7 Alternating Current

Facts that Matter

- The current which changes in magnitude and direction both with time is called alternating current and the corresponding emf is called alternating emf.
- When a coil is rotated in uniform magnetic field with angular speed ω, the induced emf

$$\varepsilon = NBA\omega \sin \omega t$$

or $$\varepsilon = \varepsilon_0 \sin \omega t \quad ...(i)$$

$\Rightarrow$ $$\frac{\varepsilon}{R} = \frac{\varepsilon_0}{R} \sin \omega t$$

or $$I = I_0 \sin \omega t \quad ...(ii)$$

Eqs (*i*) and (*ii*) are the equation of alternating emf and alternating current respectively.

There equation can be represented by graph as shown in Fig. 7.1

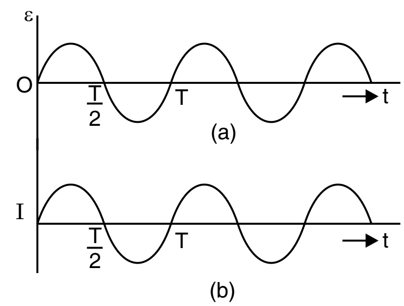

Fig. 7.1

- The average value of alternating current is given by

$$I_{av} = \int_0^T \frac{I_0 \sin \omega t\, dt}{\int_0^T dt}$$

$$= \frac{I_0 \left[-\frac{\cos \omega t}{\omega}\right]_0^T}{[t]_0^T}$$

$$= \frac{I_0}{T}\left[\frac{T}{2\pi}\left(-\cos \frac{2\pi}{T}(T) + \cos 0°\right)\right]$$

$$= \frac{I_0}{2\pi}(-1+1)$$

$$= 0$$

- The average value of a.c. for half cycle,

$$I_{av} = \int_0^{T/2} \frac{I_0 \sin \omega t\, dt}{\int_0^{T/2} dt}$$

$$= \frac{2I_0}{T}\left[\frac{1}{\omega}(-\cos \omega t)_0^{T/2}\right]$$

$$= \frac{2I_0}{T}\left[\frac{T}{2\pi}\left(-\cos\frac{2\pi}{T}(T/2)\right)+\cos 0°\right]$$

$$= \frac{2I_0}{2\pi}(2) = \frac{2I_0}{\pi}$$

- **Root Mean Square Value of AC**

$\because \quad I = I_0 \sin \omega t$

$\therefore \quad I^2 = I_0^2 \sin^2 \omega t$

$$= I_0^2 \frac{1}{2}(1-\cos 2\omega t)$$

The mean value of I^2,

$$I^2_{\text{mean}} = \frac{\int_0^T I_0^2 \frac{1}{2}(1-\cos 2\omega t)\, dt}{\int_0^T dt}$$

$$= \frac{I_0^2}{2T}\left[t - \frac{\sin \omega t}{2\omega}\right]_0^T$$

$$= \frac{I_0^2}{2T}\left[(T-0) - \frac{1}{2\omega}\left(\sin\frac{2\pi}{T}.T - \sin 0\right)\right]$$

$$= \frac{I_0^2}{2T}\left[T - \frac{1}{2\omega}(0)\right]$$

$$I^2_{\text{rms}} = \frac{I_0^2}{2}$$

Taking the root of I^2_{mean},

$$I_{\text{rms}} = \sqrt{\frac{I_0^2}{2}} = \frac{I_0}{\sqrt{2}}$$

or $$\boxed{I_{\text{rms}} = \frac{I_0}{\sqrt{2}}}$$

or $$\boxed{I_0 = I_{\text{rms}}\sqrt{2}}$$

Similarly $$\boxed{\varepsilon_{\text{rms}} = \frac{\varepsilon_0}{\sqrt{2}}}$$

or $$\boxed{\varepsilon_0 = \varepsilon_{\text{rms}}\sqrt{2}}$$

- **Alternating Current Circuits**

An alternating current circuit may have three elements separately or simultaneously. These three elements are

(*i*) Resistance (*R*) (*ii*) Inductance (*L*) (*iii*) Capacitance (*C*)

- **Phasor Diagram**

A phasor diagram is a rotating vector about origin with angular velocity ω. The vertical component of a phasor is sinusoidal varying quantity which represents the ε_0, ε or I_0, I on y-axis and ωt on x-axis as shown in Fig. 7.2

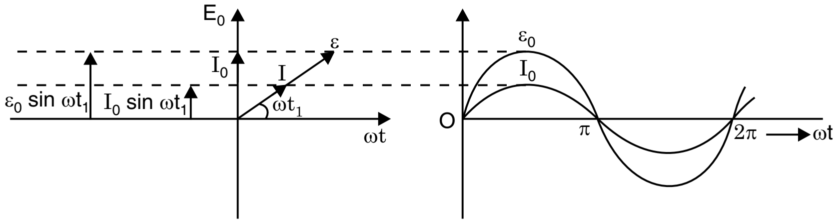

Fig. 7.2

- **Circuit Containing Resistance Only**

Let the voltage $\qquad \varepsilon = \varepsilon_0 \sin \omega t \qquad ...(i)$

is applied across a circuit containing resistance only. The current in the circuit will be

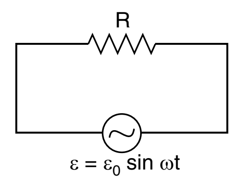

Fig. 7.3

$$\frac{\varepsilon}{R} = \frac{\varepsilon_0}{R} \sin \omega t$$

or $\qquad I = I_0 \sin \omega t \qquad ...(ii)$

where $\frac{\varepsilon}{I}$ or $\frac{\varepsilon_0}{I_0}$ is the resistance of the circuit.

Eqs. (*i*) and (*ii*) show that there is no phase difference between current and voltage in the circuit. The phasor diagram variation of voltage and corresponding current with time in shown in Fig. 7.4.

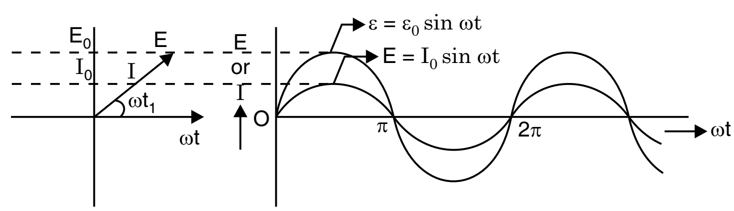

Fig. 7.4

- **Circuit containing Inductor only**

Let a voltage $\qquad \varepsilon = \varepsilon_0 \sin \omega t \qquad ...(i)$

is applied across the circuit containing inductance L only. The current in the circuit will be

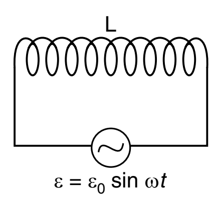

Fig. 7.5

$$\int dI = \int \frac{\varepsilon_0}{L} \sin \omega t \qquad \left(\because \ \varepsilon = L\frac{dI}{dt}\right)$$

or $$I = \frac{-\varepsilon_0}{\omega L} \cos \omega t$$

or $$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right) \qquad ...(ii)$$

where $$I_0 = \frac{\varepsilon_0}{\omega L}$$

or $\dfrac{\varepsilon_0}{I_0} = \omega L = (X_L)$ is the resistance offered by inductor known as inductive reactance.

Eqs. (*i*) and (*ii*) show that there is a phase difference of $\pi/2$ between voltage and current. The voltage leads the current $\pi/2$ as shown in phasor diagram 7.6.

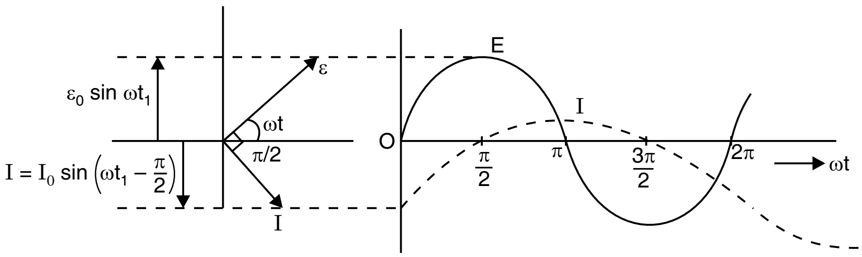

Fig. 7.6

- **Circuit Containing Capacitor Only**

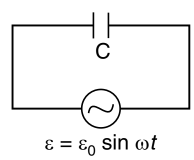

Fig. 7.7

Let a voltage $$\varepsilon = \varepsilon_0 \sin \omega t \qquad ...(i)$$

is applied across a circuit containg capacitance C only. The current in the circuit will be

$$I = \frac{dq}{dt}$$

$$= \frac{d}{dt}(C\varepsilon)$$

$$= \frac{d}{dt}(C\varepsilon_0 \sin \omega t)$$

or $$I = \omega C\varepsilon_0 \cos \omega t$$

or $$I = I_0 \sin(\omega t + \pi/2) \qquad ...(ii)$$

where $$I_0 = \omega C\varepsilon_0 \quad \text{or} \quad E_0 = \frac{I_0}{\omega C} = I_0 X_C$$

or $\dfrac{\varepsilon_0}{I_0} = \dfrac{1}{\omega C} - (X_C)$ is the resistance offered by capacitor known as capacitive reactance.

Eqs. (*i*) and (*ii*) show that there is a phase difference of $\pi/2$ between voltage and current. The current leads the voltage by $\pi/2$ as shown in Fig. 7.8.

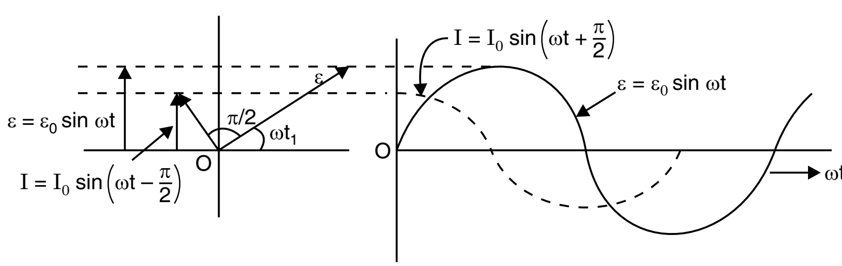

Fig. 7.8

- **Circuit Containing Inductor and Resistor**

Writing the equation of voltage and current for inductor and resistor respectively,

$$I = I_0 \sin \omega t$$

and $$\varepsilon = \varepsilon_0 \sin \omega t$$

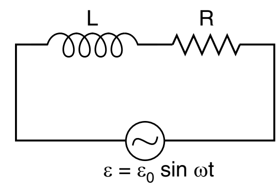

Fig. 7.9

For inductor only,

$$I = I_0 \sin \omega t$$

and $$\varepsilon = \varepsilon_0 \sin (\omega t + \pi/2)$$

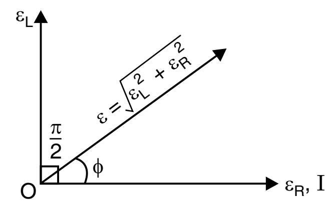

Fig. 7.10

The current in inductor and resistor remains same because they are connected in series but the voltage across inductor and resistor differ in phase by $\pi/2$. The voltage across the inductor is $\pi/2$ ahead the voltage across resistor as shown in Fig. 7.10. The net voltage of the circuit is $\varepsilon = \sqrt{\varepsilon_L^2 + \varepsilon_R^2}$ and there is a phase difference of ϕ between net voltage and current.

The corresponding phasor diagram can also be made in terms of resistances as shown in Fig. 7.11. The net resistance of $L - R$ circuit called inductive reactance is given by

$$X_L = \sqrt{R^2 + X_L^2}$$

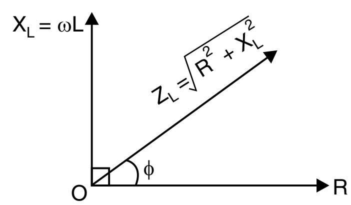

Fig. 7.11

And the current in the circuit,

$$I = \frac{\sqrt{\varepsilon_R^2 + \varepsilon_L^2}}{\sqrt{R^2 + X_L^2}}$$

The phase difference between net voltage and current,

$$\phi = \tan^{-1}\left(\frac{\varepsilon_L}{\varepsilon_R}\right)$$

or $$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

- **Circuit Containing Capacitor and Resistor**

Writing the equations for current and voltage for resistor and capacitor respectively.

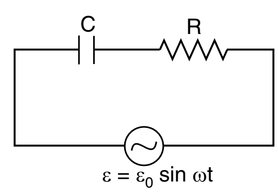

Fig. 7.12

For resistance only

$$I = I_0 \sin \omega t$$

and $$\varepsilon = \varepsilon_0 \sin \omega t$$

For capacitor only

$$I = I_0 \sin \omega t$$

and $$\varepsilon = \varepsilon_0 \sin (\omega t - \pi/2)$$

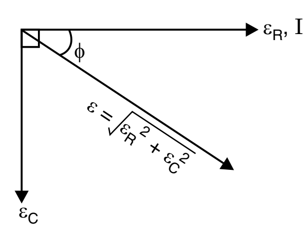

Fig. 7.13

$\because$ C and R are in series, hence the current them is same but the voltage across resistor is $\pi/2$ ahead of the voltage across capacitor which in shown in Fig. 7.13.

And net voltage of the circuit is given by

$$\varepsilon = \sqrt{\varepsilon_0^2 + \varepsilon_R^2}$$

The corresponding phaser diagram in terms of capacitive reactance and resistance can be made as shown in the Fig. 7.14

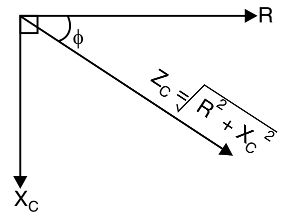

Fig. 7.14

The net resistance called capacitive impedance is given by

$$Z_C = \sqrt{R^2 + X_C^2}$$

Thus, current in the circuit $$I = \frac{\sqrt{\varepsilon_R^2 + \varepsilon_C^2}}{\sqrt{R^2 + X_C^2}}$$

And phase difference between net voltage and current,

$$\phi = \tan^{-1}\left(\frac{\varepsilon_C}{\varepsilon_R}\right) \quad \text{or} \quad \phi = \tan^{-1}\left(\frac{X_C}{R}\right)$$

- **Circuit Containing Inductor, Capacitor and Resistor**

Writing the equations of current and voltage for all element of the circuit respectively. For resistance only

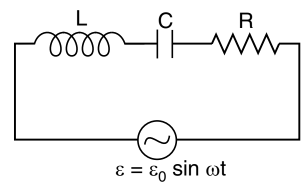

Fig. 7.15

$$I = I_0 \sin \omega t$$

and $$\varepsilon = \varepsilon_0 \sin \omega t$$

For inductor only

$$I = I_0 \sin \omega t$$

and $$\varepsilon = \varepsilon_0 \sin (\omega t + \pi/2)$$

For capacitor only

$$I = I_0 \sin \omega t$$

and $$\varepsilon = \varepsilon_0 \sin (\omega t - \pi/2)$$

∵ All elements of the circuit are in series therefore, current in all remains same but the voltage across all differ in phase which is shown in Fig. 7.15

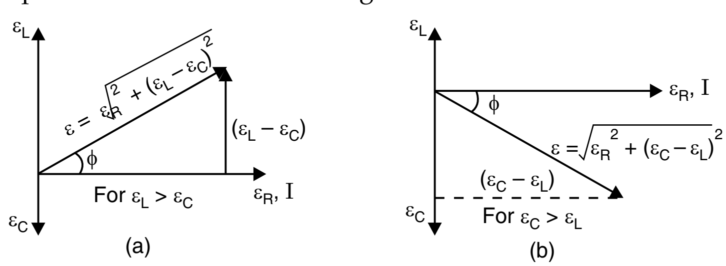

Fig. 7.16

The corresponding phasor diagram in terms the resistance of the elements can also be drawn as shown in Fig. 7.16

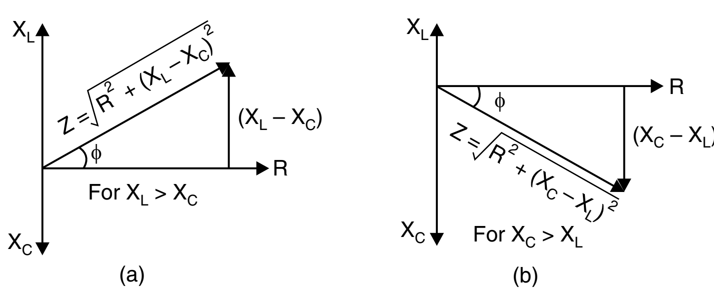

Fig. 7.17

The net voltage of the circuit is given by

$$\varepsilon = \sqrt{\varepsilon_R^2 + (\varepsilon_L \sim \varepsilon_C)^2}$$

and the net resistance offered by all the elements present in the circuit called impedance is given by

$$z = \sqrt{R^2 + (X_L \sim X_C)^2}$$

Thus, current in the circuit,

$$I = \frac{\varepsilon}{z} = \frac{\sqrt{\varepsilon_R^2 + (\varepsilon_C \sim \varepsilon_C)^2}}{\sqrt{R^2 + (X_L \sim X_C)^2}}$$

The phase difference between net voltage and current

$$\phi = \tan^{-1}\frac{(\varepsilon_L - \varepsilon_C)}{\varepsilon_R} \text{ for } \varepsilon_L > \varepsilon_C$$

or $$\phi = \tan^{-1}\left(\frac{X_L \sim X_C}{R}\right)$$

Also $$\phi = \tan^{-1}\left(\frac{\varepsilon_C \sim \varepsilon_L}{R}\right) \text{ for } \varepsilon_C > \varepsilon_L$$

or $$\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

For $X_L > X_C$, $\varepsilon_L > \varepsilon_C$ and voltage will lead the current. For $X_C > X_L$ and current will lead the voltage.

