6 Work, Energy and Power

Facts that Matter

- Work is said to be done when a force applied on the body displaces the body through a certain distance in the direction of applied force.

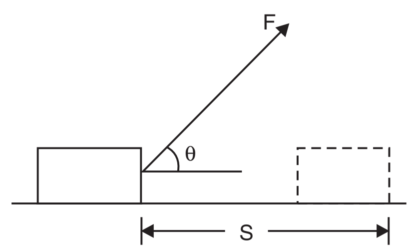
It is measured by the product of the force and the distance moved in the direction of the force, *i.e.,*

$$W = F \cdot S$$

- If an object undergoes a displacement 'S' along a straight line while acted on a force F that makes an angle θ with S as shown.

The work done W by the agent is the product of the component of force in the direction of displacement and the magnitude of displacement.

i.e., $$W = FS\cos\theta = \vec{F}\cdot\vec{S}$$



- Work done is a scalar quantity measured in newtonmetre.

Its dimension is $[ML^2T^{-2}]$.

(1 newton-metre = 1 joule)

- Following are some significant points about work done, derived from the definition given above.
  - (*i*) Work done by a force is zero if displacement is perpendicular to the force (θ = 90°).
  - (*ii*) Work done by the force is positive if angle between force and displacement is acute (θ < 90°).
  - (*iii*) Work done by the force is negative if angle between force and displacement is obtuse (θ > 90°).
- If the applied force varies with time/position, the work done is given by:

$$W = \int \vec{F}\cdot d\vec{s}$$

- If we plot a graph between force applied and the displacement, then work done can be obtained by finding the area under the *F-s* graph.
- If a spring is stretched or compressed by a small distance from its unstretched configuration, the spring will exert a force on the block given by

$F = -kx$, where x is compression or elongation in spring. k is a constant called spring constant whose value depends inversely on unstretched length and the nature of material of spring.

> The negative sign indicates that the direction of the spring force is opposite to x, the displacement of the free-end.

Work done by spring when block is displaced by x_0 is given by

$$W = -\int_0^{x_0} kx\,dx = -\frac{1}{2}kx_0^2$$

Work done by an agent in giving an elongation or compression of x_0 is given as $\frac{1}{2}kx_0^2$.

- **Energy**

The energy of a body is its capacity to do work. Anything which is able to do work is said to possess energy. Energy is measured in the same unit as that of work, namely, Joule.

Mechanical energy is of two types: Kinetic energy and Potential energy.

- **Kinetic Energy**

The energy possessed by a body by virtue of its motion is known as its kinetic energy.

For an object of mass m and having a velocity v, the kinetic energy is given by:

$$\text{K.E. or K} = \frac{1}{2}mv^2$$

- **Potential Energy**

The energy possessed by a body by virtue of its position or condition is known as its potential energy.

There are two common forms of potential energy: gravitational and elastic.

→ Gravitational potential energy of a body is the energy possessed by the body by virtue of its position above the surface of the earth.

It is given by

$$\text{(U) P.E.} = mgh$$

where m → mass of a body

g → acceleration due to gravity on the surface of earth.

h → height through which the body is raised.

→ When an elastic body is displaced from its equilibrium position, work is needed to be done against the restoring elastic force. The work done is stored up in the body in the form of its elastic potential energy.

If an elastic spring is stretched (or compressed) by a distance 'x' from its equilibrium position, then its elastic potential energy is given by

$$U = \frac{1}{2}kx^2,$$

where, k → force constant of given spring

- **Work-Energy Theorem**

According to work-energy theorem, the work done by a force on a body is equal to the change in kinetic energy of the body.

$$W = \text{Change in K.E. of a body} = \Delta\,(K.E.)$$

Δ(K.E.) → The difference between the final and initial kinetic energies of the body.

- Energy and momentum are related by, $E = \frac{p^2}{2m}$, where m is the mass.

- **The Law of Conservation of Energy**

According to the law of conservation of energy, the total energy of an isolated system does not change. Energy may be transformed from one form to another but the total energy of an isolated system remains constant.

Energy can neither be created, nor destroyed.

- Besides mechanical energy, the energy may manifest itself in many other forms. Some of these forms are: thermal energy, electrical energy, chemical energy, visual light energy, nuclear energy etc.

- **Equivalence of Mass and Energy**

According to Einstein, mass and energy are inter-convertible. That is, mass can be converted into energy and energy can be converted into mass.

The energy (E) equivalent to mass m is given by the relation

$$E = mc^2 \quad \text{where,} \quad c = 3 \times 10^8 \text{ ms}^{-1}\text{, velocity of light in vacuum or air.}$$

- When a body moves with a velocity v, comparable to the velocity of light 'C' its mass m is given by

$$m = \frac{m_0}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}, \text{ where } m_0 \text{ its rest mass.}$$

- **Power**

It is the rate of doing work, *i.e.,* the work done per unit time.

$$p = \frac{dW}{dt} = \vec{F}.\vec{v} = Fv \cos \alpha$$

where α is the angle between the force F and the velocity v.

- Power is a scalar quantity. Its *SI* unit is watt, 1 watt = 1 Js^{-1}. The dimensional formula of power is $[M^1L^2T^{-3}]$.

- Other commonly used units of power are:

$$1 \text{ kilowatt} = 1 \text{ KW} = 10^3 \text{ W}$$

$$1 \text{ megawatt} = 1 \text{ MW} = 10^3 \text{ KW} = 10^6 \text{ W}$$

$$1 \text{ horse power (hp)} = 746 \text{ watt} = 0.746 \text{ KW.}$$

- **Collision**

Collision is defined as an isolated event in which two or more colliding bodies exert relatively strong forces on each other for a relatively short time.

Collision between particles have been divided broadly into two types:

(*i*) Elastic collisions (*ii*) Inelastic collisions

- **Elastic Collision**

A collision between two particles or bodies is said to be elastic if both the linear momentum and the kinetic energy of the system remain conserved.

Example: Collisions between atomic particles, atoms, marble balls and billiard balls.

- **Inelastic Collision**

A collision is said to be inelastic if the linear momentum of the system remains conserved but its kinetic energy is not conserved.

Example: When we drop a ball of wet putty on to the floor then the collision between ball and floor is an inelastic collision.

- Collision is said to be one dimensional, if the colliding particles, move along the same straight line path both before as well as after the collision.

- In one dimensional elastic collision, the relative velocity of approach before collision is equal to the relative velocity of separation after collision.

- If two particles of mass m_1 and m_2 moving with velocities $\vec{u}_1$ and $\vec{u}_2$ respectively collide head on such that $\vec{v}_1$ and $\vec{v}_2$ be their respective velocities after collision, then,

$$\vec{v}_1 = \frac{(m_1 - m_2)\,\vec{u}_1 + 2m_2\vec{u}_2}{(m_1 + m_2)} \text{ and } \vec{v}_2 = \frac{2m_1\vec{u}_1 + (m_2 - m_1)\vec{u}_2}{(m_1 + m_2)}$$

- **Coefficient of Restitution or Coefficient of Resilience**

Coefficient of restitution is defined as the ratio of relative velocity of separation after collision to the relative velocity of approach before collision.
It is represented by '*e*'.

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

- **Elastic and Inelastic Collisions in Two Dimensions**

Let us consider two bodies *A* and *B* moving with their initial velocities $\vec{u}_1$ and $\vec{u}_2$ respectively is a two dimensional plane. If they collide with each other and still moving with certain velocities $\vec{v}_1$ and $\vec{v}_2$ respectively after the collision, then the collision is known as two dimensional (or an oblique) collision Fig. (*i*).
When the collision is elastic the total kinetic energy of the two bodies before the collision is equal to the total energy of the bodies after the collision. It means, the kinetic energy is conserved is case of elastic collision.
The kinetic energy is not conserved in case of inelastic collision.

When the body *A* is moving with a velocity of $\vec{u}_1$ and the body *B* is at rest i.e. $\vec{u}_2 = 0$, then Fig. (*ii*) after the collision, let θ be the angle known as scattering angle made by the body with its initial direction Fig. (*iii*) and the body *B* moves with an angle of ϕ with its initial direction. This angle is known as 'angle of recoil'.
According to the law of conservation of momentum

$$m_1u_1 + 0 = m_1v_1 \cos\theta + m_2v_2 \cos\phi$$

(*i*)

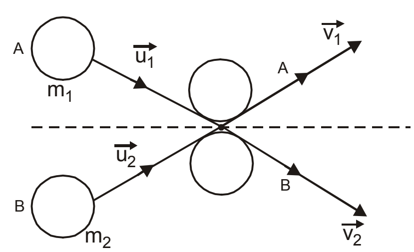


fig (*i*)

and $$0 = m_1v_1 \sin\theta - m_2v_2 \sin\phi \qquad (ii)$$

As the collision is perfectly elastic,

Total K.E. before the collision = Total K.E. after the collision.

$$\frac{1}{2}m_1u_1^2 + 0 = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 \quad (iii)$$

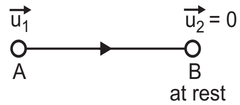


Fig. (*ii*)

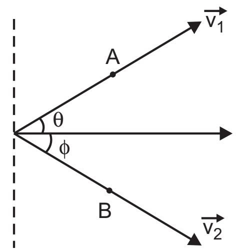


Fig. (*iii*)

Special Case : If $m_1 = m_2$ then the above equation (*i*), (*ii*) and (*iii*) are reduced to

$$u_1 = v_1 \cos\theta + v_2 \cos\phi \quad (iv)$$

$$0 = v_1 \sin\theta - v_2 \sin\phi \quad (v)$$

and $$u_1^2 = V_1^2 + V_2^2 \quad (vi)$$

From eq. (*iv*) and (*vi*) we get,

$$(V_1 \cos\theta + V_2 \cos\phi)^2 = V_1^2 + V_2^2$$

$$\Rightarrow V_1^2\cos^2\theta + V_2^2\cos^2\phi + 2V_1V_2\cos\theta\cos\phi = V_1^2 + V_2^2$$

$$\Rightarrow 2V_1V_2 \cos\theta \cos\phi = V_1^2 - V_1^2\cos^2\theta + V_2^2 - V_2^2\cos^2\phi$$

$$\Rightarrow 2V_1V_2 \cos\theta \cos\phi = V_1^2(1-\cos^2\theta) + V_2^2(1-\cos^2\phi)$$

$$\Rightarrow 2V_1V_2\cos\theta \cos\phi = V_1^2\sin^2\theta + V_2^2\sin^2\phi \quad (vii)$$

$$\Rightarrow 2V_1V_2 \cos\theta \cos\phi = V_1^2\sin^2\theta + V_1^2\sin^2\theta$$

$$\Rightarrow 2V_1V_2 \cos\theta \cos\phi = 2V_1^2\sin^2\theta$$

$$\Rightarrow V_2 \cos\theta \cos\phi = V_1 \sin^2\theta$$

$$\therefore \cos\theta = \left(\frac{V_1}{V_2}\right)\left(\frac{\sin^2\theta}{\cos\phi}\right) \quad (viii)$$

Now $$\cos(\theta + \phi) = \cos\theta \cos\phi - \sin\theta \sin\phi$$

$$= \left(\frac{V_1}{V_2}\right)\left(\frac{\sin^2\theta}{\cos\phi}\right)\cdot\cos\phi - \sin\theta\cdot\left(\frac{V_1}{V_2}\right)\sin\theta \quad \text{[From } (v)\text{]}$$

$$= \frac{V_1}{V_2}\sin^2\theta - \frac{V_1}{V_2}\sin^2\theta = 0$$

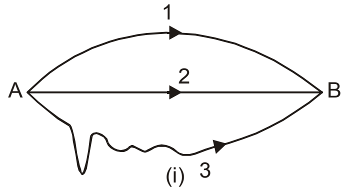
$$\therefore \theta + \phi = \frac{\pi}{2}$$

So, for a special case the two bodies of equal mass, make right angle between their directions after the collision.

- **Non-conservative Forces**

A force is said to be non-conservative if the work done in moving from one point to another depends upon the the path followed.

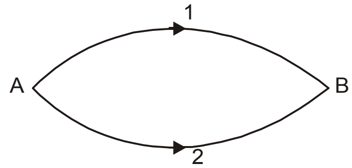
Let W_1 be the work done in moving from A to B following the path 1. W_2 through the path 2 and W_3 through the path 3. Fig. (*i*).



(i)

∴ For non-conservative forces $W_1 \neq W_2 \neq W_3$

Also in figure (*ii*) the net work done by or against the non-conservative forces is not zero i.e. $W_1 \neq W_2$.



(ii)

Mathematically $\int F \times dS \neq 0$

Examples of non-conservative forces are :

(*i*) Force of friction (*ii*) Viscus force

Low of conservation of energy holds goods for both conservative and non-conservative forces.

- **IMPORTANT TABLES**

TABLE 6.1 Average power consumption in some common activities

	Activity	*Power (watt)*
1.	Heart beat	1.2
2.	Sleeping	75
3.	Slow walking	200
4.	Cycling	500

TABLE 6.2 Some typical kinetic energy values

S.No.	*Object*	*mass (kg)*	*Speed (ms^{-1})*	*K.E. (J)*
1.	Stone dropped from 10 *m*.	0.5	14	≈ 50
2.	Rain drop at terminal speed	3.5×10^{-5}	9	≈ 1.4×10^{-3}
3.	Air molecule	10^{-26}	500	≈ 10^{-21}
4.	Running athlete	70	10	3.5×10^{3}
5.	Bullet	5×10^{-2}	200	10^{3}
6.	Car	2000	20	4×10^{5}

TABLE 6.3 Energy associated with some important phenomena

S.No.	*Phenomenon*	*Energy (J)*
1.	Energy required to break one bond in DNA	$\approx 10^{-20}$
2.	Energy of an electron in an atom	$\approx 10^{-18}$
3.	Energy of a proton in a nucleus	$\approx 10^{-13}$
4.	Energy associated with discharge of a single neutron	$\approx 10^{-10}$
5.	Energy spent in turning a page	$\approx 10^{-3}$
6.	Work done by human heart beat	≈ 0.5
7.	Daily food intake of a human adult	$\approx 10^{7}$
8.	Energy released in burning 1 litre of gasoline	$\approx 3 \times 10^{7}$
9.	K.E. of a jet aircraft	$\approx 10^{9}$
10.	Energy released in burning 1000 kg of coal	$\approx 3 \times 10^{10}$
11.	Energy release of 15 megaton fusion bomb	$\approx 10^{17}$
12.	Annual solar energy incident on earth	$\approx 5 \times 10^{24}$
13.	Rotational energy of earth	$\approx 10^{29}$
14.	Energy released in a supernova explosion	$\approx 10^{44}$
15.	Rotational energy of Milky way	$\approx 10^{52}$
16.	Big Bang	$\approx 10^{68}$

TABLE 6.4 Some Important Physical Quantities, Symbols, Dimensions and Units

Physical Quanity	*Symbols*	*Dimensions*	*Units*
Work	W	$[ML^2T^{-2}]$	Joule
Kinetic energy	K	$[ML^2T^{-2}]$	Joule
Potential energy	$V(x)$	$[ML^2T^{-2}]$	Joule
Mechanical energy	E	$[ML^2T^{-2}]$	Joule
Spring constant	k	$[MT^{-2}]$	Nm^{-1}
Power	P	$[ML^2T^{-3}]$	Watt

NCERT TEXTBOOK QUESTIONS SOLVED

6.1. *The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative:*

(a) Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket,

(b) Work done by gravitational force in the above case,

(c) Work done by friction on a body sliding down an inclined plane,

(d) Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity,

(e) Work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

Sol. Work done, $W = \vec{F}.\vec{S} = FS \cos \theta$

(*a*) Work done 'positive', because force is acting in the direction of displacement *i.e.,* $\theta = 0°$.

