4 Motion in a Plane

Facts that Matter

- Motion in a plane is called as motion in two dimensions *e.g.,* projectile motion, circular motion etc. For the analysis of such motion our reference will be made of an origin and two co-ordinate axes X and Y.

• Scalar and Vector Quantities

Scalar Quantities. The physical quantities which are completely specified by their magnitude or size alone are called scalar quantities.

Examples. Length, mass, density, speed, work, etc.

Vector Quantities. Vector quantities are those physical quantities which are characterised by both magnitude and direction.

Examples. Velocity, displacement, acceleration, force, momentum, torque etc.

• Characteristics of Vectors

Following are the characteristics of vectors:

(*i*) These possess both magnitude and direction.

(*ii*) These do not obey the ordinary laws of Algebra.

(*iii*) These change if either magnitude or direction or both change.

(*iv*) These are represented by bold-faced letters or letters having arrow over them.

• Unit Vector

A unit vector is a vector of unit magnitude and points in a particular direction. It is used to specify the direction only. Unit vector is represented by putting a cap (∧) over the quantity.

The unit vector in the direction of $\vec{A}$ is denoted by $\hat{A}$ and defined by

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{A} \text{ or } \vec{A} = A\,\hat{A}$$

• Equal Vectors

Vectors $\vec{A}$ and $\vec{B}$ are said to be equal if $|\vec{A}| = |\vec{B}|$ as well as their directions are same.

• Zero Vector

A vector with zero magnitude and an arbitrary direction is called a zero vector. It is represented by $\vec{O}$ and also known as null vector.

• Negative of a Vector

The vector whose magnitude is same as that of $\vec{a}$ but the direction is opposite to that of $\vec{a}$ is called the negative of $\vec{a}$ and is written as $-\vec{a}$.

• Parallel Vectors

$\vec{A}$ and $\vec{B}$ are said to be parallel vectors if they have same direction, and may or may not have equal magnitude $\left(\vec{A} \parallel \vec{B}\right)$. If the directions are opposite, then $\vec{A}$ is anti-parallel to $\vec{B}$.

• Coplanar Vectors

Vectors are said to be coplanar if they lie in the same plane or they are parallel to the same plane, otherwise they are said to be non-coplanar vectors.

• Displacement Vector

The displacement vector is a vector which gives the position of a point with reference to a point other than the origin of the co-ordinate system.

Displacement vector $\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$.

• Parallelogram Law of Vector Addition

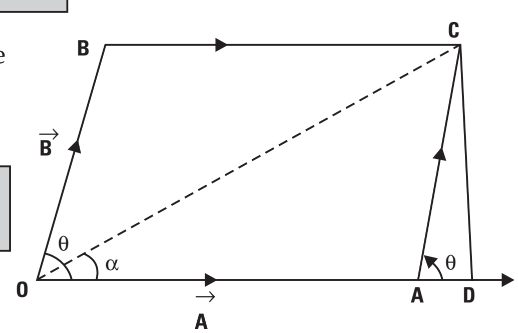
If two vectors, acting simultaneously at a point, can be represented both in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, then the resultant is represented completely both in magnitude and direction by the diagonal of the parallelogram passing through that point.

If $\vec{A}$ and $\vec{B}$ be two adjacent sides of a parallelogram, inclined at angle θ, then the magnitude of resultant vector is given as

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Direction of resultant $\vec{R}$. Let α be the angle made by resultant $\vec{R}$ with vector $\vec{A}$. Then

$$\alpha = \tan^{-1}\frac{B\sin\theta}{A + B\cos\theta}$$

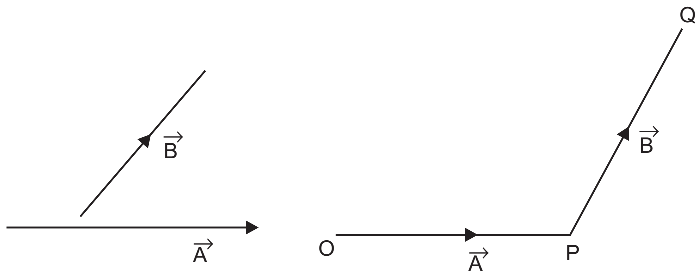


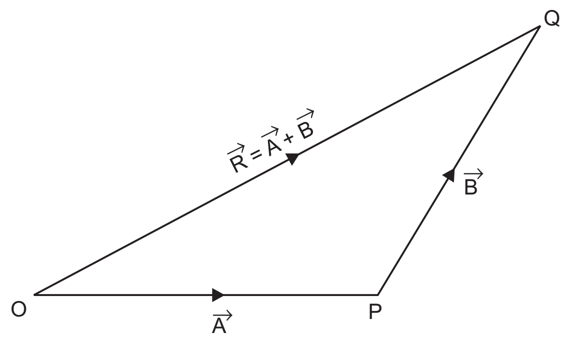
• Triangle Law of Vector Addition

If two vectors are represented both in magnitude and direction by the two sides of a triangle taken in the same order, then the resultant of these vectors is represented both in magnitude and direction by the third side of the triangle taken in the opposite order.

If two vectors $\vec{A}$ and $\vec{B}$ are to be added according to the triangle law of vector addition, then assume

$$\overrightarrow{OP} = \vec{A} \text{ and } \overrightarrow{PQ} = \vec{B}.$$





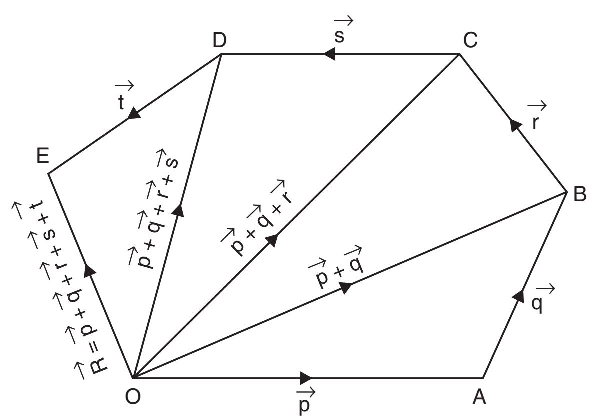
The sum of the resultant of $\vec{A}$ and $\vec{B}$ is represented by the vector $\overrightarrow{OQ}$ (joining tail of $\overrightarrow{OP}$ to the head of $\overrightarrow{PQ}$.

Hence, $\quad \overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$

$$\overrightarrow{OQ} = \vec{A} + \vec{B} \quad \text{(Resultant vector)}$$

- **Polygon Law of Vector Addition**

If a number of vectors are represented both in magnitude and direction by the sides of a polygon taken in the same order, then the resultant vector is represented both in magnitude and direction by the closing side of the polygon taken in the opposite order.



Resultant $\vec{R} = \vec{p} + \vec{q} + \vec{r} + \vec{s} + \vec{t}$

- **Properties of Vector Addition**

Vector addition has following properties:

(*i*) It obeys commutative law

If $\vec{a}$ and $\vec{b}$ are any two vectors,

then $\quad \vec{a} + \vec{b} = \vec{b} + \vec{a}$

(*ii*) It obeys associative law

If $\vec{a}, \vec{b}$ and $\vec{c}$ are any three vectors then

$$\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$$

(*iii*) **It obeys distributive property**

If $\vec{a}$ and $\vec{b}$ are two vectors and λ is a real number then, $\lambda(\vec{a}+\vec{b}) = \lambda\vec{a}+\lambda\vec{b}$

- Equilibriant vector is a vector which balances two or more than two vectors acting simultaneously at a point. It is equal in magnitude and opposite in direction to the resultant vector of given vectors.

$$\vec{R'} \text{ (equilibriant vector)} = -\vec{R} = -\left(\vec{A}+\vec{B}+.....\right)$$

- If vector $\vec{A}$ is multiplied by a real number λ then it gives a vector $\vec{B}$ whose magnitude is λ times the magnitude of the vector $\vec{A}$ and whose direction is the same or opposite depending upon whether λ is positive or negative.
- Subtraction of vector can be defined in terms of addition of two vectors.

If $\vec{P}$ and $\vec{Q}$ two vectors are to be subtracted then we take them as follows:

$$\vec{P}-\vec{Q} = \vec{P}+\left(-\vec{Q}\right)$$

- Vector subtraction is non-commutative and non-associative.

$$\Rightarrow \quad \vec{A}-\vec{B} \neq \vec{B}-\vec{A}$$

$$\Rightarrow \quad \vec{A}-\left(\vec{B}-\vec{C}\right) \neq \left(\vec{A}-\vec{B}\right)-\vec{C}$$

• Resolution of Vectors

It is a process of splitting a single vector into two or more vectors in different directions which together produce the same effect as is produced by the single vector alone.

The vectors into which the given single vector is splitted are called component of vectors. In fact, the resolution of a vector is just opposite to composition of vectors.

(*i*) If a vector $\vec{A}$ makes an angle θ with *x*-axis then magnitude of its rectangular components in *x*-*y* plane are given by $A_x = A\cos\theta$ and $A_y = A\sin\theta$, where,

$$A = \sqrt{A_x^2 + A_y^2}$$

(*ii*) If a vector $\vec{A}$ lie in free space and subtends an angle α with *x*-axis, angle β with *y*-axis and angle γ with *z*-axis then the magnitudes of its rectangular components along the three axes are given as $A_x = A\cos\alpha$, $A_y = A\cos\beta$ and $A_z = A\cos\gamma$.

where $$A = \sqrt{A_x^2 + B_y^2 + C_z^2}$$

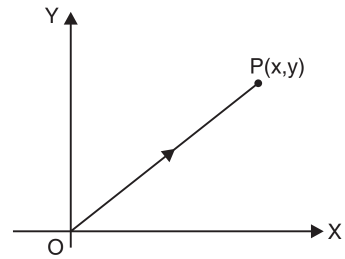
(*iii*) A vector $\vec{A}$ may be expressed in terms of its rectangular components as:

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

where, $\hat{i}, \hat{j}$ and $\hat{k}$ are the unit vectors along *x, y* and *z* axis respectively.

If the components of a given vector are perpendicular to each other, then they are called rectangular components.

• Position Vector



Position vector is a vector to represent any position of a body. The straight line joining the origin and the point represents the position vector. It is represented by both magnitude and direction.

It is represented by $\vec{r} = \overrightarrow{OP} = x\hat{i} + y\hat{i}$ where $\hat{i}$ and $\hat{j}$ are the unit vectors along x and y axis respectively.

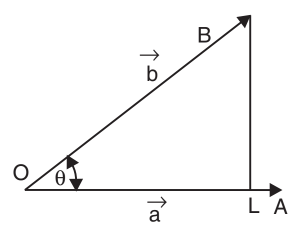
If position vector $\vec{r}$ is in three dimensions, then it is given by $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ where, $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors along x, y and z co-ordinates respectively.

$$\left|\vec{r}\right| = \sqrt{x^2 + y^2 + z^2}$$

• Multiplication of Vectors

(*i*) **Scalar product (Dot product).** Scalar product of two vectors is defined as the product of the magnitude of two vectors with cosine of smaller angle between them.

It is always a scalar, so it is called as scalar product.



$$\vec{A}\cdot\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta$$

Geometrically, $\vec{a}\cdot\vec{b}$ = (Mod of $\vec{a}$) (Projection of b on $\vec{a}$)

(*ii*) **Vector product (Cross product).** The cross or vector product of two vectors $\vec{A}$ and $\vec{B}$ is defined as,

$$\vec{A}\times\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\sin\theta\,\hat{n}, \quad \text{where}$$

θ — angle between $\vec{A}$ and $\vec{B}$ taken in anti-clockwise direction.

$\hat{n}$ — unit vector in the direction perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

Geometrically, $\vec{a}\times\vec{b}$ is a vector whose modulus is the area of the parallelogram formed by the two vectors as the adjacent sides and direction is perpendicular to both $\vec{a}$ and $\vec{b}$.

• Properties of Scalar Product

(*i*) It obeys commutative law

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

(*ii*) It obeys distributive law

$$\vec{A}\cdot\left(\vec{B}+\vec{C}\right)s = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$$

(*iii*) Scalar (Dot) product of two mutually perpendicular vectors is zero *i.e.,*

$$\left(\vec{A}\cdot\vec{B}\right) = AB\cos 90° = 0$$

(*iv*) Scalar (Dot) product will be maximum when θ = 0° *i.e.,* vectors are parallel to each other.

$$\left(\vec{A}\cdot\vec{B}\right)_{max} = |A||B|$$

(*v*) It $\vec{a}$ and $\vec{b}$ are unit vectors then $\left|\vec{a}\right| = \left|\vec{b}\right| = 1$ and $\vec{a}\cdot\vec{b} = 1\cdot 1\cos\theta = \cos\theta$

(*vi*) Dot product of unit vectors $\hat{i}, \hat{j}, \hat{k}$

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$$

$$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$$

(*vii*) Square of a vector $\vec{a}\cdot\vec{a} = |a||a| \cos 0 = a^2$

(*viii*) If the two vectors $\vec{A}$ and $\vec{B}$, in terms of their rectangular components, are

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \quad \text{and} \quad \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}, \quad \text{then,}$$

$$\vec{A}\cdot\vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\cdot\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

- **Properties of Cross Product**

(*i*) Cross product of two vectors is not commutative

$$\vec{a}\times\vec{b} \neq \vec{b}\times\vec{a}$$

$$\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$$

(*ii*) Cross product is not associative

$$\vec{a}\times\left(\vec{b}\times\vec{c}\right) \neq \left(\vec{a}\times\vec{b}\right)\times\vec{c}s$$

(*iii*) Cross product obeys distributive law

$$\vec{a}\times\left(\vec{b}+\vec{c}\right) = \vec{a}\times\vec{b} + \vec{a}\times\vec{c}$$

(*iv*) If θ = 0 or π it means the two vectors are collinear.

$$\vec{a}\times\vec{b} = \vec{0}$$

and conversely, if $\vec{a}\times\vec{b} = \vec{0}$ then the vector $\vec{a}$ and $\vec{b}$ are parallel provided $\vec{a}$ and $\vec{b}$ are non-zero vectors.

(*v*) If θ = 90°, and $\hat{n}$ is the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$

$$\vec{a}\times\vec{b} = |a||b| \sin 90° \ \hat{n} = |a||b| \ \hat{n}$$

(*vi*) The vector product of any vector with itself is $\vec{0}$

$$\vec{a}\times\vec{a} = \vec{0}$$

(*vii*) If $\vec{a}\times\vec{b} = \vec{0}$, then

$$\vec{a} = 0 \quad \text{or} \quad \vec{b} = 0 \quad \text{or} \quad \vec{a} \parallel \vec{b}$$

(*viii*) If $\vec{a}$ and $\vec{b}$ are unit vectors, then $\vec{a}\times\vec{b} = 1 \cdot 1 \sin\theta \ \hat{n} = \sin\theta \ \hat{n}$

(*ix*) Cross product of unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$

$$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$$

$$\hat{i}\times\hat{j} = k = -\hat{j}\times\hat{j}$$

$$\hat{j}\times\hat{k} = \hat{i} = -\hat{k}\times\hat{j}$$

$$\hat{k}\times\hat{i} = \hat{j} = -\hat{i}\times\hat{k}$$

(*x*) If the two vectors $\vec{A}$ and $\vec{B}$ in terms of their rectangular components are

$$\vec{A} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$$

$$\vec{B} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$$

$$\vec{A}\times\vec{B} = \left(a_1\hat{i} + b_1\hat{j} + c_1\hat{k}\right)\times\left(a_2\hat{i} + b_2\hat{j} + c_2\hat{k}\right)$$

It can be found by the determinant method

i.e.,
$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \hat{i}\ (b_1\ c_2 - b_2\ c_1) - \hat{j}\ (a_1\ c_2 - a_2\ c_1) + \hat{k}\ (a_1\ b_2 - a_2\ b_1)$$

- For motion in a plane, velocity is defined as:

$$\vec{v} = \frac{\vec{r_2} - \vec{r_1}}{t_2 - t_1} = \frac{\left(x_2\ \hat{i} + y_2\ \hat{j}\right) - \left(x_1\ \hat{i} + y_1\ \hat{j}\right)}{\left(t_2 - t_1\right)} = \frac{x_2 - x_1}{t_2 - t_1}\hat{i} + \frac{y_2 - y_1}{t_2 - t_1}\hat{j} = v_x\ \hat{i}\ \cdot$$

and $v = \sqrt{a_x^2 + a_y^2}$.

- For motion in a plane, acceleration is defined as

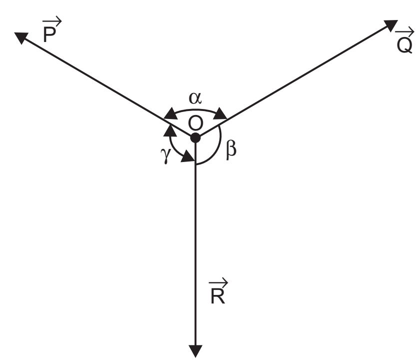
$$\vec{a} = \frac{\vec{v_2} - \vec{v_1}}{t_2 - t_1} = \frac{\left(v_{x_2}\hat{i} + v_{y_2}\hat{j}\right) - \left(v_{x_1}\hat{i} + v_{y_1}\hat{j}\right)}{\left(t_2 - t_1\right)} = \left(\frac{v_{x_2} - v_{x_1}}{t_2 - t_1}\right)\hat{i} + \left(\frac{v_{y_2} - v_{y_1}}{t_2 - t_1}\right)\hat{j}$$

and $a = \sqrt{a_x^2 + a_y^2}$.

