6 Electromagnetic Induction

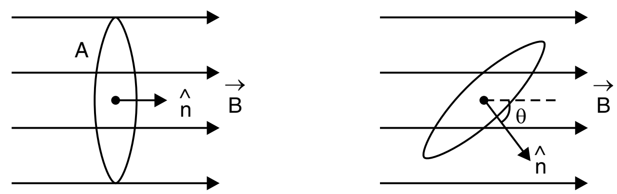
Facts that Matter

- **Magnetic Flux**

It is the total number of magnetic field lines passing through a given area. Mathematically it is given by

$$Q = \int \vec{B} . \vec{ds}$$
$$= B . A \cos \theta$$

where *ds* is elementary area and *B* is the intensity of magnetic field and θ is the angle between magnetic field and normal drawn to the surface.



(a) $\phi = BA \cos 0° = BA$ (b) $\phi = BA \cos \theta$

Fig. 6.1

- The unit of magnetic flux is weber (*Wb*) or Tm^{-1}. Its C.G.S. unit is maxwell

$$1\text{Wb} = 10^8 \text{ max}$$

- To calculate magnetic flux the normal is always drawn outward.

- **Electromagnetic Induction**

Experimently Faraday observed that when there is a change in magnetic flux linked with a circuit an induced emf is produced in the circuit. The induced emf persists only as long as there is change or cutting of flux.

According to Faraday where there is a change in magnetic flux linked with a circuit an induced emf is developed. The induced emf developed is directly proportional to the negative rate of change in magnetic flux.

$$\varepsilon = -\frac{d\phi}{dt}$$

This is called Faraday's law of electromagnetic induction. Negative sign shows that the induced emf opposes the cause of production.

- If *R* be the resistance of the circuit, then

$$\varepsilon = IR = \frac{dq}{dt} . R$$

or $$dq = -\frac{d\phi}{R}$$

dq is the induced charge which does not depend upon rate of charge in magnetic flux but it depends on net change in magnetic flux.

- **Production of Induced EMF: By changing area:**

(*i*) Let there be a rectangular loop $ABCD$ whose side BC can move over BA and CD sides is placed in uniform magnetic field of strength B.

If side BC of length l is moved from BC to $B'C'$, then change in area in time dt,

$$dA = lvdt \quad (BB' = vdt)$$

∴ The induced emf,

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt} BA\cos\theta$$

$$= \frac{Blvdt}{dt} \qquad (\theta = 180° \quad \therefore \quad \cos\theta = 180°)$$

or $$\boxed{\varepsilon = Blv}$$

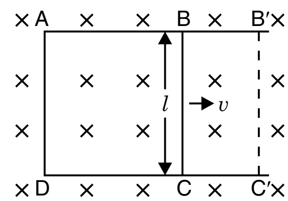


Fig. 6.2

(*ii*) Let a rods of length is rotated in uniform magnetic field of strength B with angular speed w. The change in area is the area swept by rods in one rotation = πl^2

∴ induced emf across the ends of the rods,

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt} BA\cos\theta$$

$$= \frac{B\pi l^2}{2\pi/w} \qquad \left(\cos\theta = -1,\ dt = \frac{2\pi}{w}\right)$$

or $$\boxed{\varepsilon = \frac{1}{2} Bwl^2}$$

or $$\boxed{\varepsilon = \frac{1}{2} Blv} \qquad (wl = v)$$

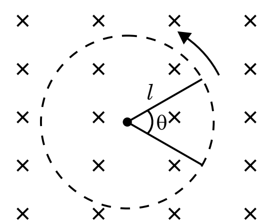


Fig. 6.3

(*iii*) Similarly of a disc of radius R is rotated in uniform magnetic field B with angular speed w, then the induced emf.

$$\boxed{\varepsilon = \frac{1}{2} Bwl^2} \quad \text{or} \quad \boxed{\varepsilon = \frac{1}{2} Blv}$$

By Rotating the coil:

Let a coil of area A is rotated in uniform magnetic field of strength with angular speed, then the induced emf,

$$\varepsilon = -\frac{d\phi}{dt}$$

$$= -\frac{d}{dt} BA\cos\theta$$

$$= -\frac{d}{dt} BA\cos wt \qquad (\because \quad \theta = wt)$$

or $$\varepsilon = BAw \sin wt$$

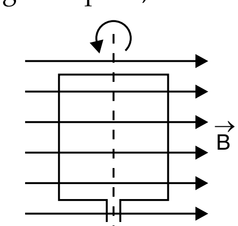


Fig. 6.4

If N be the number of turns in the coil, then induced emf,

$$\varepsilon = NBAw \sin wt$$

or

$$\varepsilon = \varepsilon_0 \sin wt$$

where $w_0 = NBAw$ the maximum emf developed in the coil.

Motional EMF

Let a metallic rod of length l is moved in uniform magnetic field of intensity B downward with velocity v.

The free electrons present in the metal rod experience a magnetic force $F_B = evB$ and move toward one end of the rod due to which one end of the rod becomes negative and the other end relatively positive. Correspondingly an electric field also developed directed from positive end to the negative end. The electrons present in the rod also experience an electric force in the direction opposite to the direction of magnetic force. In equilibrium

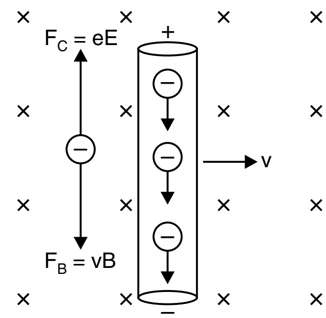


Fig. 6.5

$$eE = evB$$

or

$$\frac{\varepsilon}{l} = vB \qquad (\because \quad \varepsilon = El)$$

or induced emf developed due to motion of rod in magnetic field called motional emf,

$$\boxed{\varepsilon = Blv}$$

- **Direction of Induced EMF**

Direction of induced emf is determined by Fleming's right hand rule according to which when we open our right hand keeping thumb, for finger and middle finger mutually perpendicular to each other having direction of field along the fore finger, direction of motion along the thumb, the direction of induced current will be along the middle finger (Fig. 6.6).

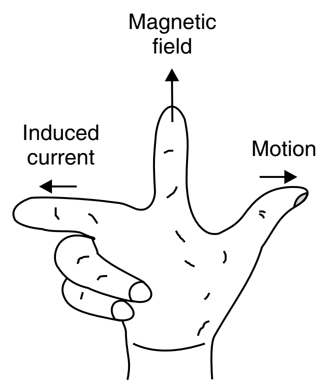


Fig. 6.6

- **Lenz's Rule**

It states that when there is a change in magnetic flux linked with a circuit, the induced emf is developed in the circuit which opposes the causes of its production. When a magnet is brought near to a circuit the magnetic flux changes and induced current is produced in the circuit. Hence the mechanical energy is converted into electrical energy. This induced current produces magnetic field which opposes the motion of the magnet as shown in Fig. 6.7. Here electrical energy converts into magnetic energy and then magnetic energy converts into mechanical energy again. Hence Lenz's rule is in accordance of conservation law of energy.

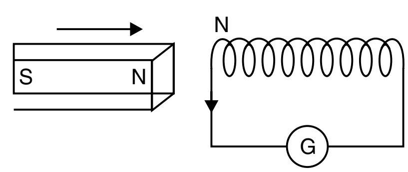


Fig. 6.7

If north pole of a magnet is brought near to a loop of oppose the motion of the north pole of the magnet, the loop behaves like a north pole. And according to right hand palm rule, the direction of current in the loop will be anticlockwise. Similarly when north pole is taken away from the loop, the loop behaves like a south pole and direction of current in the loop will be clockwise. Similarly in case of south pole.

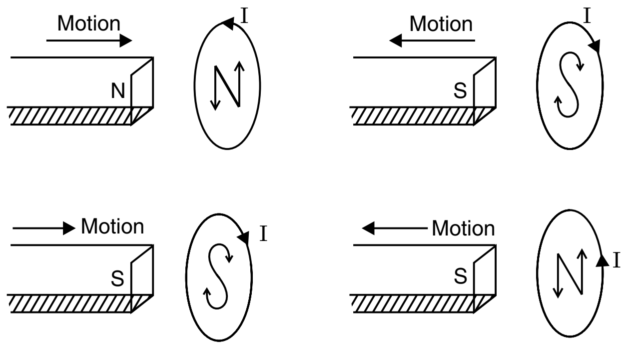


Fig. 6.8

- **Force Acting on Rod Moving in Magnetic Field**

The force on current carrying conductor of l in uniform magnetic field is given by

$$F = IBl \sin \theta$$

$$= IBl \qquad (\because \sin \theta = 1)$$

or
$$F = \frac{\varepsilon}{R} . Bl \qquad (\varepsilon = \text{induced emf})$$

$$= \frac{(Blv)\,Bl}{R}$$

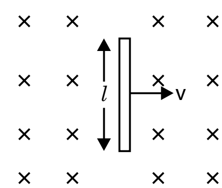


Fig. 6.9

or
$$\boxed{F = \frac{B^2 l^2 v}{R}}$$

- Power dissipated under the force F by an object moving with velocity is given by

$$P = Fv$$

$$= \frac{(B^2 l^2 v)}{R} \text{V}$$

or
$$\boxed{P = \frac{B^2 l^2 v^2}{R}}$$

- **Self Induction**

When there is a change in current in a circuit, the magnetic field associated with current also changes and correspondingly the flux linked with the circuit changes due to which an induced emf is developed in the circuit. This phenomenon of producing the induced emf due to change in current in the same circuit is called **self induction.**

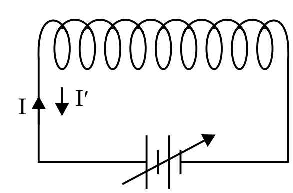


Fig. 6.10

In self induction the change in magnetic flux is directly proportional to the change in current.

$$d\phi \propto dI$$

or
$$d\phi = LdI$$

where L is the coefficient of self induction. Its S.I. unit is henery.

Applying Faraday's law, the induced emf,

$$\varepsilon = -\frac{d\phi}{dt}$$

or $$\varepsilon = -L\frac{dI}{dt}$$

or $$L = -\frac{\varepsilon}{\frac{dI}{dt}}$$

$$Df = -\frac{dI}{dt} = | AS^{-1}, \text{ then } L = \varepsilon$$

Thus, *coefficient of self induction is defined as the induced emf produced in a circuit in which current changes at the rate of one ampere per second.*

- **Coefficient of Self Induction of a Solenoid**

Let I be the change in current in a solenoid of N turns, radius R and length L, the induced emf developed,

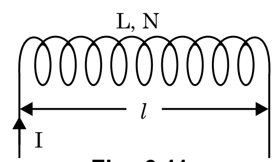


Fig. 6.11

$$\varepsilon = -N\frac{dd}{dt}$$

$$= -\frac{Nd}{dt}BA\cos\theta$$

($\because$ $\cos\theta = 1$, angle between field direction and area vector is zero)

$$\varepsilon = -\frac{Nd}{dt}\left(\frac{N\mu_0 I}{l}\right)\pi R^2 = -\frac{n^2\mu_0\pi R^2}{l}\cdot\frac{dI}{dt}$$

$\because$ $$\varepsilon = -L\frac{dI}{dt}$$

$\therefore$ $$-L\frac{dI}{dt} = -\frac{N^2\mu_0\pi R^2}{l}\cdot\frac{dI}{dt}$$

or $$\boxed{L = \frac{N^2\mu_0\pi R^2}{l}}$$

or $$L = \mu_0 n^2 l\pi R^2$$

$$L = \mu_r\mu_0 n^2 l\pi R^2$$

If magnetic substance of μ_r is placed inside solenoid.

