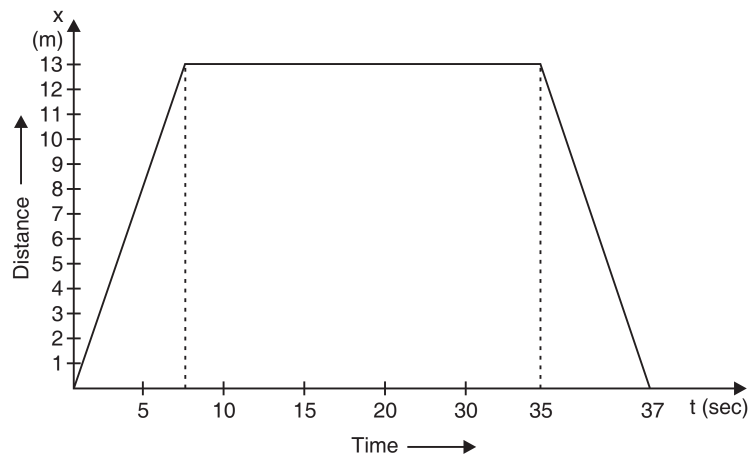
Along time-axis, one division equals 1 hour.

Along positive-axis, one division equals 0.5 km.

3.4. *A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1 m long and requires 1 s. Plot the x-t graph of his motion. Determine graphically and otherwise how long the drunkard takes to fall in a pit 13 m away from the start.*

Sol. Since the man steadily moves forward as the time progresses so the following graph will represent his motion till he covers 13 m. In 5 s he moves through a distance of 5 m and then in next 3 s comes back by 3 m.

Thus in 8 s he covers only 2 m, as shown in the graph he would fall in the pit in 37 s.



As pointed out earlier, the man covers 2 m in 8 s so, he will cover 8 m in 32 s. But at the end in 5 s he would cover another 5 m *i.e.*, 32 s + 5 s = 37 s, he would cover 8 m + 5 m = 13 m.

Thus, he would fall in the pit in 37^{th} second.

3.5. *A jet airplane travelling at the speed of 500 km h^{-1} ejects its products of combustion at the speed of 1500 km h^{-1} relative to the jet plane. What is the speed of the latter with respect to an observer on the ground?*

Sol. Velocity of jet airplane *w.r.t* observer on ground

$$= 500 \text{ km/h}.$$

If $\vec{v}_j$ and $\vec{v}_0$ represent the velocities of jet and observer respectively, then

$$v_j - v_0 = 500 \text{ km h}^{-1}$$

Similarly, if $\vec{v}_c$ represents the velocity of the combustion products *w.r.t* jet plane, then

$$v_c - v_j = -1500 \text{ km/h}$$

The negative sign indicates that the combustion products move in a direction opposite to that of jet.

Speed of combustion products *w.r.t.* observer

$$= v_c - u_0 = (v_c - v_j) + (v_j - v_0) = (-1500 + 500) \text{ km h}^{-1}$$
$$= -1000 \text{ km h}^{-1}.$$

3.6. *A car moving along a straight highway with speed of 126 km h^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop?*

Sol. Given $u = 126 \text{ km/h} = 126 \times \frac{5}{18} \text{ m/s} = 35 \text{ m/s}$

$$S = 200 \text{ m and } v = 0$$

As $v^2 - u^2 = 2as$

$\therefore$ $0 - (35)^2 = 2a \times 200$

$\Rightarrow$ $$a = \frac{-(35)^2}{400} = -3.06 \text{ m/s}^2$$

Also, $$v = u + at$$

$\Rightarrow$ $$t = \frac{v-u}{a} = \frac{0-35}{-3.06} = 11.4 \text{ s.}$$

3.7. *Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 km h^{-1} in the same direction, with A ahead of B. The driver of B decides to overtake A and accelerates by 1 ms^{-2}. If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between them?*

Sol. Here length of train A = length of train B = l = 400 m. As speed of both trains u = 72 km h^{-1} = 20 ms^{-1} in same direction, hence their relative velocity $u_{BA} = 0$.

Let initial distance between the two trains be 'S' then train B covers the distance (S + 2l) = (S + 800) m in time t = 50 s when accelerated with a uniform acceleration $a = 1 \text{ m/s}^2$.

$\therefore$ $$(S + 800) = u_{AB} \times t + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 1 \times (50)^2 = 1250 \text{ m}$$

$\Rightarrow$ $$S = 1250 - 800 = 450 \text{ m}$$

and initial distance between guard of train B from driver of train A = 450 + 800 = 1250 m.

3.8. *On a two-lane road, car A is travelling with a speed of 36 km h^{-1}. Two cars B and C approach car A in opposite directions with a speed of 54 km h^{-1} each. At a certain instant, when the distance AB is equal to AC, both being 1 km, B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident?*

Sol. Speed of car A, $$v_A = 36 \text{ km/h} = 36 \times \frac{5}{18} = 10 \text{ m/s}$$

Speed of car B, $$v_B = 54 \text{ km/h} = 54 \times \frac{5}{18} = 15 \text{ m/s}$$

Relative speed of car A w.r.t car C = v_{AC} = (10 + 15) ms^{-1} = 25 ms^{-1}

Relative speed of car B w.r.t car A = v_{BA} = (15 – 10) ms^{-1} = 5 m/s

Time taken by car C to cover distance AC,

$$t = \frac{1000}{v_{AC}} = \frac{1000}{25} = 40\text{s}$$

If a is the acceleration, then

$$S = ut + \frac{1}{2}at^2$$

$\Rightarrow$ $$1000 = 5 \times 40 + \frac{1}{2}a \times (40)^2$$

$\Rightarrow$ $$a = \frac{1000-200}{800} = 1 \text{ m/s}^2.$$

3.9. *Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T minute. A man cycling with a speed of 20 km h^{-1} in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. What is the period T of the bus service and with what speed (assumed constant) do the buses ply on the road?*

Sol. Let v_b be the speed of each bus. Let v_c be the speed of cyclist.

Relative velocity of the buses plying in the direction of motion of cyclist is $v_b - v_c$.

The buses playing in the direction of motion of the cyclist go past him after every 18 minute *i.e.,* $\frac{18}{60}$h.

$$\therefore \quad \text{Distance covered is } (v_b - v_c) \times \frac{18}{60}.$$

Since a bus leaves after every T minute therefore distance is also equal to $v_b \times \frac{T}{60}$.

$$\therefore \quad (v_b - v_c) \times \frac{18}{60} = v_b \times \frac{T}{60}. \quad ...(1)$$

Relative velocity of the buses plying opposite to the direction of motion of the cyclist is $v_b + v_c$. In this case, the buses go past the cyclist after every 6 minute.

$$\therefore \quad (v_b + v_c) \times \frac{6}{60} = v_b \times \frac{T}{60} \quad ...(2)$$

Dividing (1) by (2), we get $\frac{(v_b - v_c)18}{(v_b + v_c)6} = 1$

On simplification $v_b = 2v_c$

But $v_c = 20 \text{ km h}^{-1}$

$\therefore$ $v_b = 40 \text{ km h}^{-1}$

From equation (1),

$$(40 - 20) \times \frac{18}{60} = 40 \times \frac{T}{60}$$

On simplification, $\boldsymbol{T}$ **= 9 minutes.**

3.10. *A player throws a ball upwards with an initial speed of 29.4 ms*$^{-1}$.

(a) What is the direction of acceleration during the upward motion of the ball?

(b) What are the velocity and acceleration of the ball at the highest point of its motion?

(c) Choose the x = 0 m and t = 0 s to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of x-axis, and give the signs of position, velocity and acceleration of the ball during its upward, and downward motion.

(d) To what height does the ball rise and after how long does the ball return to the player's hands? (Take g = 9.8 m s^{-2} *and neglect air resistance).*

Sol. (*a*) The direction of acceleration during the upward motion of the ball is vertically downward.

(*b*) At the highest point, velocity of ball is zero but acceleration ($g = 9.8 \text{ ms}^{-2}$) in vertically downward direction.

(*c*) If we consider highest point of ball motion as $x = 0$, $t = 0$ and vertically downward direction to be $+ve$ direction of x-axis, then (*i*) during upward motion of ball before reaching the highest point position (as well as displacement) $x = +ve$, velocity $v = -ve$ and acceleration $a = g = +ve$.

(*ii*) during the downward motion of ball after reaching the highest point, x, v and $a = g$ all the three quantities are positive.

(*d*) During upward motion

$$u = -29.4 \text{ ms}^{-1},\ a = 9.8 \text{ ms}^{-2},\ v = 0$$

As $v^2 - u^2 = 2\,a\,S \Rightarrow 0 - (29.4)^2 = 2 \times 9.8 \text{ x } S$

$$\Rightarrow \quad S = \frac{-(29.4)^2}{2 \times 9.8} = -44.1 \text{ m}$$

Also $v = u + at \Rightarrow v - u = at$

$$\Rightarrow \quad 0 - (-29.4) = 9.8\,t$$

$$\text{or} \quad t = \frac{29.4}{9.8} = 3 \text{ s}$$

Total time = 3 + 3 = 6 s [$\because$ time of ascent = time of descent]

3.11. *Read each statement below carefully and state with reasons and examples, if it is true or false; A particle in one-dimensional motion*

(a) with zero speed at an instant may have non-zero acceleration at that instant.

(b) with zero speed may have non-zero velocity.

(c) with constant speed must have zero acceleration,

(d) with positive value of acceleration must be speeding up.

Sol. (*a*) True. Consider a ball thrown up. At the highest point, speed is zero but the acceleration is non-zero.

(*b*) False. If a particle has non-zero velocity, it must have speed.

(*c*) True. If the particle rebounds instantly with the same speed, it implies infinite acceleration which is physically impossible.

(*d*) False. True only when the chosen position direction is along the direction of motion.

3.12. *A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one tenth of its speed. Plot the speed-time graph of its motion between t = 0 to 12 s.*

Sol. $u = 0,\ a = 10 \text{ ms}^{-2},\ S = 90 \text{ m},\ t = ?,\ v = ?$

Using $v^2 - u^2 = 2as$, $v^2 - (0)^2 = 2 \times 10 \times 90$

$$\Rightarrow \quad v = 30\sqrt{2} \text{ m/s}$$

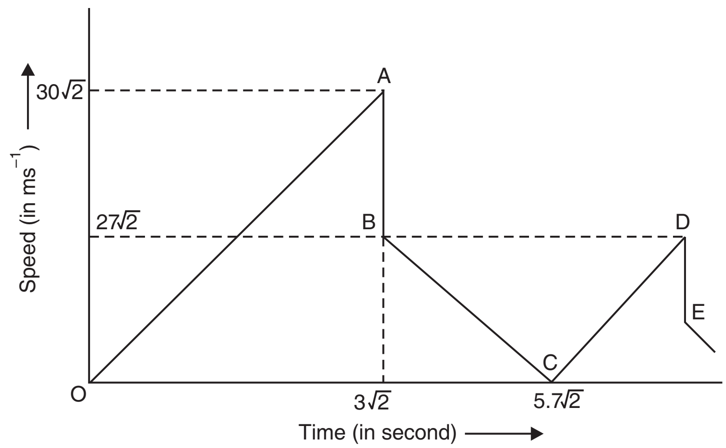
Again, using $S = ut + \frac{1}{2}at^2$, $90 = 0 \times t + \frac{1}{2} \times 10t^2$

$$\Rightarrow \quad t = \sqrt{18} \text{ s} = 3\sqrt{2} \text{ s}$$

$$\text{Rebound velocity} = \frac{9}{10} \times 30\sqrt{2} \text{ ms}^{-1} = \sqrt{2} \text{ ms}^{-1}$$

Time taken to reach highest point $= \frac{27\sqrt{2}}{10}\text{s} = 2.7\sqrt{2}$ s

Total time = $(3\sqrt{2} + 2.7\sqrt{2})$ s = $5.7\sqrt{2}$ s



OA represents the vertically downward motion after the ball has been dropped from a height of 90 m. The ball reaches the floor with a velocity of $30\sqrt{2}$ ms^{-1} after having been in motion for $3\sqrt{2}$ s. The vertical straight portion AB represents the loss of $\frac{1}{10}$ th of speed. *BC* represents the vertically upward motion after first rebound. The ball reaches? the highest point in $2.7\sqrt{2}$ s. The total time from the beginning is $3\sqrt{2} + 2.7\sqrt{2}$ *i.e.,* $5.7\sqrt{2}$ s. *C* represents the highest point reached after first rebound. *CD* represents the vertically downward motion. *D* represents the situation when the ball again reaches the floor. *DE* represents the loss of speed.

3.13. *Explain clearly, with examples, the distinction between:*

(a) Magnitude of displacement (sometimes called distance) over an interval of time, and the total length of path covered by a particle over the same interval;

(b) Magnitude of average velocity over an interval of time, and the average speed over the same interval. (Average speed of a particle over an interval of time is defined as the total path length divided by the time interval). Show in both (a) and (b) that the second quantity is either greater than or equal to the first. When is the equality sign true? [For simplicity, consider one-dimensional motion only].

Sol. (*a*) Suppose a particle goes from point *A* to *B* along a straight path and returns to *A* along the same path. The magnitude of the displacement of the particle is zero, because the particle has returned to its initial position. The total length of path covered by the particle is *AB* + *BA* = *AB* + *AB* = 2*AB*. Thus, the second quantity is greater than the first.

(*b*) Suppose, in the above example, the particle takes time *t* to cover the whole journey. Then, the magnitude of the average velocity of the particle over time-interval *t* is

$$\frac{\text{Magnitude of displacement}}{\text{Time-interval}} = \frac{0}{t} = 0$$

While the average speed of the particle over the same time-interval is

$$\frac{\text{Total path length}}{\text{Time-interval}} = \frac{2\,AB}{t}$$

Again, the second quantity (average speed) is greater than the first (magnitude of average velocity).

Note: In both the above cases, the two quantities are equal if the particle moves from one point to another along a straight path in the same direction only.