(*b*) Work done is negative, because force is acting against the displacement *i.e.,* $\theta = 180°$.

(*c*) Work done is negative, because force of friction is acting against the displacement *i.e.,* $\theta = 180°$.

(*d*) Work done is positive, because body moves in the direction of applied force *i.e.,* $\theta = 0°$.

(*e*) Work done is negative, because the resistive force of air opposes the motion *i.e.,* $\theta = 180°$.

6.2. *A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction = 0.1. Compute the*

(a) Work done by the applied force in 10 s

(b) Work done by friction in 10 s

(c) Work done by the net force on the body in 10 s

(d) Change in kinetic energy of the body in 10 s and interpret your results.

Sol. (*a*) We know that $\mu_k = \dfrac{\text{frictional force}}{\text{normal reaction}}$

$\therefore$ frictional force $= \mu_k \times$ normal reaction

$= 0.1 \times 2$ kg *wt* $= 0.1 \times 2 \times 9.8$ N $= 1.96$ N

net effective force $= (7 - 1.96)$ N $= 5.04$ N

acceleration $= \dfrac{5.04}{2}$ ms^{-2} $= 2.52$ ms^{-2}

distance, $s = \dfrac{1}{2} \times 2.52 \times 10 \times 10 = 126$ m

work done by applied force $= 7 \times 126$ J $= 882$ J

(*b*) Work done by friction $= 1.96 \times 126 = -246.96$ J

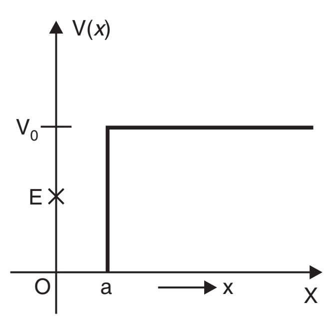
(*c*) Work done by net force $= 5.04 \times 126 = 635.04$ J

(*d*) Change in the kinetic energy of the body

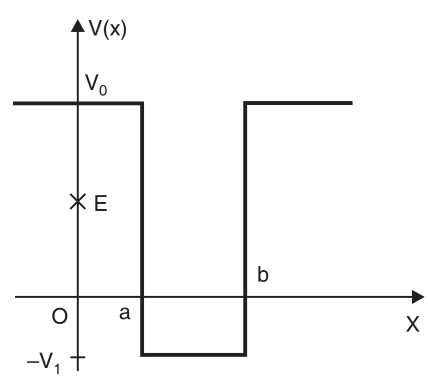
= work done by the net force in 10 seconds

= 635.04 J (This is in accordance with work-energy theorem).

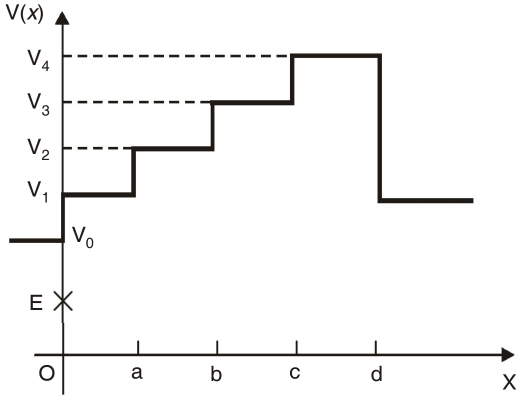
6.3. *Given figures are examples of some potential energy functions in one dimension. The total energy of the particle is indicated by a cross on the ordinate axis. In each case, specify the regions, if any, in which the particle cannot be found for the given energy. Also, indicate the minimum total energy the particle must have in each case. Think of some physical contexts for which these potential energy shapes are relevant.*



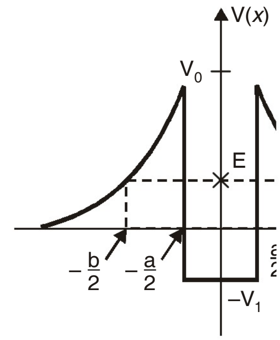
(a)



(b)



(c)



(d)

Sol. We know that total energy E = K.E. + P.E. or K.E. = E – P.E. and kinetic energy can never be negative.

The object can not exist in the region, where its K.E. would become negative.

(*a*) In the region between $x = 0$ and $x = a$, potential energy is zero. So, kinetic energy is positive. In the region $x > a$, the potential energy has a value greater than E. So, kinetic energy will be negative in this region. Thus the particle cannot be present in the region $x > a$.

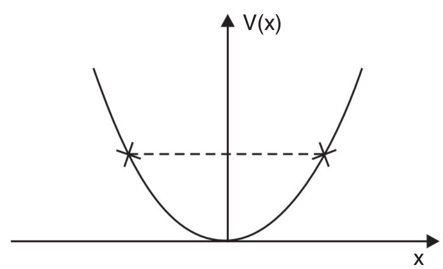
The minimum total energy that the particle can have in this case is zero.

(*b*) Here P.E. > E, the total energy of the object and as such the kinetic energy of the object would be negative. Thus object cannot be present in any region of the graph.

(*c*) Here $x = 0$ to $x = a$ and $x > b$, the P.E. is more than E, so K.E. is negative. The particle can not exist in these portions.

(*d*) The object can not exist in the region between $x = -\frac{b}{2}$ to $x = -\frac{a}{2}$ and $x = \frac{-a}{2}$ to $x = \frac{-b}{2}$. Because in this region P.E. > E.

6.4. *The potential energy function for a particle executing linear simple harmonic motion is given by V (x) = $kx^2/2$, where k is the force constant of the oscillator. For k = 0.5 Nm^{-1}, the graph of V (x) versus x is shown in Fig. Show that a particle of total energy 1 J moving under this potential must 'turn back' when it reaches x = ± 2 m.*



Sol. Here, force constant $k = 0.5\ Nm^{-1}$ and total energy of particle E = 1J. The particle can go up to a maximum distance x_m, where its total energy is transformed into elastic potential energy.

$$\frac{1}{2}kx_m^2 = E \Rightarrow x_m = \sqrt{\frac{2E}{K}} = \sqrt{\frac{2\times 1}{0.5}} = \sqrt{4} = \pm 2\text{m}.$$

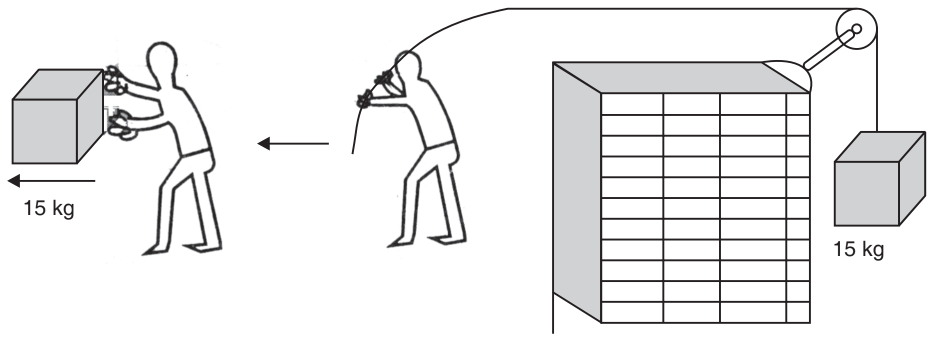
6.5. *Answer the following:*

(a) The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained? The rocket or the atmosphere?

(b) Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why?

(c) *An artificial satellite orbiting the earth in very thin atomosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then does its speed increase progressively as it comes closer and closer to the earth?*

(d) *In Fig. (i), the man walks 2 m carrying a mass of 15 kg on his hands. In Fig. (ii), he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end. In which case is the work done greater?*



Sol. (*a*) Heat energy required for burning of casing of rocket comes from the rocket itself. As a result of work done against friction the kinetic energy of rocket continuously decreases and this work against friction reappears as heat energy.

(*b*) This is because gravitational force is a conservative force. Work done by the gravitational force of the sun over a closed path in every complete orbit of the comet is zero.

(*c*) As an artificial satellite gradually loses its energy due to dissipation against atmospheric resistance, its potential decreases rapidly. As a result, kinetic energy of satellite slightly increases *i.e.,* its speed increases progressively.

(*d*) In Fig. (*i*), force is applied on the mass, by the man in vertically upward direction but distance is moved along the horizontal.

$\therefore \quad \theta = 90°. \quad W = Fs \cos 90° = \text{zero}$

In Fig. (*ii*), force is applied along the horizontal and the distance moved is also along the horizontal. Therefore, $\theta = 0°$.

$$W = Fs \cos \theta = mg \times s \cos 0°$$

$$W = 15 \times 9.8 \times 2 \times 1 = 294 \text{ joule.}$$

Thus, work done in (*ii*) case is greater.

6.6. *Point out the correct alternative:*

(a) *When a conservative force does positive work on a body, the potential energy of the body increases/decreases/remains unaltered.*

(b) *Work done by a body against friction always results in a loss of its kinetic/potential energy.*

(c) *The rate of change of total momentum of a many particle system is proportional to the external force/sum of the internal forces of the system.*

(d) *In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum/total energy of the system of two bodies.*

Sol. (*a*) Potential energy of the body decreases, because the body in this case goes closer to the centre of the force.

(*b*) Kinetic energy, because friction does its work against motion.

(*c*) Internal forces can not change the total or net momentum of a system. Hence the rate of change of total momentum of many particle system is proportional to the external force on the system.

(*d*) In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum/total energy of the system of two bodies.

6.7. *State if each of the following statements is true or false. Give reasons for your answer.*

(*a*) *In an elastic collision of two bodies, the momentum and energy of each body is conserved.*

(*b*) *Total energy of a system is always conserved, no matter what internal and external forces on the body are present.*

(*c*) *Work done in the motion of a body over a closed loop is zero for every force in nature.*

(*d*) *In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system.*

Sol. (*a*) False, the total momentum and total energy of the system are conserved.

(*b*) False, the external force on the system may increase or decrease the total energy of the system.

(*c*) False, the work done during the motion of a body over a closed loop is zero only when body is moving under the action of a conservative force (such as gravitational or electrostatic force). Friction is not a conservative force hence work done by force of friction (or work done on the body against friction) is not zero over a closed loop.

(*d*) True, usually in an inelastic collision the final kinetic energy is always less than the initial kinetic energy of the system.

6.8. *Answer carefully, with reasons:*

(a) In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls (i.e., when they are in contact)?

(b) Is the total linear momentum conserved during the short time of an elastic collision of two balls?

(c) What are the answers to (a) and (b) for an inelastic collision?

(d) If the potential energy of two billiard balls depends only on the separation distance between their centres, is the collision elastic or inelastic? (Note, we are talking here of potential energy corresponding to the force during collision, not gravitational potential energy).

Sol. (*a*) In this case total kinetic energy is not conserved because when the bodies are in contact during elastic collision even, the kinetic energy is converted into potential energy.

(*b*) Yes, because total momentum conserves as per law of conservation of momentum.

(*c*) The answers remain unchanged.

(*d*) It is a case of elastic collision because in this case the forces will be of conservative nature.

6.9. *A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to*

(i) $t^{1/2}$ *(ii)* t *(iii)* $t^{3/2}$ *(iv)* t^2

Sol. (*ii*) From $v = u + at$

$v = 0 + at = at$

As power, $p = F \times v$

$\therefore$ $p = (ma) \times at = ma^2t$

Since m and a are constants, therefore, $p \propto t$.

6.10. *A body is moving unidirectionally under the influence of a source of constant power. Its displacement in time t is proportional to*

(i) $t^{1/2}$ *(ii)* t *(iii)* $t^{3/2}$ *(iv)* t^2

Sol. (*ii*) p = force × velocity

$[p] = [F]\,[v] = [MLT^{-2}]\,[LT^{-1}] \Rightarrow [p] = [ML^2T^{-3}]$

or L^2T^{-3} = constant $\Rightarrow \dfrac{L^2}{T^2}$ = constant

$\therefore$ $L^2 \propto T^3 \Rightarrow L \propto T^{3/2}$.

6.11. *A body constrained to move along the z-axis of a coordinate system is subject to a constant force* ***F*** *given by*

$$\mathbf{F} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}N$$

where $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}},$ *are unit vectors along the x–, y– and z-axis of the system respectively. What is the work done by this force in moving the body a distance of 4 m along the z-axis?*

Sol. Since the body is displaced 4 m along z-axis only,

$\therefore$ $\vec{S} = 0\hat{i} + 0\hat{j} + 4\hat{k}$

Also $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$

Work done, $W = \vec{F}.\vec{S} = (-\hat{i} + 2\hat{j} + 3\hat{k}).(0\hat{i} + 0\hat{j} + 4\hat{k})$

$= 12\ (\hat{k}.\hat{k})$ Joule = 12 Joule.

6.12. *An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV, and the second with 100 keV. Which is faster, the electron or the proton? Obtain the ratio of their speeds. (electron mass = 9.11 × 10^{-31} kg, proton mass = 1.67 × 10^{-27} kg, 1 eV = 1.60 × 10^{19}J).*

Sol. Here $K_e = 10\ keV$ and $K_p = 100\ keV$

$m_e = 9.11 \times 10^{-31}$ kg and $m_p = 1.67 \times 10^{-27}$ kg

As $K = \dfrac{1}{2}mv^2$ or $v = \sqrt{\dfrac{2K}{m}}$,

Hence, $\dfrac{v_e}{v_p} = \sqrt{\dfrac{K_e}{K_p} \times \dfrac{m_p}{m_e}} = \sqrt{\dfrac{10\ keV}{100\ keV} \times \dfrac{1.67 \times 10^{-27}\ \text{kg}}{9.11 \times 10^{-31}\ \text{kg}}} = 13.54$

$\Rightarrow$ $v_e = 13.54\ v_p$.

Thus, electron is travelling faster.

6.13. *A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height, it attains its maximum (terminal) speed, and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10* ms^{-1}*?*

Sol. Here, $r = 2\ \text{mm} = 2 \times 10^{-3}$m.

Distance moved in each half of the journey, $s = \frac{500}{2} = 250$ m.