• Lami's Theorem

Lami's theorem states, ''If a particle under the simultaneous action of three forces is in equilibrium, then each force has a constant ratio with the sine of the angle between the other two forces.''

If three forces $\vec{P}, \vec{Q}$ and $\vec{R}$ are acting on a particle O in directions given by angles α, β and γ, then, the particle O is in equilibrium, when



$$\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin\alpha}$$

• Projectile Motion

The projectile is a general name given to an object that is given an initial inclined velocity and which subsequently follows a path determined by the gravitational force acting on it and by the frictional resistance of the air. The path followed by a projectile is called its trajectory.

Equation of projectile motion. The general case of projectile motion corresponds to that of an object that has been given an initial velocity *u* at some angle θ above (or below) the horizontal. The horizontal and vertical displacements *x* and *y* are given by

$$u_x = u \cos \theta$$
$$u_y = u \sin \theta$$

Then the equation of trajectory of a projectile is given as

$$y = (\tan \theta)\, x - \frac{g}{2\,(u \cos \theta)^2}\, x^2$$

The above equation is in the form of $y = ax + bx^2$ where a and b are constants. This is a equation of a parabola. Thus the trajectory of a projectile is parabolic.

Time of Flight. The time taken by a projectile to return to its initial elevation after projection is known as its time of flight (T). It is given by

$$T = \frac{2u \sin \theta}{g}$$

Horizontal Range. The maximum horizontal distance between the point of projection and the point on the horizontal plane where the projectile hits is called horizontal range.

$$R = \frac{u^2 \sin 2\theta}{g}$$

The range of the projectile will be maximum if sin 2θ is maximum (*i.e.,* 1).

$$\sin 2\theta = 1$$

or $$2\theta = 90° \Rightarrow \theta = 45°$$

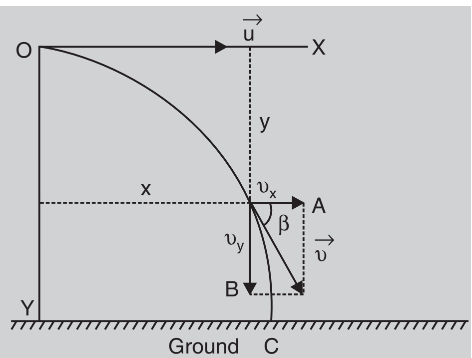
Thus the projectile has maximum range if it is projected at an angle of 45° with the horizontal.

$$R_{max} = \frac{u^2}{g}$$

Maximum Height. The maximum vertical distance travelled by the projectile during its journey is called the maximum height attained by the projectile.

It is given by $$H = \frac{u^2 \sin^2 \theta}{2g}$$

Projectile Given Horizontal Projection:



(*i*) Equation of path $y = kx^2$, which is a parabola

(*ii*) Time of flight $T = \sqrt{\frac{2h}{g}}$

(*iii*) Horizontal Range $R = u\sqrt{\frac{2h}{g}}$

(*iv*) Velocity at any time t is $v = \sqrt{u^2 + g^2t^2}$ and angle made by resultant velocity with horizontal $\beta = \tan^{-1}\left(\frac{gt}{u}\right)$.

(*v*) Velocity of projectile when it hits the ground $v = \sqrt{u^2 + 2gh}$.

- **Angular Displacement**

Angular displacement of the object moving around a circular path is defined as the angle traced out by the radius vector at the centre of the circular path in a given time.

$$\theta \text{ (angle)} = \text{arc/radius}$$

$\theta \rightarrow$ the magnitude of angular displacement. It is expressed in radians (rad).

- **Angular Velocity**

Angular velocity of an object in circular motion is defined as the time rate of change of its angular displacement.

It is denoted by ω and is measured in radians per second (rad.s^{-1}).

$$\omega = \frac{\text{angular displacement}}{\text{Time}} = \frac{\theta}{t} = \frac{d\theta}{dt}$$

- **Angular Acceleration**

Angular acceleration of an object in circular motion is defined as the time rate of change of its angular velocity.

It is denoted by 'α' and measured in rad s^{-2}.

$$\alpha = \frac{\text{angular velocity change}}{\text{time taken}} = \frac{d\omega}{dt}$$

- For uniform angular acceleration α, the equations of motion can be modified as,

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

- **Uniform Circular Motion**

When a body moves in a circular path with a constant speed, then the motion of the body is known as uniform circular motion.

The time taken by the object to complete one revolution on its circular path is called time period.

For circular motion, the number of revolutions completed per unit time is known as the frequency (ν). Unit of frequency is 1 Hertz (1 Hz). It is found that

$$\nu \cdot T = 1 \quad \text{or} \quad \nu = \frac{1}{T}$$

- The relation between angular velocity, frequency and time period is given by

$$\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi\nu$$

- **Centripetal Acceleration**

To maintain a particle in its uniform circular motion a radially inward acceleration should be continuously maintained. It is known as the centripetal acceleration.

$$a_c = \frac{v^2}{r} = r\omega^2 = \frac{r \cdot 4\pi^2}{T^2} = r \cdot 4\pi^2 \cdot \omega^2$$

- **IMPORTANT TABLES**

TABLE 4.1 Comparison of Equations of Motion of Linear and Circular Motions

S.No.	*Linear Motion*	*Circular Motion*
1.	$S = ut$	$\theta = \omega_0 t$
2.	$S = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
3.	$v = u + at$	$\omega = \omega_0 + \alpha t$
4.	$v^2 - u^2 = 2aS$	$\omega^2 - \omega_0^2 = 2\alpha\theta$

TABLE 4.2

Physical Quantity	*Symbol*	*Dimensions*	*Unit*
Position vector	$\mathbf{r}$	[L]	m
Displacement	$\Delta\mathbf{r}$	[L]	m
Velocity		$[LT^{-1}]$	m s^{-1}
(*a*) Average	$\overline{\mathbf{V}}$		
(*b*) Instantaneous	$\mathbf{V}$		
Acceleration		$[LT^{-2}]$	m s^{-2}
(*a*) Average	$\bar{\boldsymbol{a}}$		
(*b*) Instantaneous	$\boldsymbol{a}$		
Projectile motion			
(*a*) Time of max. height	t_m	[T]	s
(*b*) Max. height	h_m	[L]	m
(*c*) Horizontal range	R	[L]	m
Circular motion			
(*a*) Angular speed	ω	$[T^{-1}]$	rad/s
(*b*) Centripetal acceleration	a_c	$[LT^{-2}]$	m s^{-2}

NCERT TEXTBOOK QUESTIONS SOLVED

4.1. *State, for each of the following physical quantities, if it is a scalar or a vector: volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.*

Sol. **Scalars:** Volume, mass, speed, density, number of moles, angular frequency.

Vectors: Acceleration, velocity, displacement, angular velocity.

4.2. *Pick out the two scalar quantities in the following list: force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, relative velocity.*

Sol. Work and current are the scalar quantities in the given list.

4.3. *Pick out the only vector quantity in the following list:*

Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.

Sol. Impulse.

4.4. *State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful:*

(a) adding any two scalars, (b) adding a scalar to a vector of the same dimensions, (c) multiplying any vector by any scalar, (d) multiplying any two scalars, (e) adding any two vectors, (f) adding a component of a vector to the same vector.

Sol. (*a*) No, because only the scalars of same dimensions can be added.

(*b*) No, because a scalar cannot be added to a vector.

(*c*) Yes, multiplying a vector with a scalar gives the scalar (number) times the vector quantity which makes sense and one gets a bigger vector. For example, when acceleration $\vec{A}$ is multiplied by mass *m*, we get a force $\vec{F} = m\vec{A}$.

(*d*) Yes, two scalars multiplied yield a meaningful result, for example multiplication of rise in temperature of water and its mass gives the amount of heat absorbed by that mass of water.

(*e*) No, because the two vectors of same dimensions can be added.

(*f*) Yes, because both are vectors of the same dimensions.

4.5. *Read each statement below carefully and state with reasons, if it is true or false:*

(a) The magnitude of a vector is always a scalar.

(b) Each component of a vector is always a scalar.

(c) The total path length is always equal to the magnitude of the displacement vector of a particle.

(d) The average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time.

(e) Three vectors not lying in a plane can never add up to give a null vector.

Sol. (*a*) True, magnitude of the velocity of a body moving in a straight line may be equal to the speed of the body.

(*b*) False, each component of a vector is always a vector, not scalar.

(*c*) False, total path length can also be more than the magnitude of displacement vector of a particle.

(*d*) True, because the total path length is either greater than or equal to the magnitude of the displacement vector.

(*e*) True, this is because the resultant of two vectors will not lie in the plane of third vector and hence cannot cancel its effect to give null vector.

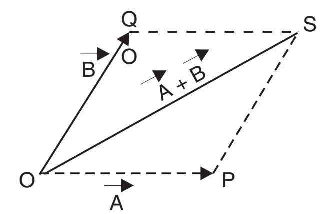
4.6. *Establish the following inequalities geometrically or otherwise:*

(a) $\left|\vec{A}+\vec{B}\right| \le \left|\vec{A}\right| + \left|\vec{B}\right|$ *(b)* $\left|\vec{A}+\vec{B}\right| \ge \left|\left|\vec{A}\right| - \left|\vec{B}\right|\right|$

(c) $\left|\vec{A}-\vec{B}\right| \le \left|\vec{A}\right| + \left|\vec{B}\right|$ *(d)* $\left|\vec{A}-\vec{B}\right| \ge \left|\left|\vec{A}\right| - \left|\vec{B}\right|\right|$

When does the equality sign above apply?

Sol. Consider two vectors $\vec{A}$ and $\vec{B}$ be represented by the sides $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ of a parallelogram OPSQ. According to parallelogram law of vector addition; $(\vec{A}+\vec{B})$ will be represented by $\overrightarrow{OS}$ as shown in Fig. Thus



$$OP = |\vec{A}|, \quad OQ = PS = |\vec{B}|$$

and $$OS = |\vec{A}+\vec{B}|$$

(*a*) **To prove** $|\vec{A}+\vec{B}| \le |\vec{A}|+|\vec{B}|$

We know that the length of one side of a triangle is always less than the sum of the lengths of the other two sides. Hence from Δ OPS, we have

$$OS < OP + PS \quad \text{or} \quad OS < OP + OQ \quad \text{or} \quad |\vec{A}+\vec{B}| < |\vec{A}|+|\vec{B}| \qquad ...(i)$$

If the two vectors $\vec{A}$ and $\vec{B}$ are acting along the same straight line and in the same direction

then $$|\vec{A}+\vec{B}| = |\vec{A}|+|\vec{B}| \qquad ...(ii)$$

Combining the conditions mentioned in (*i*) and (*ii*) we have

$$|\vec{A}+\vec{B}| \le |\vec{A}|+|\vec{B}|$$

(*b*) **To prove** $|\vec{A}+\vec{B}| \ge \left||\vec{A}|-|\vec{B}|\right|$

From Δ OPS, we have $OS + PS > OP$ or $OS > |OP - PS|$ or $OS > |OP - OQ|$

...(*iii*) ($\because PS = OQ$)

The modulus of ($OP - PS$) has been taken because the L.H.S. is always positive but the R.H.S. may be negative if $OP < PS$. Thus from (*iii*) we have.

$$|\vec{A}+\vec{B}| > \left||\vec{A}|-|\vec{B}|\right| \qquad ...(iv)$$

If the two vectors $\vec{A}$ and $\vec{B}$ are acting along a straight line in opposite directions, then

$$|\vec{A}+\vec{B}| = \left||\vec{A}|-|\vec{B}|\right| \qquad ...(v)$$

Combining the conditions mentioned in (*iv*) and (*v*) we get.

$$|\vec{A}+\vec{B}| \ge \left||\vec{A}|-|\vec{B}|\right|$$

(*c*) **To prove** $|\vec{A}-\vec{B}| \le |\vec{A}|+|\vec{B}|$

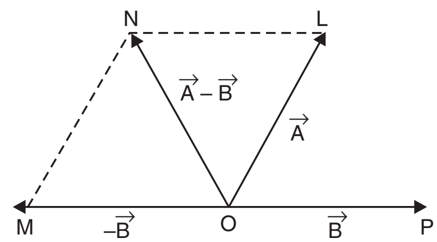
In fig. $\overrightarrow{OL}$ and $\overrightarrow{OM}$ represents vectors $\vec{A}$ and $\vec{B}$ respectively. Here $\overrightarrow{ON}$ represents $\vec{A}-\vec{B}$.

Consider the Δ OMN,

$$ON < MN + OM$$

or $$|\vec{A}-\vec{B}| < |\vec{A}|+|-\vec{B}|$$

or $\left|\vec{A}-\vec{B}\right| < \left|\vec{A}\right|+\left|\vec{B}\right|$ ...(*vi*)



When $\vec{A}$ and $\vec{B}$ are along the same straight line, but point in the opposite direction, then

$$\left|\vec{A}-\vec{B}\right| = \left|\vec{A}\right|+\left|\vec{B}\right| \quad ...(vii)$$

Combining equation (*vi*) and (*vii*), we get

$$\left|\vec{A}-\vec{B}\right| \le \left|\vec{A}\right|+\left|\vec{B}\right|$$

(*d*) **To prove** $\left|\vec{A}-\vec{B}\right| \ge \left|\left|\vec{A}\right|-\left|\vec{B}\right|\right|$

Let us consider the Δ OMN,

$ON + OM > MN$ or $ON > |MN - OM|$

Since $MN = OL$ ∴ $ON > |OL - OM|$

or $\left|\vec{A}-\vec{B}\right| > \left|\left|\vec{A}\right|-\left|\vec{B}\right|\right|$ ...(*viii*)

When $\vec{A}$ and $\vec{B}$ are along the same straight line and point in the same direction, then

$$\left|\vec{A}-\vec{B}\right| = \left|\vec{A}\right|-\left|\vec{B}\right| \quad ...(ix)$$

Combining equations (*viii*) and (*ix*), we get

$$\left|\vec{A}-\vec{B}\right| \ge \left|\left|\vec{A}\right|-\left|\vec{B}\right|\right|$$

4.7. *Given* $\vec{a}+\vec{b}+\vec{c}+\vec{d} = \vec{0}$, *which of the following statements are correct:*

(a) $\vec{a},\vec{b},\vec{c}$ *and* $\vec{d}$ *must each be a null vector,*

(b) *The magnitude of* $(\vec{a}+\vec{c})$ *equals the magnitude of* $(\vec{b}+\vec{d})$.

(c) *The magnitude of* $\vec{a}$ *can never be greater than the sum of the magnitudes of* $\vec{b},\vec{c}$ *and* $\vec{d}$.

(d) $\vec{b}+\vec{c}$ *must lie in the plane of* $\vec{a}$ *and* $\vec{d}$ *if* $\vec{a}$ *and* $\vec{d}$ *are not collinear, and in the line of* $\vec{a}$ *and* $\vec{d}$, *if they are collinear?*

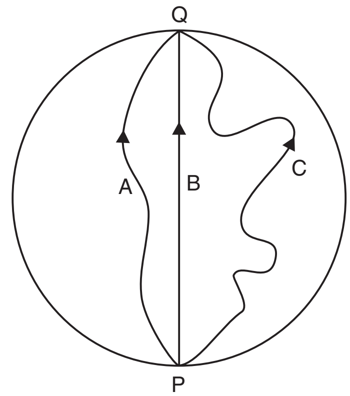
Sol. (*a*) This statement is not correct. Each need not be a null vector. Even when $\vec{a} = -\vec{b}$ and $\vec{c} = -\vec{d}$, they can form a null vector.

(*b*) This statement is correct. When $\left|\vec{a}+\vec{c}\right| = \left|\vec{b}+\vec{d}\right|$, the addition may be a null vector, if $\vec{a}+\vec{c} = -(\vec{b}+\vec{d})$ and are collinear.

(*c*) This statement is correct. Let $\left|\vec{a}\right| \ngtr \left|\vec{b}+\vec{c}+\vec{d}\right|$. If it is true the vector sum cannot be zero. Even if $\vec{b},\vec{c},\vec{d}$ form a triangle, the vector sum $\vec{b}+\vec{c}+\vec{d} = 0$ but then $\vec{a}+\vec{b}+\vec{c}+\vec{d}$ is not zero.

(*d*) This statement is correct. If $\vec{b}+\vec{c}$ do not lie in the plane of $\vec{a}+\vec{d}$, the vector sum $(\vec{a}+\vec{b})+(\vec{c}+\vec{d})$ is not zero because the addends will have different magnitude and different direction.

4.8. *Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in Fig. What is the magnitude of the displacement vector for each? For which girl is this equal to the actual length of path skate?*



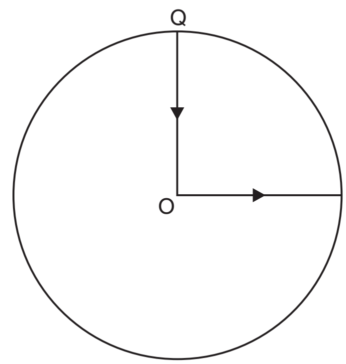
Sol. Displacement for each girl = $\overrightarrow{PQ}$.