- **Circuit Containing Inductor and Capacitor**

Writing the equations for voltage and current for inductor and capacitor respectively. For inductor only

$$I = I_0 \sin \omega t$$

and $$\varepsilon = \varepsilon_0 \sin(\omega t + \pi/2)$$

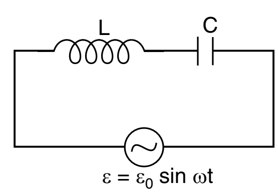

Fig. 7.18

For capacitor only

$$I = I_0 \sin \omega t$$

and $$\varepsilon = \varepsilon_0 \sin(\omega t - \pi/2)$$

∵ Inductor and capacitor are in series

∴ The current in them remains same but voltage across inductor and across capacitor differ in phase by π. The phase difference is shown in Fig. 7.18.

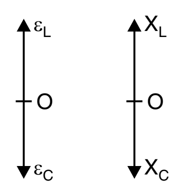

Fig. 7.19

The net voltage of the circuit is given by

$$\varepsilon = \varepsilon_L \sim \varepsilon_C$$

and the net resistance or impedance of the circuit,

$$Z_{LC} = X_L \sim X_C$$

Thus, current in the circuit,

$$I = \frac{\varepsilon_L \sim \varepsilon_C}{X_L \sim X_C}$$

For $X_L = X_C$, the current will be infinitely large and impedance of the circuit becomes zero. For this no energy of the circuit will dissipate in the form of heat and it will remain conserved. However during the charging of the capacitor energy will store in the form of electrical energy in capacitor *i.e.,* $\frac{1}{2}\frac{q^2}{C}$ or $\frac{1}{2}CV^2$ and during the discharging of capacitor energy will store in the form of magnetic energy in inductor *i.e.,* $\frac{1}{2}LI^2$.

Thus, electric charge oscillates between electric field of capacitor and magnetic field of inductor. These oscillations are called $L - C$ oscillation.

For $L - C$ oscillations $X_L = X_C$

or $$\omega L = \frac{1}{\omega C}$$

or $$\omega^2 = \frac{1}{LC}$$

or $$\omega = \frac{1}{\sqrt{LC}}$$

or $$2\pi\nu = \frac{1}{\sqrt{LC}}$$

or $$\nu = \frac{1}{2\pi\sqrt{LC}}$$

V is the frequency of L-C oscillation and the circuit is called *resonance circuit*.

- For L-C-R resonance circuit, the frequency of oscillation will be

$$\nu = \frac{1}{2\pi\sqrt{LC}}$$

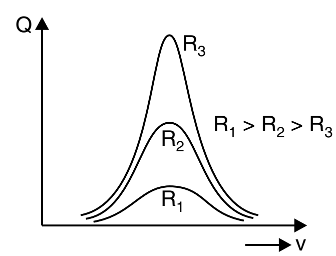

Fig. 7.20

but *quality factor i.e.,* the measure of the sharpness of the resonance will decrease as shown in Fig. 7.19

- *Quality factor* is defined as the ratio of voltage appeared across inductor or capacitor to voltage of source.

$$\therefore \quad Q = \frac{(X_L I) \text{ or } (X_C)\, I}{I_Z}$$

$$= \frac{X_L}{Z} \text{ or } \frac{X_C}{Z}$$

$$= \frac{\omega L}{R} \text{ or } \frac{1}{\omega CR} \qquad \text{(At resonance } X_L = X_C \text{ and } Z = R)$$

$$= \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} \text{ or } \frac{1}{\frac{1}{\sqrt{LC}} \cdot CR}$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}}$$

- For $X_L = X_C$, the circuit is called resistive circuit and current in the circuit remains maximum.
- For resonance circuit, $X_L = X_C$ the impedance becomes equal to resistance and phase difference between current and voltage remains zero.

- **Energy Stored in Inductor**

The amount of work done to oppose the change in magnetic flux linked with the inductor stores in the form of magnetic energy in the inductor.

$$\therefore \quad U = \int \varepsilon \, . \, dq$$

$$= \int \frac{L\,dF}{dt} \, . \, dq$$

$$= \int L\,I dI$$

or

$$\boxed{U = \frac{1}{2} L I^2}$$

- **Average Power in AC Circuit**

$$\because \quad P = \varepsilon I$$

$$= \varepsilon_0 \sin(\omega t + \phi). \sin \omega t$$

where ϕ is the difference between voltage and current.

$$\therefore \quad P = \varepsilon_0 I_0 [\sin \omega t \cos \phi + \cos \omega t \sin \phi] \sin \omega t$$

$$= \varepsilon_0 I_0 [\sin^2 \omega t \cos \phi + \sin \omega t \sin \phi \, . \cos \omega t]$$

$$= \frac{\varepsilon_0 I_0}{2} [(1 - \sin 2\omega t) \cos \phi + \sin 2\omega t \sin \phi]$$

Thus, average power

$$P_{av} = \frac{\varepsilon_0 I_0}{2} \frac{\int_0^T [\cos \phi (1 - \sin 2\omega t) \, dt + \sin \phi \sin 2\omega t \, dt]}{\int_0^T dt}$$

$$= \frac{\varepsilon_0 I_0}{2T} \left[\cos \phi \int_0^T dt - \int_0^T \sin 2\omega t \, dt + \sin \phi \int_0^T \sin 2\omega t \, dt \right]$$

$$= \frac{\varepsilon_0 I_0}{2T} [(\cos \phi (T) - 0) + \sin \phi (0)]$$

$$= \frac{\varepsilon_0 I_0 \cos \phi (T)}{2T}$$

$$= \frac{\varepsilon_0 I_0 \cos \phi}{2}$$

or

$$P_{av} = \frac{\varepsilon_0}{\sqrt{2}} . \frac{I}{\sqrt{2}} . \cos \phi$$

or

$$P_{av} = \varepsilon_{rms} I_{rms} \cos \phi$$

or

$$\boxed{P_{av} = \varepsilon_{rm} I_{rms} . \left(\frac{R}{Z}\right)}$$

• A.C. Generator

It is a device which converts mechanical energy into electrical energy. According to the principle of A.C. generator, when a coil is rotated in a magnetic field, the magnetic flux linked with coil changes and induced current is produced in the coil. It consists of a coil placed in radial magnetic field. The ends of the coil are connected across the load with the help of carbon brushes and slip rings as shown in Fig. 7.20.

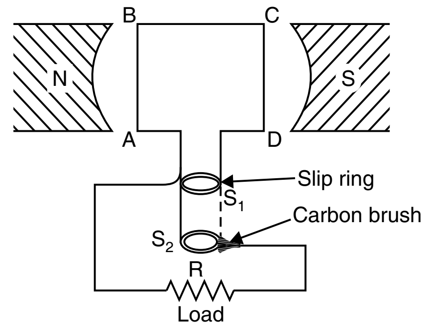

Fig. 7.21

When coil *ABCD* is rotated in magnetic field, the flux linked with coil changes and an induced current is produced in the coil which is drawn out across the load with the help of slip rings S_1, S_2 and carbon brushes.

When coil rotates the flux changes from zero to maximum for one forth rotation and then reduces to zero for next forth rotation and so on. And according Fleming right hand rule the direction of current also changes for each half of the rotation as shown in Fig. 7.21 (*a*) and (*b*).

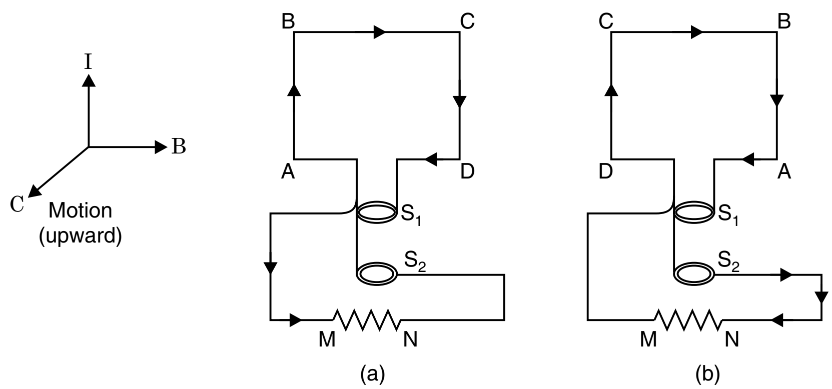

Fig. 7.22

Thus, the current produced changes in magnitude and direction both and called *a.c. current* and the device is called a.c. generator. The variation of current can be represented by the equation $I = I_0 \sin \omega t$ and by graph shown in Fig. 7.22.

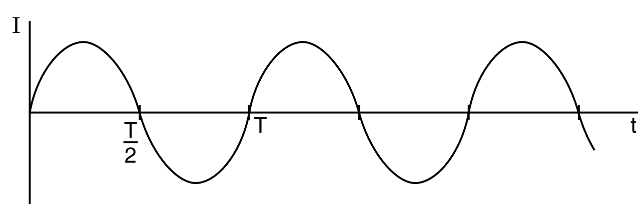

Fig. 7.23

• Transformer

It is a device which is used to change the current or voltage in the circuit. It is based on the principle of mutual induction. It consists of two coils, one connected to the source is called primary coil and the other connected to the real is called secondary coil as shown in Fig. 7.23.

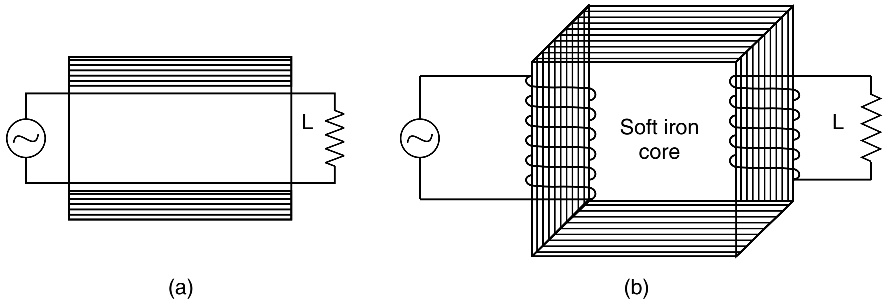

Fig. 7.24 Two arrangements for winding of primary and secondary coil in a transformer (*a*) two coils on top of each other (*b*) two coils on separats limbs of the coil.

When alternating voltage applied to the primary coil changes, the magnetic flux linked with the secondary coil changes and an induced emf is developed across the secondary coil and vice-versa. The emf developed in secondary coil is given by

$$\varepsilon_s = -M\frac{dI_p}{dt}$$

and the emf of primary coil is given by

$$\varepsilon_p = -M\frac{dI_s}{dt}$$

Therefore, power associated with primary coil and secondary coil is given as

$$\varepsilon_p I_p = -M\frac{dI_s}{dt}.I_p$$

and $$\varepsilon_s I_s = -M\frac{dI_p}{dt}.I_s \qquad ...(ii)$$

From Eqs. (*i*) and (*ii*)

$$\varepsilon_p I_p = \varepsilon_s I_s \text{ (For no loss of energy)}$$

or $$\frac{\varepsilon_s}{\varepsilon_p} = \frac{I_P}{I_s}$$

$\because$ $\varepsilon \propto N$ (N is the number of turns in the coil)

$\therefore$ $$\frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_P}$$

$\Rightarrow$ $$\frac{\varepsilon_s}{\varepsilon_P} = \frac{N_s}{N_P} = \frac{I_P}{I_s} \quad \begin{bmatrix} N_s = \text{number of turns in secondary} \\ N_p = \text{number of turns in primary} \end{bmatrix}$$

- If $N_s > N_p$

  $\varepsilon_s > \varepsilon_p$

  and $I_s < I_p$

  This type of transformer is called step-up transformer which increases the voltage and decreases the current in secondary coil.

- If $N_s < N_p$

  $\varepsilon_s < \varepsilon_p$

  and $I_s > I_p$

  This type of transformer is called step-down transformer which decreases the voltage and increases the current in the secondary coil.

• Eddy Currents

When a metallic block is placed in a changing magnetic field, the flux linked with the coil changes and induced currents are produced in the block. The direction of these currents is circular in nature, hence, these are called eddy current. The currents are also called *Focault current*. (Fig. 7.24).

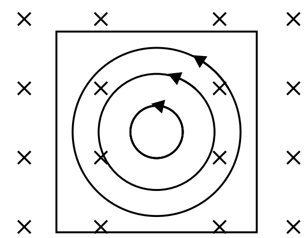

Fig. 7.25

- These currents are common in the (*i*) core of transformer (*ii*) induction furnance (*iii*) magnetic jeckes and (*iv*) magnetic breaks.
- In the core of the transformer some part of electrical energy dissipates in the form of heat due to production of eddy currents in soft iron core of transformer.
- To minimise the eddy currents in the core of transformer, the core is made laminated
- Electrical energy is always transmitted at high voltage for long transmission.
- At high voltage the current is reduced and correspondingly the power loss (I^2R) is reduced.

QUESTIONS FROM TEXTBOOK

7.1. *A 100 Ω resistor is connected to a 220 V, 50 Hz ac supply.*

(a) What is the rms value of current in the circuit?

(b) What is the net power consumed over a full cycle?

Sol. Given, $R = 100\ \Omega,\quad V = 220\ \text{V}_{eff},\quad f = 50$ Hz

$\therefore\quad \omega = 2\pi f = 2 \times 3.14 \times 50 = 314$

(*a*) Using the relation

$$I_{eff} = \frac{V_{eff}}{R}$$

Putting values,

$$I_{eff} = \frac{220}{100} = 2.2\ \text{A}$$

(*b*) Power consumed $=$ current × voltage $= I_{eff} \times V_{eff}$

$= 2.2 \times 200 = 484$ watt.

7.2. *(a) The peak voltage of an ac supply is 300 V. What is the rms voltage?*

(b) The rms value of current in an ac circuit is 10 A. What is the peak current?

Sol. Given, $E_0 = 300$ V, $I_{rms} = 10$ A (*a*) Using relation

$$E_{rms} = \frac{E_0}{\sqrt{2}}$$

$$= \frac{300}{\sqrt{2}} = \frac{300}{1.414} = 212.13 \text{ V}$$

(*b*)

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

or,

$$I_0 = \sqrt{2}\ I_{rms}$$

$$= \sqrt{2} \times 10 = 14.1 \text{ A.}$$

7.3. *A 44 mH inductor is connected to 220 V, 50 Hz ac supply. Determine the rms value of the current in the circuit.*

Sol. Given,

$$L = 44 \text{ mH} = 44 \times 10^{-3} \text{ H}$$

$$f = 50 \text{ Hz}, \quad E_{rms} = 220 \text{ V}$$

$$X_L = L.\omega = L \cdot 2\pi f$$

$$= 44 \times 10^{-3} \times 2 \times 3.14 \times 50$$

Now,

$$I_{rms} = \frac{E_{rms}}{X_L} = \frac{220}{44 \times 10^{-3} \times 2 \times 3.14 \times 50}$$

$$= 15.9 \text{ A.}$$

7.4. *A 60 μF capacitor is connected to a 110 V, 60 Hz ac supply. Determine the rms value of the current in the circuit.*

Sol. Given,

$$C = 60\ \mu\text{F} = 60 \times 10^{-6} \text{ F}$$

$$E_{rms} = 110 \text{ V}$$

$$f = 60 \text{ Hz}$$

$$I_{rms} = \frac{E_{rms}}{X_C} = \frac{E_{rms}}{\frac{1}{C.\omega}} = E_{rms} \cdot C \cdot 2\pi f$$

Putting the values,

$$I_{rms} = 110 \times 60 \times 10^{-6} \times 2 \times 3.14 \times 60$$

$$= 2.49 \text{ A.}$$

7.5. *In Questions 7.3 and 7.4, what is the net power absorbed by each circuit over a complete cycle. Explain your answer.*

Sol. Net power absorbed by the circuit over a complete cycle is zero. Since power is not absorbed by pure inductor or capacitor and it is only resistance which absorbs the power.

