5 Laws of Motion

Facts that Matter

- Dynamics is the branch of physics in which we study the motion of a body by taking into consideration the cause *i.e.,* force which produces the motion.

- **Force**

Force is an external cause in the form of push or pull, which produces or tries to produce motion in a body at rest, or stops/tries to stop a moving body or changes/tries to change the direction of motion of the body.

- The inherent property, with which a body resists any change in its state of motion is called inertia. Heavier the body, the inertia is more and lighter the body, lesser the inertia.
- Law of inertia states that a body has the inability to change its state of rest or uniform motion (*i.e.,* a motion with constant velocity) or direction of motion by itself.

- **Newton's Laws of Motion**

Law 1. A body will remain at rest or continue to move with uniform velocity unless an external force is applied to it.

First law of motion is also referred to as the 'law of inertia'. It defines inertia, force and inertial frame of reference.

> There is always a need of 'frame of reference' to describe and understand the motion of particle. The simplest 'frame of reference' used are known as the inertial frames.
>
> A frame of reference is known as an inertial frame if, within it, all accelerations of any particle are caused by the action of 'real forces' on that particle.
>
> When we talk about accelerations produced by 'fictitious' or 'pseudo' forces, the frame of reference is a non-inertial one.

Law 2. When an external force is applied to a body of constant mass the force produces an acceleration, which is directly proportional to the force and inversely proportional to the mass of the body.

$$\vec{F} = K\frac{d\vec{p}}{dt} = Km\vec{a}$$

where $\vec{F}$ is the net external force on the body and $\vec{a}$ its acceleration. There is no loss of generality in choosing the constant of proportionality $k = 1$. Then

$$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}$$

The SI unit of force is newton: 1 N = 1 kg ms^{-2}.

(*a*) The second law of Newton is consistent with the first law ($\vec{F} = 0$ implies $\vec{a} = 0$).

(*b*) It is a vector equation.

(*c*) It is strictly applicable to a point particle, but is also applicable to a body or a system of particles, provided we mean by $\vec{F}$ the total external force on the system and $\vec{a}$ is the acceleration of the system as a whole.

(*d*) $\vec{F}$ at a space point at a certain instant determines $\vec{a}$ at the same point and the same instant.

Law 3. ''To every action there is equal and opposite reaction force''. When a body *A* exerts a force on another body *B*, *B* exerts an equal and opposite force on *A*.

- **Linear Momentum**

The linear momentum of a body is defined as the product of the mass of the body and its velocity.

Linear momentum = mass × velocity

$$\vec{p} = m\vec{v}$$

where m is the mass of the body, $\vec{v}$ is the velocity of the body and $\vec{p}$ linear momentum. Momentum is a vector quantity having the same direction as the velocity $(\vec{v})$. Its SI unit is kg ms^{-1}.

- **Impulse**

Forces acting for short duration are called impulsive forces. Impulse is defined as the product of force and the small time interval for which it acts. It is given by

$$I = \int F\,dt$$

Impulse of a force is a vector quantity and its SI unit is 1 Nm.

— If force of an impulse is changing with time, then the impulse is measured by finding the area bound by force-time graph for that force.

— Impulse of a force for a given time is equal to the total change in momentum of the body during the given time. Thus, we have

$$I = \int_{t_1}^{t_2} \vec{F}\,dt = \vec{P}_2 - \vec{P}_1 \qquad \text{(Impulse-momentum theorem)}$$

- **Law of Conservation of Momentum**

The total momentum of an isolated system of particles is conserved.

In other words, when no external force is applied to the system, its total momentum remains constant.

i.e., If $F_{ext} = 0, \quad \sum_{i=1}^{n} P_i = \text{constant} \quad \text{or} \quad P = \text{constant}$

- Recoiling of a gun, flight of rockets and jet planes are some simple applications of the law of conservation of linear momentum.

- When a bullet of mass m_b is fired with a velocity $\vec{v}_b$, the gun of mass m_g will acquire a velocity $\vec{v}_g$ which is given by

$$\vec{v}_g = -\frac{m_b\,\vec{v}_b}{m_g}$$

The negative sign shows that direction of $\vec{v}_g$ is opposite to the direction of $\vec{v}_b$ *i.e.,* the gun recoils.

- The rocket sent up in space will acquire a velocity (v) which is given by

$$v = v_0 + u \log_e \frac{m_0}{m} \qquad \text{(Instantaneous velocity of the rocket)}$$

Usually, initial velocity of the rocket at $t = 0$ is zero *i.e.,* $v_0 = 0$

Thus, $$v = u \log_e \frac{m_0}{m}$$

→ Exhaust speed (u) of the gases.
→ Log of the ratio of initial mass (m_0) of the rocket to its mass (m) at that instant of time.

Speed of the rocket at any instant depends upon these two factors discussed above.

where,
m_0 = initial mass of the rocket and fuel.
$\vec{v}_0$ = initial velocity of rocket
m = mass of the rocket at any instant t during its flight
$\vec{v}$ = velocity acquired by the rocket at that instant w.r.t. ground

- The instantaneous acceleration of the rocket is given by

$$a = -\frac{u}{m}\frac{dm}{dt}$$

where $\frac{dm}{dt}$ is the rate at which the fuel is consumed.

- **Concurrent Forces and Equilibrium**

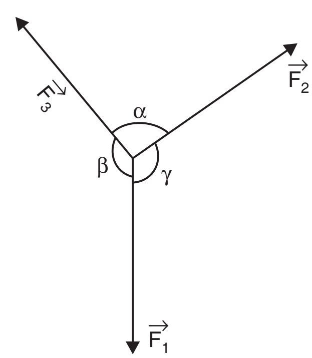
"A group of forces which are acting at one point are called concurrent forces."

Concurrent forces are said to be in equilibrium if there is no change in the position of rest or the state of uniform motion of the body on which these concurrent forces are acting.

For concurrent forces to be in equilibrium, their resultant force must be zero. In case of three concurrent forces acting in a plane, the body will be in equilibrium if these three forces may be completely represented by three sides of a triangle taken in order. If number of concurrent forces is more than three, then these forces must be represented by sides of a closed polygon in order for equilibrium.

Mathematically,

$$\sum \vec{F} = 0 \quad i.e., \quad \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ldots = 0.$$



- If three concurrent forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ act at a point and are in equilibrium, then

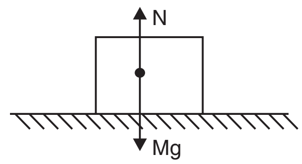
$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

where α, β and γ are angles indicated.

- **Commonly Used Forces**

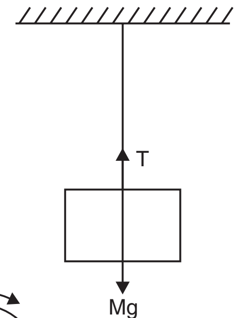
(*i*) **Weight of a body.** It is the force with which earth attracts a body towards its centre. If M is mass of body and g is acceleration due to gravity, weight of the body is Mg in vertically downward direction.

(*ii*) **Normal Force.** If two bodies are in contact a contact force arises, if the surface is smooth the direction of force is normal to the plane of contact. We call this force as Normal force.



Example. Let us consider a book resting on the table. It is acted upon by its weight in vertically downward direction and is at rest. It means there is another force acting on the block in opposite direction, which balances its weight. This force is provided by the table and we call it as normal force.

(*iii*) **Tension in string.** Suppose a block is hanging from a string. Weight of the block is acting vertically downward but it is not moving, hence its weight is balanced by a force due to string. This force is called 'Tension in string'. Tension is a force in a stretched string. Its direction is taken along the string and away from the body under consideration.



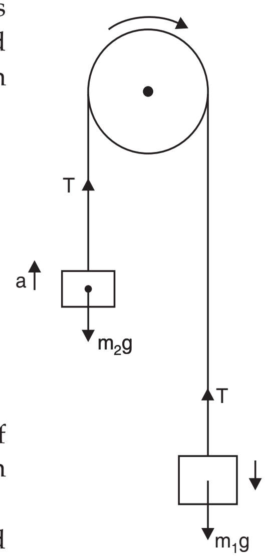
• Simple Pulley

Consider two bodies of masses m_1 and m_2 tied at the ends of an inextensible string, which passes over a light and frictionless pulley. Let $m_1 > m_2$. The heavier body will move downwards and the lighter will move upwards. Let a be the common acceleration of the system of two bodies, which is given by

$$a = \frac{(m_1 - m_2)\,g}{m_1 + m_2}$$

Tension in the string is given by

$$T = \frac{2m_1\,m_2 \times g}{m_1 + m_2}$$



• Apparent Weight and Actual Weight

– 'Apparent weight' of a body is equal to its 'actual weight' if the body is either in a state of rest or in a state of uniform motion.

– Apparent weight of a body for vertically upward accelerated motion is given as

Apparent weight =Actual weight + $Ma = M(g + a)$

– Apparent weight of a body for vertically downward accelerated motion is given as

Apparent weight =Actual weight – $Ma = M(g - a)$.

• Friction

The opposition to any relative motion between two surfaces in contact is referred to as friction. It arises because of the 'intermeshing' of the surface irregularities of the two surfaces in contact.

• Static and Dynamic (Kinetic) Friction

The frictional forces between two surfaces in contact (*i*) before and (*ii*) after a relative motion between them has started, are referred to as static and dynamic friction respectively. Static friction is always a little more than dynamic friction.

The magnitude of kinetic frictional force is also proportional to normal force.

$$f_k = \mu_k N$$

- **Limiting Frictional Force**

This frictional force acts when body is about to move. This is the maximum frictional force that can exist at the contact surface. We calculate its value using laws of friction.

Laws of Friction:

(*i*) The magnitude of limiting frictional force is proportional to the normal force at the contact surface.

$$f_{lim} \propto N \quad \Rightarrow \quad f_{lim} = \mu_s N$$

Here μ_s is a constant, the value of which depends on nature of surfaces in contact and is called as 'coefficient of static friction'. Typical values of μ ranges from 0.05 to 1.5.

(*ii*) The magnitude of limiting frictional force is independent of area of contact between the surfaces.

- **Coefficient of Friction**

The coefficient of friction (μ) between two surfaces is the ratio of their limiting frictional force to the normal force between them, *i.e.,*

$$\mu = \frac{\text{limiting frictional force}}{\text{normal reaction}} = \frac{F}{R}$$

When there is no relative motion $\mu = \mu_s$, the coefficient of static friction. When there is a relative motion $\mu = \mu_k$, the coefficient of kinetic friction.

- **Angle of Friction**

It is the angle which the resultant of the force of limiting friction F and the normal reaction R makes with the direction of the normal reaction. If θ is the angle of friction, we have

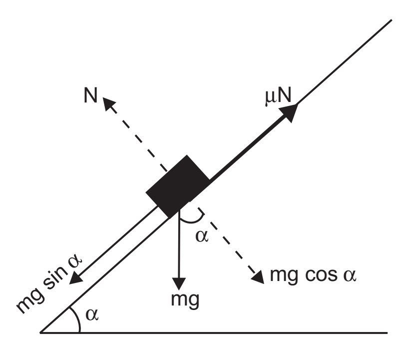
$$\tan\theta = \frac{F}{R} = \mu$$

$$\boxed{\theta = \tan^{-1}\mu}$$

- **Angle of Repose**

Angle of repose (α) is the angle of an inclined plane with the horizontal at which a body placed over it just begins to slide down without any acceleration. Angle of repose is given by

$$\alpha = \tan^{-1}(\mu)$$



- **Motion on a Rough Inclined Plane**

Suppose a motion up the plane takes place under the action of pull P acting parallel to the plane.

$$N = mg\cos\alpha$$

Frictional force acting down the plane,

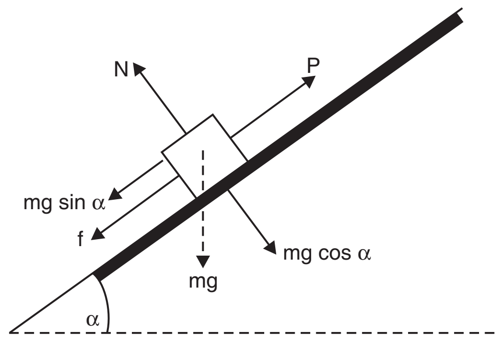
$$F = \mu N = \mu\, mg\cos\alpha$$

Applying Newton's second law for motion up the plane.

$$P - (mg \sin \alpha + f) = ma$$

$$P - mg \sin \alpha - \mu\, mg \cos \alpha = ma$$

If $P = 0$ the block may slide downwards with an acceleration a. The frictional force would then act up the plane.



$$mg \sin \alpha - F = ma$$

or, $$mg \sin \alpha - \mu\, mg \cos \alpha = ma$$

- Friction can be reduced by (*i*) making the surfaces smooth, (*ii*) lubrication, (*iii*) replacing sliding friction by rolling friction, (*iv*) using ball bearings, (*v*) streamlining the shape of the bodies.

- **Centripetal Force**

Centripetal force is the force required to move a body uniformly in a circle. This force acts along the radius and towards the centre of the circle.
It is given by

$$F = \frac{mv^2}{r} = mr\omega^2$$

where, v is the linear velocity, r is the radius of circular path and ω is the angular velocity of the body.

- **Centrifugal Force**

Centrifugal force is a force that arises when a body is moving actually along a circular path, by virtue of tendency of the body to regain its natural straight line path.
The magnitude of centrifugal force is same as that of centripetal force.

- The maximum speed with which a vehicle can negotiate a curve is $v_{max} = \sqrt{\mu rg}$, where μ is the coefficient of friction and r is the radius of the path.
- In order to help a vehicle owner to drive safely along a curved path, the outer edge of the road is slightly raised above the inner edge. The angle of banking θ of the road from horizontal is given by

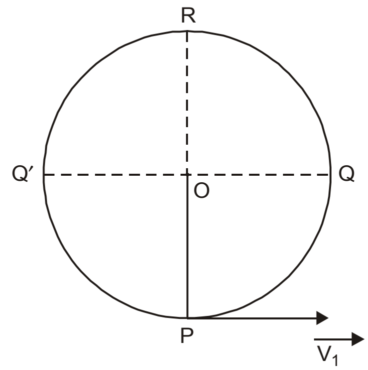
$$\tan \theta = \frac{v^2}{rg} \quad \text{or} \quad v \le \sqrt{rg \tan \theta}$$

- In a banked curve (θ) with friction, the safe velocity is given as

$$v = \left[\frac{rg\,(\mu + \tan \theta)}{1 - \mu \tan \theta}\right]^{1/2}$$

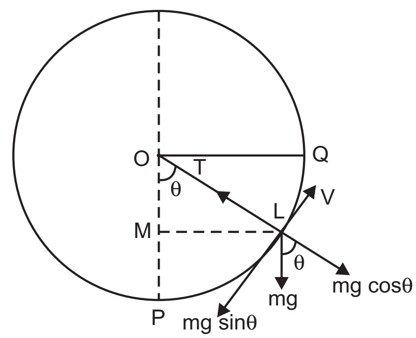
- **Motion in a Vertical Circle**

The motion of a particle in a horizontal circle is different from the motion in vertical circle. In horizontal circle, the motion is not effected by the acceleration due to gravity (g) whereas in the motion of vertical circle, the value of 'g' plays an important role. the motion in this case does not remain uniform. When the particle move up from its lowest position P, its speed continuously decreases till it reaches the heighest point of its circular path. This is due to the work done against the force of gravity. When the particle moves down the circle, its speed would keep on increasing.



(*i*)

Let us consider a particle moving in a circular vertical path of radius '*r*' and centre o tide with a string. *L* be the instantaneous position of the particle such that



(*ii*)

$$\angle POL = \theta^\circ$$

Here the following forces act on the particle of mass '*m*'.

(*i*) Its weight = mg (verticaly downwards).

(*ii*) The tension in the string *T* along *LO*.

The instantaneous velocity of the particle at *L* is $\bar{V}$ along the direction of the tangent to the circle.

Centripetal force on the particle $= \dfrac{mV^2}{r}$

$$\therefore \quad \frac{mV^2}{r} = T - mg\cos\theta \qquad \text{[From Fig. (}ii\text{)]}$$

So
$$T = \frac{mV^2}{r} + mg\cos\theta \qquad (i)$$

We can take the horizontal direction at the lowest point '*p*' as the position of zero gravitational potential energy. Now as per the principle of conservation of energy, The total energy at P =Total energy at L

$$\frac{1}{2}mV_1^2 + 0 = \frac{1}{2}mV^2 + mgh$$

$$\Rightarrow \qquad V_1^2 = V^2 + 2mgh \qquad (ii)$$

From right angled Δ OML,

$$OM = OL \cos\theta = r\cos\theta$$

$$\therefore \qquad MP = h = OP - OM = r - r\cos\theta = r(1 - \cos\theta) \qquad (iii)$$

Substituting the value of 'h' in eq. (ii) we get,

$$V_1^2 = V^2 + 2gr(1 - \cos\theta) \qquad (iv)$$

Now substituting the value of V^2 from (iv) to (i), we get

$$T = \frac{m}{r}\left[V_1^2 - 2gr(1-\cos\theta)\right] + mg\cos\theta$$

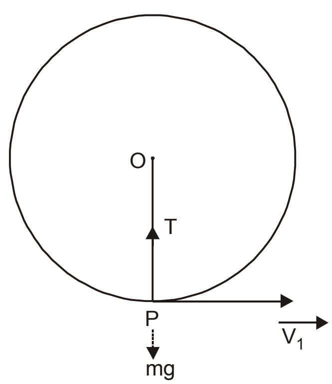
$$T = \frac{mV_1^2}{r} - 2mg(1-\cos\theta) + mg\cos\theta$$

$$= \frac{mV_1^2}{r} - 2mg + 2mg\,cos\theta + mg\,cos\theta$$

$$= \frac{mV_1^2}{r} - 2mg + 3mg\cos\theta \qquad (v)$$

From this relation, we can calculate the tension in the string at the lowest point P, mid-way point and at the highest position of the moving particle.

Case (*i*) : At the lowest point P, $\theta = 0°$



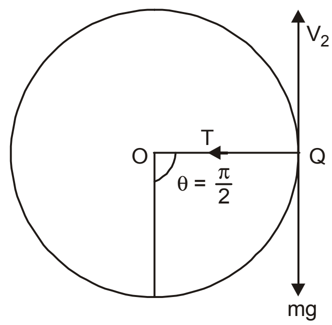
$$\therefore \qquad T_p = \frac{mV_1^2}{r} - 2mg + 3mg\cos 0° = \frac{mV_1^2}{r} - 2mg + 3mg$$

$$= \frac{mV_1^2}{r} + mg \qquad (vi)$$

Hence, the tension in the string at the lowest point P is,

$$T = \frac{mV_1^2}{r} + mg$$

Case (*ii*) : At the point 'Q' where the string in horizontal position, Where $\theta = \frac{\pi}{2}$



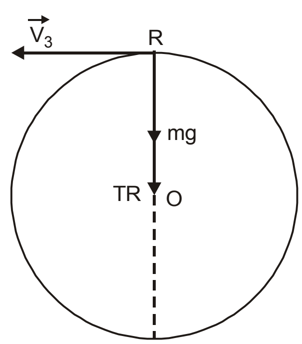
$$\therefore \quad T_Q = \frac{mV_1^2}{r} - 2mg + 3mg\cos\frac{\pi}{2} = \frac{mV_1^2}{r} - 2mg + 0$$

Hence $$T_Q = \frac{mV_1^2}{r} - 2mg \qquad (vii)$$

The change in the tension in the string at two position P and Q

$$= (T_P - T_Q) = \left(\frac{mV_1^2}{r} + mg\right) - \left(\frac{mV_1^2}{r} - 2mg\right) = 3 \text{ mg}$$

Case (*iii*) : At point R, the highest position of the moving particle where $\theta = \pi$.



$$\therefore \quad T_R = \frac{mV_1^2}{r} - 2mg + 3mg\cos\pi = \frac{mV_1^2}{r} - 2mg - 3mg$$

Here $$T_R = \frac{mV_1^2}{r} - 5mg \qquad (viii)$$

The change in the string as the particle moves from P to R.

$$= (T_P - T_R)$$

$$= \left(\frac{mV_1^2}{r} + mg\right) - \left(\frac{mV_1^2}{r} - 5mg\right) = 6mg$$

Thus we can see that the tension in the string is maximum at the lowest point P and minimum at the highest point R of the circular path.

From Eq. (*viii*) it is clear that T_R may be (*a*) positive (*b*) negative (*c*) zero, depending on the value of $\bar{V}_1$. If T_R becomes a negative number, the string would get slackened and the particle will fall down without completing the circular path. Therefore, to complete the circular path, the minimum value of T_R must be zero.

$$\therefore \quad T_{R_{(\min)}} = \frac{mV_1^2(\min)}{r} - 5mg$$

$$0 = \frac{mV_1^2(\min)}{r} - 5mg$$

$$\therefore \quad V_{1(\min)} = \sqrt{5gr} \qquad (ix)$$

When the particle completes its motion along the vertical circle, it is referred to as "Looping the Loop" for this the minimum speed at the lowest position must be $\sqrt{5gr}$.