∴ Magnitude of the displacement for each girl

$= PQ$ = diameter of circular ice ground

$= 2 \times 200 = 400$ m.

4.9. *A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO as shown in Fig. If the round trip takes 10 min, what is the (a) net displacement, (b) average velocity, and (c) average speed of the cyclist?*

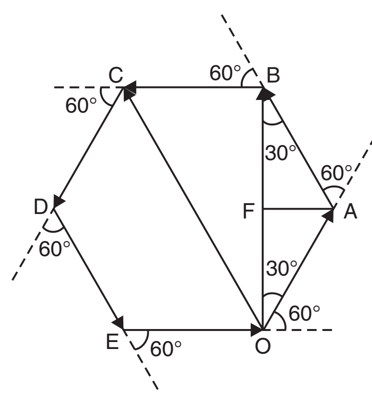


Sol. (*a*) Since both the initial and final positions are the same therefore the net displacement is zero.

(*b*) Average velocity is the ratio of net displacement and total time taken. Since the net displacement is zero therefore the average velocity is also zero.

(*c*)
$$\text{Average speed} = \frac{\text{distance covered}}{\text{time taken}}$$
$$= \frac{OP + \text{Actual distance } PQ + QO}{10 \text{ minute}}$$
$$= \frac{1\text{ km} + \frac{1}{4} \times 2\pi \times 1\text{ km} + 1\text{ km}}{10/60\ h}$$
$$= 6\left(2 + \frac{22}{14}\right)\text{km h}^{-1} = 6 \times \frac{50}{14}\text{ km h}^{-1}$$
$$= 21.43 \text{ km h}^{-1}.$$

4.10. *On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.*



Sol. (*i*) The path followed by the motorist will be a closed hexagonal path.

Suppose the motorist starts his journey from the point *O*. He takes the turn at the point *C*.

$$\text{Displacement} = \overrightarrow{OC}$$

Here $$OC = \sqrt{(OB)^2 + (BC)^2} = \sqrt{(OF + FB)^2 + (BC)^2}$$

$$= \sqrt{(500\cos 30° + 500\cos 30°)^2 + (500)^2}$$

$$= \sqrt{\left(2 \times 500 \times \frac{\sqrt{3}}{2}\right)^2 + (500)^2}$$

$$= 500\sqrt{4} = 1000 \text{ m} = 1 \text{ km}$$

Total path length = 500 m + 500 m + 500 m = 1500 m = 1.5 km

$$\frac{\text{Magnitude of displacement}}{\text{Total path length}} = \frac{1}{1.5} = \frac{2}{3} = 0.67$$

(*ii*) The motorist will take the sixth turn at O.

Displacement is zero. So, displacement vector is a null vector.

Path length is 3000 m, *i.e.,* 3 km.

Ratio of magnitude of displacement and path length is zero.

(*iii*) The motorist will take the 8^{th} turn at *B*.

Magnitude of displacement = 2 × 500 cos 30° = $500\sqrt{3}\text{m} = \frac{\sqrt{3}}{2}\text{ km} = 0.866 \text{ km}$

Path length = 8 × 500 m = 4 km

Ratio of magnitude of displacement and path length is $\frac{\sqrt{3}/2}{4}$ *i.e.,* $\frac{\sqrt{3}}{8} = 0.22$

4.11. *A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is (a) the average speed of the taxi, (b) the magnitude of average velocity? Are the two equal?*

Sol. Here, actual path length travelled, s = 23 km; Displacement = 10 km;

Time taken, $t = 28 \text{ min} = \frac{28}{60} \text{ h}$

(*a*) Average speed of taxi = $\frac{\text{actual path length}}{\text{time taken}} = \frac{23}{28/60}\text{ km/h} = 49.3 \text{ km/h}$

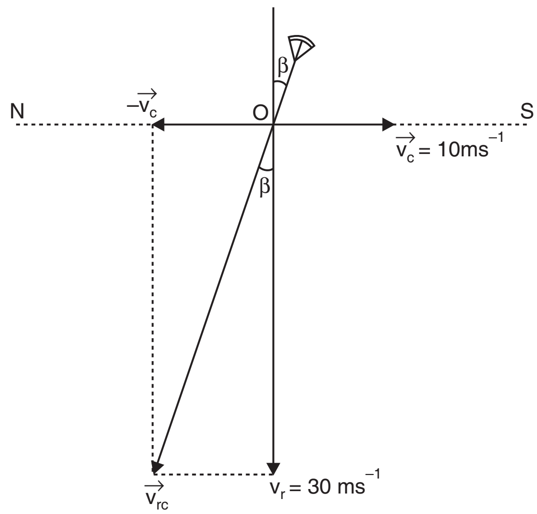
(*b*) Magnitude of average velocity = $\frac{\text{displacement}}{\text{time taken}} = \frac{10}{28/60}\text{ km/h} = 21.4 \text{ km/h}$

The average speed is not equal to the magnitude of average velocity. The two are equal for the motion of taxi along a straight path in one direction.

4.12. *Rain is falling vertically with a speed of 30 m s^{-1}. A woman rides a bicycle with a speed of 10 m s^{-1} in the north to south direction. What is the direction in which she should hold her umbrella ?*

Sol. The situation has been demonstrated in the figure below. Here $\vec{v}_r$ = 30 ms^{-1} is the rain velocity in vertically downward direction and $\vec{v}_c$ = 10 ms^{-1} is the velocity of cyclist woman in horizontal plane from north *N* to south *S*.

$\therefore$ Relative velocity of rain w.r.t. cyclist $\vec{v}_{rc}$ subtends an angle β with vertical such that



$$\tan \beta = \frac{|\vec{v}_c|}{|\vec{v}_r|} = \frac{10}{30} = \frac{1}{3}$$

$$\therefore \qquad \beta = \tan^{-1}\left(\frac{1}{3}\right) = 18° \, 26'$$

Hence, the woman should hold her umbrella at 18° 26′ south of vertical.

4.13. *A man can swim with a speed of 4.0 km h^{-1} in still water. How long does he take to cross a river 1.0 km wide if the river flows steadily at 3.0 km h^{-1} and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank?*

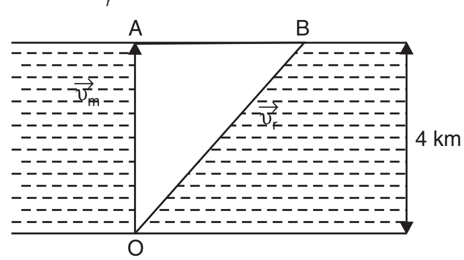
Sol. Here, $\vec{V}_m$ = 4 km h^{-1}; $\vec{v}_r$ = 3 km h^{-1};

OA = 1 km

Let t = time taken by man to reach the other bank.

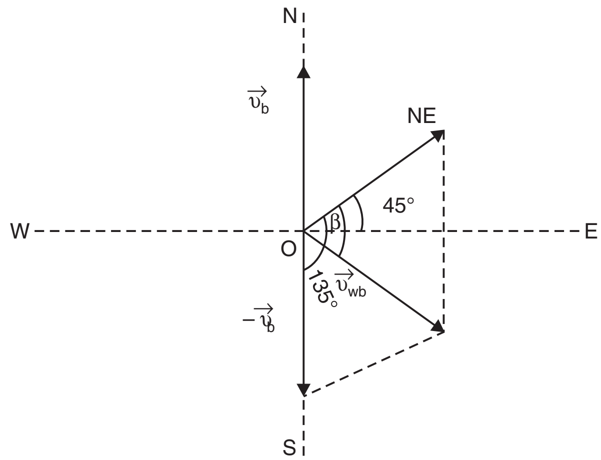
then $$t = \frac{OA}{v_m} = \frac{1}{4} = 0.25 \text{ h}$$

Distance, $AB = v_r \times t = 3 \times 0.25 = 0.75$ km.



4.14. *In a harbour, wind is blowing at the speed of 72 km/h and the flag on the mast of a boat anchored in the harbour flutters along the N-E direction. If the boat starts moving at a speed of 51 km/h to the north, what is the direction of the flag on the mast of the boat?*

Sol. When the boat is anchored in the harbour, the flag flutters along the *N-E* direction. It shows that the velocity of wind is along the north-east direction. When the boat starts moving, the flag will flutter along the direction of relative velocity of wind w.r.t. boat. Let $\vec{v}_{wb}$ be the relative velocity of wind w.r.t. boat and β be the angle between $\vec{v}_{wb}$ and $\vec{v}_w$. (see fig. below)



Now, $$\vec{v}_{wb} = \vec{v}_w + \left(-\vec{v}_b\right)$$

Here, $$\left|\vec{v}_w\right| = 72 \text{ km/h}$$

$$\left|-\vec{v}_b\right| = 51 \text{ km/h}$$

Angle between $\vec{v}_w$ and $-\vec{v}_b$ is 135° *i.e.*, $\theta = 135°$. Then

$$\tan\beta = \frac{51\sin 135°}{72 + 51\cos 135°} = \frac{51\sin 45°}{72 + 51\left(-\cos 45°\right)}$$

$$= \frac{51\times\left(1/\sqrt{2}\right)}{72 - 51\left(1/\sqrt{2}\right)} = 1.0039$$

$\therefore$ $$\beta = \tan^{-1}(1.0039) = 45.1°$$

Angle w.r.t. east direction = 45.1° – 45° = 0.1°

It means the flag will flutter almost due east.

4.15. *The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m s^{-1} can go without hitting the ceiling of the hall?*

Sol. Maximum height h_{max} = 25 m; Horizontal range, R =?

Velocity of projection, v = 40 ms^{-1}

We know that $$h_{max} = \frac{v^2\sin^2\theta}{2g}$$

or $$\sin^2\theta = \frac{25\times 2\times 9.8}{40\times 40} = 0.30625 \quad\text{or}\quad \sin\theta = 0.5534$$

$$\theta = \sin^{-1}(0.5534) = 33.6°$$

Again, $$R = \frac{v^2\sin 2\theta}{g} = \frac{40\times 40\sin 67.2°}{9.8}$$

or $$R = \frac{1600}{9.8}\times 0.9219 \text{ m} = 150.5 \text{ m}.$$

4.16. *A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball?*

Sol. $R_{max} = 100$ m;

Since $R_{max} = \frac{v^2}{g} \Rightarrow 100 = \frac{v^2}{g}$

Using equation of motion

$$v^2 - u^2 = 2as$$

Here, $v = 0$, $a = -g$, $s = R_{max} = 100$ m

$\therefore$ $(0)^2 - u^2 = 2(-g) \times s$

$\Rightarrow$ $s = \frac{1}{2}\frac{u^2}{g}$

Since $u = v$

$\therefore$ $s = \frac{1}{2}\frac{v^2}{g} = \frac{1}{2} \times 100 = 50 \text{ m}$.

4.17. *A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone?*

Sol. Here, $r = 80$ cm $= 0.8$ m;

$$\nu = \frac{14}{25} \text{ rev/s}$$

$\therefore$ $\omega = 2\pi\nu = 2 \times \frac{22}{7} \times \frac{14}{25} \text{ rad/s} = \frac{88}{25} \text{ rad} \cdot \text{s}^{-1}$

The centripetal acceleration,

$$a = \omega^2 r = \left(\frac{88}{25}\right)^2 \times 0.80 = 9.90 \text{ ms}^{-2}$$

The direction of centripetal acceleration is along the string directed towards the centre of circular path.

4.18. *An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of 900 km/h. Compare its centripetal acceleration with the acceleration due to gravity.*

Sol. Here $r = 1 \text{ km} = 10^3 \text{ m}$, $v = 900 \text{ km h}^{-1} = 900 \times \frac{5}{18} = 250 \text{ ms}^{-1}$

Centripetal acceleration $= a_c = \frac{v^2}{r} = \frac{(250)^2}{10^3} = 62.5 \text{ ms}^{-2}$

Now, $\frac{a_c}{g} = \frac{62.5}{9.8} = 6.38$.

4.19. *Read each statement below carefully and state, with reasons, if it is true or false:*

(a) The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre.

(b) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point.

(c) The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector.

Sol. (*a*) False, the net acceleration of a particle in circular motion is along the radius of the circle towards the centre only in uniform circular motion.

(*b*) True, because while leaving the circular path, the particle moves tangentially to the circular path.

(*c*) True, the direction of acceleration vector in a uniform circular motion is directed towards the centre of circular path. It is constantly changing with time. The resultant of all these vectors will be a zero vector.

4.20. *The position of a particle is given by*

$$r = 3.0t\ \hat{i}\ - 2.0t^2\ \hat{j}\ + 4.0\ \hat{k}\ m$$

where t is in seconds and the coefficients have the proper units for ***r*** *to be in metres.*

(a) *Find the* $\vec{v}$ *and* $\vec{a}$ *of the particle.*

(b) *What is the magnitude and direction of velocity of the particle at t = 2.0 s?*

Sol. Here $\vec{r}(t) = \left(3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,\hat{k}\right)\text{m}$

(*a*) ∴ $\vec{v}(t) = \dfrac{\overrightarrow{dr}}{dt} = \left(3.0\,\hat{i} - 4.0t\,\hat{j}\right)\text{m/s}s$

and $\vec{a}(t)s = \dfrac{\overrightarrow{dv}}{dt} = \left(-4.0\,\hat{j}\right)\text{m/s}^2$

(*b*) Magnitude of velocity at t = 2.0 s,

$$v_{(t\,=\,2s)} = \sqrt{(3.0)^2 + (-4.0\times 2)^2} = \sqrt{9+64} = \sqrt{73}$$
$$= 8.54\ \text{m s}^{-1}$$

This velocity will subtend an angle β from x-axis, where $\tan\beta = \dfrac{(-4.0\times 2)}{(3.0)} = -2.667.$

= -2.6667.

∴ $\beta = \tan^{-1}(-2.6667) = -69.44° = 69.44°$ from negative x-axis.

4.21. *A particle starts from the origin at t = 0 s with a velocity of 10.0* $\hat{j}$ *m/s and moves in the x-y plane with a constant acceleration of* $\left(8.0\hat{i} + 2.0\hat{j}\right) m\ s^{-2}$.

(a) *At what time is the x-coordinate of the particle 16 m? What is the y-coordinate of the particle at that time?*

(b) *What is the speed of the particle at the time?*

Sol. It is given that $\vec{r}_{(t=0s)} = 0, \vec{v}_{(0)} = 10.0\,\hat{j}\ \text{m/s}$ and $\vec{a}(t)s = \left(8.0\,\hat{i} + 2.0\,\hat{j}\right)\text{m s}^{-2}$

(*a*) It means $x_0 = 0$, $u_x = 0$, $a_x = 8.0\ \text{m s}^{-2}$ and $x = 16$ m

Using relation $s = x - x_0 = u_x t + \dfrac{1}{2}a_x t^2$, we have

$$16 - 0 = 0 + \frac{1}{2} \times 8.0 \times t^2 \Rightarrow t = 2\ \text{s}$$

∴ $y = y_0 + u_y t + \dfrac{1}{2}a_y t^2 = 0 + 10.0 \times 2 + \dfrac{1}{2} \times 2.0 \times (2)^2$

$= 20 + 4 = 24$ m

(*b*) Velocity of particle at $t = 2$ s along x-axis

$$v_x = u_x + a_x t = 0 + 8.0 \times 2 = 16.0 \text{ m/s}$$

and along y-axis $\quad v_y = u_y + a_y t = 10.0 + 2.0 \times 2 = 14.0 \text{ m/s}$

∴ Speed of particle at $t = 2$ s

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(16.0)^2 + (14.0)^2} = 21.26 \text{ m s}^{-1}.$$

4.22. *$\hat{i}$ and $\hat{j}$ are unit vectors along x and y-axis respectively. What is the magnitude and direction of the vectors $\hat{i} + \hat{j}$, and $\hat{i} - \hat{j}$? What are the components of a vector $\vec{A} = 2\hat{i} + 3\hat{j}$ along the directions of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$? [You may use graphical method]*

Sol. (*i*) $$\hat{i} + \hat{j} = \sqrt{(1)^2 + (1)^2 + 2 \times 1 \times 1 \times \cos 90^\circ} = \sqrt{2} = 1.414 \text{ units}$$

$$\tan\theta = \frac{1}{1} = 1, \quad \therefore \quad \theta = 45^\circ$$

So the vector $\hat{i} + \hat{j}$ makes an angle of 45° with x-axis.

(*ii*) $$\left|\hat{i} - \hat{j}\right| = \sqrt{(1)^2 + (2)^2 - 2 \times 1 \times 1 \times \cos 90^\circ}$$

$$= \sqrt{2} = 1.414 \text{ units}$$

The vector $\hat{i} - \hat{j}$ makes an angle of –45° with x-axis.

(*iii*) Let us now determine the component of $\vec{A} = 2\hat{i} + 3\hat{j}$ in the direction of $\hat{i} + \hat{j}$.