Power for pure inductor or capacitor circuit = $P_{av} = V_{eff}\, I_{eff} \cos\left(\pm \frac{\pi}{2}\right)$

7.6. *Obtain the resonant frequency ω_r of a series LCR circuit with L = 2.0H, C = 32 μF and R = 10 Ω. What is the Q-value of this circuit?*

Sol. Here,

$$L = 2.0 \text{ H}, \quad C = 32\ \mu\text{F} = 32 \times 10^{-6} \text{ F}$$

$$R = 10 \text{ Ohm}$$

$$\boxed{\omega_r = \frac{1}{\sqrt{LC}}} = \frac{1}{\sqrt{2.0 \times 32 \times 10^{-6}}} = \frac{10^3}{8} = 125 \text{ rad/s}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}} = \frac{1}{10}\sqrt{\frac{2}{32 \times 10^{-6}}}$$

$$Q = \frac{1000}{40} = 25.$$

7.7. *A charged 30 μF capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?*

Sol. Given, $C = 30\ \mu\text{F} = 30 \times 10^{-6}$ F

$L = 27$ mH $= 27 \times 10^{-3}$ H

$$\boxed{\omega_r = \frac{1}{\sqrt{LC}}} = \frac{1}{\sqrt{27 \times 10^{-3} \times 30 \times 10^{-6}}}$$

$$= \frac{10^4}{9} = 1.1 \times 10^3 \text{ s}^{-1}.$$

7.8. *Suppose the initial charge on the capacitor in Question 7.6 is 6 mC. What is the total energy stored in the circuit initially? What is the total energy at later time?*

Sol. Given, $Q = 6$ mC $= 6 \times 10^{-3}$ C

Since, $$\boxed{E = \frac{1}{2}\frac{Q^2}{C}} \quad \text{(energy stored)}$$

or, $$E = \frac{1}{2} \times \frac{\left(6 \times 10^{-3}\right)^2}{30 \times 10^{-6}}$$

$$= \frac{36}{60} = 0.6 \text{ J}.$$

As there is no loss of energy, the total energy remains the same.

7.9. *A series LCR circuit with R = 20 Ω, L = 1.5 H and C = 35 μF is connected to a variable-frequency 200 V ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?*

Sol. When the frequency of the *a.c.* supply is equal to the natural frequency, then

$z = R$ and $\phi = 0°$

∴ $z = 20\ \Omega$

$$I_v = \frac{E_v}{Z} = \frac{200}{20} = 10 \text{ A}$$

Average power transferred

$$P = E_v I_v \cos 0°$$
$$= 200 \times 10 \times 1 = 2000 \text{ watt}.$$

7.10. *A radio can tune over the frequency range of a portion of MW broadcast band: (800 kHz to 1200 kHz). If its LC circuit has an effective inductance of 200 μH, what must be the range of its variable capacitor?*

[Hint: For tuning, the natural frequency i.e., the frequency of free oscillations of the LC circuit should be equal to the frequency of the radiowave.]

Sol.
$$f_1 = 800 \text{ kHz} = 800 \times 10^3 \text{ Hz},$$
$$f_2 = 1200 \text{ kHz} = 1200 \times 10^3 \text{ Hz}$$
$$L = 200 \text{ μH} = 200 \times 10^{-6} \text{ H}$$

We know that the resonant frequency is given by

$$f = \frac{1}{2\pi\sqrt{LC}} \quad \text{or} \quad f^2 = \frac{1}{4\pi^2 LC}$$

or,
$$C = \frac{1}{4\pi^2 Lf^2}$$

Now,
$$C_1 = \frac{1}{4\pi^2 Lf_1^2} = \frac{49}{4\,(22)^2 \times 200 \times 10^{-6} \left(800 \times 10^3\right)^2}$$

$$C_1 = 197.73 \text{ pF.}$$

Similarly,
$$C_2 = \frac{1}{4\pi^2 Lf_2^2}$$

$$= \frac{49}{4\,(22)^2 \times 200 \times 10^{-6} \left(1200 \times 10^3\right)^2} F$$

or,
$$C_2 = 87.88 \text{ pF.}$$

The range of the variable condenser is from 87.88 pF to 197.73 pF.

7.11. *Figure shows a series LCR circuit connected to a variable frequency 230 V source. L = 5.0 H, C = 80 μF, R = 40 Ω.*

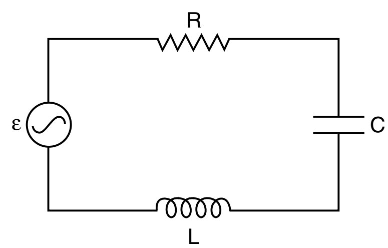

Fig. 7.26

(a) *Determine the source frequency which drives the circuit in resonance.*

(b) *Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.*

(c) *Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.*

Sol. Here,
$$L = 5.0 \text{ H}, \quad R = 40\ \Omega$$
$$C = 80 \text{ μF} = 80 \times 10^{-6} \text{ F}$$
$$E_v = 230 \text{ volt}$$
$$E_0 = \sqrt{0}\ E_v = \sqrt{2} \times 230 \text{ V}$$

(*a*) Resonance angular frequency,

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = \frac{1}{2 \times 10^{-2}} = 50 \text{ rad/sec.}$$

(*b*) Impedance $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

At resonance, $\omega L = \frac{1}{\omega C}$

$$Z = \sqrt{R^2} = R = 40\ \Omega$$

Amplitude of current at resonating frequency

$$I_0 = \frac{E_0}{z} = \frac{\sqrt{2} \times 230}{40} = 8.13 \text{ amp.}$$

$$I_v = \frac{I_0}{\sqrt{2}} = \frac{8.13}{\sqrt{2}} = 5.75 \text{ amp.}$$

(*c*) Potential drop across L

$$V_{L\,\text{rms}} = I_v\, \omega_r\, L = 5.75 \times 50 \times 5.0 = 1437.5 \text{ V}$$

Potential drop across R

$$V_{R\,\text{rms}} = I_v \times R = 5.75 \times 40 = 230 \text{ volt}$$

Potential drop across C

$$V_{C\,\text{rms}} = I_v \left(\frac{1}{\omega_r C}\right)$$

$$= 5.75 \times \frac{1}{50 \times 80 \times 10^{-6}}$$

$$= \frac{5.75}{4} \times 10^3 = 1437.5 \text{ V}$$

Potential drop across LC circuit

$$V_{LC\,\text{rms}} = V_{L\,\text{rms}} - V_{C\,\text{rms}} = 0$$

7.12. *An LC circuit contains a 20 mH inductor and a 50 μF capacitor with an initial charge of 10 mC. The resistance of the circuit is negligible. Let the instant the circuit is closed be t = 0.*

(a) What is the total energy stored initially? Is it conserved during LC oscillations?

(b) What is the natural frequency of the circuit?

(c) At what time is the energy stored

(i) completely electrical (i.e., stored in the capacitor)? (ii) completely magnetic (i.e., stored in the inductor)?

(d) At what times is the total energy shared equally between the inductor and the capacitor?

(e) If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat?

Sol. (*a*) Total initial energy

$$\boxed{E = \frac{Q_0^2}{2C}} = \frac{10^{-2} \times 10^{-2}}{2 \times 50 \times 10^{-6}} \text{ J} = 1 \text{ J}$$

This energy shall remain conserved in the absence of resistance.

(*b*) Angular frequency, $\omega = \frac{1}{\sqrt{LC}}$

$$= \frac{1}{\left(20 \times 10^{-3} \times 50 \times 10^{-6}\right)^{1/2}} \text{Hz} = 10^3 \text{ rad s}^{-1}.$$

$$v = \frac{10^3}{2\pi} \text{Hz} = 159 \text{ Hz}.$$

(*c*) $$Q = Q_0 \cos \omega t$$

or $$Q = Q_0 \cos \frac{2\pi}{T} t, \quad \text{where} \quad T = \frac{1}{v} = \frac{1}{159} \text{s} = 6.3 \text{ ms}$$

Energy stored is completely electrical at $t = 0, T/2, T, 3T/2, \ldots$

Electrical energy is zero *i.e.,* energy stored is completely magnetic at $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \ldots$

(*d*) At $$t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, \ldots \qquad \left[\because Q = Q_0 \cos \frac{\omega T}{8} = Q_0 \cos \frac{\pi}{4} = \frac{Q_0}{\sqrt{2}}\right]$$

$\therefore$ Electrical energy $= \frac{Q^2}{2C} = \frac{1}{2} \frac{Q_0^2}{2C}$, which is half of the total energy.

(*e*) R damps out the LC oscillations eventually. The whole of the initial energy 1.0 J is eventually dissipated as heat.

7.13. *A coil of inductance 0.50 H and resistance 100 Ω is connected to a 240 V 50 Hz ac supply.*

(a) What is the maximum current in the coil?

(b) What is the time lag between the voltage maximum and the current maximum?

Sol. Here,

$$L = 0.50 \text{ H}, \quad R = 100 \ \Omega$$

$$E_v = 240 \text{ V}, \quad f = 50 \text{ Hz}$$

$$\omega = 2\pi f = 100 \ \pi$$

$$E_0 = \sqrt{2} E_v = \sqrt{2} \times 240 \text{ V}$$

(*a*) $$I_0 = \frac{E_0}{\sqrt{R^2 + \omega^2 L^2}} = \frac{\sqrt{2} \times 240}{\sqrt{10^4 + (100\pi \times 0.5)^2}} = 1.82 \text{ A}$$

(*b*) In LR circuit,

If $$E = E_0 \cos \omega t, \quad I = I_0 \cos (\omega t - \phi)$$

At $t = 0$, $E = E_0$ *i.e.,* voltage is maximum.

At $t = \frac{\phi}{\omega}$, $I = I_0 \cos (\phi - \phi) = I_0 \times 1$, current is maximum

$\therefore$ Time lag between voltage maximum and current maximum $= \frac{\phi}{\omega}$

As $$\boxed{\tan \phi = \frac{\omega L}{R}} = \frac{2\pi \times 50 \times 0.50}{100}$$

$$= \frac{22}{7 \times 2} = 1.571$$

$$\phi = \tan^{-1} (1.571) = 57.5° = \frac{57.5\pi}{180} \text{ radian}$$

$\therefore$ $$\boxed{\text{Time lag} = \frac{\phi}{\omega}} = \frac{57.5\pi}{180 \times 2\pi f} = \frac{57.5}{180 \times 2 \times 50} = 3.19 \times 10^{-3} \text{ s}.$$

7.14. *Obtain the answers to (a) and (b) in Exercise 13, if the circuit is connected to a high frequency supply (240 V, 10 kHz). Hence explain statement that at very high frequency, an inductor in circuit nearly amounts to an open circuit. How does an inductor behave in a d.c. circuit after the steady state?*

Sol. Here, $L = 0.50 \text{ H}, \quad R = 100\ \Omega$

$V_{rms} = 240 \text{ V}, \quad f = 10 \text{ kHz} = 10^4 \text{ Hz}$

$\omega = 2\pi f = 2\pi \times 10^4 \text{ rad s}^{-1}$

Peak voltage, $V_0 = \sqrt{2}\ V_{rms} = \sqrt{2} \times 240 = 339.36 \text{ V}$

Maximum current,
$$I_0 = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}}$$
$$= \frac{339.36}{\sqrt{(100)^2 + \left(2\pi \times 10^4 \times 0.5\right)^2}} \text{A}$$
$$= \frac{339.36}{31416} \text{A} \qquad \text{(Neglecting } R\text{)}$$
$$= 0.01212 \text{ A} = 1.12 \times 10^{-2} \text{ A}$$

This current is much smaller than for the low frequency case (1.82 A in above question) showing that the inductive reactance is very large at high frequencies and L nearly amounts to an open circuit. In *d.c.* circuit (after steady state) $\omega = 0$.

$\therefore \qquad Z_L = \omega L = 0$

i.e., inductance L behaves like a pure inductor.

7.15. *A 100 μF capacitor in series with a 40 Ω resistance is connected to a 110 V, 60 Hz supply.*

(a) What is the maximum current in the circuit?

(b) What is the time lag between current maximum and voltage maximum?

Sol. Here, $C = 100\ \mu F = 100 \times 10^{-6} F = 10^{-4} \text{ F}, \quad R = 40\ \Omega.$

$E_v = 110 \text{ volt}, \quad E_0 = \sqrt{2}\ .\ E_v = \sqrt{2} \times 110 \text{ V}$

$v = 60 \text{ Hz.}, \quad \omega = 2\pi v = 120\pi \text{ rad/s}$

$I_0 = ?$

In RC circuit, as
$$\boxed{Z = \sqrt{R^2 + X_c^2}} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

$\therefore$
$$\boxed{I_0 = \frac{E_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}} = \frac{\sqrt{2} \times 110}{\sqrt{1600 + \frac{1}{\left(120\pi \times 10^{-4}\right)^2}}}$$

$I_0 = 3.24$ amp.

In RC circuit, voltage lags behind the current by phase angle ϕ,

where
$$\boxed{\tan\phi = \frac{1/\omega C}{R}} = \frac{1}{\omega CR} = \frac{1}{120\pi \times 10^{-4} \times 40} = 0.6628$$

$$\phi = \tan^{-1}(0.6628) = 33.5° = \frac{33.5\pi}{180} \text{ rad.}$$

$$\boxed{\text{Time lag} = \frac{\phi}{\omega}} = \frac{33.5\pi}{180 \times 120\pi} = 1.55 \times 10^{-3} \text{ sec.}$$

7.16. *Obtain the answers to (a) and (b) in Exercise 15, if the circuit is connected to 110 V, 12 kHz supply. Hence explain the statement that a capacitor is a conductor at very high frequencies. Compare this behaviour with that of a capacitor in d.c. circuit after the steady state.*

Sol. (*a*) For the high frequency,

$$\omega = 2\pi f = 2\pi \times 12 \times 10^3 \text{ rad s}^{-1}$$

$$\therefore \quad I_0 = \frac{E_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} = \frac{2E_v}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

or,

$$I_0 = \frac{\sqrt{2} \times 110}{\sqrt{1600 + \frac{1}{4\pi^2 \times 144 \times 10^6 \times 10^{-8}}}} \text{ A}$$

$$= \frac{1.414 \times 110}{\sqrt{1600 + 0.0176}} \text{ A} = \frac{1.414}{40} \text{ A}$$

$$= 3.9 \text{ A}$$

[It may be noted that the C term is negligible at higher frequencies.]

(*b*)

$$\tan\phi = \frac{1}{2\pi \times 12 \times 10^3 \times 10^{-4} \times 40} = \frac{1}{96\pi}$$

ϕ is nearly zero at high frequency.

It is clear from here that at high frequency, C acts like a conductor. For a *D.C.* circuit, after steady state has been reached, $\omega = 0$ and C amounts to an open circuit.