Example 1. *A small stone of mass 200 g is tied to one end of a string of length 80 cm. Holding the other end in hand, the stone is whirled into a vertical circle. What is the minimum speed, that needs to be imparted at the lowest point of the circular path so that the stone is just able to complete the vertical cicle? What would be the tension is the string at the lowest point of circular path.* (*Take* $g = 10\ ms^{-2}$)

Sol. The minimum speed ed. need at the lowest point so that the particle is just able to complete the vertical circle.

$$V_{\min} = \sqrt{5gr} = \sqrt{5 \times 10 \times 0.8} = 6.32\ \text{ms}^{-1}$$

Also T_1 = Tenstion in the string at the lowest point of its circular path.

$$\therefore \quad T_1 = \frac{mV_{(\min)}^2}{r} + mg = 5mg + mg$$

$$= 6mg = 6 \times 0.2 \times 10 = 12\ \text{N}$$

Example 2. *A massless string, of length 1.2 m has a breaking strength of 2 kg wt. A stone of mass 0.4 kg tied to one end of the string, in made to move in a vertical circle, by holding the other end in the hand. Can the particle describe the vertical circle? (Take* $g = 10\ m \times s^{-2}$*)*

Sol. We are given that

$T_{\max}$ = Maximum tension in the string so that it does not break.

= 2 kg wt = 2 × 10 = 20 N

Let T_1 be the tension in the string when the stone is in its lowest position of its circular path is

$$T_1 = \frac{mV_1^2}{r} + mg$$

T_1 is minimum when $\bar{V}_1 = \sqrt{5\,gr}$

$$\therefore \quad T_{1(\text{min})} = 5mg + mg = 6mg$$
$$= 6 \times 0.4 \times 10 = 24 \text{ N}$$

Thus we see that $T_{1(\text{min})}$ is more than the breaking strength of the string. Hence, the particle can not describe the vertical circle.

Example 3. *A small stone of mass 0.2 kg tied to a massless, inextensible string, is rotated in a vertical circle of radius 2 m. If the particle is just able to complete the vertical circle. What is its speed at the heighest point of its circular path? How would this speed get effected if the mass of the stone is increased by 50%?*

(Take $g = 10 \text{ ms}^{-2}$)

Sol. Let $\vec{V}_1$ be the speed of the stone at the lowest point of its vertical circle.

As the stone is just able to complete the circular path, then

$$\vec{V}_1 = \sqrt{5gr} = \sqrt{5 \times 10 \times 2} = 10 \text{ ms}^{-1}$$

Now, let $\vec{V}_2$ be the speed of the stone at the highest point of the circular path, then

$$\left|\vec{V}_2\right|^2 = \left|\vec{V}_1\right|^2 - 4gr = (10)^2 - 4 \times 10 \times 2 = 100 - 80 = 20$$

$$\therefore \quad \vec{V}_2 = \sqrt{20} = 4.47 \text{ ms}^{-1}$$

So from the above result, we can see that the value of $\vec{V}_2$ does not depend on the mass (m) of the stone. Hence $\vec{V}_2$ would remain the same when the mass of the stone is increased by 50%.

Example 4. *A particle of mass 150 g is attached to one end of a massless, inextensible string. It is made to describe a vertical circle of radius 1 m. When the string is making an angle of 48.2° with the vertical, its instantaneous speed is 2 ms^{-1}. What is the tension in the string in this position? Would this particle be able to complete its circular path? (Take $g = 10 \text{ ms}^{-2}$)*

Sol. The tension in the string when it makes angle θ with the vertical line is given by

$$T = \frac{mV^2}{r} + mg\cos\theta \quad (V \text{ is the instantaneous velocity})$$

$$T = \frac{0.15 \times (2)^2}{1} + 0.15 \times 10 \times \cos 48.2°$$
$$= (0.6 + 1.5 \times 0.67) \text{ N} = 1.6 \text{ N}$$

Now let $\vec{V}_1$ be the speed of the particle at the lowest point

$$\therefore \quad V_1^2 = V^2 + 2gr(1 - \cos\theta)$$
$$= (2)^2 + 2 \times 10 \times 1\,(1 - \cos 48.2°)$$
$$= 4 + 20(1 - 0.67) = 4 + 20 \times 0.33$$
$$= 4 + 6.6 = 10.6$$

$$\therefore \quad V_1 = \sqrt{10.6} = 3.25 \text{ ms}^{-1}$$

The minimum value of V_1 so that the particle is able to complete its vertical circle is $\sqrt{5gr}$

$$\therefore \qquad \vec{V}_{1(\min)} = \sqrt{5 \times 10 \times 1} = 7.07 \text{ m/s}$$

The value of $\vec{V}_1$ obtained above is less than this minimum speed. The particle in the given case would not be able to complete its vertical circular path.

Example 5. *A bucket containing 4 kg of water is tied to rope of length 2.5 m and rotated is a vertical circle in such a way that the water is it just does not spill over when the bucket is in its upside down position. What is the speed of the bucket at the (a) highest and (b) lowest point of its circular path. (Take g = 10 ms⁻²)*

Sol. Let $\vec{V}_1$ be the speed of the bucket at the lowest point then

$$\vec{V}_1 = \sqrt{5gr} = \sqrt{5 \times 10 \times 2.5}$$

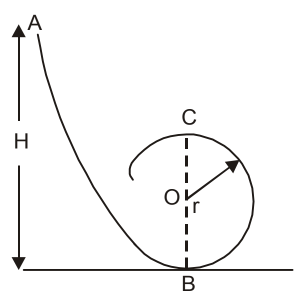
$$= \sqrt{125} = 11.18 \text{ m/s}$$

Now let $\vec{V}_2$ be the speed of the bucket at the highest point of its circular path then

$$\vec{V}_2 = \sqrt{gr} = \sqrt{10 \times 2.5} = 5 \text{ ms}^{-1}$$

Example 6. *The figure here shows a smooth 'Looping the loop' track. A particle of mass m is released from point A as shown. If H = 3r, would the particle "loop the loop"? What is the force on the circular track when the particle is a point (i) B (ii) C?*

Sol. Let $\vec{V}_B$ be the speed acquired by the particle at the lowest point B. From the law of conservation of energy, we have



Total energy at A =Total energy at B

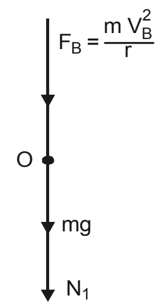
$$\therefore \qquad (0 + mg\,H) = \frac{1}{2} m V_B^2 + 0$$

$$\Rightarrow \qquad mgH = \frac{1}{2} m V_B^2$$

$$\Rightarrow \qquad mg(3r) = \frac{1}{2} m V_B^2$$

$$\therefore \qquad V_B = \sqrt{6gr}$$

So the minimum speed, needed by the particle at B, so that it can 'loop the loop' is $\sqrt{6gr}$. Since the value of V_B is more than $\sqrt{5gr}$, the particle would 'loop and loop'.



Let N_1 be the force exerted on the particle by the track. According to Newton's third law, the force exerted by the particle on the track is equal and opposite to N_1.

$$\therefore \quad N_1 = mg + \frac{mV_B^2}{r}$$

$$= mg + \frac{m \times 6gr}{r}$$

$$= mg + 6mg$$

$$= 7mg \text{ N}$$

Hence the force exerted by the particle on the track is 7 mg N directed vertically downwards.

Now the forces acting on the particle in position C, as shown in figure, the speed $\vec{V}_C$ of the particle at C is given by

$$V_C^2 = V_B^2 - 4gr$$

$$= 6gr - 4 gr$$

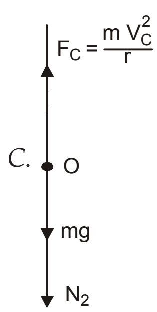
$$= 2gr$$

$$\therefore \quad N_2 = \frac{mV_2^2}{r} - mg$$

$$= 2mg - mg$$

$$= mg \text{ N}$$

Hence the particle exerts a force mg N directed outwards on the track at point C.



- **IMPORTANT TABLES**

TABLE 5.1 Same physical Quantity, Symbols, Units and Dimensions

Physical Quantity	*Symbol*	*Units*	*Dimensions*
Momentum	P	kg ms^{-1} or Ns	[MLT^{-1}]
Force	F	N	[MLT^{-2}]
Impulse	I/J	kg ms^{-1} or Ns	[MLT^{-1}]
Static friction	f_s	N	[MLT^{-2}]
Kinetic friction	f_k	N	[MLT^{-2}]

TABLE 5.2. Coefficient of Limiting and Kinetic Friction

S.No.	*Surfaces in contact*	*Coeff. of limiting friction*	*Coeff. of kinetic friction*
1.	Wood on wood	0.70	0.40
2.	Wood on leather	0.50	0.40
3.	Steel on steel (mild)	0.74	0.57
4.	Steel on steel (hard)	0.78	0.42
5.	Steel on steel (greased)	0.10	0.05

NCERT TEXTBOOK QUESTIONS SOLVED

5.1. *Give the magnitude and direction of the net force acting on*

(a) a drop of rain falling down with a constant speed,

(b) a cork of mass 10 g floating on water,

(c) a kite skilfully held stationary in the sky,

(d) a car moving with a constant velocity of 30 km/h on a rough road,

(e) a high-speed electron in space far from all material objects, and free of electric and magnetic fields.

Sol. (*a*) As the drop of rain is falling with constant speed, in accordance with first law of motion, the net force on the drop of rain is zero.

(*b*) As the cork is floating on water, its weight is being balanced by the upthrust (equal to weight of water displaced). Hence net force on the cork is zero.

(*c*) Net force on a kite skilfully held stationary in sky is zero because it is at rest.

(*d*) Since car is moving with a constant velocity, the net force on the car is zero.

(*e*) Since electron is far away from all material agencies producing electromagnetic and gravitational forces, the net force on electron is zero.

5.2. *A pebble of mass 0.05 kg is thrown vertically upwards. Give the direction and magnitude of the net force on the pebble,*

(a) during its upward motion,

(b) during its downward motion,

(c) at the highest point where it is momentarily at rest. Do your answers change if the pebble was thrown at an angle of 45° with the horizontal direction?

Ignore air resistance.

Sol. (*a*) When the pebble is moving upward, the acceleration g is acting downward, so the force is acting downward is equal to

$$F = mg = 0.05 \text{ kg} \times 10 \text{ ms}^{-2} = 0.5 \text{ N}.$$

(*b*) In this case also $F = mg = 0.05 \times 10 = 0.5$ N. (downwards).

(*c*) The pebble is not at rest at highest point but has horizontal component of velocity.

The direction and magnitude of the net force on the pebble will not alter even if it is thrown at 45° because no other acceleration except 'g' is acting on pebble.

5.3. *Give the magnitude and direction of the net force acting on a stone of mass 0.1 kg,*

(*a*) *just after it is dropped from the window of a stationary train,*

(*b*) *just after it is dropped from the window of a train running at a constant velocity of 36 km/ h,*

(*c*) *just after it is dropped from the window of a train accelerating with 1 m* s^{-2},

(*d*) *lying on the floor of a train which is accelerating with 1 m* s^{-2}, *the stone being at rest relative to the train.*

Neglect air resistance throughout.

Sol. (*a*) Mass of stone = 0.1 kg

Net force, $F = mg = 0.1 \times 10 = 1.0$ N. (vertically downwards).

(*b*) When the train is running at a constant velocity, its acceleration is zero. No force acts on the stone due to this motion. Therefore, the force on the stone is the same (1.0 N.).

(*c*) The stone will experience an additional force F' (along horizontal) *i.e.,*

$$F' = ma = 0.1 \times 1 = 0.1 \text{ N}$$

As the stone is dropped, the force F' no longer acts and the net force acting on the stone

$$F = mg = 0.1 \times 10 = 1.0 \text{ N. (vertically downwards).}$$

(*d*) As the stone is lying on the floor of the train, its acceleration is same as that of the train.

$\therefore$ force acting on the stone, $F = ma = 0.1 \times 1 = 0.1$ N.

It acts along the direction of motion of the train.

5.4. *One end of a string of length l is connected to a particle of mass m and the other to a small peg on a smooth horizontal table. If the particle moves in a circle with speed v the net force on the particle (directed towards the centre) is:*

(*i*) T, (*ii*) $T - \frac{mv^2}{l}$, (*iii*) $T + \frac{mv^2}{l}$, (*iv*) 0

T is the tension in the string. [Choose the correct alternative].

Sol. (*i*) T

The net force T on the particle is directed towards the centre. It provides the centripetal force required by the particle to move along a circle.

5.5. *A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a speed of 15 m* s^{-1}. *How long does the body take to stop?*

Sol. Here $m = 20$ kg, $F = -50$ N (retardation force)

As $F = ma$

$\Rightarrow$ $a = \frac{F}{m} = \frac{-50}{20} = -2.5 \text{ ms}^{-2}$

Using equation, $v = u + at$

Given, $u = 15 \text{ ms}^{-1}$, $v = 0$

Now, $0 = 15 + (-25)\, t$

or, $t = 0.6$ s.

5.6. *A constant force acting on a body of mass 3.0 kg changes its speed from 2.0 m* s^{-1} *to 3.5 m* s^{-1} *in 25 s. The direction of the motion of the body remains unchanged. What is the magnitude and direction of the force?*

Sol. Here, $m = 3.0$ kg, $u = 2.0$ ms^{-1}

$v = 3.5$ ms^{-1}, $t = 25$ s

As $F = ma$ or $F = m\left(\frac{v-u}{t}\right)$ $\left[\because a = \frac{v-u}{t}\right]$

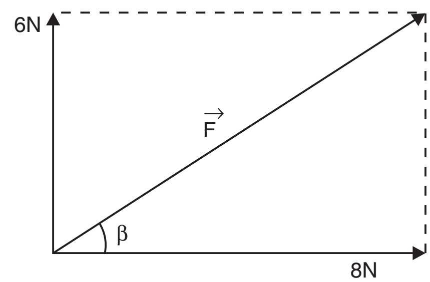
$$\Rightarrow \quad F = \frac{3.0\,(3.5-2.0)}{25} = 0.18\text{ N.}$$

The force is along the direction of motion.

5.7. *A body of mass 5 kg is acted upon by two perpendicular forces 8 N and 6 N. Give the magnitude and direction of the acceleration of the body.*

Sol. Here $m = 5$ kg

$F_1 = 8$ N and $F_2 = 6$ N



The resultant force on the body

$$F = \sqrt{F_1^2 + F_2^2} = \sqrt{8^2 + 6^2}\ N$$

$$\Rightarrow \quad F = \sqrt{64+36}\text{ N} = 10\text{ N}.$$

The acceleration, $a = \frac{F}{m}$

$$\Rightarrow \quad a = \frac{10}{5} = 2\text{ ms}^{-2}$$ in the same direction as the resultant force.

The direction of acceleration,

$$\tan\beta = \frac{6}{8} = \frac{3}{4} = 0.75$$

or $\beta = \tan^{-1}(0.75) = 37°$ with 8 N force.

5.8. *The driver of a three-wheeler moving with a speed of 36 km/h sees a child standing in the middle of the road and brings his vehicle to rest in 4.0 s just in time to save the child. What is the average retarding force on the vehicle? The mass of the three-wheeler is 400 kg and the mass of the driver is 65 kg.*

Sol. Here mass of three-wheeler $m_1 = 400$ kg, mass of driver $= m_2 = 65$ kg, initial speed of auto,

$u = 36$ km/h $= 36 \times \frac{5}{18}$ m/s $= 10$ ms^{-1}, final speed, $v = 0$ and $t = 4$s.

As acceleration, $a = \frac{v-u}{t} = \frac{0-10}{4} = -2.5$ ms^{-2}

Now $F = (m_1 + m_2)\,a = (400 + 65) \times (-2.5)$

$= -1162.5$ N $= -1.2 \times 10^3$ N.

The $-ve$ sign shows that the force is retarding force.

5.9. *A rocket with a lift-off mass 20,000 kg is blasted upwards with an initial acceleration of 5.0 ms^{-2}. Calculate the initial thrust (force) of the blast.*

Sol. Here, $m = 20000$ kg $= 2 \times 10^4$ kg

Initial acceleration $= 5$ ms^{-2}

Clearly, the thrust should be such that it overcomes the force of gravity besides giving it an upward acceleration of 5 ms^{-2}.

Thus the force should produce a net acceleration of 9.8 + 5.0 = 14.8 ms^{-2}.

Since, thrust = force = mass × acceleration

$\therefore$ $F = 2 \times 10^4 \times 14.8 = 2.96 \times 10^5$ N.

5.10. *A body of mass 0.40 kg moving initially with a constant speed of 10 m s^{-1} to the north is subject to a constant force of 8.0 N directed towards the south for 30 s. Take the instant the force is applied to be t = 0, the position of the body at that time to be x = 0, and predict its position at t = –5 s, 25 s, 100 s.*

Sol. Here $m = 0.40$ kg, $u = 10\ ms^{-1}$, $F = -8$ N (retarding force)

As $a = \frac{F}{m} = -\frac{8}{0.4} = -20\ ms^{-2}$

Also $S = ut + \frac{1}{2}at^2$

(*i*) Position at $t = -5$s

$$S = 10(-5) + \frac{1}{2} \times 0 \times (-5)^2 = -50 \text{ m}$$

(*ii*) Position at $t = 25$ s

$$S_1 = 10 \times 25 + \frac{1}{2} \times (-20) \times (25)^2 = -6000 \text{ m} = -6 \text{ km}$$

(*iii*) Position at $t = 100$ s

$$S_2 = 10 \times 30 + \frac{1}{2} \times (-20) \times (30)^2 = -8700 \text{ m}$$

At $t = 30$ s, $v = u + at$

$v = 10 - 20 \times 30 = -590\ ms^{-1}$

Now, for motion from 30 s to 100 s

$$S_3 = -590 \times 70 + \frac{1}{2}(0) \times (70)^2 = -41300 \text{ m}$$

Total distance = $S_2 + S_3 = -8700 - 41300 = -50000$ m = – 50 km.

5.11. *A truck starts from rest and accelerates uniformly at 2.0 m s^{-2}. At t = 10 s, a stone is dropped by a person standing on the top of the truck (6 m high from the ground). What are the (a) velocity, and (b) acceleration of the stone at t = 11s? (Neglect air resistance.)*

Sol. $u = 0$, $a = 2\ ms^{-2}$, $t = 10$s

Using equation, $v = u + at$, we get

$v = 0 + 2 \times 10 = 20\ ms^{-1}$

(*a*) Let us first consider horizontal motion. The only force acting on the stone is force of gravity which acts vertically downwards.

Its horizontal component is zero. Moreover, air resistance is to be neglected. So, horizontal motion is uniform motion.

$\therefore$ $v_x = v = 20\ ms^{-1}$

Let us now consider vertical motion which is controlled by force of gravity.

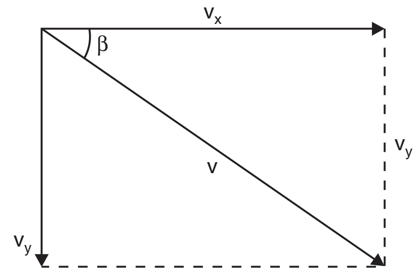
$u = 0$, $a = g = 10\ ms^{-2}$, $t = (11 - 10)$ s = 1*s*

Using $v = u + at$, $v_y = 0 + 10 \times 1 = 10\ ms^{-1}$

Resultant velocity,

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\Rightarrow \qquad v = \sqrt{20^2 + 10^2} \text{ ms}^{-1}$$
$$= \sqrt{500} \text{ ms}^{-1}$$
$$= 22.36 \text{ ms}^{-1}.$$
$$\tan \beta = \frac{v_y}{v_x} = \frac{10}{20} = \frac{1}{2} = 0.5$$

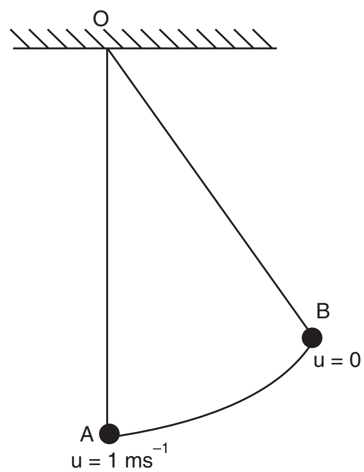


or $\beta = \tan^{-1} (0.5) = 26.56°$

or $\beta = 26° 34'$. This angle is with the horizontal.

(*b*) The moment the stone is dropped from the car, horizontal force on the stone is zero. The only acceleration of the stone is that due to gravity. This gives a vertically downward acceleration of 10 ms^{-2}. This is also the net acceleration of the stone.

5.12. *A bob of mass 0.1 kg hung from the ceiling of a room by a string 2 m long is set into oscillation. The speed of the bob at its mean position is 1 m s^{-1}. What is the trajectory of the bob if the string is cut when the bob is (a) at one of its extreme positions, (b) at its mean position ?*



Sol. Let the bob be oscillating as shown in the figure.