3.14. *A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km h^{-1}. Finding the market closed, he instantly turns and walks back home with a speed of 7.5 km h^{-1}. What is the*

(a) Magnitude of average velocity, and

(b) Average speed of the man over the interval of time (i) 0 to 30 min. (ii) 0 to 50 min. (iii) 0 to 40 min? [Note: You will appreciate from this exercise why it is better to define average speed as total path length divided by time, and not as magnitude of average velocity. You would not like to tell the tired man on his return home that his average speed was zero!]

Sol. Since $$v = \frac{S}{t} \Rightarrow t = \frac{S}{v}$$

Time taken by the man to reach market,

$$t = \frac{S}{v} = \frac{2.5}{5} = 0.5 \text{ h}$$

Time taken by the man to come back,

$$t_1 = \frac{S}{v_1} = \frac{2.5}{7.5} = 0.333 \text{ h}$$

(*i*) Average velocity (0 – 30 min) $= \frac{\Delta x}{\Delta t} = \frac{2.5}{0.5} = 5 \text{ kmh}^{-1}$

[∵ In 0.5 h, distance covered by man = 2.5 km]

(*ii*) Average velocity (0 – 50 min)

$$= \frac{(2.5 + 2.5) \text{ km}}{(0.5 + 0.333) \text{ h}} = \frac{5}{0.833} \text{kmh}^{-1} = 8 \text{ kmh}^{-1}$$

(*iii*) Average velocity (0 – 4 min) $= \frac{\Delta x}{\Delta t} = \frac{\left(2.5 - \frac{2.5}{2}\right)\text{km}}{\frac{40}{60} h} = 1.875 \text{ kmh}^{-1}$

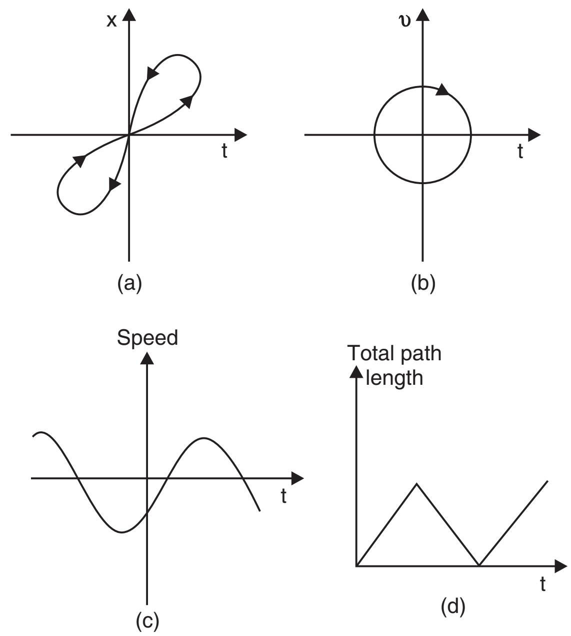
[∵ during 1st 30 min, distance covered = 2.5 km, in next 10 min, distance covered = $\frac{2.5}{2}$ km in return journey]

(*iv*) Average speed (0 – 40 min) $= \frac{\text{Total distance}}{\text{Total time}} = \frac{2.5 + \frac{2.5}{2}}{\frac{40}{60}} = 5.625 \text{ km h}^{-1}$

3.15. *In Exercises 3.13 and 3.14, we have carefully distinguished between average speed and magnitude of average velocity. No such distinction is necessary when we consider instantaneous speed and magnitude of velocity. The instantaneous speed is always equal to the magnitude of instantaneous velocity. Why?*

Sol. Instantaneous velocity is the velocity of a particle at a particular instant of time. In this case of small interval of time, the magnitude of the displacement is effectively equal to the distance travelled by the particle in the same interval of time. Therefore, there is no distinction between instantaneous velocity and speed.

3.16. *Look at the graphs (a) to (d) Fig. carefully and state, with reasons, which of these cannot possibly represent one-dimensional motion of a particle.*



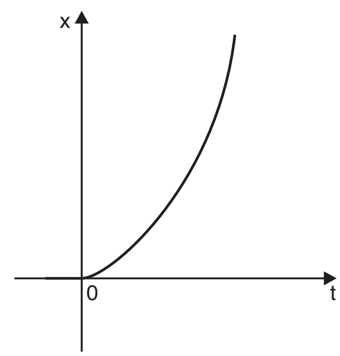
Sol. None of the four graph represent a possible one-dimensional motion. In graphs (*a*) and (*b*) motions are definitely two dimensional. Graph (*a*) represents two positions at the same time which is not possible.

In graph (*b*) opposite motion is visible at the same time.

The graph (*c*) is not correct since it shows that the particle has negative speed at a certain instant. Speed is always positive.

In graph (*d*) path length is shown as increasing as well as decreasing. Path length never decreases.

3.17. *Figure shows the x-t plot of one-dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$? If not, suggest a suitable physical context for this graph.*



Sol. It is not correct to say that the particle moves in a straight line for $t < 0$ (*i.e.*, –*ve*) and on a parabolic path for $t > 0$ (*i.e.*, + *ve*) because the *x-t* graph can not show the path of the particle.

For the graph, a suitable physical context can be the particle thrown from the top of a tower at the instant $t = 0$.

3.18. *A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1}. If the muzzle speed of the bullet is 150 ms^{-1}, with what speed does the bullet hit the thief's car? (Note: Obtain that speed which is relevant for damaging the thief's car).*

Sol. Speed of police van $= v_p = 30 \text{ kmh}^{-1} = 30 \times \dfrac{1000}{3600} \text{ ms}^{-1} = \dfrac{25}{3} \text{ ms}^{-1}$

Speed of thief's car $= v_t = 192 \text{ km h}^{-1}$

$$= 192 \times \frac{5}{18} \text{ ms}^{-1} = \frac{160}{3} \text{ ms}^{-1}$$

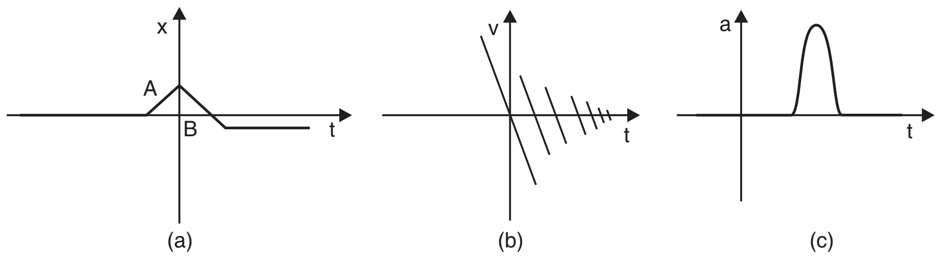
Speed of bullet, v_b = Speed of police van + speed with which bullet is actually fired

$$\therefore \quad v_b = \left(\frac{25}{3} + 150\right) \text{ms}^{-1} = \frac{475}{3} \text{ ms}^{-1}$$

Relative velocity of bullet w.r.t thief's car,

$$v_{bt} = v_b - v_t = \left(\frac{475}{3} - \frac{160}{3}\right) \text{ms}^{-1} = 105 \text{ ms}^{-1}.$$

3.19. *Suggest a suitable physical situation for each of the following graphs:*

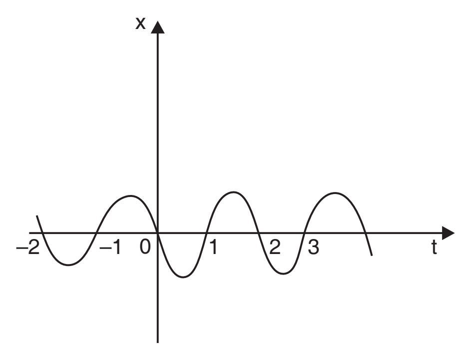


Sol. (*a*) A ball at rest on a smooth floor is kicked. It rebounds from a wall with reduced speed and moves to the opposite wall which stops it.

(*b*) The graph shows that velocity changes again and again with the passage of time and every time losing some speed. Therefore, it may represent a physical situation such as a ball falling freely (after thrown up), on striking the ground rebounds with reduced speed after each hit against the ground.

(*c*) A uniformly moving cricket ball turned back by hitting it with a bat for a very short time-interval.

3.20. *Figure gives the x-t plot of a particle executing one-dimensional simple harmonic motion. (You will learn about this motion in more detail in Chapter 14). Give the signs of position, velocity and acceleration variables of the particle at t = 0.3 s, 1.2 s, – 1.2 s.*



Sol. In *x-t* graph of Fig. showing simple harmonic motion of a particle, the signs of position, velocity and acceleration are as given below.

In S.H.M., acceleration, $a \propto -x$ or $a = -kx$.

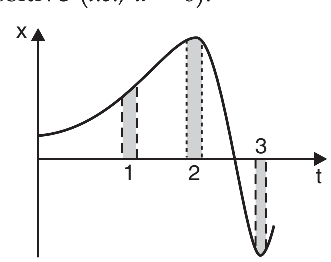
(*i*) At $t = 0.3$ s, $x < 0$ *i.e.,* x is in $-ve$ direction. Moreover, as x is becoming more negative with time, it shows that v is also $-ve$ (*i.e.,* $v < 0$). However, $a = -kx$ will be $+ve$ $(a > 0)$.

(*ii*) At $t = 1.2$ s, $x > 0$, $v > 0$ and $a < 0$.

(*iii*) At $t = -1.2$ s, $x < 0$, but here on increasing the time t, value of x becomes less negative.

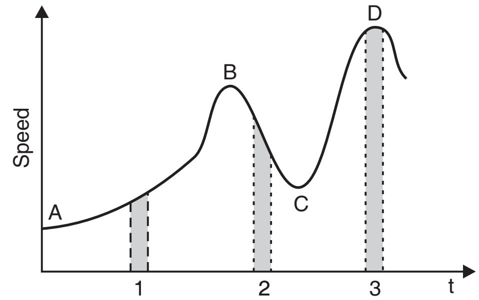
It means that v is $+ve$ (*i.e.,* $v > 0$). Again $a = -kx$ will be positive (*i.e.,* $a > 0$).

3.21. *Figure gives the x-t plot of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least? Give the sign of average velocity for each interval.*



Sol. Greater in 3, least in 2; $v > 0$ in 1 and 2, $v < 0$ in interval 3.

3.22. *Figure gives a speed-time graph of a particle in motion along a constant direction. Three equal intervals of time are shown. In which interval is the average acceleration greatest in magnitude? In which interval is the average speed greatest? Choosing the positive direction as the constant direction of motion, give the signs of v and a in the three intervals. What are the accelerations at the points A, B, C and D?*



Sol. The acceleration is greatest in magnitude in interval 2 as the change in speed in the same time is maximum in this interval.

The average speed is greatest in interval 3 (peak D is at maximum on speed axis).

The sign of v and a in the three intervals are:

$$v > 0 \text{ in } 1, 2 \text{ and } 3; \; a > 0 \text{ in } 1$$
$$a < 0 \text{ in } 2, \; a = 0 \text{ in } 3.$$

Acceleration is zero at A, B, C and D.

3.23. *A three-wheeler starts from rest, accelerates uniformly with 1 m s^{-2} on a straight road for 10 s, and then moves with uniform velocity. Plot the distance covered by the vehicle during the nth second (n = 1, 2, 3.....) versus n. What do you expect this plot to be during accelerated motion: a straight line or a parabola?*

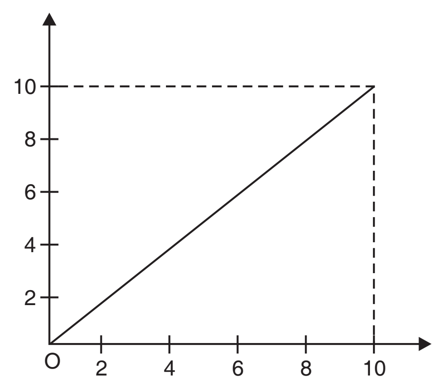
Sol. Since

$$S_{nth} = u + \frac{1}{2}a\ (2n - 1)$$

when $u = 0, a = 1 \text{ ms}^{-2}$

$$\therefore \quad S_{nth} = 0 + \frac{1}{2}(2n-1) = \frac{1}{2}(2n-1)$$

$\therefore$ For $n = 1, 2, 3$ ....



$$S_1 = \frac{1}{2}(2 \times 1 - 1) = 0.5 \text{ m}$$
$$S_2 = \frac{1}{2}(2 \times 2 - 1) = 1.5 \text{ m}$$
$$S_3 = \frac{1}{2}(2 \times 3 - 1) = 2.5 \text{ m}$$
$$S_4 = \frac{1}{2}(2 \times 4 - 1) = 3.5 \text{ m}$$
$$S_5 = \frac{1}{2}(2 \times 5 - 1) = 4.5 \text{ m}$$
$$S_6 = \frac{1}{2}(2 \times 6 - 1) = 5.5 \text{ m}$$
$$S_7 = \frac{1}{2}(2 \times 7 - 1) = 6.5 \text{ m}, \; S_8 = \frac{1}{2}(2 \times 8 - 1) = 7.5 \text{ m}$$
$$S_9 = \frac{1}{2}(2 \times 9 - 1) = 8.5 \text{ m}, \; S_{10} = \frac{1}{2}(2 \times 10 - 1) = 9.5 \text{ m}$$

3.24. *A boy standing on a stationary lift (open from above) throws a ball upwards with the maximum initial speed he can, equal to 49 m s^{-1}. How much time does the ball take to return to his hands? If the lift starts moving up with a uniform speed of 5 m s^{-1} and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands?*

Sol. When either the lift is at rest or the lift is moving either vertically upward or downward with a constant speed, we can apply three simple kinetnatic motion equations presuming $a = \pm g$ (as the case may be).