- **Coefficient of Self Induction of Straight Wire**

Let I be the change in current in a long straight wire of radius r. The induced emf,

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}BA\cos\theta$$

$$= -\frac{d}{dt}\left(\frac{\mu_0 I}{2\pi r}\right)\pi r^2 \qquad (\because \cos\theta = 1)$$

$$= -\frac{\mu_0 r}{2}\frac{dI}{dt}$$

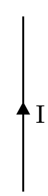


Fig. 6.12

But, $$\varepsilon = -L\frac{dI}{dt}$$

$\therefore \quad -L\frac{dI}{dt} = -\frac{\mu_0 r}{2}\cdot\frac{dI}{dt}$

$\Rightarrow \quad \boxed{L = -\frac{1}{2}\mu_0 r}$

- **Coefficient of Self induction due to a Circular Loop**

Let I be the change in current in a loop of radius R. The induced emf,

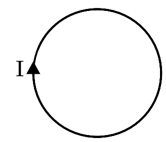


Fig. 6.13

$$\varepsilon = -\frac{d\phi}{dt}$$

or $\quad \varepsilon = -\frac{dB}{dt}A\cos\theta \qquad (\because \cos\theta = 1)$

$$= -\frac{d}{dt}\frac{\mu_0 I}{2R}(\pi R^2)$$

But $\quad \varepsilon = -L\frac{dI}{dt}$

$\therefore \quad -L\frac{dI}{dt} = -\frac{\mu_0\pi R}{2}\cdot\frac{dI}{dt} \quad$ or $\quad L = \frac{1}{2}\mu_0\pi R$

$\Rightarrow \quad \boxed{L = \frac{1}{2}N\mu_0\pi R} \qquad$ For N turns in the loop

- **Coefficient of Mutual Induction**

When two circuits are placed very close to each other and current changes in one circuit, the magnetic field associated with this current changes and the flux linked with the other circuit also changes due to which an induced emf in developed. *The phenomenon of producing induced emf in a circuit due to change in current in near the circuit is called mutual induction.*

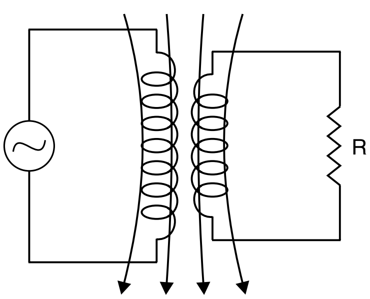


Fig. 6.14

In mutual induction change in magnetic flux linked with the circuit is directly proportional to the change in current in near by circuit.

$$d\phi \propto dI$$

$$d\theta = MdI$$

Where M is the coefficient of mutual induction. If unit is henery.

Applying the Faraday's law, induced emf,

$$\varepsilon = -\frac{d\phi}{dt}$$

or $\quad \boxed{\varepsilon = -M\frac{dI}{dt}}$

or $\quad M = -\frac{\varepsilon}{dI/dt}$

If $\qquad -\frac{dI}{dt} = 1\ AS^{-1}$

then $\qquad M = \varepsilon$

Thus, *coefficient of mutual induction is defined as the induced emf produced in a circuit due to change in current in near by circuit at the rate of one ampere per second.*

- **Coefficient of Mutual Induction between Two Solenoids**

Let there are two solenoids each of length l_1, l_2 number of turns N_1, N_2 and area of cross-section A_1 and A_2 respectively. If current I is changed in the first coil, then the induced emf developed in the second coil,

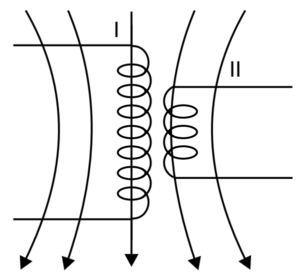


Fig. 6.15

$$\varepsilon_2 = -N\frac{d\phi_2}{dt} = \frac{dN_2}{dt}B_1A_2\cos\theta$$

$$= -\frac{dN_2}{dt}\left(\frac{N_1\mu_0 I}{l_1}\right)A_2$$

$(\because \quad \cos\theta = 1)$

$$\varepsilon_2 = \frac{-N_1 N_2 \mu_0 A_2}{l_1}\frac{dI}{dt}$$

$\because \qquad \varepsilon_2 = -M_2\frac{dI}{dt}$

$\therefore \qquad -M_2\frac{dI}{dt} = -\frac{N_1 N_2 \mu_0 A_2}{l_1}$

$\Rightarrow \qquad \boxed{M_2 = \frac{N_1 N_2 \mu_0 A_2}{l_1}}$

Similarly,

$$\boxed{M_1 = \frac{N_1 N_2 \mu_0 A_1}{l_2}}$$

For identical coils, $M_1 = M_2 = M$ (say)

or $\qquad M = \frac{N_1 N_2 \mu_0 A}{l}$

or $\qquad M^2 = \frac{N_1^2 \mu_0 A}{l}\cdot\frac{N_2^2 \mu_0 A}{l}$

$$M^2 = L_1L_2$$

or $\qquad \boxed{M = \sqrt{L_1L_2}}$

For this relation flux of one must be completely linked with the other coil.

If flux linked one coil with the other is $K\%$, then

$$\boxed{M = K\sqrt{L_1L_2}}$$

QUESTIONS FROM TEXTBOOK

6.1. *Predict the direction of induced current in the situations described by the following figs. (a) to (f).*

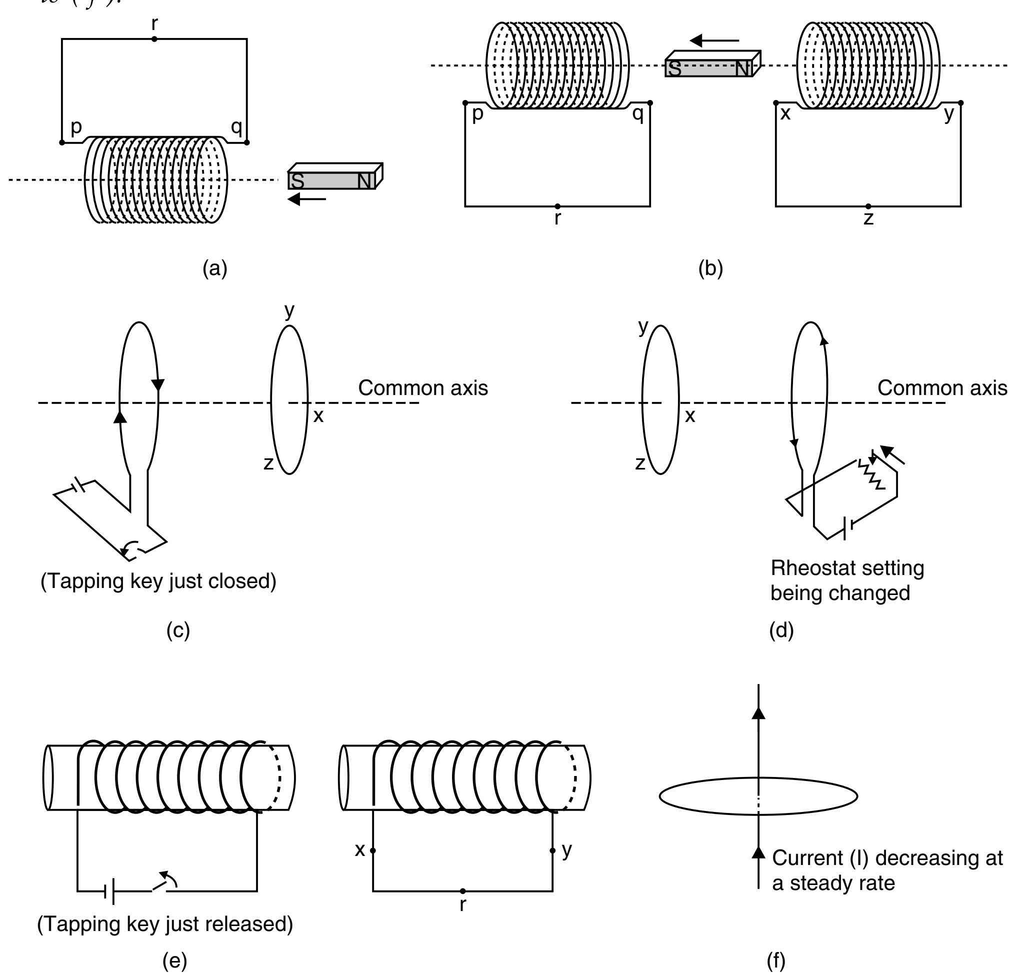


Fig. 6.16

Sol. (*a*) South pole develops at *q*, the induced current should flow clockwise. Therefore, induced current in the coil flows from *qr* to *pq*.

(*b*) Coil *pq* in this case would develop *S*-pole at *q* and coil *XY* would also develop *S* pole at *X*. Therefore, induced current in coil *pq* will be from *q* to *p* and induced current in the coil *XY* will be from *Y* to *X*.

(*c*) Induced current in the right loop will be along *XYZ*.

(*d*) Induced current in the left loop will be along *ZYX* as seen from front.

(*e*) Induced current in the right coil is from *X* to *Y*.

(*f*) Since magnetic lines of force lie in the plane of the loop, no current is induced.

6.2. *Use Lenz's law to determine the direction of induced current in the situation described by Fig.:*

(a) A wire of irregular shape turning into a circular shape;

(b) A circular loop being deformed into a narrow straight wire.

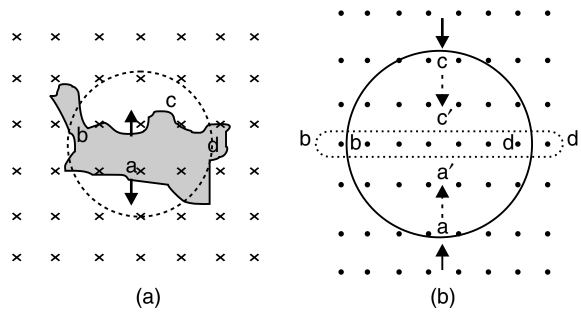


Fig. 6.17

Sol. (*a*) When a wire of irregular shape turns into a circular loop, area of the loop tends to increase. Therefore, magnetic flux linked with the loop increases. According to Lenz's law, the direction of induced current must oppose the magnetic field, for which induced current should flow along *adcba*.

(*b*) In this case, the magnetic flux tends to decrease. Therefore, induced current must support the magnetic field, for which induced current should flow along *adcba*.

6.3. *A long solenoid with 15 turns per cm has a small loop of area 2.0 cm² placed inside normal to the axis of the solenoid. If the current carried by the solenoid changes steadily from 2A to 4A in 0.1 s, what is the induced voltage in the loop while the current is changing?*

Sol. n = 15 turns/cm = 1500 turns/m; $A = 2\ \text{cm}^2 = 2 \times 10^{-4}\ \text{m}^2$; I_1 = 2A, I_2 = 4A; Δt = 0.1s

The magnetic field associated with current I_1, $\boxed{B_1 = \mu_0 n I_1}$

The magnetic field associated with current I_2, $\boxed{B_2 = \mu_0 n I_2}$

The change in the flux, $\boxed{\Delta\phi = (B_2 - B_1) A} = 4\pi \times 10^{-7} \times 1500 \times (4-2) \times 2 \times 10^{-4}$

$= 7.6 \times 10^{-7}$ weber

The induced EMF, $\boxed{|E| = \frac{\Delta\phi}{\Delta t}} = \frac{7.6\times10^{-7}}{0.1} = 7.6 \times 10^{-6}$ V

6.4. *A rectangular loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 tesla directed normal to the loop. What is the voltage developed across the cut if velocity of loop is 1 cm s⁻¹ in a direction normal to the (i) longer side (ii) shorter side of the loop? For how long does the induced voltage last in each case?*

Sol. Given,

Length of loop, l = 8 cm = 8×10^{-2} m

Breadth of loop, b = 2 cm = 2×10^{-2} m

Strength of magnetic field,

B = 0.3 T

Velocity of loop v = 1 cm/sec = 10^{-2} m/sec

Let the field be perpendicular to the plane of the paper directed inwards.