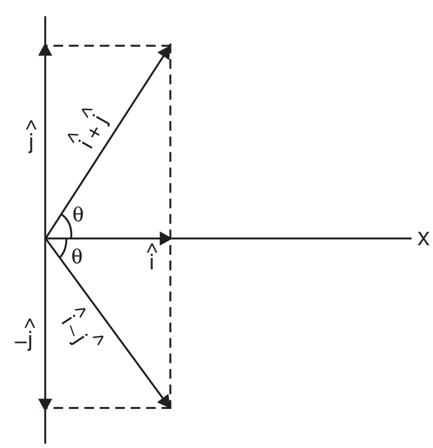
Let $\quad \vec{B} = \hat{i} + \hat{j}$

$$\vec{A} \cdot \vec{B} = AB\cos\theta = (A\cos\theta)\, B$$

So the component of $\vec{A}$ in the direction of $\vec{B} = \dfrac{\vec{A} \cdot \vec{B}}{B}$

$$= \frac{(2\hat{i} + 3\hat{j}) \cdot (\hat{i} + \hat{j})}{\sqrt{(1)^2 + (1)^2}} = \frac{2\hat{i} \cdot \hat{i} + 2\hat{i} \cdot \hat{j} + 3\hat{j} \cdot \hat{i} + 3\hat{j} \cdot \hat{j}}{\sqrt{2}} = \frac{5}{\sqrt{2}} \text{ units}$$

(*iv*) Component of $\vec{A}$ in the direction of $\hat{i} - \hat{j} = \dfrac{(2\hat{i} + 3\hat{j}) \cdot (\hat{i} - \hat{j})}{\sqrt{2}} = -\dfrac{1}{\sqrt{2}}$ units.



4.23. *For any arbitrary motion in space, which of the following relations are true:*

(a) $v_{average} = (1/2)\,[v\,(t_1) + v\,(t_2)]$

(b) $v_{average} = [r\,(t_2) - r\,(t_1)]/(t_2 - t_1)$

(c) $v\,(t) = v\,(0) + a\,t$

(d) $r\,(t) = r\,(0) + v\,(0)\,t + (1/2)\,a\,t^2$

(e) $a_{average} = [v\,(t_2) - v\,(t_1)]/(t_2 - t_1)$

(The 'average' stands for average of the quantity over the time interval t_1 to t_2)

Sol. (*b*) and (*e*) are true; others are false because relations (*a*), (*c*) and (*d*) hold only for uniform acceleration.

4.24. *Read each statement below carefully and state, with reasons and examples, if it is true or false: A scalar quantity is one that*

(a) *is conserved in a process*

(b) *can never take negative values*

(c) *must be dimensionless*

(d) *does not vary from one point to another in space*

(e) *has the same value for observers with different orientations of axes.*

Sol. (*a*) False, because kinetic energy is a scalar but does not remain conserved in an inelastic collision.

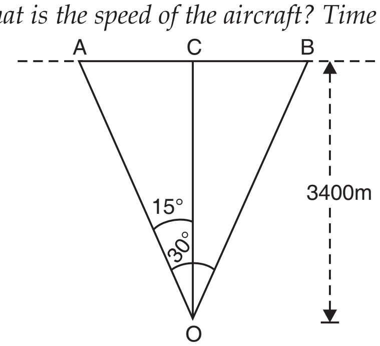
(*b*) False, because potential energy in a gravitational field may have negative values.

(*c*) False, because mass, length, time, speed, work etc., all have dimensions.

(*d*) False, because speed, energy etc., vary from point to point in space.

(*e*) True, because a scalar quantity will have the same value for observers with different orientations of axes since a scalar has no direction of its own.

4.25. *An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10 s apart is 30°, what is the speed of the aircraft? Time taken by aircraft from A to B is 10 s.*



Sol. In Fig, *O* is the observation point at the ground. *A* and *B* are the positions of air craft for which ∠AOB = 30°. Draw a perpendicular *OC* on *AB*. Here *OC* = 3400 m and ∠AOC = ∠COB = 15°.

In ΔAOC, $AC = OC \tan 15° = 3400 \times 0.2679 = 910.86$ m.

$AB = AC + CB = AC + AC = 2\,AC = 2 \times 910.86$ m

Speed of the aircraft, $v = \dfrac{\text{distance } AB}{\text{time}} = \dfrac{2 \times 910.86}{10} = 182.17 \text{ ms}^{-1} = \mathbf{182.2 \text{ ms}^{-1}}$.

4.26. *A vector has magnitude and direction.*

(i) *Does it have a location in the space?*

(ii) *Can it vary with time?*

(iii) *Will two equal vectors $\vec{a}$ and $\vec{b}$ at different locations in space necessarily have identical physical effects? Give examples in support of your answer.*

Sol. (*i*) Besides having magnitude and direction, each vector has also a location in space.

(*ii*) A vector can vary with time. As an example, velocity and acceleration vectors may vary with time.

(*iii*) Two equal vectors $\vec{a}$ and $\vec{b}$ having different locations may not have same physical effect. As an example, two balls thrown with the same force, one from earth and the other from moon will attain different 'maximum heights'.

4.27. *A vector has both magnitude and direction. Does that mean anything that has magnitude and direction is necessarily a vector? The rotation of a body can be specified by the direction of the axis of rotation and the angle of rotation about the axis. Does that make any rotation a vector?*

Sol. No. Finite rotation of a body about an axis is not a vector because finite rotations do not obey the laws of vector addition.

4.28. *Can you associate vectors with (a) the length of a wire bent into a loop (b) a plane area (c) a sphere? Explain.*

Sol. (*a*) We cannot associate a vector with the length of a wire bent into a loop. This is because the length of the loop does not have a definite direction.

(*b*) We can associate a vector with a plane area. Such a vector is called area vector and its direction is represented by a normal drawn outward to the area.

(*c*) The area of a sphere does not point in any difinite direction. However, we can associate a null vector with the area of the sphere. We cannot associate a vector with the volume of a sphere.

4.29. *A bullet fired at an angle of 30° with the horizontal hits the ground 3 km away. By adjusting its angle of projection, can one hope to hit a target 5 km away? Assume the muzzle speed to the fixed, and neglect air resistance.*

Sol. Here R = 3 km = 3000 m, $\theta = 30°$, $g = 9.8 \text{ m s}^{-2}$.

As $$R = \frac{u^2 \sin 2\theta}{g}$$

$$\Rightarrow \quad 3000 = \frac{u^2 \sin 2 \times 30°}{9.8} = \frac{u^2 \sin 60°}{9.8}$$

$$\Rightarrow \quad u^2 = \frac{3000 \times 9.8}{\sqrt{3}/2} = 3464 \times 9.8$$

Also, $$R' = \frac{u^2 \sin 2\theta'}{g} \quad \Rightarrow \quad 5000 = \frac{3464 \times 9.8 \times \sin 2\theta'}{9.8}$$

i.e., $$\sin 2\theta' = \frac{5000}{3464} = 1.44$$

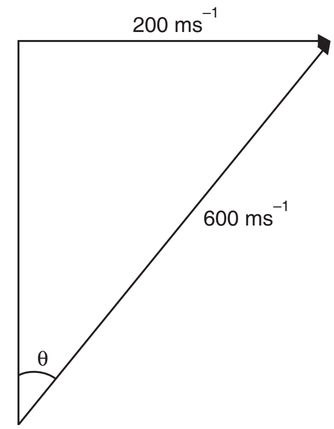
which is impossible because sine of an angle cannot be more than 1. Thus this target cannot be hoped to be hit.

4.30. *A fighter plane flying horizontally at an altitude of 1.5 km with speed 720 km h^{-1} passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed 600 m s^{-1} to hit the plane? At what minimum altitude should the pilot fly the plane to avoid being hit? (Take g = 10 m s^{-2})?*

Sol. Velocity of plane,

$$v_p = 720 \times \frac{5}{18} \text{ ms}^{-1} = 200 \text{ ms}^{-1}$$

Velocity of shell = 600 ms^{-1};



$$\sin\theta = \frac{200}{600} = \frac{1}{3}$$

or $$\theta = \sin^{-1}\left(\frac{1}{3}\right)s = 19.47°$$

This angle is with the vertical.

Let h be the required minimum height.

Using equation

$$v^2 - u^2 = 2as, \quad \text{we get}$$

$$(0)^2 - (600\cos\theta)^2 = -2 \times 10 \times h$$

or, $$h = \frac{600 \times 600\left(1 - \sin^2\theta\right)}{20}$$

$$= 30 \times 600\left(1 - \frac{1}{9}\right) = \frac{8}{9} \times 30 \times 600 \text{ m} = 16 \text{ km}.$$

4.31. *A cyclist is riding with a speed of 27 km/h. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.50 m/s every second. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?*

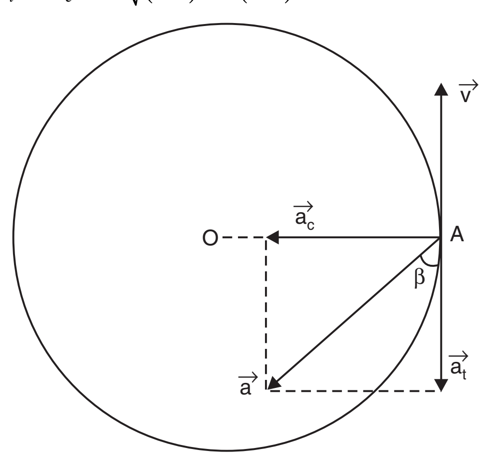
Sol. Here v = 27 km/h = $27 \times \frac{5}{18}$ m/s = 7.5 m/s, r = 80 m

and tangential acceleration $a_t = -0.50$ m/s^2

$\therefore$ Centripetal acceleration $a_c = \frac{v^2}{r} = \frac{(7.5)^2}{80}$ = 0.70 ms^{-2} (radially inwards).

Thus, as shown in fig. above, two accelerations are acting in mutually perpendicular directions. If $\vec{a}$ be the resultant acceleration, then

$$\left|\vec{a}\right| = \sqrt{a_t^2 + a_c^2} = \sqrt{(0.5)^2 + (0.7)^2} = 0.86 \text{ ms}^{-2}$$



and $$\tan\beta = \frac{a_c}{a_t} = \frac{0.7}{0.5} = 1.4$$

$\Rightarrow$ $\beta = \tan^{-1}(1.4) = 54.5°$ from the direction of negative of the velocity.

4.32. (*a*) *Show that for a projectile the angle between the velocity and the x-axis as a function of time is given by*

$$\theta(t) = tan^{-1}\left(\frac{v_{oy} - gt}{v_{ox}}\right)$$

(*b*) *Shows that the projection angle θ_0 for a projectile launched from the origin is given by*

$$\theta_0 = \tan^{-1}\left(\frac{4h_m}{R}\right)$$

where the symbols have their usual meaning.

Sol. (*a*) Let the projectile be fired at an angle θ with *x*-axis.

As θ depends on *t*, θ (*t*), at any instant

$$\tan\theta(t) = \frac{v_y}{v_x} = \frac{v_{oy} - gt}{v_{ox}} \qquad \text{(Since } v_y = v_{oy} - gt \text{ and } v_x = v_{ox}\text{)}$$

$$\Rightarrow \quad \theta(t) = \tan^{-1}\left(\frac{v_{oy} - gt}{v_{ox}}\right)$$

(*b*) Since, $$h_{\max} = \frac{u^2 \sin^2\theta}{2g}$$

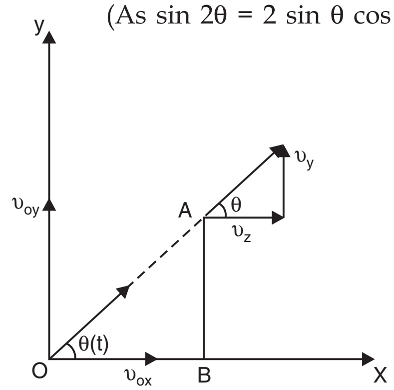
and $$R = \frac{u^2 \sin 2\theta}{g}$$

$$\Rightarrow \quad \frac{h_{\max}}{R} = \frac{u^2\sin^2\theta/2g}{u^2\sin 2\theta/g} = \frac{\tan\theta}{4} \qquad \text{(As } \sin 2\theta = 2\sin\theta\cos\theta\text{)}$$

$$\Rightarrow \quad \frac{\tan\theta}{4} = \frac{h_{\max}}{R}$$

or $$\tan\theta = \frac{4\,h_{\max}}{R}$$

or $$\theta = \tan^{-1}\left(\frac{4\,h_{\max}}{R}\right).$$



ADDITIONAL QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *Can three non-coplanar vectors give zero resultant? Why?*

Ans. No, since resultant of two vectors lie in the same plane.

Q. 2. *What is the angle between $\vec{P} \times \vec{Q}$ and $\vec{Q} \times \vec{P}$?*

Ans. The given vectors are anti-parallel.

$\therefore \quad \theta = 180°$

Q. 3. *If both speed of a body and radius of the circular path are doubled, what will be the change in centripetal force?*

Ans. Since, Centripetal force, $f = \frac{mv^2}{r}$

If speed and radius are doubled it gets doubled.

Q. 4. *Express the unit vector A in mathematical form.*

Ans. $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$

Q. 5. *If* $|\vec{A} \times \vec{B}| = \vec{A} \cdot \vec{B}$*, what is the angle between* $\vec{A}$ *and* $\vec{B}$*?*

Ans. $|\vec{A} \times \vec{B}| = \vec{A} \cdot \vec{B}$

$$AB \sin \theta = AB \cos \theta$$

∴ $\tan \theta = 1$

or $\theta = 45°$

Q. 6. *Under what condition is the magnitude of sum of two vectors minimum?*

Ans. When the two vectors are in mutually opposite directions.

Q. 7. *Under what condition is the magnitude of sum of two vectors maximum?*

Ans. When angle between $\vec{A}$ and $\vec{B}$ is zero *i.e.,* two vectors are in the same direction.

Q. 8. *Two bodies are projected at angle θ and (90° – θ) to the horizontal with the same speed. Find the ratio of their time of flight.*

Ans. $T_1 = \frac{2u \sin\theta}{g}$ and $T_2 = \frac{2u \sin(90° - \theta)}{g}$

Now, $\frac{T_1}{T_2} = \frac{2u \sin\theta}{g} \times \frac{g}{2u \sin(90° - \theta)}$

$= \sin \theta / \cos \theta = \tan \theta.$

Q. 9. *What is the angle of projection at which the* h_{max} *and range are equal?*

Ans. $\frac{u^2 \sin^2\theta}{2g} = \frac{u^2 \sin 2\theta}{g}$

$\sin^2 \theta = 2 \cdot 2 \sin \theta \cos \theta$ [∵ $\sin 2\theta = 2 \sin \theta \cos \theta$]

$\sin \theta = 4 \cos \theta$

or, $\tan \theta = 4$

$\theta = \tan^{-1} (4).$

Q. 10. *What conclusion do you draw about* $\vec{B}$*, if* $\vec{A} - \vec{B} = \vec{A} + \vec{B}$ *?*

Ans. The identity is possible only if $\vec{B} = \vec{0}$ *i.e.,* $\vec{B}$ is a null vector.

Q. 11. *Is* A^2 *scalar or a vector? Justify your answer.*

Ans. A^2 is a scalar. This is because it is the dot product of $\vec{A}$ and $\vec{A}$.

Q. 12. *If two bodies have circular path of radius* r_1 *and* r_2 *and the time taken are the same, find the ratio of the angular speed.*

Ans. Since time taken is same and $\omega = \frac{2\pi}{T}$

Thus, $\omega_1 : \omega_2 = 1 : 1.$

Q. 13. *What is the value of 'm' in* $\hat{i} + m\hat{j} + \hat{k}$ *to be unit vector?*

Ans. For unit vector $\left| \hat{i} + m\hat{j} + \hat{k} \right| = \sqrt{1 + m^2 + 1} = 0$

$\Rightarrow \quad m^2 = -2 \quad$ (m is imaginary)

Q. 14. *A body is thrown horizontally with a velocity v from a tower μ meter high. After how much time and at what distance from the base of the tower will the body strike the ground?*

Ans. Time, $t = \sqrt{2H/g}$

Horizontal distance, $x = vt = v\sqrt{2H/g}$.

Q. 15. *Can a component of a given vector be greater than the vector itself?*

Ans. Yes. For example, consider a vector $\vec{R} = (\vec{A} + \vec{B})s$, where angle θ between $\vec{A}$ and $\vec{B}$ is 180°. In such a case, the resultant vector $\vec{R}$ will be smaller than at least one of its components.

Q. 16. *For what angle between* $\vec{P}$ *and* $\vec{Q}$*, the value of* $\vec{P} + \vec{Q}$ *is maximum?*

Ans. $\left| \vec{P} + \vec{Q} \right| = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

clearly $\left| \vec{P} + \vec{Q} \right|$ is maximum when $\cos\theta = 1$

i.e., when $\theta = 0°$.

Q. 17. *What is the minimum number of coplanar vectors of different magnitudes, which may give zero resultant?*

Ans. Three. Their resultant will be zero provided they can be represented by three sides of a triangle taken in order.

Q. 18. *What is the maximum vertical height to which a baseball player can throw a ball if he can throw it to a maximum horizontal distance of 100 m?*

Ans. Maximum horizontal distance = 100 m = $\frac{v^2}{g}$

Maximum vertical height = $\frac{v^2}{2g}$

$$= \frac{1}{2} \times 100 = 50 \text{ m.}$$

Q. 19. *What is the angle between velocity vector and acceleration vector in uniform circular motion?*

Ans. 90°.

Q. 20. *A body is projected with a speed u at an angle to the horizontal to have maximum range. What is its velocity at the highest point?*

Ans. For range to be maximum θ = 45°. At the highest point, the vertical component velocity is zero but the horizontal component velocity = $u \cos\theta = u \cos 45° = \frac{u}{\sqrt{2}}$.