(*a*) When the bob is at its extreme position (say *B*), then its velocity is zero. Hence on cutting the string the bob will fall vertically downward under the force of its weight $F = mg$.

(*b*) When the bob is at its mean position (say *A*), it has a horizontal velocity of $v = 1$ ms^{-1} and on cutting the string it will experience an acceleration $a = g = 10$ ms^{-2} in vertical downward direction. Consequently, the bob will behave like a projectile and will fall on ground after describing a parabolic path.

5.13. *A man of mass 70 kg, stands on a weighing machine in a lift, which is moving*

(a) upwards with a uniform speed of 10 ms^{-1}.

(b) downwards with a uniform acceleration of 5 ms^{-2}.

(c) upwards with a uniform acceleration of 5 ms^{-2}.

What would be the readings on the scale in each case?

(d) What would be the reading if the lift mechanism failed and it hurtled down freely under gravity?

Sol. Here, $m = 70$ kg, $g = 10$ m/s^2

The weighing machine in each case measures the *reaction R i.e.,* the apparent weight.

(*a*) When the lift moves upwards with a uniform speed, its *acceleration is zero.*

$$\therefore \qquad R = mg = 70 \times 10 = 700 \text{ N}$$

(*b*) When the lift *moves downwards* with $a = 5$ ms^{-2}

$$R = m(g - a) = 70 (10 - 5) = 350 \text{ N}$$

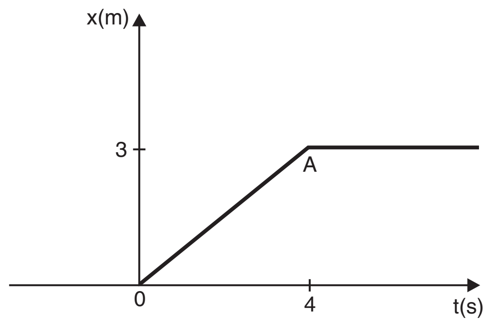
(*c*) When the lift *moves upwards* with

$$a = 5 \text{ ms}^{-2}$$
$$R = m(g + a) = 70 (10 + 5) = 1050 \text{ N}$$

(*d*) If the lift were to come down freely under gravity, downward acc. $a = g$

$\therefore \quad R = m(g - a) = m(g - g) = \text{Zero}.$

5.14. *Figure shows the position-time graph of a particle of mass 4 kg. What is the (a) force on the particle for $t < 0$, $t > 4$ s, $0 < t < 4$ s? (b) impulse at $t = 0$ and $t = 4$ s? (Consider one-dimensional motion only).*



Sol. (*a*) When $t < 0$. As this part is horizontal, thus it can be concluded that distance covered by the particle is zero and hence force on the particle is zero.

When $0 < t < 4$s. As OA has a constant slope, hence in this interval, particle moves with constant velocity $\left(\frac{3}{4} = 0.75 \text{ ms}^{-1}\right)$. Hence force on the particle is zero.

When $t > 4$s. As this portion shows that particle always remains at a distance of 3 m from the origin *i.e.,* the particle is at rest. Hence force on the particle is zero.

(*b*) Impulse at $t = 0$. Here $u = 0$, $v = 0.75$ ms^{-1}, $M = 4$ kg

$\therefore \quad \text{Impulse} = \text{total change in momentum} = Mv - Mu$

$= M(v - u) = 4(0.75 - 0) = 3 \text{ kg ms}^{-1}$

Impulse at $t = 4$s. Here $u = 0.75$ ms^{-1}, $v = 0$

$\therefore \quad \text{Impulse} = M(v - u) = 4(0 - 0.75) = -3 \text{ kg ms}^{-1}.$

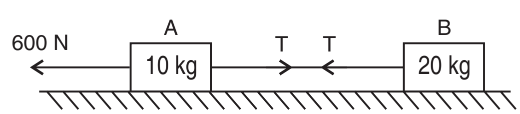
5.15. *Two bodies of masses 10 kg and 20 kg respectively kept on a smooth, horizontal surface are tied to the ends of a light string. A horizontal force F = 600 N is applied to (i) A, (ii) B along the direction of string. What is the tension in the string in each case?*

Sol. $\text{Acceleration} = \dfrac{600 \text{ N}}{10 \text{ kg} + 20 \text{ kg}} = 20 \text{ ms}^{-2}$

(*i*) When force is applied on 10 kg mass

$600 - T = 10 \times 20 \quad \text{or}$

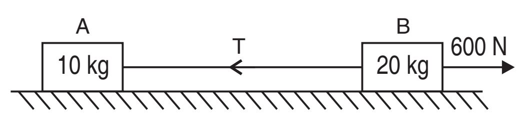
$T = 400 \text{ N}$



(*ii*) When force is applied on 20 kg mass,

$600 - T = 20 \times 20 \quad \text{or}$

$T = 200 \text{ N}$



5.16. *Two masses 8 kg and 12 kg are connected at the two ends of a light inextensible string that goes over a frictionless pulley. Find the acceleration of the masses, and the tension in the string when the masses are released.*

Sol. For block $m_2 \rightarrow m_2 g - T = m_2 a$ ...(*i*)

and for block $m_1 \rightarrow T - m_1 g = m_1 a$ ...(*ii*)

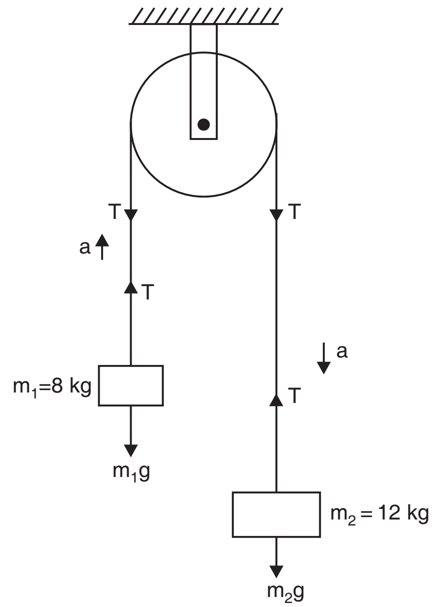
Adding (*i*) and (*ii*), we obtain

$$(m_2 - m_1)\, g = (m_2 + m_1)\, a$$

or
$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g$$

$$= \frac{12-8}{12+8} \times 10$$

$$= \frac{4 \times 10}{20} = 2 \text{ ms}^{-2}$$



Substituting value of a in equation (*ii*), we obtain

$$T = m_1 (g + a)$$
$$= 8 \times (10 + 2)$$
$$= 8 \times 12 = 96 \text{ N}.$$

5.17. *A nucleus is at rest in the laboratory frame of reference. Show that if it disintegrates into two smaller nuclei the products must move in opposite directions.*

Sol. Let m_1, m_2 be the masses of products and $\vec{v}_1, \vec{v}_2$ be their respective velocities. Therefore, total linear momentum after disintegration = $m_1\vec{v}_1 + m_2\vec{v}_2$. Before disintegration, the nucleus is at rest.

Therefore, its linear momentum before disintegration is zero.

According to the principle of conservation of linear momentum,

$$m_1\vec{v}_1 + m_2\vec{v}_2 = 0 \quad \text{or} \quad \vec{v}_2 = -\frac{m_1\vec{v}_1}{m_2}$$

Negative sign shows that $\vec{v}_1$ and $\vec{v}_2$ are in opposite directions.

5.18. *Two billiard balls, each of mass 0.05 kg, moving in opposite directions with speed 6 ms^{-1} collide and rebound with the same speed. What is the impulse imparted to each ball due to the other?*

Sol. Initial momentum of each ball before collision

$= 0.05 \times 6$ kg m s^{-1} = 0.3 kg m s^{-1}

Final momentum of each ball after collision

$= - 0.05 \times 6$ kg m s^{-1} = – 0.3 kg m s^{-1}

Impulse imparted to each ball due to the other

= final momentum – initial momentum

= – 0.3 kg m s^{-1} – 0.3 kg m s^{-1}

= – 0.6 kg m s^{-1} = **0.6 kg m s^{-1}** (in magnitude)

The two impulses are **opposite in direction.**

5.19. *A shell of mass 0.020 kg is fired by a gun of mass 100 kg. If the muzzle speed of the shell is 80 m s^{-1}, what is the recoil speed of the gun?*

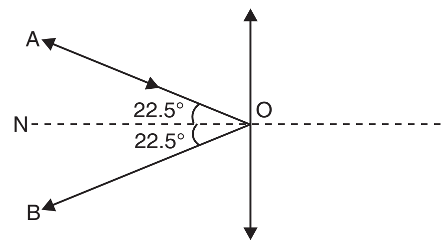
Sol. $m = 0.02$ kg, $M = 100$ kg, $v = 80$ m s^{-1}, $V = ?$

$$V = -\frac{mv}{M} = -\frac{0.020\text{ kg} \times 80\text{ m s}^{-1}}{100\text{ kg}} = -0.016\text{ m s}^{-1} = \mathbf{-1.6\text{ cm s}^{-1}}$$

Negative sign indicates that the gun moves in a direction opposite to the direction of motion of the bullet.

5.20. *A batsman deflects a ball by an angle of 45° without changing its initial speed which is equal to 54 km/h. What is the impulse imparted to the ball? (Mass of the ball is 0.15 kg.)*

Sol. Suppose the point O as the position of bat. AO line shows the path along which the ball strikes the bat with velocity u and OB is the path showing deflection such that $\angle AOB$ = 45°. Now initial momentum of ball



$$= mu \cos \theta$$

$$= \frac{0.15 \times 54 \times 1000 \times \cos 22.5}{3600}$$

$$= 0.15 \times 15 \times 0.9239 \text{ along } NO$$

Final momentum of ball = $mu \cos \theta$ along ON

$$\text{Impulse} = \text{change in momentum} = mu \cos \theta - (-mu \cos \theta)$$

$$= 2\, mu \cos \theta = 2 \times 0.15 \times 15 \times 0.9239 = 4.16 \text{ kg ms}^{-1}.$$

5.21. *A stone of mass 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with a speed of 40 rev./min in a horizontal plane. What is the tension in the string? What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 200 N?*

Sol. Here, $m = 0.25$ kg, $r = 1.5$ m

$$n = 40 \text{ rpm} = \frac{40}{60} \text{ rps} = \frac{2}{3} \text{ rps}$$

Now $T = mr\omega^2 = mr(2\pi n)^2 = 4\pi^2 mrn^2$

$$T = 4 \times \frac{22}{7} \times \frac{22}{7} \times 0.25 \times 1.5 \times \left(\frac{2}{3}\right)^2 = 6.6 \text{ N}$$

If $T_{max} = 200$ N, then from

$$T_{max} = \frac{mv_{max}^2}{r} \Rightarrow v_{max}^2 = \frac{T_{max} \times r}{m}$$

or $$v_{max}^2 = \frac{200 \times 1.5}{0.25} = 1200 \Rightarrow v_{max} = \sqrt{1200} = 34.6 \text{ ms}^{-1}.$$

5.22. *If, in Exercise 5.21, the speed of the stone is increased beyond the maximum permissible value, and the string breaks suddenly, which of the following correctly describes the trajectory of the stone after the string breaks:*

(a) the stone moves radially outwards,

(b) the stone flies off tangentially from the instant the string breaks,

(c) the stone flies off at an angle with the tangent whose magnitude depends on the speed of the particle?

Sol. (*b*) The velocity is tangential at each point of circular motion. At the time the string breaks, the particle continues to move in the tangential direction according to Newton's first law of motion.

5.23. *Explain why*

(a) a horse cannot pull a cart and run in empty space,

(b) passengers are thrown forward from their seats when a speeding bus stops suddenly,

(c) it is easier to pull a lawn mower than to push it,

(d) a cricketer moves his hands backwards while holding a catch.

Sol. (*a*) A horse by itself cannot move in space due to law of inertia and so cannot pull a cart in space.

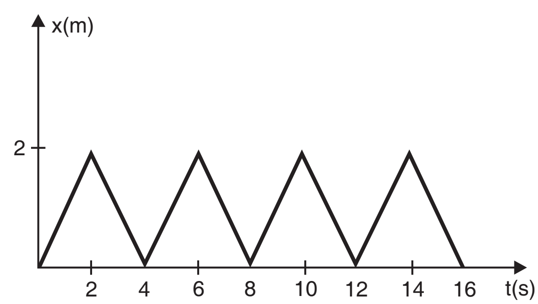
(*b*) The passengers in a speeding bus have inertia of motion. When the bus is suddenly stopped the passengers are thrown forward due to this inertia of motion.

(*c*) In the case of pull, the effective weight is reduced due to the vertical component of the pull. In the case of push, the vertical component increases the effective weight.

(*d*) The ball comes with large momentum after being hit by the batsman. When the player takes catch it causes large impulse on his palms which may hurt the cricketer. When he moves his hands backward the time of contact of ball and hand is increased so the force is reduced.

5.24. *Figure shows the position-time graph of a particle of mass 0.04 kg. Suggest a suitable physical context for this motion. What is the time between two consecutive impulses received by the particle ? What is the magnitude of each impulse?*

Sol. This graph can be of a ball rebounding between two walls situated at position 0 cm and 2 cm. The ball is rebounding from one wall to another, time and again every 2 s with uniform velocity.



Impulse. Here, Velocity $= \dfrac{\text{displacement}}{\text{time}} = \dfrac{2}{100 \times 2} = 0.01\ \text{ms}^{-1}$

Initial momentum $= mu = 0.04 \times 0.01 = 4 \times 10^{-4}\ \text{kg ms}^{-1}$

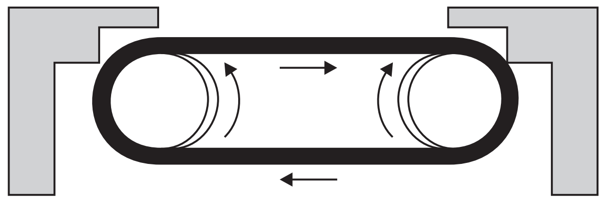
Final momentum $= mv = 0.04 \times (-\,0.01) = -\,4 \times 10^{-4}\ \text{kg ms}^{-1}$

Magnitude of Impulse = Change in momentum

$= (4 \times 10^{-4}) - (-\,4 \times 10^{-4}) = 8 \times 10^{-4}\ \text{kg ms}^{-1}$

Time between two consecutive impulses is 2 s *i.e.,* the ball receives an impulse every 2 *s*.

5.25. *Figure shows a man standing stationary with respect to a horizontal conveyor belt that is accelerating with 1 ms^{-2}. What is the net force on the man? If the coefficient of static friction between the man's shoes and the belt is 0.2, up to what acceleration of the belt can the man continue to be stationary relative to the belt? (Mass of the man = 65 kg.)*



Sol. Here acceleration of conveyor belt a = 1 ms^{-2}, μ_s = 0.2 and mass of man m = 65 kg.

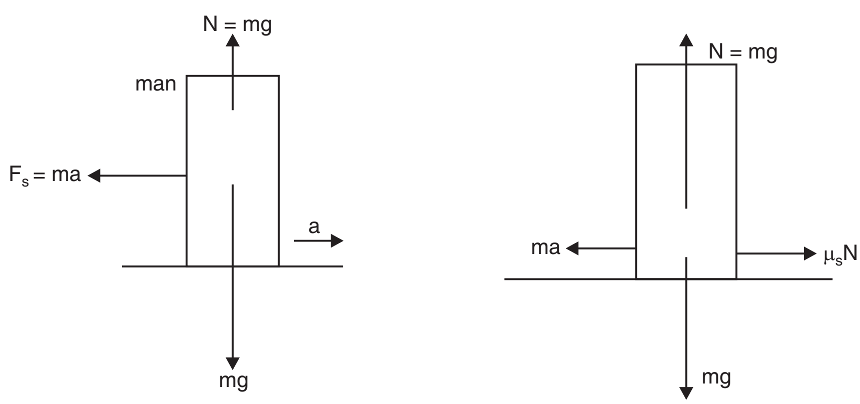
As the man is in an accelerating frame, he experiences a pseudo force $F_s = ma$ as shown in fig. (*a*). Hence to maintain his equilibrium, he exerts a force $F = -F_s = ma = 65 \times 1 =$ 65 N in forward direction *i.e.,* direction of motion of belt.

$\therefore$ Net force acting on man = 65 N (forward)

As shown in fig. (*b*), the man can continue to be stationary with respect to belt, if force of friction

$$\mu_s N = \mu_s mg = ma_{max}$$

$$a_{max} = \mu_s \cdot g = 0.2 \times 10 = 2 \text{ m s}^{-2}$$



(*a*) (*b*)

5.26. *A stone of mass m tied to the end of a string is revolving in a vertical circle of radius R. The net force at the lowest and highest points of the circle directed vertically downwards are: (choose the correct alternative).*

	Lowest Point	*Highest Point*
(a)	$mg - T_1$	$mg + T_2$
(b)	$mg + T_1$	$mg - T_2$
(c)	$mg + T_1 - (mv_1^2)/R$	$mg - T_2 + (mv_1^2)/R$
(d)	$mg - T_1 - (mv_1^2)/R$	$mg + T_1 + (mv_1^2)/R$

T_1 and v_1 denote the tension and speed at the lowest point. T_2 and v_2 denote corresponding values at the highest point.

Sol. The net force at the lowest point is ($mg - T_1$) and the net force at the highest point is ($mg + T_2$). Therefore, alternative (*a*) is correct.

Since mg and T_1 are in mutually opposite directions at lowest point and mg and T_2 are in same direction at the highest point.

5.27. *A helicopter of mass 1000 kg rises with a vertical acceleration of 15 ms^{-2}. The crew and the passengers weigh 300 kg. Give the magnitude and direction of*

(a) force on the floor by the crew and passengers,

(b) action of the rotor of the helicopter on surrounding air,

(c) force on the helicopter due to the surrounding air,

Sol. Here, mass of helicopter, $m_1 = 1000$ kg

Mass of the crew and passengers, $m_2 = 300$ kg

upward acceleration, $a = 15$ ms^{-2} and $g = 10$ ms^{-2}

(*a*) Force on the floor of helicopter by the crew and passengers = apparent weight of crew and passengers

$$= m_2 (g + a) = 300 (10 + 15) \text{ N} = 7500 \text{ N}$$

(*b*) Action of rotor of helicopter on surrounding air is obviously *vertically downwards,* because helicopter rises on account of reaction to this force. Thus, force of action

$F = (m_1 + m_2)(g + a) = (1000 + 300)(10 + 15) = 1300 \times 25 = 32500$ N

(*c*) Force on the helicopter due to surrounding air is the *reaction.* As action and reaction are equal and opposite, therefore, force of reaction, $F' = 32500$ N, *vertically upwards.*

5.28. *A stream of water flowing horizontally with a speed of 15 ms^{-1} pushes out of a tube of cross sectional area 10^{-2} m^2, and hits at a vertical wall nearby. What is the force exerted on the wall by the impact of water, assuming that it does not rebound?*

Sol. In one second, the distance travelled is equal to the velocity v.

$\therefore$ Volume of water hitting the wall per second, $V = av$ where a is the cross-sectional area of the tube and v is the speed of water coming out of the tube.

$$V = 10^{-2} \text{ m}^2 \times 15 \text{ ms}^{-1} = 15 \times 10^{-2} \text{ m}^3 \text{ s}^{-1}$$

Mass of water hitting the wall per second

$$= 15 \times 10^{-2} \times 10^3 \text{ kg s}^{-1} = 150 \text{ kg s}^{-1} \quad [\because \text{density of water} = 1000 \text{ kg m}^{-3}]$$

Initial momentum of water hitting the wall per second

$$= 150 \text{ kg s}^{-1} \times 15 \text{ ms}^{-1} = 2250 \text{ kg ms}^{-2} \text{ or } 2250 \text{ N}$$

Final momentum per second = 0

Force exerted by the wall = 0 – 2250 N = – 2250 N

$\therefore$ Force exerted on the wall = – (– 2250) N = 2250 N.

5.29. *Ten one rupee coins are put on top of one another on a table. Each coin has a mass m kg. Give the magnitude and direction of*

(a) the force on the 7th coin (counted from the bottom) due to all coins above it.

(b) the force on the 7th coin by the eighth coin and

(c) the reaction of the sixth coin on the seventh coin.

Sol. (*a*) The force on 7th coin is due to weight of the three coins lying above it. Therefore,

$$F = (3\ m)\ \text{kgf} = (3\ mg)\ \text{N}$$

where g is acceleration due to gravity. This force acts vertically downwards.

(*b*) The eighth coin is already under the weight of two coins above it and it has its own weight too. Hence force on 7th coin due to 8th coin is sum of the two forces i.e.

$$F = 2\ m + m = (3\ m)\ \text{kg f} = (3\ mg)\ \text{N}$$

The force acts vertically downwards.