Density of water, $\rho = 10^3$ kg/m^3

Mass of rain drop = volume of drop × density

$$m = \frac{4}{3}\pi r^3 \times \rho = \frac{4}{3}\times\frac{22}{7}(2 \times 10^{-3})^3 \times 10^3 = 3.35 \times 10^{-5} \text{ kg}$$

$$\therefore \quad W = mg \times s = 3.35 \times 10^{-5} \times 9.8 \times 250 = 0.082 \text{ J}$$

Note: Whether the drop moves with decreasing acceleration or with uniform speed, work done by the gravitational force on the drop remains the same.

If there was no resistive forces, energy of drop on reaching the ground.

$$E_1 = mgh = 3.35 \times 10^{-5} \times 9.8 \times 500 = 0.164 \text{ J}$$

Actual energy, $E_2 = \frac{1}{2}mv^2 = \frac{1}{2} \times 3.35 \times 10^{-5} (10)^2 = 1.675 \times 10^{-3}$ J

$\therefore$ Work done by the resistive forces, $W = E_1 - E_2 = 0.164 - 1.675 \times 10^{-3}\ W$

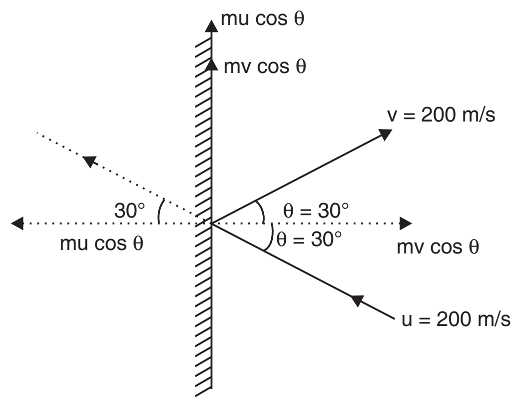
$= 0.1623$ joule.

6.14. *A molecule in a gas container hits a horizontal wall with speed 200 ms^{-1} and angle 30° with the normal, and rebounds with the same speed. Is momentum conserved in the collision? Is the collision elastic or inelastic?*

Sol. Let us consider the mass of the molecule be m and that of wall be M. The wall remains at rest due to its large mass. Resolving momentum of the molecule along x-axis and y-axis, we get

The x-component of momentum of molecule

$$= mu \cos\theta = -m\, 200 \cos 30° = -100\sqrt{3}\, m$$



y-component of the molecule

$$= mu \sin\theta = m \times 200 \times \sin 30° = 100\, m$$

Before collision: x-component of total momentum (wall + molecule)

$$= 0 + (-100\sqrt{3}\ m) = -100\sqrt{3}\, m$$

y- component of momentum (wall + molecule) = 0 + 100 m = 100 m

After collision: x-component of the momentum (wall + molecule)

$$= 0 + m\ 200 \cos 30° = 100\sqrt{3}\, m$$

and y-component $= 0 + m\ 100 \sin 30° = 100\ m$

We find that momentum of the (molecule + wall) system is conserved. The wall has a recoil momentum such that momentum of the wall + momentum of outgoing molecule equals the momentum of the incoming molecule.

Initial kinetic energy $\left(\frac{1}{2}mu^2\right)$ is the same as final K.E. $\left(\frac{1}{2}mv^2\right)$ of the molecule as

$u = v = 200$ m/s *i.e.,* thus, the collision is elastic collision.

6.15. *A pump on the ground floor of a building can pump up water to fill a tank of volume 30m³ in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump?*

Sol. Here, volume of water $= 30\ m^3$; $t = 15$ min $= 15 \times 60 = 900s$

$$h = 40\ m;\quad \eta = 30\%$$

As the density of water $= \rho = 10^3$ kg m^{-3}

$\therefore$ Mass of water pumped, m = volume × density $= 30 \times 10^3$ kg

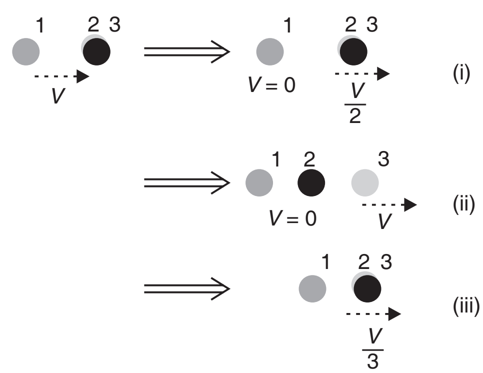
Actual power consumed or output power $p_0 = \frac{W}{t} = \frac{mgh}{t}$

$$\Rightarrow \quad p_0 = \frac{30 \times 10^3 \times 9.8 \times 40}{900} = 13070 \text{ watt}$$

If p_i is input power (required), then as

$$\eta = \frac{p_0}{p_i} \Rightarrow p_i = \frac{p_0}{\eta} = \frac{13070}{30/100} = 43567 \text{ W} = 43.56 \text{ KW}$$

6.16. *Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed V. If the collision is elastic, which of the following (Fig.) is a possible result after collision?*



Sol. Let m be the mass of each ball bearing. Before collision, total K.E. of the system

$$= \frac{1}{2}mv^2 + 0 = \frac{1}{2}mv^2$$

After collision, K.E. of the system is

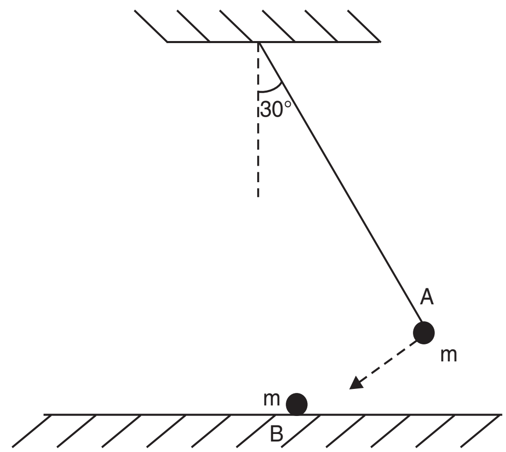
Case I, $E_1 = \frac{1}{2}(2m)\ (v/2)^2 = \frac{1}{4}mv^2$

Case II, $E_2 = \frac{1}{2}mv^2$

Case III, $E_3 = \frac{1}{2}(3m)\ (v/3)^2 = \frac{1}{6}mv^2$

Thus, case II is the only possibility since K.E. is conserved in this case.

6.17. *The bob A of a pendulum released from 30° to the vertical hits another bob B of the same mass at rest on a table as shown in Fig. How high does the bob A rise after the collision? Neglect the size of the bobs and assume the collision to be elastic.*



Sol. Since collision is elastic therefore *A* would come to rest and B would begin to move with the velocity of A.

The bob transfers its entire momentum to the ball on the table. The bob does not rise at all.

6.18. *The bob of a pendulum is released from a horizontal position. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point, given that it dissipated 5% of its initial energy against air resistance?*

Sol. On releasing the bob of pendulum from horizontal position, it falls vertically downward by a distance equal to length of pendulum *i.e.*, $h = l = 1.5$ m .

As 5% of loss in P.E. is dissipated against air resistance, the balance 95% energy is transformed into K.E. Hence,

$$\frac{1}{2}mv^2 = \frac{95}{100} \times mgh$$

$$\Rightarrow \quad v = \sqrt{2 \times \frac{95}{100} \times gh} = \sqrt{\frac{2 \times 95 \times 9.8 \times 1.5}{100}} = 5.3 \text{ ms}^{-1}$$

6.19. *A trolley of mass 300 kg carrying a sandbag of 25 kg is moving uniformly with a speed of 27 km/h on a frictionless track. After a while, sand starts leaking out of a hole on the trolley's floor at the rate of 0.05 kg s^{-1}. What is the speed of the trolley after the entire sand bag is empty?*

Sol. The system of trolley and sandbag is moving with a uniform speed. Clearly, the system is not being acted upon by an external force. If the sand leaks out, even then no external force acts. So there shall be no change in the speed of the trolley.

6.20. *A particle of mass 0.5 kg travels in a straight line with velocity $\upsilon = ax^{3/2}$, where $a = 5\ m^{-1/2}\ s^{-1}$. What is the work done by the net force during its displacement from x = 0 to x = 2 m?*

Sol. Here $m = 0.5$ kg

$$\upsilon = ax^{3/2},\ a = 5 \text{ m}^{-1/2}\text{ s}^{-1}.$$

Initial velocity at $x = 0, \upsilon_1 = a \times 0 = 0$

Final velocity at $x = 2, \upsilon_2 = a\,(2)^{3/2} = 5 \times (2)^{3/2}$

Work done = increase in K.E

$$= \frac{1}{2}m\,(\upsilon_2^2 - \upsilon_1^2) = \frac{1}{2} \times 0.5[(5 \times 2^{3/2})^2 - 0] = 50 \text{ J}.$$

6.21. *The blades of a windmill sweep out a circle of area A. (a) If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through it in time t? (b) What is the kinetic energy of the air? (c) Assume that the windmill converts 25% of the wind's energy into electrical energy, and that A = 30 m², v = 36 km/h and the density of air is 1.2 kg m⁻³. What is the electrical power produced?*

Sol. (*a*) Volume of wind flowing per second $= Av$

Mass of wind flowing per second $= Av\rho$

Mass of air passing in second $= Av\rho t$

(*b*) Kinetic energy of air $= \frac{1}{2}mv^2 = \frac{1}{2}(Av\rho t)\,v^2 = \frac{1}{2}Av^3\rho t$

(*c*) Electrical energy produced $= \frac{25}{100}\times\frac{1}{2}Av^3\rho t = \frac{Av^3\rho t}{8}$

Electrical power $= \frac{Av^3\rho t}{8t} = \frac{Av^3\rho}{8}$

Now, $A = 30 \text{ m}^2, \quad v = 36 \text{ kmh}^{-1} = 36 \times \frac{5}{18}\text{ ms}^{-1} = 10 \text{ ms}^{-1}$

and $\rho = 1.2 \text{ kg ms}^{-1}$

$\therefore$ Electrical power $= \frac{30\times10\times10\times10\times1.2}{8}$ W = 4500 W = 4.5 KW.

6.22. *A person trying to lose weight (dieter) lifts a 10 kg mass, one thousand times, to a height of 0.5 m each time. Assume that the potential energy lost each time she lowers the mass is dissipated. (a) How much work does she do against the gravitational force? (b) Fat supplies 3.8 × 10⁷J of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up?*

Sol. Here, $m = 10$ kg, $h = 0.5\ m$, $n = 1000$

(*a*) work done against gravitational force.

$$W = n(\text{mgh}) = 1000 \times (10 \times 9.8 \times 0.5) = 49000\text{J}.$$

(*b*) Mechanical energy supplied by 1 kg of fat $= 3.8 \times 10^7 \times \frac{20}{100} = 0.76 \times 10^7$ J/kg

$\therefore$ Fat used up by the dieter $= \frac{1 \text{ kg}}{0.76\times10^7} \times 49000 = 6.45 \times 10^{-3}$ kg

6.23. *A family uses 8 kW of power. (a) Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square meter. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW? (b) Compare this area to that of the roof of a typical house.*

Sol. (*a*) Power used by family, p = 8 KW = 8000 W

As only 20% of solar energy can be converted to useful electrical energy, hence, power to be supplied by solar energy $= \frac{8000\,W}{20\%} = 40000$ W

As solar energy is incident at a rate of 200 Wm^{-2}, hence the area needed

$$A = \frac{40000\ W}{200\ Wm^{-2}} = 200\ m^2$$

(*b*) The area needed is camparable to roof area of a large sized house.

6.24. *A bullet of mass 0.012 kg and horizontal speed 70 ms^{-1} strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by thin wire. Calculate the height to which the block rises. Also, estimate the amount of heat produced in the block.*

Sol. Here, $m_1 = 0.012$ kg, $u_1 = 70$ m/s

$m_2 = 0.4$ kg, $u_2 = 0$

As the bullet comes to rest with respect to the block, the two behave as one body. Let v be the velocity acquired by the combination.

Applying principle of conservation of linear momentum, $(m_1 + m_2)\, v = m_1u_1 + m_2\, u_2 = m_1u_1$

$$v = \frac{m_1u_1}{m_1+m_2} = \frac{0.012\times 70}{0.012+0.4} = \frac{0.84}{0.412} = 2.04\ ms^{-1}$$

Let the block rise to a height h.

P.E. of the combination = K.E. of the combination

$$(m_1 + m_2)\, gh = \frac{1}{2}(m_1 + m_2)\, v^2$$

$$\therefore \quad h = \frac{v^2}{2g} = \frac{2.04\times 2.04}{2\times 9.8} = 0.212 \text{ m.}$$

For calculating heat produced, we calculate energy lost (W), where

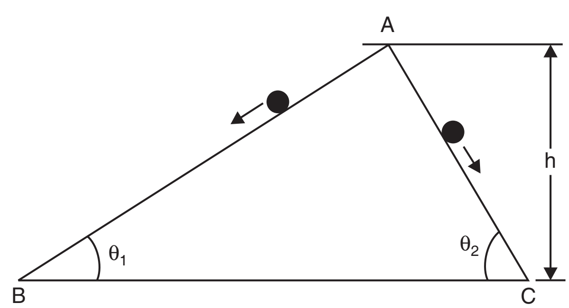
W = intial K.E. of bullet – final K.E. of combination

$$= \frac{1}{2}m_1u_1^2 - \frac{1}{2}(m_1+m_2)\, v^2 = \frac{1}{2}\times 0.012\,(70)^2 - \frac{1}{2}(0.412)\,(2.04)^2$$

W = 29.4 – 0.86 = 28.54 joule

$$\therefore \text{ Heat produced,} \quad H = \frac{W}{J} = \frac{28.54}{4.2} = 6.8 \text{ cal.}$$

6.25. *Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track (Fig). Will the stones reach the bottom at the same time? Will they reach there with the same speed? Explain. Given $\theta_1 = 30°$, $\theta_2 = 60°$, and h = 10 m, what are the speeds and times taken by the two stones?*



Sol.
$$\frac{1}{2}mv^2 = mgh, \quad v = \sqrt{2gh} = \sqrt{2\times 10\times 10}\ ms^{-1} = 14.14\ ms^{-1}$$

$$v_B = v_C = 14.14\ ms^{-1}, \quad l = \frac{1}{2}(g \sin\theta)\, t^2$$

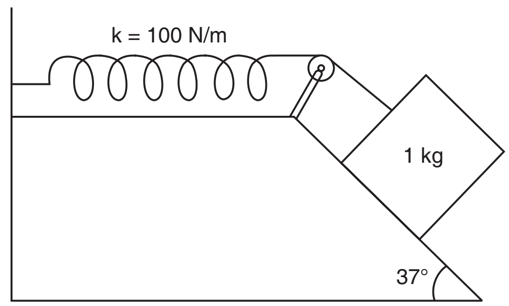
$$\sin\theta = \frac{h}{l}, \quad l = \frac{h}{\sin\theta}$$

$$\therefore \quad \frac{h}{\sin\theta} = \frac{1}{2} g \sin\theta\, t^2 \quad \text{or} \quad t = \sqrt{\frac{2h}{g}} \cdot \frac{1}{\sin\theta}$$

$$t_B = \sqrt{\frac{2\times 10}{10}} \cdot \frac{1}{\sin 30°} = 2\sqrt{2}\text{s.}$$

$$t_C = \sqrt{\frac{2\times 10}{10}} \cdot \frac{1}{\sin 60°} = \frac{2\sqrt{2}}{\sqrt{3}}\text{s.}$$