7.17. *Keeping the source frequency equal to the resonating frequency of the series LCR circuit, if the three elements L, C and R are arranged in parallel, show that the total current in the parallel LCR circuit is minimum at this frequency. Obtain the current rms value in each branch of the circuit for the elements and source specified in Exercise 11 for this frequency.*

Sol.

$$V_{rms} = 230 \text{ V}, \quad L = 5.0 \text{ H}$$

$$C = 80\ \mu\text{F} = 80 \times 10^{-6} \text{ F}, \quad R = 40\ \Omega$$

Using relation,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}}$$

$$= \frac{1}{\sqrt{400 \times 10^{-6}}} = 50 \text{ rad s}^{-1}$$

Since elements are in parallel, reactance X of L and C in parallel is given by

$$\frac{1}{X} = \frac{1}{\omega L} - \frac{1}{1/\omega C} = \frac{1}{\omega L} - \omega C$$

Impedance of R and X in parallel is given by

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2}+\frac{1}{X^2}} = \sqrt{\frac{1}{R^2}+\left(\frac{1}{\omega L}-\omega C\right)^2}$$

or,
$$\frac{1}{Z} = \frac{\sqrt{1+R^2\left(\frac{1}{\omega L}-\omega C\right)^2}}{R}$$

$$Z = \frac{R}{\left[1+R^2\left(\frac{1}{\omega L}-\omega C\right)^2\right]}$$

which is less than resistance R. At resonant frequency,

$$\omega L = \frac{1}{\omega C} \quad \text{or} \quad \omega C = \frac{1}{\omega L}$$

and
$$\left(\frac{1}{\omega L}-WC\right) = 0$$

Then, impedance $Z = R$ and will be maximum. Hence, current will be minimum at resonant frequency in the parallel LCR circuit. From Ex. 11 $L = 5H.$; $C = 80 \times 10^{-6}$ F, $R = 40\ \Omega$

$$E_{\text{rms}} = 230 \text{ V.}$$

$$(I_R)_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{230}{40} = 5.75 \text{ A.}$$

$$(I_L)_{\text{rms}} = \frac{V_{\text{rms}}}{\omega L} = \frac{230}{50\times 5} = 0.92 \text{ A.}$$

$$(I_C)_{\text{rms}} = \frac{V_{\text{rms}}}{1/\omega C} = 230 \times 50 \times 80 \times 10^{-6} = 0.92 \text{ A.}$$

Current through L and C will be in opposite phase, hence, I_{rms} in circuit will be only 5.75 A. $\left(= \frac{V_{\text{rms}}}{R}\right)$ as circuit impedance will be equal to R only.

7.18. *A circuit containing a 80 mH inductor and a 60 μF capacitor in series is connected to a 230 V, 50 Hz supply. The resistance of the circuit is negligible.*

(a) Obtain the current amplitude and rms value.
(b) Obtain the rms values of potential drop across each element.
(c) What is the average power transferred to the inductor?
(d) What is the average power transferred to the capacitor?
(e) What is the total average power absorbed by the circuit? ['Average' implies 'averaged over one cycle'.]

Sol. Here,
$$L = 80 \text{ mH} = 80 \times 10^{-3} \text{ H}$$
$$C = 60\ \mu\text{F} = 60 \times 10^{-6} \text{ F}, \quad R = 0$$
$$E_v = 230 \text{ V}, \quad E_0 = \sqrt{2}\times E_v = \sqrt{2}\times 230 \text{ V}$$
$$f = 50 \text{ Hz}, \quad \omega = 2\pi f = 100\ \pi \text{ rad/s}$$

(*a*)
$$I_0 = ?\ I_v = ?$$

$$I_0 = \frac{E_0}{\left(\omega L - \frac{1}{\omega L}\right)} = \frac{230\sqrt{2}}{\left(100\ \pi\times 80\times 10^{-3} - \frac{1}{100\ \pi\times 60\times 10^{-6}}\right)}$$

$$= \frac{230\sqrt{2}}{\left(8\pi - \frac{1000}{6\pi}\right)} = \frac{230\sqrt{2}}{-27.91} = -11.63 \text{ amp.}$$

$$I_v = \frac{I_0}{\sqrt{2}} = \frac{-11.63}{1.414} = -8.23 \text{ amp.}$$

Negative sign appears as $\omega L < \frac{1}{\omega C}$.

$\therefore$ e.m.f. lags behind the current by 90°

(*b*) Across L, $V = I_v \times \omega L = 8.23 \times 100\,\pi \times 80 \times 10^{-3} = 206.74$ volt.

Across C, $V = I_v \times \frac{1}{\omega C} = 8.23 \times \frac{1}{100\,\pi \times 60 \times 10^{-6}} = 436.84$ volt.

As voltages across L and C are 180° out of phase, therefore, they get subtracted.

That is why applied *r.m.s.* voltage = 436.84 – 206.74 = 230.1 volt.

(*c*) Average power transferred over a complete cycle by the source to inductor is always zero because of phase difference of $\pi/2$ between voltage and current through L.

(*d*) Average power transferred over a complete cycle by the source to the capacitor is also zero because of phase difference of $\pi/2$ between voltage and current through C.

(*e*) Total average power absorbed by the circuit is also, therefore zero.

7.19. *Suppose the circuit in Question 7.18 has a resistance of 15 Ω. Obtain the average power transferred to each element of the circuit, and the total power absorbed.*

Sol. Here, $R = 15\ \Omega$

$\therefore$ Impedance, $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

or, $Z = \sqrt{15^2 + \left(2\pi \times 50 \times 80 \times 10^{-3} - \frac{1}{2\pi \times 50 \times 60 \times 10^{-6}}\right)^2}$

$$= \sqrt{225 + 779.5}\ \Omega = 31.7\ \Omega$$

$\therefore$ $I_{rms} = \frac{V_{rms}}{Z} = \frac{230}{31.7} = 7.255$ A

Average power transferred to $L = E_v I_v \cos\frac{\pi}{2} = 0$

Average power transferred to $C = E_v I_v \cos\left(\frac{-\pi}{2}\right) = 0$

Average power transferred to $R = I^2_{rms} \times R$

$= (7.255)^2 \times 15$ W

$= 789.5$ W.

7.20. *A series LCR circuit with L = 0.12 H, C = 480 nF, R = 23 Ω is connected to a 230 V variable frequency supply.*

(a) What is the source frequency for which current amplitude is maximum. Obtain this maximum value.

(b) *What is the source frequency for which average power absorbed by the circuit is maximum? Obtain the value of this maximum power.*

(c) *For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency? What is the current amplitude at these frequencies?*

(d) *What is the Q-factor of the given circuit?*

Sol. (*a*) Here, $L = 0.12$ H, $R = 23\ \Omega$, $C = 480$ nF $= 480 \times 10^{-9}$ F

$$E_v = 230 \text{ volt}, \quad E_0 = \sqrt{2}\, E_v = \sqrt{2} \times 230 \text{ volt.}$$

$$I_0 = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

I_0 would be maximum, when

$$\omega_r = \omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.12 \times 480 \times 10^{-9}}} = 4166.7 \text{ rad s}^{-1}$$

$$I_0 = \frac{E_0}{R} = \frac{\sqrt{2} \times 230}{23} = 14.14 \text{ amp.}$$

(*b*) Average power absorbed by the circuit is maximum, when $I = I_0$

$$P_{av} = \frac{1}{2} I_0^2 R = \frac{1}{2} (14.14)^2 \times 23 = 2299.3 \text{ watt}$$

(*c*) The two angular frequencies for which the power transferred to the circuit is half the power at the resonant frequency,

$$\boxed{\omega = \omega_r \pm \Delta\omega}$$

When $\boxed{\Delta\omega = \frac{R}{2L}} = \frac{23}{2 \times 0.12} = 95.83 \text{ rad s}^{-1}$

$\therefore$ angular frequencies at which power transferred is half $= \omega_r \pm \Delta\omega$

$$= 4166.7 \pm 95.83 = 4262.3 \text{ and } 4070.87 \text{ rad s}^{-1}$$

current amplitude at these frequencies is

$$\frac{I_0}{\sqrt{2}} = \frac{14.14}{1.414} = 10 \text{ A.}$$

(*d*) $$\text{Q-factor} = \frac{\omega_r L}{R} = \frac{4166.7 \times 0.12}{23} = 21.74.$$

7.21. *Obtain the resonant frequency and Q-factor of a series LCR circuit with L = 3.0 H, C = 27 μF, and R = 7.4 Ω. It is desired to improve the sharpness of the resonance of the circuit by reducing its 'full width at half maximum' by a factor of 2. Suggest a suitable way.*

Sol. Given, $L = 3.0$ H, $C = 27\ \mu$F $= 27 \times 10^{-6}$ F

$R = 7.4\ \Omega$

Resonant frequency,

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{3.0 \times 27 \times 10^{-6}}} = 111 \text{ rad s}^{-1}$$

Q-factor of the circuit,

$$Q = \frac{\omega_r L}{R} = \frac{111 \times 3.0}{7.4} = 45$$

For improvement in sharpness of resonance by a factor of 2, Q should be doubled. To double Q with changing ω_r, R should be reduced to half, *i.e.,* to 3.7 Ω.

7.22. *Answer the following questions:*

(*a*) *In any a.c. circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit? Is the same true for rms voltage?*

(*b*) *A capacitor is used in the primary circuit of an induction coil.*

(*c*) *An applied voltage signal consists of a superposition of a dc voltage and an a.c. voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the dc signal will appear across C and the ac signal across L.*

(*d*) *A choke coil in series with a lamp is connected to a dc line. The lamp is seen to shine brightly. Insertion of an iron core in the choke causes no change in the lamp's brightness. Predict the corresponding observations if the connection is to an a.c. line.*

(*e*) *Why is choke coil needed in the use of fluorescent tubes with ac mains? Why can we not use an ordinary resistor instead of the choke coil?*

Sol. (*a*) Yes, the applied instantaneous voltage is equal to the algebraic sum of the instantaneous voltages across the series elements because the voltage variations across each element will follow the variations of the supply voltage at all instants. But this is not true for rms voltage because voltage across different elements may not be in phase.

(*b*) When the circuit is broken, the large induced voltage is used up in charging the capacitor. Thus sparking etc. is avoided.

(c) For high frequency, the inductive reactance for *a.c.,* $X_L = \omega L = \infty$ and capacitance of reactance $X_C = \frac{1}{\omega C} = 0$. Hence, capacitor does not offer any resistance for a.c. Thus *a.c.* components of voltage appears across L only.

Consequently, X_L for *d.c.,* $X_L = \omega L = 0$ and

$$X_C = \frac{1}{\omega C} = \infty$$

Therefore, *d.c.* components of voltage appears across C only.

(*d*) For a steady state *d.c.,* L has no effect even if it is increased by an iron core. For *a.c.,* the lamp will shine dimly because of additional impedance of the choke. It will dim further when the iron core is inserted which increases the choke's impedance.

(*e*) A choke coil is needed in the use of fluorescent tubes to reduce *a.c.* without loss of power, if we use an ordinary resistor, a.c. will reduce, but if loss of power due to heating will be there.

Power dissipated $= E_v I_v \cos \phi$

In a resistor, $\phi = 0°$

∴ Power dissipated $= E_v I_v \cos 0°$

$= E_v I_v = \text{max}$

In a choke coil, $\phi = 90°$

∴ Power dissipated $= E_v I_v \cos 90° = \text{zero}.$

7.23. *A power transmission line feeds input power at 2300 V to a step-down transformer with its primary windings having 4000 turns. What should be the number of turns in the secondary in order to get output power at 230 V?*

Sol. $V_1 = 2300$ volt $\quad n_1 = 4000$

$V_2 = 230$ volt

$$\frac{V_2}{V_1} = \frac{n_2}{n_1}$$

or, $$n_2 = n_1 \frac{V_2}{V_1} = 4000 \times \frac{230}{2300} = 400 \text{ turns.}$$

7.24. *At a hydroelectric power plant, the water pressure head is at a height of 300 m and the water flow available is 100 m³s⁻¹. If the turbine generator efficiency is 60%, estimate the electric power available from the plant (g = 9.8 ms⁻²).*

Sol. Here, $h = 300$ m

Volume of the water flowing per second = 100 m^3

Mass of water flowing per second,

$$m = 100 \times 10^3 \text{ kg} = 10^5 \text{ kg}$$

$$g = 9.8 \text{ ms}^{-2}$$

Potential energy of water fall during one second

$$= mgh = 10^5 \times 9.8 \times 300$$

$$= 29.4 \times 10^7 \text{ J.}$$

$$\text{Input power} = 294 \times 10^6 \text{ Js}^{-1}$$

$$\text{Efficiency, } \eta = \frac{\text{output power}}{\text{input power}}$$

$$\therefore \quad \text{Output power} = \eta \times \text{input power}$$

$$= 0.6 \times 294 \times 10^6$$

$$= 176.4 \times 10^6 \text{ watt} = 176.4 \text{ MW.}$$

7.25. *A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric plant generating power at 440 V. The resistance of the two wire line carrying power is 0.5 Ω per km. The town gets power from the line through a 4000-220 V step-down transformer at a sub-station in the town.*

(a) Estimate the line power loss in the form of heat.

(b) How much power must the plant supply, assuming there is negligible power loss due to leakage?

(c) Characterise the step-up transformer at the plant.

Sol. Power required ⇒ $P = 800 \text{ kW} = 800 \times 10^3$ W

Total resistance of two wire lines

$$R = 2 \times 15 \times 0.5 = 15\ \Omega.$$

Since supply is through 4000 – 230 V transformer

$$\therefore \quad E_v = 4000 \text{ volt}$$

As $$P = E_v I_v$$

$$\therefore \quad 800 \times 10^3 = 4000\ I_v$$

$$I_v = \frac{800 \times 10^3}{4000} = 200 \text{ amp}$$

(*a*) Line power loss in the form of heat

$$= I_v^2 R$$

$$= (200)^2 \times 15 = 60 \times 10^4 \text{ watt.}$$

$$= 600 \text{ kW.}$$

(*b*) If there is no power loss due to leakage,
then the essential plant supply = 800 + 600 = 1400 kW

(*c*) Voltage drop on the line = $I_v R$

$$= 200 \times 15 = 3000 \text{ volt}$$

∴ Voltage from transmission = 3000 + 4000 = 7000 V

Since the power is generated at 440 volt, the step-up transformer needed at the plant is 440 V – 7000 V.

7.26. *Do the same exercise as above with the replacement of the earlier transformer by a 40,000-200 V step-down transformer (Neglect, as before, leakage losses though this may not be a good assumption any longer because of the very high voltage transmission involved). Hence, explain why high voltage transmission is preferred?*

Sol. The rms current in the two-wire line

$$= \frac{800 \times 10^3 \text{ W}}{40000 \text{ V}} = 20 \text{ A}$$

(*a*) Line power loss = $I_v^2 R = (20)^2 \times 15 = 6000$ W = 6 kW

(*b*) Power supplied by the plant = 800 + 6 = 806 kW

(*c*) Voltage drop on the line = 20 × 15 = 300 V

Voltage output of the step-up transformer at the plant = 40000 + 300 = 40300 V

∴ The step-up transformer at the plant is 440 V – 40300 V

Power loss is (exercise 25) = $\frac{600}{1400} \times 100 = 43\%$

Power loss in this exercise = $\frac{6}{806} \times 100 = 0.74\%$

MORE QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *The instantaneous current and voltage of an a.c. circuit are given by*

$$i = 10 \sin 314\, t \text{ A and}$$
$$v = 50 \sin (314\, t + \pi/2) \text{ V.}$$

What is the power dissipation in the circuit?

Ans. Given,

$$i = 10 \sin 314\, t \text{ A}$$
$$V = 50 \sin (314\, t + \pi/2) \text{ V}$$

$$\therefore \quad i_0 = 10, \quad V_0 = 50 \quad \text{and} \quad \phi = \frac{\pi}{2}$$

Power dissipation in the circuit is given by

$$\boxed{P = E_v I_v \cos \phi}$$

$$E_v = \frac{E_0}{\sqrt{2}} \quad V_0 = \frac{V_0}{\sqrt{2}}$$

$$P = \left(\frac{50}{\sqrt{2}}\right)\left(\frac{10}{\sqrt{2}}\right)\cos\frac{\pi}{2}$$

$$= 0 \qquad \left[\because \cos\frac{\pi}{2} = 0\right]$$

Q. 2. *The number of turns in secondary coil of a transformer is 100 times the number of turns in the primary coil. What is the transformation ratio?*

Ans. Transformation ratio

$$\Rightarrow \quad k = \frac{N_s}{N_p}$$

Since $Ns = 100 \times N_p$

Thus, $k = \frac{100\,N_p}{N_p} = 100.$

Q. 3. *What do you mean by power factor? On what factors does it depend?*

Ans. The power factor is defined as the cosine of the phase angle between alternating e.m.f. and current in an *a.c.* circuit.

Power factor of *a.c.* circuit is given by

$$\cos\phi = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

It depends upon the frequency of the *a.c.* source.

Q. 4. *In a transformer with transformation ratio 0.1, 220 volt a.c. is fed to primary. What voltage is obtained across the secondary?*

Ans. Transformation ratio $= \frac{N_s}{N_p} = \frac{E_s}{E_p}$

$$0.1 = \frac{E_s}{220} \quad \therefore \quad E_s = 22 \text{ volt.}$$

Q. 5. *When a lamp is connected to an alternating voltage supply, if lights with the same brightness as when connected to a 12 V DC battery. What is the peak value of alternating voltage source?*

Ans. $V_{\text{rms}} = 12 \text{ V}$

$$V_{eff.} = V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$$

So, $V_0 = \sqrt{2}\; V_{\text{rms}} = 1.414 \times 12 = 16.97$ V.