(*c*) The sixth coin is under the weight of four coins above it.

$$\text{Reaction, } R = -F = -4\ m\ (\text{kgf}) = -(4\ mg)\ \text{N}$$

Minus sign indicates that the reaction acts vertically *upwards*, opposite to the weight.

5.30. *An aircraft executes a horizontal loop at a speed of 720 km/h with its wings banked at 15°. What is the radius of the loop?*

Sol. Here $v = 720\ \text{km/h} = 720 \times \frac{5}{18}\ \text{m/s} = 200\ \text{m/s}$ and angle of banking $\theta = 15°$

From the relation

$$\tan\theta = \frac{v^2}{rg} \quad \text{we have}$$

$$r = \frac{v^2}{g\tan\theta} = \frac{200 \times 200}{10 \times \tan 15°} = \frac{200 \times 200}{10 \times 0.2679}$$

$$\Rightarrow \qquad r = 14931\ \text{m} = 14.9\ \text{km}.$$

5.31. *A train runs along an unbanked circular track of radius 30 m at a speed of 54 km/h. The mass of the train is 10^6 kg. What provides the centripetal force required for this purpose — the engine or the rails? What is the angle of banking required to prevent wearing out of the rail?*

Sol. Here $r = 30$ m, $v = 54\ \text{km/h} = 54 \times \frac{5}{18}\ \text{m/s} = 15\ \text{m/s}$, mass of train $m = 10^6$ kg.

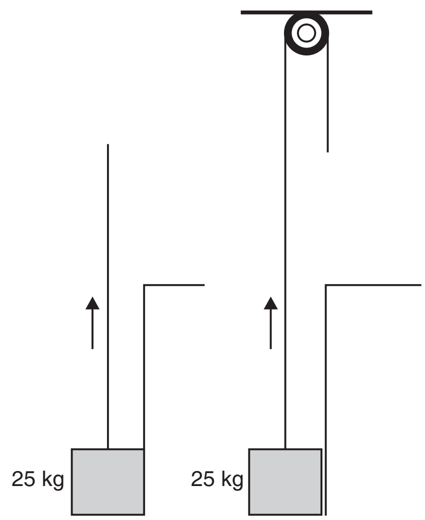
The centripetal force $F = \frac{mv^2}{r}$ for negotiating the circular track is provided by the force of lateral friction due to rails on the wheels of the train.

To prevent wearing out of rails, the angle of banking θ is given by

$$\tan\theta = \frac{v^2}{rg} = \frac{15 \times 15}{30 \times 10} = 0.75$$

$$\Rightarrow \qquad \theta = \tan^{-1}(0.75) \simeq 37°.$$

5.32. *A block of mass 25 kg is raised by a 50 kg man in two different ways as shown in Fig. What is the action on the floor by the man in the two cases? If the floor yields to a normal force of 700 N, which mode should the man adopt to lift the block without the floor yielding?*



Sol. In 1st case, man applies an upward force of 25 kg *wt*., (same as the weight of the block). According to Newton's third law of motion, there will be a downward reaction on the floor.

The action on the floor by the man

$$= 50 \text{ kg } wt. + 25 \text{ kg } wt. = 75 \text{ kg } wt. = 75 \text{ kg} \times 10 \text{ m/s}^2 = 750 \text{ N}.$$

In case II, the man applies a downward force of 25 kg *wt*. According to Newton's third law, the reaction is in the upward direction.

In this case, action on the floor by the man

$$= 50 \text{ kg } wt. - 25 \text{ kg } wt. = 25 \text{ kg } wt. = 25 \text{ kg} \times 10 \text{ m/s}^2 = 250 \text{ N}.$$

Therefore, the man should adopt the second method.

5.33. *A monkey of mass 40 kg climbs on a rope (Fig.) which can stand a maximum tension of 600 N. In which of the following cases will the rope break: the monkey*

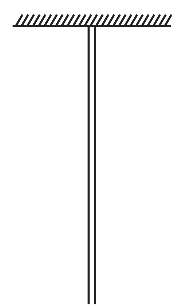
(a) climbs up with an acceleration of 6 ms^{-2}

(b) climbs down with an acceleration of 4 ms^{-2}

(c) climbs up with a uniform speed of 5 ms^{-1}

(d) falls down the rope nearly freely under gravity?

(Ignore the mass of the rope).



Sol. (*a*) When the monkey climbs up with an acceleration *a*, then

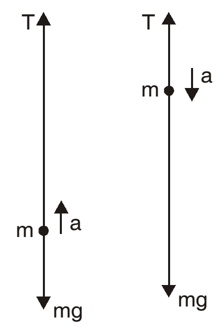
$$T - mg = ma$$

where T represents the tension (figure *a*).

$$\therefore \quad T = mg + ma = m(g + a)$$

$$\text{or} \quad T = 40 \text{ kg } (10 + 6) \text{ ms}^{-2} = 640 \text{ N}$$

But the rope can withstand a maximum tension of 600 N. So the rope will break.



(*a*) (*b*)

(*b*) When the monkey is climbing down with an acceleration, then

$$mg - T = ma \qquad \text{(Figure (}b\text{)}$$

$\Rightarrow$ $T = mg - ma = m(g - a)$

or $T = 40 \text{ kg} \times (10 - 4) \text{ ms}^{-2} = 240 \text{ N}$

The rope will not break.

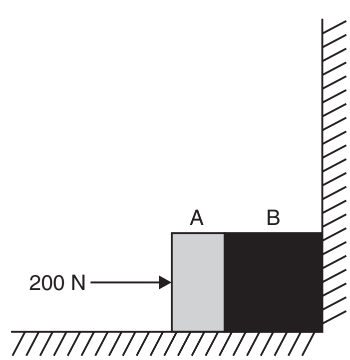
(*c*) When the monkey climbs up with uniform speed, then

$$T = mg = 40 \text{ kg} \times 10 \text{ ms}^{-2} = 400 \text{ N}$$

The rope will not break.

(*d*) When the monkey is falling freely, it would be a state of weightlessness. So, tension will be zero and the rope will not break.

5.34. *Two bodies A and B of masses 5 kg and 10 kg in contact with each other rest on a table against a rigid wall (Fig.). The coefficient of friction between the bodies and the table is 0.15. A force of 200 N is applied horizontally to A. What are (a) the reaction of the partition (b) the action-reaction forces between A and B? What happens when the wall is removed? Does the answer to (b) change, when the bodies are in motion? Ignore the difference between* μ_s *and* μ_k.



Sol. (*i*) When the wall exists and blocks *A* and *B* are pushing the wall, there can't be any motion *i.e.,* blocks are at rest. Hence, (*a*) reaction of the partition = – (force applied on A) = 200 N towards left.

(*b*) action reaction forces between A and B are 200 N each. A presses B towards right with an action force 200 N and B exerts a reaction force on A towards left having magnitude 200 N.

(*ii*) When the wall is removed, motion can take place such that net pushing force provides the acceleration to the block system. Hence, taking kinetic friction into account, we have

$$200 - \mu (m_1 + m_2) g = (m_1 + m_2) a$$

$$\Rightarrow \quad a = \frac{200 - \mu (m_1 + m_2) g}{(m_1 + m_2)} = \frac{200 - 0.15 \times (5+10) \times 10}{(5+10)}$$

$$= \frac{200 - 22.5}{15} = \frac{177.5}{15} = 11.8 \text{ ms}^{-2}$$

∴ If force exerted by A on B be F_{BA}, then considering equilibrium (or free body diagram) of only block A, we have

$$200 - f_{k_1} = m_1 a + F_{BA} \quad \text{or} \quad 200 - \mu m_1 g = m_1 a + F_{BA}$$

$$\Rightarrow \quad F_{BA} = 200 - \mu m_1 g - m_1 a = 200 - (0.15 \times 5 \times 10) - (5 \times 11.8)$$

$$= 200 - 7.5 - 59$$

$$= 200 - 66.5 = 133.5 \text{ N} \simeq 1.3 \times 10^2 \text{ N towards right}$$

∴ Force exerted on A by B $F_{AB} = - F_{BA} = 1.3 \times 10^2$ N towards left.

5.35. *A block of mass 15 kg is placed on a long trolley. The coefficient of static friction between the block and the trolley is 0.18. The trolley accelerates from rest with 0.5 m s^{-2} for 20 s and then moves with uniform velocity. Discuss the motion of the block as viewed by (a) a stationary observer on the ground, (b) an observer moving with the trolley.*

Sol. (*a*) Force experienced by block, $F = ma = 15 \times 0.5 = 7.5$ N

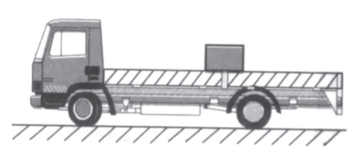
Force of friction, $F_f = \mu\, mg = 0.18 \times 15 \times 10 = 27$ N.

i.e., force experienced by block will be less than the friction.

So the block will not move. It will remain stationary w.r.t. trolley for a stationary observer on ground.

(*b*) The observer moving with trolley has an accelerated motion *i.e.,* he forms non-inertial frame in which Newton's laws of motion are not applicable. The box will be at rest relative to the observer.

5.36. *The rear side of a truck is open and a box of 40 kg mass is placed 5 m away from the open end as shown in Fig. The coefficient of friction between the box and the surface below it is 0.15. On a straight road, the truck starts from rest and accelerates with 2 m s^{-2}. At what distance from the starting point does the box fall off the truck? (Ignore the size of the box).*



Sol. Force experienced by box, $F = ma = 40 \times 2 = 80$ N

Frictional force $F_f = \mu mg = 0.15 \times 40 \times 10 = 60$ N.

Net force $= F - F_f = 80 - 60 = 20$ N.

Backward acceleration produced in the box, $a = \frac{20}{40}\left(\frac{\text{Net force}}{m}\right)$

$$\Rightarrow \quad a = 0.5 \text{ ms}^{-2}$$

If t is time taken by the box to travel $s = 5$ metre and fall off the truck, then from

$$S = ut + \frac{1}{2} at^2$$

$$5 = 0 \times t + \frac{1}{2} \times 0.5\, t^2$$

$$t = \sqrt{\frac{5 \times 2}{0.5}} = 4.47 \text{ s.}$$

If the truck travels a distance x during this time, then again from

$$S = ut + \frac{1}{2} at^2$$

$$x = 0 \times 4.47 + \frac{1}{2} \times 2\,(4.47)^2 = 19.98 \text{ m.}$$

5.37. *A disc revolves with a speed of $33\frac{1}{3}$ rpm and has a radius of 15 cm. Two coins are placed at 4 cm and 14 cm away from the centre of the record. If the coefficient of friction between the coins and record is 0.15, which of the coins will revolve with the record?*

Sol. If the coin is to revolve with the record, then the force of friction must be enough to provide the necessary centripetal force.

$$\therefore \quad mr\,\omega^2 \le \mu_s\, mg \quad \text{or} \quad r \le \frac{\mu_s\, mg}{m\,\omega^2} \quad \text{or} \quad r \le \frac{\mu_s\, g}{\omega^2}$$

$$\text{frequency} = 33\frac{1}{3}\text{ rpm} = \frac{100}{3}\text{ rpm} = \frac{100}{3 \times 60}\text{ rps}$$

The problems in which centripetal force is obtained from force of friction, start with the following equation:

$$m\, r\omega^2 \le \mu_s\, mg$$

$$\omega = 2\pi \times \frac{100}{3 \times 60}\text{ rad s}^{-1} = \frac{10}{9}\pi \text{ rad s}^{-1}$$

$$\frac{\mu_s\, g}{\omega^2} = \frac{0.15 \times 10}{\left(\frac{10}{9}\pi\right)^2}\text{ m} = 0.12 \text{ m} = 12 \text{ cm}$$

The condition ($r \le 12$ cm) is satisfied by the coin placed at 4 cm from the centre of the record. So, **the coin at 4 cm will revolve with the record.**

5.38. *You may have seen in a circus a motorcyclist driving in vertical loops inside a 'death well' (a hollow spherical chamber with holes, so the spectators can watch from outside). Explain clearly why the motorcyclist does not drop down when he is at the uppermost point, with no support from below. What is the minimum speed required at the uppermost position to perform a vertical loop if the radius of the chamber is 25 m?*

Sol. When the motorcyclist is at the highest point of the death-well, the normal reaction R on the motorcyclist by the ceiling of the chamber acts downwards. His weight mg also acts downwards. These two forces are balanced by the outward centrifugal force acting on him.

$$\therefore \qquad R + mg = \frac{mv^2}{r} \qquad ...(1)$$

Here v is the speed of the motorcyclist and m is the mass of the motorcyclist (including the mass of the motor cycle). Because of the balancing of the forces, the motorcyclist does not fall down.

The minimum speed required to perform a vertical loop is given by equation (1) when $R = 0$.

$$\therefore \qquad mg = \frac{mv^2_{\text{min}}}{r} \quad \text{or} \quad v^2_{\text{min.}} = gr$$

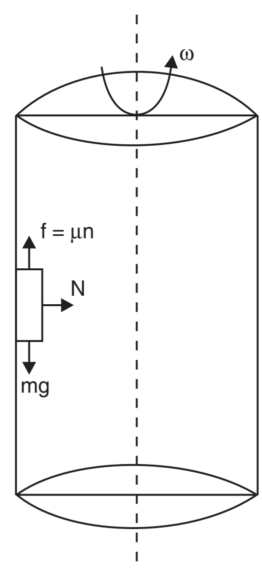
or $$v = \sqrt{gr} = \sqrt{10 \times 25} \text{ m s}^{-1} = 15.8 \text{ m s}^{-1}$$

5.39. *A 70 kg man stands in contact against the inner wall of a hollow cylindrical drum of radius 3 m rotating about its vertical axis with 200 rev/min. The coefficient of friction between the wall and his clothing is 0.15. What is the minimum rotational speed of the cylinder to enable the man to remain stuck to the wall (without falling) when the floor is suddenly removed?*

Sol. $R = 3$ m, $\omega = 200 \text{ rev/min} = 2 \times \frac{22}{7} \times \frac{200}{60}$ rad/s

$$= \frac{440}{21} \text{ rad/s}$$

and $$\mu = 0.15$$



As shown in the figure, the normal reaction (N) of the wall on the man acts in the horizontal direction towards the axis of the cylinder while the force of friction (f) acts vertically upwards. The required centripetal force will be provided by the horizontal reaction N of the wall on the man, *i.e.*,

$$N = \frac{mv^2}{R} = m\omega^2 R$$

The frictional force f acting vertically upwards will be balanced by the weight of the man. Hence, the man remains stuck to the wall after the floor is removed if $mg \leq$ limiting frictional force f_e (or μN)

or if $$mg \leq \mu\, m\omega^2 R$$

or $$g \leq \mu\, \omega^2 R$$

or $$\mu\, \omega^2 R \geq g \quad \text{or} \quad \omega \geq \frac{g}{R\mu}$$

Hence, for minimum rotational speed of the cylinder

$$\omega^2 = \frac{g}{\mu R} = \frac{10}{0.15 \times 3} = 22.2$$

$\Rightarrow$ $$\omega = \sqrt{22.2} = 4.7 \text{ rad/s.}$$

5.40. *A thin circular loop of radius R rotates about its vertical diameter with an angular frequency ω. Show that a small bead on the wire loop remains at its lowermost point for $\omega \le \sqrt{g/R}$. What is the angle made by the radius vector joining the centre to the bead with the vertical downward direction for $\omega = \sqrt{2g/R}$? Neglect friction.*

Sol. Let the radius vector joining the bead to the centre of the wire make an angle θ with the vertical downward direction. If N is normal reaction, then from fig,

$$mg = N \cos \theta \quad ...(i)$$

$$mr\omega^2 = N \sin \theta \quad ...(ii)$$

or $$m (R \sin \theta)\, \omega^2 = N \sin \theta$$

or $$mR\omega^2 = N$$

From equation (i), $mg = mR\omega^2 \cos \theta$

or $$\cos \theta = \frac{g}{R\omega^2} \quad ...(iii)$$

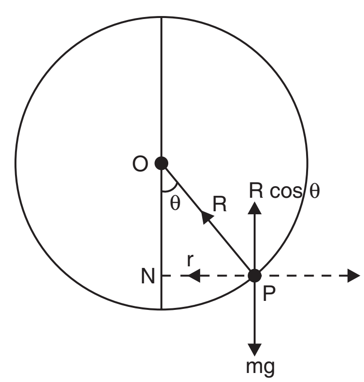
As $|\cos \theta| \le 1$, therefore bead will remain at its lowermost point for

$$\frac{g}{R\omega^2} \le 1 \quad \text{or} \quad \omega \le \sqrt{\frac{g}{R}}$$

When $$\omega = \sqrt{\frac{2g}{R}}$$ from equation (iii),

$$\cos \theta = \frac{g}{R}\left(\frac{R}{2g}\right) = \frac{1}{2}$$

$\therefore$ $$\theta = 60°.$$



QUESTIONS BASED ON SUPPLEMENTARY CONTENTS

Q. 1. *A stone of mass 0.2 kg is tied to one end of a string of length 80 cm. Holding the other end, the stone is whiled into a vertical circle. What is the minimum speed of the stone at the lowest point so that it just completes the circle. What is the tension in the string at the lowest point of the circular path? (g = 10 ms^{-2})*

Sol.

Here $$m = 0.2 \text{ kg}$$

$$r = 80 \text{ cm} = 0.8 \text{ m}$$

$\therefore$ V_{min} at the lowest point so that the stone is just able to complete the circle.

$$= \sqrt{5gr} = \sqrt{5 \times 10 \times 0.8} = \sqrt{40} = 6.32 \text{ ms}^{-1}$$

Now tension in the string at the lower point of the circular path = $\frac{mV_{min}^2}{r} + mg$

$$= 5mg + mg \qquad \left[\because \frac{mV_{min}^2}{r} = 5mg\right]$$

$$= 6mg = 6 \times 0.2 \times 10 = 12 \text{ N.}$$

Q. 2. *A particle of mass 100 mg is moving in a circular vertical path of radius 2 m. The particle is just 'looping the loop'. What is the speed of particle and the tension in the string at the highest point of the circular path? (g = 10 ms^{-2})*

Sol. Here,
$$m = 100 \text{ g} = 0.1 \text{ kg}$$
$$r = 2 \text{ m}$$

∴ Minimum speed of the particle at the highest point for just looping the loop

$$= \sqrt{gr} = \sqrt{10 \times 2} = \sqrt{20} = 4.47 \text{ ms}^{-1}$$

Tension in the string at highest point

$$= \frac{mV_1^2}{r} - 5mg = 5mg - 5mg = 0$$

Hence the tension in the string at the highest point is zero.

Q. 3. *A particle of mass 0.2 kg attached to a massless string is moving in a vertical circle of radius 1.2 m. It is imparted a speed of 8 ms^{-1} at the lowest point of its circular path. Does the particle complete the vertical circle? What is the change is tension in the string when the particle moves from the position where the string is vertical to the position where the string is horizontal?*

Sol. Here
$$m = 0.2 \text{ kg}$$
$$r = 1.2 \text{ m}$$
$$V_1 = 8 \text{ ms}^{-1}$$

∴ Min speed required to the particle at the lowest point of the circular path

$$= \sqrt{5gr} = \sqrt{5 \times 10 \times 1.2}$$
$$= \sqrt{60} = 7.7 \text{ ms}^{-1}$$

As the speed given to the particle at the lowest point of the circular path is 8 ms^{-1} which is more than 7.7 ms^{-1}, therefore, the particle will certainly complete the circle.

Now the change of tension in the string at the lowest point and the point where the string is horizontal

$$= 3 \text{ mg} = 3 \times 0.2 \times 10 = 6 \text{ N}$$

Q. 4. *A particle of mass 200 g is whirled into a vertical circle of radius 80 cm using a massless string. The speed of particle when the string makes an angle 60° with the vertical line is 1.5 ms^{-1}. What is the tension in the string is this position?*

Sol. Here,
$$m = 200 \text{ g} = 0.2 \text{ kg}$$
$$r = 80 \text{ cm} = 0.8 \text{ m}$$

$$V = 1.5 \text{ ms}^{-1}$$

$$\theta = 60°$$

$\therefore$ Required tension in the string

$$T = \frac{mV^2}{r} + mg\cos\theta = \frac{0.2 \times (1.5)^2}{0.8} + 0.2 \times 10 \times \cos 60°$$

$$= \frac{2.25}{4} + 0.2 \times 10 \times \frac{1}{2} = 0.56 + 1 = 1.56 \text{ N}$$

ADDITIONAL QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *Which physical quantity is found from the area under force-time graph?*

Ans. Impulse.

Q. 2. *Two objects having different masses have same momentum. Which one of them will move faster?*

Ans. Object with smaller mass.

Q. 3. *Write the units of momentum and impulse.*

Ans. kg ms^{-1} and Ns.

Q. 4. *An athelete runs some distance before taking a jump. Explain why?*

Ans. Running provides greater momentum which helps in jumping through a longer distance.

Q. 5. *What happens to coefficient of friction, when weight of body is doubled?*

Ans. Coefficient of friction is not affected as it depends only on material and nature of surfaces in contact.