Q. 21. *A cannon on a level plane is aimed at an angle β above the horizontal and a shell is fired with a muzzle velocity v towards a vertical cliff a distance x away. At what height y from the bottom the shell would hit the cliff?*

Ans. $y = x \tan \beta - \dfrac{gx^2}{2v^2 \cos^2 \beta}$.

Q. 22. *Can a vector be zero if one of its components is non-zero?*

Ans. A vector cannot be zero if one of its components is non-zero.

Q. 23. *Can a vector be non-zero if one of its components is zero?*

Ans. A vector can be non-zero even if one of its components is zero.

Q. 24. *What will be the effect on the horizontal range of a projectile when its initial speed is doubled keeping its angle of projection same?*

Ans. As $R \propto u^2$

Thus range will become 4 times.

Q. 25. *The velocity at the maximum height of a projectile is half its initial velocity of projection u. Find its range on the horizontal plane.*

Ans. $u \cos \theta = \dfrac{u}{2}$

$$\Rightarrow \quad \cos \theta = \frac{1}{2} \quad \Rightarrow \quad \theta = 60°$$

As, Horizontal range $= R = \dfrac{u^2 \sin 2\theta}{g}$

$$\Rightarrow \quad R = \frac{u^2 \sin 2 \times 60°}{g} = \frac{u^2 \sin 120°}{g} = \frac{\sqrt{3}\, u^2}{2g}.$$

Q. 26. *Give an example where centripetal acceleration is acting.*

Ans. Earth revolving around the sun. Here, centripetal acceleration is being provided by gravitational attraction between the sun and the Earth.

Q. 27. *A projectile of mass m is fired with velocity v at an angle θ with the horizontal. What is the change in momentum as it rises to the highest point of the trajectory?*

Ans. Change in momentum

$$= mg \times \frac{v \sin \theta}{g} = mv \sin \theta.$$

Q. 28. *Is the maximum height attained by projectile largest when its horizontal range is maximum?*

Ans. No; horizontal range is maximum when θ = 45° and maximum height attained by projectile is largest when θ = 90°.

Q. 29. *Name two physical quantities which remain constant for a particle in uniform circular motion.*

Ans. Speed and kinetic energy.

Q. 30. *At which point of the trajectory is the speed of motion minimum?*

Ans. Minimum speed is at the highest point, since the vertical component of velocity is zero.

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *If R be the horizontal range for inclination θ and h be the maximum height reached by the projectile, show that maximum range is given by* $\frac{R^2}{8h} + 2h$.

Ans. Horizontal range of the projectile is

$$R = \frac{u^2 \sin 2\theta}{g}$$

Maximum height attained by the projectile is

$$h = \frac{u^2 \sin^2 \theta}{2g}$$

$$\therefore \frac{R^2}{8h} + 2h$$

$$= \frac{u^2 (\sin 2\theta)^2}{g^2} \times \frac{2g}{8u^2 \sin^2 \theta} + \frac{2u^2 \sin^2 \theta}{2g}$$

$$= \frac{u^2 (2 \sin \theta \cos \theta)^2}{4g \sin^2 \theta} + \frac{u^2 \sin^2 \theta}{g}$$

$$= \frac{u^2 \cos^2 \theta}{g} + \frac{u^2 \sin^2 \theta}{g}$$

$$= \frac{u^2}{g}\left(\cos^2 \theta + \sin^2 \theta\right) = \frac{u^2}{g} = R_{max}.$$

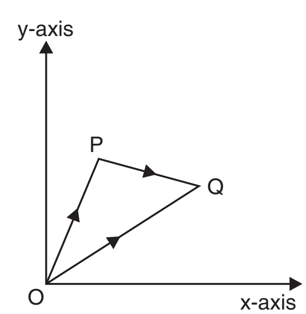
Q. 2. *If the position vectors of P and Q be respectively* $\left(\hat{i} + 3\hat{j} - 7\hat{k}\right)$ *and* $\left(5\hat{i} - 2\hat{j} + 4\hat{k}\right)$, *find* $\overrightarrow{PQ}$.

Ans. Let O be the origin

Given $\overrightarrow{OP} = \hat{i} + 3\hat{j} - 7\hat{k}$

$\overrightarrow{OQ} = 5\hat{i} - 2\hat{j} + 4\hat{k}$

By triangle law of vector addition,



$$\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$$

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$

$$= \left(5\hat{i} - 2\hat{j} + 4\hat{k}\right) - \left(\hat{i} + 3\hat{j} - 7\hat{k}\right)$$

$$= \left(4\hat{\boldsymbol{i}} - 5\hat{\boldsymbol{j}} + 11\hat{\boldsymbol{k}}\right)$$

Q. 3. *Derive a relation for the time taken by a projectile to reach the highest point and the maximum height attained.*

Ans. Consider a projectile projected at an angle θ to the horizontal with velocity u, the horizontal and vertical components initially with velocity $u \cos \theta$ and $u \sin \theta$ respectively. Vertical velocity at highest point is zero, due to gravity acting vertically downwards.

Using, $v = u + at$

we have, $0 = u \sin\theta - gt \Rightarrow t = \dfrac{u \sin\theta}{g}$

The time to reach topmost point, $t = \dfrac{u \sin\theta}{g}$

Using $v^2 = u^2 + 2as$

We have, $0 = u^2 \sin^2\theta - 2g\,h_{max} \Rightarrow h_{max} = \dfrac{u^2 \sin^2\theta}{2g}$.

Q. 4. *Two bodies are thrown with the same initial velocity at angles α and $(90° - \alpha)$ with the horizontal. What will be the ratio of (i) maximum heights attained by them and (ii) of horizontal ranges?*

Ans. Horizontal range, $R = \dfrac{u^2}{g}\sin 2\theta$ and Max. height, $H = \dfrac{u^2 \sin^2\theta}{2g}$

Case (i) when $\theta = \alpha$; $R_1 = \dfrac{u^2}{g}\sin 2\alpha$ and $H_1 = \dfrac{u^2 \sin^2\alpha}{2g}$

Case (ii) When $\theta = (90° - \alpha)$;

$$R_2 = \frac{u^2 \sin 2(90° - \alpha)}{g} = \frac{u^2 \sin 2\alpha}{g}$$

and
$$H_2 = \frac{u^2 \sin^2(90 - \alpha)}{g} = \frac{u^2 \cos^2\alpha}{g}$$

$$\therefore \quad \frac{H_1}{H_2} = \frac{\sin^2\alpha}{\cos^2\alpha} = \tan^2\alpha \quad \text{and} \quad \frac{R_1}{R_2} = 1.$$

Q. 5. *A bullet P is fired from a gun when the angle of elevation of the gun is 30°. Another bullet Q is fired from the gun when the angle of elevation is 60°. The vertical height attained in the second case is x times the vertical height attained in the first case. What is the value of x?*

Ans.
$$h_2 = \frac{u^2 \sin^2 60°}{2g} = \frac{3u^2}{8g} \quad \text{and} \quad h_1 = \frac{u^2 \sin^2 30°}{2g} = \frac{u^2}{8g}$$

Clearly, $h_2 = 3\,h_1$

$\therefore$ $x = 3$.

Q. 6. *A person aims a gun at a bird from a point at a horizontal distance of 100 m. If the gun can impart a speed of 500 ms^{-1} to the bullet, at what height above the bird must he aim his gun in order to hit it?*

Ans. Horizontal distance, $x = 100$ m

velocity, $v = 500$ ms^{-1}

Time taken to travel this distance, $t = \dfrac{x}{v} = \dfrac{100}{500} = \dfrac{1}{5}$ s

Vertical distance travelled by the bullet in time $\dfrac{1}{5}$s is

$$y = u_{oy}t + \frac{1}{2}gt^2 = 0 + \frac{1}{2} \times 10 \times \frac{1}{25} = \frac{1}{5}\text{ m} = 20\text{ cm}.$$

If he directly aims at the bird, the bullet will hit 20 cm below the bird.

Therefore, the gun must be aimed at 20 cm above the position of the bird.

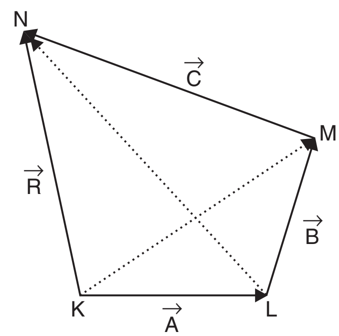
Q. 7. *Show that the vector addition is associative.*

Or

Show that $(\vec{A}+\vec{B})+\vec{C} = \vec{A}+(\vec{B}+\vec{C})$.

Ans. To show that vector addition is associative, we consider addition of three vectors $\vec{A}, \vec{B}$ and $\vec{C}$ in two different manners. Let us first add $\vec{A}$ and $\vec{B}$ to obtain a vector $\overrightarrow{KM}$ and then add $\vec{C}$ to it so as to get the resultant vector $\overrightarrow{KN}$. It means that

$$(\vec{A}+\vec{B})+\vec{C} = \overrightarrow{KN} = \vec{R} \quad ...(i)$$



Again we add $\vec{B}$ and $\vec{C}$ to obtain a vector $\overrightarrow{LN}$. Now to vector $\vec{A}$ add $\overrightarrow{LN}$ so as to get a resultant $\overrightarrow{KN} = \vec{R}$ as shown in Fig. It means that

$$(\vec{A})+(\vec{B}+\vec{C}) = \overrightarrow{KL}+\overrightarrow{LN} = \overrightarrow{KN} = \vec{R} \quad ...(ii)$$

From (*i*) and (*ii*), it is clear that

$$(\vec{A}+\vec{B})+\vec{C} = \vec{A}+(\vec{B}+\vec{C})$$

That is the vector addition is associative.

Q. 8. *Which is greater; the angular velocity of the hour hand of a watch or angular velocity of earth around its own axis?*

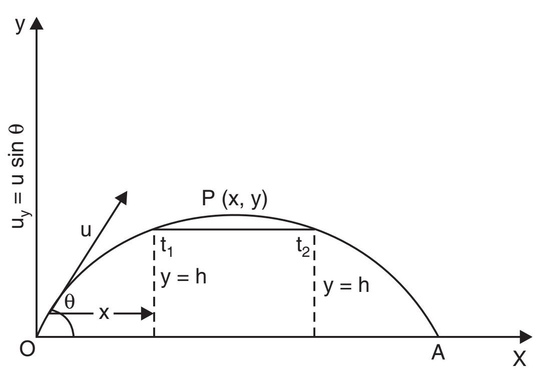
Ans. Time period for hour hand of watch, T_h = 12 h

and for earth, T_e = 24 h

Now, Angular velocity, $\omega = \dfrac{2\pi}{T}$

$$\therefore \quad \frac{\omega_h}{\omega_e} = \frac{T_e}{T_h} = \frac{24}{12} = 2 \quad \text{or} \quad \omega_h = 2\,\omega_e.$$

Q. 9. *Show that there are two values of time for same height during the course of flight of a projectile and the sum of these times is equal to the total time of flight.*



Ans. We know, the vertical distance travelled by a projectile in time t is given by,

$$y = u \sin\theta \times t - \frac{1}{2}gt^2$$

If h be the height of point p, then for $y = h$,

we have $$h = u \sin\theta \times t - \frac{1}{2}gt^2$$

or $$\frac{1}{2}gt^2 - u\sin\theta \times t + h = 0 \quad \text{or} \quad t^2 - \frac{2u\sin\theta}{g}\cdot t + \frac{2h}{g} = 0$$

This equation is quadratic in t and has two roots t_1 and t_2. Thus there are two values of time for which the height of the projectile is same during flight of projectile.

$$t = \frac{\frac{2u\sin\theta}{g} \pm \sqrt{\left(\frac{2u\sin\theta}{g}\right)^2 - \frac{8h}{g}}}{2}$$

$$t = \frac{u\sin\theta}{g} \pm \sqrt{\frac{u^2\sin^2\theta}{g^2} - \frac{2h}{g}}$$

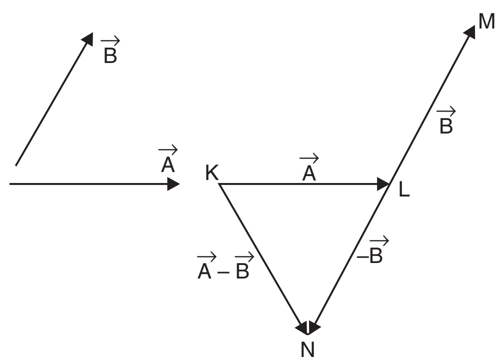
$$\therefore \quad t_1 = \frac{u\sin\theta}{g} + \sqrt{\frac{u^2\sin^2\theta}{g^2} - \frac{2h}{g}} \quad \text{and} \quad t_2 = \frac{u\sin\theta}{g} - \sqrt{\frac{u^2\sin^2\theta}{g^2} - \frac{2h}{g}}$$

Now, $t_1 + t_2 = \frac{2u\sin\theta}{g}$ (Time during the course of flight).

Thus, the sum of times for the same height is equal to the total time of flight.

Q. 10. *Briefly discuss subtraction of vectors.*

Ans. Subtraction of vectors is a special case of vector addition. Subtraction of $\vec{B}$ from $\vec{A}$ may be considered as addition of $(-\vec{B})$ *i.e.,* negative of $\vec{B}$ to vector $\vec{A}$.



$$\therefore \quad \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

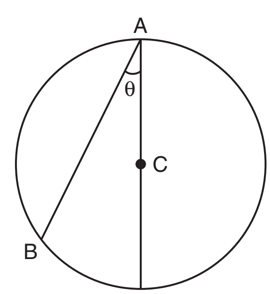
Therefore, we first draw $\vec{A}$ and $\vec{B}$. Now draw a vector having same magnitude as of $\vec{B}$ but in opposite direction. It is $(-\vec{B})$. Sum of $\vec{A}$ and $(-\vec{B})$s gives the requisite result.

In Fig.

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) = \overrightarrow{KL} + (-\overrightarrow{LM}) = \overrightarrow{KL} + \overrightarrow{LN} = \overrightarrow{KN}.$$

Q. 11. *A glass marble slides from rest from the topmost point of a vertical circle of radius r along a smooth chord. Does the time of descent depend upon the chord chosen?*

Ans. Consider a chord AB which makes an angle θ with the vertical. C is the centre of the circle. Let t be the time taken by the glass marble to slide along AB.



Then, $AB = 0 \times t + \frac{1}{2} g \cos\theta \times t^2$

or $t^2 = \frac{AB}{g\cos\theta}$

$= \frac{2 \times 2r\cos\theta}{g\cos\theta} = \frac{4r}{g}$

or $t = 2\sqrt{r/g}$

Clearly, the time of descent does not depend upon θ. In other words, the time of descent does not depend upon the choice of chord.

Q. 12. *A body is projected in horizontal direction with a uniform velocity from top of tower. Show that the path is parabola.*

Ans. Let the body be projected horizontally with a velocity u, from the top of a tower of height h.

Time taken to reach the ground, $t = \sqrt{2h/g}$.

Since the initial vertical velocity is zero and there is no acceleration in the horizontal.

Thus, $x = ut$

$$x = u\sqrt{2h/g} \quad i.e., \quad h = x^2 \cdot \frac{g}{2u^2}$$

As $h \propto x^2$, the path is a parabola.

Q. 13. *There are two displacement vectors, one of magnitude 3 metres and the other of 4 metres. How would the two vectors be added so that the magnitude of the resultant vector be (a) 7 metres (b) 1 metre and (c) 5 metres?*

Ans. The magnitude of resultant $\vec{R}$ of two vectors $\vec{A}$ and $\vec{B}$ is given by

$$R = \sqrt{A^2 + B^2 + 2\,AB\cos\theta}\,; \quad \text{where} \quad A = 3 \text{ m and } B = 4\text{m}.$$

Now $$R = \sqrt{3^2 + 4^2 + 2 \times 3 \times 4 \cos\theta}.$$

(*i*) R will be 7 m if $\theta = 0°$ (*ii*) R will be 1 metre if $\theta = 180°$ and (*iii*) R will be 5 m if $\theta = 90°$.

Q. 14. *Can any of the rectangular components of a given vector have magnitude greater than the vector itself? Explain.*

Ans. **No ;** The rectangular components of a vector $\vec{A}$ has values $A\cos\theta$ and $A\sin\theta$. Since the values of $\cos\theta$ and $\sin\theta$ can never be greater than one, hence the value of any rectangular components of a vector can never be greater than the given vector.

