7 System of Particles and Rotational Motion

Facts that Matter

- A rigid body is a body with a perfectly definite and unchanging shape. The distances between all pairs of particles of such a body do not change.

- **Centre of Mass**

For a system of particles, the centre of mass is defined as that point where the entire mass of the system is imagined to be concentrated, for consideration of its translational motion.

If all the external forces acting on the body/system of bodies were to be applied at the centre of mass, the state of rest/motion of the body/system of bodies shall remain unaffected.

- The centre of mass of a body or a system is its balancing point. The centre of mass of a two-particle system always lies on the line joining the two particles and is somewhere in between the particles.

If there are two particles of masses m_1 and m_2 having position vectors $\vec{r}_1$ and $\vec{r}_2$, then the position vector of the centre of mass is given by

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

Special Note: If the masses are of equal magnitude the centre of mass lies at the mid-point of the line joining them. If the masses are unequal, centre of mass is closer to the heavier body.

- For a system of n particles of masses m_1, m_2, m_3 .... m_n and their respective position vectors $\vec{r}_1$, $\vec{r}_2, \vec{r}_3$ .... $\vec{r}_n$; the position

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots\ldots\ldots + m_n\vec{r}_n}{m_1 + m_2 + \ldots\ldots\ldots m_n} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{\sum_{i=1}^{n} m_i}$$

- The co-ordinates of the centre of mass of an n-particle system is given as:

$$X = \frac{m_1x_1 + m_2x_2 + \ldots\ldots\ldots\ldots m_nx_n}{m_1 + m_2 + \ldots\ldots\ldots\ldots m_n} = \frac{\sum_{i=1}^{n} m_ix_i}{\sum_{i=1}^{n} m_i} = \frac{1}{M}\sum_{i=1}^{n} m_ix_i$$

where $\sum_{i=1}^{n} m_i = M$, mass of system.

$$Y = \frac{m_1 y_1 + m_2 y_2 + ... + m_n y_n}{m_1 + m_2 + ... + m_n} = \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i} = \frac{1}{M} \sum_{i=1}^{n} m_i y_i$$

$$Z = \frac{m_1 z_1 + m_2 z_2 + ... + m_n z_n}{m_1 + m_2 + ... + m_n} = \frac{\sum_{i=1}^{n} m_i z_i}{\sum_{i=1}^{n} m_i} = \frac{1}{M} \sum_{i=1}^{n} m_i z_i$$

- **Motion of centre of Mass**

The centre of mass of a system of particles moves as if the entire mass of the system were concentrated at the centre of mass and all the external forces were applied at that point.

Velocity of centre of mass of a system of two particles, m_1 and m_2 with velocity v_1 and v_2 is given by,

$$V_{\text{cm}} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$$

Acceleration of centre of mass, a_{cm} of a two body system is given by

$$a_{\text{cm}} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2}{m_1 + m_2}$$

- If no external force acts on the body, then the centre of mass will have constant momentum. Its velocity is constant and acceleration is zero, *i.e.,* MV_{cm} = constant.

- **Vector Product or Cross Product of two vectors**

The vector product or cross product of two vectors $\vec{A}$ and $\vec{B}$ is another vector $\vec{C}$, whose magnitude is equal to the product of the magnitudes of the two vectors and sine of the smaller angle between them.

If θ is the smaller angle between $\vec{A}$ and $\vec{B}$, then

$$\vec{A} \times \vec{B} = \vec{C} = AB \sin\theta\, \hat{C}$$

where $\hat{C}$ is a unit vector in the direction of $\vec{C}$. The direction of $\vec{C}$ or $\hat{C}$ (*i.e.,* vector product of two vectors) is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$ and pointing in the direction of advance of a right handed screw when rotated from $\vec{A}$ to $\vec{B}$.