Q. 6. *What is the angle of friction between two surfaces in contact, if coefficient of friction is* $\sqrt{3}$ *?*

Ans. Since $\tan\theta = \mu = \sqrt{3}$

$$\theta = \tan^{-1}\left(\sqrt{3}\right) \Rightarrow \theta = 60°$$

Q. 7. *What is the angle between frictional force and instantaneous velocity of the body moving over a rough surface?*

Ans. The angle is 180°, because force of friction always opposes the relative motion.

Q. 8. *Is a force required to maintain a body in its state of uniform motion along a straight line?*

Ans. No, force is not required to maintain a body in its state of uniform motion along a straight line.

Q. 9. *What is the effect on the acceleration of a particle if the net force on the particle is doubled?*

Ans. Since, $a = \frac{F}{m}$. On doubling the force, the acceleration will also be doubled.

Q. 10. *What is the apparent weight felt by a person in an elevator, when it is accelerating: (i) upward (ii) downward?*

Ans. (*i*) Apparent weight = $m(g + a)$ (*ii*) Apparent weight = $m(g - a)$.

Q. 11. *Is earth an inertial frame of reference?*

Ans. Since earth rotates on its own axis and also revolves around the sun, there will be acceleration associated. So earth cannot be taken as inertial frame of reference.

Q. 12. *A bus weighing 1000 kg is at rest on the bus stand. What is the linear momentum of the bus?*

Ans. Linear momentum, $\vec{p} = m\vec{v}$. As $\vec{v} = 0$, therefore $\vec{p} = 0$.

Q. 13. *Which law of motion is involved in rocket propulsion?*

Ans. Newton's third law of motion.

Q. 14. *Why does a heavy rifle not kick as strongly as a light rifle using the same cartridges ?*

Ans. The recoil speed of rifle $V = \frac{mv}{M}$ is inversely proportional to its mass. So for a heavy rifle the kick is less stronger.

Q. 15. *What is the effect on the direction of the centripetal force when the revolving body reverses its direction of motion?*

Ans. The centripetal force will be directed towards the centre of the circle. This fact does not depend upon the sense of rotation of the particle.

Q. 16. *A retarding force is applied to stop a motor car. If the speed of the motor car is doubled, how much more distance will it cover before stopping under the same retarding force ?*

Ans. Since, $S \propto v^2$, therefore motor car will cover a distance four times longer than before.

Q. 17. *When are the two given bodies said to have same inertial mass?*

Ans. If on applying same force on the bodies, same acceleration is produced, then their inertial masses are same.

Q. 18. *On what factors does the thrust on a rocket depend?*

Ans. The upward thrust on a rocket depends on exhaust speed of the gases w.r.t. the rocket and rate at which mass of the exhaust gases escapes.

Q. 19. *What do you mean by normal reaction?*

Ans. It is the reaction due to the surface on which the body moves. It acts perpendicular to the surface of contact.

Q. 20. *What provides the centripetal force to a car taking a turn on a level road?*

Ans. The force of friction between the road and the tyres of car.

Q. 21. *At which place on earth, is the centripetal force maximum?*

Ans. Since $F = \frac{mv^2}{r}$, so at the pole, the value of F is maximum.

Q. 22. *Calculate the force acting on a body whose linear momentum changes from 20 kg ms^{-1} to 40 kg ms^{-1} in 10 s.*

Ans. Force = rate of change of linear momentum = $\frac{20}{10}$ = 2 N.

Q. 23. *What provides the centripetal force in the following cases?*

(i) Electron revolving around the nucleus.

(ii) Earth revolving around the sun.

Ans. (*i*) Electrostatic force (*ii*) Gravitational force

Q. 24. *A body is describing a vertical circle of radius r. What are the values of its minimum speed at the bottom and top of the vertical circle?*

Ans. At bottom, $v = \sqrt{5gr}$ and at the top $v = \sqrt{gr}$.

Q. 25. *What are the units and dimensions of coefficient of friction?*

Ans. Coefficient of friction is a unitless and dimensionless term.

Q. 26. *The linear momentum of the body can change only in the direction of applied force. Comment.*

Ans. The statement is correct. It is in accordance with Newton's second law of motion.

Q. 27. *Rubber tyres are preferred over steel tyres. Why?*

Ans. Rubber tyres are preferred because coefficient of friction between rubber and road is less than that between steel and road.

Q. 28. *How does air friction affect the maximum height of a projectile?*

Ans. The maximum height is reduced.

Q. 29. *Rockets can move in air-free space but jet plane cannot. Why?*

Ans. Jet planes use atmospheric oxygen for burning fuel but rockets carry their own fuel and oxygen and do not depend on atmospheric oxygen.

Q. 30. *Name the forces which have shortest range and the longest range.*

Ans. Nuclear forces have the shortest range and gravitational forces have the longest range.

Q. 31. *It is easier to roll a barrel than to pull it along the road. Why?*

Ans. It is easier to roll a barrel because at the time of rolling we require to apply a force in order to overcome rolling friction and value of rolling friction is much less than sliding friction.

Q. 32. *Which of the following is a scalar quantity? Inertia, force and linear momentum.*

Ans. Inertia of linear motion is measured by mass of the body, which is a scalar quantitiy.

Q. 33. *Can a body in linear motion be in equilibrium?*

Ans. Yes, provided the vector sum of the forces acting upon the body is zero.

Q. 34. *A stone, when thrown on a glass window, smashes the window pane to pieces. But a bullet fired from a gun passes through making a hole. Why?*

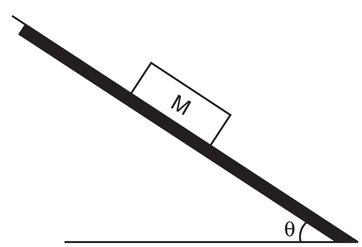
Ans. A stone gains lesser momentum compared to a bullet. Window pane can create enough resistance to the stone, and not to the bullet.

Q. 35. *Why are curved roads generally banked?*

Ans. Curved roads are generally banked so as to help in providing centripetal force needed to balance the centrifugal force, arising due to circular motion on the curved road.

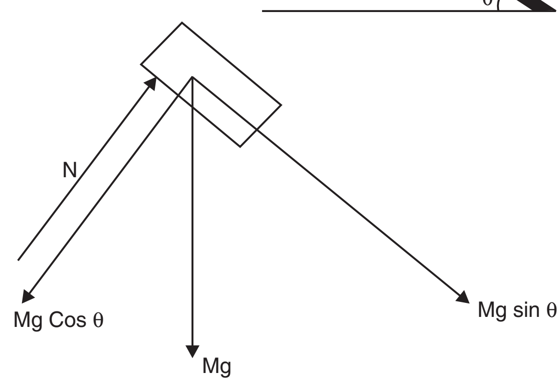
II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *A block of mass M is placed on a frictionless, inclined plane of angle θ, as shown in the figure. Determine the acceleration of the block after it is released. What is force exerted by the incline on the block?*



Ans. When the block is released, it will move down the incline. Let its acceleration be *a*. As the surface is frictionless, so the contact force will be normal to the plane. Let it be *N*.

Here, for the block we can apply equation for motion along the plane and equation for equilibrium perpendicular to the plane.



i.e., $Mg \sin \theta = Ma \Rightarrow a = g \sin \theta$

Also, $Mg \cos \theta - N = 0 \Rightarrow N = Mg \cos \theta$.

Q. 2. *A body of mass 500 g tied to a string of length 1 m is revolved in the vertical circle with a constant speed. Find the minimum speed at which there will not be any slack on the string. Take g = 10 m s^{-2}.*

Ans. The tension T in the string will provide the necessary centripetal force $\frac{m v^2}{r}$

i.e., $$T = \frac{m v^2}{r}$$

Here $m = 500 \text{ g} = \frac{1}{2} \text{kg} ; r = 1 \text{ m}$

$$\therefore \quad T = \frac{1}{2} v^2 \text{ N} \quad ...(1)$$

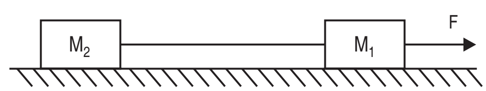
There will not be slack if $T \geq$ weight of the body

i.e., $$T \geq mg \quad \text{or} \quad \frac{1}{2} v^2 \geq \frac{1}{2} \times 10$$

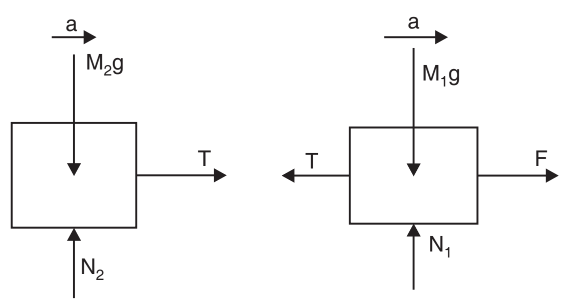
$$v^2 \geq 10 \quad \text{or} \quad v \geq \sqrt{10} \text{ m s}^{-1}$$

So the minimum speed = $\sqrt{10} \text{ m s}^{-1}$ = 3.162 m s^{-1}.

Q. 3. *A light, inextensible string as shown in figure connects two blocks of mass M_1 and M_2. A force F as shown acts upon M_1. Find acceleration of the system and tension in string.*



Ans. Here as the string is inextensible, acceleration of two blocks will be same. Also, string is massless so tension throughout the string will be same. Contact force will be normal force only.



Let acceleration of each block is a, tension in string is T and contact force between M_1 and surface is N_1 and contact force between M_2 and surface is N_2.

Applying Newton's second law for the blocks;

For M_1, $F - T = M_1 a$ ...(*i*)

$M_1 g - N_1 = 0$ ...(*ii*)

For M_2, $T = M_2 a$ ...(*iii*)

$M_2 g - N_2 = 0$ ...(*iv*)

Solving equations (*i*) and (*iii*)

$$a = \frac{F}{M_1 + M_2} \quad \text{and} \quad T = \frac{M_2 F}{M_1 + M_2}.$$

Q. 4. *Show that the total linear momentum of an isolated system of interacting particles is conserved.*

Ans. Consider two bodies A and B, with inital momenta $\vec{p}_A$ and $\vec{p}_B$ respectively. Let the two bodies collide, get apart and have final momenta $\vec{p'}_A$ and $\vec{p'}_B$ respectively. By the second law of motion:

Change in momentum of body A, $\vec{p'}_A - \vec{p}_A = \vec{F}_{AB}\,\Delta t$ ...(*i*)

where $\vec{F}_{AB}$ is the force acting on A due to action of B for a time Δt.

Similarly change in momentum of body B, $\vec{p'}_B - \vec{p}_B = \vec{F}_{BA}\,\Delta t$ ...(*ii*)

Here time Δt, the time for which two bodies A and B are in contact and interact, is same for both the forces.

Moreover, from third law of motion $\vec{F}_{AB} = -\vec{F}_{BA}$

Hence, adding (*i*) and (*ii*), we obtain

$$\left(\vec{p'}_A - \vec{p}_A\right) + \left(\vec{p'}_B - \vec{p}_B\right)s = \vec{F}_{AB}\,\Delta t + \vec{F}_{BA}\,\Delta t = -\vec{F}_{BA}\,\Delta t + \vec{F}_{BA}\,\Delta t = 0$$

$$\Rightarrow \qquad \vec{p'}_A + \vec{p'}_B = \vec{p}_A + \vec{p}_B$$

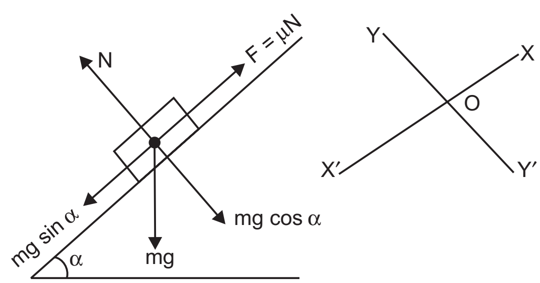
which shows that the total final momentum of the isolated system is exactly same as its initial momentum. Thus, it is proved that total momentum of an isolated system remains conserved.

Q. 5. *A 20 kg box is gently placed on a rough inclined plane of inclination 30° with horizontal. The coefficient of sliding friction between the box and the plane is 0.4. Find the acceleration of the box down the incline.*

Ans. In solving inclined plane problems, the x and y directions along which the forces are to be considered, may be taken as shown. The components of weight of the box are

(*i*) $mg \sin \alpha$ acting down the plane and

(*ii*) $mg \cos \alpha$ acting perpendicular to the plane.



$$N = mg \cos \alpha$$

$$mg \sin \alpha - \mu N = ma$$

$$mg \sin \alpha - \mu\, mg \cos \alpha = ma$$

$$a = g \sin \alpha - \mu g \cos \alpha = g(\sin \alpha - \mu \cos \alpha)$$

$$= 9.8\left(\frac{1}{2} - 0.4 \times \frac{\sqrt{3}}{2}\right) = 4.9 \times 0.3072 = 1.505 \text{ m/s}^2$$

The box accelerates down the plane at 1.505 m/s^2.

Q. 6. *A hammer of mass 1 kg strikes on the head of a nail with a velocity of 10 m s^{-1}. It drives the nail 1 cm into a wooden block. Calculate the force applied by the hammer and the time of impact.*

Ans. Here mass of hammer M = 1 kg, when hammer strikes the nail with a velocity of 10 m s^{-1} and as mass of nail is extremely small, hence nail also starts moving with same velocity. Thus, for nail u = 10 m s^{-1}, v = 0 and s = 1 cm = 0.01 m.

Using the relation $v^2 - u^2 = 2$ as, we get

$$(0)^2 - (10)^2 = 2 \times a \times (0.01)$$

$$\Rightarrow \quad a = -\frac{10 \times 10}{2 \times 0.01} = -5 \times 10^3 \text{ m s}^{-2}$$

and using relation $v = u + at$, we have

$$0 = 10 - 5 \times 10^3 . t$$

$$\Rightarrow \quad t = \frac{10}{5 \times 10^3} = 2 \times 10^{-3} \text{ s or 2 ms}$$

$\therefore$ Force exerted by the hammer on the nail = $\dfrac{\Delta p}{\Delta t} = \dfrac{Mu - 0}{\Delta t} = \dfrac{1 \times 10}{2 \times 10^{-3}} = 5 \times 10^3$ N.

Q. 7. *A bird is sitting on the floor of a closed glass cage and the cage is in the hand of a girl. Will the girl experience any change in the weight of the cage when the bird (i) starts flying in the cage with a constant velocity (ii) flies upwards with acceleration (iii) flies downwards with acceleration?*

Ans. In a closed glass cage, air inside is bound with the cage. Therefore,

(*i*) there would be no change in weight of the cage if the bird flies with a constant velocity.

(*ii*) the cage becomes heavier, when bird flies upwards with an acceleration.

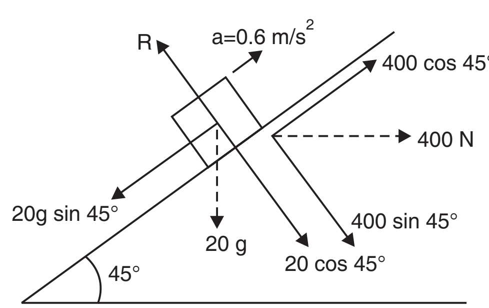
(*iii*) the cage appears lighter, when bird flies downwards with an acceleration.

Q. 8. *Is a 'single isolated force' possible in nature?*

Ans. A single isolated force is not possible. This follows from Newton's third law of motion, according to which to every action, there is an equal and opposite reaction. So, the forces must always exist in pairs. When we talk of a single force, we are considering only one aspect of mutual interaction.

Q. 9. *A force of 400 N acting horizontal pushes up a 20 kg block placed on a rough inclined plane which makes an angle of 45° with the horizontal. The acceleration experienced by the block is 0.6 m/s^2. Find the coefficient of sliding friction between the box and incline.*

Ans. The horizontally directed force 400 N and weight 20 kg of the block are resolved into two mutually perpendicular components, parallel and perpendicular to the plane as shown.



N =20 g cos 45° + 400 sin 45° = 421.4 N

The frictional force experienced by the block F = μN = μ × 421.4 = 421.4 μN.

As the accelerated motion is taking place up the plane.

400 cos 45° – 20g sin 45° – f = 20a

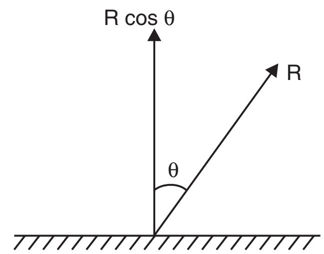
$$\frac{400}{\sqrt{2}} - \frac{20 \times 9.8}{\sqrt{2}} - 421.4\mu = 20a = 20$$

$$\mu = \left(\frac{400}{\sqrt{2}} - \frac{196}{\sqrt{2}} - 12\right) \times \frac{1}{421.4} = \frac{282.8 - 138.6 - 12}{421.4} = \mathbf{0.3137}$$

The coefficient of sliding friction between the block and the incline = **0.3137**

Q. 10. *When walking on ice, one should take short steps rather than long steps. Why?*

Ans. Let R represent the reaction offered by the ground. The vertical component $R \cos \theta$ will balance the weight of the person and the horizontal component $R \sin \theta$ will help the person to walk forward.



Now, normal reaction $= R \cos \theta$

Friction force $= R \sin \theta$

Coefficient of friction,

$$\mu = \frac{R \sin \theta}{R \cos \theta} = \tan \theta$$

In a long step, θ is more. So $\tan \theta$ is more. But μ has a fixed value. So, there is danger of slipping in a long step.

Q. 11. *In a circus, the diameter of globe of death is 30 m. From what minimum height must a cyclist start in order to roll down the inclined and go round the globe successfully?*

Ans. Diameter of globe = 30 m

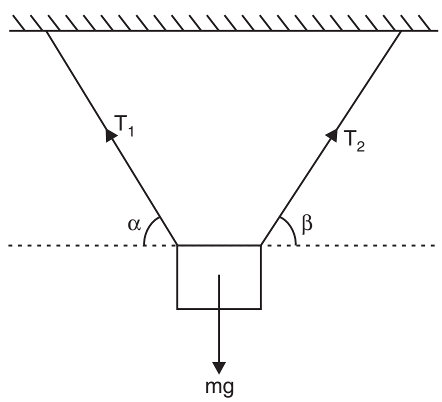
Radius of globe, r = 15 m

Let 'h' be the minimum height from which the cyclist after rolling down an incline will acquire velocity = $\sqrt{2gh}$

For looping the loop, the minimum velocity at the lowest point should be $\sqrt{5gr}$.

$$\therefore \quad \sqrt{5gr} = \sqrt{2gh} \quad \text{or} \quad h = \frac{5r}{2} = \frac{5 \times 15}{2} = 37.5 \text{ m}.$$

Q. 12. *A body of mass m is suspended by two strings making angles α and β with the horizontal as shown in Fig. Calculate the tensions in the two strings.* (*3 marks*)



Ans. Considering components of tensions T_1 and T_2 along the horizontal and vertical directions, we have

$$-T_1 \cos \alpha + T_2 \cos \beta = 0 \quad \text{or}$$

$$T_1 \cos \alpha = T_2 \cos \beta \qquad ...(i)$$

and $$T_1 \sin \alpha + T_2 \sin \beta = m\,g \qquad ...(ii)$$

From (*i*) $T_2 = \dfrac{T_1 \cos\alpha}{\cos\beta}$ and substituting it in (*ii*), we get

$$T_1 \sin\alpha + \left(\frac{T_1 \cos\alpha}{\cos\beta}\right)\sin\beta = mg \quad \text{or} \quad T_1\left[\frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\beta}\right] = mg$$

or $$T_1 \frac{\sin(\alpha+\beta)}{\cos\beta} = mg \quad \Rightarrow \quad T_1 = \frac{mg\cos\beta}{\sin(\alpha+\beta)}$$

and hence $$T_2 = \frac{T_1\cos\alpha}{\cos\beta} = \frac{mg\cos\beta}{\sin(\alpha+\beta)}\cdot\frac{\cos\alpha}{\cos\beta} = \frac{mg\cos\alpha}{\sin(\alpha+\beta)}$$

Q. 13. *The driver of a truck travelling with a velocity v suddenly notices a brick wall in front of him at a distance d. Is it better for him to apply brakes or to make a circular turn without applying brakes in order to just avoid crashing into the wall? Why?*

Ans. In applying brakes, suppose F_B is the force required to stop the truck in distance (d)

$$\therefore \quad F_B \times d = \frac{1}{2}mv^2 \quad \text{or} \quad F_B = \frac{mv^2}{2d}$$

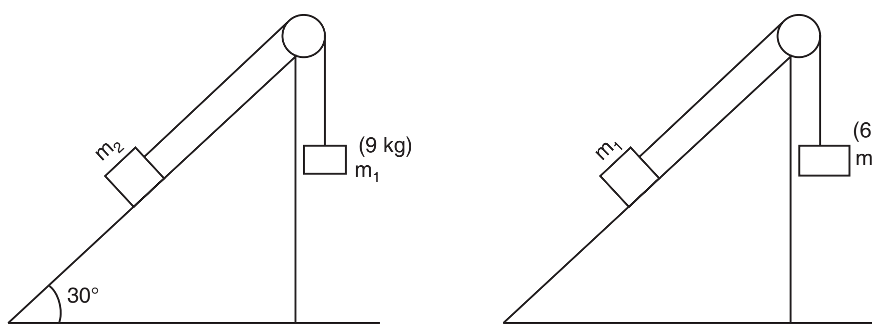
In taking a turn of radius d, the force required is

$$F_T = \frac{mv^2}{d} = 2F_B \quad \text{or} \quad F_B = \frac{1}{2}F_T$$

Therefore, it is better to apply brakes.