In present case $u = 49$ ms^{-1} (upward) $a = g = 9.8$ ms^{-2} (downward)

If the ball returns to boy's hands after a time t, then displacement of ball relative to boy is zero *i.e.*, $s = 0$. Hence, using equation $s = ut + \frac{1}{2}at^2$, we have

$$0 = 49 \pm -\frac{1}{2} \times 9.8 \times t^2$$

$$\Rightarrow \quad 4.9\,t^2 - 49t = 0 \Rightarrow t = 0 \text{ or } 10 \text{ s}$$

As $t = 0$ is physically not possible, hence time $t = 10$ s.

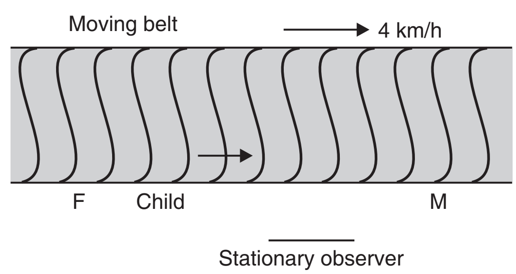
3.25. *On a long horizontally moving belt (Fig.), a child runs to and fro with a speed 9 km h^{-1} (with respect to the belt) between his father and mother located 50 m apart on the moving belt. The belt moves with a speed of 4 km h^{-1}. For an observer on a stationary platform outside, what is the*

(a) Speed of the child running in the direction of motion of the belt?

(b) Speed of the child running opposite to the direction of motion of the belt?

(c) Time taken by the child in (a) and (b)?

Which of the answers alter if motion is viewed by one of the parents?



Sol. Speed of child with respect to belt = 9 km h^{-1}

Speed of belt = 4 km h^{-1}

(*a*) When the child runs in the direction of motion of the belt, then speed of child w.r.t. stationary observer = (9 + 4) km h^{-1} = 13 km h^{-1}.

(*b*) When the child runs opposite to the direction of motion of the belt, then speed of child w.r.t. stationary observer = (9 – 4) km h^{-1} = 5 km h^{-1}

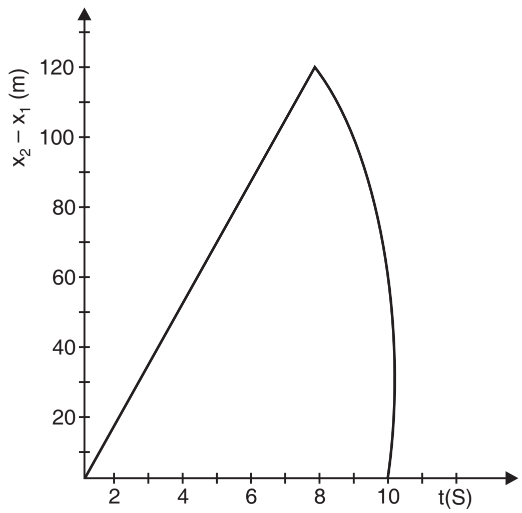
(*c*) Speed of child w.r.t. either parent = 9 km h^{-1}

Distance to be covered = 50 m = 0.05 km

$$\text{Time} = \frac{0.05 \text{ km}}{9 \text{ km h}^{-1}} = 0.0056 \text{ h} \approx 20 \text{ S}$$

If the motion is viewed by one of the parents, then the answers to (*a*) and (*b*) are altered but answer to (*c*) remains unaltered.

3.26. *Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 ms^{-1} and 30 ms^{-1}. Verify that the graph shown in Fig. correctly represents the time variation of the relative position of the second stone with respect to the first. Neglect air resistance and assume that the stones do not rebound after hitting the ground. Take g = 10 ms^{-2}. Give the equations for the linear and curved parts of the plot.*



Sol. For first stone,

$$x(0) = 200 \text{ m},\ v(0) = 15 \text{ ms}^{-1},\ a = -10 \text{ ms}^{-2}$$

$$x_1(t) = x(0) + v(0)\,t + \frac{1}{2}\,a\,t^2$$

$$x_1(t) = 200 + 15t - 5t^2$$

When the first stone hits the ground, $x_1(t) = 0$

$\therefore \quad -5t^2 + 15t + 200 = 0$

On simplification, $t = 8$ s

For second stone, $x(0) = 200$ m, $v(0) = 30$ ms^{-1}, $a = -10$ ms^{-2}

$$x_2(t) = 200 + 30t - 5t^2$$

When this stone hits the ground, $x_2(t) = 0$

$\therefore \quad -5t^2 + 30t + 200 = 0$

Relative position of second stone w.r.t. first is given by

$$x_2(t) - x_1(t) = 15t$$

Since there is a linear relationship between $x_2(t) - x_1(t)$ and t, therefore the graph is a straight line.

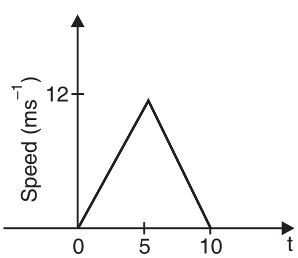
For maximum separation, $t = 8$ s

So maximum separation is 120 m

After 8 second, only the second stone would be in motion. So, the graph is in accordance with the quadratic equation.

3.27. *The speed-time graph of a particle moving along a fixed direction is shown in Fig. Obtain the distance traversed by the particle between (a) t = 0 s to 10 s. (b) t = 2 s to 6 s.*

What is the average speed of the particle over the intervals in (a) and (b)?



Sol. (*a*) Distance travelled by the particle between t = 0 s to 10 s

$$= \text{area of } \Delta OAB = \frac{1}{2} \text{ base} \times \text{height}$$

$$= \frac{1}{2} \times 10 \times 12 = 60\text{m}$$

$\therefore$ Average speed of particle $v_{av} = \dfrac{60 \text{ m}}{10 \text{ s}} = 6 \text{ ms}^{-1}$

(*b*) The distance traversed by the particle between

$t = 2$ s to $t = 6$ s

= distance from 2 to 5 s (S_1) + distance in 6th second (S_2)

Now, $u = 0,\ t = 5,\ v = 12\ \text{ms}^{-1}$

$\therefore$ Acceleration for 0 – 5 s, $a = \dfrac{v-u}{t} = \dfrac{12-0}{5}\ \text{ms}^{-2} = 2.4\ \text{ms}^{-2}$

$\therefore$ Distance covered from 2 to 5 s = distance covered in 5 s – distance covered in 2 s

$$S_1 = \frac{1}{2}a(5)^2 - \frac{1}{2}a(2)^2 = \frac{1}{2}\times 2.4\times\left[(5)^2-(2)^2\right] = 25.2\ \text{m.}$$

For motion from 5 to 10 s, $u = +12\ \text{ms}^{-1}$ and $a = -2.4\ \text{ms}^{-2}$ and interval $t = 5$ s to $t = 6$ s means $n = 1$ for this motion.

$\therefore$ Distance covered in 6th second $S_2 = u + \dfrac{1}{2}a(2n-1)s$

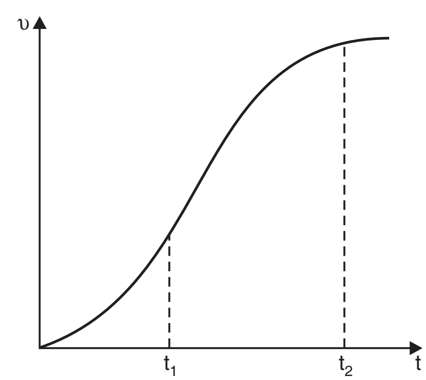
$$= 12 - \frac{2.4}{2}(2\times 1 - 1) = 10.8\ \text{m}$$

$\therefore$ Total distance covered from $t = 2$ s to 6 s = $S_1 + S_2$

$= 25.2 + 10.8 = 36$ m

and average speed $= \dfrac{36\ \text{m}}{(6-2)\ \text{s}} = 9\ \text{ms}^{-1}$.

3.28. *The velocity-time graph of a particle in one-dimensional motion is shown below. Which of the following formulae are correct for describing the motion of the particle over the time interval from t_1 to t_2?*



(*a*) $x(t_2) = x(t_1) + v(t_1)(t_2 - t_1) + \frac{1}{2}a(t_2 - t_1)^2$

(*b*) $v(t_2) = v(t_1) + a(t_2 - t_1)$

(*c*) $a_{average} = [x(t_2) - x(t_1)]/(t_2 - t_1)$

(*d*) $a_{average} = [v(t_2) - v(t_1)]/(t_2 - t_1)$

(*e*) $x(t_2) = x(t_1) + v_{av}(t_2 - t_1) + \frac{1}{2}a_{av}(t_2 - t_1)^2$

(*f*) *$x(t_2) - x(t_1)$ = Area under the v-t curve bounded by t-axis and the dotted lines.*

Sol. (*c*), (*d*), (*f*).

As it is evident from the shape of *v-t* graph that acceleration of the particle is not uniform between time intervals t_1 and t_2. (since the given *v-t* graph is not straight). The equations (*a*), (*b*) and (*e*) represent uniform acceleration.

ADDITIONAL QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *Can a moving body have relative velocity zero with respect to another body? Give an example.*

Ans. Yes, two cyclists moving with same velocity in the same direction.

Q. 2. *Define displacement of a particle.*

Ans. The change in the position co-ordinates of a particle over a given period of time is called the displacement of the particle.

Q. 3. *What is the shape of displacement-time graph for uniform linear motion?*

Ans. A straight line inclined to the time axis.

Q. 4. *Write the expression for distance covered in n^{th} second by a uniformly accelerated body.*

Ans. If a is the uniform acceleration

then, $$s = u + \frac{1}{2} a \left(2n - 1\right)$$

where u is the initial velocity.

Q. 5. *Under what condition will the distance and displacement of a moving object will have the same magnitude?*

Ans. Distance and displacement will have the same magnitude when the object moves along a straight line without change in its direction.

Q. 6. *What is meant by 'point object' in physics?*

Ans. An object is said to be point object if its dimensions are negligible as compared to the distance travelled by it. For example, an aeroplane which flies from Delhi to London.

Q. 7. *What is the significance of the slope of x-t graph?*

Ans. Slope of x-t graph provides velocity of motion. The nature of motion is identified by the shape of the graph.

Q. 8. *Give an example of uniformly accelerated linear motion.*

Ans. Motion of a body under gravity.

Q. 9. *Can a particle have acceleration at an instant if its velocity is zero at that instant? Give example.*

Ans. Yes, it is possible. In motion under gravity at the highest point of its motion velocity is zero but acceleration $a = g$ in downward direction.

Q. 10. *Write two uses of v-t graph.*

Ans. (*i*) slope of v-t graph gives acceleration.

(*ii*) area under v-t graph gives displacement.

Q. 11. *Define non-uniformly accelerated motion.*

Ans. A body has non-uniformly accelerated motion if it moves with variable acceleration.

Q. 12. *Two particles A and B are moving along the same straight line. B is ahead of A. Velocities remaining unchanged, what would be the effect on the magnitude of relative velocity if A is ahead of B?*

Ans. There will be no effect on the magnitude of relative velocity.

Q. 13. *What is the relative velocity of two bodies having equal velocities?*

Ans. When two bodies have equal velocities $\left(i.e., \vec{v}_a = \vec{v}_b = \vec{v}\right)$, then their relative velocity is zero *i.e.*, $\vec{v}_{ab} = \vec{v}_a - \vec{v}_b = \vec{v} - \vec{v} = 0$.

Q. 14. *What does speedometer record: the average speed or the instantaneous speed?*

Ans. The speedometer measures the instantaneous speed.

Q. 15. *Can Earth be regarded as a 'point object' if only the orbital motion of Earth around the sun is considered?*

Ans. Yes. This is because the size of the Earth is very small as compared to the size of the orbit of the Earth around the sun.

Q. 16. *If position of a particle at instant t is given by $x = t^3$, find acceleration of the particle.*

Ans. Given, $x = t^3$

$$\therefore \quad v = \frac{dx}{dt} = \frac{d}{dt}\left(t^3\right) = 3t^2$$

Now, acceleration $(a) = \frac{dv}{dt} = \frac{d}{dt}\left(3t^2\right) = 6t$.

Q. 17. *A particle is moving in a straight line. Is it possible for it to maintain the motion in the same direction while the acceleration is in the reverse direction?*

Ans. Yes, due to acceleration in reverse direction the velocity starts decreasing with time but the direction of motion is maintained till the velocity is reduced to zero.

Q. 18. *Why the speed of an object can never be negative?*

Ans. Speed is distance covered per unit time. Since distance cannot be negative therefore speed cannot be negative.

Q. 19. *What is common between the two graphs shown in Figs. (a) and (b)?*

Ans. Both represent negative velocity.

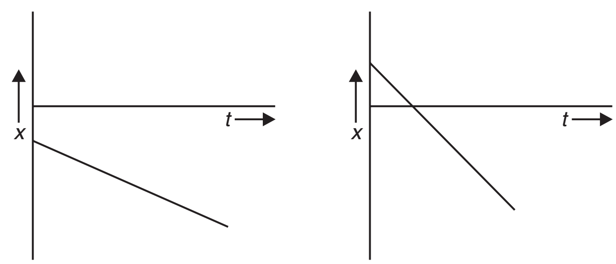
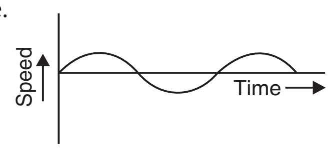


Fig. (a) *Fig. (b)*

Q. 20. *Is the speed-time graph shown in Fig possible?*

Ans. No. Speed cannot be negative.



Q. 21. *When a body accelerates by α t, what is the velocity after time 't', when it starts from rest?*

Ans. $a = \alpha t$ i.e., $\int dv = \alpha \int t\, dt$

$$v = \frac{\alpha t^2}{2}.$$

Q. 22. *A ball dropped from height h reaches the ground in t s. After what time the ball was passing through a point at a height h/2?*

Ans.
$$t = \sqrt{2\frac{h}{g}}$$

$$t' = \sqrt{\frac{2}{g}\cdot\left(\frac{h}{2}\right)} = \sqrt{\frac{h}{g}} \quad \Rightarrow \quad t' = \frac{t}{2}.$$

Q. 23. *Define uniformly accelerated motion.*

Ans. A body has uniformly accelerated motion if it moves with constant acceleration.