Q. 6. *The number of turns in the secondary coil of a transformer is 500 times that in primary. What power is obtained from the secondary when power fed to the primary is 10 W?*

Ans. If there is no loss of energy, then the output power will be 10 W.

Q. 7. *What is the power dissipated in an a.c. circuit in which voltage and current are given by*

$$V = 230 \sin(\omega t + \pi/2) \quad \text{and} \quad I = 10 \sin \omega t?$$

Ans. Phase difference between V and $I = \pi/2$

$\therefore \qquad P_{av} = V_{eff} \, I_{eff} \cos \phi$

$$= V_{eff} \, . \, I_{eff} \cos \frac{\pi}{2} = 0 \qquad \left[\cos \frac{\pi}{2} = 0\right]$$

Q. 8. *A small dc motor operates at 110 V dc. What is back emf when its efficiency is maximum?*

Ans. $E = \frac{110}{2} \text{ V} = 55 \text{ V}.$

Q. 9. *In a series LCR circuit, the voltage across an inductor, capacitor and resistor are 20 V, 20 V and 40 V respectively. What is the phase difference between the applied voltage and the current in the circuit?*

Ans.

$$\boxed{\tan \phi = \frac{V_L - V_C}{V_R}}$$

$$= \frac{20 - 20}{40} = 0$$

$\therefore$ Phase difference $\phi = 0°$.

Q. 10. *In an L-R circuit reactance and resistance are equal. Calculate phase by which voltage differ current?*

Ans.

$$\boxed{\tan \phi = \frac{X_L}{R}} \Rightarrow \phi = 45°$$

Q. 11. *Find the capacitance of the capacitor that have a reactance of 100 Ω when used with an a.c source of frequency 5/π kHz.*

Ans.

$$\boxed{X_C = \frac{1}{\omega C}}$$

$$\Rightarrow \qquad C = \frac{1}{\omega X_C} = \frac{1}{2\pi f X_C}$$

or,

$$C = \frac{1}{2\pi \times \frac{5}{\pi} \times 1000 \times 100} = 1 \, \mu\text{F}.$$

Q. 12. *Power factor can often be improved by the use of capacitor of appropriate capacitance in the circuit. Justify.*

Ans. Power factor $\cos \theta = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$. As the value of C is changed, the value of Z also changes, hence power factor can be improved with the help of appropriate capacitance in the circuit.

Q. 13. *A bulb and a capacitor are connected in series to an a.c. source of variable frequency. How will the brightness of the bulb change on increasing the frequency of the a.c. source? Give reason.*

Ans. As the frequency of the a.c. source increases, the capacitive reactance decreases ($X_C \propto 1/f$). More current flows through the circuit. So the bulb glows with more brightness.

Q. 14. *The power factor of an a.c. circuit is 0.5. What will be the phase difference between voltage and current in this circuit?*

Ans. Power factor, $\cos \phi = 0.5 = \frac{1}{2} = \cos 60°$.

The phase difference between voltage an current is 60° or $\frac{\pi}{3}$ radian.

Q. 15. *Peak value of emf of an a.c. source is E0. What is its rms value?*

Ans. $$E_{rms} = \frac{E_0}{\sqrt{2}}$$

Q. 16. *The peak value of an a.c. circuit supply is 300 V. What is the r.m.s. voltage?*

Ans. r.m.s. $$\text{voltage} = \frac{\text{Peak voltage}}{\sqrt{2}}$$
$$= \frac{300}{\sqrt{2}} = 221.2 \text{ V.}$$

Q. 17. *Is a motor starter a variable R or L or?*

Ans. Variable *R*.

Q. 18. *What is iron loss in a transformer?*

Ans. It is loss of energy in the form of heat in iron core of a transformer.

Q. 19. *What is the phase difference between voltage across an inductor and a capacitor in an a.c. circuit?*

Ans. 180°.

Q. 20. *What happens to the power dissipation if the value of electric current passing through a conductor of constant resistance is doubled?*

Ans. When current is doubled, the dissipation increases four times because

$$P \propto I^2.$$

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. (*i*) *State the law that gives the polarity of the induced emf.*

(*ii*) *A 15.0 μF capacitor is connected to 220 V, 50 Hz source. Find the capacitive reactance and the rms current.*

Ans. (*i*) Faraday's law of electromagnetic induction:

→ Induced e.m.f. is produced whenever magnetic flux linked with a circuit changes.

→ As long as the change in the magnetic flux continues, the induced e.m.f. lasts.

→ The magnitude of the induced e.m.f. is directly proportional to the rate of change of the magnetic flux linked with the circuit.

(*ii*) Given, $C = 15.0 \ \mu\text{F} = 15 \times 10^{-6}$ F

$V = 220$ V, $f = 50$ Hz

Capacitive reactance, $X_C = \frac{1}{2\pi fc}$

or $$X_C = \frac{1}{2 \times 3.14 \times 50 \times 15 \times 10^{-6}}$$
$$= \frac{10^6}{314 \times 15} = \frac{1000000}{4710} = 212.31 \ \Omega$$

$$\therefore \quad I_{rms} = \frac{V}{X_C} = \frac{220}{212.31} = 1.03 \text{ A.}$$

Q. 2. *An a.c. voltage, V = Vm sin ωt, is applied across a*

(i) series RC circuit in which the capacitative impedance is 'a' times the resistance in the circuit.

(ii) series RL circuit in which the inductive impedance is 'b' times the resistance in the circuit. Calculate the value of the power factor of the circuit in each case.

Ans. (*i*) Power factor in *RC* circuit is

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_C^2}}$$

Here $X_C = aR$

$$\therefore \quad \cos\phi = \frac{R}{\sqrt{R^2 + a^2R^2}} = \frac{1}{\sqrt{1+a^2}}$$

(*ii*) Power factor in *RL* circuit is

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_L^2}}$$

Here, $X_L = bR$

$$\therefore \quad \cos\phi = \frac{R}{\sqrt{R^2 + b^2R^2}} = \frac{1}{\sqrt{1+b^2}}.$$

Q. 3. *An inductor 200 μH, capacitor 500 μF, resistor 10 Ω are connected in series with a 100 V, variable frequency a.c. source. Calculate the*

(i) frequency at which the power factor of the circuit is unity

(ii) current amplitude at this frequency

(iii) Q-factor

Ans. (*i*) Power foactor

$$\cos\theta = \frac{R}{Z} = 1$$

So $R = Z$

$$R = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$R^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC} \quad \text{or} \quad \omega = \frac{1}{\sqrt{LC}}$$

$$2\pi\nu = \frac{1}{\sqrt{LC}}$$

$$\nu = \frac{1}{2\pi\sqrt{LC}}$$

$$\omega_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{\sqrt{200\times10^{-6}\times500\times10^{-6}}} = 3.16 \times 10^{-3} \text{ rad s}^{-1}$$

$$\omega_0 = 3.16 \times 10^{-3} \text{ rad/s}$$

(ii) The current amplitude at this frequency, $I_0 = \frac{V}{R} = \frac{100}{10} = 10$ A

(iii) The Q-factor, $Q = \frac{X_L}{R} = \frac{\omega_0 L}{R} = \frac{3.16 \times 10^{-3} \times 200 \times 10^{-6}}{10} = 6.32 \times 10^{-8}$

Q. 4. *Prove that an ideal inductor does not dissipate power in an a.c. circuit.*

Or

Derive an expression for the self-inductance of a long air cored solenoid of length l and number of turns N.

Ans. The instantaneous EMF is given by $E = E_0 \sin \omega t$

The instantaneous current in the inductor is given by $I = I_0 \cos \omega t$

The instantaneous power in the inductor is given by

$$P = EI = E_0 I_0 \cos \omega t \sin \omega t = \frac{E_0 I_0}{2} \sin 2\omega t$$

The average power over the complete cycle $P_{av} = \frac{E_0 I_0}{2} \langle \sin 2\omega t \rangle = 0$

Hence, an ideal inductor does not dissipate power.

Q. 5. *State the condition under which the phenomenon of resonance occurs in a series LCR circuit. Plot a graph showing variation of current with frequency of a.c. source in a series LCR circuit.*

Ans. Resonance occurs in a series of LCR circuit when

$$X_L = X_C.$$

The graph showing the variations of current with frequency of $a.c.$ source in a series LCR circuit is given below.

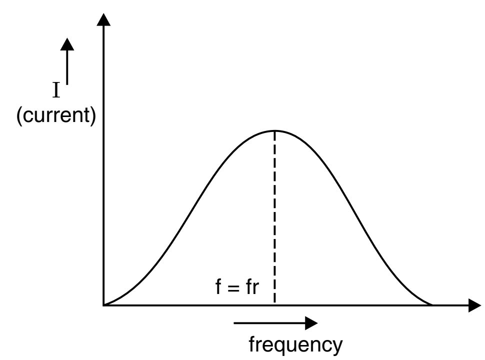

Fig. 7.27

Q. 6. *An alternating voltage of frequency f is applied across a series LCR circuit. Let fr be the resonance frequency for the circuit. Will the current in the circuit lag, lead or remain in phase with the applied voltage when (i) f > fr, (ii) f < fr? Explain your answer in each case.*

Ans. $X_L = 2\pi f L$ and $X_C = \frac{1}{2\pi f C}$

(i) When $f > f_r$, X_L is large and X_C is small. The circuit is inductive. So current lags behind the applied voltage.

(ii) When $f < f_r$, X_L is small and X_C is large.

The circuit is capacitive. The current leads the voltage in phase.

Q. 7. *In the following circuit, calculate,*

(i) the capacitance 'c' of the capacitor if the power factor of the circuit is unity, and

(ii) also calculate the Q-factor of the circuit.

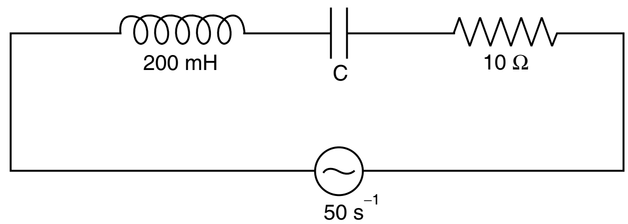

Fig. 7.28

Ans. (*i*) Power factor, $\cos \phi = \dfrac{R}{Z}$ or $Z = R$ [For power factor unity $\cos \theta = 1$]

$\therefore \quad X_C = X_L$ or $\dfrac{1}{2\pi f C} = 2\pi f L$

or, $$C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4 \times 9.87 \times (50)^2 \times 200 \times 10^{-3}}$$

$$= 5 \times 10^{-5} \text{ F}$$

or, $C = 50\ \mu\text{F}$.

(*ii*) $$\text{Q-factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q = \frac{1}{10}\sqrt{\frac{200 \times 10^{-3}}{5 \times 10^{-5}}} = 6.32.$$

Q. 8. *A capacitor and a resistor are connected in series with an a.c. source. If the potential difference across C, R are 120 V, 90 V respectively and if the r.m.s. current of the circuit is 3A, calculate the (i) impedance, (ii) power factor of the circuit.*

Ans. Given, $I_{\text{rms}} = 3$ A

$V_R = 90$ V, $V_C = 120$ V

$$E_{\text{rms}} = \sqrt{V_R^2 + V_C^2}$$

$$= \sqrt{90^2 + 120^2} = \sqrt{22500} = 150 \text{ V}$$

(*i*) Impedance, $$Z = \frac{E_{\text{rms}}}{I_{\text{rms}}} = \frac{150}{3} = 50\ \Omega$$

(*ii*) Power factor, $$\cos \phi = \frac{V_R}{E_{\text{rms}}} = \frac{90}{150} = 0.6$$

Q. 9. *If the voltage in a.c. circuit is represented by the equation,*

$$V = \mathbf{220\sqrt{2}}\ \sin (314\ t - \phi)$$

Calculate (a) peak and rms value of the voltage, (b) average voltage, (c) frequency of a.c.

Ans. (*a*) For *a.c.* voltage,

$$V = V_0 \sin (\omega t - \phi)$$

The peak value

$$V_0 = 220\sqrt{2} = 311 \text{ V}$$

The rms value of voltage

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}; \quad V_{\text{rms}} = 220 \text{ V}$$

(*b*) Average voltage in full cycle is zero. Average voltage in half cycle is

$$V_{av} = \frac{2}{\pi} V_0 = \frac{2}{\pi} \times 311 = 198.17 \text{ V}$$

(c) As $\omega = 2\pi f, \quad 2\pi f = 314$

i.e., $f = \dfrac{314}{2 \times \pi} = 50 \text{ Hz}$

Q. 10. *Distinguish between the terms 'effective value' and 'peak value' of an alternating current. An alternating current from a source is represented by*

$$I = 10 \sin (314\, t)$$

Write the corresponding values of:

(i) its 'effective value'

(ii) frequency of the source.

Ans. Effective value of *a.c.* : The value of direct current which produces the same heating effect in a given resistor as is produced by the given alternating current when passed for the same time is termed as effective value of *a.c.*

Peak value of *a.c.* : The maximum value attained by an alternating current in either of its half cycle is called its peak value.

$$I_{eff} = \frac{I_0}{\sqrt{2}}$$

Given $I = 10 \sin (314\, t)$

Comparing with $I = I_0 \sin 2\pi f t$, we get

$$I_0 = 10 \text{ A}$$

(*i*) $I_{\text{eff}} = \dfrac{I_0}{\sqrt{2}} = 0.707 \times 10 = 7.07 \text{ A}.$

(*ii*) $2\pi f = 314$

$$f = \frac{314}{2\pi} = \frac{314}{2 \times 3.14} = 50 \text{ H}_z.$$

Q. 11. *A potential of* $E = 50 \sin \left(200\, \pi t + \dfrac{\pi}{4}\right)$ *is applied across a resistor of 10 Ω resistance. Find*

(i) rms value of potential

(ii) frequency of a.c

(iii) initial phase

(iv) rms value of current

Ans. (*i*) $E_{RMS} = \dfrac{E_0}{\sqrt{2}} = \dfrac{50}{\sqrt{2}} = 25\sqrt{2} \text{ V}$

(*ii*) Frequency, $v = \frac{\omega}{2\pi} = \frac{200\pi}{2\pi} = 100\ \text{H}_z$

(*iii*) Initial phase = $\frac{\pi}{4}$

(*iv*) $I_{RMS} = \frac{1}{\sqrt{2}}\frac{E_0}{R} = \frac{1}{\sqrt{2}} \times \frac{50}{10} = 5\sqrt{2}\ \text{A}$

Q. 12. *An a.c. voltage E = E0 sin ωt is applied across an inductor L. Obtain an expression for current I.*

Ans. Let a pure inductance *L* connected across a source of alternating emf given by

$$E = E_0 \sin \omega t$$

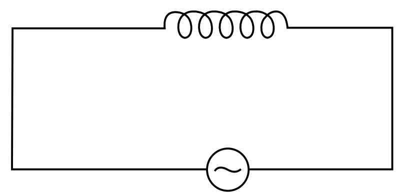

Fig. 7.29

The rate of change of current in the circuit is $\frac{dI}{dt}$. (If *I* is the current through the circuit).

∴ Instantaneous induced emf across inductance = $-L\frac{dI}{dt}$ (By Kirchhoff's law)

∴ The net emf is

$$E - L\frac{dI}{dt} = 0$$

$$E = L\frac{dI}{dt}$$

or, $$dI = \frac{E}{L}dt = \frac{E_0}{L}\sin \omega t\ dt$$

or, $$\int dI = \frac{E_0}{L}\int \sin \omega t\ dt$$

or, $$I = -\frac{E_0}{\omega L}\cos \omega t = -I_0 \cos \omega t$$

where $I_0 = \frac{E_0}{\omega L}$ ⇒ Peak value of alternating current.

Here ωL has the units of resistance and it is called inductive reactance. It is denoted by X_L.