Q. 14. *A body m_1 of mass 9 kg and another body m_2 of mass 6 kg are connected by a light inextensible string. Consider a smooth inclined plane of inclination 30° over which one of them can be placed while the other hangs vertically and freely. Show that m_1 will drag m_2 up the whole length of the plane in half the time that m_2 hanging vertically would take to draw m_1 up the plane.*

Ans.



Case (*i*): Let a_1 be the acceleration of the system when 9 kg mass hangs freely and T the tension in the string.

$$M_1 g - T = m_1 a_1$$

$$T - m_2 g \sin 30° = m_2 a_1$$

$$\Rightarrow \quad g(m_1 - m_2 \sin 30°) = a_1(m_1 + m_2)$$

$$\Rightarrow \quad a_1 = \frac{g\left(9 - 6 \times \frac{1}{2}\right)}{15} = \frac{6g}{15} = \frac{2g}{5}$$

Case (*ii*): Let a_2 be the acceleration of the system when 6 kg mass hangs freely and T' the tension in the string.

$$m_2 g - T' = m_2 a_2$$

$$T' - m_1 g \sin 30° = m_1 a_2$$

$$\Rightarrow \quad g(m_2 - m_1 \sin 30°) = a_2 (m_2 + m_1)$$

$$\Rightarrow \quad g\left(6 - 9 \times \frac{1}{2}\right) = a_2 (6 + 9) \quad \Rightarrow \quad a_2 = \frac{3g}{30} = \frac{g}{10}$$

If S is the length of the plane,

In case (*i*), $$S = \frac{1}{2} a_1 t_1^2$$

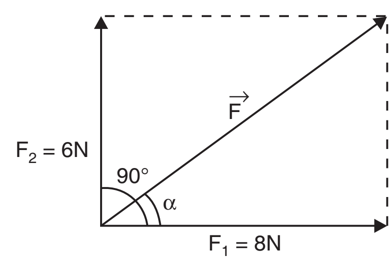
In case (*ii*), $$S = \frac{1}{2} a_2 t_2^2 \quad \Rightarrow \quad a_1 t_1^2 = a_2 t_2^2$$

$$\Rightarrow \quad \frac{t_1}{t_2} = \sqrt{\frac{a_2}{a_1}} = \sqrt{\frac{g/10}{2g/5}} = \sqrt{\frac{1}{4}} \quad \Rightarrow \quad t_1 : t_2 = 1 : 2.$$

Q. 15. *Two mutually perpendicular forces of 8 N and 6 N acts on the same body of mass 10 kg. Calculate (i) net force acting on the body, (ii) magnitude of the acceleration of the body and (iii) direction of acceleration of the body.*

Ans. Here $m = 10$ kg, $F_1 = 8$ N,

$F_2 = 6$ N, $\theta = 90°$



(*i*) Net force acting on the body is

$$F = [F_1^2 + F_2^2 + 2 F_1 F_2 \cos\theta]^{1/2}$$

$$= [F_1^2 + F_2^2]^{1/2}$$

$$[\because \cos\theta = \cos 90° = 0]$$

$$= [8^2 + 6^2]^{1/2} = 10 \quad \text{or} \quad F = 10 \text{ N}$$

(*ii*) Now $F = m a$

$$\therefore \quad a = \frac{F}{m} = \frac{10 \text{ N}}{10 \text{ Kg}} = 1 \text{ ms}^{-2}$$

(*iii*) Let α be the angle made by resultant force (F) or the acceleration with F_1

$$\therefore \quad \tan\alpha = \frac{F_2}{F_1} = \frac{6}{8} = \frac{3}{4} = 0.7500 \quad \text{or} \quad \alpha = 36° 53'$$

$\therefore$ Magnitude of acceleration = 1 ms^{-2} and it makes an angle of 36° 53′ with 8 N force.

Q. 16. *An object weighing 70 kg is kept in a lift. Find its weight as recorded by a spring balance when the lift (a) moves upwards with a uniform velocity of 5 ms^{-1}, (b) moves upwards with a uniform acceleration of 2.2 ms^{-2}, (c) moves downwards with a uniform acceleration of 2.8 ms^{-2} and (d) falls freely under gravity.*

Ans. (*a*) When the lift is moving upwards with a uniform velocity 5 ms^{-1} (acceleration is zero), the reaction R or the pressure on the base is

$$R = mg = 70 \times 9.8 \text{ N} = 686 \text{ N}$$

(*b*) When the lift is moving upwards with a uniform acceleration of 2.2 ms^{-2}, the reaction R' or the pressure on the base increases and is given by

$$R' = m(g + a) = 70\ (9.8 + 2.2) \text{ N} = 840 \text{ N}$$

(*c*) When the lift descends with a uniform acceleration of 2.8 ms^{-2}, the reaction R'' is given by

$$R'' = m(g - a) = 70\,(9.8 - 2.8)\text{ N} = 490\text{ N}$$

(*d*) When the lift falls freely under gravity, the reaction R''' is given by

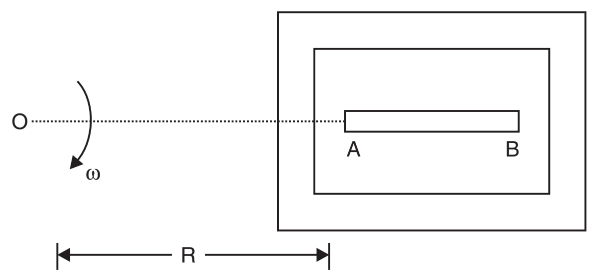
$$R''' = m(g - g) = 0$$

i.e., the object appears to have become weightless.

Q. 17. *Give one argument in favour of the fact that frictional force is a non-conservative force.*

Ans. The direction of the frictional force is opposite to the direction of motion. When a body is moved, say from *A* to *B* and then back to *A*, work is required to be done both during forward and backward motion. So, the net work done in a round trip is not zero. Hence, the frictional force is a non-conservative force.

Q. 18. *A table with smooth horizontal surface is fixed in a cabin that rotates with angular speed ω in a circular path of radius R. A smooth groove AB of length L (<< R) is made on the surface of table as shown in figure.*



A small particle is kept at the point A in the groove and is released to move, find the time taken by the particle to reach the point B.

Ans. Let us analyse the motion of particle with respect to table which is moving with cabin with an angular speed of ω. Along *AB* centrifugal force of magnitude $m\omega^2 R$ will act at *A* on the particle which can be treated as constant from *A* to *B* as $L << R$.

∴ acceleration of particle along *AB* with respect to cabin $a = \omega^2 R$ (constant)

Required time '*t*' is given by

$$S = ut + \frac{1}{2}at^2 \Rightarrow L = 0 + \frac{1}{2} \times \omega^2 R t^2 \Rightarrow t = \sqrt{\frac{2L}{\omega^2 R}}$$

Q. 19. *A cricket ball of mass 150 g is moving with a velocity of 12 ms^{-1} and is hit by a bat so that the ball is turned back with a velocity of 20 ms^{-1}. The force of the blow acts for 0.01 s. Find the average force exerted on the ball by the bat.*

Ans. The impulse of the force exerted by the bat is given by the change in the momentum of the ball. Now

Initial momentum of the ball = $\frac{150}{1000} \times 12$ kg ms^{-1} = 1.8 kg ms^{-1}

Final momentum of the ball = $-\frac{150}{1000} \times 20$ kg ms^{-1} = – 3.0 kg ms^{-1}

Change in the momentum of the ball = [1.8 – (– 3.0)] kg ms^{-1} = 4.8 kg ms^{-1}

This equals the impulse of the force exerted by the bat. Since

$$\text{Impulse} = \text{force} \times \text{time}$$

we have

$$\text{Average force exerted} = \frac{\text{Impulse}}{\text{time}} = \frac{4.8 \text{ kg ms}^{-1}}{0.01\text{s}} = 480 \text{ kg ms}^{-2} = 480 \text{ N.}$$

Q. 20. *The barrel of a gun is 1 m long and it fires a bullet of mass 0.05 kg with a muzzle velocity of 400 ms⁻¹. Find (i) the acceleration, (ii) the force, and (iii) the impulse given to the bullet by the gun.*

Ans. Here mass of bullet, $m = 0.05$ kg, initial velocity of bullet before firing $u = 0$, length of barrel of gun, moving through which the bullet is accelerated $s = 1$ m, final muzzle velocity of bullet $v = 400 \text{ ms}^{-1}$.

(*i*) Using the relation $v^2 - u^2 = 2as$, we have

$$(400)^2 - (0)^2 = 2 \times a \times 1$$

$$\Rightarrow \quad a = \frac{400 \times 400}{2 \times 1} = 8 \times 10^4 \text{ ms}^{-2}$$

(*ii*) Force $F = ma = 0.05 \times 8 \times 10^4 = 4000$ N

(*iii*) Impulse given to the bullet by the gun, J = change in momentum of bullet

$$= m(v - u) = 0.05 \times (400 - 0) = 20 \text{ Ns.}$$

Q. 21. *A nucleus is at rest. All of a sudden it splits into two small nuclei. What is the angle at which these two nuclei fly apart?*

Ans. Let M = mass of nucleus at rest

∴ Momentum of the nucleus before disintegration = $M \times 0 = 0$

Let m_1 and m_2 be the mass of the two smaller nuclei and v_1 and v_2 be their velocities.

∴ Momentum of the nucleus after disintegration = $m_1 v_1 + m_2 v_2$.

According to the law of conservation of linear momentum $m_1 v_1 + m_2 v_2 = 0$

or $\quad m_1 v_1 = - m_2 v_2$.

The –ve sign shows that the velocities v_1 and v_2 must be opposite sign *i.e.,* the two products must be emitted in opposite direction. Thus, the angle between two nuclei is 180°.

Q. 22. *State the law of conservation of momentum. Establish the same for a 'n' body system.*

Ans. When no external force acts on a system the momentum will remain conserved. Consider a system of a n bodies of masses m_1, m_2, m_3 ....., m_n. If p_1, p_2, p_3, ..., p_n are the momentum associated then, the rate of change of momentum with the system,

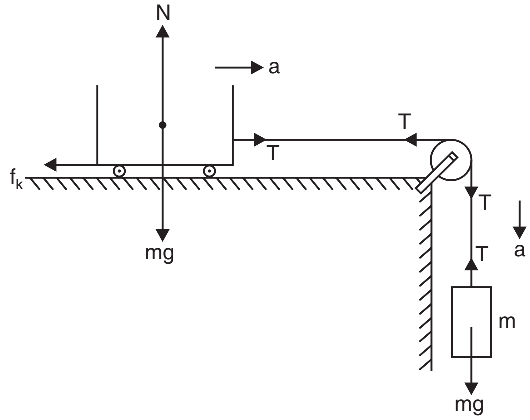
$$\frac{dp}{dt} = \frac{dp_1}{dt} + \frac{dp_2}{dt} + \frac{dp_3}{dt} + \ldots.. + \frac{dp_n}{dt} = \frac{d}{dt}(p_1 + p_2 + p_3 + \ldots.. + p_n)$$

If no external force acts, $\frac{dp}{dt} = 0$

∴ p = constant, *i.e.,* $p_1 + p_2 + p_3 + \ldots.. + p_n$ = constant.

Q. 23. *A trolley of mass 20 kg rests on a horizontal surface. A massless string tied to the trolley passes over a frictionless pulley and a load of 5 kg is suspended from other end of string. If coefficient of kinetic friction between trolley and surface be 0.1, find the acceleration of trolley and tension in the string. (Take g = 10 m s^{-2})*

Ans. The free body diagram has been shown in Fig. below



Here M = 20 kg, m = 5 kg and μ_k = 0.1

Here net pulling force

$$F = mg - f_k = mg - \mu_k.N$$
$$= mg - \mu_k.Mg = 5 \times 10 - 0.1 \times 20 \times 10$$
$$= 50 - 20 = 30 \text{ N}$$

∴ Acceleration of the system

$$a = \frac{F}{(m+M)}$$
$$= \frac{30 \text{ N}}{(5+20) \text{ kg}} = 1.2 \text{ m s}^{-2}$$

∴ Tension in string $T = mg - ma = 5 \times 10 - 5 \times 1.2 = 50 - 6 = 44$ N.

Q. 24. *A particle moves in a circle of radius 20 cm. Its linear speed at any time is given by v = 2t where v is in m/s and t is in seconds. Find the radial and tangential accelerations at t = 3 seconds and hence calculate the total acceleration at this time.*

Ans. The linear speed at 3 seconds is

$$v = 2 \times 3 = 6 \text{ m/s}$$

The radial acceleration at 3 seconds

$$= \frac{v^2}{r} = \frac{6 \times 6}{0.2} = 180 \text{ m/s}^2$$

The tangential acceleration is given by

$$\frac{dv}{dt} = 2, \quad \text{since} \quad v = 2t$$

∴ tangential acceleration is 2 m/s^2.

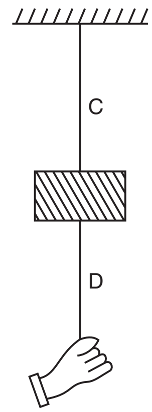
$$\text{Total acceleration} = \sqrt{a_r^2 + a_t^2} = \sqrt{180^2 + 2^2} = \sqrt{32400 + 4} = \sqrt{32404} \text{ ms}^{-2}.$$

Q. 25. *A car of mass 1000 kg moving with a speed of 30 m s^{-1} collides with the back of a stationary lorry of mass 9000 kg. (Fig.). Calculate the speed of the vehicles immediately after the collision if they remain jammed together.*

Ans. Using conservation of momentum,

$$(1000 + 9000)\ V = 1000 \times 30 + 9000 \times 0 \quad \text{or} \quad V = \frac{1000 \times 30}{10000} \text{ m s}^{-1} = 3 \text{ m s}^{-1}.$$

Q. 26. *A block is supported by a cord C from a rigid support, and another cord D is attached to the bottom of the block. If you give a sudden jerk to D, it will break. But if you pull on D steadily, C will break. Why?*



Ans. String *C* breaks because *C* is stretched more than *D*. This is because *C* was already in stretched state due to large weight. When *D* is given a jerk, the load will receive only a small acceleration due to its large mass. Thus, *C* will not be further stretched but *D* will exceed the safe limit and break.

Q. 27. *A helicopter of mass 500 kg rises with a vertical acceleration of 10 ms^{-2}. The weight of pilot is 60 kg. Give the magnitude and direction of*

(i) force on the floor of the helicopter by the pilot

(ii) action of the rotor of the helicopter on the surrounding air

(iii) force on the helicopter due to the surrounding air.

(Take g = 10 ms^{-2}).

Ans. (*i*) Force on the floor by the pilot

$$= mg + ma = m(g + a) = 60\ (10 + 10) = 1200 \text{ N (downward)}$$

(*ii*) Force of helicopter on the surrounding air $= (m_1 + m_2)(g + a)$

$$= (500 + 60)\ (10 + 10) = 11200 \text{ N (downwards)}$$

(*iii*) According to Newton's third law of motion, action and reaction are equal and opposite.

∴ Force on the helicopter due to surrounding air = 11200 N (upwards).

Q. 28. *State the laws of limiting friction. Hence define coefficient of friction.*

Ans. The laws of limiting friction are as follows:

(1) The value of limiting friction depends on the nature of the two surfaces in contact and on the state of their smoothness.

(2) The force of friction acts tangential to the surfaces in contact in a direction opposite to the direction of relative motion.

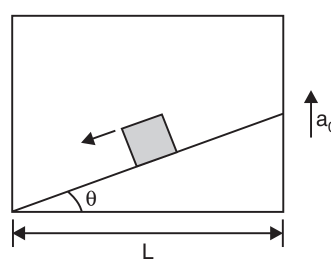
(3) The value of limiting friction is directly proportional to the normal reaction between the two given surfaces.

(4) For any two given surfaces and for a given value of normal reaction the force of limiting friction is independent of the shape and surface area of surfaces in contact.

Coefficient of limiting friction for two given surfaces in contact is defined as the ratio of the force of limiting friction f_l between them and the force of normal reaction *N*.

$$\therefore \quad \mu_l = \frac{f_l}{N}$$

Q. 29. *A block slides down from top of a smooth inclined plane of elevation θ fixed in an elevator going up with an acceleration a_0. The base of incline has length L. Find the time taken by the block to reach the bottom.*



Ans. Let us solve the problem in the elevator frame. The free body force diagram is shown. The forces are

(*i*) *N* normal to the plane

(*ii*) *mg* acting vertically down

(*iii*) ma_0 (pseudo force).

If *a* is the acceleration of the body with respect to incline, taking components of forces parallel to the incline

$$mg \sin\theta + ma_0 \sin\theta = ma$$

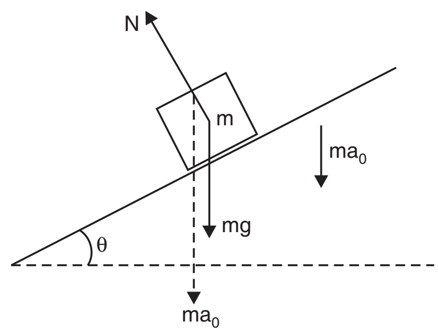
$$\therefore \quad a = (g + a_0) \sin\theta$$

This is the acceleration with respect to elevator.

The distance travelled is $\frac{L}{\cos\theta}$. If t is the time for reaching the bottom of incline

$$\frac{L}{\cos\theta} = 0 + \frac{1}{2}(g + a_0) \sin\theta \cdot t^2$$

$$t = \left[\frac{2L}{(g + a_0)\sin\theta\cos\theta}\right]^{1/2}$$



Q. 30. *A car is moving in a circular horizontal track of radius 10 m with constant speed of 10 m/s. A plumb bob is suspended from roof by a light rigid rod of length 1 m. Find the angle made by the rod with the track.*

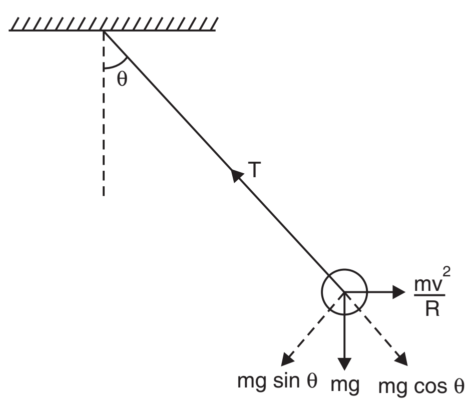
Ans. The different forces acting on the bob are shown in figure. Resolving the forces along the length and perpendicular to the rod, we have

$$mg \cos\theta + \frac{mv^2}{R} \sin\theta = T$$

$$mg \sin\theta = \frac{mv^2}{R} \cos\theta$$

Now, $\tan\theta = \frac{v^2}{Rg} = \frac{(10)^2}{10 \times 10} = 1$

$\Rightarrow$ $\tan\theta = 1 \Rightarrow \theta = \tan^{-1}(1) = 45°$.



III. LONG ANSWER TYPE QUESTIONS

Q. 1. *State Newton's second law of motion. How does it help to measure force? Also state the units of force.*

Ans. Newton's second law of motion states that the rate of change of momentum of a rigid body is directly proportional to the force applied on it.

The law implies that when a bigger force is applied on a body of given mass, its linear momentum changes faster and vice-versa. The momentum will change in the direction of the applied force.

Let, m = mass of a body,

$\vec{v}$ = velocity of the body

∴ The linear momentum of the body

$$\vec{p} = m\vec{v} \qquad ...(i)$$

Now, suppose $\vec{F}$ = external force applied on the body in the direction of motion of the body.

$\Delta\vec{p}$ = a small change in linear momentum of the body in a small time Δt.

Rate of change of linear momentum of the body = $\dfrac{\Delta\vec{p}}{\Delta t}$

According to Newton's second law,

$$\frac{\Delta\vec{p}}{\Delta t} \propto \vec{F} \quad \text{or} \quad \vec{F} \propto \frac{\Delta\vec{p}}{\Delta t} \quad \text{or} \quad \vec{F} = k\frac{\Delta\vec{p}}{\Delta t} \qquad ...(ii)$$

where k is a constant of proportionality.