6.26. *A 1 kg block situated on a rough incline is connected to a spring with spring constant 100 Nm^{-1} as shown in Figure. The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that the spring has negligible mass and the pulley is frictionless.*



Sol. From the above figure,

$$R = mg \cos\theta$$

$$F = \mu R = \mu\, mg \cos\theta$$

Net force on the block down the incline

$$= mg \sin\theta - F = mg \sin\theta - \mu\, mg \cos\theta$$

$$= mg (\sin\theta - \mu \cos\theta)$$

Here distance moved $x = 10 \text{ cm} = 0.1 \text{ m}$

In equilibrium,

work done = Potential energy of stretched spring

$$mg (\sin\theta - \mu\cos\theta)\, x = \frac{1}{2} kx^2$$

or $\quad 2mg (\sin\theta - \mu\cos\theta) = kx$

or $\quad 2 \times 1 \text{ kg} \times 10\text{ms}^{-2} (\sin 37° - \mu \cos 37°) = 100 \times 0.1 \text{ m}$

or $\quad 20(0.601 - \mu \times 0.798) = 10$

or $\quad 0.601 - 0.798\mu = \frac{10}{20} = 0.5$

or $\quad -0.798\mu = 0.5 - 0.601 = -0.101$

or $\quad \mu = \frac{-0.101}{-0.798} = \frac{101}{798} = 0.126$

Hence $\quad \mu = \mathbf{0.126}$

6.27. *A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with a uniform speed of 7 ms^{-1}. It hits the floor of the elevator (length of elevator = 3 m) and does not rebound. What is the heat produced by the impact? Would your answer be different if the elevator were stationary ?*

Sol. P.E. of bolt = mgh = 0.3 × 9.8 × 3 = 8.82 J

The bolt does not rebound. So the whole of the energy is converted into heat. Since the value of acceleration due to gravity is the same in all inertial system, therefore the answer will not change even if the elevator is stationary.

6.28. *A trolley of mass 200 kg moves with a uniform speed of 36 km h^{-1} on a frictionless track. A child of mass 20 kg runs on the trolley from one end to the other (10 m away) with a speed of 4 ms^{-1} relative to the trolley in a direction opposite to the trolley's motion, and jumps out of the trolley. What is the final speed of the trolley? How much has the trolley moved from the time the child begins to run?*

Sol. Let there be an observer travelling parallel to the trolley with the same speed. He will observe the initial momentum of the trolley of mass M and child of mass m as zero. When the child jumps in opposite direction, he will observe the increase in the velocity of the trolley by Δv.

Let u be the velocity of the child. He will observe child landing at velocity $(u - \Delta v)$

Therefore, initial momentum = 0

Final momentum = $M\Delta v - m(u - \Delta v)$

Hence, $M\Delta v - m(u - \Delta v) = 0$

Whence $\Delta v = \dfrac{mu}{M+m}$

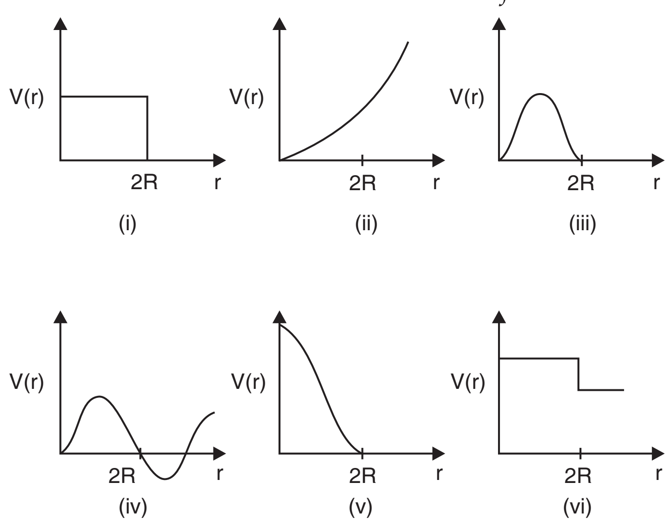
Putting values $\Delta v = \dfrac{4 \times 20}{20+220} = 0.36$ ms^{-1}

∴ Final speed of trolley is 10.36 ms^{-1}.

The child take 2.5 s to run on the trolley.

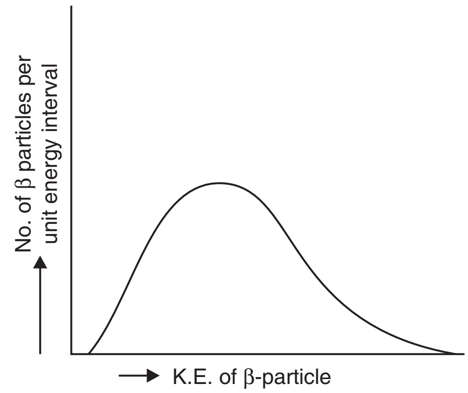
Therefore, the trolley moves a distance = 2.5 × 10.36 m = 25.9 m.

6.29. *Which of the following potential energy curves in Fig. cannot possibly describe the elastic collision of two billiard balls? Here r is distance between centres of the balls.*



Sol. The potential energy of a system of two masses varies inversely as the distance (r) between them *i.e.,* V $(r) \propto \dfrac{1}{r}$. When the two billiard balls touch each other, P.E. becomes zero *i.e.,* at r = R + R = 2R; V (r) = 0. Out of the given graphs, curve (v) only satisfies these two conditions. Therefore, all other curves cannot possibly describe the elastic collision of two billiard balls.

6.30. *Consider the decay of a free neutron at rest: $n \longrightarrow p + e^-$. Show that the two body decay of this type must necessarily give an electron of fixed energy, and therefore, cannot account for the observed continuous energy distribution in the β-decay of a neutron or a nucleus, Fig.*



Sol. Let the masses of the electron and proton be m and M respectively. Let v and V be the velocities of electron and proton respectively. Using law of conservation of momentum.

Momentum of electron + momentum of proton = momentum of neutron

$$\therefore \qquad mv + MV = 0 \Rightarrow V = -\frac{m}{M}v$$

Clearly, the electron and the proton move in opposite directions. If mass Δm has been converted into energy in the reaction, then

$$\frac{1}{2}mv^2 + \frac{1}{2}MV^2 = \Delta m \times c^2 \quad \text{or} \quad \frac{1}{2}mv^2 + \frac{1}{2}M\left[-\frac{m}{M}\right]^2 v^2 = \Delta mc^2$$

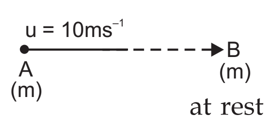
$$\text{or} \quad \frac{1}{2}mv^2\left[1+\frac{m}{M}\right] = \Delta mc^2 \quad \text{or} \quad v^2 = \frac{2M\Delta\, mc^2}{m\,(M+m)}$$

Thus, it is proved that the value of v^2 is fixed since all the quantities in right hand side are constant. It establishes that the emitted electron must have a fixed energy and thus we cannot account for the continuous energy distribution in the β-decay of a neutron.

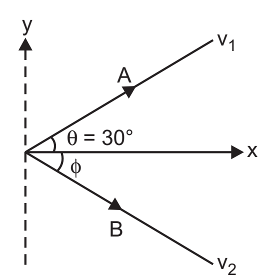
QUESTIONS BASED ON SUPPLEMENTARY CONTENTS

Q. 1. *A and B are two particles having the same mass 'm'. A is moving along x-axis with a speed of 10 ms^{-1} and B is at rest. After undergoing a perfectly elastic collision with B, particle A get scattered through an angle of 30°. What is the direction of the motion of B and the speeds of A and B after the collision?*

Sol.



Before Collision



After collision

The masses of two bodies are same and the collision is perfectly elastic.

$$\therefore \quad \theta + \phi = 90°$$

$$30° + \phi = 90°$$

$$\therefore \quad \phi = 90° - 30° = 60°$$

According to law of conservation of momentum, for x-component

$$u = v_1 \cos 30° + v_2 \cos 60°$$

$$10 = \frac{\sqrt{3}}{2}v_1 + \frac{1}{2}v_2$$

$$\therefore \quad \sqrt{3}v_1 + v_2 = 20 \qquad (i)$$

For y-cmponent, we get

$$0 = V_1 \sin 30° - V_2 \sin 60°$$

$$0 = V_1 \times \frac{1}{2} - V_2 \times \frac{\sqrt{3}}{2}$$

$$\therefore \quad V_1 = \sqrt{3}V_2 \qquad (ii)$$

Solving Eq. (*i*) and (*ii*) we get

$$\sqrt{3} \times \sqrt{3}V_2 + V_2 = 20$$

$$3V_2 + V_2 = 20$$

$$4V_2 = 20$$

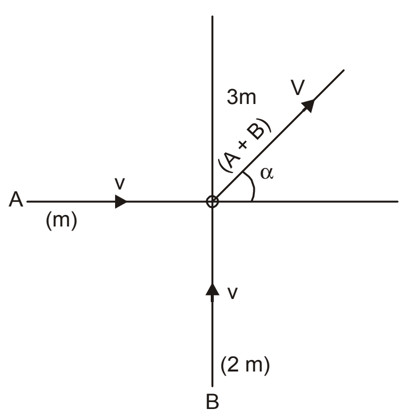
$$\therefore \quad V_2 = 5 \text{ m/s} \quad \text{and} \quad V_1 = \sqrt{3} \times 5$$

$$V_1 = 5 \times 1.732$$

$$V_1 = 8.66 \text{ m/s}$$

Q. 2. *Two particles A and B of masses m and 2m, are moving along the X and Y-axes, respectively with the same speed of V. They collide at the origin and coalesce into one body after the collision. What is the velocity of the coulesced mass? What is the loss of energy during this collission?*

Sol. Let a be the angle of scattering of the coalesced mass ($m + 2m$) i.e. $3m$ and V be the velocity after the collision at the origin.



∴ According to law of conservation of momentum, for x-component.

$$mv = 3mV \cos \alpha$$

$$v = 3V \cos \alpha \qquad (i)$$

For y-component

$$2mv = 3mV \sin \alpha$$

$$2v = 3V \sin \alpha \qquad (ii)$$

Dividing eq. (ii) by eq. (i) we get

$$\tan \alpha = \frac{2mv}{mv} = 2$$

$$\therefore \quad \alpha = \tan^{-1}(2) = 63.4°$$

Squaring eq. (i) and eq. (ii) we get,

$$v^2 + (2v)^2 = (3V \cos \alpha)^2 + (3V \sin \alpha)^2$$

$$\Rightarrow \quad v^2 + 4v^2 = 9V^2 \cos^2 \alpha + 9V^2 \sin^2 \alpha$$

$$\Rightarrow \quad 5v^2 = 9V^2$$

$$\therefore \quad V^2 = \frac{5}{9}v^2 \quad \text{or} \quad V = \frac{\sqrt{5}}{3}v \qquad (iii)$$

Now the K.E. before the collision $= \frac{1}{2}mv^2 + \frac{1}{2}(2m)v^2 = \frac{3}{2}mv^2$

and the K.E. after the collision $= \frac{1}{2}(3m)V^2 = \frac{1}{2} \times 3m \times \frac{5}{9}v^2$ [From (iii)]

$$= \frac{5}{6}mv^2$$

∴ Loss of K.E. during the collission $= \frac{3}{2}mv^2 - \frac{5}{6}mv^2 = \frac{2}{3}mv^2$

Q. 3. *A billard ball A moving with an initial speed of 1 ms^{-1} undergoes a perfectly elastic collision with another identical ball B at rest. A is scattered through an angle of 30°. What is the angle of recoil of B? What is the speed of ball A after the collision?*

Sol. Masses of the two balls are same and the collision is perfectly elastic

$$\therefore \quad \theta + \phi = \frac{\pi}{2}$$

$$30° + \phi = 90°$$

$$\therefore \quad \phi = 90° - 30° = 60°$$

Hence, the angle of recoil is 60°

Now according to the law of conservation of momentum, for x-component,

$$u = v_1 \cos 30° + v_2 \cos 60°$$

$$1 = \frac{\sqrt{3}}{2}v_1 + \frac{1}{2}v_2$$

$$\therefore \quad \sqrt{3}v_1 + v_2 = 2 \qquad (i)$$

For y-component,

$$0 = v_1 \sin 30° - v_2 \sin 60°$$

$$0 = \frac{1}{2}v_1 - \frac{\sqrt{3}}{2}v_2$$

$$\therefore \quad v_1 = \sqrt{3}v_2 \quad \text{or} \quad v_2 = \frac{v_1}{\sqrt{3}} \qquad (ii)$$

From eq. (*i*) and eq. (*ii*) we get

$$\sqrt{3}v_1 + \frac{v_1}{\sqrt{3}} = 2$$

$$\frac{4}{\sqrt{3}}v_1 = 2$$

$$\therefore \quad v_1 = 2 \times \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}\text{ms}^{-1}$$

Hence the speed of A after the collision is $\frac{\sqrt{3}}{2}$ ms^{-1}.

Q. 4. *Two identical balls A and B undergo a perfectly elastic two dimensional collision. Initially A is moving with a speed of 10 ms^{-1} and B is at rest. Due to collision A is scattered through angle of 30°. What are the speed of A and B after the collision?*

Sol. The balls A and B are identical

$\therefore$ masses are same, $\theta = 30°$

$\therefore \quad \phi = 90° - 30° = 60°$

Initially $v_1 = 10$ ms^{-1} and $v_2 = 0$

$\therefore$ According to law of conservation of momentum for x-component.

$$u = v_1 \cos 30° + v_2 \cos 60°$$

$$10 = v_1 \times \frac{\sqrt{3}}{2} + v_2 \times \frac{1}{2}$$

$$\therefore \quad \sqrt{3}v_1 + v_2 = 20 \qquad (i)$$

For y-component,

$$0 = v_1 \sin 30° - v_2 \sin 60°$$

$$0 = v_1 \times \frac{1}{2} - v_2 \times \frac{\sqrt{3}}{2}$$

$$\therefore \quad v_1 = \sqrt{3}v_2 \qquad (ii)$$

From eq. (*i*) and (*ii*) we get

$$\sqrt{3} \times \sqrt{3}v_2 + v_2 = 20$$

$$\Rightarrow \quad 3v_2 + v_2 = 20$$

$$\Rightarrow \quad 4v_2 = 20$$

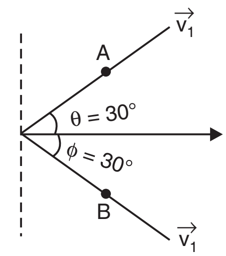
$$\therefore \quad v_2 = 5 \text{ ms}^{-1}$$

From eq. (*ii*) $\quad v_1 = \sqrt{3} \times 5 = 5\sqrt{3}\,\text{ms}^{-1}$

Here, the velocities of A and B are $5\sqrt{3}$ ms^{-1} and 5 ms^{-1} respectively.