- Some important properties of cross-product are as follows:
  - (*a*) For parallel as well as anti-parallel vectors (*i.e.,* when θ = 0° or 180°), the cross-product is zero.
  - (*b*) The magnitude of cross-product of two perpendicular vectors is equal to the product of the magnitudes of the given vectors.
  - (*c*) Vector product is anti-commutative *i.e.,* $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
  - (*d*) Vector product is distributive *i.e.,* $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$
  - (*e*) $\vec{A} \times \vec{B}$ does not change sign under reflection *i.e.,* $(-\vec{A}) \times (-\vec{B}) = \vec{A} \times \vec{B}$

(*f*) For unit orthogonal vectors, we have

$$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0, \quad \hat{i}\times\hat{j} = \hat{k}, \quad \hat{j}\times\hat{k} = \hat{i} \quad \text{and} \quad \hat{k}\times\hat{i} = \hat{j}$$

Moreover $\hat{j}\times\hat{i} = -\hat{k}, \quad \hat{k}\times\hat{j} = -\hat{i} \quad \text{and} \quad \hat{i}\times\hat{k} = -\hat{j}$

(*g*) In terms of components $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$.

• The angular velocity of a body or a particle is defined as the ratio of the angular displacement of the body or the particle to the time interval during which this displacement occurs.

$$\omega = \frac{d\theta}{dt}$$

The direction of angular velocity is along the axis of rotation. It is measured in radian/sec and its dimensional formula is $[M^0L^0T^{-1}]$.

The relation between angular velocity and linear velocity is given by

$$\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$$

• The angular acceleration of a body is defined as the ratio of the change in the angular velocity to the time interval.

$$\text{Angular acceleration} = \frac{\text{Change in angular velocity}}{\text{time taken}}$$

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

The unit of angular acceleration is rad s^{-2} and dimensional formula is $[M^0L^0L^{-2}]$.

• Torque

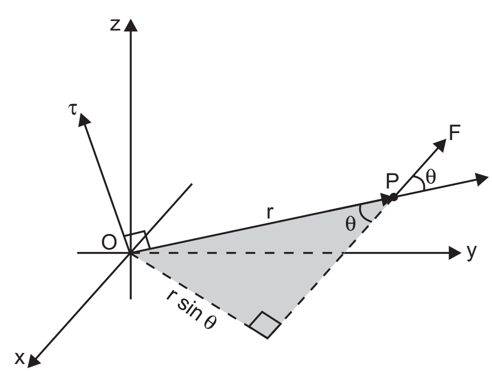
Torque is the moment of force. Torque acting on a particle is defined as the product of the magnitude of the force acting on the particle and the perpendicular distance of the application of force from the axis of rotation of the particle.

Torque or moment of force = force × perpendicular distance

$$\vec{\tau} = \vec{r} \times \vec{F} = rF \sin\theta \ \hat{n}$$

where θ is smaller angle between $\vec{r}$ and $\vec{F}$; $\hat{n}$ is unit vector along $\vec{r}$.

It is measured in Nm and has dimensions of $[ML^2T^{-2}]$.



• Angular Momentum

The angular momentum (or moment of momentum) about an axis of rotation is a vector quantity, whose magnitude is equal to the product of the magnitude of momentum and the perpendicular distance of the line of action of momentum from the axis of rotation and its direction is perpendicular to the plane containing the momentum and the perpendicular distance.

It is given by

$$\vec{L} = \vec{r} \times \vec{p}$$

SI unit of angular momentum is kg m^2s^{-1} and its dimensional formula is $[M^1L^2T^{-1}]$.

● Geometrically, the angular momentum of a particle is equal to twice the product of its mass and the areal velocity, *i.e.*,

$$L = 2\text{ m} \times \frac{dA}{dt}$$

● Torque (τ) and angular momentum are corelated as:

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

● If no net external torque acts on a system then the total angular momentum of the system remains conserved. Mathematically, if $\vec{\tau}_{ext} = \vec{0}$, then

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \ldots\ldots + \vec{L}_n = \text{a constant}$$

● Axis of Rotation

A rigid body is said to be rotating if every point mass that makes it up, describes a circular path of a different radius but the same angular speed. The circular paths of all the point masses have a common centre. A line passing through this common centre is the axis of rotation.

● A rigid body is said to be in equilibrium if under the action of forces/torques, the body remains in its position of rest or of uniform motion.

For translational equilibrium, the vector sum of all the forces acting on a body must be zero. For rotational equilibrium, the vector sum of torques of all the forces acting on that body about the reference point must be zero. For complete equilibrium, both these conditions must be fulfilled.