Taking the limit $\Delta t \to 0$, the term $\dfrac{\Delta\vec{p}}{\Delta t}$ becomes the derivative or differential coefficient of $\vec{p}$ w.r.t. time t. It is denoted by $\dfrac{d\vec{p}}{dt}$.

$$\therefore \qquad \vec{F} = k\frac{d\vec{p}}{dt}$$

Using eqn (i), $$\vec{F} = k\frac{d}{dt}(m\vec{v}) = km\frac{d\vec{v}}{dt}$$

$$\vec{F} = km\vec{a} \qquad ...(iii)$$

where $\vec{a} = \dfrac{d\vec{v}}{dt}$ represents acceleration of the body.

The value of constant of proportionality k depends on the units adopted for measuring the force.

Now, putting $k = 1$

$$\vec{F} = m\vec{a},$$ This gives mean of measuring force.

Units of Force: Force in SI units is measured in 'newton' or N. From the relation $\vec{F} = m\vec{a}$, we can see that a newton force is that force which produces 1 ms^{-2} acceleration in a body of mass 1 kg.

1 newton = 1 kilogram × 1 metre/second^2

$$\Rightarrow \qquad 1 \text{ N} = 1 \text{ kg} \times 1 \text{ ms}^{-2} = 1 \text{ kg ms}^{-2}$$

In CGS system, force is measured in 'dyne'.

$$1 \text{ dyne} = 1 \text{ gram} \times 1 \text{ cm s}^{-2} = 1 \text{ g cm s}^{-2}$$

Since $1 \text{ N} = 1 \text{ kg ms}^{-2} = 1000 \text{ g} \times 100 \text{ cm s}^{-2} = 10^5 \text{ g cm s}^{-2} = 10^5$ dyne.

$$\Rightarrow \qquad 1\text{N} = 10^5 \text{ dyne.} \quad \text{or} \quad 1 \text{ dyne} = 10^{-5} \text{ N.}$$

Q. 2. *Two bodies with masses 10 kg and 12 kg are connected by a light inextensible string passing over a smooth fixed pulley. Find (a) the velocity at the end of 3 s, (b) the distance covered in 3 s, (c) if, at the end of 3 s the string is cut, find the distance moved by the bodies in the next 6 s.*

Ans. Let T be the tension in the string and a the common acceleration of the two masses (Fig.). Writing the equations of motion of the two masses, we have

$$12 \times g - T = 12 \times a \quad \text{and} \quad T - 10 \times g = 10 \times a$$

Adding these equations, we get

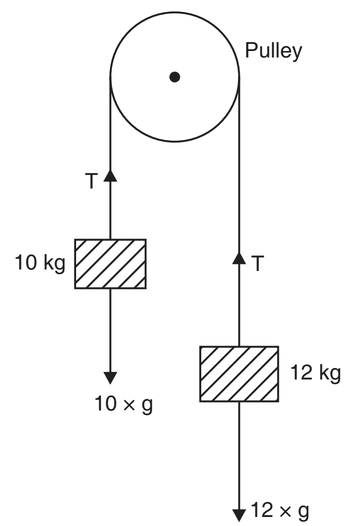
$$a = \frac{9}{11} = \frac{9.8}{11} = 0.89 \text{ ms}^{-2}$$

(*a*) Since the bodies start from rest, their velocity at the end of 3 s is

$$v_3 = 0 + 0.89 \times (3)^2 = 2.67 \text{ ms}^{-1}$$

(*b*) The distance moved by each body in 3s is

$$s_3 = 0 + \frac{1}{2} \times 0.89 \times (3)^2 = 4.005 \text{ m}$$



(*c*) At the end of 3s, the string is cut. The bodies now fall freely under gravity. We now have for the 10 kg mass

$$u = 2.67 \text{ ms}^{-1} \text{ upwards}$$
$$a = 9.8 \text{ ms}^{-2} \text{ downwards}$$
$$t = 6 \text{ s}$$

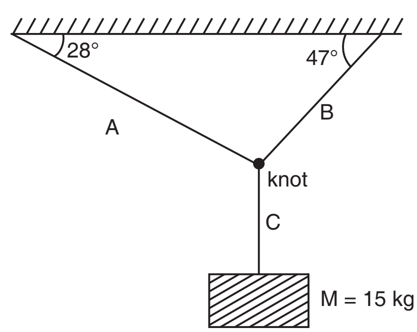
$$\therefore \qquad s = -2.67 \times 6 + \frac{1}{2} \times 9.8 \times (6)^2 = 160.38 \text{ m}$$

For the 12 kg mass

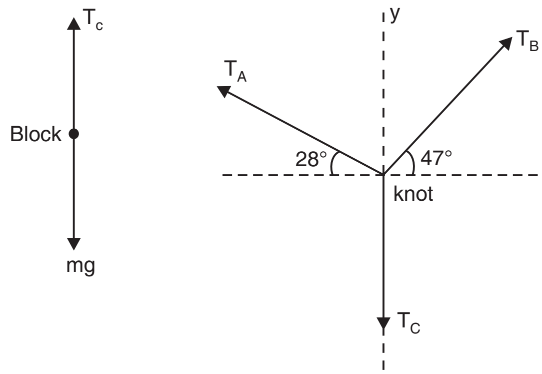
$$u = 2.67 \text{ ms}^{-1} \text{ downwards}$$
$$a = 9.8 \text{ ms}^{-2} \text{ downwards}$$
$$t = 6 \text{ s}$$

$$\therefore \qquad s = \left(2.67 \times 6 + \frac{1}{2} \times 9.8 \times 6^2\right) = 192.4 \text{ m}$$

Q. 3. *A block of mass 15 kg hangs from three chords as shown in figure. What are the tensions in the chords?*



Ans. The free-body diagram of the given problem is shown in the figure (*a*).



$$mg = 15 \times 9.8 = 147 \text{ N}$$

Since block is at rest, so $T_C = mg = 15 \times 9.8 = 147$ N

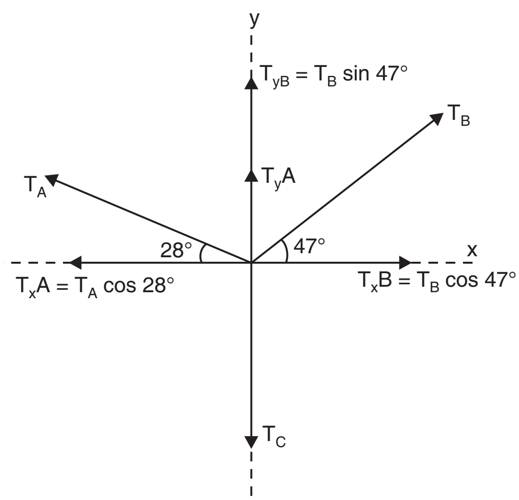


Fig. (*b*)

Resolve T_A and T_B into x and y components. x-component of T_A is given by

$$T_{xA} = -T_A \cos 28° = -T_A\,(0.8830) = -0.8830\,T_A$$

y-component of T_A is given by

$$T_{yA} = T_A \sin 28° = 0.4690\,T_A$$

Similarly, x-component of T_B is given by

$$T_{xB} = T_B \cos 47° = 0.6820\,T_B$$

and y-component of T_B is given by

$$T_{yB} = T_B \sin 47° = 0.7310\,T_B.$$

These components are represented as shown in fig. (*b*).

Since the knot is in equilibrium, so

(*i*) Σ x-components of tensions = 0

i.e., $T_{xA} + T_{xB} = 0$

or $-0.8830\,T_A + 0.6820\,T_B = 0$

or $T_A = \dfrac{0.6820}{0.8830} T_B = 0.772\ T_B$ ...(*i*)

(*ii*) Σ *y*-components of tensions = 0

i.e., $T_{yA} + T_{yB} - T_C = 0$

or $0.4690\ T_A + 0.7310\ T_B - 147 = 0$

or $0.4690\ T_A + 0.7310\ T_B = 147$

Substituting the value of equation (*i*), we get

$0.4690 \times 0.772\ T_B + 0.7310\ T_B = 147$

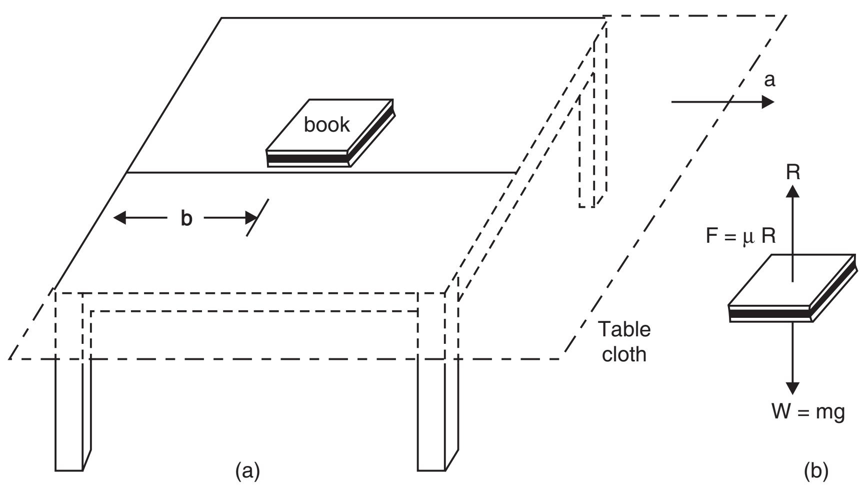
or $1.093\ T_B = 147$ or $T_B = \dfrac{147}{1.093} = 134.5\ \text{N}$

Now substituting the values of T_B in eqn. (*i*), we get

$T_A = 0.772 \times 134.5\ \text{N} = 103.8\ \text{N}$

Thus, $T_A = 103.8$ N; $T_B = 134.5$ N and $T_C = 147$ N.

Q. 4. *A book rests on a table cloth spread over a table as shown in Fig. The centre of the book is at a distance of b from the edge of the table at time t = 0; the table cloth is suddenly pulled at this instant with an acceleration a. Show that the cloth will slip from under the book if a > μ g, where μ is the coefficient of sliding friction between the table cloth and the book. Assuming this condition to be satisfied, calculate (i) time instant, (ii) the velocity, and (iii) the distance of the centre of the book from the edge of the table when the edge of the table cloth passes over the centre of the book.*



Ans. The forward pulling of the table cloth will tend to push the book backwards. The force of friction on the book will, therefore, be in the forward direction and will have a magnitude

$$F = \mu R = \mu m g$$

where m is the mass of the book. The acceleration of the book will, therefore, be

$$\frac{F}{m} = \frac{\mu m g}{m} = \mu g$$

Thus if, $\mu g < a$, the acceleration of the table cloth, the cloth will slip from under the book. At the instant (say $t = t$), the edge of the table cloth comes under the centre of the book, the distance of the table cloth edge and the centre of the book from the edge of the table must be the same. Now the distance moved by the edge of the table cloth (the initial distance from the edge of the table is zero) in a time t is

$$x' = 0 + \frac{1}{2} a t^2 \quad \text{(the table cloth is at rest at } t = 0)$$

The distance moved by the centre of the book in time t is

$$x' = b + \frac{1}{2} (\mu g) t^2 \quad \text{(acceleration of the book} = \mu g)$$

The time instant t is, therefore, given by

$$\frac{1}{2} a t^2 = \frac{1}{2} \mu g t^2 + b \quad \text{or} \quad t^2 (a - \mu g) = 2b$$

or

$$t = \sqrt{\frac{2b}{(a - \mu g)}}$$

The velocity of the book and the distance of its centre from the edge of the table at this instant are given by

$$v = 0 + \mu g \sqrt{\frac{2b}{(a - \mu g)}} = \sqrt{\frac{2b\mu^2 g^2}{(a - \mu g)}}$$

and

$$s = b + \frac{1}{2} \mu g \frac{2b}{(a - \mu g)} = b \left\{ 1 + \frac{\mu g}{(a - \mu g)} \right\} = \frac{ab}{(a - \mu g)}$$

Q. 5. *State Newton's third law of motion. Discuss its consequences.*

Ans. Newton's third law of motion states that for any action, there is equal and opposite reaction.

So, if a body applies a force F_{12} on body 2 (action), then body 2 also applies a force F_{21} on body 1 but in opposite direction, then

$$F_{21} = -F_{12}$$

In terms of magnitude

$$|F_{21}| = |F_{12}|$$

It is very important to note that F_{12} and F_{21} though are equal in magnitude and opposite in direction yet act on different points or else no motion will be possible.

For example, hands pull up a chest expander (spring) and spring in turn exerts force on the arms. A football pressed reacts on the foot with the same force and so on.

The most important consequence of the third law of motion is the law of conservation of linear momentum and its application in collision problems.

Since $\quad F_{12} = -F_{21} \quad$ and $\quad F = m \dfrac{\Delta v}{\Delta t}$

$$\therefore \quad m_1 \frac{\Delta v_1}{\Delta t} = -m_2 \frac{\Delta v_2}{\Delta t}$$

Here Δt is the time for which the bodies come in contact during impact. This is same for the two bodies of masses m_1 and m_2 and having velocity changes Δv_1 and Δv_2 respectively.

Therefore,

$$m_1 \Delta v_1 = m_2 \Delta v_2$$

or $m_1 \Delta v_1 + m_2 \Delta v_2 = 0$

Let u_1, u_2 and v_1, v_2 be the initial and final velocities of the two masses before and after collision, then,

$$m_1 (v_1 - u_1) = - m_2 (v_2 - u_2)$$

or $\quad m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

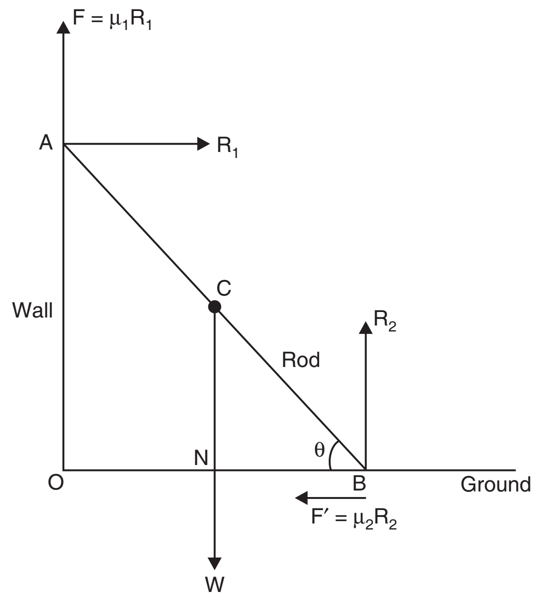
Momentum before impact = Momentum after impact.

(This is known as the law of conservation of momentum).

Q. 6. *A uniform rod is made to lean between a rough vertical wall and the ground. Show that the least angle at which the rod can be leaned without slipping is given by*

$$\theta = tan^{-1}\left(\frac{1 - \mu_1 \mu_2}{2 \mu_2}\right)$$

where μ_1 and μ_2 stand for the coefficient of friction between (i) the rod and the wall and (ii) the rod and the ground.



Ans. Figure shows the various forces acting on the rod AB when it is leaning between the wall and the ground without slipping.

Since the rod is in equilibrium, the net force as well as the net torque on it must each be zero. Considering the forces acting on the rod: we have

$$R_1 + (- F') = 0 \text{ or } R_1 = \mu_2 R_2$$

and $R_2 + F + (- W) = 0$ or $R_2 + \mu_1 R_1 = W$

Considering next the moments of all forces about A, we have

$$R_2 \times OB = W \times ON + \mu_2 R_2 \times OA$$

or $\quad R_2 \times AB \cos\theta = W \times \frac{AB\cos\theta}{2} + \mu_2 R_2 \times AB \sin\theta$

or $\quad \left(R_2 - \frac{W}{2}\right)\cos\theta = (\mu_2 R_2) \sin\theta$

or $\quad \tan\theta = \frac{\left(R_2 - \frac{W}{2}\right)}{\mu_2 R_2}$

Now $\quad W = R_2 + \mu_1 R_1 = R_2 + \mu_1 (\mu_2 R_2) = R_2 (1 + \mu_1 \mu_2)$

$\therefore \quad R_2 - W/2 = R_2 - \frac{R_2}{2}(1 + \mu_1\mu_2) = \frac{R_2}{2} - \frac{R_2}{2}\mu_1 \mu_2 = \frac{R_2}{2}(1 - \mu_1 \mu_2)$

$\therefore \quad \tan\theta = \frac{R_2}{2}\frac{(1 - \mu_1 \mu_2)}{\mu_2 R_2} = \frac{(1 - \mu_1 \mu_2)}{2\mu_2}$

or $\quad \theta = \tan^{-1}\left\{\frac{1 - \mu_1 \mu_2}{2 \mu_2}\right\}$

IV. MULTIPLE CHOICE QUESTIONS

1. If the tension in the cable supporting an elevator is equal to the weight of the elevator, the elevator may

(*a*) going up with uniform speed

(*b*) going down with non-uniform speed

(*c*) going up with increasing speed

(*d*) going down with increasing speed

2. The dimension of Impulse is

(*a*) $[MLT^{-2}]$ (*b*) $[MLT^{-1}]$ (*c*) $[MLT^{-3}]$ (*d*) $[MLT]$

3. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. Force P is applied at one end of the rope. The force which the rope exerts on the block is:

(*a*) $\frac{P}{M-m}$ (*b*) $\frac{PM}{m+M}$ (*c*) $\frac{P}{M(m+M)}$ (*d*) $\frac{Pm}{M-m}$

4. A particle of mass 5 kg is pulled along a smooth horizontal surface by a horizontal string. The acceleration of the particle is 10 ms^{-2}. The tension in the string is

(*a*) 2 N (*b*) 50 N (*c*) 15 N (*d*) 10 N

5. A rectangular body is held at rest by pressing it against a vertical wall. Which of the following is generally true ?

(*a*) It will be easier to hold the body if the surfaces in contact are smooth.

(*b*) Pressing force required is smaller than weight mg of the body.

(*c*) Pressing force required is greater than weight mg of the body.

(*d*) The required pressing force is independent of friction between surfaces in contact.

6. Action and reaction

(*a*) act on two different objects (*b*) have equal magnitude

(*c*) have opposite directions (*d*) all are correct

7. *A* force of 200 N is required to push a car of mass 500 kg slowly at constant speed on a level road. If a force of 500 N is applied, the acceleration of the car (in m s^{-2}) will be

(*a*) zero (*b*) 0.2 (*c*) 0.6 (*d*) 1.0.

8. A block of mass m is placed on a smooth inclined plane of inclination θ with the horizontal. The force exerted by the plane on the block has a magnitude

(*a*) $mg \cos \theta$ (*b*) $mg \tan \theta$ (*c*) $mg/\cos \theta$ (*d*) mg

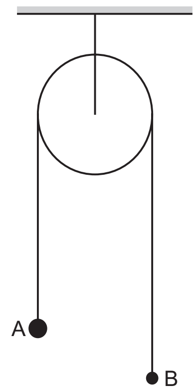
9. An insect is crawling up on the concave surface of a fixed hemispherical bowl of radius R. If the coefficient of friction is $\frac{1}{3}$ then the height up to which the insect can crawl is nearly,

(*a*) 5% of R (*b*) 6% of R (*c*) 6.5% of R (*d*) 7.5% of R

10. Water is poured from a height of 10 m into an e mpty barrel at the rate of 1 litre per second. If the weight of the barrel is 10 kg, the weight indicated at time t = 60 s will be

(*a*) 71.4 kg (*b*) 68.6 kg (*c*) 70.0 kg (*d*) 84.0 kg.

11. The pulley in the diagram is smooth and light. The masses of *A* and *B* are 5 kg and 2 kg. The acceleration of the system is



(*a*) g (*b*) $\frac{7}{3}g$

(*c*) $\frac{3}{7}g$ (*d*) $\frac{1}{7}g$

12. The physical quantity which is equal to the change in momentum of a body is known as

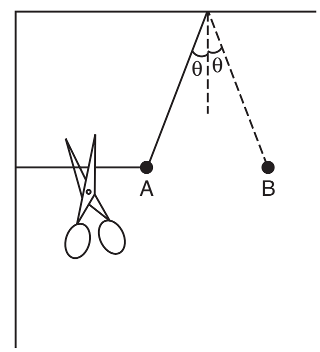
(*a*) acceleration (*b*) Impulse

(*c*) reaction (*d*) force

Ans. **1.**—(*a*) **2.**—(*b*) **3.**—(*b*) **4.**—(*b*) **5.**—(*c*)
6.—(*d*) **7.**—(*c*) **8.**—(*c*) **9.**—(*a*) **10.**—(*a*)
11.—(*c*) **12.**—(*b*)

V. QUESTIONS ON HIGH ORDER THINKING SKILLS (HOTS)

Q. 1. *A ball is held at rest at position A in Fig., by two light strings. The horizontal string is cut and the ball starts swinging as a pendulum. Point B is the farthest to the right where the ball goes as it swings back and forth. What is the ratio of the tension in the supporting string in position B to its value at A before the horizontal string was cut?*



Ans. In the first case, ball is in equilibrium. So, the net force on the body in any direction should be zero.