Q. 5. *A and B are two identical balls. A moving with a speed of 6 ms^{-1} along the positive x-axis, undergoes a collision with B, intially at rest. After collision each ball moves and the directions making of ±30° with the x-axis. What are the speeds of A and B after the collision? Is the collision perfectly elastic?*

Sol. Both the balls A and B are identical



∴ Masses are same

Here $u_1 = 6\ ms^{-1},\ \theta = 30°,\ \phi = 30°$

$u_2 = 0$

According to the law of conservation of momentum.

For x-component, $u = v_1 \cos 30° + v_2 \cos 30°$

$$6 = v_1 \times \frac{\sqrt{3}}{2} + v_2 \times \frac{\sqrt{3}}{2}$$

$$\therefore \quad v_1 + v_2 = \frac{12}{\sqrt{3}} \quad (i)$$

For y-component,

$$0 = v_1 \sin 30° - v_2 \sin 30°$$

$$\therefore \quad v_1 - v_2 = 0 \quad (ii)$$

From (i) and (ii) we get

$$v_1 = v_2 = 2\sqrt{3}\ ms^{-1}$$

Hence, the velocities of the two bodies after collision are same i.e. $2\sqrt{3}\ m \cdot s^{-1}$

(ii) For perfectly elastic collision

$$\theta + \phi = 90°$$

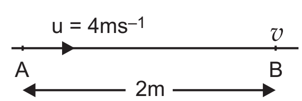
But here $\theta = 30°$ and $\phi = 30°$

$$\therefore \quad \theta + \phi \neq 90°$$

Here the collision is not perfectly elastic.

Q. 6. *A particle of mass 0.1 kg has an initial speed of 4 ms^{-1} at a point A on a rough horizontal road. The coefficient of friction, between the object and the road is 0.15. The particle moves to a point B at a distance of 2 m from A. What is the speed of the particle B? (Take g = 10 ms^{-2})*

Sol. Speed of the particle at A



Let the speed of the particle at B be $\vec{v}$ ms^{-1}.

The particle has to do the work against the force of friction.

∴ The force is non-conservative.

So the work done W = force × distance travelled

$= \mu\ mg \times S = 0.15 \times 0.1 \times 10 \times 2 = 0.3$ J

Now change in K.E. $= \frac{1}{2} m[(u^2) - V^2] = \frac{1}{2} \times 0.1 \left[(u)^2 - V^2\right]$

$\therefore \quad \frac{1}{2}\times 0.1\left[(u)^2 - V^2\right] = 0.3$

$\Rightarrow \quad 0.05[16 - V^2] = 0.3$

$\Rightarrow \quad 16 - V^2 = 6$

$\Rightarrow \quad V^2 = 16 - 6 = 10$

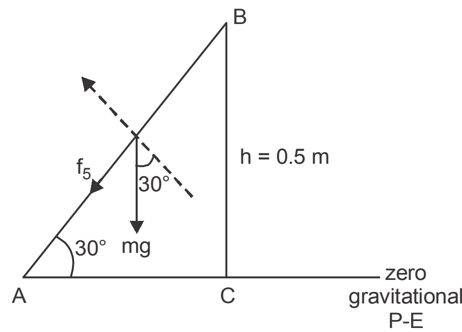
$\therefore \quad V = \sqrt{10} = 3.16 \text{ ms}^{-1}$

Q. 7. *A particle of mass 0.2 kg, has an initial speed of 5 ms⁻¹ at the bottom of a rough inclined plane of inclination 30° and vertical height 0.5 m. What is the speed of the particle as it reaches the top of the inclived plane? [Take* $\mu = \frac{1}{\sqrt{3}}$, $g = 10 \text{ ms}^{-2}$ *]*

Sol. $\sin 30° = \frac{h}{AB}$

$\frac{1}{2} = \frac{0.5}{AB}$

$\therefore \quad AB = 1 \text{ m}$



Force of friction between the particle and the inclined plane

$= \mu N$

$= \mu \times mg \cos\theta$

$= \frac{1}{\sqrt{3}}\times 0.2\times 10\times \cos 30°$

$= \frac{1}{\sqrt{3}}\times 0.2\times 10\times \frac{\sqrt{3}}{2} = 1 \text{ N}$

$\therefore$ The work done by the particle in moving from A to B against the force of friction

$= \mu \; mg \cos\theta \times AB$

$= 1 \text{ N} \times 1 \text{ m} = 1\text{J}$

Let $\vec{v}$ be the velocity of the particle at B

$\therefore \quad w = \text{change in energy from } A \text{ to } B$

$1\ J = \left[\frac{1}{2}\times 0.2\times (5)^2 + 0\right] - \left[\frac{1}{2}\times 0.2\times V^2 + 0.2\times 10\times 0.5\right]$

$\Rightarrow \quad 1 = 2.5 - [0.1\ V^2 + 1]$

$\Rightarrow \quad 1 = 2.5 - 0.1\ V^2 - 1$

$\Rightarrow \quad 0.1\ V^2 = 1.5 - 1$

$\Rightarrow \quad 0.1\ V^2 = 0.5$

$\Rightarrow \quad V^2 = 5$

$\Rightarrow \quad V = \sqrt{5} = 2.24 \text{ ms}^{-1}$

ADDITIONAL QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *What is the significance of the –ve sign in W = – mgd?*

Ans. Negative sign indicates that the work is done against the force.

Q. 2. *Write the work energy theorem.*

Ans. Work done by a force is the change in kinetic energy associated with the body.

Q. 3. *When an air bubble rises in water, what happens to its potential energy?*

Ans. Potential energy of air bubble decreases, because work is done by upthrust on the bubble.

Q. 4. *What is the elastic potential energy stored in a spring?*

Ans. Energy stored in a spring is $\frac{1}{2}kx^2$.

Q. 5. *Why is the work done by centripetal force zero?*

Ans. Because centripetal force is perpendicular to the displacement of the body.

Q. 6. *What is 1 eV?*

Ans. Kinetic energy acquired by an electron to pass through a potential difference of 1 volt.

i.e., $1 \text{ eV} = 1.6 \times 10^{-19} \text{ C} \times 1 \text{ volt} = 1.6 \times 10^{-19} \text{ J}$.

Q. 7. *Does potential energy of a spring decrease/increase when it is compressed or stretched ?*

Ans. When a spring is compressed or stretched, potential energy of the spring increases in both the cases. This is because work is done by us in compression as well as stretching.

Q. 8. *A mass 'm' collides with another mass '2m' and sticks to it. What is the nature of the collision?*

Ans. Whenever a mass collides and gets stuck to the other mass, the collision is said to be inelastic.

Q. 9. *Can kinetic energy of a body be negative? Can potential energy be negative?*

Ans. Kinetic energy of a body cannot be negative because it is given by $\frac{1}{2}mv^2$ and is always positive whether v is *–ve* or *+ve*. The gravitational potential energy of a body may be negative or positive.

Q. 10. *Name the parameter which is a measure of the degree of elasticity of a body.*

Ans. Coefficient of restitution.

Q. 11. *Write down characteristics of elastic collision.*

Ans. (*i*) Kinetic energy of the system remains conserved.

(*ii*) Linear momentum of the system remains conserved.

Q. 12. *What should be the angle between the force and the displacement for maximum and minimum work?*

Ans. For maximum work, $\theta = 0°$ and for minimum work, $\theta = 90°$.

Q. 13. *What is the nature of force involved in the winding of a watch?*

Ans. As the energy is recoverable therefore the force is conservative force.

Q. 14. *A light body and a heavy body have the same kinetic energy. Which one will have greater momentum?*

Ans. Since $k = \frac{p^2}{2m}$ or $p = \sqrt{2mk}$

As k is same for both bodies, $p \propto \sqrt{m}$, *i.e,* heavier body has more momentum than the lighter body.

Q. 15. *Give an example of negative work.*

Ans. Work done against the gravitational force.

Q. 16. *A spring is stretched. Is the work done by the stretching force positive or negative?*

Ans. Positive work. Both the force and the displacement are in the same direction.

Q. 17. *When a ball is thrown up. The magnitude of its momentum first decreases and then increases. Does this violate the conservation of momentum principle?*

Ans. The momentum of the system remains constant. Here the system includes: ball, earth and air molecules.

Q. 18. *What happens when two identical objects moving in mutually opposite directions suffer an elastic collision?*

Ans. The objects mutually exchange their velocities as a consequence of collision.

Q. 19. *Give the characteristics of inelastic collision.*

Ans. (*i*) Kinetic energy does not remain conserved.

(*ii*) Linear momentum of the system remains conserved.

Q. 20. *What happens to internal energy, when temperature of body increases?*

Ans. Increases.

Q. 21. *Give the conditions under which a force is called conservative force.*

Ans. Any force is called conservative force if,

(*i*) Work done against is independent of path.

(*ii*) Work done in a closed path is zero.

Q. 22. *Is work done by a non conservative force always negative? Comment.*

Ans. No. For example, work done by a non-conservative force like friction is zero, so long as the body does not start moving. Again when friction causes motion, work done by friction is positive.

Q. 23. *What is a headon collision?*

Ans. Collision in which the colliding particles move along the same straight line path before as well as after the collision, is called headon collision.

Q. 24. *What is an oblique collision?*

Ans. Collision is said to be oblique, if the colliding particles do not move along the same straight line path.

Q. 25. *Where is the speed of a swinging pendulum maximum?*

Ans. At the bottom of the swing.

Q. 26. *The momentum of an object is doubled. How does its K.E. change?*

Ans. K.E. becomes four times.

Q. 27. *If mechanical work is done on a body, will its kinetic energy increase or decrease?*

Ans. The kinetic energy of the body will increase when work is done on it in accordance with the work-energy theorem.

Q. 28. *Moutain roads rarely go straight up but wind up gradually. Why?*

Ans. Frictional force f is given by $f = \mu\, mg \cos\theta$. If the roads go straight up the angle of slope θ would be large and frictional force will be less and vehicles may slip.

Q. 29. *What is a variable force?*

Ans. Force whose either magnitude or direction or both change.

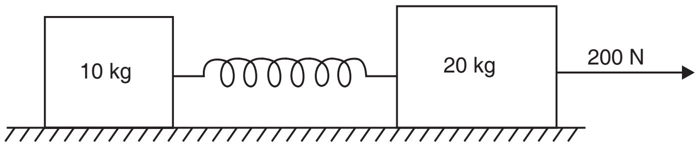
Q. 30. *It is possible to have a situation when $E - U < 0$?*

Ans. No. Since $E = K + U. \Rightarrow K = E - U$

As K is never negative, $(E - U)$ is never less than zero.

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *Two masses 10 kg and 20 kg are connected by a massless spring. A force of 200 N acts on 20 kg mass. At the instant when the 10 kg mass has an acceleration 12 m/s², what will be the energy stored in the spring? (Given k = 2400 N/m).*



Ans. Since $F = ma$

$\Rightarrow$ $F = 10 \times 12 = 120$ N

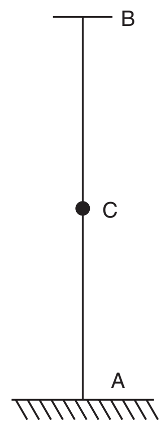
Also, $F = kx = 2400\,x$

$\therefore$ $x = \frac{1}{20}$

Energy stored in the spring, $E = \frac{1}{2}kx^2$

$\Rightarrow$ $E = \frac{1}{2} \times 2400 \times \left(\frac{1}{20}\right)^2$ or $E = \frac{1}{2}\times 2400\times\frac{1}{400} = 3$ J.

Q. 2. *A body of mass 2kg is at rest at a height of 10 m above the ground. Calculate its potential energy and kinetic energy after it has fallen through half the height. Also find the velocity at this instant.*



Ans. Total energy at B = kinetic energy + potential energy

$= 0 + mgh = 2 \times 9.8 \times 10 = 196$ J

As it descends half the height, it loses potential energy which is given by

$= mg\frac{h}{2} = \frac{1}{2}mgh = 98$ J

$\therefore$ its potential energy at $C = (196 - 98) = 98$ J

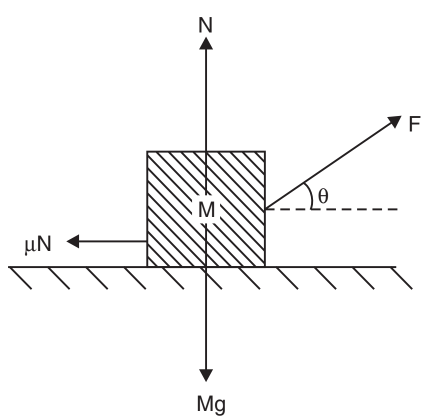
The loss of potential energy = gain in kinetic energy

$= 196 - 98 = 98$ J

But K.E. $= \frac{1}{2}mv^2$

$\therefore$ $\frac{1}{2}\times 2\times v^2 = 98 \Rightarrow v^2 = 98$ or $v = 7\sqrt{2}$ m/s.

Q. 3. *A block of mass M is pulled along a horizontal surface by applying a force at an angle θ with horizontal. Coefficient of friction between block and surface is μ. If the block travels with uniform velocity, find the work done by this applied force during a displacement d of the block.*



Ans. The forces acting on the block are shown in Figure. As the block moves with uniform velocity the forces add up to zero.

$\therefore$ $F\cos\theta = \mu N$ ...(*i*)

$F\sin\theta + N = Mg$ ...(*ii*)

Eliminating N from equations (*i*) and (*ii*),

$F\cos\theta = \mu(Mg - F\sin\theta)$

$$F = \frac{\mu Mg}{\cos\theta + \mu\sin\theta}$$

Work done by this force during a displacement d

$$W = F.\ d\cos\theta = \frac{\mu Mgd\cos\theta}{\cos\theta + \mu\sin\theta}$$

Q. 4. *A particle is moving in a circular path of radius r with constant speed. Due to change in the direction of motion of the particle continuously, the velocity of the particle is changing. But the kinetic energy of the particle remains the same. Explain why.*

Ans. Kinetic energy is given by

$$E = \frac{1}{2}mv^2 = \frac{1}{2}m(\vec{v}.\vec{v})$$

Since $\vec{v}.\vec{v} = v^2$, a scalar quantity, so it is the speed which is taken into account while calculating the kinetic energy of the particle. As the speed is constant, so kinetic energy of the particle will also remain constant.