∴ $$I = -I_0 \cos \omega t = I_0 \sin(\omega t - \pi/2).$$

Q. 13. *A resistor of resistance R, an inductor of inductance L and a capacitor of capacitance C all are connected in series with an a.c. supply. The resistance of R is 16 ohm and for a given frequency, the inductive reactance of L is 24 ohm and capacitive reactance of C is 12 ohm. If the current in the circuit is 5 amp., find*

(a) the potential difference across R, L and C

(b) the impedance of the circuit

(c) the voltage of a.c. supply

(d) phase angle

Ans. (*a*) Potential difference across resistance

$$V_R = iR = 5 \times 16 = 80 \text{ volt}$$

Potential difference across inductance

$$V_L = i \times (\omega L) = 5 \times 24 = 120 \text{ volt}$$

Potential difference across condenser

$$V_C = i \times \left(\frac{1}{\omega C}\right) = 5 \times 12 = 60 \text{ volt}$$

(*b*) The impedance of the circuit is given as

$$Z = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$$

$$= \sqrt{\left[(16)^2 + (24 - 12)^2\right]} = 20 \text{ ohm}$$

(*c*) The voltage of *a.c.* supply is given by

$$V = iz = 5 \times 20 = 100 \text{ volt}$$

(*d*) Phase angle

$$\phi = \tan^{-1}\left[\frac{\omega L - \left(\frac{1}{\omega C}\right)}{R}\right]$$

$$= \tan^{-1}\left[\frac{24 - 12}{16}\right]$$

$$= \tan^{-1}(0.75) = 36°46'.$$

Q. 14. *A series circuit consists of a resistance of 15 ohms, an inductance of 0.08 henry and a condenser of capacity 30 microfarad. The applied voltage has a frequency of 500 radian/s. Does the current lead or lag the applied voltage and by what angle.*

Ans. Here, $X_L = \omega L = 500 \times 0.08 = 40 \text{ ohm}$

and $$\frac{1}{\omega C} = \frac{1}{500 \times \left(30 \times 10^{-6}\right)}$$

$$X_C = 66.7 \text{ ohm}$$

$$\tan\phi = \frac{\left[\omega L - \left(\frac{1}{\omega C}\right)\right]}{R}$$

$$= \frac{40 - 66.7}{15} = -1.78$$

$$\phi = -60.65°$$

Thus the current leads the applied voltage by 60.65°.

Q. 15. *A 100 volt a.c. source of frequency 500 hertz is connected to LCR circuit with L = 8.1 millihenry, C = 12.5 micro farad and R = 10 ohm, all connected in series. Find the potential difference across the resistance.*

Ans. The impedance of *LCR* circuit is given by

$$Z = \sqrt{\left[R^2 + (X_L - X_C)^2\right]}$$

where $X_L = \omega L = 2\pi f L$

$= 2 \times 3.14 \times 500 \times (8.1 \times 10^{-3})$

$= 25.4$ ohm

and $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

$$= \frac{1}{2 \times 3.14 \times 400 \times \left(12.5 \times 10^{-6}\right)}$$

$= 25.4$ ohm

$\therefore \quad Z = \sqrt{\left[(10)^2 + (25.4 - 25.4)^2\right]} = 10$ ohm

$\therefore \quad I_{rms} = \dfrac{E_{rms}}{Z} = \dfrac{100 \text{ volt}}{10 \text{ ohm}} = 10$ amp.

Potential difference across resistance

$V_R = I_{rms} \times R = 10$ amp $\times$ 10 ohm

$= 100$ volt.

Q. 16. *An LCR series circuit with 100 Ω resistance is connected to an a.c. source of 200 V and angular frequency 300 radians per second. When only the capacitance is removed, the current lags behind the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°. Calculate the current and power dissipated in LCR circuit.*

Ans. $\tan 60° = \dfrac{\omega L}{R}$

or, $\tan 60° = \dfrac{1/\omega C}{R}$

$\because \quad \omega L = \dfrac{1}{\omega C}$

Impedance of circuit,

$$Z = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]} = R$$

Current in the circuit,

$I_0 = \dfrac{V_0}{Z} = \dfrac{V_0}{R} = \dfrac{200}{100}$

$= 2$ amp.

Average power, $P = \dfrac{1}{2} V_0 I_0 \cos \phi$

But, $\tan \phi = \dfrac{\omega L - (1/\omega C)}{R} = 0$ $(\cos \phi = 1)$

Now, $P = \dfrac{1}{2} \times 200 \times 2 \times 1 = 200$ watt.

Q. 17. *A resistance of 10 ohm is joined in series with an inductance of 0.5 henry. What capacitance should be put in series with the combination to obtain the maximum current? What will be the potential difference across the resistance, inductance and capacitor? The current is being supplied by 200 volts and 50 cycles per second mains.*

Ans. The current in the circuit would be maximum when $X_L = X_C$

$$\omega L = \frac{1}{\omega C} \quad \text{or} \quad C = \frac{1}{\omega^2 L}$$

$$\therefore \quad C = \frac{1}{(2\pi f)^2 L} = \frac{1}{(2 \times 3.14 \times 50)^2 \times 0.5} = 20.24 \times 10^{-6} \text{ farad.}$$

Here, $\omega L = \dfrac{1}{\omega C}$. So the impedance z of the circuit

$$Z = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]} = R = 10 \text{ ohm}$$

$$I = \frac{E}{R} = \frac{200}{10} = 20 \text{ amp.}$$

Potential difference across resistance

$$V_R = I \times R = 20 \times 10 = 200 \text{ volt}$$

Potential difference across inductance

$$V_L = \omega L \times I = (2\pi \times 50 \times 0.5) \times 20$$
$$= 3142 \text{ volt.}$$

Potential difference across condenser

$$V_C = \frac{1}{\omega C} = I \times \omega L = 3142 \text{ volt.}$$

III. LONG ANSWER TYPE QUESTIONS

Q. 1. *Derive an expression for the average power consumed in a series LCR circuit connected to a.c., source in which phase difference between the voltage and the current in the circuit is ϕ.*

Ans. Average power in *LCR* circuit:

Let the alternating e.m.f. applied to an *LCR* circuit is

$$E = E_0 \sin \omega t \qquad ...(i)$$

If alternating current developed lags behind the applied e.m.f. by a phase angle ϕ then

$$I = I_0 \sin (\omega t - \phi)$$

Total work done over a complete cycle is

$$W = \int_0^T EI \, dt$$

$$= \int_0^T E_0 \sin \omega t . I_0 \sin (\omega t - \phi) \, dt$$

$$= E_0 I_0 \int_0^T \sin \omega t \sin (\omega t - \phi)\, dt$$

$$= \frac{E_0\, I_0}{2} \int_0^T 2 \sin \omega t \sin (\omega t - \phi)\, dt$$

$$= \frac{E_0\, I_0}{2} \int_0^T \left[\cos (\omega t \pm \omega t + \phi) - \cos (\omega t + \omega t - \phi)\right] dt$$

[$\because$ 2 sin A sin B = cos (A – B) – cos (A + B)]

or, $$W = \frac{E_0\, I_0}{2} \int_0^T \left[\cos \phi - \cos (2\omega t - \phi)\right] dt$$

or, $$W = \frac{E_0\, I_0}{2} \left[t \cos \phi - \frac{\sin (2\, \omega t - \phi)}{2\omega}\right]_0^T$$

$$= \frac{E_0\, I_0}{2}\ [T \cos \phi]$$

$$W = \frac{E_0\, I_0}{2} \cdot \cos \phi\ .\ T$$

$\therefore$ Average power in LCR circuit over a complete cycle is

$$P = \frac{W}{T} = \frac{E_0\, I_0}{2} \cos \phi = \frac{E_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cos \phi$$

$\therefore$ $$P = E_v\, I_v \cos \phi.$$

Q. 2. *A current of 4 A flows in a coil when connected to a 12 V d.c. source. If the same coil is connected to a 12 V, 50 rad/s, a.c. source, a current of 2.4 A flows in the circuit. Determine the inductance of the coil. Also find the power developed in the circuit if a 2500 μF condenser is connected in series with the coil.*

Ans. When the coil is connected to a *d.c.* source, its resistance R is given by

$$R = \frac{V}{I} = \frac{12}{4} = 3\,\Omega$$

When it is connected to *a.c.* source, the impedance Z of the coil is given by

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{12}{2.4} = 5\,\Omega$$

For a coil, $$Z = \sqrt{\left[R^2 + (\omega L)^2\right]}$$

$\therefore$ $$5 = \sqrt{\left[(3)^2 + (50L)^2\right]}$$

or, $25 = [(3)^2 + (50L)^2]$

Solving we get $L = 0.08$ henry

When the coil is connected with a condenser in series, the impedance Z' is given by

$$Z' = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$$

$$= \left[(3)^2 + \left(50 \times 0.08 - \frac{1}{50 - 2500 \times 10^{-6}}\right)^2\right]^{1/2} = 5 \text{ ohm}$$

Power developed $P = V_{rms} \times I_{rms} \times \cos\theta$

where $\cos\phi = R/Z' = 3/5 = 0.6$

$\therefore \quad P = 12 \times 2.4 \times 0.6 = 17.28$ watt

Q. 3. *An a.c. source of voltage V = Vm sin ωt is connected, one-by-one, to three circuit elements X, Y and Z. It is observed that the current flowing in them,*

(i) is in phase with applied voltage for element X.

(ii) lags the applied voltage, in phase, by π/2 for element Y.

(iii) leads the applied voltage, in phase, by π/2 for element Z.

Identify the three circuit elements.

Find an expression for the (a) current flowing in the circuit, (b) net impedance of the circuit, when the same a.c. source is connected across a series combination of the elements X, Y and Z.

(c) If the frequency of the applied voltage is varied, set up the condition of frequency when the current amplitude in the circuit is maximum. Write the expression for this current amplitude.

Ans. (*i*) Circuit element X is resistance R.

(*ii*) Circuit element Y is capacitance C.

(*iii*) Circuit element Z is capacitance L.

(*a*)
$$I = I_m \sin \omega t \quad (\text{For } R)$$
$$I = I_m \sin(\omega t - \pi/2) \quad (\text{For } L)$$
$$I = I_m \sin(\omega t + \pi/2) \quad (\text{For } C)$$

(*b*) Let a resistance R, capacitance C and inductance L be connected in series to a source of alternating e.m.f., as shown in figure (*a*). Since R, L and C are in series, therefore, current at any instant through three elements has the same amplitude and phase. Let it be given as

$$I = I_0 \sin \omega t$$

However, voltage across each element bears a different phase relationship with the current. Now,

(*i*) The maximum voltage across R is

$$\vec{V}_R = \vec{I}_0 R$$

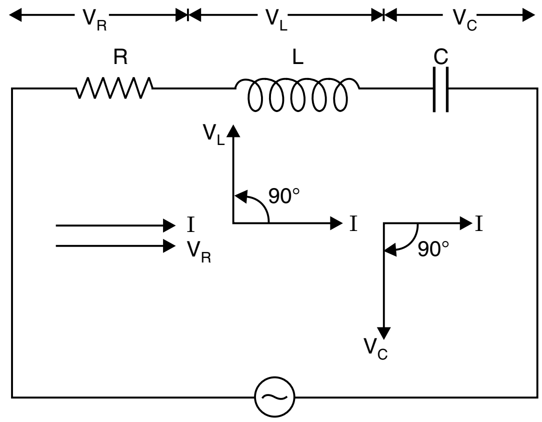

Fig. 7.30

In fig. (*b*), current phasor $\vec{I}_0$ is represented along *OX*.

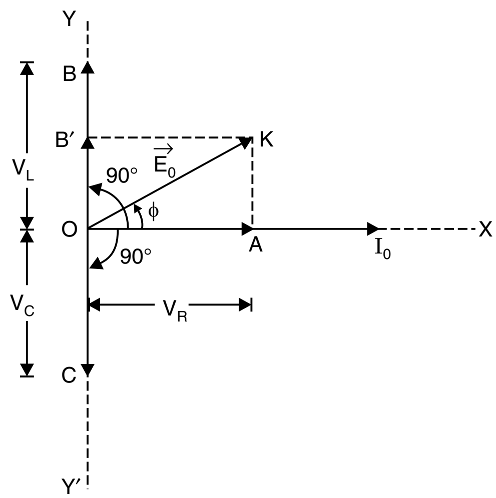

Fig. 7.31

As $\vec{V}_R$ is in phase with current, it is represented by the vector $\vec{O}A$, along *OX*.

(*ii*) The maximum voltage across *L* is

$$\vec{V}_L = \vec{I}_0 X_L$$

As voltage across the inductor leads the current by 90°, it is represented by $\overrightarrow{OB}$ along *OY*, 90° ahead of $\vec{I}_0$.

(*iii*) The maximum voltage across *C* is

$$\vec{V}_C = \vec{I}_0 X_C$$

As voltage across the capacitor lags behind the alternating current by 90°, it is represented by $\overrightarrow{OC}$ rotated clockwise through 90° from the direction of $\vec{I}_0 . \overrightarrow{OC}$ is along *OY*′.

As the voltage across *L* and *C* have a phase difference of 180°, the net reactive voltage is $\left(\vec{V}_L - \vec{V}_C\right)$, assuming that $\vec{V}_L > \vec{V}_C$.

In figures (*a*) and (*b*). 13, it is represented by $\overrightarrow{OB'}$. The resultant of $\overrightarrow{OA}$ and $\overrightarrow{OB'}$ is the diagonal $\overrightarrow{OK}$ of the rectangle *OAKB′*. Hence the vector sum of $\vec{V}_R, \vec{V}_L$ and $\vec{V}_C$ is phasor $\vec{E}_0$ represented by $\overrightarrow{OK}$, making an angle ϕ with current phasor $\vec{I}_0$

As $$OK = \sqrt{OA^2 + OB'^2}$$

$$\therefore \quad E_0 = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{(I_0 R)^2 + (I_0 X_L - I_0 X_C)^2}$$

$$E_0 = I_0 \sqrt{R^2 + (X_L - X_C)^2}$$

The total effective resistance of *RLC* circuit is called Impedance of the circuit. It is represented by *Z*, where

$$Z = \frac{E_0}{I_0} = \sqrt{R^2 + (X_L - X_C)^2}$$

(*c*) When the current amplitude in the circuit is maximum then $X_L = X_C$.

$$2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

$$f_0 = \frac{1}{2\pi \sqrt{LC}}$$

where f_0 is called the resonant frequency.

Q. 4. *A 20 volts 5 watt lamp is used in a.c. main of 220 volts 50 c.p.s. calculate the*

(i) capacitance of capacitor.

(ii) inductance of inductor, to be put in series to run the lamp.

(iii) what pure resistance should be included in place of the above device so that the lamp can run on its voltage?

(iv) which of the above arrangements will be more economical and why?

Ans. The current required by the lamp

$$I = \frac{\text{wattage}}{\text{voltage}} = \frac{5}{20} = 0.25 \text{ amp.}$$

The resistance of the lamp

$$R = \frac{\text{voltage}}{\text{current}} = \frac{20}{0.25} = 80 \text{ ohm}$$

So for proper running of the lamp, the current through the lamp should be 0.25 amp.

(*i*) When the condenser *C* is placed in series with lamp, then

$$Z = \sqrt{\left[R^2 + \left(\frac{1}{\omega C}\right)^2\right]}$$

The current through the circuit

$$I = \frac{200}{\sqrt{\left[R^2 + \left(\frac{1}{\omega C}\right)^2\right]}} = 0.25$$

$$\text{or, } \frac{200}{\sqrt{(80)^2 + \left(\frac{1}{4\pi^2 \times 50^2 + C^2}\right)}} = 0.25$$

Solving it for C, we get

$$C = 4.0 \times 10^{-6} \text{ F} = 4.0 \ \mu\text{F}$$

(*ii*) When inductor L henry is placed in series with the lamp, then

$$Z = \sqrt{\left[R^2 + (\omega L)^2\right]}$$

$$\text{or, } \frac{200}{\sqrt{\left[R^2 + (\omega L)^2\right]}} = 0.25$$

$$\text{or, } \frac{200}{\sqrt{\left[(80)^2 + \left(4\pi^2 \times 50^2 \times L^2\right)\right]}} = 0.25$$

Solving it for L, we get $L = 2.53$ henry.

(*iii*) When resistance r ohm is placed in series with lamp of resistance R, then

$$\frac{200}{R + r} = 0.25 \quad \text{or} \quad \frac{200}{80 + r} = 0.25$$

$$\Rightarrow \quad r = 720 \text{ ohms}$$

(*iv*) It will be more economical to use inductance or capacitance in series with the lamp to run it as it consumes no power while there would be dissipation of power when resistance is inserted in series with the lamp.

Q. 5. *An emf V0 sin ωt is applied to a circuit which consists of a self-inductance L of negligible resistance in series with a variable capacitor C. The capacitor is shunted by a variable resistance R. Find the value of C for which the amplitude of the current is independent of R.*

Ans. To make the problem easy, let us make use of phasor algebra. The complex impedance, of the circuit as shown in the figure.