Q. 24. *Define one dimensional motion.*

Ans. A particle moving along a straight line or a path is said to undergo one dimensional motion.

Q. 25. *Explain the difference between uniform velocity and variable velocity.*

Ans. If a body travels equal displacements in equal intervals of time, then the velocity of body is uniform velocity. On the other hand, if the body covers unequal displacements in equal intervals of time, then its velocity is variable velocity.

Q. 26. *Write two important points to distinguish displacement from distance.*

Ans. Length of actual path covered between the initial and final points is distance while the length of the shortest path between initial and final points is displacement.

The magnitude of displacement can be both positive and negative while distance is always positive.

II. SHORT ANSWER TYPE QUESTIONS

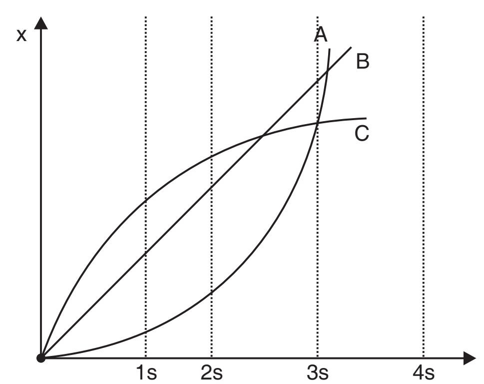
Q. 1. *The position-time-graph in figure depicts the journey of three bodies A, B and C.*

(a) At 1s, which has the greatest velocity?

(b) At 2s, which has travelled the farthest?

(c) When A meets C, is B moving faster or slower than A?

(d) Is there any time at which the velocity of A is equal to that of B?



Ans. (*a*) B

(*b*) C

(*c*) Slower

(*d*) Yes, in the interval 2 to 3 s

Q. 2. *A certain automobile manufacturer claims that its super-delux sports car will accelerate from rest to a speed of 42.0 ms^{-1} in 8.0 s. Under the important assumption that the acceleration is constant,*

(a) Determine the acceleration of car in ms^{-2}.

(b) Find the distance the car travels in 8.0 s.

(c) Find the distance the car travels in 8th second.

Ans. (*a*) We are given that $u = 0$ and velocity after 8 s is 42 m/s, so we can use $v = u + at$ to find acceleration

$$a = \frac{v-u}{t} = \frac{42.0-u}{8.0} = 5.25 \text{ ms}^{-2}$$

(*b*) distance travelled in 8.0 s,

we can use, $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 5.25 \times 8^2 = 168$ m

(*c*) distance travelled in 8th second,

we have, $S_n = u + (2n-1)\frac{a}{2} = (2 \times 8 - 1) \times \frac{5.25}{2} = 39.375$ m.

Q. 3. *The velocity of a particle is given by equation $v = 4 + 2(c_1 + c_2 t)$, where c_1 and c_2 are constant. Find the initial velocity and acceleration of the particle.*

Ans. Given equation of velocity,

$$v = 4 + 2(c_1 + c_2 t) \Rightarrow v = (4 + 2c_1) + 2c_2 t$$

Compare the above equation with equation of motion

$$v = u + at$$

Initial velocity, $u = 4 + 2c_1$

Acceleration of the particle $= 2c_2$.

Q. 4. *Two trains of lengths 109 m and 91 m are moving in opposite directions with velocities 34 km h^{-1} and 38 kmh^{-1} respectively. In what time the two trains will completely cross each other? Choose the most logical reference point for time measurement.*

Ans. Relative speed $= (34 + 38)\ kmh^{-1} = 72\ kmh^{-1}$

$$= 72 \times \frac{5}{18}\ ms^{-1} = 20\ ms^{-1}$$

Total distance $= (109 + 91)\ m = 200\ m$

$$\text{Time} = \frac{200\ m}{20\ ms^{-1}} = 10\ s.$$

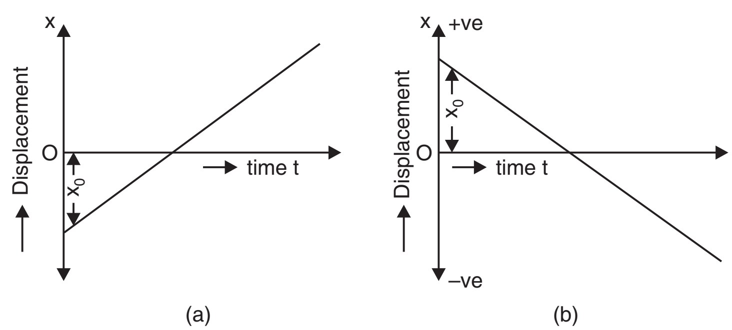
Q. 5. *An object is moving with uniform velocity v along a straight line. What will be the shape of displacement-time graph if:*

(a) $x_0 = -ve$, $v = +ve$, (b) $x_0 = +ve$, $v = -ve$

Here, x_0 represents the position of the particle at time $t = 0$.

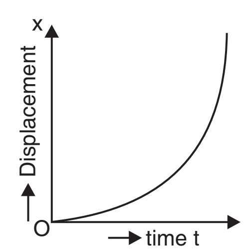
Ans. Displacement-time graphs are shown in Fig. In Fig. (*a*) x_0 is *–ve* but velocity v is *+ve* *i.e.*, slope of curve is *+ve*.

In Fig. (*b*), x_0 is + *ve* but slope of line and hence velocity is negative.



Q. 6. *Draw displacement-time graph for a uniformly accelerated motion. What is its shape?*

Ans. Displacement-time graph for a uniformly accelerated motion has been shown in adjoining Fig. The graph is parabolic in shape.



Q. 7. *A particle moves along a semicircular path of radius R in time t with constant speed. For the particle calculate*

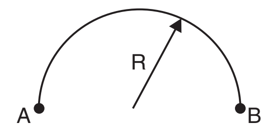
(i) distance travelled

(ii) displacement

(iii) average speed

(iv) average velocity

(v) average acceleration



Ans. (*i*) Distance = length of path of particle = $AB = \pi R$

(*ii*) Displacement = minimum distance between initial and final point = $AB = 2R$

(*iii*) Average speed, $v = \dfrac{\text{Distance}}{\text{Time}} = \dfrac{\pi R}{t}$

(*iv*) Average velocity $= \dfrac{2R}{t}\left(\dfrac{\text{Displacement}}{\text{Time}}\right)$

(*v*) Average acceleration $= \dfrac{\text{Change in velocity}}{\text{Time taken}} = \dfrac{2v}{t} = \dfrac{2\pi R}{t^2}$

Q. 8. *A body covers half of its journey with a speed of 40 m/s and other half with a speed of 60 m/s. What is the average speed during the whole journey?*

Ans. Average speed $= \dfrac{\text{Total distance}}{\text{Time taken}}$

Let x be the distance to be covered

$$\therefore \quad \text{average speed} = \frac{x}{\dfrac{x}{2v_1} + \dfrac{x}{2v_2}}$$

where, $\dfrac{x}{2v_1}$ = time taken to cover first half of the distance

$\dfrac{x}{2v_2}$ = time taken to cover second half of the distance

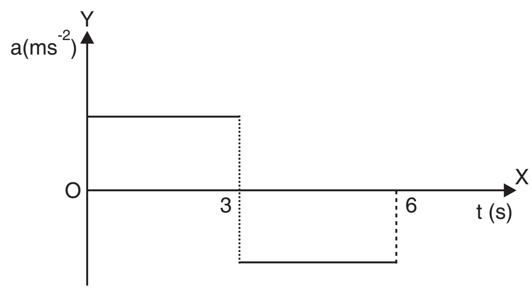
$$\text{Now,} \quad \text{average speed} = \frac{x}{\dfrac{x(v_1 + v_2)}{2v_1 v_2}} = \frac{x \times 2v_1 v_2}{x(v_1 + v_2)} = \frac{2v_1 v_2}{v_1 + v_2} s$$

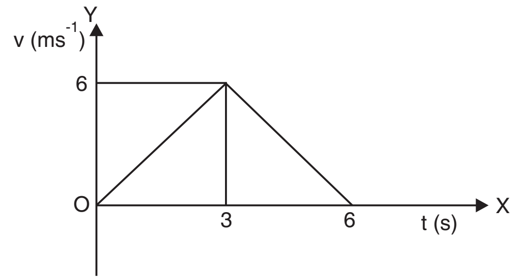
$$\Rightarrow \quad V_{av} = \frac{2 \times 40 \text{ m/s} \times 60 \text{ m/s}}{100 \text{ m/s}} = 48 \text{ ms}^{-1}$$

Q. 9. *At $t = 0$ a particle is at rest at origin. Its acceleration is 2 ms^{-2} for the first 3s and –2 ms^{-2} for the next 3 s. Plot the acceleration versus time, velocity versus time and position versus time graph.*

Ans. We are given that for first 3s acceleration is 2 ms^{-2} and for next 3s acceleration is 2 ms^{-2}. Hence acceleration time graph is as shown in the figure.

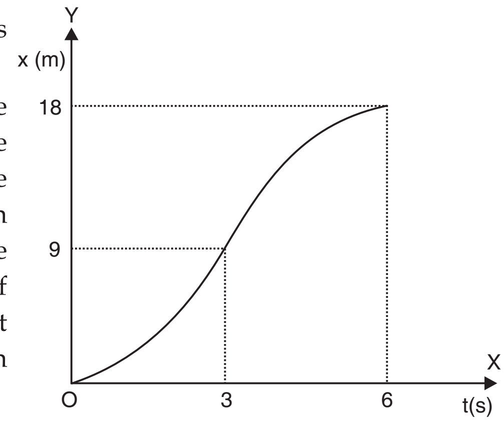
The area enclosed between *a-t* curve and *t*-axis gives change in velocity for the corresponding interval. Also at $t = 0$, $v = 0$, hence final velocity at $t = 3$ s will increase to 6 ms^{-1}. In next 3 s the velocity will decrease to zero. Hence the velocity time-graph is as shown in figure.





Note that v-t curves are taken as straight line as acceleration is constant.

Now for displacement time curve, we will use the fact that area enclosed between v-t curve and time axis gives displacement for the corresponding interval. Hence displacement in first three seconds is 9 m and in next three seconds is 9 m. Also the x-t curve will be of parabolic nature as motion is with constant acceleration. Therefore x-t curve is as shown in figure below.



Q. 10. *The distance travelled by a body is proportional to the square of time. What type of motion this body has?*

Ans. Here, $x \propto t^2$ or $x = kt^2$

where, k is constant of proportionality.

Now, $v = \dfrac{dx}{dt} = \dfrac{d}{dt}\left(kt^2\right) = 2kt$ and, $a = \dfrac{dv}{dt} = \dfrac{d}{dt}\left(2kt\right) = 2k$ (constant)

Thus, the body has uniform accelerated motion.

Q. 11. *Establish the kinematic equation $s = ut + \frac{1}{2}at^2$ from velocity-time graph for a uniformly accelerated motion.* *(3 marks)*

Ans. Let AB be a velocity-time graph for uniformly accelerated motion with initial velocity u at time $t = 0$ and acceleration of the particle under motion being given by

$$a = \tan\theta = \frac{BD}{AD}$$

We know that area under the v-t graph gives the value of displacement during that time.

$\therefore$ Displacement of particle in time t will be

$$s = \text{area under } v\text{-}t \text{ graph} = \text{area } OABC$$

$$= \text{Area of rectangle OADC + area of triangle ADB}$$

$$= OA \times OC + \frac{1}{2}AD \times DB = u \times t + \frac{1}{2}(AD) \times \left(\frac{AD \times DB}{AD}\right)$$

$$= ut + \frac{1}{2}(AD)^2 \times \left(\frac{DB}{AD}\right)s = ut + \frac{1}{2}t^2 a$$

$\therefore$ $$s = ut + \frac{1}{2}at^2.$$

Q. 12. *Two bodies of different masses m_1 and m_2 are dropped from two different heights 'a' and 'b'. What is the ratio of time taken by the two bodies to drop through these distances?*

Ans. Let t_1 and t_2 are the time taken by two bodies of masses m_1 and m_2 to drop from heights 'a' and 'b' respectively.