Q. 5. *Two springs have force constants K_1 and K_2 ($K_1 > K_2$). On which spring is more work done when they are stretched by the same force?*

Ans. $$K_1 = \frac{F}{x_1} \quad \text{and} \quad K_2 = \frac{F}{x_2}$$

Since $K_1 > K_2$

$\therefore$ $x_1 < x_2$

$$W_1 = \frac{1}{2}K_1x_1^2 \quad \text{and} \quad W_2 = \frac{1}{2}K_2x_2^2$$

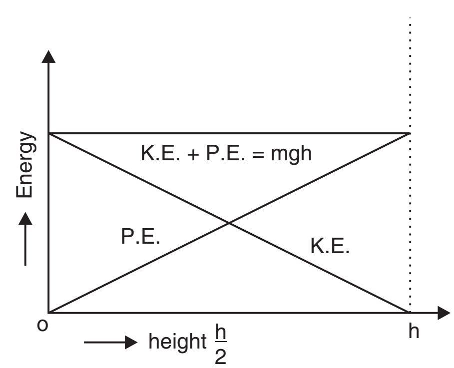
$$\frac{W_1}{W_2} = \frac{\frac{1}{2}K_1x_1^2}{\frac{1}{2}K_2x_2^2} = \frac{\left(\frac{F}{x_1}\right)\times x_1^2}{\left(\frac{F}{x_2}\right)\times x_2^2} = \frac{x_1}{x_2}$$

As $x_1 < x_2$

$\therefore$ $W_1 < W_2$ or $W_2 > W_1$

Q. 6. *Draw a graph showing variation of potential energy, kinetic energy and the total energy of a body freely falling on Earth from a height h.*

Ans. Graphs depicting variation of (*i*) gravitational potential energy (P.E.), (*ii*) kinetic energy (K.E.), and (*iii*) the total sum of potential and kinetic energies for a freely falling body are as shown in adjoining Fig. From the graphs, it is clear that:



(*a*) Gravitational potential energy decreases as the body falls downwards and is zero at the Earth.

(*b*) Kinetic energy increases as the body falls downwards and is maximum when the body just strikes the ground.

(*c*) The sum of kinetic and potential energies remains constant at all points during its free fall.

Q. 7. *A person slowly lifts a block of mass m through a vertical height h, and then walks horizontally a distance d while holding the block. Determine work done by the person.*

Ans. The man slowly lifts the block, therefore he must be applying a force equal to the weight of the block, *mg*, the work done during the vertical displacement is *mgh*, since the force is in the direction of displacement. The work done by the person during the horizontal displacement of the block is zero. Since the applied force during this process is perpendicular to displacement. Therefore total work done by the man is *mgh*.

Q. 8. *Two bodies A and B having masses m_A and m_B respectively have equal kinetic energies. If p_A and p_B are their respective momenta, then prove that the ratio of momenta is equal to the square root of ratio of respective masses.*

Ans. Let v_A and v_B be the velocities of A and B respectively.

Since their kinetic energies are equal,

$$\therefore \quad \frac{1}{2}m_A v_A^2 = \frac{1}{2}m_B v_B^2 \qquad ...(i)$$

$$\text{or} \quad m_A v_A^2 = m_B v_B^2 \quad \text{or} \quad (m_A\, v_A)\, v_A = (m_B\, v_B)\, v_B$$

$$\text{or} \quad p_A v_A = p_B v_B \Rightarrow \frac{p_A}{p_B} = \frac{v_B}{v_A} \qquad ...(ii)$$

From equation (*i*), $\frac{v_A^2}{v_B^2} = \frac{m_B}{m_A}$ or $\frac{v_A}{v_B} = \sqrt{\frac{m_B}{m_A}}$

From equation (*ii*), $\frac{p_A}{p_B} = \sqrt{\frac{m_A}{m_B}}$

Q. 9. *A body of mass 2kg is initially at rest. A constant force of 5 N acts on it for 10s. Calculate the average power of the force.*

Ans. Here, $m = 2\text{kg}, \quad u = 0, \quad F = 5N, \quad t = 10\text{s},$

∴ Acceleration produced in the body

$$a = \frac{F}{m} = \frac{5}{2} = 2.5 \text{ ms}^{-2}$$

Velocity attained by the body after 10s is

$$v = u + at = 0 + 2.5 \times 10 = 25 \text{ ms}^{-1}$$

$$\therefore \quad \text{Work done} = \text{change in K.E.} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2} \times 2 \times 625 - 0 = 625 \text{ J}$$

$$\therefore \quad p = \frac{W}{t} = \frac{625}{10}$$

$$\Rightarrow \quad p = 62.5 \text{ Js}^{-1} = 62.5\ W.$$

Q. 10. *A body of mass M at rest is struck by a moving body of mass m. Show that the fraction of the initial kinetic energy of moving mass m transferred to the struck body is* $\frac{4Mm}{(m+M)^2}$.

Ans. Here, $m_1 = m, m_2 = M u_1 = u$ (say) and $u_2 = 0$.

∴ Velocity of the struck body after collision $v_2 = \frac{2m_1u_1 + (m_2 - m_1)\, u_2}{(m_1 + m_2)}$

$$\Rightarrow \quad v_2 = \frac{2mu}{(m+M)}$$

$\therefore$ Initial K.E. of moving body before collision $K = \frac{1}{2}m_1u_1^2 = \frac{1}{2}mu^2$ and energy transferred by moving body to struck body during collision

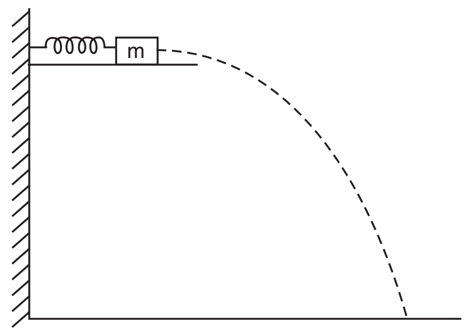
$$= \text{Final K.E. of struck body } K' = \frac{1}{2}m_2v_2^2 = \frac{1}{2}M.\left[\frac{2mu}{m+M}\right]^2 = \frac{2Mm^2u^2}{(m+M)^2}.$$

$$\therefore \quad \frac{\text{K.E. transferred by moving body during collision}}{\text{Initial K.E. of moving body}} = \frac{K'}{K} = \frac{\frac{2Mm^2u^2}{(m+M)^2}}{\frac{1}{2}mu^2} = \frac{4mM}{(m+M)^2}$$

$$\Rightarrow \quad K' = \frac{4mM}{(m+M)^2}.K.$$

Q. 11. *A small block of mass 'm' is pressed against a horizontal spring fixed at one end to compress the spring through 5.0 cm. When released the block moves horizontally till it leaves the spring. Where will it hit the ground at a distance 2m below the slab?*

$$\left[k = 100\,\frac{N}{m} \text{ and } m = 100\ g\right]$$



Ans. Here $\quad \frac{1}{2}kx^2 = \frac{1}{2}mv^2$

or $\quad v = \sqrt{\frac{k}{m}x^2} = \sqrt{\frac{100}{100}\times\frac{25\times10^{-4}}{10^{-3}}}\text{ms}^{-1} = \sqrt{2.5}\ \text{ms}^{-1}$

Height $= 2m$; $t = \sqrt{\frac{2h}{g}} = \sqrt{0.4}$

The horizontal length covered $= \sqrt{0.4}\times\sqrt{2.5}$ m $= 1$ m.

Q. 12. *If stretch in a spring of force constant k is doubled, calculate*

(a) ratio of final to initial force in the spring.

(b) ratio of elastic energies stored in the two cases.

(c) work done in changing to the state of double stretch.

Ans. (*a*) For a given spring, $F = kx$

$$\therefore \quad \frac{F_2}{F_1} = \frac{kx_2}{kx_1} = \frac{2x}{x} = 2.$$

(*b*) For a given spring, $U = \frac{1}{2}kx^2$

$$\frac{U_2}{U_1} = \frac{\frac{1}{2}kx_2^2}{\frac{1}{2}kx_1^2} = \frac{(2x)^2}{x^2} = 4$$

(*c*) Since work done in stretching the spring is stored in the spring in the form of elastic potential energy of the spring, therefore,

$$W = U_2 - U_1 = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$

$$\Rightarrow \qquad W = \frac{1}{2}k\left[(2x)^2 - x^2\right] = \frac{3}{2}kx^2.$$

Q. 13. *A body of mass 3 kg makes an elastic collision with anthor body at rest and continues to move in the original direction with a speed equal to one-third of its original speed. Find the mass of the second body.*

Ans. Here, $m_1 = 3$ kg

Let $u_1 = x$ ms^{-1}

and $m_2 = m$ kg, $u_2 = 0$, $v_1 = \frac{x}{3}$ms^{-1}

Since collision is elastic, so both momentum and K.E. remain conserved.

According to law of conservation of linear momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{or} \quad 3x + 0 = \frac{3x}{x_3} + mv_2$$

or $\quad mv_2 = 2x \quad$ ...(1)

According to the law of conservation of K.E.,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

or $\quad \frac{1}{2}\times 3x^2 + 0 = \frac{1}{2}\times 3\frac{x^2}{9} + \frac{1}{2}m\,v_2^2 \quad$ or $\quad mv_2^2 = \frac{8x^2}{3} \quad$ ...(2)

Dividing (2) by (1), $v_2 = \frac{4x}{3}$

Put this value in eqn. (1). we get

$$m \times \frac{4x}{3} = 2x \quad \text{or} \quad m = \frac{3}{2} = 1.5 \text{ kg.}$$

Thus, mass of second body is 1.5 kg.

Q. 14. *What are the conditions so that transfer of kinetic energy is maximum during a collision ?*

Ans. For maximum transfer of kinetic energy during a collision, following conditions should be fulfilled.

(*a*) The collision should be a head on collision.

(*b*) The collision should be perfectly elastic.

(*c*) The target body should be at rest.

(*d*) The mass of the striking body and the target body should be exactly same.

Q. 15. *A sphere of mass 'm' moving with a velocity u hits another stationary sphere of same mass at rest. If e is the coefficient of restitution. Find the ratio of the velocities of two spheres after the collision.*

Ans. According to law of conservation of momentum,

$$mu = mv_1 + mv_2$$

or $\quad v_1 + v_2 = u \quad$ ...(*i*)

Also $\quad v_1 - v_2 = -eu \quad$ ...(*ii*)

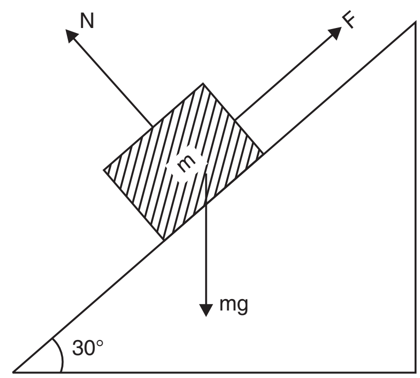
Solving eqn. (*i*) and (*ii*), we get

$$v_1 = \frac{u(1-e)}{2} \quad \text{and} \quad v_2 = \frac{u(1+e)}{2}$$

The ratio of velocities, $\frac{v_1}{v_2} = \left(\frac{1-e}{1+e}\right)$.

Q. 16. *A block of mass 2 kg is pulled up on a smooth incline of angle 30° with horizontal. If the block moves with an acceleration of 1 m/s², find the power delivered by the pulling force at a time 4 seconds after motion starts. What is the average power delivered during these four seconds after the motion starts?*

Ans. The forces acting on the block are shown in the figure.



Resolving forces parallel to incline,

$$F - mg \sin \theta = ma$$

$$\Rightarrow \quad F = mg \sin \theta + ma$$
$$= 2 \times 9.8 \times \sin 30° + 2 \times 1$$
$$= 11.8 \text{ N}$$

The velocity after 4 seconds = $u + at$

$$= 0 + 1 \times 4 = 4 \text{ m/s}$$

Power delivered by force at t = 4 seconds

= Force × velocity

= 11.8 N × 4 s = 47.2 W

The displacement during 4 seconds is given by

$$v^2 = u^2 + 2as \quad \Rightarrow \quad v^2 = 0 + 2 \times 1 \times s$$

or $\quad s = 8$ m

work done in 4 seconds = Force × distance = 11.8 × 8 = 94.4 J

$\therefore$ average power delivered $= \frac{\text{work done}}{\text{time}} = \frac{94.4}{4} = 23.6$ W.

Q. 17. *Two masses, one 'n' times as heavy as the other, have equal kinetic energy. What is the ratio of their momenta?*

Ans. Since $\quad p = \sqrt{2mE_k} \quad$ or $\quad E_k = \frac{p^2}{2m}$

As E_k is constant

$\therefore \quad p \propto \sqrt{m}$

Clearly, $\quad \frac{p_1}{p_2} = \frac{\sqrt{nm}}{\sqrt{m}} = \frac{\sqrt{n}}{1} \quad \Rightarrow \quad p_1 : p_2 = \sqrt{n} : 1.$

Q. 18. *A ball bounces to 80% of its original height. What fraction of its mechanical energy is lost in each bounce?*

Ans. Let the ball fall from a height, h then

K.E. of ball at the time of just striking the ground = P.E. of ball at height h

$\Rightarrow \quad K = mgh$

Similarly on rebounding the ball moves to a maximum height h', then K.E. of ball on rebounding

$$K' = \text{P.E. of ball at a height } h' = mgh'$$

$\therefore$ Loss of K.E. due to the rebounce $K - K' = mgh - mgh' = mg\,(h - h')$

$$= mg\left(h - \frac{80}{100}h\right) = mgh \times (0.2)$$

$\therefore$ Fractional loss in K.E. of ball in each rebounce $= \dfrac{K-K'}{K} = \dfrac{mgh \times (0.2)}{mgh} = 0.2$

$$= 0.2 \times 100\% = 20\%.$$

Q. 19. *An electric fan of mass 2 kg falls from the ceiling of a lift moving down with uniform speed of 2 ms⁻¹. It hits the floor of the lift (length of the lift = 2m) and does not rebound. How much heat will be produced by the impact?*

Ans. Mass of the fan, $m = 2$ kg

Length of the elevator, $h = 2\ m$.

The kinetic energy acquired by the fan during its fall is:

$$E = mgh = 2 \times 9.8 \times 2 = 39.2 \text{ J}$$

The amount of heat produced will also be 39.2 J, as in accordance with the law of conservation of energy, whole of the kinetic energy will be converted into heat.

Q. 20. *Two ball bearings of mass m each, moving in opposite directions with equal speed v, collide head on with each other. Predict the outcome of the collision, assuming it to be perfectly elastic.*

Ans. Here, $m_1 = m_2 = m$

$u_1 = v, \quad u_2 = -v$

Velocities of two balls after perfectly elastic collision between them are

$$v = \frac{(m_1 - m_2)\,u_1 + 2m_2u_2}{m_1 + m_2}$$

$$v_1 = \frac{0 + 2m\,(-v)}{m + m} = -v$$

$$v_2 = \frac{(m_2 - m_1)\,u_2 + 2m_1u_1}{m_1 + m_2} = \frac{0 + 2mv}{2m} = v$$

After collision, the two ball bearings will move with same speeds, but their direction of motion will be reversed.