The magnitude of induced emf,

$$\boxed{\varepsilon = B.l.v}$$

$= 0.3 \times 8 \times 10^{-2} \times 10^{-2}$

$= 2.4 \times 10^{-4}$ V

Time for which induced e.m.f. will last is equal to the time taken by the coil to move outside the field is

$$I = \frac{\text{distance travelled}}{\text{velocity}} = \frac{2 \times 10^{-2}}{10^{-2}\,\text{m}} = 2 \text{ sec.}$$

(*ii*) The conductor is moving outside the field normal to the shorter side.

$$b = 2 \times 10^{-2} \text{ m}$$

∴ The magnitude of induced emf is

$$\boxed{\varepsilon = B.b.v}$$

$$= 0.3 \times 2 \times 10^{-2} \times 10^{-2}$$

$$= 0.6 \times 10^{-4} \text{ V}$$

Time, $$t = \frac{\text{distance travelled}}{\text{velocity}} = \frac{8 \times 10^{-2}}{10^{-2}} = 8 \text{ sec.}$$

6.5. *A 1.0 m long conducting rod rotates with an angular frequency of 400 rad s^{-1} about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the e.m.f. developed between the centre and the ring.*

Sol. Here, $l = 1$ m. $\omega = 400\ s^{-1}$, $B = 0.5$ T, $e = ?$

Note that linear velocity of one end of rod is zero and linear velocity of other end is ($l\,\omega$). Average linear velocity

$$v = \frac{0 + l\omega}{2} = l\,\omega/2. \qquad (\because \quad v = r\omega)$$

$$\therefore \qquad e = Blv = B\,l\,\frac{(l\,\omega)}{2} = \frac{B\,l^2\omega}{2} = \frac{0.5 \times 1^2 \times 400}{2} = 100 \text{ V}$$

6.6. *A circular coil of radius 8.0 cm and 20 turns rotates about its vertical diameter with an angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of magnitude 3×10^{-2} T. Obtain the maximum and average e.m.f. induced in the coil. If the coil forms a closed loop of resistance 10 Ω, calculate the maximum value of current in the coil. Calculate the average power loss due to Joule heating. Where does this power come from?*

Sol. Flux through each turn,

$$\boxed{\phi = \vec{B} \cdot \vec{A}} = BA \cos\theta$$

or $$\phi = B\,\pi r^2 \cos(\omega t)$$

For N turns, $$\boxed{\phi_T = NB\,\pi r^2 \cos(\omega t)}$$

The induced e.m.f, $$|\varepsilon| = \frac{d\phi_T}{dt}$$

$$= \frac{d\left[NB\,\pi r^2 \cos(\omega t)\right]}{dt}$$

or, $$\boxed{|\varepsilon| = NB\,\pi r^2\,\omega \sin(\omega t)}$$

The maximum e.m.f,

$$\varepsilon_0 = NB\,\pi r^2\,\omega$$

$$= 20 \times 50 \times \pi \times 64 \times 10^{-4} \times 3.0 \times 10^{-2}$$

$$= 0.603 \text{ V}$$

The average e.m.f, over a cycle = 0

The maximum current,

$$I_0 = \frac{\varepsilon}{R} = \frac{0.603}{10} = 0.0603 \text{ A}$$

Power loss, $P = \frac{1}{2}E_0 I_0 = \frac{1}{2} \times 0.603 \times 0.0603 = 0.018 \text{ W}$

The induced current causes a restoring torque in the coil. An external source is responsible for the supply of energy for this torque. So we can say that source of this power is the external rotor.

6.7. *A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 ms^{-1} at right angles to the horizontal component of the earth's magnetic field 0.30 × 10^{-4} Wb m^{-2}.*

(a) What is the instantaneous value of the e.m.f. induced in the wire?

(b) What is the direction of the e.m.f.?

(c) Which end of the wire is at higher electrical potential?

Sol. Here, $l = 10$ m, $v = 5.0$ ms^{-1}, $B = 0.30 \times 10^{-4}$ T

(*a*) $\boxed{e = B\,l\,v} = 0.30 \times 10^{-4} \times 10 \times 5.0 = 1.5 \times 10^{-3}$ V.

(*b*) According to Fleming's right hand rule, the direction of induced e.m.f. is from west to east.

(*c*) West end of the wire must be at higher electric potential.

6.8. *Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average e.m.f. of 200 V induced, give an estimate of the self-inductance of the circuit?*

Sol. Here, $\frac{dI}{dt} = \frac{(I_2 - I_1)}{t} = \frac{0\cdot0 - 5\cdot0}{0\cdot1} = -50 \text{ As}^{-1}$

$e = 200$ V, L =?

As $\boxed{|e| = L\left|\frac{dI}{dt}\right|}$ $\therefore$ $L = \frac{|e|}{|dI/dt|} = \frac{200}{50} = 4$H.

6.9. *A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change in flux linkage with the other coil?*

Sol. Given, $M = 1.5$ H

$I = 20$ A

$t = 0.5$ s

Using formula, $\phi = MI$

$= 1.5 \times 20$

$Q = 30$ H.

6.10. *A jet plane is travelling towards west at the speed of 1800 km/h. What is the voltage difference developed between the ends of the wing having a span of 25 m, if the Earth's magnetic field at the location has a magnitude of 5 × 10^{-4}T and the dip angle is 30°.*

Sol. Here, $v = 1800 \text{ km h}^{-1} = \frac{1800 \times 1000}{60 \times 60} = 500$ m/s

Earth's field, $B = 5.0 \times 10^{-4}$ T

Angle of dip, $\delta = 30°$

Length of wing $= 25$ m

The vertical component (B_v) of earth's field is normal to both wings and the direction of motion.

$$\therefore \quad \boxed{B_v = B \sin \delta}$$

$$= 5.0 \times 10^{-4} \sin 30°$$

$$B_v = 5 \times 10^{-4} \times \frac{1}{2} = 2.5 \times 10^{-4} \text{ T}$$

Induced e.m.f. produced

$$\boxed{B_v = B_v.v.l} = 2.5 \times 10^{-4} \times 500 \times 25$$

$$= 3.125 \text{ V.}$$

6.11. *Suppose the loop in Exercise 4 is stationary but the current feeding the electromagnet that produces the magnetic field is gradually reduced so that the field decreases from its initial value of 0.3 T at the rate of 0.02 T s^{-1}. If the cut is joined and the loop has a resistance of 1.6 Ω, how much power is dissipated by the loop as heat? What is the source of this power?*

Sol. Using formula, $|\varepsilon| = A\dfrac{dB}{dt} \qquad E = \dfrac{d\theta}{dt} \qquad (\because \quad \theta = B.A.)$

or, $|\varepsilon| = 8 \times 10^{-2} \times 2 \times 10^{-2} \times 0.02$ V

$$= 3.2 \times 10^{-5} \text{ V}$$

$$(I_0) \text{ Induced current} = \frac{|\varepsilon|}{R} = \frac{3.2 \times 10^{-5}}{1.6} \text{ A} = 2 \times 10^{-5} \text{ A}$$

$$\text{Power loss} = I_0^2 R = (2 \times 10^{-5})^2 \times 1.6 \text{ W}$$

$$= 6.4 \times 10^{-10} \text{ A.}$$

6.12. *A square loop of side 12 cm with its sides parallel to X and Y-axes is moved with a velocity of 8 cm s^{-1} in the positive x-direction in an environment containing a magnetic field in the positive z-direction. The field is neither uniform in space nor constant in time. It has a gradient of 10^{-3} T cm^{-1} along the negative x-direction (i.e., it increases by 10^{-3} T cm^{-1} as one moves in negative x-direction), and it is decreasing in time at the rate of 10^{-3} Ts^{-1}. Determine the direction and magnitude of the induced current in the loop if its resistance is 4.5 m Ω.*

Sol.

$$A = (12 \times 10^{-2}) = 144 \times 10^{-4} \, m^2$$

$$v = 8 \text{ cm/s} = 8 \times 10^{-2} \text{ m/s}$$

$$\frac{dB}{dt} = 10^{-3} \text{ T/sec}$$

$$\frac{dB}{dx} = 10^{-3} \text{ T/cm} = 10^{-1} \text{ T/m}$$

Induced e.m.f. due to change of magnetic field B with time t

$$\varepsilon_1 = \frac{dQ}{dt} = \frac{dB.A}{dt} = \frac{A.dB}{dt} = 144 \times 10^{-4} \times 10^{-3}$$

$$\varepsilon_1 = 144 \times 10^{-7} \text{ V} \qquad \text{...(I)}$$

Induced e.m.f. due to change of magnetic field B. with distance (x)

$$\varepsilon_2 = \frac{dQ}{dt} = \frac{d}{dt}BA = A \, . \, \frac{dB}{dt}$$

$$= A \cdot \frac{dB}{dx} \cdot \frac{dx}{dt} = 144 \times 10^{-4} \times 10^{-1} \times 8 \times 10^{-2}$$

$$\varepsilon_2 = 1152 \times 10^{-7}$$

Total e.m.f. $= \varepsilon_1 + \varepsilon_2 = 144 \times 10^{-7} + 1152 \times 10^{-7}$

$$= 1296 \times 10^{-7} \text{ V}$$

$$\varepsilon = 129.6 \times 10^{-6} \text{ V}$$

$$R = 4.5 \text{ milli ohm}$$

Induced current $= \frac{\varepsilon}{R} = \frac{129.6 \times 10^{-6}}{4.5 \times 10^{-3}} \cong 2.9 \times 10^{-2}\ A.$

6.13. *It is desired to measure the magnitude of field between the poles of a powerful loud speaker magnet. A small flat search coil of area 2 cm^2 with 25 closely wound turns, is positioned normal to the field direction, and then quickly snatched out of the field region. Equivalently, one can give it a quick 90° turn to bring its plane parallel to the field direction. The total charge flown in the coil (measured by a ballistic galvanometer connected to coil) is 7.5 mC. The combined resistance of the coil and the galvanometer is 0.50 Ω. Estimate the field strength of magnet.*

Sol. $\phi_1 = B.A. \qquad \phi_2 = 0$ (Outside)

$$E = -\text{N} \cdot \frac{d\phi}{dt}$$

$$I.R = -N \frac{(\phi_2 - \phi_1)}{(t_2 - t_1)}$$

$$\frac{q}{t} R = -N \frac{(0 - B.A)}{t}$$

$$qR = +NBA$$

$$B = \frac{qR}{NA}$$

$$= \frac{7.5 \times 10^{-3} \times .50}{25 \times 2 \times 10^{-4}} = \frac{75 \times 25 \times 10^{-3+4}}{25 \times 2 \times 1000}$$

$$= -75 \times 10^{1-3}$$

$$= 0.75 \text{ Weber/m}^2.$$

6.14. *Figure shows a metal rod PQ resting on the smooth rails AB and positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the rod = 15 cm, B = 0.50 T, resistance of the closed loop containing the rod = 9.0 mΩ. Assume the field to be uniform.*

(a) Suppose K is open and the rod is moved with a speed of 12 cm s^{-1} in the direction shown. Give the polarity and magnitude of the induced emf.

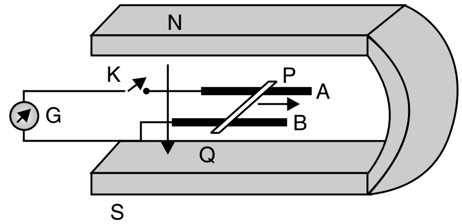


Fig. 6.18

(*b*) *Is there an excess charge built up at the ends of the rods when K is open? What if K is closed ?*

(*c*) *With K open and the rod moving uniformly, there is no net force on the electrons in the rod PQ even though they do experience magnetic force due to the motion of the rod. Explain.*

(*d*) *What is the retarding force on the rod when K is closed?*

(*e*) *How much power is required (by an external agent) to keep the rod moving at the same speed (= 12 cm s^{-1}) when K is closed? How much power is required when K is open?*

(*f*) *How much power is dissipated as heat in the closed circuit? What is the source of this power ?*

(*g*) *What is the induced emf in the moving rod if the magnetic field is parallel to the rails instead of being perpendicular?*

Sol. (*a*) The magnitude of the induced emf is given by

$$\varepsilon = B\,v\,l = 0.50 \times 0.12 \times 0.15 \text{ volt}$$
$$= 9 \times 10^{-3} \text{ volt} = 9 \text{ mV}$$

P is positive end and Q is negative end.

(*b*) Yes. When k is closed, the excess charge is maintained by the continuous flow of current.

(*c*) Magnetic force $[F_m = -e\,(v + B)]$ is cancelled by the electric force $[F_e = eE]$ set up due to the excess charge of opposite signs at the ends of the rod.

(*d*) Induced current, $I = \dfrac{e}{R} = \dfrac{9 \times 10^{-3}}{9 \times 10^{-3}} = 1\text{A}$

Retarding force on the rod $F = B\,I\,l = 0.5 \times 1 \times 0.15 = 7.5 \times 10^{-2}$ N

(*e*) Power expended by an external agent against the above retarding force to keep the rod moving uniformly at 12 cm/s

$$P = F.\,v$$
$$P = 75 \times 10^{-3} \times 12 \times 10^{-2} \text{ W} = 9.0 \times 10^{-3} \text{ W}$$

(*f*) Power dissipated as heat = $I^2 R = 1^2\,(9 \times 10^{-3}) = 9 \times 10^{-3}$ W

Source of this power external agent which keeps rod in motion, against magnetic retarding force.

(*g*) When the permanent magnet is rotated to a vertical position, the field becomes parallel to rails. The motion of rod will not cut across the lines of magnetic field and hence no e.m.f. is induced.