Now,

Using equation of motion

$$h = ut + \frac{1}{2}at^2$$

$$u = 0 \text{ and } a = g$$

$$a = \frac{1}{2}g\,t_1^2 \quad \Rightarrow \quad t_1 = \sqrt{\frac{2a}{g}}$$

$$b = \frac{1}{2}g\,t_2^2 \quad \Rightarrow \quad t_2 = \sqrt{\frac{2b}{g}}$$

$$\therefore \quad \frac{t_1}{t_2} = \sqrt{\frac{2a/g}{2b/g}} \quad \Rightarrow \quad \frac{t_1}{t_2} = \sqrt{\frac{2a}{2b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$t_1 : t_2 = \sqrt{a} : \sqrt{b}.$$

Q. 13. *A ship is moving at a speed of 56 km h^{-1}. One second later, it is moving at a speed of 58 km h^{-1}. What is its acceleration?*

Ans. Here,

Initial speed, $u = 56 \text{ km h}^{-1} = 56 \times \frac{5}{18} \text{ ms}^{-1} = \frac{140}{9} \text{ ms}^{-1} = 15.55 \text{ ms}^{-1}$

Final speed, $v = 58 \text{ km h}^{-1} = 58 \times \frac{5}{18} \text{ ms}^{-1} = \frac{145}{9} \text{ ms}^{-1} = 16.11 \text{ ms}^{-1}$

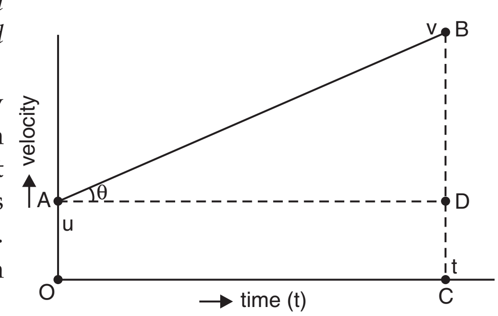
Time taken = 1 s

Using equation of motion

$$v = u + at \quad \Rightarrow \quad a = \frac{v-u}{t}$$

$$\Rightarrow \quad a = \left(\frac{16.11 - 15.55}{1}\right) = 0.56 \text{ ms}^{-2}, \quad a = 0.56 \text{ ms}^{-2}.$$

Q. 14. *Establish the kinematic equation $v^2 - u^2 = 2as$ from velocity-time graph for a uniformly accelerated motion.*



Ans. The velocity-time graph for uniformly accelerated motion has been shown in Fig. with initial velocity at $t = 0$ as u and final velocity at time t as v. Then area under the v-t graph gives the value of total displacement in the given time. Hence, displacement of moving particle in time t

= area of trapezium Δ ABC

$$s = \frac{1}{2}(OA + CB) \times OC, \quad = \frac{1}{2}(u + v) \times t$$

However, from definition of acceleration, we know that

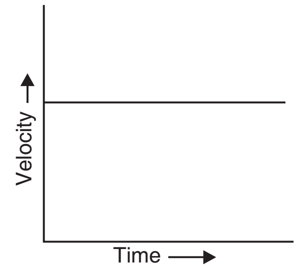
$$a = \frac{v-u}{t} \text{ or } t = \frac{v-u}{a}$$

Substituting this value of time t in equation (i), we get

$$\text{Displacement} \quad s = \frac{1}{2}(u+v) \times \frac{v-u}{a} \text{ or } \frac{(v^2-u^2)}{2a}$$

$$\Rightarrow \quad 2as = v^2 - u^2 \text{ or } v^2 = u^2 + 2as$$

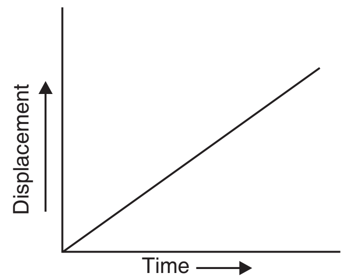
Q. 15. *Velocity time graph of a moving object is shown below. What is the acceleration of the object? Also draw displacement-time graph for the motion of the object.*



Ans. The given graph shows that the velocity of the object is constant. That is, the velocity of the object is not changing, so the acceleration of the object is zero. Since the acceleration of an object is given by

$$a = \frac{\text{Change in velocity}}{\text{Time taken}}$$

Displacement-time graph for the motion of the object is shown in the figure above.



Q. 16. *A body is thrown up with a velocity of 78.4 ms^{-1}. Find how high will it rise and how much time will it take to return to its point of projection.*

Ans. Initial velocity, $u = 78.4 \text{ ms}^{-1}$

Let the body reach to the maximum height 'h'.

The velocity at maximum height (final velocity), $v = 0$

From the equation

$$v^2 - u^2 = 2gh$$

$$v^2 = (78.4)^2 = 2(-9.8) \times \text{h}$$

$$h = \frac{78.4 \times 78.4}{2 \times 9.8} \text{ m} = 313.6 \text{ m}$$

Now,

Using equation, $v = u + gt$

$$0 = 78.4 + (-9.8)\, t$$

$$t = \frac{78.4}{9.8} \text{ s} = 8 \text{ s.}$$

Total time taken to return to the point of projection = Time taken in ascent + Time taken in descent

= 2 × 8 = 16 s. (Time taken in ascent = time taken in descent)

Q. 17. *Why does a person sitting in one train think that the other train is at rest, when both trains are moving on parallel tracks with the same speed and in the same direction?*

Ans. This is because the relative velocity of the train in which the person is sitting w.r.t. the other train is zero.

For example, let the velocity of A train

$$= 60 \text{ km/hr due east}$$

and velocity of B train $= 60$ km/hr due east

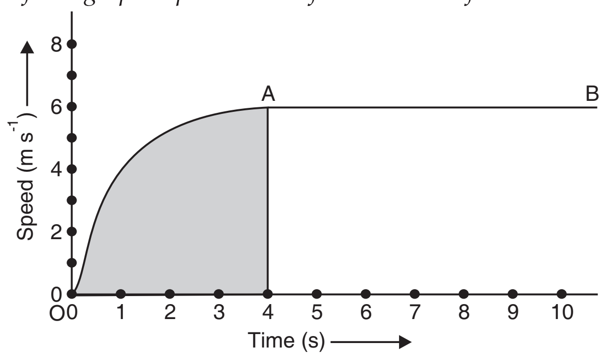
$\therefore$Relative velocity of A train w.r.t. B train

$$= 60 - 60 = 0$$

Q. 18. *The speed time graph for a car is shown in Fig.*

(a) Find how far does the car travel in the first 4 seconds. Shade the area on the graph that represents the distance travelled by the car during the period.

(b) Which part of the graph represents uniform motion of the car?



Ans. (*a*) Shaded part represents distance travelled in 4 seconds.

Area of shaded part $= \dfrac{6 \text{ ms}^{-1} \times 4 \text{ s}}{2} = 12\, m$

(*b*) Straight part *AB* represents a uniform motion of the car.

Q. 19. *A body moving with a uniform acceleration describes 12 m in 3^{rd} second of its motion and 20 m in the 5^{th} second. Find the velocity after 10 seconds.*

Ans. Let the initial velocity of the body 'u' and acceleration 'a'.

Using equation for n^{th} second motion

$$S_{n\text{th}} = u + \frac{1}{2} a (2n - 1)$$

$$S_{3\text{rd}} = u + \frac{1}{2} a (2 \times 3 - 1)$$

$$12 = u + \frac{5}{2} a \qquad ...(i)$$

$$S_{5\text{th}} = u + \frac{1}{2} a (2 \times 5 - 1)$$

$$20 = u + \frac{9}{2} a \qquad ...(ii)$$

By solving equations (*i*) and (*ii*), we get

$$a = 4 \text{ ms}^{-2} \text{ and } u = 2 \text{ ms}^{-1}$$

Now,

Using equation

$$v = u + at$$

$$v \text{ after 10 second} = 2 + 4 \times 10 \text{ ms}$$

$$v = 42 \text{ ms}^{-1}.$$

Q. 20. *A car travels 30 km, at a uniform speed of 40 km h⁻¹ and the next 30 km at a uniform speed of 20 km h⁻¹. Find its average speed.*

Ans. Here,

In the first case,

Distance travelled, $x_1 = 30$ km

Speed of car, $v_1 = 40 \text{ km h}^{-1}$

Time, t_1 = ? (to be calculated)

From relation, $\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$

We have, $\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$ *i.e.,* $t_1 = \dfrac{x_1}{v_1}$

Substituting various values, we get,

$$t_1 = \frac{30 \text{ km}}{40 \text{ km h}^{-1}} = \frac{3}{4}\text{ h or } 0.75 \text{ h}$$

In second case,

$$x_2 = 30 \text{ km}$$

$$v_2 = 20 \text{ km h}^{-1}$$

$$t_2 = ? \text{ (to be calculated)}$$

As, $t_2 = \dfrac{x_2}{v_2}$

∴ Substituting various values, we get,

$$t_2 = \frac{30 \text{ km}}{20 \text{ km h}^{-1}} = \frac{3}{2}\text{ h or } 1.5 \text{ h}$$

Now total distance travelled,

$$x = x_1 + x_2 = (30 + 30) \text{ km} = 60 \text{ km}$$

Total time taken $t = t_1 + t_2 = \left(\dfrac{3}{4} + \dfrac{3}{2}\right)\text{h}$

or $(0.75 + 1.5)\, h = \dfrac{9}{4}\text{ h or } 2.25\, h$

∴ $\text{Average speed} = \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

Substituting various values, we get,

$$v_{av} = \frac{60 \text{ km}}{(9/4)\text{ h}} = \frac{60 \times 4}{9}\text{ km h}^{-1} = \frac{80}{3}\text{ km h}^{-1}$$

or $\mathbf{v_{av} = 26.7 \text{ km h}^{-1}}$ **Ans.**

Q. 21. *What causes variation in velocity of a particle?*

Ans. The velocity of a particle changes due to either of the following three causes:

(*i*) Change in magnitude of velocity,

(*ii*) Change in direction of motion only, and

(*iii*) Change in magnitude as well as direction of the motion.

Q. 22. *A car travels first half of a length S with velocity v_1. The second half is covered with velocities v_2 and v_3 for equal time intervals. Find the average velocity of the motion.*

Ans. Average velocity, $v = \dfrac{\text{Total displacement}}{\text{Total time taken}}$

Time taken to cover first half of the length

$$= \frac{S}{2v_1}$$

Let time taken to cover second half of the length

$$= 2t$$

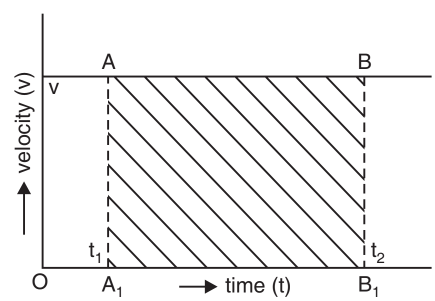
Thus, $$v = \frac{S}{\dfrac{S}{2v_1} + 2t}$$

Second half is divided equally into two parts with equal time.

$\therefore$ $$\frac{S}{2} = v_2t + v_3t = (v_2 + v_3)\,t \quad \text{or} \quad 2t = \frac{S}{(v_2 + v_3)}$$

Now, $$v = \frac{S}{\dfrac{S}{2v_1} + \dfrac{S}{(v_2 + v_3)}} \Rightarrow v = \frac{2v_1(v_2 + v_3)}{(v_2 + v_3 + 2v_1)}.$$

Q. 23. *How does the velocity-time graph for uniform motion give a geometrical way of calculating the displacement covered during a given time t?*



Ans. Consider velocity-time graph for uniform motion along a straight path. The graph is a straight line parallel to the time axis as shown in following Fig.

Let A and B be two points on velocity-time graph corresponding to the instants t_1 and t_2. As the motion is uniform, hence, $AA_1 = BB_1 = v$.

$\therefore$ Area under v-t graph between t_1 and t_2 = area ABB_1A_1

$$= AA_1 \times A_1B_1 = v\,(t_2 - t_1)$$

But velocity is defined as $v = \dfrac{\text{Displacement}}{\text{Time}} = \dfrac{x_2 - x_1}{t_2 - t_1}$

$\therefore$ $$v\,(t_2 - t_1) = x_2 - x_1$$

$\therefore$ $$\text{area } ABB_1A_1 = (x_2 - x_1)$$

Hence, displacement of a particle in time interval $(t_2 - t_1)$ is numerically equal to the area under velocity-time graph between the instants t_1 and t_2.

Q. 24. *If a body travels half its total path in its last second of its fall from rest, calculate the time and height of its fall.*

Ans. Let T = total time taken by the body to fall down

h = total height of the fall

Using $h = ut + \frac{1}{2}gt^2$, we have

$$h = 0 + \frac{1}{2}gt^2 = \frac{1}{2}gt^2 \quad ...(i)$$

Now, $t = (T - 1)$, $h = h/2$

$$\therefore \quad \frac{h}{2} = 0 + \frac{1}{2}g\,(T-1)^2$$

$$h = g\,(T - 1)^2 \quad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$\frac{1}{2}gT^2 = g\,(T - 1)^2$$

or $T^2 = 2\,(T - 1)^2$ or $T^2 - 4T + 2 = 0$

$\therefore$ $T = 2 \pm 1.414$ or $T = 3.414$ s or 0.586 s

Since T cannot be less than 1

$\therefore$ $T = 3.414$ s

From equation (*i*) $h = \frac{1}{2} \times 9.8 \times (3.414)^2 = 57.11$ m.

Q. 25. *A body covers 200 cm in the first 2 seconds and 220 cm in the next two seconds. What will be its velocity at the end of 7 seconds? Also find the displacement in 7 seconds.*

Ans. Using the equation of motion

$$S = ut + \frac{1}{2}at^2$$

For first 2 seconds,

$$200 = 2u + \frac{1}{2}a\,(2)^2\,s$$

$$200 = 2u + 2a \quad \text{or} \quad u + a = 100 \quad ...(i)$$

For first 4 seconds,

$$420 = 4u + \frac{1}{2}a \times (4)^2$$

$$420 = 4u + 8a \quad \text{or} \quad u + 2a = 105 \quad ...(ii)$$

By solving the equation (i) and (ii), we get

$$u = 95 \text{ cm s}^{-1} \text{ and } a = 5 \text{ cm s}^{-2}$$

Displacement in 7 seconds,

$$7u + \frac{1}{2}a\,(7)^2 = 7 \times 95 + \frac{1}{2} \times 0.5 \times (7)^2$$

$$= 665 \text{ cm} + 12.25 \text{ cm} = 677.5 \text{ cm}$$

Velocity at the end of 7 seconds,

$$v = u + at$$

$$v = 95 + 5 \times 7 = 130 \text{ cm s}^{-1}$$

Q. 26. *A bus starts with a constant acceleration 1 ms^{-2}. At the same time a car moving with a constant velocity of 5 ms^{-1} overtakes the bus. (i) How far from the starting point, the bus overtakes the car and (ii) How fast the bus was moving at the time of overtake?*

Ans. Initial velocity of bus, $u = 0$. Let the bus overtakes the car after time t.

∴ Distance travelled by bus in time t,

$$S_b = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}t^2 = \frac{t^2}{2} \qquad (\because a = 1 \text{ ms}^{-2})$$

Distance travelled by car moving with constant velocity ($a = 0$),

$$S_c = ut + \frac{1}{2}at^2 = 5t \qquad (\because u = 5 \text{ ms}^{-1} \text{ and } a = 0)$$

Since $S_b = S_c$

∴ $\dfrac{t^2}{2} = 5t$ or $t = 10$ s

∴Distance travelled by bus when it overtakes car,

$$S_b = ut + \frac{1}{2}at^2 = 0 \times 10 + \frac{1}{2} \times 1 (10)^2 s = 50 \text{ m.}$$

Speed of bus, $v = u + at = 0 + 1 \times 10 = 10 \text{ ms}^{-1}$.