● Couple

Two equal and opposite forces acting on a body but having different lines of action, form a couple. The net force due to a couple is zero, but they exert a torque and produce rotational motion.

● Moment of Inertia

The rotational inertia of a rigid body is referred to as its moment of inertia.

The moment of inertia of a body about an axis is defined as the sum of the products of the masses of the particles constituting the body and the square of their respective perpendicular distance from the axis.

It is given by

$$I = m_1r_1^2 + m_2r_2^2 + \ldots\ldots\ldots\ldots + m_nr_n^2 = \sum_{i=1}^{n} m_ir_i^2,$$

where m_i is the mass and r_i the distance of the i^{th} particle of the rigid body from the axis of rotation. It is measured in kg m^2 and has the dimension of $[ML^2]$.

● Radius of Gyration

The distance of a point in a body from the axis of rotation, at which if whole of the mass of the body were supposed to be concentrated, its moment of inertia about the axis of rotation would be the same as that determined by the actual distribution of mass of the body is called radius of gyration.

If we consider that the whole mass of the body is concentrated at a distance K from the axis of rotation, then moment of inertia I can be expressed as

$$I = MK^2$$

where M is the total mass of the body and K is the radius of gyration. It is given as

$$K = \sqrt{\left(\frac{r_1^2 + r_2^2 + ... + r_n^2}{n}\right)}$$

• Theorem of Parallel Axes

According to this theorem, the moment of inertia I of a body about any axis is equal to its moment of inertia about a parallel axis through centre of mass, I_{cm}, plus Ma^2 where M is the mass of the body and 'a' is the perpendicular distance between the axes, *i.e.*,

$$I = I_{cm} + Ma^2$$

• Theorem of Perpendicular Axes

According to this theorem, the moment of inertia I of the body about a perpendicular axis is equal to the sum of moments of inertia of the body about two axes at right angles to each other in the plane of the body and intersecting at a point where the perpendicular axis passes, *i.e.*,

$$I_z = I_x + I_y$$

• A body in rotatory motion possesses rotational kinetic energy given by:

$$\text{Rotational } K.E. = \frac{1}{2} I \omega^2.$$

• In terms of moment of inertia of a body, its angular momentum is defined as the product of moment of inertia and angular velocity *i.e.*,

$$\vec{L} = I \vec{\omega}$$

• Torque may be defined as the produce of moment of inertia and the angular acceleration *i.e.*,

$$\vec{\tau} = I \vec{\alpha}$$

• Rolling Motion

The combination of rotational motion and the translational motion of a rigid body is known as rolling motion.

The kinetic energy associated with a body rolling is the sum of the translational and rotational kinetic energies, *i.e.*, $K.E$ of rolling $= \frac{1}{2} Mv^2 + \frac{1}{2} I \omega^2$

• When a body rolls down an inclined plane (θ) without slipping, the velocity on reaching the ground is,

$$v = \sqrt{\frac{2gh}{1 + \frac{K^2}{r^2}}},$$

where h is the vertical height of inclined plane and K is the radius of gyration of the rolling body.

• The acceleration of a body rolling down an inclined plane is given by

$$a = \frac{g \sin \theta}{\left(1 + \frac{K^2}{r^2}\right)}.$$

- **Law of Conservation of Angular Momentum**

According to the law of conservation of angular momentum, if there is no external couple acting, the total angular momentum of a rigid body or a system of particles is conserved.

If the moment of inertia of the body changes from I_1 to I_2 due to the change of the distribution of mass of the body, then angular velocity of the body changes from $\vec{\omega}_1$ to $\vec{\omega}_2$, such that

$$I_1 \vec{\omega}_1 = I_2 \vec{\omega}_2 \quad \text{or} \quad I_1\omega_1 = I_2\omega_2.$$

- **IMPORTANT TABLES**

TABLE 7.1

Quantity	*Symbol*	*Dimensions*	*Units*	*Remarks*
Angular velocity	ω	$[T^{-1}]$	rad s^{-1}	$\vec{v} = \vec{\omega} \times \vec{r}$
Angular Momentum	L	$[ML^2T^{-1}]$	J s	$\vec{L} = \vec{r} \times \vec{p}$
Torque	τ	$[ML^2T^{-2}]$	N m	$\vec{\tau} = \vec{r} \times \vec{F}$
Moment of inertia	I	$[ML^2]$	kg m^2	$I = \sum m_i r_i^2$