6.15. *An air-cored solenoid with length 30 cm, area of cross-section 25 cm^2 and number of turns 500, carries a current 2.5 A. The current is suddenly switched off in a brief time of 10^{-3} s. How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid.*

Sol. Given, $l = 0.30$ m, $A = 25 \times 10^{-4}$ m^2

$N = 500$, $I = 2.5$ A

Initial magnetic flux, $\boxed{\phi_1 = NBA} = N\left(\dfrac{\mu_0\,NI}{l}\right)A$

$$\phi_1 = \frac{\mu_0\,N^2 IA}{l}$$

or, $$\phi_1 = 4 \times \frac{22}{7} \times 10^{-7} \times 500 \times 500 \times 2.5 \times 25 \times 10^{-4} \times \frac{1}{0.30}$$

or, $$\phi_1 = 6.55 \times 10^{-3} \text{ Wb}$$

Final magnetic flux, $\phi_2 = 0$

Change of flux, $\Delta \phi_B = 0 - 6.55 \times 10^{-3} = - 6.55 \times 10^{-3}$ Wb

Corresponding time interval, $\Delta t = 10^{-3}$ s

Average e.m.f. induced across the open switch

$$= -\frac{\Delta\phi_B}{\Delta t} = -\frac{-6.55 \times 10^{-3}}{10^{-3}} = 6.55 \text{ V.}$$

6.16. (*a*) *Obtain an expression for the mutual inductance between a long straight wire and a square loop of side a as shown in Fig.*

(*b*) *Now assume that the straight wire carries a current of 50 A and the loop is moved to the right with a constant velocity, v = 10 m/s; calculate the induced emf in the loop at the instant when x = 0.2m. Take a = 0.1 m and assume that the loop has a large resistance.*

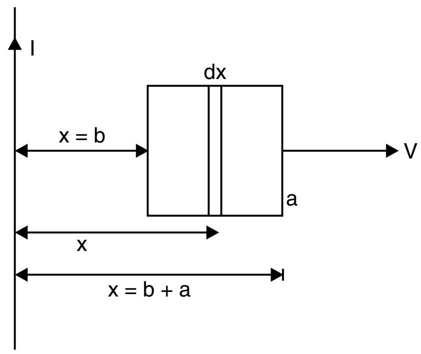


Fig. 6.19

Sol. Consider a strip of width dx (of the square loop) at a distance x from the wire carrying current. Magnetic field due to current carrying wire at a distance x from the wire is $\boxed{B = \frac{\mu_0 I}{2\pi x}}$

Small amount of magnetic flux associated with the strip

$$d\phi = B\, dA = \frac{\mu_0 I}{2\pi x}(a\, dx)$$

Magnetic flux linked with the square loop

$$\phi = \frac{\mu_0 Ia}{2\pi}\int_{x=b}^{x=a+b}\frac{dx}{x} = \frac{\mu_0 Ia}{2\pi}\left[\log_e x\right]_{x=b}^{x=a+b}$$

$$\phi = \frac{\mu_0 Ia}{2\pi}\log_e\left(\frac{a+b}{b}\right) = \frac{\mu_0 Ia}{2\pi}\log_e\left(\frac{a}{b}+1\right)$$

As $\boxed{\phi = MI}$

$\therefore$
$$MI = \frac{\mu_0 Ia}{2\pi}\log_e\left(\frac{a}{b}+1\right)$$

$$M = \frac{\mu_0 a}{2\pi}\log_e\left(\frac{a}{b}+1\right)$$

(*a*) Induced *emf* in the loop

$$e = B\, l\, v = \left(\frac{\mu_0 I}{2\pi x}\right) l\, v = \frac{4\pi \times 10^{-7} \times 50}{2\pi \times 0.2} \times 0.1 \times 10 = 5 \times 10^{-5} \text{ volt}$$

MORE QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *Magnetic field lines can be entirely confined within the core of a toroid, but not within a straight solenoid. Why?*

Ans. Because the magnetic field induction outside the toroid is zero.

Q. 2. *How does the self-inductance of a coil change when an iron rod is introduced in the coil?*

Ans. The self-inductance shall increase, since $L \propto \mu_r$.

Q. 3. *A cylindrical bar magnet is kept along the axis of a circular coil. Will there be a current induced in the coil if the magnet is rotated about its axis? Give reasons.*

Ans. No. Because $\phi = NBA = \text{constant}$

$$\therefore \quad e = \frac{d\phi}{dt} = 0; \quad i = 0$$

Q. 4. *Define magnetic flux. Give its SI unit.*

Ans. The total number of magnetic lines of force crossing any surface in the magnetic field is termed as magnetic flux.

$$\therefore \quad \phi = \text{BA} \cos \theta$$

Its SI unit is weber.

Q. 5. *Two identical loops, one of copper and another of constantan are removed from a magnetic field within the same time interval. In which loop will the induced current be greater?*

Ans. Induced current will be greater in copper loop because of its smaller resistance than that of constantan loop.

Q. 6. *Self-inductance of an air core inductor increases from 0.01 mH to 10 mH on introducing an iron core into it. What is the relative permeability of the core used?*

Ans. $\mu_r = \dfrac{L_2}{L_1} \Rightarrow \mu_r = \dfrac{10}{0.01} = 1000.$

Q. 7. *Give the direction in which the induced current flows in the coil mounted on an insulating stand when a bar magnet is quickly moved along the axis of the coil from one side to the other as shown in the figure.*

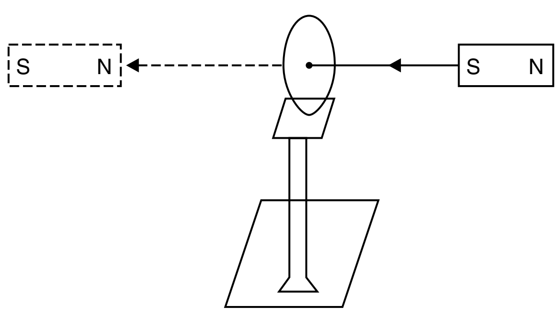


Fig. 6.20

Ans. (*i*) If seen from the right hand side, current flows clockwise when *S*-pole moves towards the coil.

(*ii*) If seen from the right hand side, current flows anticlockwise when *N*-pole moves away from the coil.

Q. 8. *A vertical metallic pole falls down through the plane of magnetic meridian. Will any e.m.f. be induced between its ends?*

Ans. No, because the pole intercepts neither H nor V.

Q. 9. *Define one henry.*

Ans. If a rate of change of current of 1 ampere per second induces an e.m.f. of volt in a coil then the inductance of coil is one henry.

Q. 10. *The electric current passing through a wire in the direction from Q to P is decreasing. What is the direction of induced current in the metallic loop kept above the wire as shown in the figure?*

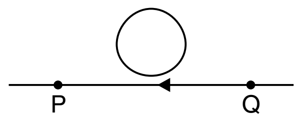


Fig. 6.21

Ans. Clockwise.

Q. 11. *When current in a coil changes with time, how is the back emf induced in the coil related to it?*

Ans. The back emf, $$\varepsilon = -L\frac{dI}{dt}$$

where, I is the current and L is the self-inductance.

Q. 12. *Could a current be induced in a coil by rotating a magnet inside the coil? If so, how?*

Ans. Yes, by holding the magnet along the axis of the coil and turning the magnet about the diameter of the coil.

Q. 13. *The south pole of a magnet is brought near a conducting loop. What is the direction of induced current as seen by a person on the other side of the loop?*

Ans. Anticlockwise.

Q. 14. *The induced emf is sometimes called 'back emf' why?*

Ans. Because induced emf opposes the current due to the source of emf.

Q. 15. *Will an induced current be always produced whenever there is change of magnetic flux linked with a coil?*

Or

Does change in magnetic flux induce emf or current?

Ans. The induced current will be produced only if the circuit is closed. However, the induced emf will be definitely produced.

Q. 16. *How does the mutual inductance of a pair of coils change when the number of turns in each coil is decreased?*

Ans. It will decrease. [$\because M \propto N_1N_2$]

Q. 17. (*a*) *When a magnet falls through a vertical coil, will its acceleration be different from the 'acceleration due to gravity'?*

(*b*) *Why does the acceleration of a magnet falling through a long solenoid decrease?*

Ans. (*a*) Yes. This is because the motion of the magnet will be opposed in accordance with Lenz's law.

(*b*) This is due to the opposition offered by induced emf.

Q. 18. *A vertical metallic pole falls down through the plane of magnetic meridian. Will any emf be induced between its ends?*

Ans. No. When $\vec{l}$ and $\vec{v}$ are parallel, no emf is induced.

Q. 19. *A train is moving with uniform velocity from north to south. Will any induced emf appear across the ends of the axle?*

Ans. Yes. The vertical component of earth's magnetic field shall induce emf.

Q. 20. *A ring is fixed to the wall of a room. When south pole of a magnet is brought near the ring. What shall be the direction of induced current in the ring?*

Ans. The induced current is clockwise as seen from the side of the magnet.

Q. 21. *Two inductors L_1 and L_2 sufficient distance apart are connected (i) in series (ii) in parallel. What is their equivalent inductance?*

Ans. In series combination,

$$L_s = L_1 + L_2$$

In parallel combination,

$$L_p = \frac{L_1 L_2}{L_1 + L_2}.$$

Q. 22. *When is magnetic flux linked with a coil held in a magnetic field zero?*

Ans. When plane of coil is along the field.

Q. 23. *Why the coil of a dead beat galvanometer is wound on a metal frame?*

Ans. The eddy currents will quickly bring the coil to rest.

Q. 24. *Write three factors on which the self-inductance of a coil depends.*

Ans. (*i*) Area of cross-section, (*ii*) Number of turns

(*iii*) Permeability of material of the core.

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *A large circular coil, of radius R, and a small circular coil, of radius r are put in vicinity of each other. If the coefficient of mutual induction, for this pair, equals 1 mH, what would be the flux linked with the large coil when a current of 0.5A flows through the small coil?*

When the current in the smaller coil falls to zero, what would be its effect in the larger coil?

Ans. Here, $M = 1 \text{ mH} = 1 \times 10^{-3}$ H

and $I = 0.5$ A

$\therefore$ $\boxed{\phi = MI} \Rightarrow 10^{-3} = M \times 0.5$

or, $\phi = \dfrac{10^{-3}}{0.5} = \dfrac{10 \times 10^{-3}}{5} = 2 \times 10^{-3}$ wb/Am

If the current through the small coil falls to zero then the induced current in the larger coil becomes zero.

Q. 2. *A jet plane is travelling west at 450 m/s. If the horizontal component of earth's magnetic field at that place is 4×10^{-4} tesla and the angle of dip is 30°, find the emf induced between the ends of wings having a span of 30 m.*

Ans. Here, $v = 450$ m/s; $l = 30$ m; $B = 4 \times 10^{-4}$ T, $\theta = 30°$

Induced e.m.f., $\boxed{\varepsilon = B\,l\,v \sin\theta}$

$= 4 \times 10^{-4} \times 30 \times 450 = 5.4$ V

The direction of the wing does not affect this e.m.f.

Q. 3. *Figure shows a bar magnet M falling under gravity through an air cored coil C. Plot a graph showing variation of induced e.m.f. (E) with time (t). What does the area enclosed by the E–t curve depict?*

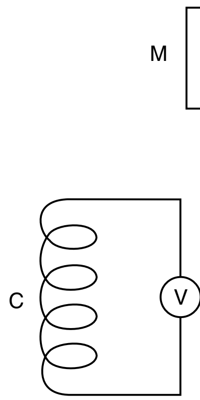


Fig. 6.22

Ans. The area of E–t graph represents the work done per unit induced current in the circuit.

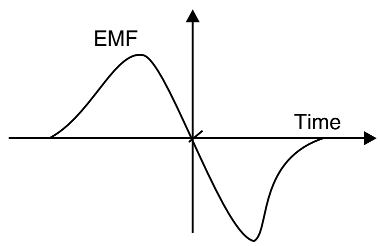


Fig. 6.23

Q. 4. *How is the mutual inductance of a pair of coils affected when:*

(i) separation between the coils is increased?

(ii) the number of turns of each coil is increased?

(iii) A thin iron sheet is placed between the two coils, other factors remaining the same? Explain your answer in each case.

Ans. (*i*) Mutual inductance (M) decreases because the quantity of flux linking to a coil due to the other one will decrease.

(*ii*) M increases because as the number of turns increase, the overall flux density also increases and hence the mutual inductance will also increase.

(*iii*) M increases because iron is ferromagnetic in nature hence, it will increase the flux density.

Q. 5. *A coil of number of turns N, area A, is rotated at a constant angular speed ω, in a uniform magnetic field B, and connected to a resistor R. Deduce expressions for:*

(i) maximum emf induced in the coil. *(ii) power dissipation in the coil.*

Ans. The flux linking to the coil,

$$\phi = NBA \sin(\omega t)$$

The induced EMF, $E = \frac{d\phi}{dt} \frac{d(NBA \sin \omega t)}{dt} = NBA\ \omega \cos \omega t$

(*i*) The maximum EMF, $E_0 = NBA\ \omega$

(*ii*) The power dissipated in the coil, $P = \frac{E_{rms}^2}{R} = \frac{E_0^2}{2R} = \frac{(NAB\ \omega)^2}{2R}$

Q. 6. *A conducting rod of length 1m is moved in a magnetic field of magnitude B with velocity v such that the arrangement is mutually perpendicular. Prove that the emf induced in the rod is |E| = Blv.*

Ans. Let us assume a rectangular loop LMNO is placed in a uniform magnetic field B.