Q. 27. *Two particles begin to fall freely from rest from the top of a tower within a gap of 1 s. How long after the first particle begins to fall, the two particles be 15 m apart? (Given g = 10 ms^{-2})*

Ans. Let the first particle takes time t to reach a position co-ordinate 15 m below the second particle. Obviously, the second body has fallen under gravity for a time $(t-1)$ s only. Hence

$$15 = y_1 - y_2 = \frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2 = gt - \frac{g}{2}$$

or $$15 = 10t - 5 \Rightarrow t = \frac{15+5}{10} = 2 \text{ s.}$$

Q. 28. *A ball A is just dropped from a height h. Simultaneously, another ball B is thrown vertically upwards from ground with a speed $\sqrt{gh}$. After what time will they meet and at what height?*

Ans. Let the balls A and B meet at a height h' from the ground at time t. Then, displacement of ball A in time $t = (h - h')$.

∴ For ball A $$(h - h') = \frac{1}{2}gt^2 \qquad ...(i)$$

For ball B $$h' = +\sqrt{gh \cdot t} - \frac{1}{2}gt^2 \qquad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$h = \sqrt{gh \cdot t} \Rightarrow t = \sqrt{\frac{h}{g}}$$

Substituting the value of t in (*ii*), we have

$$h' = \sqrt{gh} \cdot \sqrt{\frac{h}{g}} - \frac{1}{2}g \cdot \left(\frac{h}{g}\right) = h - \frac{h}{2} = \frac{h}{2}.$$

Q. 29. *A motor boat covers the distance between two spots on the river in time of 8 hours and 12 hours downstream and upstream respectively. What is the time required for the boat to cover this distance in still water?*

Ans. Time taken in downstream,

$$8 = \frac{S}{v_r + v_b}$$

Time taken in upstream,

$$12 = \frac{S}{v_b - v_r}$$

Given, $$v_r + v_b = \frac{S}{8}, v_b - v_r = \frac{S}{12}$$

By solving the equations, we get

$$v_b = \frac{S}{2}\left(\frac{1}{8} + \frac{1}{12}\right) \quad \text{or,} \quad v_b = \frac{S}{2} \times \frac{20}{96} \quad \text{or,} \quad v_b = \frac{10S}{96}$$

Now, $$v_r = \frac{10\,S}{96} - \frac{S}{12} = \frac{2\,S}{96}$$

In still water, only the velocity is to be considered.

$\therefore$ time taken in still water for covering length S is,

$$t = \frac{S}{v_b} = \frac{S \times 96}{10\,S} = 9.6 \text{ seconds}.$$

Q. 30. *Differentiate between average and instantaneous velocity.*

Ans. **Average velocity:** Average velocity is the displacement divided by the time interval in which the displacement occurs.

$$\vec{v}_{av} = \frac{\Delta \vec{x}}{\Delta t}$$

Instantaneous velocity: Instantaneous velocity is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{dx}{dt}.$$

Q. 31. *Give three important characteristics of displacement.*

Ans. Three important characteristics of displacement are:

(*i*) Displacement is a vector quantity having both magnitude as well as direction.

(ii) Displacement of a particle between two given positions is unique and is the shortest path through which particle may go from its initial to final position.

(*iii*) Displacement is independent of the choice of origin to the co-ordinate system.

Q. 32. *A body, starts from rest and accelerates uniformly, find the ratio of the displacement in, (a) One, two and three seconds, (b) First, second and third second.*

Ans. (*a*) Length covered in t seconds = $\frac{1}{2}at^2$.

Since initial velocity is zero.

$\therefore$ Ratio of length covered in one, two and three seconds is 1: 4: 9.

(*b*) Length covered in t^{th} second = $\frac{a}{2}(2t-1)$

$\because$ $u = 0$

$\therefore$ Ratio of length covered in first, second and third second is

1: 3: 5.

Q. 33. *A stone is dropped from the top of a tall cliff and 'n' second later another stone is thrown vertically downwards with a velocity of 'u' m/s. How far below the top of the cliff will the second stone overtake the first?* *(3 marks)*

Ans. The second stone will 'catch up' with the first stone when the distance covered by it in $(t - n)$ second will equal the distance covered by the first stone in t second.

Now distance covered by the first stone in t second = $\frac{1}{2}gt^2$ and distance covered by the second stone in $(t\text{-}n)$ second.

$$= u(t-n) + \frac{1}{2}g(t-n)^2$$

$$\therefore \quad \frac{1}{2}gt^2 = u(t-n) + \frac{1}{2}g(t-n)^2 s$$

$$\text{or} \quad \frac{1}{2}g\left[t^2-(t-n)^2\right]s = u(t-n) \quad \text{or} \quad \frac{1}{2}g\left[(2t-n)n\right] = u(t-n)$$

$$\text{or} \quad gnt - \frac{1}{2}gn^2 = ut - un \quad \text{or} \quad t(gn-u) = \left(\frac{1}{2}gn-u\right)n$$

$$\text{or} \quad t = \frac{n\left(\frac{1}{2}gn-u\right)}{(gn-u)}$$

The distance covered by the first stone in this time is

$$h = \frac{1}{2}gt^2 = \frac{1}{2}g\left[\frac{n\left(\frac{1}{2}gn-u\right)}{(gn-u)}\right]^2$$

Thus the second stone will overtake the first at distance

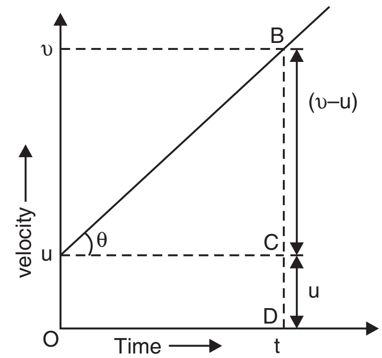
$$\frac{1}{2}g\left[\frac{n\left(\frac{gn}{2}-u\right)}{(gn-u)}\right]^2 \text{ below the top of the cliff.}$$

III. LONG ANSWER TYPE QUESTIONS

Q. 1. *State the kinematic equations for uniformly accelerated motion.*

Ans. For uniformly accelerated motion, we can derive some simple equations that relate displacement(x), time taken(t), Initial velocity(u), final velocity(v) and acceleration(a).

(*i*) **Velocity attained after time *t*:** The velocity-time graph for positive constant acceleration of a particle is shown in the figure.



Let u be the initial velocity of the particle at $t = 0$ and v is the final velocity of the particle after time t. Consider two points A and B on the curve corresponding to $t = 0$ and $t = t$ respectively.

Draw BD perpendicular to time axis. Also draw AC perpendicular to BD.

$\therefore \quad OA = CD = u;$

$BC = (v - u)$ and $OD = t$

Now slope of v-t graph = acceleration (a)

$$\therefore \quad a = \text{slope of } v - t \text{ graph} = \tan\theta = \frac{BC}{AC} = \frac{BC}{OD} \quad [\because AC = OD]$$

$$\therefore \quad a = \frac{v-u}{t} \quad \text{or} \quad v - u = at$$

$$\boxed{v = u + at}$$

(*ii*) **Distance travelled in time *t*:**

Let x_0 = position of the particle at $t = 0$ from the origin.

x = position of the particle at $t = t$ from the origin.

$\therefore \quad (x - x_0) = S$ = distance travelled by the particle in the time interval $(t - 0) = t$

We know, distance travelled by a particle in the given time

interval = area under velocity-time graph

$\therefore \quad (x - x_0)$ = Area $OABD$ (see fig. above)

= Area of trapezium $OABD$

$$= \frac{1}{2}[\text{Sum of parallel sides x perpendicular distance between parallel sic}$$

$$= \frac{1}{2}(OA + BD) \times AC = \frac{1}{2}(u + v) \times t$$

Since $v = u + at$

$$\therefore \quad (x - x_0) = \frac{1}{2}(u + u + at) \times t = \frac{1}{2}(2u + at) \times t = ut + \frac{1}{2}at^2$$

Since $x - x_0 = S$

$$\therefore \quad \boxed{S = ut + \frac{1}{2}at^2}$$

(*iii*) **Velocity attained after travelling a distance *S*:**

We know, distance travelled by a particle in time t is equal to the area under velocity-time graph. Therefore, the distance (s) travelled by a particle during time interval t is given by

S = Area under v-t graph (see fig.) or

S = area of trapenium $OABD$

$$= \frac{1}{2}(\text{sum of parallel sides}) \times \text{perpendicular distance between these para}$$

or $$S = \frac{1}{2}(OA + BD) \times ACs \quad ...(i)$$

Now, acceleration, a = slope of $v - t$ graph

or $$a = \frac{BC}{AC} = \frac{BD - CD}{AC} = \frac{v-u}{AC} \quad \text{or} \quad AC = \left(\frac{v-u}{a}\right)s \quad ...(ii)$$

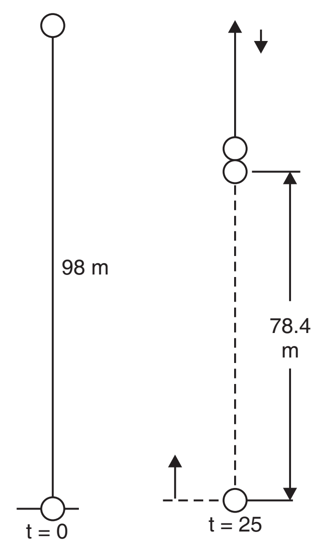
Also, $OA = u$ and $BD = v$ ...(*iii*)

Using equations (*ii*) and (*iii*) in equation (*i*), we get

$$S = \frac{1}{2}(v+u)\frac{(v-u)}{a} = \frac{v^2 - u^2}{2a}s \quad \text{or,} \quad \boxed{v^2 - u^2 = 2aS}$$

Q. 2. *A ball of mass 100 g is projected vertically upwards from the ground with a velocity of 49 m/ s. At the same time another identical ball is dropped from a height of 98 m to fall freely along the same path as followed by the first ball. After sometime, the two balls collide and stick together and finally fall together. Find the time of flight of the masses.*

Ans. We first find *when* and *where* the two balls collide. Let them collide at an instant t seconds after they start their respective motion. Clearly the two balls are at the same height above the ground at this instant.



The height of the first ball after t seconds = $49\,t - 1/2 \times 9.8t^2$ = $4.9\,t\,(10 - t)$

Also the height of the second ball after t seconds = 98 – downward distance moved by it in t seconds.

$$= 98 - \frac{1}{2} \times 9.8\,t^2 = 4.9\left(20 - t^2\right)s$$

$\therefore$ $$4.9\,t\,(100 - t) = 4.9\,(20 - t^2)$$

or $$10t - t^2 = 20 - t^2 \text{ or } t = 2 \text{ s}$$

The balls thus collide two seconds after the start of their motion. Their velocities at this instant are

First ball: $v_1 = (49 - 9.8 \times 2)$ m/s

$= 29.4$ m/s directed upwards

Second ball: $v_2 = (0 + 9.8 \times 2)$ m/s = 19.6 m/s directed downwards

If v is the velocity of the combined mass of the two balls after they stick together following their collision, we have, by principle of conservation of momentum.

$$200 \times v = 100 \text{ x } 29.4 - 100 \text{ x } 19.6$$

$\therefore$ $$v = 4.9 \text{ m/s}$$

The 'combined mass' thus moves upward, after collision with a velocity of 4.9 m/s. Its height above the ground at this instant is (considering the position of either of the two balls before collision)

$$\left(98-\frac{1}{2}\times 9.8\times 2^2\right)\text{m} = (98 - 19.6)\ \text{m} = 78.4\ \text{m}$$

We can now find the time t' taken by the 'combined mass' of the two balls to fall to ground. We have for this 'combined mass',

$$u = 4.9\ \text{m/s},\ s = -78.4\ \text{m},\ a = -g = -9.8\ \text{ms}^{-2}$$

$$\therefore \quad -78.4 = 4.9\ t' + 1/2\ (-9.8)\ t'^2 \quad \text{or} \quad t'^2 - t' - 16 = 0$$

$$\therefore \quad t' = \frac{1\pm\sqrt{1+64}}{2} = \frac{1\pm 8.06}{2}$$

$$= 4.532\ \text{s (leaving out the negative solution)}$$

The 'combined mass' thus takes 4.53 s to fall to the ground. Since the balls collided 2 s after they started their motion, their total time of flight is (2 + 4.53) s = 6.53 s.