$\therefore \sum \vec{F}$ in the vertical direction $= \vec{0}$

$$\therefore \quad T_1 \cos\theta = mg \Rightarrow T_1 = \frac{mg}{\cos\theta}$$

In the second case, the ball is not in equilibrium. However the net force along the string is zero.

$$\therefore \quad T_2 = mg\cos\theta$$

Now, $$\frac{T_2}{T_1} = \cos^2\theta.$$

Q. 2. *A piece of ice slides down a 45° incline in twice the time it takes to slide down a frictionless 45° incline. What is the coefficient of friction between the ice and the incline ?*

Ans. Here $\theta = 45°;\ S_1 = S_2;\ u = 0$

On the rough incline, $a_1 = g(\sin\theta - \mu\cos\theta)$

t_1 = time taken

On the frictionless incline, $a_2 = g\sin\theta$

t_2 = time taken

From $$S = ut + \frac{1}{2}at^2$$

$$S_1 = 0 + \frac{1}{2}g\,(\sin\theta - \mu\cos\theta)\,t_1^2 \quad \text{and} \quad S_2 = 0 + \frac{1}{2}g\,\sin\theta \cdot t_2^2$$

As $\qquad S_1 = S_2$

$$\therefore \quad \frac{1}{2} g \, (\sin\theta - \mu\cos\theta)\, t_1^2 = \frac{1}{2} g \, \sin\theta \cdot t_2^2$$

$$\frac{\sin\theta - \mu\cos\theta}{\sin\theta} = \frac{t_2^2}{t_1^2} = \frac{t_2^2}{(2t_2)^2} = \frac{1}{4}$$

$$1 - \mu\cot\theta = \frac{1}{4} \quad \text{or} \quad \mu\cot\theta = 1 - \frac{1}{4}$$

$$\Rightarrow \quad \mu\cot\theta = \frac{3}{4} \quad \Rightarrow \quad \mu = \frac{3}{4\cot\theta}.$$

Q. 3. *A man weighing 60 kg is sitting in a lift which is moving vertically with an acceleration of 2 ms^{-2}. Prove that the reaction on the base of the lift is greater when it is ascending than when it is descending. (Given $g = 9.8\ ms^{-2}$).*

Ans. We know, when the lift accelerates upward, the reaction R_1 on the base is given by

$$R_1 = Mg + Ma = M(g + a) \quad \Rightarrow \quad R_1 = 60\,(9.8 + 2) = 60 \times 11.8 = 708 \text{ N}.$$

When the lift accelerates downward with acceleration a, the reaction R_2 on the base given by

$$R_2 = Mg - Ma = M(g - a) \quad \Rightarrow \quad R_2 = 60\,(9.8 - 2) = 60 \times 7.8 = 468 \text{ N}$$

Therefore, $\quad R_1 > R_2$.

Q. 4. *A box of wood is placed on a 30° slope. If the coefficient of friction be 0.1, what is the downward acceleration of the wooden box?*

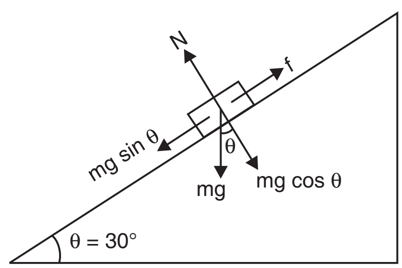
Ans. Here $\theta = 30°$ and $\mu = 0.1$

As shown in following figure, net accelerating force along the inclined plane is

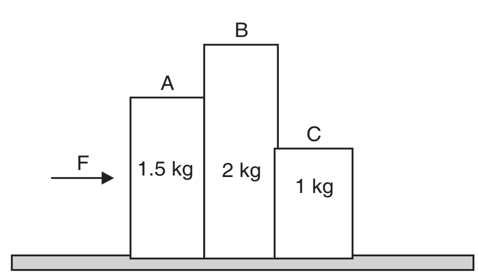
$F = m\,g\sin\theta - f = m\,g\sin\theta - \mu N = m\,g\sin\theta - \mu\, mg\cos\theta$

But $F = ma$

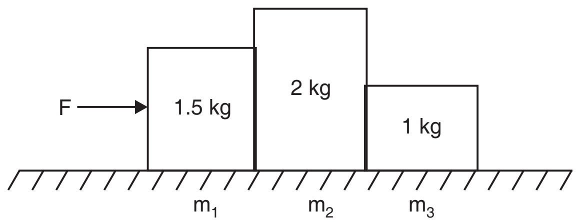
$\Rightarrow a = g\,(\sin\theta - \mu\cos\theta) = 9.8 \times (\sin 30° - 0.1 \times \cos 30°) \simeq 4 \text{ m s}^{-2}$.



Q. 5. *In the system of three blocks A, B and C shown in figure, (i) how large a force F is needed to give the blocks an acceleration of 3 m/s^2, if the coefficient of friction between blocks and table is 0.27 (ii) how large a force does the block A exert on the block B?*



Ans. (*i*) Let a be the acceleration of the system to right. All the three frictional forces $f_1 = \mu m_1 g$, $f_2 = \mu m_2 g$ and $f_3 = \mu m_3 g$ will be directed to the left as the motion of bodies is to the right. Hence, for the whole system



$$F - \mu m_1 g - \mu m_2 g - \mu m_3 g = (m_1 + m_2 + m_3)\, a$$

$$F = (m_1 + m_2 + m_3)(a + \mu g)$$

$$= (1.5 + 2 + 1)(3 + 0.2 \times 9.8) = \mathbf{22.3\ N}$$

(*ii*) The force exerted by the 1.5 kg block on the 2 kg block $= F - m_1 (a + \mu g)$

$$= 22.3 - 1.5\,(3 + 0.2 \times 9.8) = 22.3 - 7.44 = \mathbf{14.86\ N}$$

Q. 6. *If 28×10^{23} molecules of a gas strike a surface of area 14 cm^2 normally per second with velocity of 500 ms^{-1} and rebound in the opposite direction with the same speed find the pressure exerted by the gas on the surface if mass of each molecule is 5×10^{-23} g.*

Ans. Let the direction in which the molecules rebound after striking the surface be taken as positive.

Momentum of each molecule after striking the surface

$$= mv_2 = 5 \times 10^{-26}\ \text{kg} \times 500\ \text{ms}^{-1}$$

Momentum of each molecule before striking the surface

$$= mv_1 = 5 \times 10^{-26}\ \text{kg} \times (-500)$$

28×10^{23} molecules strike the surface per second.

∴ Change in momentum of the molecules due to striking the surface in 1 second = $28 \times 10^{23}\,[(5 \times 10^{-26} \times 500) - 5 \times 10^{-26}\,(-500)]$ kg ms^{-1}

$$= 28 \times 10^{23} \times 5 \times 10^{-26} \times 1000\ \text{kg ms}^{-1}$$

$$= 140\ \text{kg ms}^{-1}$$

∴ Rate of change of momentum

$$= \frac{140\ \text{kg ms}^{-1}}{1\ \text{s}} = 140\ \text{kg ms}^{-2}$$

But rate of change of momentum is equal to the impressed force.

∴ Force exerted by the surface on the molecule

$$= 140\ \text{kg ms}^{-2} = 140\ \text{N}.$$

By Newton's third law of motion, this must also be the magnitude of the force exerted by the molecules on the surface.

∴ Force exerted by the molecules on the surface

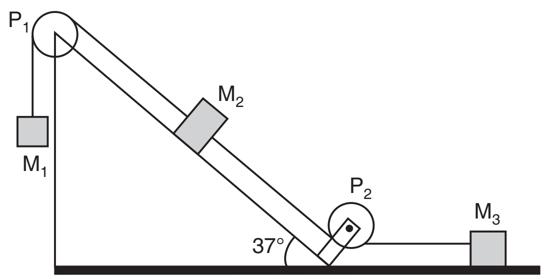
$$= 140\ \text{N}$$

Area of the surface = 14 cm^2 = 14×10^{-4} m^2

∴ Pressure on the surface

$$= \frac{\text{Force}}{\text{Area}} = \frac{140 \text{ N}}{14 \times 10^{-4} \text{ m}^2} = 10^5 \text{ N m}^{-2}.$$

Q. 7. *Masses M_1, M_2 and M_3 are connected by light strings which pass over pulleys P_1 and P_2 as shown. The masses move such that the string between P_1 and P_2 is parallel to incline and the string between P_2 and M_3 is horizontal, $M_2 = M_3 = 4kg$. The coefficient of kinetic friction between masses and the surface is 0.25. The angle of inclination of plane is 37° to the horizontal. If the mass M_1 moves downwards with uniform velocity, find M_1 and the tension in the horizontal string. Given $g = 9.8$ m/s² and sin 37° = 3/5.*

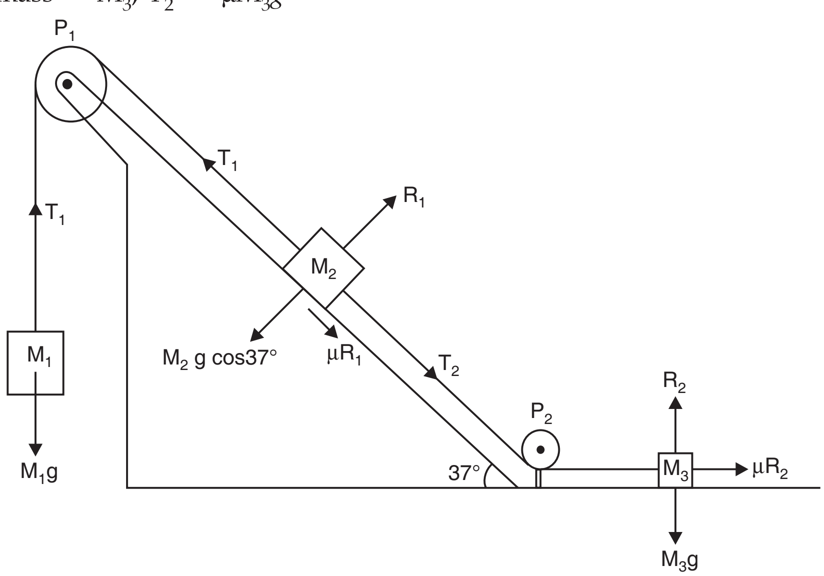


Ans. Considering the mass M_1, since it moves down with uniform speed

$$T_1 = M_1 g \qquad ...(i)$$

For mass M_2, $T_1 = T_2 + \mu M_2 g \cos 37° + M_2 g \sin 37°$ ...(*ii*)

For mass M_3, $T_2 = \mu M_3 g$ ...(*iii*)



Substituting for T_1 and T_2 from (*i*) and (*iii*) respectively in (*ii*),

$$M_1 g = \mu M_3 g + \mu M_2 g \cos 37° + M_2 g \sin 37°$$
$$M_1 = \mu M_3 + \mu M_2 \cos 37° + M_2 \sin 37°$$

$$\cos 37° = \frac{4}{5}; \quad \sin 37° = \frac{3}{5}; \quad \mu = \frac{1}{4}$$

$$\therefore \quad M_1 = \frac{1}{4}\times 4+\frac{1}{4}\times 4\times\frac{4}{5}+4\times\frac{3}{5} = 1+\frac{4}{5}+\frac{12}{5} = \mathbf{4.2\ kg}$$

$$T_2 = \mu M_3 g = \frac{1}{4}\times 4\times 9.8 = \mathbf{9.8\ N}$$

Q. 8. *A block is kept on a horizontal table. The table is undergoing S.H.M. of frequency 3 Hz in a horizontal plane. The coefficient of static friction between block and the table surface is 0.72. Find the maximum amplitude of the table at which the block does not slip on the surface.*

Ans. The table is executing S.H.M. Let at any instant, the table travels to the right with an acceleration a. Various forces acting on the block are:

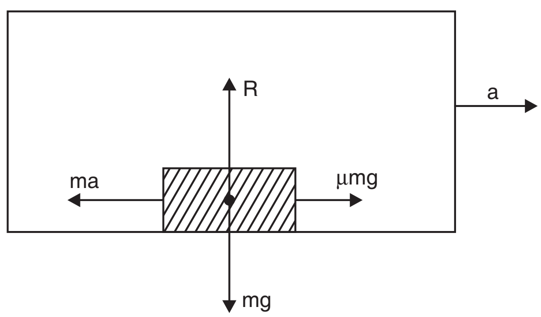
(*i*) Weight (mg) downward (*ii*) Normal reaction (R) upward

(*iii*) Force on the block to the left side = ma

(*iv*) Frictional force to the right side = $\mu\, mg$.

The block will not slip on the surface of table if

$$\mu\, mg \geq ma \quad \text{or} \quad \mu g = a_{max}$$



Since motion is S.H.M., so $a_{max} = \omega^2 A$,

where A = amplitude of S.H.M. or $a_{max} = (2\pi\nu)^2 A = 4\pi^2 \nu^2 A$

$\therefore \quad 4\pi^2 \nu^2 A = \mu g$

$$\text{or} \quad A = \frac{\mu g}{4\pi^2\nu^2} = \frac{0.72\times 10}{4\times(3.14)^2\times 9} = \frac{7.2}{354.9} = 0.02\ \text{m}.$$

Q. 9. *Assuming the length of a chain to be L and coefficient of static friction μ, calculate the maximum length of the chain which can be held outside a table without sliding.*

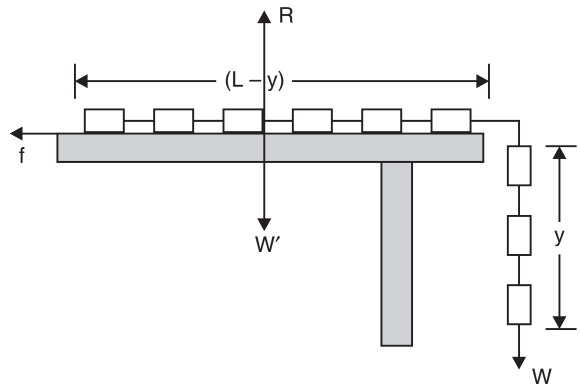
Ans. Let y be the maximum length of the chain, which can be held outside the table without sliding.

Length of the chain on the table = $(L - y)$

Weight of this part of chain, $W' = \frac{M}{L}(L - y)\, g$

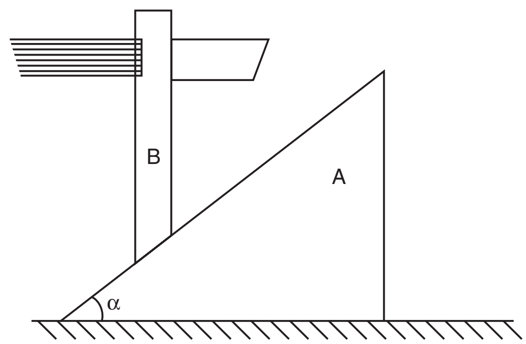
Weight of hanging part of chain = $W = \frac{M}{L} y\, g$

For equilibrium, according to Fig.
force of friction (f) = wt. of hanging part of chain



$$\mu R = W$$
$$\mu W' = W$$
$$\mu \cdot \frac{M}{L}(L-y)\, gs = \frac{M}{L} y\, g$$
$$\mu Mg - \mu \frac{M y g}{L} = \frac{M}{L} yg$$
$$\mu Mg = \frac{M}{L} y\, g(1+\mu)$$
$$y = \frac{\mu L}{(1+\mu)}$$

Q. 10. *Find the acceleration of rod B and wedge A in the arrangement shown in figure, if the ratio of the mass of wedge to that of rod equals n and there is no friction between any contact surfaces.*

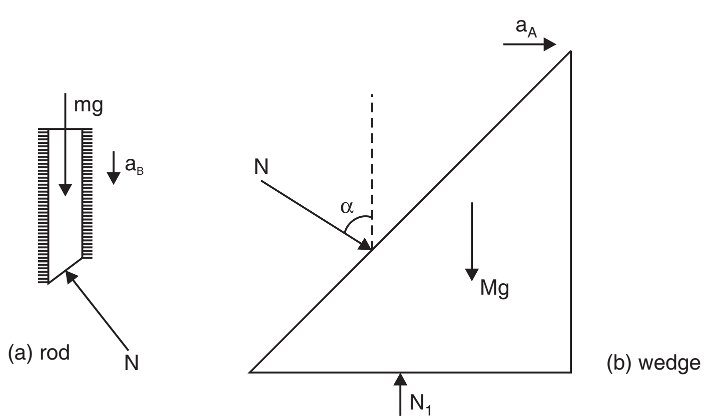


Ans. Let m be the mass of rod B and M that of wedge

Given, $$\frac{M}{m} = n$$

If acceleration of rod B and wedge A are a_A and a_B then

$$a_B = a_A \tan \alpha$$



Writing equation for motion,

for rod, $mg - N \cos \alpha = ma_B$ ...(*ii*)

for wedge, $N \sin \alpha = Ma_A$ ...(*iii*)

Solving equation (*i*), (*ii*) and (*iii*)

$$a_B = \frac{g \tan \alpha}{\tan \alpha + n \cot \alpha}, \quad a_A = \frac{g}{\tan \alpha + n \cot \alpha}.$$

IV. VALUE-BASED QUESTIONS

Q. 1. *Vipul was driving on the road with his old Grandmother. She was sitting on the front seat with him. When Vipul was about to reach his destination, he stopped the engine and did not apply the brakes. Even then the car was running on the road for sometimes. His grandmother surprised and asked her grandson the reason of the car running without the engine on. Vipul was the student of Science studying is class XI. He explained his grandmother that it is only the momentum due to which the car is going on:*

(i) What values Vipul exhibit here?

(ii) What is momentum and on which factors it depends?

Ans. (*i*) The values displayed by Vipul were intelligent, helping nature, awareness and sympathy.

(*ii*) Momentum of a body is defined as the product of its mass and the velocity with which it is moving.

Momentum = Mass × Velocity

Momentum of a body depends upon it mass and the velocity.

Q. 2. *Ramu a student of class VI was going on foot with his grandfather on a road. On the way, Ramu observed that when the road is turning in a circular mode, the inner side of the road is inclined slightly inwards and a motor cyclist was also inclined inwards which passing through that turn. Ramu got surprised and asked his grandfather who is a retired science teacher that why did the cyclist not fall from the motorcycle when he was inclined innerwards. His grandfather explained him about the centripetal and centrifugal forces when a body is moving in a circular path.*

(i) What are the values that Ramu displayed?

(ii) Define centripetal force. A cyclist speeding at 18 km/hr on a level road takes a sharp circular turn of radius 3 m without reducing the speed. The coefficient of static friction is 0.1. Will the cyclist slip while taking the turn?

Ans. (*i*) Learner, enthusiastic, curious and sharp observing nature.

(*ii*) The force on the body towards the centre while it is moving is a circular path.

The condition for the cyclist not to slip is

$$V^2 \leq \mu_s \times R \times g$$

$$V^2 \leq 0.1 \times 3 \times 9.8$$

$$V^2 = 2.94 \text{ m}^2/\text{s}^2$$

But the speed of the cyclist is

$$18 \text{ km/hr} = 5 \text{ m/s}$$

$$\therefore \quad V^2 = 25 \text{ m}^2/\text{s}^2$$

∴ The condition is not obeyed

∴ The cyclist will slip.

Q. 3. *Ashwani aged 5 years was crossing the river with his father in a boat. When they reached the bank of the river, they were asked to get down from the boat. Ashwin observed that when his father jumped out of the boat, the boat also moved in opposite direction. Ashwani got this point in his mind and asked his father the reason behind it. His father was a Professor in University. He very well explained the Newton's third Law of Motion. He also gave another example to his son Ashwani. His father put a loaded gun with its but on his shoulder and fired. He showed his son that his shoulder was also pushed back by the gun.*

(*i*) *What value of his father displayed?*

(*ii*) *What are the three Newtons Laws of Motion.*

Ans. (*i*) **Values are :** Explanation, helping nature, awareness and intelligence.

(*ii*) **Newton's First Law of Motion :** Every body continues to be at rest or in motion until or unless it is compelled by an external force.

Newton's Second Law of Motion : The rate of change of momentum of a body is directly proportional to the force applied and in the same direction as the force is applied.

Newton's Third Law of Motion : To every action there is equal and opposite reaction.

TEST YOUR SKILLS

1. Several forces are acting simultaneously on a body. In which direction will it move?
2. Define angle of friction.
3. Define the term inertia.
4. Action and reaction are equal and opposite. Why do they not balance each other?
5. How is friction reduced in a fast moving vehicle?
6. A particle of mass 3 kg slides down a smooth plane inclined at are sin $\frac{1}{3}$ to the horizontal. What is the acceleration of the particle?
7. A cyclist is going round a circular track with a speed of 10 ms^{-1}. If the radius of the track is 80 m, calculate the angular speed of the cyclist.
8. What is the meaning of banking of curves? Why do we need it?
9. A machine gun has a mass of 20 kg. It fires 35g bullets at the rate of 400 bullets per second with a speed of 400 ms^{-1}. What force must be applied to keep the gun in position?
10. Briefly explain static friction, limiting friction and kinetic friction. How do they vary with the applied force?

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