Suppose at any instant, length $ON = x$

Flux through the loop, $\phi = Blx$ [$\because$ max. flux, $\phi = BA$]

Induced emf, $$E = -\frac{d\phi}{dt} = -\frac{d}{dt}Blx$$

or, $$E = -Bl\frac{dx}{dt} = -Blv$$

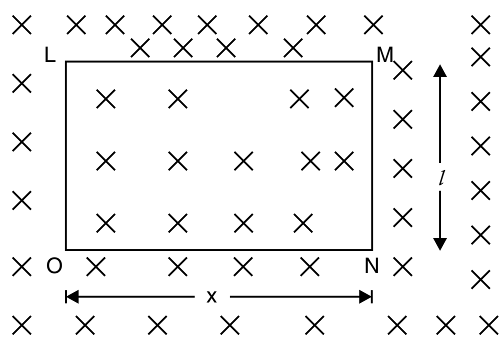


Fig. 6.24

where $\frac{dx}{dt} = -v$, is the velocity of conductor MN.

Q. 7. *The loops in the figure move into or out of the field which is along the inward normal to the plane of the paper. Indicate the direction of currents in loops 1, 2, 3, 4.*

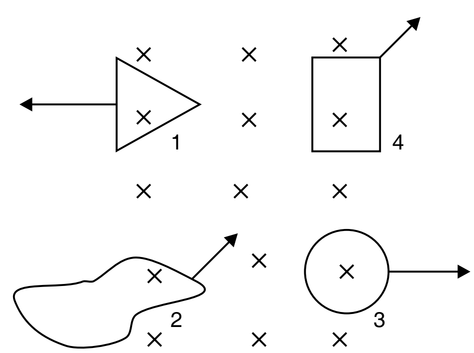


Fig. 6.25

Ans. In 1, flux decreases and so induced current must be clockwise to increase the flux. Due to the same reason currents in 3 and 4 are clockwise but in 2 current must be anticlockwise as flux is increasing.

Q. 8. *In the figure given below, a bar magnet moving towards the right or left induces an e.m.f. in the coils (1) and (2). Find, giving reason, the directions of the induced currents through the resistors AB and CD when the magnet is moving (a) towards the left, and (b) towards the right.*

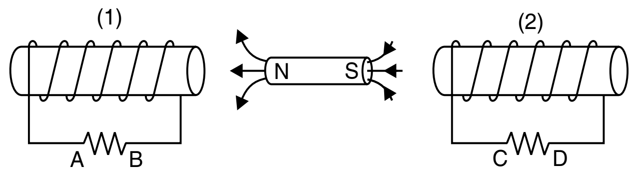


Fig. 6.26

Ans. (*a*) When bar magnet is moved towards left, the anticlockwise current will be in loop (1) and (2) {facing bar magnet} following Lenz's law. Thus, the direction in loop (1) will be from A to B and D to C.

(b) When bar magnet is moved towards right, the clockwise current will be in loop (1) and (2) {facing bar magnet} following Lenz's law. Thus, the direction in the loop (1) will be from B to A and C to D.

Q. 9. *The figure shows two identical rectangular loops (1) and (2) placed on a table along with a straight long current carrying conductor between them.*

(i) What will be the directions of the induced currents in the loops when they are pulled away from the conductor with same velocity v?

(ii) Will the e.m.f. induced in the two loops be equal? Justify your answer.

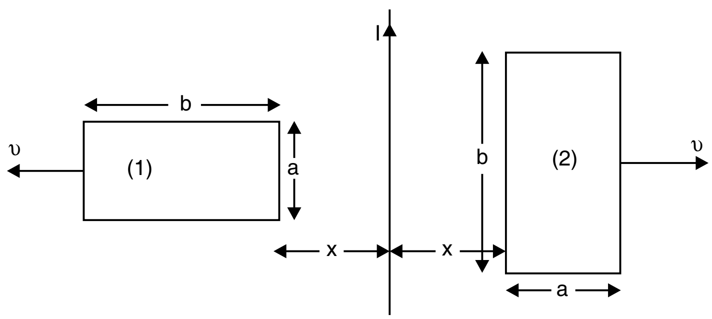


Fig. 6.27

Ans. (*i*) The direction of induced current will be such that it tends to maintain the original flux. So induced current flows anticlockwise in loop 1 and clockwise in loop 2.

(*ii*) No, the emfs induced in the two loops will not be equal.

Q. 10. *A 0.5 m long metal rod PQ completes the circuit as shown in the figure. The area of the circuit is perpendicular to the magnetic field of flux density 0.15 T. If the resistance of the total circuit is 3 Ω, calculate the force needed to move the rod in the direction as indicated with a constant speed of 2 m s*$^{-1}$.

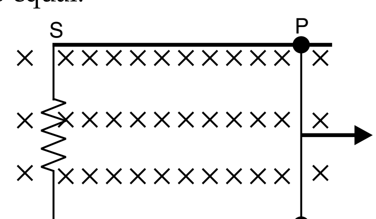


Fig. 6.28

Ans. Consider coil PQRS with its arm PQ movable as shown in the figure. A magnetic field is applied normal to the surface of the coil. The area of the coil, $\Delta S = l \times x$

$$\phi = B \, \Delta \, S = Blx$$

The rate of change of magnetic flux linked with the coil is given by

$$\frac{d\phi}{dt} = \frac{d\,(Blx)}{dt}$$

$$= Bl\frac{dx}{dt} = Blv$$

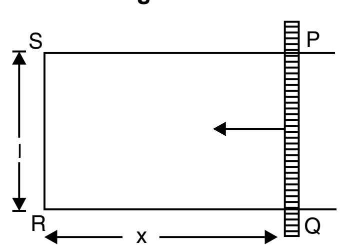


Fig. 6.29

If e is the induced emf produced, then

$$e = -\frac{d\phi}{dt} = Blv \qquad \left[\because x \text{ is decreasing, } \frac{d(x)}{dt} \text{ is } -Ve\right]$$

Let R be the resistance of movable arm PQ of the rectangular conductor. Taking the resistance of other arms as negligibly small, the current in the loop is given by,

$$I = \frac{\varepsilon}{R} = \frac{Blv}{R} \qquad ...(I)$$

The arm PQ moves with the speed v. The power required to move it is given by,

$$F = BIL$$

$$\therefore \quad F = \frac{Bl^2v}{R} \qquad \text{(From } I\text{)}$$

$$\Rightarrow \quad F = \frac{B^2l^2v}{R} = \frac{(0.15)^2 \times (0.5)^2 \times 2}{3}$$

$$= 3.75 \times 10^{-3} \text{ N.}$$

Q. 11. *Two circular coils, one of radius r and the other of radius R are placed coaxially with their centres coinciding. For R >> r, obtain an expression for the mutual inductance of the arrangement.*

Ans. Suppose a current I_2 flows through the outer circular coil. Magnetic field at the centre of the coil is

$$B_2 = \frac{\mu_0 I_2}{2R}$$

Field B_2 may be considered constant over the cross-sectional area of the inner smaller coil. Hence

$$\phi = B.A.$$

$$\phi_1 = \pi r^2 B_2 = \frac{\mu_0 \pi r^2 I_2}{2R} = MI_2$$

$$\therefore \quad M = \frac{\phi_1}{I_2} = \frac{\mu_0 \pi r^2}{2R}$$

Q. 12. *A coil of inductance 2 mH carrying a current 2A is given. If the current is reversed in 0.01 seconds, how much back e.m.f. is produced?*

Ans. The current reverses in 0.01 s, this means the time period is 0.01/2 = 0.005s.

Induced EMF, $$\varepsilon = -L\frac{dI}{dt}$$

$$= 2 \times 10^{-3} \times \frac{2}{0.005}$$

$$= 0.8 \text{ V}$$

Q. 13. *The two rails of a railway track, insulated from each other, and the ground, are connected to a millivoltmeter. What is the reading of the voltmeter when a train travels at a speed of 180 km h–1 along the track? Given vertical component of earth's magnetic field = 0.2 × 10–4 T and the separation between the rails = 1 m.*

Ans. The induced emf generated is given by

$$\boxed{\varepsilon = -\frac{d\phi}{dt}} = -\frac{d}{dt}(B\,A) = -B\frac{dA}{dt}$$

where A is the area and B, the magnetic field. If l is the distance between the rails and v, the speed of the train, then

$$\frac{dA}{dt} = l\,v$$

$$e = -B\,l\,v$$

Here $l = 1$ m and $B = 0.2 \times 10^{-4}$ T. Thus we have

$$v = \frac{180 \times 1000}{3600} = 50 \text{ m s}^{-1}$$

$$|e| = 0.2 \times 10^{-4} \times 1 \times 50 = 1 \times 10^{-3} \text{ V} = 1 \text{ m V}$$

Hence, the millivoltmeter will read 1 mV.

Q. 14. *A coil of inductance 0.25 H is connected to 18 V battery. Calculate the rate of growth of current?*

Ans. Given, $L = 0.25$ H

$e = 18$ V

Using formula, $$\boxed{e = L\frac{dI}{dt}}$$

$$\frac{dI}{dt} = \frac{e}{L} = \frac{18}{0.25} = 72 \text{ As}^{-1}.$$

Q. 15. *Two coils have mutual inductance of 1.5 henry. If current in primary circuit is raised to 5 ampere in one millisecond after closing the circuit, what is the e.m.f. induced in the secondary?*

Ans. Given, $dI = 5$ ampere

$M = 1.5$ henry

$dt = 1$ milli sec $= 10^{-3}$ sec.

Using formula, $$\boxed{\varepsilon = M\frac{dI}{dt}}$$

$$\therefore \quad \varepsilon = 1.5 \times \frac{5}{10^{-3}} = 7.5 \times 10^{3} \text{ V}$$

III. LONG ANSWER TYPE QUESTIONS

1. (*a*) *State Lenz's law. Give one example to illustrate this law. ''The Lenz's law is a consequence of the principle of conservation of energy.'' Justify this statement.*

(*b*) *Deduce an expression for the mutual inductance of two long coaxial solenoids but having different radii and different number of turns.*

Ans. (*a*) According to Lenz's law, the direction of the induced current (caused by induced emf) is always such as to oppose the change causing it.

$$\boxed{\varepsilon = -k\frac{d\phi}{dt}}$$

where k is a positive constant. The negative sign expresses Lenz's law. It means that the induced emf is such that, if the circuit is closed, the induced current opposes the change in flux.

(*b*) Figure shows a coil of N_2 turns and radius R_2 surrounding a long solenoid of length l_1, radius R_1 and number of turns N_1.

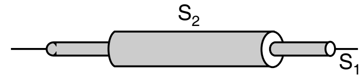


Fig. 6.30

To calculate M between them, let us assume a current i_1 through the inner solenoid S_1

There is no magnetic field outside the solenoid and the field inside has magnitude,

$$B = \mu_0 \left(\frac{N_1}{l_1}\right) i_1$$

and is directed parallel to the solenoid's axis. The magnetic flux ϕ_{B2} through the surrounding coil is, therefore,

$$\phi_{B2} = B\,(\pi R_1^2) = \frac{\mu_0 N_1\, i_1}{l_1}\pi R_1^2$$

Now, $$M = \frac{N_2 \phi_{B2}}{i_1} = \left(\frac{N_2}{i_1}\right)\left(\frac{\mu_0 N_1 i_1}{l_1}\right)\pi R_1^2 = \frac{\mu_0 N_1 N_2 \pi R_1^2}{l_1},\ M = \frac{\mu_0 N_1 N_2 \pi R_1^2}{l_1}$$

Notice that M is independent of the radius R_2 of the surrounding coil. This is because solenoid's magnetic field is confined to its interior.

Q. 2. *What is induced emf? Write Faraday's law of electromagnetic induction. Express it mathematically. A conducting rod of length 'l', with one end pivoted, is rotated with a uniform angular speed 'ω' in a vertical plane, normal to a uniform magnetic field 'B'. Deduce an expression for the emf induced in this rod.*

Ans. Whenever the magnetic flux linked with a closed circuit changes, an emf is set up across it which lasts only so long as the change in flux is taking place. This emf is called induced emf.

According to Faraday's law of electromagnetic induction, the magnitude of induced emf is equal to the rate of change of magnetic flux linked with the closed circuit (or coil).

Mathematically,

$$E = -N\frac{d\phi_B}{dt}$$

where N is the number of turns in the circuit and ϕ_B is the magnetic flux linked with each turn.