TABLE 7.2 Analogy between linear motion and rotational motion

	Linear Motion		*Rotational Motion*
1.	Distance/displacement (s)	**1.**	Angle or angular displacement (θ)
2.	Linear velocity $v = \frac{ds}{dt}$	**2.**	Angular velocity $\omega = \frac{d\theta}{dt}$
3.	Linear acceleration $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$	**3.**	Angular acceleration $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
4.	Mass (m)	**4.**	Moment of inertia (I)
5.	Linear momentum $P = mv$	**5.**	Angular momentum $L = I\omega$
6.	Force $F = ma$	**6.**	Torque $\tau = I\,\alpha$
7.	Also, force $F = \frac{dp}{dt}$	**7.**	Also, torque $\tau = \frac{dL}{dt}$
8.	Translational K.E $= \frac{1}{2}mv^2 = \frac{p^2}{2m}$	**8.**	Rotational $K.E = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$
9.	Work done, $W = Fs$	**9.**	Work done, $W = \tau\,\theta$
10.	Power $P = Fv$	**10.**	Power $= \tau\,\omega$
11.	Linear momentum of a system is conserved when no external force acts on the system. (Principle of conservation of linear momentum)	**11.**	Angular momentum of a system is conserved when no external torque acts on the system. (Principle of conservation of angular momentum)
12.	Equations of Translational motion (*i*) $v = u + at$ (*ii*) $s = ut + \frac{1}{2}at^2$ (*iii*) $v^2 - u^2 = 2as$, where the symbols have their usual meaning.	**12.**	Equations of Rotational motion (*i*) $\omega_2 = \omega_1 + \alpha t$ (*ii*) $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$ (*iii*) $\omega_2^2 - \omega_1^2 = 2\,\alpha\,\theta$, where the symbols have their usual meaning.
13.	Distance travelled in nth second $S_{nth} = u + \frac{a}{2}(2n-1)$	**13.**	Angle traced in nth second $\theta_{nth} = \omega_1 + \frac{\alpha}{2}(2n-1)$

TABLE 7.3 Moment of Inertia of some bodies of regular shape

S. No.	*Body*	*Axis*	*Moment of Inertia*
1.	Uniform rod of length l	perpendicular to rod through its centre	$\frac{1}{12} M l^2$
2.	Uniform rectangular lamina of length l and breadth b	perpendicular to lamina and through its centre	$M\left(\frac{l^2+b^2}{12}\right)$
3.	Uniform circular ring of radius R	perpendicular to its plane and through the centre	MR^2
4.	Uniform circular ring of radius R	Diameter	$MR^2/2$
5.	Uniform circular disc of radius R	perpendicular to its plane and through the centre	$\frac{1}{2}MR^2$
6.	Uniform circular disc of radius R	Diameter	$\frac{1}{4}MR^2$
7.	Hollow cylinder of radius R	Axis of cylinder	MR^2
8.	Solid cylinder of radius R	Axis of cylinder	$\frac{1}{2}MR^2$
9.	Hollow sphere of radius R	Diameter	$\frac{2}{3}MR^2$
10.	Solid sphere of radius R	Diameter	$\frac{2}{5}MR^2$

NCERT TEXTBOOK QUESTIONS SOLVED

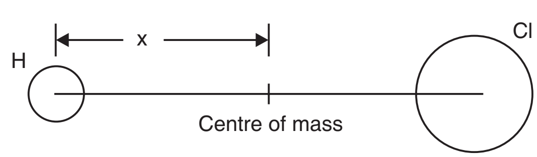
7.1. *Give the location of the centre of mass of a (i) sphere, (ii) cylinder, (iii) ring, and (iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body?*

Ans. In all the four cases, as the mass density is uniform, centre of mass is located at their respective geometrical centres.

No, it is not necessary that the centre of mass of a body should lie on the body. For example, in case of a circular ring, centre of mass is at the centre of the ring, where there is no mass.