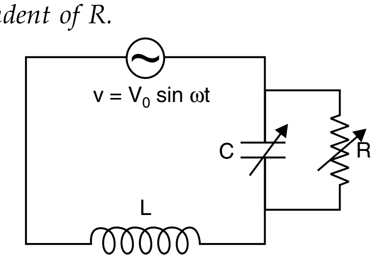

Fig. 7.32

$$Z = j\omega L + Z'$$

where Z' is complex impedance due to C and R in parallel and is given by

$$\frac{1}{Z'} = \frac{1}{R} + j\omega C = \frac{1 + j\omega CR}{R}$$

$$\text{or, } \quad Z' = \frac{R}{1 + j\omega CR} = \frac{R\left(1 - j\omega CR\right)}{1 + \omega^2 C^2 R^2}$$

$$\therefore \quad Z = j\omega L + \frac{R\left(1 - j\omega CR\right)}{1 + \omega^2 C^2 R^2}$$

$$= \frac{R}{1 + \omega^2 C^2 R^2} + j\left(\omega L - \frac{\omega C R^2}{1 + \omega^2 C^2 R^2}\right)$$

The magnitude of Z is thus given by

$$Z = \sqrt{\left[\frac{R^2}{(1+\omega^2C^2R^2)^2} + \left(\omega L - \frac{\omega CR^2}{1+\omega^2C^2R^2}\right)^2\right]}$$

or
$$Z^2 = \frac{R^2}{(1+\omega^2C^2R^2)^2} + \omega^2L^2 + \frac{\omega^2C^2R^4}{(1+\omega^2C^2R^2)^2} - \frac{2\omega^2LCR^2}{1+\omega^2C^2R^2}$$

$$= \frac{R^2 - 2\omega^2LCR^2}{1+\omega^2C^2R^2} + \omega^2L^2$$

The peak value of current will be independent of R, if Z or Z^2 is also independent of R. It is possible when

$$R^2 - 2\omega^2 LCR^2 = 0, \quad \text{or} \quad C = \frac{1}{2\omega^2 L}$$

Q. 6. *An alternating emf is applied across a capacitor. Show mathematically that current in it leads the applied emf by a phase angle of π/2. What is its capacitive reactance ? Draw a graph showing the variation of capacitive reactance with the frequency of the a.c. source.*

Ans. Let an alternating emf, $E = E_0 \sin \omega t$ is applied across a capacitor of capacitance C. The current flowing in the circuit transfers charge to the plates of the capacitor due to which a potential difference develops across its plates. Also, assume that q be the charge on each plate of the capacitor at any instant t. Therefore, potential difference across the plates of capacitor, $V = \frac{q}{C}$.

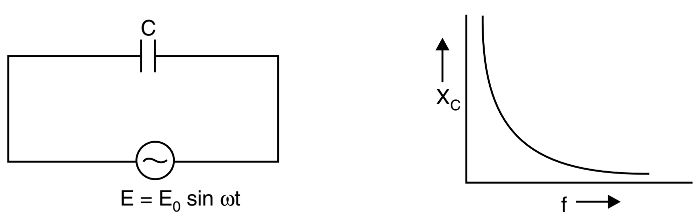

Fig. 7.33 Variation of capacitive reactance with frequency

At every instant potential difference V must be equal to the applied emf.

i.e., $$V = \frac{q}{C} = E = E_0 \sin \omega t \quad ...(i) \; [\because V = E = E_0 \sin \omega t]$$

or, $$q = CE_0 \sin \omega t$$

∴ Instantaneous current,

$$I = \frac{dq}{dt} = \frac{d}{dt}(CE_0 \sin \omega t) = CE_0 \cos \omega t \cdot \omega$$

or, $$I = \frac{E_0}{\frac{1}{\omega C}} \sin\left(\omega t + \frac{\pi}{2}\right) \quad ...(ii) \left[\because \sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta\right]$$

The current is maximum, *i.e.,* $I = I_0$ when

$$\sin\left(\omega t + \frac{\pi}{2}\right) = 1$$

From equation (*i*), $I_0 = \dfrac{E_0}{\dfrac{1}{\omega C}} \times 1 = \dfrac{E_0}{\dfrac{1}{\omega C}}$ ...(*iii*)

Putting in equation (*ii*), we get

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \quad ...(iv)$$

Comparing equations (*i*) and (*iii*), we see that in an *a.c.* circuit containing capacitor only, current leads the emf by a phase angle of $\pi/2$.

Comparing (*iii*) with $I_0 = E_0/R$, we find $1/\omega C$ is the effective resistance of the capacitive *a.c.* circuit. It is called capacitive reactance.

$$\chi_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}.$$

Q. 7. *What is meant by root mean square value of alternating current?*
Derive an expression for r.m.s. value of alternating current.

Ans. *Root mean square (r.m.s.) or virtual value of a.c. :* It is that steady current, which when passed through a resistance for a given time will produce the same amount of heat as the alternating current does in the same resistance and in the same time. It is denoted by I_{rms} or I_V.

Derivation of r.m.s. value of current: The instantaneous value of *a.c.* passing through a resistance R is given by

$$I = I_0 \sin \omega t$$

The alternating current changes continuously with time. Suppose that the current through the resistance remains constant for an infinitesimally small time dt.

Then, small amount of heat produced the resistance R in time dt is given by

$$dH = I^2 R\, dt = (I_0 \sin \omega t)^2 R\, dt = I_0^2 R \sin^2 \omega t\, dt$$

The amount of heat produced in the resistance in time $T/2$ is

$$H = \int_0^{T/2} I_0^2 R \sin^2 \omega t\, dt = I_0^2 R \int_0^{T/2} \frac{1 - \cos 2\omega t}{2}\, dt$$

or, $$H = \frac{I_0^2 R}{2}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^{T/2}$$

or, $$H = \frac{I_0^2 R}{2}\left[\frac{T}{2} - \frac{\sin 2\omega t \cdot \frac{T}{2}}{2\omega} - 0\right]$$

or, $$H = \frac{I_0^2 R}{2}\left[\frac{T}{2} - \frac{\sin 2 \cdot \frac{2\pi}{T} \cdot \frac{T}{2}}{2\omega}\right]$$

or, $$H = \frac{I_0^2 R}{2}\left[\frac{T}{2} - \frac{\sin 2\pi}{2\omega}\right]$$

or, $$H = \frac{I_0^2 R}{2} \cdot \frac{T}{2} \quad ...(i)\ [\because \sin 2\pi = 0]$$

If I_{rms} be the r.m.s. value of *a.c.*, then by definition,

$$H = I^2_{rms} R \frac{T}{2} \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$I^2_{rms} R \frac{T}{2} = \frac{I_0^2 R}{2} \cdot \frac{T}{2}$$

or, $$I^2_{rms} = \frac{I_0^2}{2}$$

or, $$I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707 I_0.$$

Q. 8. *A series LCR circuit is connected to an a.c. source of 220 V – 50 Hz. If the readings of voltmeters across resistor, capacitor and inductor are 65 V, 415 V and 204 V; and R = 100 Ω, calculate: (i) current in the circuit; (ii) value of L, (iii) value of C and (iv) capacitance required to produce resonance with the given inductor L.*

Ans. Given, $E_V = 200$ V, $f = 50$ Hz

$R = 100\ \Omega$, $V_R = 65$ V, $V_C = 415$ V, $V_L = 204$ V

(*i*) If I_V is the current in the circuit, then

$$V_R = I_V \times R$$

$$65 = I_V \times 100$$

or, $$I_V = 0.65 \text{ A.}$$

(*ii*) $$V_L = I_V X_L$$

or, $$X_L = \frac{V_L}{I_V} = \frac{204}{0.65} = 313.85\ \Omega$$

$$X_L = \omega L = 2\pi f L = 313.85$$

$$L = \frac{313.85}{2\pi f} = \frac{313.85}{2 \times 3.14 \times 50}$$

or, $$L = 1.0 \text{ H.}$$

(*iii*) $$V_C = I_V X_C$$

$$X_C = \frac{V_C}{I_V} = \frac{415}{0.65} = 638.5\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Now, $$C = \frac{1}{2\pi f\ X_C} = \frac{1}{2 \times 3.14 \times 50 \times 638.5}$$

or, $$C = 4.99 \times 10^{-6} \text{ F}$$

(*iv*) Consider C' be the capacitance that would produce resonance with $L = 1.0$ H, then

$$f = \frac{1}{2\pi \sqrt{LC'}}$$

$$C' = \frac{1}{4\pi^2 f^2 L}$$

$$C' = \frac{1}{4 \times (3.14)^2 \times (50)^2 \times 1}$$

$$= 10.1 \times 10^{-6} \text{ F} = 10.1 \ \mu\text{F}.$$

QUESTIONS ON HIGH ORDER THINKING SKILLS (HOTS)

Q. 1. *A 12 ohm resistance and an inductance of 0.05/π henry with negligible resistance are connected in series. Across the end of this circuit is connected a 130 volt alternating voltage of frequency 50 cycles/second. Calculate the alternating current in the circuit and potential difference across the resistance and that across the inductance.*

Ans. The impedance of the circuit is given by

$$Z = \sqrt{(R^2 + \omega^2 L^2)} = \sqrt{\left[R^2 + (2\pi f L)^2\right]}$$

$$= \sqrt{\left[(12)^2 + \{2 \times 3.14 \times 50 \times (0.05/3.14)\}^2\right]} = \sqrt{(144 + 25)} = 13 \text{ ohm}$$

Current in the circuit, $i = \frac{E}{Z} = \frac{130}{13} = 10$ amp.

Potential difference across resistance,

$$V_R = iR = 10 \times 12 = 120 \text{ volt}$$

Inductive reactance of coil, $X_L = \omega L = 2\pi f L$

$$\therefore \quad X_L = 2\pi \times 50 \times \left(\frac{0.05}{\pi}\right) = 5 \text{ ohm.}$$

Potential difference across inductance

$$V_L = i \times X_L = 10 \times 5 = 50 \text{ volt.}$$

Q. 2. *A circuit draws a power of 550 W from a 220 V – 50 Hz source. The power factor of the circuit is 0.8. A current in the circuit lags behind the voltage. Show that a capacitor of about* $\frac{1}{42\pi} \times 10^{-2}$ *F will have to be connected in the circuit to bring its power factor to unity.*

Ans.

$$\boxed{P = I_V E_V \cos\phi}$$

$$I_V = \frac{P}{E_V \cos\phi} = \frac{550}{220 \times 0.8} \text{ A} = 3.125 \text{ A}$$

$$\boxed{R = \frac{P}{I_V^2}} = \frac{550}{(3.125)^2} = 56.3 \ \Omega$$

Now using $\boxed{\tan\phi = \frac{\omega L}{R}}$, we get

$$\omega L = 42 \ \Omega$$

Again, $\boxed{\omega L = \frac{1}{\omega C}}$ [For power factor one] or $$C = \frac{1}{\omega(\omega L)}$$

$$= \frac{1}{100\pi \times 42} \text{ F}$$

$$= \frac{1}{42\pi} \times 10^{-2} \text{ F}.$$

Q. 3. *A capacitor, resistor of 5 Ω and an inductor of 50 mH are in series with an a.c. source marked 100 V, 50 Hz. It is found that the voltage is in phase with the current. Calculate the capacitance of the capacitor and the impedance of the circuit.*

Ans. Since the voltage is in phase with the current, therefore, it is a case of resonance. The circuit is purely resistive. So, impedance, $Z = R = 5\ \Omega$.

Again, $$\boxed{f = \frac{1}{2\pi\sqrt{LC}}} \quad \text{or,} \quad C = \frac{1}{4\pi^2 L f^2}$$

or $$C = \frac{49}{4 \times 484 \times 50 \times 10^{-3} \times 50 \times 50} \text{ F}$$

$$= \frac{49}{4 \times 484 \times 125} \text{ F} = 2.02 \times 10^{-4} \text{ F}$$

Q. 4. (a) *A coil of self-inductance 0.16 H is connected to a condenser of capacity 0.81 μF. What should be the frequency of alternating current that should be applied so that there is resonance in the circuit? Resistance of circuit is negligible.*

(b) *If the initial charge on capacitor in part (a) is 6 mC, what is the total initial energy stored in the circuit?*

Ans. (*a*) $$\boxed{\nu = \frac{1}{2\pi\sqrt{LC}}}$$

$$= \frac{1}{2 \times 3.14\sqrt{0.16 \times 0.81 \times 10^{-6}}} \text{ Hz}$$

$$= \frac{1000}{6.28 \times 0.36} \text{ Hz} = 442.3 \text{ Hz}$$

(*b*) $$\boxed{U = \frac{Q^2}{2C}} = \frac{36 \times 10^{-6}}{2 \times 0.81 \times 10^{-6}} \text{ J} = 22.2 \text{ J}$$

Q. 5. *A LCR circuit has L = 10 mH, R = 3 ohm and C = 1 μF connected in series to an a.c. source of the voltage 15 V. Calculate current amplitude and the average power dissipated per cycle at a frequency that is 10% lower than the resonant frequency.*

Ans. Resonant frequency, $$\boxed{\omega_r = \frac{1}{\sqrt{LC}}}$$

Here, $L = 10 \text{ mH} = 10 \times 10^{-3} \text{ H}$

and $C = 1\ \mu\text{F} = 1 \times 10^{-6} \text{ F}$

$\therefore \qquad \omega_r = \sqrt{\left[\dfrac{1}{(10 \times 10^{-3})(1 \times 10^{-6})}\right]} = 10^4/\text{second.}$

Now, 10% less frequency will be

$$\omega = 10^4 - 10^4 \times \frac{10}{100} = 9 \times 10^3/\text{second.}$$

At this frequency,

$$\boxed{X_L = \omega L} = 9 \times 10^3 \times (10 \times 10^{-3}) = 90 \text{ ohm}$$

$$\boxed{X_C = \frac{1}{\omega C}} = \frac{1}{(9 \times 10^3)(1 \times 10^{-6})} = 111.11 \text{ ohm}$$

$$\therefore \qquad \boxed{Z = \sqrt{\left[R^2 + (X_L - X_C)^2\right]}}$$

$$= \sqrt{\left[(3)^2 + (90 - 111.11)^2\right]} = 21.32 \text{ ohm}$$

Current amplitude,

$$\boxed{I_0 = \frac{E_0}{Z}} = \frac{15}{21.32} = 0.704 \text{ amp.}$$

Average power, $\boxed{P = \dfrac{1}{2} E_0 I_0 \cos \phi}$

where $\boxed{\cos \phi = \dfrac{R}{Z}} = \dfrac{3}{21.32} = 0.141$

$$P = \frac{1}{2} \times 15 \times 0.704 \times 0.141 = 0.744 \text{ watt.}$$

Q. 6. *In a series LCR circuit, voltages across an inductor, a capacitor and a resistor are 30 V, 30 V and 60 V respectively. What is the phase difference between applied voltage and current in the circuit?*

Ans. As $V_L = V_C = 30$ V, therefore,

$$\boxed{\tan \phi = \frac{V_L - V_C}{V_R}} = 0 \quad \therefore \quad \phi = 0°$$

i.e., there is no phase difference between applied voltage and current in the circuit.

Q. 7. *Give expression for average value of a.c. voltage $V = V_0 \sin \omega t$ over time interval $t = 0$ to $t = \dfrac{\pi}{\omega}$.*

Ans. As $t = \dfrac{\pi}{\omega} = \dfrac{1}{2} \cdot \dfrac{2\pi}{\omega} = \dfrac{1}{2} T$, therefore, we wish to know average value of *a.c.* voltage over first half cycle $(0 \to T/2)$. It is $\boxed{E_{av} = \dfrac{2V_0}{\pi}}$.

Q. 8. *An a.c., source of angular frequency ω is fed across a resistor R and a capacitor C in series. The current registered is i. If now the frequency of the source is changed to ω/3 (but maintaining the same voltage), the current in the circuit is found to be halved. Calculate the ratio of reactance of resistance at the original frequency.*

Ans. At angular frequency ω, the current in *R-C* circuit is given by

$$i_{\text{rms}} = \frac{E_{rms}}{\sqrt{\left[R^2 + \left(\frac{1}{\omega^2 C^2}\right)\right]}} \qquad ...(i)$$

When frequency is changed to ω/3, the current is halved. Thus

$$\frac{i_{rms}}{2} = \frac{E_{rms}}{\sqrt{\left[R^2 + \frac{1}{(\omega/3)^2 C^2}\right]}} = \frac{E_{rms}}{\sqrt{\left[R^2 + \left(\frac{9}{\omega^2 C^2}\right)\right]}} \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$\frac{1}{\sqrt{\left[R^2 + \left(\frac{1}{\omega^2 C^2}\right)\right]}} = \frac{2}{\sqrt{\left[R^2 + \left(\frac{9}{\omega^2 C^2}\right)\right]}}$$

Solving this equation, we get

$$3R^2 = \frac{5}{\omega^2 C^2}$$

Hence, the ratio of reactance to resistance is $\dfrac{\left(\frac{1}{\omega C}\right)}{R} = \sqrt{\left(\frac{3}{5}\right)}$.