Suppose the conducting rod completes one revolution in time T. Then

$$\text{Change in flux} = B \times \text{Area swept} = B \times \pi l^2$$

$$\text{Induced emf} = \frac{\text{Change in flux}}{\text{Time}}$$

$$\varepsilon = \frac{B \times \pi l^2}{T}$$

But $$T = \frac{2\pi}{\omega}$$

∴ $$\varepsilon = \frac{B \times \pi l^2}{2\pi/\omega} = \frac{1}{2}Bl^2\omega.$$

Q. 3. *Two long parallel horizontal rails, distance d apart and each having a resistance λ per unit length, are joined at one end by a resistance R. A perfectly conducting rod MN of mass m is free to slide* *along the rails without friction (see Fig.). There is a uniform magnetic field of induction B normal to the plane of the paper and directed into the paper. A variable force F is applied to the rod MN such that as the rod moves, constant current flows through R.*

(i) Find the velocity of the rod and the applied force F as function of the distance x of the rod from R.

(ii) What fraction of the work done per second by F is converted into heat?

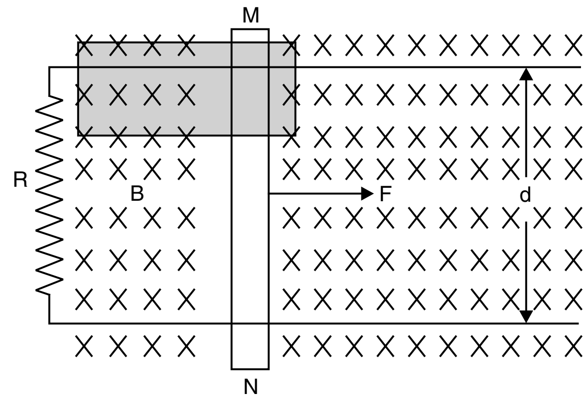


Fig. 6.31

Ans. Let the distance from R to MN be x. Then the area of the loop between MN and R is xd and the magnetic flux linked with the loop is $B\,x\,d$. As the rod moves, the emf induced in the loop is given by

$$|\varepsilon| = \frac{d}{dt}(B\,x\,d) = B\,d\,\frac{dx}{dt} = B\,v\,d$$

where v is the velocity of MN. The total resistance of the loop between R and MN is $R + 2\lambda x$. The current in the loop is given by

$$i = \frac{|e|}{R + 2\lambda x} = \frac{Bvd}{R + 2\lambda x}$$

(i) Force acting on the rod,

$$F = iBd = \frac{B^2d^2}{R + 2\lambda x}v$$

$$\therefore \qquad m\frac{dv}{dt} = \frac{B^2d^2}{R + 2\lambda x}\cdot\frac{dx}{dt}$$

$$\text{or,} \qquad dv = \frac{B^2d^2}{m}\cdot\frac{dx}{R + 2\lambda x}$$

On integrating both sides, we get

$$v = \frac{B^2d^2}{2\lambda m}\ \text{In}\left(\frac{R + 2\lambda x}{R}\right)$$

$$\text{and} \qquad \text{force} = \frac{B^2d^2}{R + 2\lambda x}\cdot\frac{B^2d^2}{2\lambda m}\ \text{In}\left(\frac{R + 2\lambda x}{R}\right).$$

(ii) Work done per second $= Fv$

Heat produced per second $= i^2 (R + 2\lambda x)$

$$= \left(\frac{Bvd}{R + 2\lambda x}\right)^2 (R + 2\lambda x)$$

$$= \left(\frac{B^2 d^2 v}{R + 2\lambda x}\right) \cdot v$$

$$= F.v$$

Thus, the ratio of heat produced to work done is 1. The entire work done by F per second is converted into heat.

Q. 4. *A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. (a) What is the flux linking the bigger loop if a current of 2.0 A flows through the smaller loop? (b) Obtain the mutual inductance of the two loops.*

Ans. We know from the considerations of symmetry that $M_{12} = M_{21}$. Direct calculation of flux linking the bigger loop due to the field by the smaller loop will be difficult to handle. Instead, let us calculate the flux through the smaller loop due to a current in the bigger loop. The smaller loop is so small in area that one can take the simple formula for field B on the axis of the bigger loop and multiply B by the small area of the loop to calculate flux without much error. Let 1 refer to the bigger loop and 2 the smaller loop. Field B_2 at 2 due to I_1 in 1 is

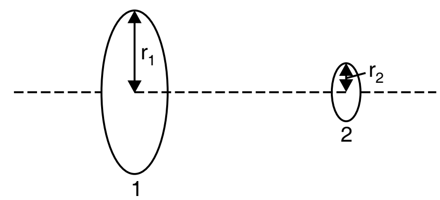


Fig. 6.32

$$B_{21} = \frac{\mu_0 I_1 r_1^2}{2\left(x^2 + r_1^2\right)^{3/2}}$$

Here x is distance between the centres.

$$\phi_{21} = B_{21} \times A_2$$

$$\phi_{21} = B_2 \pi r_2^2 = \frac{\pi\mu_0 r_1^2 r_2^2}{2\left(x^2 + r_1^2\right)^{3/2}} I_1$$

But $\phi_{21} = M_{21} I_1$

$$\therefore \quad M_{21} = \frac{\pi\mu_0 r_1^2 r_2^2}{2\left(x^2 + r_1^2\right)^{3/2}} = M_{12}$$

$$\therefore \quad \phi_{12} = M_{12} I_2 = \frac{\pi\mu_0 r_1^2 r_2^2}{2\left(x^2 + r_1^2\right)^{3/2}} I_2$$

$$M_{12} = M_{21} = \frac{\pi\mu_0 r_1^2 r_2^2}{2(x^2 + r_1^2)^{3/2}}$$

Using the given data

$$M_{12} = M_{21} = 4.55 \times 10^{-11} \text{ H}$$

$$\phi_1 = 9.1 \times 10^{-11} \text{ Wb}$$

Q. 5. *The current in a coil of self-inductance L = 2H is increasing according to the law i = 2 sin t^2. Find the amount of energy spent during the period when the current changes from 0 to 2 ampere.*

Ans. Let the current be 2 ampere at $t = t^2$.

Then $$2 = 2\sin t^2 \Rightarrow t = \sqrt{\pi/2}$$

When the instantaneous current is i, the self induced emf is $L\dfrac{di}{dt}$. If dq amount of charge is displaced in time dt then elementary work done $= L\left(\dfrac{di}{dt}\right) dq = L\dfrac{di}{dt} idt = Lidi$

$$W = \int_0^{\tau} Li\,di = \int_0^{\tau} L2\sin t^2\, d\left(2\sin t^2\right)$$

$$W = \int_0^{\tau} 8L\sin t^2 \cos t^2\, t\,dt = 4L\int_0^{\tau} \sin 2t^2\, t\,dt$$

Let $$\theta = 2t^2\ d\theta = 4t\ dt$$

$\therefore$ $$\text{The integral} = 4L\int \frac{\sin\theta\, d\theta}{4}$$
$$= L(-\cos\theta)$$
$$= -L\cos 2t^2$$

or, $$W = -L\left[\cos 2t^2\right]_0^{\sqrt{\pi}/2}$$
$$= 2L = 2 \times 2 = 4 \text{ joules.}$$

Q. 6. *An infinitesimally small bar magnet of dipole moment $\vec{M}$ is pointing and moving with the speed v in the x-direction. A small closed circular conducting loop of radius a and of negligible self-inductance lies in the y-z plane with its centre at x = 0, and its axis coinciding with the x-axis. Find the force opposing the motion of the magnet, if the resistance of the loop is R. Assume that the distance x of the magnet from the centre of the loop is much greater than a.*

Ans. Field due to the bar magnet at distance x (near the loop)

$$B_a = \frac{\mu_0}{4\pi}\cdot\frac{2M}{x^3} \qquad \text{(axial line)}$$

$\Rightarrow$ Flux linked with the loop: $\phi = BA = \pi a^2 \cdot \dfrac{\mu_0}{4\pi}\cdot\dfrac{2M}{x^3}$

Emf induced in the loop: $$e = -\frac{d\phi}{dt} = \frac{\mu_0}{4\pi}\cdot\frac{6\pi Ma^2}{x^4}\cdot\frac{dx}{dt}$$
$$= \frac{\mu_0}{4\pi}\cdot\frac{6\pi Ma^2}{x^4}v.$$

$\Rightarrow$ Induced current: $$i = \frac{e}{R} = \frac{\mu_0}{4\pi}\cdot\frac{6\pi Ma^2}{Rx^4}v.$$

Let F = force opposing the motion of the magnet

Power due to the opposing force = Heat dissipated in the coil per second

$$\Rightarrow \qquad Fv = i^2 R \quad \Rightarrow \quad F = \frac{i^2 R}{v} = \left(\frac{\mu_0}{4\pi}\right)^2 \times \left(\frac{6\pi Ma^2}{Rx^4}\right)^2 xv^2 \times \frac{R}{v}$$

$$F = \frac{9}{4}\left(\frac{\mu_0^2 M^2 a^4 v}{Rx^8}\right).$$

Q. 7. *(a)* *Show that the energy stored in an inductor i.e., the energy required to build current in the circuit from zero to I is* $\frac{1}{2}$ *LI^2, where L is the self-inductance of the circuit.*

(b) *Extend this result to a pair of coils of self-inductances L_1 and L_2 and mutual inductance M. Hence obtain the inequality $M^2 < L_1 L_2$.*

Ans. (*a*) Energy spent by the source to increase current from i to $i + di$ in time dt in an inductor

$$= L\frac{di}{dt} \times i \times dt$$

$$= Li\, di$$

Energy required to increase current from 0 to I

$$E = \int_0^I Li\, di = L\left[\frac{i^2}{2}\right]_0^I$$

$$E = L\left[\frac{I^2}{2} - 0\right] = \frac{1}{2}LI^2$$

(*b*) The energy stored in the two inductors is independent of the manner of building up current in the coils. Let $I_2 = 0$ initially and also suppose that the current be built up from 0 to I_1 in coil 1. As discussed in part (*a*) of this problem, the required energy is

$$E = \frac{1}{2} L_1 I_1^2. \qquad ...(I)$$

Let us now build up current in coil 2. Let the current increase from i_2 to $i_2 + di_2$ in time dt.

Work done for coil (2)

$$= i_2 L_2 \frac{di_2}{dt} dt$$

But this change in i_2 causes a flux change in 1 and induces emf in 1

$$= M\frac{di_2}{dt}$$

Work done in time dt to maintain current I_1

$$\text{W.D. in 1} = I_1 M \frac{di_2}{dt} dt$$

Total work done in increasing current from 0 to I_2 in coil 2 and for maintaining current I_1 in coil (1)

$$= L_2 \int_0^{I_2} i_2\, di_2 + MI_1 \int_0^{I_2} di_2$$

$$E_2 = \frac{1}{2} L_2 I_2^2 + MI_1 I_2 \quad ...(II)$$

Total energy stored in a pair of coupled coils

$$E = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + MI_1I_2 \quad \text{from } (I, II)$$

$$= \frac{1}{2} L_1 \left(I_1^2 + \frac{2M}{L_1} I_1 L_2 + \frac{M^2}{L_1^2} I_2^2 \right) + \frac{1}{2} L_2 I_2^2 - \frac{1}{2} \frac{M^2}{L_1} I_1^2$$

$$= \frac{1}{2} L_1 \left(I_1 + \frac{M}{L_1} I_2 \right)^2 + \frac{1}{2} \left(L_2 - \frac{M^2}{L_1} \right) I_2^2$$

In order to that the energy be non-negative for all values of I_1 and I_2 (including those values for which the first term is zero), a necessary and sufficient condition is that

$$L_2 > \frac{M^2}{L_1} \quad i.e., \quad M^2 < L_1 L_2.$$

Q. 8. *A current of 10 A is flowing in a long straight wire situated near a rectangular coil. The two sides, of the coil, of length 0.2 m are parallel to the wire. One of them is at a distance of 0.05 m and the other is at a distance of 0.10 m from the wire. The wire is in the plane of the coil. Calculate the magnetic flux through the rectangular coil. If the current decays uniformly to zero in 0.02 s, find the emf induced in the coil and indicate the direction in which the induced current flows.*

Ans. Consider a strip of width dr at a distance r from the straight wire.