Q. 21. *The displacement x of a particle moving in one dimension under the action of a constant force is related to time by the equation*

$$t = \sqrt{x} + 3$$

where x is in meter and t in second. Calculate the work done by the force in the first 6 second.

Ans. We have $t = \sqrt{x} + 3$ or $\sqrt{x} = t - 3$

$\therefore \quad x = (t - 3)^2$

The velocity of the particle is given by

$$v = \frac{dx}{dt} = 2\,(t - 3) \text{ ms}^{-1}$$

∴ The acceleration of the particle is given by

$$a = \frac{dv}{dt} = 2 \text{ ms}^{-2}$$

Force required to produce this acceleration is given by

$$F = m \times a = m \times 2 = 2\,m$$

where m is the mass of the particle.

Distance travelled by the particle in first 6 seconds

$$x = (6 - 3)^2 = 9 \text{ m}$$

Hence work done = $Fx = (2m \times 9) = 18\,m$ **Joule**

Q. 22. *A bullet of mass 20 g is moving with a speed of 150 m s*$^{-1}$*. It strikes a target and is brought to rest after piercing 10 cm into it. Calculate the average force of resistance offered by the target.*

Ans. Here m = 20 g = 0.02 kg, u = 150 m s^{-1}, v = 0 and s = 10 cm = 0.1 m

According to work-kinetic energy theorem, we have

$$K - K' = W = Fs$$

$$\therefore \quad \frac{1}{2}mu^2 - 0 = Fs \Rightarrow F = \frac{mu^2}{2s} = \frac{0.02 \times (150)^2}{2 \times 0.1} = 2250 \text{ N.}$$

Q. 23. *A group of clouds at a height of 500 m above the earth burst and cause enough rainfall to cover an area of* 10^6 *m*^{2} *with a depth of 2 cm. How much work would have been done in raising water to the height of clouds?*

Ans. Area; $A = 10^6 \text{ m}^2$

Depth, d = 2cm = 2×10^{-2} m and S = 500 m

∴ Volume of water, $V = Ad$

$= 10^6 \times 2 \times 10^{-2} = 2 \times 10^4 \text{ m}^3$

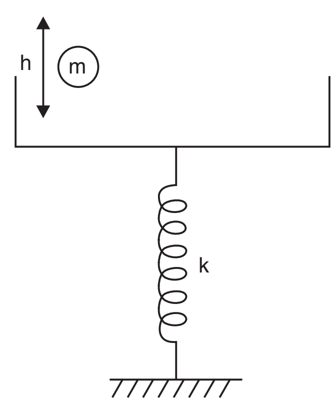
Density of water, $\rho = 10^3 \text{ kg m}^{-3}$

∴ mass of water, $m = V\rho = 2 \times 10^4 \times 10^3 = 2 \times 10^7$ kg

Weight of water, $W = mg = 2 \times 10^7 \times 9.8 = 19.6 \times 10^7$ N = F

Work done = $F \times S = 19.6 \times 10^7 \times 500 = 9.8 \times 10^{10}$ J.

Q. 24. *A mass m is placed on a platform from a height 'h'. The platform is attached to a spring whose other end is fixed to the ground. Find the compression in the spring, if the spring constant is k.*



Ans. Drop in potential energy = Energy stored in spring

or $$mg\,(h + x) = \frac{1}{2}kx^2$$

or $$2\,mgh + 2\,mgx = kx^2$$

or $$kx^2 - 2\,mgx - 2\,mgh = 0$$

$$x = \frac{2mg \pm \sqrt{4m^2g^2 - 4k.2mgh}}{2k}$$

$$\Rightarrow \quad x = \frac{mg}{k} \pm \frac{mg}{k}\sqrt{1 - \frac{2kh}{mg}}.$$

Q. 25. *A block of mass m moving with speed v compresses a spring through a distance x before its speed is halved. What is the value of spring constant?*

Ans. Initial Kinetic energy $= \frac{1}{2}mv^2$

Final Energy $= \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}kx^2$

By the principle of conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}\frac{mv^2}{4} + \frac{1}{2}kx^2$$

$$\therefore \qquad K = \frac{3mv^2}{4x^2}.$$

III. LONG ANSWER TYPE QUESTIONS

Q. 1. *Define conservative and non-conservative forces. Give example and properties of conservative forces.*

A 5 kg rifle fires a 5g bullet with a speed of 500 ms^{-1}. What kinetic energy is acquired (i) by the bullet and (ii) by the rifle? (iii) Find the the ratio of the distance which the rifle moves backward while the bullet is in the barrel to the distance the bullet moves forward.

Ans. **Conservative force:** A force is said to be conservative if work done by or against the force in moving a body depends only on the initial and final positions of the body and not on the nature of path followed between the initial and final positions.

Non-conservative forces: A force is said to be non-conservative if work done by or against the force in moving a body depends upon the path between the initial and final positions.

Examples of Conservative Forces

(*i*) Gravitational force, not only due to the Earth but in its general form as given by the universal law of gravitation, is a conservative force.

(*ii*) Elastic force in a stretched or compressed spring is a conservative force.

(*iii*) Electrostatic force between two stationary electric charges is a conservative force.

(*iv*) Magnetic force between two magnetic poles is a conservative force.

Properties of Conservative Forces

(*i*) Work done by or against a conservative force depends only on the initial and final positions of the body.

(*ii*) Work done by or against a conservative force does not depend upon the nature of the path between initial and final positions of the body.

(*iii*) Work done by or against a conservative force in a round trip is zero.

(*iv*) The work done by a conservative force is completely recoverable.

Numerical:

Mass of rifle, $M = 5$ kg;

Mass of bullet, $m = 5\text{g} = 5 \times 10^{-3}$ kg

Speed of bullet, $v = 500 \text{ ms}^{-1}$

(*i*) Kinetic energy acquired by bullet,

$$K_1 = \frac{1}{2}mv^2 = \frac{1}{2} \times 5 \times 10^{-3} \times (500)^2 = 625 \text{ J}$$

(*ii*) According to the law of conservation of momentum (As no external force acts on the bullet or rifle)

$$0 = MV + mv$$

$$\therefore \quad V \text{ (speed of rifle)} = -\frac{mv}{M} = -\frac{5 \times 10^{-3} \times 500}{5}$$

$$\Rightarrow \quad V = -0.5 \text{ ms}^{-1}.$$

–ve sign shows that rifle recoils back.

$$\therefore \text{ Kinetic energy of rifle} = \frac{1}{2}MV^2 = \frac{1}{2} \times 5 \times (-0.5)^2 = 0.625 \text{ J}$$

(*iii*) Since $mv + \text{MV} = 0$

$$\text{or} \quad m\frac{dx_1}{dt} + M\frac{dx_2}{dt} = 0 \qquad \left[\begin{aligned} \because v &= \frac{dx_1}{dt} = \frac{\text{Distance moved by bullet}}{\text{time}} \\ V &= \frac{dx_2}{dt} = \frac{\text{Distance moved by rifle}}{\text{time}} \end{aligned}\right]$$

$$\therefore \quad mdx_1 + Mdx_2 = 0$$

$$\text{or} \quad \left|\frac{dx_2}{dx_1}\right| = \frac{m}{M} = \frac{5 \times 10^{-3}}{5} = \frac{1}{1000}.$$

Q. 2. *A hammer of mass M drops from a height h. It strikes a nail placed vertically on the ground and drives it into the ground through a distance D. Calculate (i) the average resistance offered by the ground, assuming that the hammer and nail remain stuck together after impact, (ii) the time for which the nail is in motion and (iii) the loss in kinetic energy in impact.*

Ans. The hammer of mass M falls freely under gravity through a distance. Let v be the speed acquired by the hammer when it strikes the nail. Obviously

$$v = \sqrt{2gh} \qquad \text{...(1)}$$

On impact, the hammer and nail are stuck together. Let v be the speed of the combination after impact. The law of conservation of linear momentum gives

$$Mv = (M + m)v' \qquad \text{...(2)}$$

where m is the mass of the nail.

Let F_G be the average resistance (or resistive force) exerted by the ground. The net upward force F on the hammer-nail combination is

$$F = F_G - (M + m)g$$

The combination moves through a distance d against this net upward force. Obviously, the work done against the force F in a distance d must equal the kinetic energy the combination had just after impact. Hence

$$\frac{1}{2}(M+m)v'^2 = F \times d = [F_G - (M + m)\text{g}] \times d \qquad \text{...(3)}$$

Using Eqs. (1) and (2), we can rewrite Eq. (3) as

$$\frac{1}{2}(M+m)\left[\frac{Mv}{M+m}\right]^2 = [F_G - (M + M)\ g] \times d$$

or $$\frac{1}{2}\frac{M^2}{M+m} \times 2\,g\,h = [F_G - (M + m)\ g] \times d.$$

or $$F_G = \frac{M^2 g\,h}{(M+m)\,d} + (M+m)\,g \qquad (4)$$

(*ii*) Let the hammer-nail combination be moving for a time Δt, before coming to rest. If ΔP is the change in momentum, then from Newton's second law, we have

$$\frac{\Delta P}{\Delta t} = F_G - (m + M)\ g$$

also $$|\Delta P| = (M + m)\ v' - 0 = (M + m)\ \frac{Mv}{(M+m)} = Mv$$

$$\Delta t = \frac{\Delta P}{F_G - (M+m)\,g} = \frac{Mv}{\left\{\frac{M^2}{M+m)}\frac{g\,h}{d} + (M+m)\,g - (M+m)\,g\right\}}$$

$$= \frac{(M+m)\,d}{M^2\ g\,h}Mv = \frac{(M+m)\,d}{M\,g\,h}\sqrt{2\,g\,h} = \left[\frac{M+m}{M}\right]d\sqrt{\frac{2}{gh}}$$

(*iii*) The kinetic energy of the hammer-nail combination just before the impact is $\frac{1}{2}Mv^2$ and after impact, it is $\frac{1}{2}(M+m)\,v'^2$. Therefore,

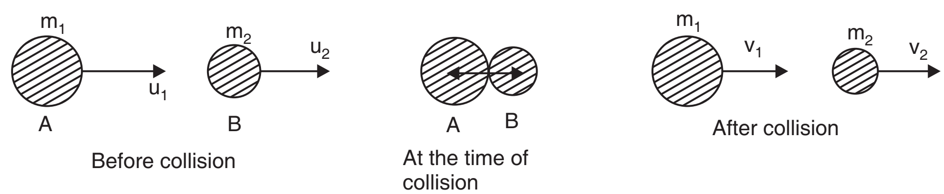
$$\text{Loss in kinetic energy} = \frac{1}{2}M\,v^2 - \frac{1}{2}(M+m)\,v'^2 = \frac{1}{2}Mv^2 - \frac{1}{2}\frac{M^2v^2}{(M+m)}$$

$$= \frac{1}{2}\frac{Mm}{(M+m)}v^2$$

Q. 3. *What do you mean by elastic collision? For an elastic head on collision, find expressions for final velocities of the bodies after collision.*

Ans. Elastic collision is one in which both momentum and energy are conserved.

Consider two bodies *A* and *B* of masses m_1 and m_2 moving in a straight line with velocities u_1 and u_2 respectively ($u_1 > u_2$). After collision, let their velocities change to v_1 and v_2 respectively. If the centres of two colliding bodies are in one line, then the collision is said to be head-on-collision.



Total linear momentum of the system before collision = $m_1u_2 + m_2u_2$

Total linear momentum of the system after collision = $m_1v_1 + m_2v_2$

As collision is elastic, so the linear momentum and kinetic energy of the system are conserved.

According to the law of conservation of linear momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad ...(1)$$

or $$m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad ...(2)$$

According to the law of conservation of kinetic energy,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2u_2^2 \quad ...(3)$$

or $$m_1(u_1^2 - u_1^2) = m_2(v_2^2 - u_2^2)$$

or $$m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2) \quad ...(4)$$

Dividing eqn. (4) by eqn. (2), we get

$$u_1 + v_1 = v_2 + u_2 \quad ...(5)$$

$$u_1 + u_2 = v_2 + v_1 \quad ...(6)$$

From eqn. (6) $$v_2 = u_1 - u_2 + v_1 \quad ...(7)$$

Substituting the value of v_2 in eqn. (2), we get

$$m_1(u_1 - v_1) = m_2[u_1 - 2u_2 + v_1]$$

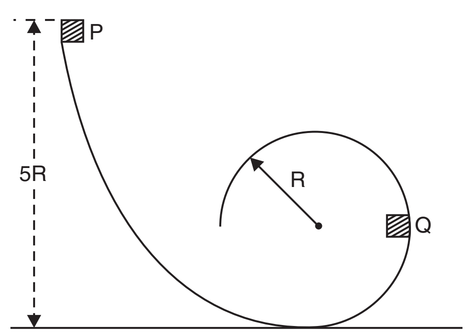
or $$v_1 = \frac{2m_2u_2 + u_1(m_1 - m_2)}{(m_1 + m_2)}$$

Substituting value of v_1 in eqn. (7), we get

$$v_2 = \frac{2m_1u_1 + u_2(m_2 - m_1)}{(m_1 + m_2)}$$

Q. 4. *A small block of mass m slides along the frictionless loop-to-loop track shown in the Figure. (a) If it starts from rest at P what is the resultant force acting on it at Q? (b) At what height above the bottom of loop should the block be released so that the force it exerts against the track at the top of the loop equals its weight?*

Ans. (*a*) Point Q is at a height R above the ground. Thus, the difference in height between points P and Q is $4R$. Hence, the difference in gravitational potential energy of the block between these points = $4\ mgR$.



Since the block starts from rest at P its kinetic energy at Q is equal to its change in potential energy. By the conservation of energy.

$$\therefore \quad \frac{1}{2}mv^2 = 4mgR$$

$$v^2 = 8gR$$

At Q, the only forces acting on the block are its weight mg acting downward and the force N of the track on block acting in radial direction. Since the block is moving in a circular path, the normal reaction provides the centripetal force for circular motion.

$$N = \frac{mv^2}{R} = \frac{m \times 8gR}{R} = \mathbf{8\ mg}$$

The loop must exert a force on the block equal to eight times the block's weight.

(*b*) For the block to exert a force equal to its weight against the track at the top of the loop,

$$\frac{mv'^2}{R} = 2mg \quad \text{or} \quad v'^2 = 2gR$$

$$\therefore \qquad mgh = \frac{1}{2}mv'^2$$

$$h = \frac{v'^2}{2g} = \frac{2gR}{2g} = R$$

The block must be released at a height 3*R* above the bottom of the loop.