Q. 15. *The time of flight of an object projected at an angle θ is T. What will be its time of flight, if the object be projected at an angle (90 – θ)°?*

Ans. When an object is projected with an initial velocity u at an angle θ with horizontal, its time of flight is given by

$$T = \frac{2u\sin\theta}{g} \qquad ...(i)$$

When the same object is projected with the same initial velocity at an angle $(90 - \theta)°$, the time of flight is given by

$$T' = \frac{2u}{g}\sin(90 - \theta) = \frac{2u}{g}\cos\theta \qquad ...(ii)$$

Dividing (*ii*) by (*i*), we find that

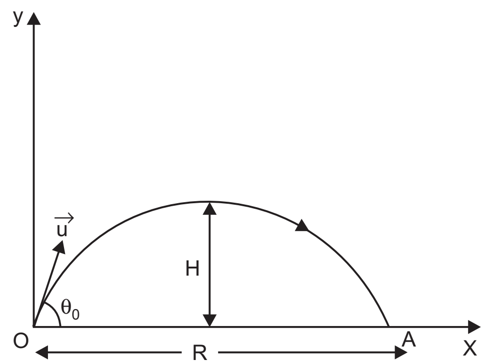
$$\frac{T'}{T} = \frac{\cos\theta}{\sin\theta} = \cot\theta \quad \Rightarrow \quad T' = T\cot\theta.$$

Q. 16. *Show that the projection angle θ_0 for a projectile launched from the origin is given by*

$$\theta_0 = \tan^{-1}\left(\frac{4H}{R}\right)$$

where, H is the maximum height attained by the projectile and R is the range of the projectile. *(3 marks)*

Ans. The path followed by a projectile projected at an angle θ_0 with velocity $\vec{u}$ is shown in figure. The maximum height attained by the projectile is given by



$$H = \frac{u^2 \sin^2 \theta_0}{2g} \qquad ...(i)$$

The range of the projectile is given by

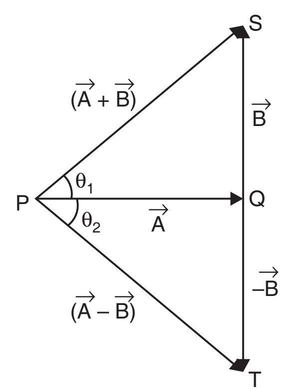
$$R = \frac{u^2 \sin 2\theta_0}{g} = \frac{2u^2 \sin\theta_0 \cos\theta_0}{g} \qquad ...(ii)$$

Dividing eqn. (*i*) by eqn. (*ii*), we get

$$\tan \theta_0 = \frac{4H}{R} \Rightarrow \theta_0 = \tan^{-1}\left(\frac{4H}{R}\right).$$

Q. 17. *Two vectors $\vec{A}$ and $\vec{B}$ are of equal lengths (A = B) and mutually perpendicular. Show by vector diagram that their vector sum $\left(\vec{A}+\vec{B}\right)$s and vector difference $\left(\vec{A}-\vec{B}\right)$ will be of the same length and mutually perpendicular.*

Ans. Draw $\left(\overrightarrow{PQ}\right) = \vec{A}$ and from the arrow head of $\vec{A}$, draw $\left(\overrightarrow{QS}\right) = \vec{B}$, of the same length (*i.e., QS = PQ*) and perpendicular to $\vec{A}$ Fig. Now $\left(\overrightarrow{PS}\right)$ will represent $\left(\vec{A}+\vec{B}\right)$. Here,



$$\tan \theta_1 = \frac{QS}{PQ} = 1 \quad \text{or} \quad \theta_1 = 45°$$

Now draw $\left(\overrightarrow{QT}\right) = -\vec{B}$, where $QT = QS$. Now $\left(\overrightarrow{PT}\right)$ will represent $\left(\vec{A}-\vec{B}\right)$. Here,

$$\tan \theta_2 = \frac{QT}{PQ} = 1 \quad \text{or} \quad \theta_2 = 45°.$$

On measuring, the lengths of $\left(\vec{A}+\vec{B}\right)$ and $\left(\vec{A}-\vec{B}\right)$ come out to be the same and angle between them $(\theta_1 + \theta_2) = 45° + 45° = 90°$.

Q. 18. *A ball is thrown from a point with a speed v_0 at an angle of projection θ. From the same point and at the same instant, a person starts running with a constant speed $\frac{v_0}{2}$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection?*

Ans. Yes. The person will be able to catch the ball if the horizontal component of the velocity of the ball is equal to the speed of the person.

$$\therefore \qquad u_0 \cos \theta = \frac{v_0}{2} \Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = 60°.$$

Q. 19. *A particle is projected with a velocity of 40 m s^{-1}. After 2s it just crosses a vertical pole of height 20 m. Calculate the angle of projection and the horizontal range.* *(3 marks)*

Ans. It is given that $u = 40$ m s^{-1}, vertical distance travelled in 2s is 20 m.

Using the relation $y = ut \sin \theta - \frac{1}{2} g t^2$, we have

$$20 = 40 \sin \theta \times 2 - \frac{1}{2} \times 10 \times (2)^2 = 80 \sin \theta - 20$$

$$\Rightarrow \quad 80 \sin \theta = 20 + 20 = 40 \quad \Rightarrow \quad \sin \theta = \frac{40}{80} = \frac{1}{2}$$

$$\therefore \quad \theta = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

$\therefore$ Horizontal range of projectile, $R = \dfrac{u^2 \sin 2\theta}{g}$

$$= \frac{(40)^2 \times \sin(2 \times 30°)}{10} = \frac{1600 \times \sin 60°}{10} = 160 \times \frac{\sqrt{3}}{2} = 138.6 \text{ m.}$$

Q. 20. *Prove that the vectors $\vec{A} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{B} = \hat{i} + \hat{j} + \hat{k}$ are mutually perpendicular.*

Ans.

$$\vec{A} \cdot \vec{B} = \left(2\hat{i} - 3\hat{j} + \hat{k}\right) \cdot \left(\hat{i} + \hat{j} + \hat{k}\right)$$

$$AB \cos \theta = (2)(1) + (-3)(1) + (1)(1) = 0$$

$$AB \cos \theta = 0 \quad (\text{as } A \neq 0, B \neq 0)$$

$$\therefore \quad \theta = 90°$$

or, the vectors $\vec{A}$ and $\vec{B}$ are mutually perpendicular.

Q. 21. *A fighter jet makes a loop of 1000 m with a speed of 250 m s^{-1}. Compare its centripetal acceleration with the acceleration due to gravity.*

Ans. Here $r = 1000$ m

$v = 250$ m s^{-1}

Centripetal acceleration is given by

$$a = \frac{v^2}{r} = \frac{250 \times 250}{1000} = 62.5 \text{ m s}^{-2}$$

Acceleration due to gravity, $g = 9.8$ m s^{-2}

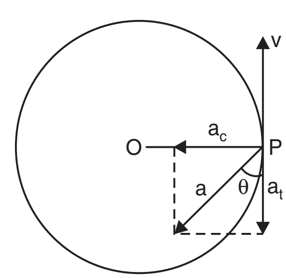
$$\therefore \quad \frac{\text{Centripetal acceleration}}{\text{Acceleration due to gravity}} = \frac{62.5}{9.8} = 6.4$$

Q. 22. *A cyclist moving with a velocity of 7.5 m s^{-1} approaches a U-turn of radius 80 m. He applies brakes to slow down his speed at a rate of 0.5 ms^{-2}. Calculate the acceleration of the cyclist on the turn.*

Ans. $v = 7.5$ m s^{-1}, $r = 80$ m

Centripetal acceleration is

$$a_c = \frac{v^2}{r} = \frac{7.5 \times 7.5}{80} = 0.7 \text{ m s}^{-2}$$



When the cyclist applies brakes at P of the circular turn, then the tangential acceleration will act opposite to the velocity

i.e., $a_t = 0.5$ m s^{-2}

$\therefore$ Net acceleration, $a = \sqrt{a_c^2 + a_t^2} = \sqrt{(0.7)^2 + (0.5)^2} = 0.86 \text{ m s}^{-2}$

Let θ be the angle made by net acceleration with the velocity of the cyclist, then

$$\tan\theta = \frac{a_c}{a_t} = \frac{0.7}{0.5} = 1.4$$

$\therefore$ $\theta = \tan^{-1}(1.4) = 54° - 27'$

Q. 23. *A man can jump on moon about six times as high as on the earth. Why?*

Ans. We know that value of g on surface of moon is nearly $\frac{1}{6}$th of its value at earth. We also know that maximum height which a man can cover is given by $h = \frac{u^2 \sin^2\theta}{2g}$

i.e., $h \propto \frac{1}{g}$

Therefore, $\frac{h_{moon}}{h_{earth}} = \frac{g_{earth}}{g_{moon}} = \frac{g}{g/6} = 6$ or $h_{moon} = 6\, h_{earth}$.

Q. 24. *What would be the effect on a vector if all its components are reversed in direction?*

Ans. Consider a vector $\vec{F}$.

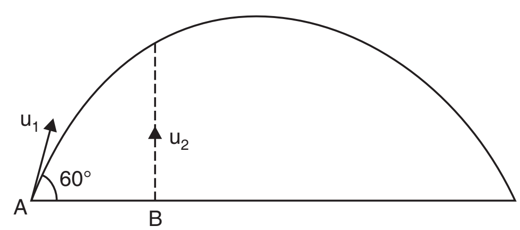
$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

When x, y and z components are reversed, we get

$$F_x(-\hat{i}) + F_y(-\hat{j}) + F_z(-\hat{k}) = -\left[F_x\hat{i} + F_y\hat{j} + F_z\hat{k}\right] = -\vec{F}$$

Therefore, the vector itself is reversed.

Q. 25. *A body is projected with the velocity u_1 from the point A as shown in figure. At the same time another body is projected vertically upwards with the velocity u_2 from the point B. What should be the value of u_1/u_2 for both the bodies to collide?*



Ans. The two bodies will collide, if they reach at a point acquiring the same vertical distance in the same time. Therefore

$$y = u_1 \sin 60° \times t - \frac{1}{2}gt^2 = u_2 t - \frac{1}{2}gt^2$$

$$\Rightarrow \quad u_1 \sin 60° \times t = u_2\, t \quad \text{or} \quad \frac{u_1}{u_2} = \frac{1}{\sin 60°} = \frac{2}{\sqrt{3}}$$

Q. 26. *The equation of trajectory of an oblique projectile is*

$$y = \sqrt{3}\, x - \frac{gx^2}{2}$$

What is the initial velocity and the angle of projection of the projectile?

Ans. Comparing the given equation with the standard equation of the trajectory of an oblique projectile,

$$y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$$

we get $\tan\theta = \sqrt{3} \Rightarrow \theta = 60°$

Also, $v^2\cos^2\theta = 1$

$$\Rightarrow \quad v^2 = \frac{1}{\cos^2\theta} \quad \text{or} \quad v^2 = \frac{1}{\cos^2 60°} = 4 \quad \Rightarrow \quad v = 2 \text{ m s}^{-1}.$$

Q. 27. *Establish a relation between angular velocity and time period.*

Ans. We know that angular velocity $\omega = \dfrac{\Delta\theta}{\Delta t}$.

For motion with uniform angular velocity, in one complete revolution $\Delta\theta = 2\pi$ radian and $\Delta t = T$ s, hence

$$\omega = \frac{2\pi}{T} \quad \text{or} \quad T = \frac{2\pi}{\omega}.$$

Q. 28. *Define centripetal acceleration. Give two examples.*

Ans. Acceleration needed for a particle to undergo uniform circular motion is called 'centripetal acceleration'. It is directed along the radius of circular path towards its centre. Two common examples are :

(*i*) An electron revolving around the nucleus of an atom in a uniform circular motion experiences a centripetal acceleration on account of Coulombian electrostatic force on electron due to nucleus.

(*ii*) A satellite revolving around the earth in a circular orbit experiences a centripetal acceleration on account of gravitational force due to the earth.

Q. 29. *Calculate the angular speed of the seconds hand of a clock. If the length of the seconds hand is 4 cm, calculate the speed of the tip of the seconds hand.*

Ans. Seconds hand of a clock completes one rotation in 60 s *i.e.*

$$T = 60 \text{ s}, \quad \theta = 2\pi \text{ rad}$$

$\therefore$ Angular speed, $\omega = \dfrac{\theta}{T} = \dfrac{2\pi \text{ rad}}{60 \text{ s}} = \dfrac{\pi}{30} \text{ rad s}^{-1}$

Length of the seconds hand, $R = 4$ cm.

$\therefore$ Speed of the tip of second's hand is

$$v = \omega R = \frac{\pi}{30} \times 4 = \frac{2\pi}{15} \text{ cm s}^{-1}.$$

Q. 30. *Two particles located at a point begin to move with velocities 4 ms^{-1} and 1 ms^{-1} horizontally in opposite directions. Determine the time when their velocity vectors become perpendicular. Assume that the motion takes place in a uniform gravitational field of strength g.*

Ans. Velocity of first particle at time $t = 4\hat{i} - gt\,\hat{j}$

Velocity of second particle at time $t = -\hat{i} - gt\,\hat{j}$

Since the dot product of two perpendicular vectors is zero therefore

$$\left(4\hat{i} - gt\,\hat{j}\right)\cdot\left(-\hat{i} - gt\,\hat{j}\right)s = 0 \quad \Rightarrow \quad -4 + g^2 t^2 = 0$$

or $\quad g^2 t^2 = 4 \quad$ or $\quad gt = 2$

$\Rightarrow \quad t = \dfrac{2}{g}.$

III. LONG ANSWER TYPE QUESTIONS

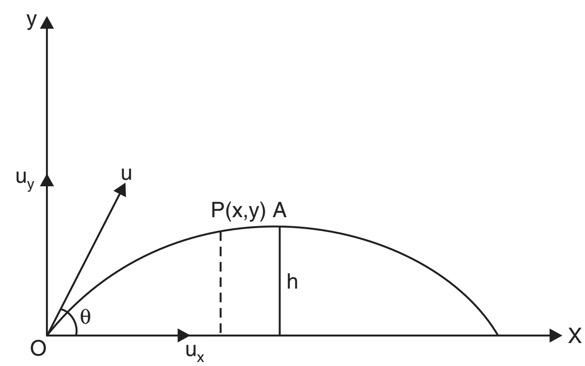
Q. 1. *A body is projected with some initial velocity making an angle θ with the horizontal. Show that its path is a parabola. Find the maximum height attained, time for maximum height, horizontal range, maximum horizontal range and the time of flight.*

Ans. Let the body be projected with velocity u inclined at angle θ with the horizontal. The horizontal and vertical components of velocity and acceleration are

$$u_x,\ a_x \quad \text{and} \quad u_y,\ a_y$$

where $\quad u_x = u\cos\theta, \quad u_y = u\sin\theta, \quad a_x = 0, \quad a_y = -g$

g is the acceleration due to gravity.



The coordinates of O are (0, 0) considering horizontal motion.

The position of the body after time t has coordinate (x, y);

where $\quad x(t) = x_0 + u_x t + \dfrac{1}{2} a_x t^2$

Substituting for various factors

$$x(t) = 0 + u\cos\theta\cdot t + \frac{1}{2}\times 0\times t^2 \quad \text{or} \quad x(t) = u\cos\theta\cdot t$$

$$t = \frac{x(t)}{u\cos\theta} \qquad ...(i)$$

Considering the vertical motion

$$y(t) = y(0) + u_y + \frac{1}{2} a_y t^2$$

or $\quad y(t) = 0 + u\sin\theta\cdot t - \dfrac{1}{2} gt^2$

or $\quad y(t) = u\sin\theta\cdot t - \dfrac{1}{2} gt^2 \qquad ...(ii)$

Substituting for t from equation (*i*) in equation (*ii*), we get

$$y(t) = u \sin\theta \left(\frac{x(t)}{u\cos\theta}\right) - \frac{1}{2} g \left(\frac{x(t)}{u\cos\theta}\right)^2 s$$

$$\Rightarrow \quad y(t) = x(t)\tan\theta - \frac{1}{2} g \left(\frac{x^2(t)}{u^2\cos^2\theta}\right) \qquad ...(iii)$$

This is an equation of parabola. Thus, the path of a projectile is a parabola.

Maximum height attained. At the maximum height, the vertical component of velocity becomes zero. Now using the equation of motion.

$$h = \frac{v_y^2 - u_y^2}{2a_y}$$

We have maximum height

$$\therefore \quad h_{max} = \frac{0^2 - (u\sin\theta)^2}{2(-g)} \quad \text{or} \quad H = \frac{u^2\sin^2\theta}{2g} \qquad ...(iv)$$

Time for maximum height. Using equation of motion $v = u + at$

or $\quad v_x = u_x + a_y t$

we have $\quad 0 = u \sin\theta - gt \quad \text{or} \quad t = \frac{u\sin\theta}{g} \qquad ...(v)$

Horizontal range. Let the horizontal range be x. Since there is no acceleration in the horizontal direction so

$$x = x(0) + u_x t + \frac{1}{2} a_x t^2$$

As $x(0) = 0$, $u_x = u\cos\theta$, $a_x = 0$ and it is the total time of the flight which is twice the time for maximum height because body takes same time in rising to and falling from the highest point.

Hence, $\quad t = \frac{2u\sin\theta}{g}$

$$\therefore \quad x = 0 + u\cos\theta \cdot t = u\cos\theta\left(\frac{2u\sin\theta}{g}\right)$$

or $\quad x = \frac{u^2}{g}(2\sin\theta\cos\theta) \quad \Rightarrow \quad x = \frac{u^2}{g}\sin 2\theta \qquad ...(vi)$

Maximum horizontal range. From equation (*vi*) for x to be maximum, the value of $\sin 2\theta$ should be maximum which is 1,

hence $\quad x_{max} = \frac{u^2}{g} \qquad ...(vii)$

For this x_{max}, $\quad \sin 2\theta = 1 \quad \Rightarrow \quad \theta = 45°$

Therefore, the horizontal range will be maximum if the angle of projection is 45° or $\frac{\pi}{4}$ radians.