Q. 3. *Derive the three basic kinematic equations by calculus method.*

Ans. (*i*) **Velocity attained by a particle after time *t*:**

Let dv be the change in velocity of the particle in time dt. Therefore, the acceleration of the particle is given by

$$a = \frac{dv}{dt} \quad \text{or} \quad dv = a\ dt$$

By integrating both sides, we get

$$\int dv = \int a\ dt \quad \text{or} \quad \int dv = a\int dt \quad \text{or} \quad v = at + k \qquad ...(i)$$

where k is constant of integration.

when $t = 0,\ v = u$

Putting these values in equation (*i*), we get

$$k = u$$

Now putting the value of k in equation (*i*), we get

$$\boxed{v = u + at}$$

(*ii*) **Displacement of the particle after time *t*:**

Let dx be the displacement of the particle in time dt. Therefore, the velocity of the particle is given by

$$v = \frac{dx}{dt} \quad \text{or} \quad dx = vdt$$

Since $v = u + at$

$$\therefore \quad dx = (u + at)\ dt$$

Integrating both sides, we get

$$\int dx = \int (u+at)\ dt \quad \text{or} \quad \int dx = \int u\ dt + \int at\ dt$$

$$x = u\int dt + a\int t\ dt \qquad [\because u \text{ and } a \text{ are constants}]$$

or $$x = ut + a\frac{t^2}{2} + k \qquad ...(ii)$$

where k is constant of proportionality

where $t = 0, x = x_0$

$\therefore$ from equation (*ii*), we get

$$x = x_0 + ut + \frac{1}{2}at^2$$

or $$x - x_0 = ut + \frac{1}{2}at^2$$

since $x - x_0 = S$, displacement of the particle in the time interval t.

$$\therefore \quad \boxed{S = ut + \frac{1}{2}at^2}$$

(*iii*) **Velocity attained by a particle after travelling a distance S:**

We know, $$v = \frac{dx}{dt}$$

Multiplying and dividing R.H.S. by dv, we get

$$v = \frac{dx}{dt}\cdot\frac{dv}{dv} = \frac{dx}{dv}\cdot\frac{dv}{dt}$$

As $$\frac{dv}{dt} = a \text{ (acceleration)}$$

$$\therefore \quad v = a\frac{dx}{dv} \quad \text{or} \quad v\,dv = a\,dx$$

Integrating both sides, we get

$$\int v\,dv = \int a\,dx = a\int dx \quad \text{or} \quad \frac{v^2}{2} = ax + k \qquad ...(i)$$

when $x = 0, \; v = u$

Then, from eqn. (*i*), $k = \frac{u^2}{2}$

Putting the value of k in eqn. (*i*), we get

$$\frac{v^2}{2} = ax + \frac{u^2}{2} \quad \text{or} \quad \frac{v^2}{2} - \frac{u^2}{2} = ax \quad \text{or} \quad v^2 - u^2 = 2ax$$

If $x = S$, then

$$\boxed{v^2 - u^2 = 2aS.}$$

Q. 4. *An object starts from rest and covers a total distance X in the following manner:*
It first has a uniform acceleration a_1 for some time t_1, moves with the speed acquired at the end of t_1 for some distance and is then given a uniform retardation a_2 so that it is again at rest at the end of the journey. Show that the journey is covered in least time if the body is accelerated for a time of $\left[\frac{2\,X\,a_2}{a_1\,(a_1 + a_2)}\right]^{\frac{1}{2}}$ *and this minimum time is* $\left[2\,X\left(\frac{1}{a_1} + \frac{1}{a_2}\right)\right]^{\frac{1}{2}}$.

Ans. Let x_1 be the distance travelled by the object in t_1 second (starting from rest with uniform acceleration a_1). Then v, the speed acquired after travelling distance x_1, is

$$v = a_1 t_1 \quad \text{...(1)}$$

Also $$2 a_1 x_1 = v^2 - 0^2 = v^2$$

$\therefore$ $$x_1 = \frac{v^2}{2 a_1} \quad \text{...(2)}$$

Let x_2 and x_3 denote the distance travelled in the second and third leg of the journey of the particle extending over time t_2 and t_3 respectively,

Then, $$x_2 = v t_2 \quad \text{...(3)}$$

and $$-2 a_2 x_3 = 0^2 - v^2 = -v^2 \quad \text{...(4)}$$

Also $$0 = v - a_2 t_3 \quad \text{or} \quad v = a_2 t_3 \quad \text{...(5)}$$

The total time t of the journey is

$$t = t_1 + t_2 + t_3 = \frac{v}{a_1} + \frac{x_2}{v} + \frac{v}{a_2} \quad \text{...(6)}$$

Also $$X = x_1 + x_2 + x_3 = \frac{v^2}{2 a_1} + x_2 + \frac{v^2}{2 a_2}$$

$\therefore$ $$x_2 = X - \frac{v^2}{2}\left(\frac{1}{a_1} + \frac{1}{a_2}\right)s \quad \text{...(7)}$$

From Eqns. (6) and (7) we have

$$t = \frac{v}{a_1} + \frac{X}{v} - \frac{v}{2}\left(\frac{1}{a_1} + \frac{1}{a_2}\right) + \frac{v}{a_2} = \frac{X}{v} + \frac{v}{2}\left(\frac{1}{a_1} + \frac{1}{a_2}\right) \quad \text{...(8)}$$

Using Eqns. (8) and (1), we have

$$t = \frac{X}{a_1 t_1} + \frac{a_1 t_1}{2}\left(\frac{1}{a_1} + \frac{1}{a_2}\right) \quad \text{...(9)}$$

For a particular value of X, t is least if,

$$\frac{dt}{dt_1} = 0 \quad \text{...(10)}$$

Differentiating Eqn. (9), we get, for least t

$$\frac{dt}{dt_1} = -\frac{X}{a_1 t_1^2} + \frac{a_1}{2}\left(\frac{1}{a_1} + \frac{1}{a_2}\right) = 0$$

or $$\frac{X}{a_1 t_1^2} = \frac{a_1 (a_1 + a_2)}{2 a_1 a_2} \quad \text{or} \quad t_1 = \left[\frac{X \cdot 2 a_2}{a_1 (a_1 + a_2)}\right]^{1/2} s$$

Corresponding to this values of t_1, we get from Eqn. (9),

$$t = \left[2 X\left(\frac{1}{a_1} + \frac{1}{a_2}\right)\right]^{1/2}$$

IV. MULTIPLE CHOICE QUESTIONS

1. The displacement x of a particle varies with time according to the relation $x = \frac{a}{b}\left(1 - e^{-bt}\right)$. Then

(*a*) At $t = \frac{1}{b}$, the displacement of the particle is nearly (2/3) (a/b).

(*b*) The particle cannot reach a point at a distance x from its starting position if $x > a/b$.

(*c*) The velocity and acceleration of the particle at t = 0 are a and $-ab$ respectively.

(*d*) The particle will come back to its starting point as $t \to \infty$.

2. The displacement of an object at any instant is given by $x = 30 + 20\,t^2$, where x is in metres and t in seconds.

The acceleration of the object will be

(*a*) 40 ms^{-2} (*b*) 50 ms^{-2} (*c*) 30 ms^{-2} (*d*) zero

3. A particle of mass 'm' moving with a velocity v strikes a stationary particle of mass $2m$ and sticks to it. The speed of the system will be:

(*a*) $\frac{v}{2}$ (*b*) $2v$ (*c*) $\frac{v}{3}$ (*d*) $3v$

4. Distance-time graph of a body at rest is

(*a*) parallel to time-axis (*b*) parallel to distance-axis

(*c*) inclined to time-axis (*d*) perpendicular to both axes.

5. The area under the velocity time graph between any two instants $t = t_1$ and $t = t_2$ gives the distance covered in a time $\delta t = t_2 - t_1$.

(*a*) only if the particle moves with a uniform acceleration.

(*b*) only if the particle moves with a uniform velocity.

(*c*) only if the particle moves with an acceleration increasing at a uniform rate.

(*d*) in all cases irrespective of whether the motion is one of uniform velocity, or of uniform acceleration or of variable acceleration.

6. Which of the following is not a vector quantity?

(*a*) acceleration (*b*) velocity (*c*) speed (*d*) displacement

7. When the distance travelled by a body is directly proportional to the time, the body is said to have a

(*a*) zero speed (*b*) uniform acceleration

(*c*) zero velocity (*d*) uniform speed

8. Area under velocity time graph represents

(*a*) acceleration (*b*) displacement (*c*) retardation (*d*) average speed

9. In case of a moving body

(*a*) displacement > distance (*b*) displacement < distance

(*c*) displacement ≥ distance (*d*) displacement ≤ distance

10. Identify one dimensional motion out of the following:

(*a*) A honey bee dancing in air

(*b*) A teacher writing on a blackboard

(*c*) A scooterist speeding on a level road

(*d*) A kite flying in sky.

Ans. **1.**—(*a*), (*b*) and (*c*) **2.**—(*a*) **3.**—(*c*) **4.**—(*a*)
5.—(*d*) **6.**—(*c*) **7.**—(*d*) **8.**—(*b*) **9.**—(*d*)
10.—(*c*)

V. QUESTIONS ON HIGH ORDER THINKING SKILLS (HOTS)

Q. 1. *If x, y and z are distances moved by a particle moving with a constant acceleration during l^{th}, m^{th} and n^{th} second of its motion respectively. Show that,*

$$x\,(m - n) + y\,(n - l) + z\,(l - m) = 0$$

Ans. Distance covered in l^{th} second,

$$x = u + \frac{a}{2}(2l - 1) \quad ...(i)$$

Distance covered in m^{th} second,

$$y = u + \frac{a}{2}(2m - 1) \quad ...(ii)$$

Subtracting equation (*ii*) from (*i*)

$$x - y = \frac{a}{2} \times 2\,(l - m) = a\,(l - m)$$

or, $$(x - y) \times z = a\,(l - m) \times z \quad ...(iii)$$

Distance covered in n^{th} second,

$$z = u + \frac{a}{2}(2n - 1) \quad ...(iv)$$

Subtracting equation (*iv*) from (*ii*), we get

$$y - z = \frac{a}{2} \times 2\,(m - n) = a\,(m - n)$$

or $$(y - z) \times x = a\,(m - n) \times x \quad ...(v)$$

and $$z - x = a\,(n - l) \text{ or } (z - x)\,y = a\,(n - l)\,y \quad ...(vi)$$

By adding equations (*iii*), (*v*) and (*vi*), we get

$z\,(x - y) + x\,(y - z) + y\,(z - x) = a\,[(l - m)\,z + (m - n)\,x + (n - l)\,y]$

$\Rightarrow \quad 0 = a\,[(m - n)\,x + (n - l)\,y + (l - m)\,z]$

or, $x\,(m - n) + y\,(n - l) + z\,(l - m) = 0$.

Q. 2. *A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β to come to rest. If t is the total time elapsed, then calculate.*

(a) the maximum velocity attained by the car, and (b) the total distance travelled by the car.

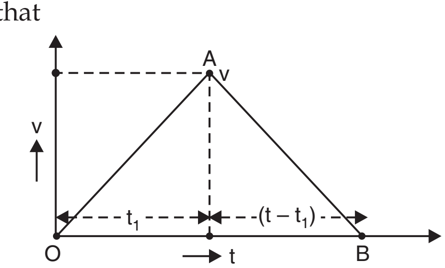
Ans. (*a*) Let the car accelerate at a rate α for time t_1 and attain a maximum velocity v. Then the car decelerates at a constant rate β for remaining time $(t - t_1)$ and again comes to rest. Then from adjoining figure, it is clear that

$$v = \alpha t_1 = \beta\,(t - t_1)$$

$$\therefore \quad t_1 = \frac{v}{\alpha} \quad \text{and} \quad (t - t_1) = \frac{v}{\beta}$$

Adding these two, we have

$$t = \frac{v}{\alpha} + \frac{v}{\beta} = v\left(\frac{\alpha + \beta}{\alpha\beta}\right)$$



$$\Rightarrow \qquad v = \frac{\alpha\beta t}{(\alpha+\beta)}$$

(*b*) Total distance travelled by the car s = area OAB = $\frac{1}{2}(t)\times(v)$

$$\therefore \qquad s = \frac{1}{2}t\times\frac{\alpha\beta t}{(\alpha+\beta)} = \frac{1}{2}\frac{\alpha\beta}{(\alpha+\beta)}t^2.$$

Q. 3. *An object is moving along x-axis in such a way that its displacement is given by*

$$x = 30 + 20\ t^2$$

where x is in metres and t is in seconds.

(a) Find the velocity and acceleration.

(b) What are the initial position and the velocity of the object?

Ans. Given $\qquad x = 30 + 20\ t^2 \qquad$ ...(*i*)

(*a*) Differentiating eqn. (*i*) w.r.t. '*t*', we get

$$\frac{dx}{dt} = v = \frac{d}{dt}\left(30+20\,t^2\right) = \frac{d}{dx}(30)+\frac{d}{dt}\left(20\,t^2\right) = 0 + 20 \times 2t$$

$$v = 40\,t \text{ ms}^{-1} \qquad \text{...}(ii)$$

Now, differentiating eqn. (*ii*) w.r.t. '*t*', we get

$$\frac{dv}{dt} = a = \frac{d}{dt}(40\,t)s = 40 \times 1 = 40 \text{ ms}^{-2}$$

(*b*) Put $t = 0$ in eqn. (*i*), we get the initial position

∴ Initial position, $x_0 = 30$ m

Putting $t = 2$ in eqn. (*ii*), we get the initial velocity

∴ Initial velocity, $u = 40 \times 0 = 0.$

Q. 4. *A particle moves in a straight line such that its displacement at any time is given by $s^2 = t^2 + 1$. Find (a) velocity, (b) acceleration as a function of s.*

Ans. (*a*) $\qquad s^2 = t^2 + 1$

Differentiating with respect to time, we get

$$2s\frac{ds}{dt} = 2t \quad \text{or} \quad sv = t \quad \text{or} \quad v = \frac{t}{s}$$

(*b*) Differentiating again, with respect to time, we get

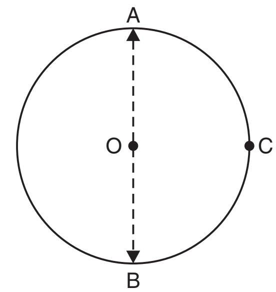
$$s \cdot a + v \cdot v = 1$$

$$\text{or} \qquad a = \frac{1-v^2}{s} = \frac{1}{s}-\frac{v^2}{s} = \frac{1}{s}-\frac{t^2}{s^3} = \frac{s^2-t^2}{s^3} = \frac{1}{s^3}$$

$$\text{or,} \qquad a = \frac{1}{s^3}.$$

Q. 5. *The minute hand of a wall clock is 10 cm long. Find its displacement and the distance covered from 12.00 noon to 12.30 p.m.*

Ans. Length of minute hand = radius of circle described by minute hand r = 10 cm = 0.1 m.



From 12.00 noon to 12.30 p.m., the tip of minute hand covers a net displacement equal to the diameter of circle. Hence,

Displacement $\overrightarrow{AOB} = 2 \times r = 2 \times 0.1 \text{ m} = 0.2 \text{ m}$

During the same time total distance covered by tip of minute hand = semicircular path $ACB = \pi r = 3.14 \times 0.1 = 0.31$ m.