IV. MULTIPLE CHOICE QUESTIONS

1. Equal masses (*m* each) are attached at the two ends of a string passing over two pulleys. Another mass is attached at the centre of the string. In order that there is no sag in the string, this mass should be

(*a*) *m* (*b*) *m*/2 (*c*) 2 *m* (*d*) Zero

2. The work done by all the forces (external and internal) on a system equals the change in

(*a*) total energy (*b*) kinetic energy (*c*) potential energy (*d*) none of these

3. A heavy stone is thrown from a cliff of height *h* with a speed *v*. The stone will hit the ground with maximum speed if it is thrown

(*a*) vertically downward (*b*) vertically upward

(*c*) horizontally (*d*) the speed does not depend on the initial direction

4. Two bodies of masses *m* and 4 *m* are moving with equal kinetic energy. The ratio of their linear momenta is

(*a*) 1 : 4 (*b*) 4 : 1 (*c*) 1 : 2 (*d*) 1 : 1

5. Two bodies of masses *m* and 4 *m* are moving with equal linear momentum. The ratio of their kinetic energies is

(*a*) 1 : 4 (*b*) 4 : 1 (*c*) 1 : 1 (*d*) 1 : 2

6. The work done by the external forces on a system equals the change in

(*a*) total energy (*b*) kinetic energy

(*c*) potential energy (*d*) none of these

7. What is the dimensions of power:

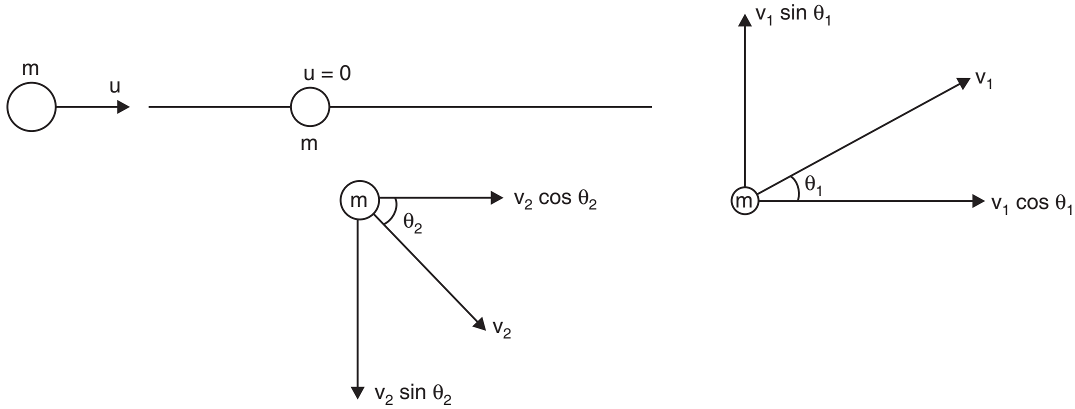
(*a*) $[MLT^{-2}]$ (*b*) $[ML^2T]$ (*c*) $[ML^2T^2]$ (*d*) $[MLT^{-3}]$

Ans. **1.**—(*d*) **2.**—(*b*) **3.**—(*d*) **4.**—(*c*) **5.**—(*b*)
6.—(*a*) **7.**—(*d*)

V. QUESTIONS ON HIGH ORDER THINKING SKILLS (HOTS)

Q. 1. *Two identical masses, one at rest and the other moving, undergo elastic oblique collision. Prove that they will move at right angles to each other after collision.*

Ans. Let the identical masses be m. Let the velocity at their motion initially be u and zero. Since momentum is a vector, we have



$$mu = mv_1 \cos\theta_1 + mv_2 \cos\theta_2$$

i.e., $$u = v_1 \cos\theta_1 + v_2 \cos\theta_2 \quad ...(i)$$

and $$0 = v_1 \sin\theta_1 - v_2 \sin\theta_2 \quad ...(ii)$$

based on the conservation of momentum.

Since collision is elastic, kinetic energy is conserved.

$$\therefore \quad \frac{1}{2}mu^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$\therefore \quad u^2 = v_1^2 + v_2^2 \quad ...(iii)$$

Squaring and adding (*i*) and (*ii*), we have,

$$u^2 = v_1^2 + v_2^2 + 2v_1v_2 (\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2)$$

or $$u^2 = v_1^2 + v_2^2 + 2v_1v_2 \cos(\theta_1 + \theta_2)$$

Using equation (*iii*), we have

$$2 v_1v_2 \cos(\theta_1 + \theta_2) = 0$$

$$\therefore \quad \cos(\theta_1 + \theta_2) = 0 \quad \text{or} \quad \theta_1 + \theta_2 = \pi/2$$

i.e., the masses move at right angles after the collision.

Q. 2. *A bullet of mass m moving with a velocity v is embedded into a block of mass M suspended by a thread. As a result of this collision, the block alongwith the bullet rises to a height h. Prove that velocity of bullet was* $\left(\frac{m+M}{m}\right)\sqrt{2gh}$.

Ans. Let v_1 be velocity of the system of block and the bullet. Applying principle of conservation of linear momentum, $(m + M) v_1 = mv$

$$v_1 = \frac{mv}{(m+M)} \quad ...(i)$$

As K.E. of the system = P.E. of the system at height h

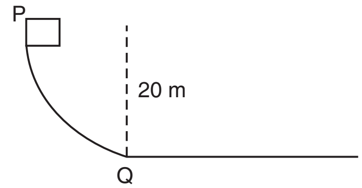
$$\therefore \quad \frac{1}{2}(m+M) v_1^2 = (m + M) gh$$

$$v_1 = \sqrt{2gh}$$

From (*i*),

$$v = \frac{(m+M)\,v_1}{m} = \frac{(m+M)\sqrt{2\,gh}}{m}$$

Q. 3. *A block of mass 200 g is released from P which slides down without friction till it reaches a point Q of a circular path of radius 2.0 m. Find (i) the velocity of the block at point Q and (ii) the coefficient of friction if the block comes to rest 2.0 m from Q, assuming the horizontal part of the path is rough. Take g = 10 ms*$^{-2}$.



Ans. (*a*) P.E. of block at P is converted into K.E. of the block at Q

$$\therefore \quad mgh = \frac{1}{2}mv^2$$

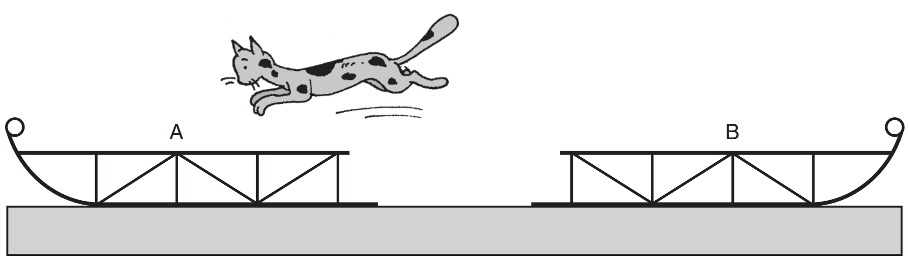
$$\text{or} \quad v = \sqrt{2gh} = \sqrt{2\times10\times2} = \sqrt{40} = 6.32 \text{ ms}^{-1}$$

(*ii*) When the block comes to rest after travelling a distance, s = 2.0 m from Q, then according to work – energy theorem,

Work done against friction = change in K.E.

$$\text{i.e,} \quad \mu\, mgs = \frac{1}{2}mv^2 \quad \text{or} \quad \mu = \frac{v^2}{2gs} = \frac{40}{2\times10\times2} = 1$$

Q. 4. *Two 22.7 kg ice sleds A and B are placed a short distance apart, one directly behind the other, as shown in Fig. A 3.63 kg cat, standing on one sled, jumps across to the other and immediately back to the first. Both jumps are made at a speed of 3.05 ms*$^{-1}$ *relative to the ice. Find the final speeds of the two sleds.*



Ans. Total momentum imparted to B

$$= 2 \times 3.63 \times 3.05 \text{ kg ms}^{-1}$$

$$\text{Velocity of B} = \frac{2\times3.63\times3.05}{22.7}\text{ ms}^{-1} = \mathbf{0.975 \text{ ms}^{-1}}$$

Velocity of A when the cat jumps away from A

$$= \frac{3.63\times3.05}{22.7}\text{ ms}^{-1} = 0.4877 \text{ ms}^{-1}$$

When the cat comes back to A,

$$\text{Velocity of A} = \frac{22.7\times0.4877+3.63\times3.05}{22.7+3.63}\text{ ms}^{-1} = \mathbf{0.841 \text{ ms}^{-1}}$$

Q. 5. *An object of mass 0.4 kg moving with a velocity of 4 ms^{-1} collides with another object of mass 0.6 kg moving in same direction with a velocity of 2 ms^{-1}. If the collision is prefectly inelastic, what is the loss of K.E. due to impact?*

Ans. Here $m_1 = 0.4$ kg, $u_1 = 4$ ms^{-1}, $m_2 = 0.6$ kg and $u_2 = 2$ ms^{-1}.

$\therefore$ Total K.E. of system before collision $K_i = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$

$$= \frac{1}{2}\times(0.4)\times(4)^2 + \frac{1}{2}\times(0.6)\times(2)^2$$

$$= 3.2 + 1.2 = 4.4 \text{ J}$$

As collision is perfectly inelastic, the common velocity after collision v is given by

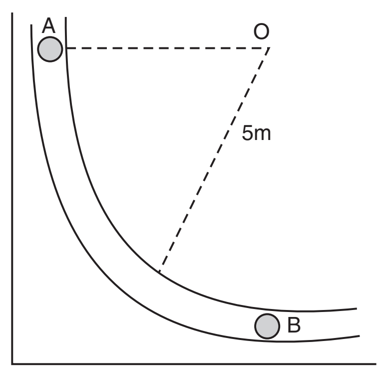
$$v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} = \frac{0.4\times4 + 0.6\times2}{0.4 + 0.6} = 2.8 \text{ ms}^{-1}$$

$\therefore$ Total K.E. of system after collision $K_f = \frac{1}{2}(m_1 + m_2)\,v^2 = \frac{1}{2}\times(0.4 + 0.6)\times(2.8)^2$

$$= 3.92 \text{ J}$$

$\therefore$ Loss in K.E. $\Delta K = K_i - K_f = 4.4 - 3.92 = 0.48$ J.

Q. 6. *A ball moves along a curved path of radius 5m as shown in figure. It starts from point A and reaches point B. If there is no force of friction between the ball and surface of the path, then find the normal force that acts on the ball at the bottom (B) of the curved path.*



Ans. K.E. of ball at A = P.E. of ball at B

i.e., $\frac{1}{2}mv^2 = mgh$ or $v^2 = 2gR$ $\quad(\because h = R)$

Centripetal acceleration of the ball, $a = \frac{v^2}{R} = \frac{2\,gR}{R} = 2g$.

$\therefore$ Centripetal force on the ball, $f = ma = 2\,mg$.

Now, Let $W = mg$ be the mass of the ball acting vertically downward at point B and R is the normal reaction or force acting on the ball in the upward direction.

$\therefore$ $f = R - W$ or $R = f + W = 2\,mg + mg = 3mg$

V. VALUE-BASED QUESTIONS

Q. 1. *Tejveer is the medical student in Meerut. His grandfather went to Meerut to visit his grandson. In the evening his grandfather asked him about the heartbeats and pulse. He also eager to know what is the difference between the heart beats of mammals relative to monkey. Tejveer being a medical student explained all about it very well.*

(i) What values of Tejveer displayed here?

(ii) What is the scale factor of human relative to monkey?

(iii) What is the monkey's heart rate?

Ans. (*i*) Values are : Intelligence, dedication, loving, social and cooperative.

(*ii*) Human heart rate is about 70 beats/min and scale factor is 2.5.

(*iii*) Monkey's heart rate = $70 \times 2.5 = 175$.

Q. 2. *Madhu is doing her Engineering in IIT Kanpur in Electrical Engineering stream. Her mother reached there and at night both are discussing about the comets. Her mother who is also a science teacher in a school in Delhi asked her daughter "comets move around the sun in elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity. Yet the work done by the gravitational force over every complete orbit of the comet is zero, why?*

Madhu had very much interest in astronomy. She explained this reason to his mother very well. "This is because gravitational force is a conservative force. Work done by gravitational force of the sun over a closed path in every complete orbit of the comet is zero.

(i) What values are displayed by Madhu?

(ii) What is in her mother's mind to ask such a question?

Ans. (*i*) Values are : Very intelligent, cooperative, apathy and well explanatory.

(*ii*) Madhu's mother wanted to test the knowledge of her daughter to ask such a technical question.

Q. 3. *Ramesh and Dinesh are two friends went to meet his physics teacher. The teacher asked Ramesh to lift a 20 kg weight vertically upwards to a hight of 5 m and Dinesh asked to pull 50 kg of weight by a rope over a pulley from the depth of 10 m of a well. Prizes were set to give to one who will do more work. Ramesh claimed that he has done more work than Dinesh because I have not used any kind of lever. The teacher told the Ramesh that he has done no work but Dinesh has done the work. So the prize is given to Dinesh.*

(i) What values of the teacher are exhibited here?

(ii) Why Ramesh did not do any work?

Ans. (*i*) The teacher wanted to know the physics knowledge of the two students. He wanted to put awareness in them.

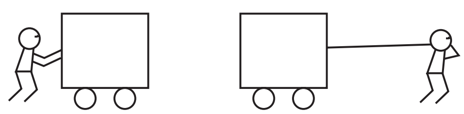
(*ii*) *Work* $W = F \times d \cos \theta$

Here $\theta = 90°$ and $\cos 90° = 0$

$\therefore \quad W = F \times d \times 0 = 0 \quad \therefore$ No work was done

TEST YOUR SKILLS

1. What do you understand by work done ? An aeroplane runs for 500 metres after the touchdown and comes to a stop. The force on the aeroplane due to tarmac on the aeroplane is 1000 N? How much work does the aeroplane do on the tarmac ?
2. It is said that no work is done, when force and displacement are mutually perpendicular. Do you agree with this statement ? Give reasons for your answer.
3. A horse pulls a carriage on a rough road. The horse applies a force of 200 N for 15 m. After that the horse gets tired and applied force reduces linearly with distance to 50N. The total distance covered by the carriage is 30 m. Plot the force applied by the horse and the frictional force, which is 50 N. What is the work done by two forces over the distance of 30 m?
4. What is the difference between completely elastic and inelastic collisions?
5. A raindrop of mass 3.5×10^{-5} is moving at a terminal speed of 9 ms^{-1}. What is its kinetic energy ?
6. What is the theory employed, while simulating accidents by car manufactures ? A car of mass 500 kg is moving with a speed of 54 kmh^{-1} on a smooth surface. The car collides with a horizontally mounted collision spring. The spring constant is 6.25×10^2 Nm^{-1}. What is the maximum compression of the spring ?
7. A billiards player wants to sink the target ball. Masses of both cue and target balls are equal. The pocket makes an angle of 53° with the target ball. What should be the angle of the cue ball? Assume the collision to be elestic and friction and rotational motions to be negligible.
8. A man pushes a trolley for some distance. Another man pulls the same trolley for the same distance. Which of the two men do more work ? Why ?



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