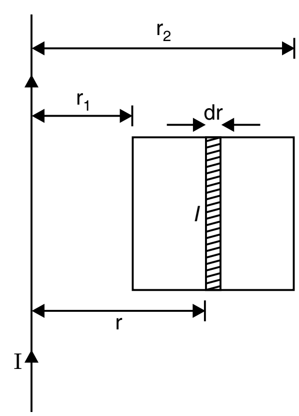


Fig. 6.33

Magnetic field at the location of the strip due to the wire,

$$B = \frac{\mu_0 I}{2\pi r}$$

Area of strip, $dA = ldr$

Magnetic flux linked with the strip,

$$d\phi_B = BdA = \frac{\mu_0 I}{2\pi r} ldr$$

Total magnetic flux linked with the coil,

$$d\phi_B = \frac{\mu_0 Il}{2\pi} \frac{dr}{r}$$

$$\int dQ_B = \frac{\mu_0 Il}{2\pi} \int_{r_1}^{r_2} \frac{dl}{r}$$

$$Q_B = \frac{\mu_0 Il}{2\pi} [\log_e r_2 - \log_e r_1]$$

$$Q_B = \frac{\mu_0 I \,.\, l}{2\pi} [\log r]_{r_1}^{r_2}$$

$$Q_B = \frac{\mu_0 Il}{2\pi} \log_e \frac{r_2}{r_1}$$

Substituting values,

$$\phi_B = \frac{4\pi \times 10^{-7} \times 10 \times 0.2}{2\pi} \log\left[\frac{0.10}{0.05}\right]$$
$$= 4 \times 10^{-7} \log_e 2$$
$$= 4 \times 0.693 \times 10^{-7} \text{ Wb}$$
$$= 2.77 \times 10^{-7} \text{ Wb}$$

Induced e.m.f,

$$|E| = -\frac{d\phi_B}{dt} = \frac{2.77 \times 10^{-7}}{0.02} \text{ V}$$
$$= 1.39 \times 10^{-5} \text{ V}$$

Magnetic field, due to wire, at the location of the coil is perpendicular to the plane of the coil and directed inwards. When current is reduced to zero, this magnetic field decreases. To oppose this decrease, induced current shall flow clockwise, so that its magnetic field is also perpendicular to the plane of the coil and downward.

QUESTIONS ON HIGH ORDER THINKING SKILLS (HOTS)

Q. 1. *A coil ACD of N turns and radius R carries a current I ampere and is placed on a horizontal table. K is a small conducting ring of radius r placed at a distance yo from the centre of and vertically above the coil ACD. Find an expression for the e.m.f. established when the ring k is allowed to fall freely. Express the e.m.f. in terms of speed.*

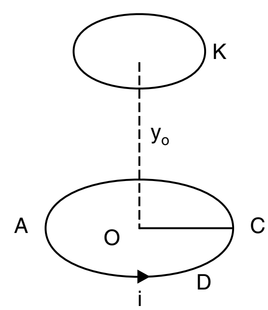


Fig. 6.34

Ans. The field along the axis of the coil *ACD* is given by

$$\frac{\mu_0}{4\pi} \cdot \frac{2\pi\, nIR^2}{\left(R^2 + y^2\right)^{3/2}}$$

Since the ring is small it may be assumed that the induction through it is uniform and is equal to that on the axis.

∴ The magnetic flux linked with it is

$$\phi = BA = \frac{\mu_0}{4\pi} \cdot \frac{2\pi\, nIR^2}{\left(R^2 + y^2\right)^{3/2}} \times \pi r^2$$
$$= \frac{\mu_0}{2} \cdot \frac{\pi nIR^2 r^2}{\left(R^2 + y^2\right)^{3/2}}.$$

Q. 2. (a) *A toroidal solenoid with an air core has an average radius of 0.15 m, area of cross-section 12×10^{-4} m² and 1200 turns. Obtain the self-inductance of the toroid. Ignore field variation across the cross-section of the toroid.*

(b) *A second coil of 300 turns is wound closely on the toroid above. If the current in the primary coil is increased from zero to 2.0 A in 0.05 s, obtain the induced emf in the secondary coil.*

Ans. (a)
$$\boxed{B = \mu_0 n_1 I} = \frac{\mu_0 N_1 I}{l} = \frac{\mu_0 N_1 I}{2\pi r}$$

Total magnetic flux, $\phi_B = N_1 BA = \dfrac{\mu_0 N_1^2 IA}{2\pi r}$

But $\boxed{\phi_B = LI}$

$\therefore$ $L = \dfrac{\mu_0 N_1^2 A}{2\pi r}$

or $L = \dfrac{4\pi \times 10^{-7} \times 1200 \times 1200 \times 12 \times 10^{-4}}{2\pi \times 0.15}$ H

$= 2.3 \times 10^{-3}\ H = 2.3$ mH

(b) $|E| = \dfrac{d}{dt}(\phi_2)$, where ϕ_2 is the total magnetic flux linked with the second coil.

$$|E| = \frac{d}{dt}(N_2 BA) = \frac{d}{dt}\left[N_2 \frac{\mu_0 N_1 I}{2\pi r} A\right]$$

or $|E| = \dfrac{\mu_0 N_1 N_2 A}{2\pi r} \dfrac{dI}{dt}$

or $|E| = \dfrac{4\pi \times 10^{-7} \times 1200 \times 300 \times 12 \times 10^{-4} \times 2}{2\pi \times 0.15 \times 0.05}$ V

$= 0.023$ V

Q. 3. *Self-induction of an air core inductor increases from 0.01 mH to 10 mH on introducing an iron core into it. What is the relative permeability of the core used?*

Ans. Here, $L_0 = 0.01$ mH $= 10^{-5}$ H

$L = 10$ mH $= 10^{-2}$ H

$$\boxed{\mu_r = \frac{L}{L_0}} = \frac{10^{-2}}{10^{-5}} = 10^3 = 1000$$

Q. 4. *A wheel with 10 metallic spokes each 0.5 m long is rotated with angular speed of 120 revolutions per minute in a plane normal to the earth's magnetic field. If the earth's magnetic field at the given place is 0.4 gauss, find the EMF induced between the axle and the rim of the wheel.*

Ans. The area covered by an angle θ,

$$A = \pi r^2 \frac{\theta}{2\pi} = \frac{r^2}{2}\theta$$

The induced EMF, $|E| = \dfrac{d\phi}{dt} = \dfrac{d\,(BA)}{dt}$

$$= B\frac{r^2}{2}\frac{d\theta}{dt} = B\frac{r^2}{2}\omega$$

$$v = 120 \text{ rev/min} = 2 \text{ rev/s}$$

$$\omega = 2\pi v = 4\pi \text{ rad/s}$$

$$r = 0.5 \text{ m}$$

$$B = 0.4 \text{ G} = 0.4 \times 10^{-7} \text{ T}$$

$$\boxed{E = \frac{1}{2}Br^2w}$$

$$\therefore \quad E = \frac{2\pi \times 2 \times 0.4 \times 10^{-4} \times (0.5)^2}{2}$$

$$= 6.28 \times 10^{-5} \text{ V}$$

Each spoke will act as a parallel source of EMF. Hence, the EMF will be 6.28×10^{-5} V.

Q. 5. *How is mutual inductance of a pair of coils affected when*

(i) separation between the coils is increased,

(ii) the number of turns of each coil is increased,

(iii) a thin iron sheet is placed between the two coils, other factors remaining the same. Explain your answer in each case.

Ans. (*i*) When separation between the coils is increased, magnetic flux linked with secondary coil decreases. Therefore, mutual inductance (M) of pair of coils decreases.

(*ii*) When number of turns of each coil is increased. M increases, because

$$\boxed{M = \frac{\mu_0 N_1 N_2 A}{l}.}$$

(*iii*) As $\boxed{M \propto \mu_r,}$ therefore, mutual inductance will increase on placing a thin iron sheet between the two coils.

Q. 6. *A small flat search coil of area 2 cm2 with 25 closely wound turns, is positioned normal to the field direction, and then quickly rotated by 90°. The total charge flown in the coil is 7.5 mC. The resistance of the rod is 0.50 Ω. Estimate the field strength of magnetic field.*

Ans. The total charge flowing through the coil,

$$q = \int_{t_1}^{t_2} I\,dt$$

$$= \frac{1}{R}\int_{t_1}^{t_2} E\,dt$$

where, induced EMF, $|E| = N\frac{d\phi}{dt} \quad \Leftrightarrow \quad E\,dt = Nd\phi$

Hence, $$q = \frac{N}{R}\int_{\phi_1}^{\phi_2} d\phi = \frac{N}{R}(\phi_1 - \phi_2) \quad ...(I)$$

Given, $N = 25, \quad q = 7.5 \times 10^{-3} \text{ C}, \quad \phi_2 = 0, \quad R = 0.50\ \Omega$

Putting these values in equations (*i*), we get

$$\phi_1 = 1.5 \times 10^{-4} \text{ Wb}, \quad A = 2.0 \times 10^{-4} \text{ m}^2$$

$$Q = \frac{qR}{N} \qquad \text{(From } I\text{)}$$

$$\therefore \qquad B = \frac{\phi_1 - \phi_2}{A} = 0.75 \text{ T.}$$

Q. 7. *The figure shows a conductor of length l = 0.5 m and resistance r = 0.5 ohm sliding without friction at a velocity v = 2 m/s over two conducting parallel rods ab and cd lying in a horizontal plane. A resistance R = 2.5 Ω connects the ends b and c. A vertical uniform magnetic field of induction B = 0.6 T exists over the region. Determine (i) the current in the circuit, (ii) the force in the direction of motion to be applied to the conductor for the latter to move with the velocity v and (iii) the thermal power dissipated by the circuit. Neglect the resistance of the guiding rods ab and cd.*

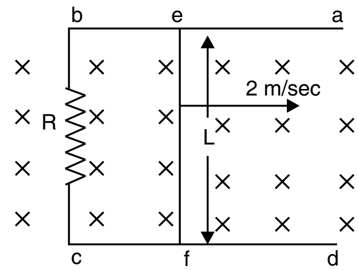


Fig. 6.35

Ans. The conductor *ef* moves with a velocity *v* perpendicular to a uniform magnetic induction *B* and hence induces an emf $E = Blv$.

The resistance of the circuit = $(R + r)$

(*i*) Hence, the current in the circuit $= \frac{Blv}{R+r}$

$$= \frac{(0.6)(0.5)(2)}{(2.5+0.5)}$$

$$= 0.2 \text{ A}$$

(*ii*) The power spend in the system

$$P = \frac{E^2}{R}$$

$$F.v = \frac{(Blv)^2}{(R+r)}$$

$$F = \frac{B^2 l^2 v}{R+r} = \frac{(0.6)^2}{3 \times 2} = 0.06 \text{ N}$$

A force of 0.06 N is required to maintain the motion of the conductor

(*ii*) The power generated $\qquad F.v = \frac{(Blv)^2}{(R+r)}$

$$= 0.06 \times 2 = 0.12 \text{ W}$$

Q. 8. *An electromagnet has stored 648 J of magnetic energy, when a current of 9 A exists in its coils. What average emf is induced if the current is reduced to zero in 0.45 s ?*

Ans. Given, $\qquad$ energy $U = 648$ J, $\quad I = 9$A

$$dI = 9 - 0 = 9\text{A}, \quad dt = 0.45 \text{ s}$$

Using formula,

$$\boxed{U = \frac{1}{2} LI^2}$$

$$648 = \frac{1}{2} \times L\,(9)^2$$

or, $$L = \frac{648 \times 2}{9 \times 9} = 16 \text{ H}$$

Since, $$\boxed{e = \frac{L\, dI}{dt}}$$

∴ $$e = \frac{16\,(9)}{0.45} = 320 \text{ V.}$$

Q. 9. *Two different coils have self-inductances L_1 = 8 mH and L_2 = 2 mH. At a certain instant, the current in the two coils is increasing at the same constant rate and the power supplied to the two coil is the same. Find the ratio of (a) induced voltage (b) current and (c) energy stored in the two coils at that instant?*

Ans. From $$\boxed{e = L\frac{dI}{dt}}, \quad \frac{e_1}{e_2} = \frac{L_1}{L_2} = \frac{8}{2} = 4$$

Since, $$\boxed{P = eI} = \text{constant}$$

$$\frac{dI_1}{dt} = \frac{dI_2}{dt}$$

$$P_1 = P_2 = \boldsymbol{P}$$

$$e_1 I_1 = e_2 I_2$$

∴ $$\frac{I_1}{I_2} = \frac{e_2}{e_1} = \frac{1}{4}$$

$$\frac{I_1}{I_2} = \frac{e_2}{e_1}$$

$$\boxed{U = \frac{1}{2} LI^2}$$

∴ $$\frac{U_1}{U_2} = \frac{\frac{1}{2}L_1}{\frac{1}{2}L_2}\left(\frac{I_1}{I_2}\right)^2 = \frac{8}{2}\left(\frac{1}{4}\right)^2 = \frac{1}{4}.$$

Q. 10. *A square loop of wire of side 5 cm is lying on a horizontal table. An electromagnet above and to one side of the loop is turned on, causing a uniform magnetic field that is downwards at an angle of 30° to the vertical, as shown in the fig. The magnetic induction is 0.50 T. Calculate the average induced emf in the loop, if the field increase from zero to its final value is 0.2 s.*

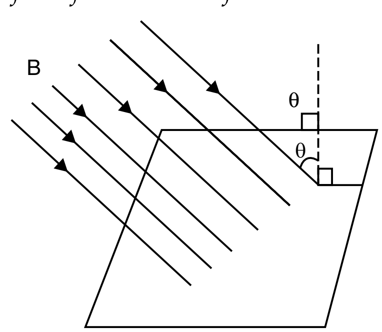


Fig. 6.36

Ans. Area of the loop, $A = (5 \times 10^{-2})^2 \text{ m}^2$

$= 2.5 \times 10^{-3} \text{ m}^2$

Magnetic flux, $\boxed{\phi = AB \cos \theta}$

where $B \cos \theta$ is the component of magnetic induction perpendicular to the plane of the loop.