Time of flight of the projectiles. The projectile after completing its flight returns back to the same horizontal level from which it was projected. Therefore, the vertical displacement in the whole flight is zero. Considering vertical motion.

$$y(t) = y(0) + u_y t + \frac{1}{2} a_y t^2$$

Now $\quad y(t) = 0, \quad y(0) = 0, \quad u_y = u \sin\theta, \quad a_y = -g$

Then $\quad 0 = 0 + u \sin\theta \cdot T - \frac{1}{2} g T^2$

$$\Rightarrow \quad T\left(u \sin\theta - \frac{1}{2} gT\right)s = 0$$

Therefore $\quad T = 0$

and $\quad u \sin\theta - \frac{1}{2} gT = 0 \Rightarrow u \sin\theta = \frac{1}{2} gT$

or $\quad gT = 2u \sin\theta$

$$T = \frac{2u \sin\theta}{g} \qquad ...(viii)$$

Equation (*viii*) gives the total time of flight. This is twice the time for maximum height.

Q. 2. *Mathematically describe the motion of a particle along a plane (or along a two-dimensional plane).*

Ans. Consider motion of a particle in x-y plane.

Displacement. Let the particle be at position A having position vector $\vec{r_1}$ at time t_1 where $\vec{r_1} = \left(x_1\hat{i} + y_1\hat{j}\right)$. At time t_2, the object reaches point B having position vector $\vec{r_2}$ where $\vec{r_2} = \left(x_2\,\hat{i} + y_2\,\hat{j}\right)$.

∴ Displacement of the particle during the time $(t_2 - t_1)$ will be

$$\vec{r} \text{ or } \overrightarrow{r_{12}} = \vec{r_2} - \vec{r_1} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}, \text{ where } \left|\vec{r}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Velocity. If motion be uniform then uniform velocity may be defined as the displacement covered per unit time *i.e.,*

$$\text{Velocity } \vec{v} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r_2} - \vec{r_1}}{t_2 - t_1} = \frac{(x_2 - x_1)}{(t_2 - t_1)}\hat{i} + \frac{(y_2 - y_1)}{(t_2 - t_1)}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

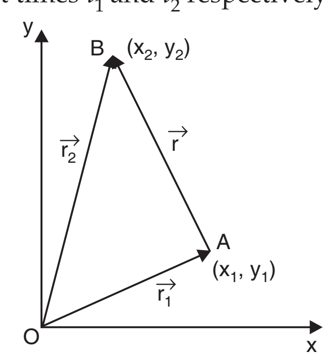
where $v_x = \dfrac{x_2 - x_1}{t_2 - t_1}$, $v_y = \dfrac{y_2 - y_1}{t_2 - t_1}$ and $\left|\vec{v}\right| = \sqrt{v_x^2 + v_y^2}$.

Acceleration. If velocity of particle is variable then time rate of change of velocity is the acceleration $\vec{a}$. If $\vec{v}_1$ and $\vec{v}_2$ be the velocities of the particle at times t_1 and t_2 respectively and acceleration be uniform, then

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

$$= \frac{\left(v_{2x}\,\hat{i} + v_{2y}\,\hat{j}\right) - \left(v_{1x}\,\hat{i} + v_{1y}\,\hat{j}\right)}{(t_2 - t_1)}$$

$$= \frac{(v_{2x} - v_{1x})}{(t_2 - t_1)}\hat{i} + \frac{(v_{2y} - v_{1y})}{(t_2 - t_1)}\hat{j}$$



$$= a_x \hat{i} + a_y \hat{j}$$

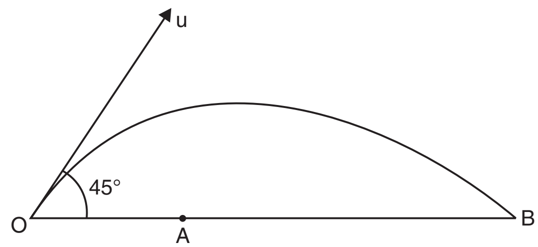
where $a_x = \dfrac{v_{2x} - v_{1x}}{t_2 - t_1}$, $a_y = \dfrac{v_{2y} - v_{1y}}{t_2 - t_1}$ and $\left|\vec{a}\right| = \sqrt{a_x^2 + a_y^2}$.

Q. 3. *A gun kept on a straight horizontal road is used to hit a car travelling along the same road away from the gun with a uniform speed of 72 km/h. The car is at a distance of 500 m from the gun when the gun is fired at an angle of 45° to the horizontal. Find*

(i) the distance of the car from the gun when the shell hits it, and

(ii) the speed of projection of the shell from the gun.

Ans. The gun and the car are at O and A respectively at $t = 0$ (fig below). Let us say that at $t = t$, the shell and the car reach B simultaneously so that the shell hits the car when it is at a distance OB from the gun. Let u be the speed of projection of the shell from the gun. Then the initial horizontal component of the velocity of the shell = $u \cos 45° = \dfrac{u}{\sqrt{2}}$ and the initial vertical component of the velocity of the shell = $u \sin 45° = \dfrac{u}{\sqrt{2}}$.



Time of flight of the shell = $\dfrac{2\left(u/\sqrt{2}\right)}{g} = \sqrt{2}\,(u/g)$.

The car takes this time to cover the distance AB while the shell covers the distance OB in this time. But

$$OB = OA + AB = 500 + AB$$

Also $$OB = \frac{u}{\sqrt{2}} \cdot \frac{\sqrt{2}\,u}{g} = \frac{u^2}{g}$$

and $$AB = 20 \times \sqrt{2}\left(\frac{u}{g}\right) = 20\sqrt{2}\,\frac{u}{g} \qquad (\because\ 72 \text{ km/h} = 20 \text{ ms}^{-1})$$

$$\therefore \quad \frac{u^2}{g} = 500 + 20\sqrt{2}\,\frac{u}{g}$$

or $u^2 - 20\sqrt{2}\,u - 500 \times 9.8 = 0$

or $u^2 - 20\sqrt{2}\,u - 4900 = 0$

or $$u = \frac{20\sqrt{2} \pm \sqrt{400 \times 4 + 4 \times 4900}}{2} \text{ ms}^{-1} = \left(10\sqrt{2} \pm \sqrt{5300}\right) \text{ms}^{-1}$$

$$= 10\left[\sqrt{2} \pm \sqrt{53}\right] \text{ms}^{-1} = 10\ [1.414 + 7.280] \text{ ms}^{-1} = 86.94 \text{ ms}^{-1}$$

This is the speed of projection of the shell from the gun. The distance of the car from the gun when the shell hits it is OB where

$$OB = \frac{u^2}{g} = \frac{(86.94)^2}{9.8} \text{ m} \approx 771.3 \text{ m}.$$

Q. 4. *A particle is thrown over a triangle from one end of a horizontal base that grazing the vertex falls on the other end of the base. If α and β be the base angles and θ the angle of projection; prove that : $\tan\theta = \tan\alpha + \tan\beta$*

Ans. The statement in the question is shown in the diagram,

$$\tan\alpha = \frac{y}{x} \text{ and } \tan\beta = \frac{y}{MA} = \frac{y}{R-x}, \text{ where } R \text{ is horizontal range.}$$

$$\therefore \quad \tan\alpha + \tan\beta = \frac{y}{x} + \frac{y}{R-x} = \frac{(R-x+x)\,y}{x(R-x)} = \frac{yR}{x(R-x)}$$

$$\text{or} \quad \tan\alpha + \tan\beta = \frac{yR}{x(R-x)} \qquad ...(i)$$

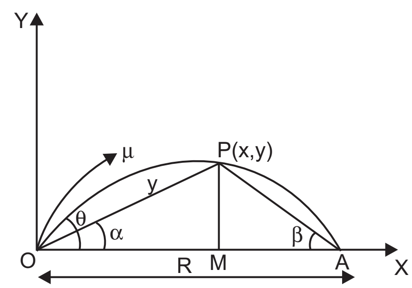
Again,

$$x = (u\cos\theta)\,t \qquad ...(ii)$$

$$y = (u\sin\theta)\,t - \frac{1}{2}gt^2 \qquad ...(iii)$$

From (*ii*) and (*iii*), we have

$$y = x\tan\theta\left[1 - \frac{xg}{2u^2\cos^2\theta\tan\theta}\right]$$



Putting,

$$R = \frac{2u^2\sin\theta\cos\theta}{g}$$

we get

$$y = x\tan\theta\left[1 - \frac{xg}{2u^2\cos\theta\sin\theta}\right]$$

$$= x\tan\theta\left[1 - \frac{x}{R}\right]$$

$$\text{or} \quad \frac{y}{x} = \tan\theta\left(\frac{R-x}{R}\right) \qquad ...(iv)$$

Putting (*iv*) in (*i*), we get

$$\tan\alpha + \tan\beta = \frac{yR}{x(R-X)} = \tan\theta$$

$$\therefore \quad \tan\alpha + \tan\beta = \tan\theta.$$

Q. 5. *Write the expression for the magnitude and direction of the resultant of two vectors inclined at an angle θ. Discuss special cases when value of θ is (i) 0°, (ii) 180° and (iii) 90°.*

Ans. Let two vectors $\vec{A}$ and $\vec{B}$ be acting simultaneously at a particle, inclined at an angle θ from one another. The magnitude of resultant vector $\vec{R}$ is given by

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

If the resultant vector subtends an angle β from the direction of $\vec{A}$, then

$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

There are three special cases which are as follows:

(*i*) If $\theta = 0°$ *i.e.,* vectors $\vec{A}$ and $\vec{B}$ are acting in same direction, then $\cos\theta = \cos 0° = 1$ and $\sin\theta = \sin 0° = 0$. Hence,

$$R = \sqrt{A^2 + B^2 + 2AB(1)} = A + B$$

and $$\tan\beta = \frac{B \times (0)}{A + B(1)} = 0 \quad \text{or} \quad \beta = \tan^{-1}(0) = 0°$$

Thus the magnitude of resultant vector is equal to the sum of the magnitudes of $\vec{A}$ and $\vec{B}$ and the resultant vector acts in the direction of $\vec{A}$ or $\vec{B}$.

(*ii*) If $\theta = 180°$, *i.e.,* vectors $\vec{A}$ and $\vec{B}$ are acting in mutually opposite direction, then $\cos\theta = \cos 180° = -1$ and $\sin\theta = \sin 180° = 0$.

$$\therefore \quad R = \sqrt{A^2 + B^2 + 2AB(-1)} = \sqrt{A^2 + B^2 - 2AB} = (A - B)$$

and $$\tan\beta = \frac{B \times (0)}{A + B \times (-1)} = 0 \quad \text{or} \quad \beta = \tan^{-1}(0) = 0° \text{ or } 180°.$$

Hence the magnitude of resultant vector is equal to the difference of the magnitudes of $\vec{A}$ and $\vec{B}$ and the resultant vector acts in the direction of bigger of two vectors $\vec{A}$ or $\vec{B}$.

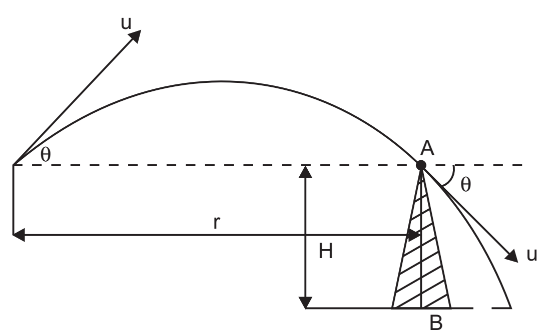
(*iii*) If $\theta = 90°$ *i.e.,* vectors $\vec{A}$ and $\vec{B}$ are in mutually perpendicular directions, then $\cos\theta = \cos 90° = 0$ and $\sin\theta = \sin 90° = 1$

$$\therefore \quad R = \sqrt{A^2 + B^2 + 2AB \times (0)} = \sqrt{A^2 + B^2} \quad \text{and} \quad \tan\beta = \frac{B \times (1)}{A + B \cdot (0)} = \frac{B}{A}$$

Thus, the magnitude of resultant vector is $R = \sqrt{A^2 + B^2}$ and it subtends an angle β from $\vec{A}$ such that $\tan\beta = \frac{B}{A}$.

Q. 6. *A ball is thrown from a point in level with and at a horizontal distance r from the top of a tower of height H. How must the speed and angle of projection of the ball be related to r in order that the ball may just go grazing past the top edge of the tower? At what horizontal distance x from the foot of the tower does the ball hit the ground ? For a given speed of projection, obtain an equation for finding the angle of projection so that x is at a minimum.*

Ans. Let AB be the tower of height H and O the point of projection at a horizontal distance r from A as shown in Fig. Let u and θ be the speed and the angle of projection of the ball.



The ball will go just grazing past the top edge of the tower if r equals the horizontal range of the projectile, *i.e.,* if

$$r = u\cos\theta \cdot \frac{2u\sin\theta}{g} = \frac{u^2\sin 2\theta}{g}$$

Thus r, u and θ must be related according to the relation

$$g\,r = u^2 \sin 2\theta$$

At point A, the velocity of the ball is again u at an angle θ to the horizontal. The horizontal distance x at which the ball strikes the ground from the foot of the tower is the horizontal distance covered with a horizontal velcoity $u \cos \theta$ in the time the ball falls vertically through a distance H starting with an initial vertically downward velocity $u \sin \theta$ and having a vertically downward acceleration g. If t is this time, we have

$$H = (u \sin \theta)\ t + \frac{1}{2} g t^2$$

or $$g t^2 + 2 u \sin \theta\ t - 2H = 0$$

or $$t = \frac{-2 u \sin\theta \pm \sqrt{4 u^2 \sin^2 \theta + 8 g H}}{2g}$$

or $$t = \frac{-2 u \sin\theta \pm 2\sqrt{u^2 \sin^2 \theta + 2 g H}}{2g}$$

$$= \frac{1}{g}\left[-u \sin\theta \pm \sqrt{u^2 \sin^2 \theta + 2 g H}\right]$$

Taking only the positive sign, we have

$$t = \frac{1}{g}\left[\sqrt{u^2 \sin^2 \theta + 2 g H} - u \sin\theta\right]$$

Thus $$x = u \cos \theta\ t = \frac{u \cos\theta}{g}\left[\sqrt{u^2 \sin^2 \theta + 2 g H} - u \sin\theta\right]$$

The angle of projection θ for which x is minimum for a given value of u is given by

$$\frac{dx}{d\theta} = 0$$

Thus, $$\frac{u \cos\theta}{g}\left[\frac{u^2\ 2 \sin\theta \cos\theta}{\sqrt{u^2 \sin^2\theta + 2 g H}} - u \cos\theta\right]$$

$$+\left[\sqrt{u^2 \sin^2 \theta + 2 g H} - u \sin\theta\right]\left(\frac{-u \sin\theta}{g}\right) = 0$$

or $$\frac{u^2 \sin 2\theta \cos\theta}{g\sqrt{u^2 \sin^2\theta + 2g H}} - \frac{u \cos^2\theta}{g} - \frac{\sin\theta}{g}\sqrt{u^2 \sin^2\theta + 2 g H} + \frac{u \sin^2\theta}{g} = 0$$

or $$\frac{u^2 \sin 2\theta \cos\theta}{\sqrt{u^2 \sin^2\theta + 2 gH}} - \sin\theta\sqrt{u^2 \sin^2\theta + 2 g H} - u \cos 2\theta = 0$$

or $u^2 \sin 2\theta \cos\theta - \sin\theta\,(u^2 \sin^2\theta + 2 g H) - u \cos 2\theta \sqrt{u^2 \sin^2\theta + 2 g H} = 0$

The angle of projection for which x is minimum is a solution of this equation.

Q. 7. *Define angular velocity and angular acceleration. The total speed V_1 of a projectile at its greatest height is $\sqrt{\frac{6}{7}}$ of its speed V_2 when it is at half its greatest height. Show that the angle of projection is 30°.*

Ans. For the definition of angular velocity and angular acceleration, see text.

Numerical. Velocity at highest point = $u \cos \theta = V_1$ (given)

$$h_{max} = \frac{u^2 \sin^2 \theta}{2g}$$

Vertical velocity at $\frac{h_{max}}{2} = V_{2y} = \sqrt{u^2 \sin^2 \theta - 2g \frac{h_{max}}{2}}$

or $$V_{2y} = \sqrt{u^2 \sin^2 \theta \left(1 - \frac{1}{2}\right)} = \frac{u \sin \theta}{\sqrt{2}}$$

$$V_{2x} = u \cos \theta$$

$$V_2 = \sqrt{V_{2x}^2 + V_{2y}^2} = \sqrt{u^2 \cos^2 \theta + \frac{u^2 \sin^2 \theta}{2}}$$

Given, $$V_1 = \sqrt{\frac{6}{7}} V_2$$

$$\therefore \quad \frac{u \cos \theta}{u\sqrt{\cos^2 \theta + \frac{\sin^2 \theta}{2}}} = \sqrt{\frac{6}{7}}$$

Squaring both the sides

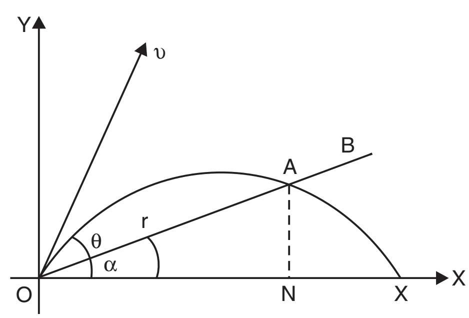
$$\frac{\cos^2 \theta}{\cos^2 \theta + \frac{\sin^2 \theta}{2}} = \frac{6}{7} \quad \text{or} \quad \frac{1}{1 + \frac{\tan^2 \theta}{2}} = \frac{6}{7}$$

or $$1 + \frac{\tan^2 \theta}{2} = \frac{7}{6} \quad \Rightarrow \quad \frac{\tan^2 \theta}{2} = \frac{7}{6} - 1 = \frac{1}{6}$$

or $$\tan \theta = \sqrt{\frac{2}{6}} = \frac{1}{\sqrt{3}} \quad \therefore \quad \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ .$$

Q. 8. *A projectile, launched with a speed v at an angle θ to the horizontal, hits a plane inclined at an angle α to the horizontal (α < θ) and passing through the point of launching. Obtain an expression for the range r of the projectile on this inclined plane. When does the projectile hit this plane?*

Ans. Let *OAB* be the inclined plane making an angle α to the horizontal and let the projectile hit it at a point *A* where *OA* = *r* (Fig.).