Q. 9. *C.O.S. is a change over switch. Work out current in the circuit shown in figure, when switch is connecting (i) 1 and 2, (ii) 1 and 3, and the circuits are resonating.*

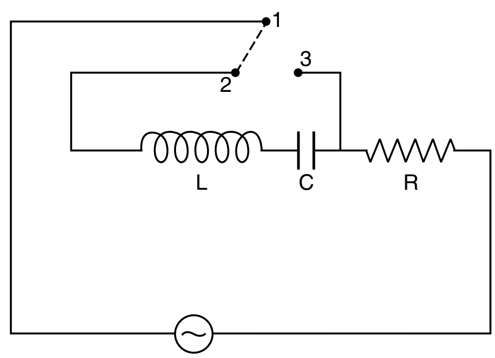

Fig. 7.34

Ans. (*i*) When switch is connecting 1 and 2, *L, C, R* are in series. *At resonance,*

$$Z = R$$

$$\therefore \qquad I_v = \frac{E_v}{Z} = \frac{E_v}{R}$$

(*ii*) When switch is connecting 2 and 3, the circuit becomes parallel resonance circuit. At resonance, $I_v = 0$.

Q. 10. *Sketch a graph showing the variation of impedance of LCR circuit with the frequency of applied voltage.*

Ans. The impedance of *LCR* circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{R^2 + \left(2\pi vL - \frac{1}{2\pi vC}\right)^2}$$

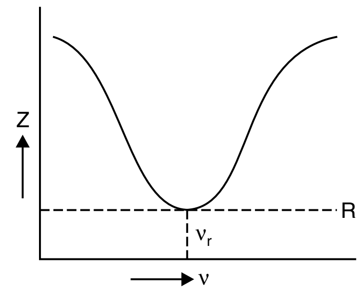

Fig. 7.35

The variation of Z with v is as shown in figure At $v = v_r$; $X_L = X_C$;

$$Z = R = \text{minimum.}$$

Q. 11. *A circuit is set-up by connecting L = 100 mH, C = 5 μF and R = 100 Ω in series. An alternating emf of $(150\sqrt{2})$ volt, $\frac{500}{\pi}$ Hz is applied across this combination. Calculate the impedance of the circuit. What is the average power dissipated in (a) the resistor (b) the capacitor (c) the inductor and (d) the complete circuit?*

Ans.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$R = 100\ \Omega,$$

$$X_L = \omega L = 2\pi fL = 2\pi \times \frac{500}{\pi} \times 100 \times 10^{-3} = 100\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{2\pi\left(\frac{500}{\pi}\right)\times 5\times 10^{-6}}$$

or, $$X_C = 200\ \Omega$$

$$Z = \sqrt{(100)^2 + (100 - 200)^2} = 141.4\ \Omega$$

$$I = \frac{V}{Z} \quad \text{or,} \quad I = \frac{150\sqrt{2}}{100\sqrt{2}} = 1.5\ \text{A}$$

(*a*) Power consumed in resistor is I^2R = 1.5 × 1.5 × 100 = W = 225 W.
(*b*) Power consumed in capacitor is zero.
(*c*) Power consumed in inductor is zero.
(*d*) Power consumed in circuits same as power consumed in resistor *i.e.*, 225 W.

Q. 12. *(a) Out of the two arrangements given below, for winding of primary and secondary coils in a transformer, which arrangement do you think will have higher efficiency and why?*

(b) Show that, in ideal transformer, when the voltage is stepped up by a certain factor, the current gets stepped down by the same factor.

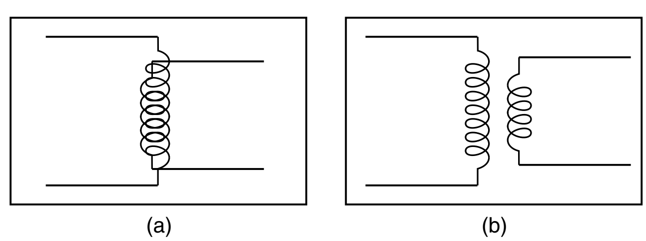

Fig. 7.36

(c) State any two causes of energy loss in transformer.

Ans. (*a*) Arrangement (*a*) will have higher efficiency because leakage flux in this arrangement is minimum.

(*b*) For an ideal transformer,

$$\frac{\varepsilon_s}{\varepsilon_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}$$

$$\therefore \qquad \varepsilon_s = \left(\frac{N_s}{N_p}\right)\varepsilon_p$$

$$\text{and} \qquad I_s = \left(\frac{N_p}{N_s}\right)I_p$$

$\Rightarrow$ Voltage is stepped by the factor $\left(\frac{N_s}{N_p}\right)$ and current is by the factor of $\left(\frac{N_p}{N_s}\right)$ or stepped down by the same factor *i.e.,* $\left(\frac{N_s}{N_p}\right)$.

(*c*) Two causes of energy loss in transformer are

(*i*) leakage of flux

(*ii*) eddy loss.

Q. 13. *In a series LCR a.c. circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltage across the series elements of the circuit? Is the same true for r.m.s. voltage?*

Ans. Yes, $\varepsilon = \varepsilon_L + \varepsilon_C + \varepsilon_R$

But it is not true for r.m.s. voltage. In case of r.m.s. voltage

$$\varepsilon = \sqrt{\varepsilon_R^2 + (\varepsilon_L \sim \varepsilon_C)^2}$$

Q. 14. *An electric heater is connected, turn by turn, to a d.c. and a.c. sources of equal voltages. Will the rate of heat production be same in the two cases? Explain.*

Ans. The rate of heat production in both the cases will be same. The rate of heat produced in a.c. and d.c. is resistance depended which have same behaviour for a.c. and d.c.

Q. 15. *A resistance of 400 Ω and a capacitor of resistance 200 Ω are connected in series to a 220 V, 50 Hz a.c. source. If the current in the circuit is 0.49 ampere find the (i) voltage across the resistor and capacitor (ii) value of inductance required so that voltage and current are in same phase.*

Ans. (*i*) The voltage across resistor,

$$\begin{aligned}\varepsilon_R &= IR \\ &= 0.49 \times 400 \\ &= 196 \text{ volt.}\end{aligned}$$

The voltage across capacitor,

$$\begin{aligned}\varepsilon_C &= IX_C \\ &= 0.49 \times 200 \\ &= 98 \text{ volt.}\end{aligned}$$

(*ii*) For voltage and current in same phase

$$X_L = X_C$$

$$\therefore \quad X_L = 200\ \Omega$$

$$\text{and} \quad WL = 200$$

$$\text{or} \quad 2\pi\nu L = 200$$

$$\text{or} \quad L = \frac{200}{2\pi\nu}$$

$$= \frac{200}{2 \times 3.14 \times 50}\ \text{H}$$

$$= \frac{200}{314}\ \text{H}.$$

MULTIPLE CHOICE QUESTIONS

1. At resonance, in a series LCR circuit, which relation does not hold?

(*a*) $\omega = \frac{1}{LC}$ (*b*) $\omega = \frac{1}{\sqrt{LC}}$

(*c*) $L\omega = \frac{1}{C\omega}$ (*d*) $C\omega = \frac{1}{L\omega}$

2. The average power dissipation in a pure capacitor in ac circuit is

(*a*) $\frac{1}{2}CV^2$ (*b*) CV^2 (*c*) $2CV^2$ (*d*) Zero

3. In series *LCR* a.c. circuit, the phase angle between current and voltage is

(*a*) Any angle between *O* and $\pm\frac{\pi}{2}$ (*b*) $\frac{\pi}{2}$

(*c*) π (*d*) Any angle between *O* and $\frac{\pi}{2}$

4. For high frequency capacity offers

(*a*) more resistance (*b*) less resistance (*c*) zero resistance (*d*) none of these

5. The phase difference between the current and voltage at resonance is

(*a*) 0 (*b*) $\pi/2$ (*c*) π (*d*) $-\pi$

6. If the current is halved in a coil then the energy stored is how much times the previous value

(*a*) 1/2 (*b*) 1/4 (*c*) 2 (*d*) 4

7. Phase difference between voltage and current in a capacitor in ac circuit is

(*a*) π (*b*) $\pi/2$ (*c*) 0 (*d*) $\pi/3$

8. A capacitor of capacity *C* has reactance *X*. If capacitance and frequency are doubled, the reactance would be

(*a*) $4X$ (*b*) $X/2$ (*c*) $X/4$ (*d*) $2X$

9. An electric bulb marked 40 W and 200 V is used in a circuit of supply voltage 100 V. Its power would be

(*a*) 40 W (*b*) 10 W (*c*) 20 W (*d*) 100 W

10. In an ideal inductor $L = 4H$ and $\omega = 100$ rad/s. The power developed is

(*a*) $V_0 I_0$ (*b*) $V_0 I_0/2$ (*c*) $2V_0 I_0$ (*d*) 0

11. In a series *LR* circuit, $X_L = 3R$. Now a capacitor with $X_C = R$ is added in series. The ratio of new to old power factor is

(*a*) $\sqrt{2}$ (*b*) $1/\sqrt{2}$ (*c*) 2 (*d*) 1

12. What is self inductance of a coil which produces 5 V, when current in it changes from 3 A to 2 A in one millisecond?

(*a*) 5000 H (*b*) 5 mH (*c*) 50 H (*d*) 5 H

13. A 220 V, 100 W bulb is connected across a 110 V main supply. The power consumed will be

(*a*) 250 W (*b*) 500 W (*c*) 1000 W (*d*) 750 W

14. The current in a series *LCR* circuit will be maximum, then ω is

(*a*) as large as possible

(*b*) equal to natural frequency of *LCR* system

(*c*) $\sqrt{LC}$

(*d*) $\sqrt{LCR}$

15. A 500 mH coil carries a current of 2 A. The energy stored in the coil is

(*a*) 0.1 J (*b*) 0.05 J (*c*) 10 J (*d*) 0.5 J

Answers

1. (*a*)	**2.** (*d*)	**3.** (*a*)	**4.** (*b*)	**5.** (*a*)
6. (*b*)	**7.** (*b*)	**8.** (*c*)	**9.** (*b*)	**10.** (*d*)
11. (*a*)	**12.** (*b*)	**13.** (*a*)	**14.** (*b*)	**15.** (*a*).

TEST YOUR SKILLS

1. What is a choke coil? Why is it preferred to resistance in a.c. circuits? In fig. (*a*), (*b*) and (*c*) are shown three a.c. circuits with equal current. If the frequency of e.m.f. be increased, then what will be the effect on the current flowing in them? Explain with reasons.

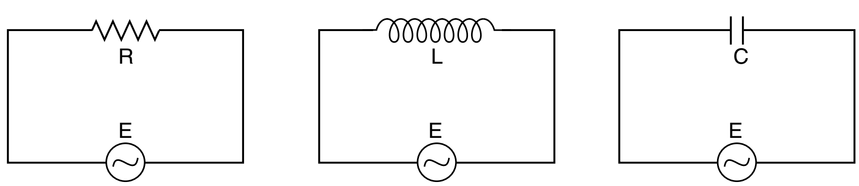

Fig. 7.37

2. Draw the variation of the following with the frequency of the a.c. source.

(*i*) reactance of an inductor

(*ii*) reactance of a capacitor.

3. Distinguish the terms 'average values' and 'rms value' of an alternating current. The instantaneous current from an a.c. source is $I = 5 \sin (314t)$ ampere. What are the average and rms values of current?

4. Given below are two electric circuits *A* and *B*.

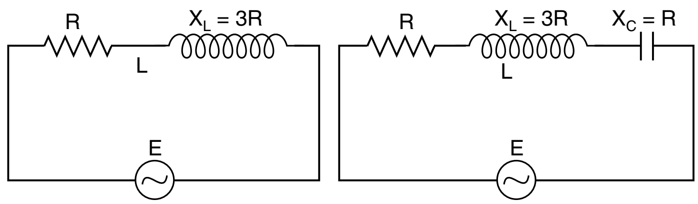

Fig. 7.38

Calculate the ratio of power factor of the circuit *B* to the power factor of circuit *A*.

5. In an a.c. circuit the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin (\omega t - \pi/2)$. What will be the power consumption in the circuit?

6. An ideal coil of 10H is connected in series with a resistance of 5 Ω and a battery of 5 V. 2 second after the connection is made, what will be the current flowing in the circuit?

7. A transformer with efficiency 80% works at 4 KW and 100 V. If the secondary voltage is 200 V. Then what will be primary and secondary currents?

8. The instantaneous voltage through a device of impedance 20 Ω is $e = 80 \sin 100 \pi t$. What is effective value of current?

9. The instantaneous voltage from an a.c. source is given $E = 300 \sin 314t$. What is the r.m.s. voltage of the source?

10. For a given a.c. circuit, distinguish between resistance, reactance and impedance. An a.c. source of frequency 50 Hz is connected to a 50 mH inductor and a bulb. The bulb glows with some brightness. Calculate the capacitance of the capacitor to be connected in series with the circuit so that the bulb glows with maximum brightness.

11. The output voltage of ideal transformer, connected to a 240 V a.c. mains is 24 V, when this transformer is used to light a bulb with rating 24 V, 24 W. Calculate the current into the primary coil of the circuit.

12. Prove that an ideal capacitor, in an a.c. circuit does not dissipate power.

13. Derive an expression for the impedance of an a.c. circuit consisting of an inductor and a resistor.

14. An inductor 200 mH, capacitor 500 μF, resistor 10 Ω are connected in series with a 100 V, variable frequency a.c. source. Calculate the

(*i*) frequency at which the power factor of the circuit is unity

(*ii*) current amplitude at this frequency.

(*iii*) *Q*-factor.

15. An inductor of unknown value, a capacitor of 100 μF and a resistor of 10 Ω are connected in series to a 200 V, 50 Hz a.c. source. It is found that the power factor of the circuit is unity. Calculate the inductance of the inductor and the current amplitude.

16. When a large circular coil of radius *R*, is kept in the neighbourhood of a small circular coil of radius *r*, the coefficient of mutual induction, for the given pair, equals 5 mH. What current must flow through the larger coil to cause a flux of 0.25×10^{-3} Wb to be linked with the smaller coil?

If this current falls to zero what would be the effect in the smaller coil?

17. An a.c. source of voltage $V = V_0 \sin \omega t$ is connected, one-by-one, to three circuit elements *X*, *Y* and *Z*. It is observed that the current flowing in them.

(*i*) is in phase with the applied voltage for element *X*.

(*ii*) lags the applied voltage, in phase, by $\pi/2$ for element *Y*.

(*iii*) leads the applied voltage, in phase by $\pi/2$ for element *Z*.

Identify the three circuit elements.

Find an expression for the (*a*) current flowing in the circuit, (*b*) net impedance of the circuit, when the same a.c. source is connected across a series combination of the elements *X*, *Y* and *Z*.

If the frequency of the applied voltage is varied, set up the condition of the frequency when then current amplitude in the circuit is maximum. Write the expression for this current amplitude.

18. What does the term 'phasors' in a.c. circuit analysis mean?

An a.c. source of voltage $V = V_0 \sin \omega t$ is applied across a pure inductor of inductance *L*. Obtain an expression for the current, *i*, flowing in the circuit. Also draw the

(*i*) phasor diagram

(*ii*) graphs of *V* and *i* versus ωt for this circuit.

19. The instantaneous current and voltage of an a.c. circuit are given by

$i = 10 \sin 300\, t$A and $V = 200 \sin 300\, t$V.

What is the power dissipation in the circuit?

20. The circuit arrangement given shows that when an a.c. passes through the coil *A* the current starts flowing in the coil *B*.

(*i*) State the underlying principle involved.

(*ii*) Mention two factors on which the current produced in the coil *B* depends.

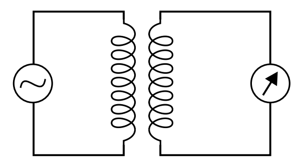

Fig. 7.39

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