Q. 6. *A particle located at $x = 0$ at $t = 0$ starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$. How does the displacement of the particle vary with time ?*

Ans. Given $\quad v = \alpha\sqrt{x}$

Since $\quad v = \dfrac{dx}{dt}$

$$\therefore \quad \frac{dx}{dt} = \alpha\sqrt{x} \quad \text{or} \quad \frac{dx}{\sqrt{x}} = \alpha\, dt \quad \text{or} \quad x^{-1/2}\, dx = \alpha\, dt$$

By integrating both sides, we get

$$\int x^{-1/2}\, dx = \int \alpha\, dt \quad \text{or} \quad \frac{x^{1/2}}{1/2} = \alpha\, t \quad \text{or} \quad x^{1/2} = \frac{1}{2}\alpha\, t$$

Squaring both sides, we get

$$x = \frac{\alpha^2\, t^2}{4}$$

$$\therefore \quad x \propto t^2.$$

Q. 7. *A particle located at $x = 0$ at time $t = 0$ starts moving along the positive x direction with a velocity v that varies as $v = \alpha\sqrt{x}$. How do the displacement, velocity and acceleration of the particle vary with time? What is the average velocity of the particle over the first s metres of its path?*

Ans. We know that the instantaneous velocity of the particle is given by

$$v = \frac{dx}{dt}$$

Since $\quad v = \alpha\sqrt{x}$

we have $\quad \dfrac{dx}{dt} = \alpha\sqrt{x} \quad \text{or} \quad \dfrac{dx}{\sqrt{x}} = \alpha\, dt$

Integrating from $t = 0$ $(x = 0)$ to $t = t$ $(x = x)$

we have $\quad \displaystyle\int_0^x x^{-1/2}\, dxs = \alpha \int_0^t dt$

$$\therefore \quad \left| \frac{x^{1/2}}{1/2} \right|_0^x = \alpha\, t \quad \text{or} \quad x = \frac{\alpha^2\, t^2}{4}$$

The time dependence of the velocity is obtained by differentiating both sides of this relation w.r.t. time t. Thus

$$v = \frac{dx}{dt} = \frac{\alpha \cdot 2\,t}{4} = \frac{\alpha^2}{2}\,t$$

The velocity x of the particle is thus increasing in direct proportion to time.

Similarly the time dependence of acceleration is obtained by differentiating both sides of this relation w.r.t. 't'. Thus

$$a = \frac{dv}{dt} = \frac{\alpha^2}{2}$$

The particle is thus moving with a constant acceleration.

To find the average velocity over the first s metre, we assume that the time taken to cover this distance is T. Using

$$x = \frac{\alpha^2\,t^2}{4}$$

we get $$s = \frac{\alpha^2\,T^2}{4} \quad \text{or} \quad T = \frac{2\sqrt{s}}{\alpha}$$

The average velocity v_{av} ($= s/T$) is, therefore

$$v_{av} = \left(\frac{\alpha}{2}\sqrt{s}\right).$$

Q. 8. *The acceleration experienced by a boat, after its engine is cut off, given by,* $\frac{dv}{dt} = -kv^3$, *where k is a constant. If* v_0 *is the magnitude of velocity at cut off (t = 0), find the magnitude of the velocity at a time t after the cut off.*

Ans. $$\frac{dv}{dt} = -kv^3$$

Integrating both sides, we get

$$\int \frac{dv}{v^3} = -k\int dt \quad \text{or} \quad -\frac{1}{2v^2} = -kt + c$$

At $$t = 0, \quad v = v_0$$

$$\therefore \quad c = -\frac{1}{2v_0^2}$$

$$\therefore \quad -\frac{1}{2v^2} = -kt - \frac{1}{2v_0^2} \quad \text{or} \quad 2v^2 = \frac{2v_0^2}{\left(2v_0^2\,kt + 1\right)}s \quad \text{or} \quad v = \sqrt{\frac{v_0^2}{\left(2v_0^2\,kt + 1\right)}}$$

Q. 9. *A 100 m sprinter uniformly increases his speed from rest at the rate of 1 ms*$^{-2}$ *up to* $\frac{3}{4}$*th of the total run and then covers the balance* $\frac{1}{4}$ *th run with uniform speed. How much time does he take to complete the race?*

Ans. Here total distance covered $s = 100$ m, $u = 0$

(*a*) For first $\frac{3}{4}$th of the run *i.e.,* $s_1 = \frac{3}{4}\text{s} = \frac{3}{4}\times 100\text{ m} = 75$ m, $a = +1$ m s^{-2}. If time for this part of run be t_1, then using the equation $s = ut + \frac{1}{2}at^2$, we have

$$75 = 0 + \frac{1}{2} \times 1 \times t_1^2 \quad \text{or} \quad t_1^2 = 75 \times 2 = 150$$

$$\Rightarrow \quad t_1 = \sqrt{150} = 12.25 \text{ s}$$

and final velocity $v = u + at_1 = 0 + 1 \times 12.25 = 12.25 \text{ ms}^{-1}$.

(*b*) For remaining run *i.e.,* $s_2 = s - s_1 = 100 - 75 = 25$ m, uniform velocity $v = 12.25 \text{ ms}^{-1}$

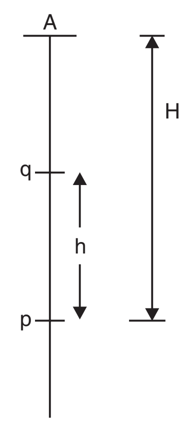
$\therefore$ Time for this run $t_2 = \frac{s_2}{v} = \frac{25}{12.25} = 2.04$ s

$\therefore$ Total time taken by the sprinter to complete the race

$$t = t_1 + t_2 = 12.25 \text{ s} + 2.04 \text{ s} = 14.29 \text{ s}.$$

Q. 10. *An object is thrown vertically upward with some speed. It crosses two points p and q which are separated by h metre. If t_p is the time between p and the highest point and coming back and t_q is the time between q and the highest point and coming back, relate acceleration due to gravity, t_p, t_q and h.*

Ans. Let H = distance between point p and the highest point A. This distance H travelled while falling from A top is given by



$$\therefore \quad H = \frac{1}{2} g \left(\frac{t_p}{2}\right)^2 = \frac{g\, t_p^2}{8} \quad \text{...(}i\text{)}$$

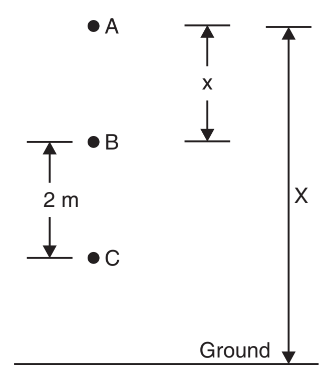
Also, $$H - h = \frac{1}{2} g \left(\frac{t_q}{2}\right)^2 = \frac{g\, t_q^2}{8} \quad \text{...(ii)}$$

Now, From eqns. (*i*) and (*ii*), we get

$$h = \frac{g\, t_p^2}{8} - \frac{g\, t_q^2}{8} = \frac{g}{8}\left(t_p^2 - t_q^2\right).$$

Q. 11. *A perfectly elastic rubber ball is dropped from the top of a building. A man standing in front of a window 2 m high notes that the ball takes a time of 0.2s in crossing the window. The ball strikes the ground suffering a perfectly elastic collision and reappears at the bottom of the window during its upward journey again after 2 seconds. What is (1) the height of the building and (2) the height of the bottom of the window above the ground? Take $g = 10 \text{ ms}^{-2}$.*

Ans. Let A be the top of the building at a height X above the ground. BC is the window of height 2m. Let t_1 be the time taken by the ball to fall from A to B. Then the time taken to fall through a distance $(x + 2)$ m is $(t_1 + 0.2)$ s. Then



$$x = \frac{1}{2} \times 10 \times t_1^2 \quad \text{...(1)}$$

and $$x + 2 = \frac{1}{2} \times 10 \times (t_1 + 0.2)^2 \quad \text{...(2)}$$

From Eqns. (1) and (2), we have

$$5\, t_1^2 + 2 = 5\, t_1^2 + 0.2 + 2\, t_1$$

or $$1.8 = 2\, t_1$$

$\therefore \qquad t_1 = 0.9 \text{ s}$

$\therefore \qquad x = \frac{1}{2} \times 10 \times (0.9)^2 = 4.05 \text{ m}$

Let v_1 be the speed acquired by the ball as it reaches the bottom C of the window. Then

$$v_1 = 0 + 10 \times 1.1 = 11 \text{ ms}^{-1}$$

During the downward journey the ball experiences an acceleration equal to g and strikes the ground with speed V. Since it undergoes a perfectly elastic collision its speed reverses. The time taken in falling from C to ground to back is 2 s. Therefore, time taken to fall from C to ground must be 1 s. Hence

$$X - (x + 2) = 11 \times 1 + \frac{1}{2} \times 10 \ (1)^2 \quad \text{or} \quad X - 6.05 = 11 + 5 = 16$$

$\therefore \qquad X = 22.05 \text{ m}$

The height of the building is 22.05 m and the bottom of the window is at height of 16 m [= (22.05 – 4.05) m] from the ground.

Q. 12. *In a car race, car A takes a time t seconds less than the car B and passes the finishing point with a velocity v more than that of the car B. If the cars start from rest and travel with constant acceleration* a_1 *and* a_2 *respectively, show that*

$$v = t\sqrt{a_1 a_2}.$$

Ans. For A: a_1, $t_2 - t$ and $v_2 + v$ be the acceleration, time taken and final velocity.

For B: a_2, t_2 and v_2 be the acceleration, time and final velocity.

But length travelled is same.

$\Rightarrow \qquad a_1(t_2 - t)^2 = a_2 t_2^2$

Solve for $\qquad t = t_2\left(1 - \sqrt{\frac{a_2}{a_1}}\right)$

Using, $\qquad v = u + at$

we have, $\qquad v_2 + v = a_1 \ (t_2 - t)$

and $\qquad v_2 = a_2 \ t_2$

$$v = a_1 \ (t_2 - t) - a_2 t_2$$

$$= (a_1 - a_2) \ t_3 - a_1 t$$

Using value of t in $v = (a_1 - a_2) \ t_2 - a_1 t$

we have, $\qquad v = t\sqrt{a_1 a_2}$.

VI. VALUE-BASED QUESTIONS

Q. 1. *Dinesh and Praveen are two classmates playing on the bank of a lake. They are throwing stones in the lake. Dinesh is a science student and Praveen is from commerce stream. Every time Dinesh threw the stone far away than Praveen. Praveen could not understand the actual reason. At last Praveen requested Dinesh, "What is the technique behind it, why I could not throw far". Dinesh explained about the minimum angle of projection which gives the maximum distances covered by the body.*

(i) What values are shown by Dinesh?

(ii) What is the minimum angle of projection?

Ans. (*i*) Intelligence, helping, social and cooperative.

(*ii*) Maximum Range : $R = \dfrac{u^2 \sin 2\theta}{g}$

For maximum range sin 2θ must have maximum value

$$\therefore \quad \sin 2\theta = 1$$

$$2\theta = 90°$$

$$\therefore \quad \theta = 45°$$

∴ If a body is projected at an angle of 45°, it will cover a maximum distance.

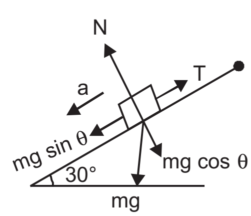
Q. 2. *Sushil lived on the first floor of a building. One day he and his grandfather was at home. Suddenly Sushil observed that there is dense smell all around his flat and found that the fire is coming from his neighbour's flat. The people were crying and under tension. There was a rush on the stair case. He was very worried and thinking how he could save his grandfather. Sushil had an idea. He took a turban cloth and suspended it on the ground and asked a person to hold the other end of the cloth tightly and asked his grand father to gently sit on the turban cloth. He comfortably reached the ground.*

(i) What values and qualities exhibited by Suresh?

(ii) What is the principle which was employed by Suresh?

(iii) What will be the tension in the cloth inclined at an angle of 30° from horizontal when a person of mass 50 kg falls through it with on acceleration of 2 m/s².

Ans. (*i*) Utmost caring, affection with his grandfather, presence of mind and courageous are the values exhibited by Suresh



(*ii*) $T = \text{mg} \sin \theta - ma$

$= m (g \sin \theta - a)$

(*iii*) Here $m = 50 \text{ kg}, \theta = 30°, a = 2\text{m/s}^2, g = 10 \text{ m/s}^2$

$\therefore$ Tension $T = m[g \sin \theta - a]$

$= 50 [10 \times \sin 30° - 2]$

$= 50 \times 3$

$= 150 \text{ N}$

TEST YOUR SKILLS

1. What is 'total path length'? Define.

2. Define uniform velocity.

3. Define acceleration and give expression of acceleration.

4. When is the average velocity over an interval of time becomes equal to instantaneous velocity ?

5. A ball is thrown up from the surface of earth and falls back on it. Taking origin at ground and upward as positive draw

(a) *x-t* graph

(b) *v-t* graph and

(c) *a-t* graph

6. A particle starts with a velocity of 6 m/s and moves with uniform acceleration of 3 m/s^2. Find its velocity after 10 seconds.

7. A stone is dropped from a height of 61.25 m from a balloon which rises up from the ground. Find the velocity of the balloon at that moment if the stone reaches the ground 5 seconds after it was dropped.

8. A point moves with a deceleration $\alpha \sqrt{v}$ in a straight line. (α is a constant). At time $t = 0$, the velocity is v_0. What distance it traverses before it comes to rest. What will be the time consumed ?

9. Derive the expression for the distance travelled in n^{th} second of a uniformly accelerated motion.

10. What is the difference between speed and velocity? Show that slope of displacement-time graph is equal to the velocity of uniform motion.

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