At the instant the projectile hits the inclined plane, its horizontal and vertical displacements are *ON* (= $r \cos \alpha$) and *NA* (= $r \sin \alpha$) respectively. Now the time taken to cover a horizontal distance $r \cos \alpha$ is clearly $r \cos \alpha / v \cos \theta$. The vertical distance moved in this time being $r \sin \alpha$, we have

$$r \sin \alpha = (v \sin \theta)\left(\frac{r \cos \alpha}{v \cos \theta}\right) - \frac{1}{2} g \left(\frac{r \cos \alpha}{v \cos \theta}\right)^2$$

or $$r (\sin \alpha - \tan \theta \cos \alpha) = -\frac{1}{2} g \frac{\cos^2 \alpha}{v^2 \cos^2 \theta} r^2$$

$$\therefore \qquad r = \frac{2v^2\cos^2\theta}{g\cos^2\alpha}[\tan\theta\cos\alpha - \sin\alpha] = \frac{2v^2\cos^2\theta}{g\cos^2\alpha}\left[\frac{\sin\theta\cos\alpha - \cos\theta\sin\alpha}{\cos\theta}\right]$$

$$= \frac{2v^2\cos\theta}{g\cos^2\alpha}\sin(\theta-\alpha)$$

Thus the range of the projectile on the inclined plane is

$$\frac{2v^2\cos\theta}{g\cos^2\alpha}\sin(\theta-\alpha)$$

The time at which the projectile hits the inclined plane being ($r\cos\alpha/v\cos\theta$), we have

$$t = \frac{r\cos\alpha}{v\cos\theta} = \frac{2v^2\cos\theta}{g\cos^2\alpha}\sin(\theta-\alpha)\frac{\cos\alpha}{v\cos\theta} \quad \text{or} \quad t = \frac{2v\sin(\theta-\alpha)}{g\cos\alpha}$$

A confirmation of these results is obtained by putting $\alpha = 0$ for which case we get

$$r = \frac{2v^2\cos\theta\sin\theta}{g} = \frac{v^2\sin 2\theta}{g} \quad \text{and} \quad t = \frac{2v\sin\theta}{g}$$

the usual expressions for the range and time of flight of a projectile.

IV. MULTIPLE CHOICE QUESTIONS

1. If $\vec{a}_1$ and $\vec{a}_2$ are two non collinear unit vectors and if $\left|\vec{a}_1 + \vec{a}_2\right| = \sqrt{3}$, then the value of $\left(\vec{a}_1 - \vec{a}_2\right)\cdot\left(2\vec{a}_1 + \vec{a}_2\right)$ is

(*a*) 2 (*b*) $\frac{3}{2}$ (*c*) $\frac{1}{2}$ (*d*) 1

2. The sum of magnitudes of two forces acting at a point is 18 units and the magnitude of their resultant is 12 units. The resultant is at 90° with the force of the smaller magnitude. The magnitude of the individual forces is

(*a*) 5, 12 (*b*) 5, 13 (*c*) 6, 14 (*d*) none of these

3. If the resultant of three forces $\vec{F}_1 = p\hat{i} + 3\hat{j} - \hat{k}$, $\vec{F}_2 = -5\hat{i} + 2\hat{k}$, and $\vec{F}_3 = 6\hat{i} - \hat{k}$ acting on a particle has a magnitude equal to 5 units, then the value of p is

(*a*) – 6 (*b*) – 4 (*c*) 3 (*d*) 4

4. A vector is of magnitude $10\sqrt{3}$ units and making equal angles with the positive direction of x, y and z axis is

(*a*) $10\left(\hat{i} + \hat{j} + \hat{k}\right)$ (*b*) $10\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

(*c*) $10\left(-\hat{i} - \hat{j} - \hat{k}\right)$ (*d*) $10\left(\hat{i} - \hat{j} + \hat{k}\right)$

5. A body is projected horizontally with a velocity of 4 ms^{-1}. The velocity of the body after 0.7 s is nearly (take g = 10 ms^{-2})

(*a*) 10 ms^{-1} (*b*) 8 ms^{-1} (*c*) 19.2 ms^{-1} (*d*) 11 ms^{-1}

6. A particle moves on a given line with a constant speed v. At a certain time it is at a point P on its straight line path. O is fixed point. The value of $\left(\overrightarrow{OP} \times \vec{v}\right)$ is (where y is perpendicular distance from O to given line)

(*a*) $-y\,v\hat{k}$ (*b*) $-2y\,v\hat{k}$ (*c*) $-3y\,v\hat{k}$ (*d*) none

7. From the top of a tower of height 40 m, a ball is projected upwards with a speed of 20 m/s at an angle of elevation of 30°. The ratio of the total time taken by the ball to hit the ground to its time of flight (time taken to come back to the same elevation) is (Take g = 10 m/s^2)

(*a*) 2 : 1 (*b*) 3 : 1 (*c*) 3 : 2 (*d*) 1.5 : 1

8. A boy aims a gun at a target from a point, at a horizontal distance of 100 m. If the gun can impart a horizontal velocity of 500 ms^{-1} to the bullet, the height above the target where he must aim his gun, in order to hit it is (Take g = 10 ms^{-2})

(*a*) 20 cm (*b*) 10 cm (*c*) 50 cm (*d*) 100 cm

9. At the top of the trajectory of a projectile, the directions of its velocity and accelerations are

(*a*) parallel to each other (*b*) anti-parallel to each other

(*c*) perpendicular to each other (*d*) inclined to each other at an angle of 45°

10. A plane is indined at an angle of 30° with horizontal. The magnitude of component of a vector $\vec{A} = -10\,\hat{k}$ perpendicular to this plane is (here z-direction is vertically upwards)

(*a*) $5\sqrt{2}$ (*b*) $5\sqrt{3}$ (*c*) 5 (*d*) 2.5

Ans. **1.**—(*c*) **2.**—(*b*) **3.**—(*c*) **4.**—(*a*) **5.**—(*b*)
6.—(*a*) **7.**—(*a*) **8.**—(*a*) **9.**—(*d*) **10.**—(*b*)

V. QUESTIONS ON HIGH ORDER THINKING SKILLS (HOTS)

Q. 1. *The acceleration associated with a mass 'm' moving in a circular path is to be found. It is given that the velocity at any instant is v = krt, where k is a constant. Classify the motion and find acceleration.*

Ans. Given $v = krt$

Since velocity changes with time, the motion in circular path involves tangential acceleration.

So it is a non-uniform circular motion.

$$a_r = \frac{v^2}{r} = k^2 rt^2 \quad \text{or} \quad a_k = \frac{dv}{dt} = kr$$

Net acceleration = $a_n = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(k^2rt^2\right)^2 + (kr)^2} = kr\sqrt{1 + k^2t^4}$.

Q. 2. *An aeroplane is flying in a horizontal direction with a velocity of 600 km/hr and at a height of 1960 m. When it is vertically above the point A on the ground, a body is dropped from it. The body strikes the ground at point B. Calculate the distance AB.*

Ans. Velocity of aeroplane in horizontal direction is

$$v_{ox} = 600 \text{ km/hr} = 600 \times \frac{5}{18} \text{ ms}^{-1} = \frac{500}{3} \text{ ms}^{-1}$$

This velocity remains constant throughout the flight of the body.

$$v_{oy} = 0 \quad \text{and} \quad y = h = 1960 \text{ m}$$

Let t = the time taken by the body to reach the ground

Now $y = v_{oy}\, t + \frac{1}{2} g t^2$

Here, $y = h = 1960$ m; $v_{oy} = 0$ (initial vertical velocity)

$\therefore$ $1960 = \frac{1}{2} \times 9.8\, t^2$

$\therefore$ $t = \sqrt{\frac{1960}{4.9}}\text{ s} = \sqrt{400}\text{ s} = 20\text{ s}$

Distance travelled by the body in the horizontal direction

$$= v_{ox}\, t = \frac{500}{3} \times 20 = \frac{10{,}000}{3} = 3333 \text{ m} = 3.333 \text{ km}$$

$\therefore \qquad AB = 3.333$ km.

Q. 3. *When two vectors $\vec{A}$ and $\vec{B}$ inclined at angle θ act on a body, the resultant is $(2k+1)\sqrt{A^2+B^2}$. When the vectors are inclined at an angle (90° – θ), the resultant is $(2k-1)\sqrt{A^2+B^2}$, prove that*

$$\tan\theta = \frac{k-1}{k+1}.$$

Ans. As $\qquad R^2 = A^2 + B^2 + 2AB\cos\theta.$

In first case,

$$(2k+1)^2 (A^2+B^2) = A^2 + B^2 + 2AB\cos\theta$$

or $\qquad 2AB\cos\theta = (A^2+B^2)[(2k+1)^2 - 1] = (A^2+B^2)[2k(2k+2)] \qquad ...(i)$

In second case,

$$(2k-1)^2 (A^2+B^2) = A^2 + B^2 + 2AB\cos(90° - \theta) = A^2 + B^2 + 2AB\sin\theta$$

$$2AB\sin\theta = (A^2+B^2)[(2k-1)^2 - 1] = (A^2+B^2)[2k(2k-2)] \qquad ...(ii)$$

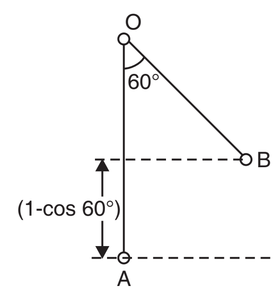
Dividing (*ii*) by (*i*), we have

$$\tan\theta = \frac{2k-2}{2k+2} = \frac{k-1}{k+1}.$$

Q. 4. *A body is suspended by a string of length 1 m and is projected horizontally with velocity 4 m/s. Calculate the tangential and radial accelerations when the string rises by 60° from its initial position. Also find the difference in velocity.*

Ans. $l = 1$ m.

By applying conservation of energy, we have



$$mg(1 - \cos 60°) = \frac{1}{2} m v_B^2$$

or $\qquad v_B = 2g\left(1 - \frac{1}{2}\right) = g = 10 \text{ ms}^{-2}$

Centripetal acceleration at $B = \frac{v_B^2}{r} = \frac{g^2}{r}$

At B, tangential acceleration $= g \sin 60° = \frac{\sqrt{3}}{2} g$

Velocity difference $(v_B - v_A) = 10 \text{ ms}^{-1} - 4 \text{ ms}^{-1} = 6 \text{ ms}^{-1}$.

Q. 5. *Can there be two vectors where the resultant is equal to either of them?*

Ans. Yes, when two vectors of same magnitude inclined at an angle of 120°, then resultant is equal to either of them.

Let x be the magnitude of each of two vectors which make an angle of 120°, then resultant is equal to either of them.

Now, $\qquad R = \sqrt{x^2 + x^2 + 2x \cdot x \cos 120°}$

$\Rightarrow \qquad R = \sqrt{2x^2 - x^2} = \sqrt{x^2} = x \qquad \left[\because \cos 120° = -\frac{1}{2}\right]$

Q. 6. *The sum of the magnitude of two forces acting at a point is 18 N and the magnitude of their resultant is 12 N. If the resultant is at 90° with the force of smaller magnitude, what are the magnitude of forces?*

Ans. Let A and B be the two forces acting at a point and θ be the angle between them. Then

$$A + B = 18 \quad ...(i)$$

and $$A^2 + B^2 + 2\,AB\cos\theta = 12^2 = 144 \quad ...(ii)$$

If A is the smaller force, then as per question

$$\tan 90° = \frac{B\sin\theta}{A + B\cos\theta} \quad \text{or} \quad \infty = \frac{B\sin\theta}{A + B\cos\theta}$$

or $A + B\cos\theta = 0$ or $\cos\theta = -A/B$

From (*ii*), $A^2 + B^2 + 2\,AB\,(-A/B) = 144$

or $B^2 - A^2 = 144$ or $(B - A)(B + A) = 144$

or $$(B - A) = \frac{144}{(B + A)} = \frac{144}{18} = 8 \quad ...(iii)$$

Solving (*i*) and (*ii*)

$$A = 5\text{ N} \quad \text{and} \quad B = 13\text{ N}.$$

Q. 7. *Show that the horizontal range of a projectile is same for angles of projection (45 + α)° and (45 – α)°.*

Ans. For angle of projection (45 + α)°, the horizontal range is

$$R_1 = \frac{u^2}{g}\sin 2(45 + \alpha)° = \frac{u^2}{g}\sin(90 + 2\alpha)° = \frac{u^2}{g}\cos 2\alpha \quad ...(i)$$

and for an angle of projection (45 – α)°, the horizontal range is

$$R_2 = \frac{u^2}{g}\sin 2(45 - \alpha)° = \frac{u^2}{g}\sin(90 - 2\alpha)° = \frac{u^2}{g}\cos 2\alpha \quad ...(ii)$$

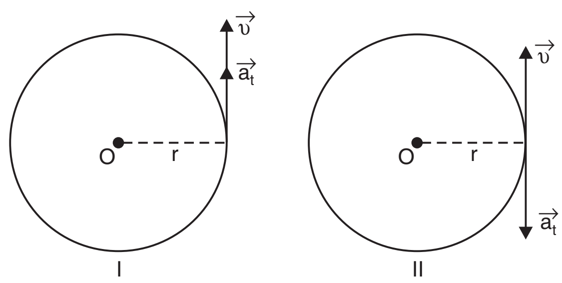
By comparing eqn. (*i*) and (*ii*), we find

$$R_1 = R_2.$$

Q. 8. *Can there be motion in two dimensions with an acceleration in only one dimension?*

Ans. Yes, the projectile motion is a motion in two dimensions with an acceleration in only one dimension *i.e.,* in vertical direction only.

Q. 9. *Instantaneous velocity and tangential acceleration of two particles moving in circular paths of radius r are shown in figure I and II. Which of the particles is speeding up and which one is slowing down?*



Ans. Particle moving in a circular path I is speeding up because the direction of both $\vec{v}$ and $\vec{a}_t$ are same. Particle moving in a circular path II is slowing down because the directions of both $\vec{v}$ and $\vec{a}_t$ are opposite to each other.

Q. 10. *If the horizontal range of projectile be a and the maximum height attained by it is b, then prove that the velocity of projectile is* $\left[2g\left(b+\frac{a^2}{16b}\right)\right]^{1/2}$.

Ans. Maximum height = $b = \frac{u^2 \sin^2 \theta}{2g}$ or $\sin^2 \theta = \frac{2bg}{u^2}$

Horizontal range, $a = \frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin\theta \cos\theta}{g}$

$\Rightarrow \quad 2 \sin \theta \cos \theta = \frac{ag}{u^2}$ or $4 \sin^2 \theta \cos^2 \theta = \frac{a^2 g^2}{u^4}$

$\Rightarrow \quad 4 \sin^2 \theta (1 - \sin^2 \theta) = \frac{a^2 g^2}{u^4}$ ...(*i*)

or $\quad 4\left(\frac{2bg}{u^2}\right)\left[1-\frac{2bg}{u^2}\right] = \frac{a^2g^2}{u^4}$ or $\frac{8bg}{u^2} - \frac{16b^2g^2}{u^4} = \frac{a^2g^2}{u^4}$

$\Rightarrow \quad a^2 g^2 + 16 b^2 g^2 = u^2 \, 8 \, bg$

or $\quad u^2 = \frac{a^2g^2 + 16b^2g^2}{8bg}$ or $u = \left[2g\left(b+\frac{a^2}{16b}\right)\right]^{1/2}$.

TEST YOUR SKILLS

1. What is the difference between 'triangle law of addition' and 'parallelogram law of addition' of vectors?
2. What is the difference between position and displacement vectors? Explain with the help of suitable diagrams.
3. A speedboat is moving towards east at 30 kmh^{-1}. The current is moving at 10 kmh^{-1} in the direction 45° south of west. What is the resultant velocity of the speedboat?
4. What is the difference between average velocity and instantaneous velocity?
5. When an object is in motion in one dimension, what are the relative directions at velocity and acceleration of the object? How does it differ with relative directions of velocity and acceleration, when the motion is in three dimensions?
6. What do you understand by relative velocities of two objects in motion? A train is moving at a speed of go kmh^{-1}. A man is walking towards the train, at a speed of 18 kmh^{-1}. If the train is 350 m long, how much time it takes the train to completely cross the man?
7. What do you understand by centripetal acceleration?

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