Therefore, $\phi = 2.5 \times 10^{-3} \times 0.50 \times \cos 30°$

$= 1.0825 \times 10^{-3}$ Wb

Average induced emf is

$$\boxed{e = \frac{\Delta\phi}{\Delta t}} = \frac{1.0825 \times 10^{-3}}{0.2}$$

$= 5.4 \times 10^{-3}$ V

The induced emf is in an anticlockwise direction around the loop during the time when the downward magnetic flux is increasing.

Q. 11. *The motion of a copper plate is damped when it is allowed to oscillate between the pole pieces of a magnet. State the cause of this damping.*

Ans. This damping is due to the eddy current developed in the copper plate. When copper plate oscillates between the pole pieces of the magnet, the flux linked with copper plate changes and eddy currents (induced current) are developed which opposes the cause of the production.

Q. 12. *A long solenoid of 20 turns per cm has a small loop of area 4 cm² placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 4A to 6A in 0.25, what is the (average) induced emf in the loop while the current is changing?*

Ans. The induced emf,

$$\varepsilon = -\frac{d\phi}{dt}$$

$$\varepsilon = \frac{d}{dt}(BA \cos \phi) \qquad (\because \cos \phi = 1)$$

$$= \frac{20 \times 100 \times 4 \times 10^{-4}}{0.2}$$

$= 4$ volt.

Q. 13. *A conducting U-tube can slide inside another U-tube maintaining electrical contact between the tubes. The magnetic field is perpendicular to the plane of paper and is directed inward. Each tube moves towards the other at constant speed v. Find the magnitude of induced emf across the ends of the tube in terms of magnetic field B, velocity v and width of the tube l.*

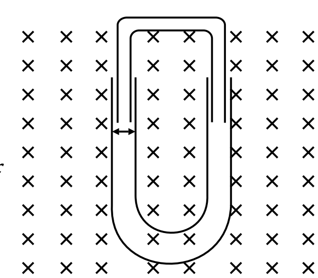


Fig. 6.37

Ans. Relative velocity of the tube of width $l = v - (-v) = 2v$

$\therefore$ induced emf,

$$\varepsilon = Bl\,(2v)$$

$= 2\,Blv.$

Q. 14. *If the number of turns of solenoid is doubled, keeping the other factors constant, how does the self-induction of the solenoid change?*

Ans. $\because$ Coefficient of self induction,

$$L = \frac{N\mu_0 A}{l}$$

$\therefore$ When l is made doubled, the coefficient of self induction becomes half.

Q. 15. *A rectangular loop and a circular loop are moving out of a same magnetic field to a field free region with constant velocity.*

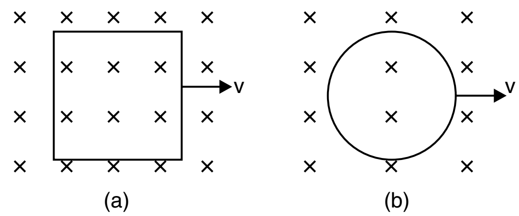


Fig. 6.38

It is given that the field is normal to the plane of both the loops. Draw the expected shape of the graphs. Showing the variation of flux with time in both the cases. What is the cause of the difference in the shape of the two graphs?

Ans.

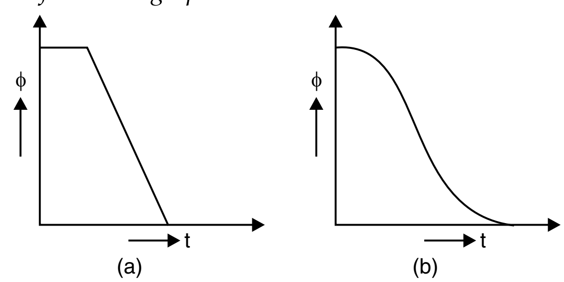


Fig. 6.39

MULTIPLE CHOICE QUESTIONS

1. In a pure inductive circuit with a.c. source, the current lags behind emf by

(*a*) π (*b*) 2π (*c*) $\frac{\pi}{2}$ (*d*) $\frac{\pi}{4}$

2. A step up transformer operates on a 230 volt line and a load current of 2 ampere. The ratio of the primary and secondary windlings is 1 : 25. The current in the primary is

(*a*) 25 amp (*b*) 50 amp (*c*) 15 amp (*d*) 12.5 amp

3. Which of the following quantities remain constant in a step-down transformer?

(*a*) Current (*b*) Voltage (*c*) Power (*d*) None of these

4. A long solenoid has 800 turns per metre. A current of 1.6 A flows through it. The magnetic induction at the end of solenoid on its axis is

(*a*) 16×10^{-4} T (*b*) 0.8×10^{-4} T (*c*) 4×10^{-4} T (*d*) 4×10^{-4} T

5. An inductance L having a resistance R is connected to an alternating source of angular frequency w. The quality factor (0) of the inductance is

(*a*) $\left(\frac{wL}{R}\right)^2$ (*b*) $\left(\frac{R}{wL}\right)^{1/2}$ (*c*) $\frac{wL}{R}$ (*d*) $\frac{R}{wL}$

6. What is the unit of self inductance of a coil?

(*a*) volt $s^{-1}A^{-1}$ (*b*) $volt^{-1}A$ (*c*) volt $s^{-1}A^{-2}$ (*d*) volt $s^{-1}A^{-1}$

7. Two inductors of inductance L each are connected in series with opposite magnetic fluxes. What is the resultant inductance?

(*a*) Zero (*b*) L (*c*) $2L$ (*d*) $3L$

8. The current flowing in a step down transformer 220 V to 22 V having impedance 220 Ω is
(*a*) 1 A (*b*) 0.1 A (*c*) 1 mA (*d*) 0.1 MA

9. For a coil having $L = 2\ mH$, current flows at the rate of $10^{-3}\ A/s$. The emf induced is
(*a*) 2 V (*b*) 1 V (*c*) 4 V (*d*) 3 V.

10. When a *d.c* motor operates at 200 V, its initial current is 5 *A*, but when it runs at maximum speed, the current is only 3*A*. What is its back emf?
(*a*) 120 V (*b*) 200 V (*c*) 80 V (*d*) Zero

11. The coefficient of mutual inductance, when magnetic flux changes by 2×10^{-2} Wh and current changes by 0.01 *A* is
(*a*) 2 H (*b*) 4 H (*c*) 3 H (*d*) 28 H

12. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon
(*a*) the rates at which currents are changing in the two coils
(*b*) relative position and orientation of the two coils
(*c*) the materials of the wires of the coils
(*d*) the currents in the two coils

13. When current changes from + 2 A to – 2 A in 0.05 sec, an emf of 8 V is induced in a coil. The coefficient of self inductance of the coil is
(*a*) 0.2 H (*b*) 0.4 H (*c*) 0.8 H (*d*) 0.1 H

14. The magnetic flux through a circuit of resistance *R* changes by an amount $\Delta\phi$ in a time Δt. Then the total quantity of electric charge *Q* that passes any point in the circuit during the time Δt is represented by
(*a*) $Q = \frac{\Delta\phi}{\Delta t}$ (*b*) $Q = R\,\frac{\Delta\phi}{\Delta t}$ (*c*) $Q = \frac{1}{R}\frac{\Delta\phi}{\Delta t}$ (*d*) $Q = \frac{\Delta\phi}{R}$

15. An emf of 100 mV is induced in a coil when current in another near by coil becomes 10 A from 0 in 0.1 S. The coefficient of mutual induction between the two coils will be
(*a*) 1 mH (*b*) 10 mH (*c*) 100 mH (*d*) 1000 mH

Answers

1. (*c*)	**2.** (*b*)	**3.** (*c*)	**4.** (*b*)	**5.** (*c*)
6. (*d*)	**7.** (*c*)	**8.** (*b*)	**9.** (*a*)	**10.** (*c*)
11. (*a*)	**12.** (*b*)	**13.** (*d*)	**14.** (*d*)	**15.** (*a*).

TEST YOUR SKILLS

1. Two circular coils, one of radius *r* and other of radius *R* are placed coaxially with their contres coinciding. For $R >> r$, obtain an expression for the mutual inductance of the arrangement.
2. A circular coil of *n* turns and radius *R*, is kept normal to a magnetic field, given by $B = B_0$ cost. Deduce an expression for emf induced in this coil. State the rule which helps to detect the direction of induced current.
3. Derive an expression for (*i*) induced emf and (*ii*) induced current, when conductor of length '*l*' is moved with a uniform velocity '*v*' normal to a uniform magnetic field *B*. Assume the resistance of the conductor to be *R*.
4. A solenoid with an iron and a bulb are connected to a d.c. source. How does the brightness of the bulb change, when the iron core is removed from the solenoid?

5. What is induced emf? Write Faraday's law of electromagnetic induction Express it mathematically. A conducting rod of length l, with one end pivoted, is rotated with a uniform angular speed 'ω' in a vertical plane, normal to uniform magnetic field 'B'. Deduce an expression for the emf. induced in this rod.

6. A rectangular conductor, LMNO, is placed in a uniform magnetic field, B, directed perpendicular to the plane of the conductor. Obtain an expression, for the emf induced in the arm MN, when the arm is moved towards the left with a speed v. Use the above expression to find the emf induced between the ends of an axel, of length, L, of a railway carriage travelling on level ground with a speed v. Assume the value of earth's magnetic field at the place, to be B and the angle of dip to be θ.

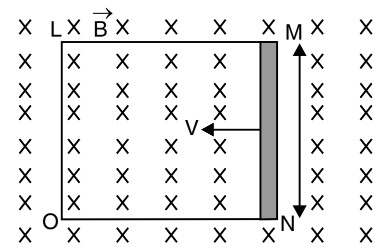


Fig. 6.40

7. The figure 6.41. shows the variation of a alternating emf with time. What is the average value of emf for shaded portion of graph?

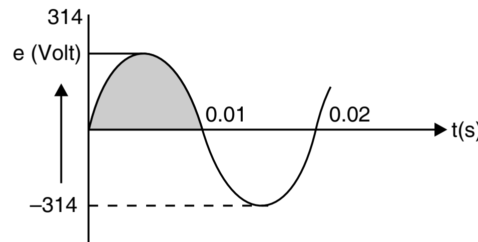


Fig. 6.41

8. Name the phenomenon associated with the production of back emf in a coil due to change of electric current through the coil itself. Name and define the S.I. unit for measuing this characteristic of the coil.

9. How does electric field produced by change in magnetic flux different as the electric field produced by static charge?

10. Why does change in magnetic flux always not produced induced current?

11. What will happen in the loop shown in figure 6.42? If the loop is placed in increasing magnetic field?

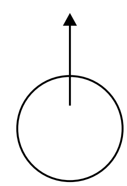


Fig. 6.42

12. How does induced charge in a circuit depend upon the change in magnetic field?

13. Does the induced current always oppose the main current in the circuit? If not justify your answer?

14. There are two solenoids made up of two wires of equal lengths. If ratio of the diameter of the solenoid is 1 : 2. Then what is the ratio of their coefficient of mutual induction?

15. If resistance R in circuit 6.43. (*i*) is decreased, find the direction of current in circuit (2).

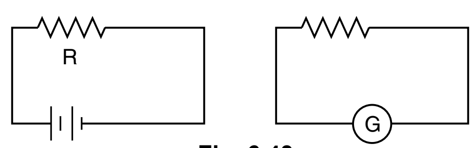


Fig. 6.43

16. In figure 6.44, a coil made of copper suspended from a fixed support is oscillating freely. A magnetic dipole is brought near the coil and held stationary. What will happen to the motion of the coil?

17. A vertical pole falls down through the plane of the magnetic meridian. Will any emf be produced between the ends? Give reason of your answer.

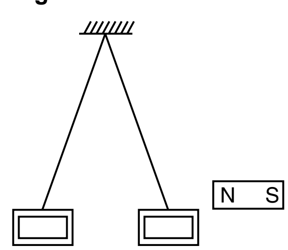


Fig. 6.44

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