7.2. *In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 Å (1 Å = 10^{-10} m). Find the approximate location of the CM of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus.*

Ans. Let us choose the nucleus of the hydrogen atom as the origin for measuring distance.



Mass of hydrogen atom, m_1 = 1 unit (say)

Since cholorine atom is 35.5 times as massive as hydrogen atom,

$\therefore$ mass of cholorine atom, m_2 = 35.5 units

Now, $x_1 = 0$ and $x_2 = 1.27$ Å $= 1.27 \times 10^{-10}$ m

Distance of centre of mass of HCl molecule from the origin is given by

$$X = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{1 \times 0 + 35.5 \times 1.27 \times 10^{-10}}{1 + 35.5} \text{ m}$$

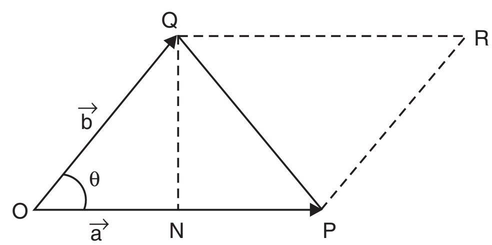
$$= \frac{35.5 \times 1.27}{36.5} \times 10^{-10} \text{ m} = 1.235 \times 10^{-10} \text{m} = 1.235 \text{ Å}$$

7.3. *A child sits stationary at one end of a long trolley moving uniformly with a speed V on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of the CM of the (trolley + child) system?*

Ans. When the child gets up and runs about on the trolley, the speed of the centre of mass of the trolley and child remains unchanged irrespective of the manner of motion of child. It is because here child and trolley constitute one single system and forces involved are purely internal forces. As there is no external force, there is no change in momentum of the system and velocity remains unchanged.

7.4. *Show that the area of the triangle contained between the vectors $\vec{a}$ and $\vec{b}$ is one half of the magnitude of $\vec{a} \times \vec{b}$.*

Ans. Let $\vec{a}$ be represented $\overrightarrow{OP}$ and $\vec{b}$ be represented by $\overrightarrow{OQ}$. Let $\angle POQ = \theta$, Fig.



Complete the ∥ gm *OPRQ*. Join *PQ*.

Draw $QN \perp OP$

In ΔOQN, $\sin\theta = \frac{QN}{OQ} = \frac{QN}{b}$

$QN = b \sin\theta$

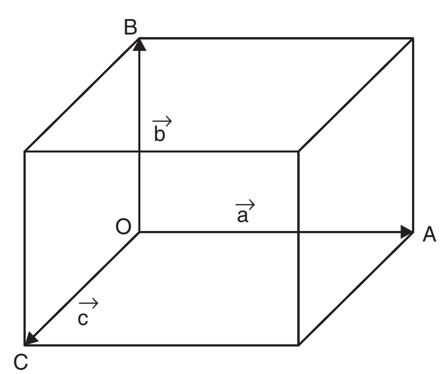
Now, by definition, $|\vec{a} \times \vec{b}| = ab \sin\theta = (OP)(QN)$

$$= \frac{2(OP)(QN)}{2} = 2 \times \text{area of } \Delta OPQ$$

$\therefore$ area of $\Delta OPQ = \frac{1}{2}|\vec{a} \times \vec{b}|$, which was to be proved.

7.5. *Show that $\vec{a}.(\vec{b} \times \vec{c})$ is equal in magnitude to the volume of the parallelopiped formed on the three vectors, $\vec{a}$, $\vec{b}$ and $\vec{c}$.*

Ans. Let *a* parallelopiped be formed on the three vectors.



$\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$

and $\overrightarrow{OC} = \vec{c}$

Now, $\vec{b} \times \vec{c} = bc \sin 90° \ \hat{n} = bc\ \hat{n}$

where $\hat{n}$ is unit vector along $\overrightarrow{OA}$ perpendicular to the plane containing $\vec{b}$ and $\vec{c}$.

Now $\vec{a}.(\vec{b} \times \vec{c}) = \vec{a}.\ bc\ \hat{n}$

$= (a)(bc) \cos 0°$

$= a\ b\ c$

which is equal in magnitude to the volume of the parallelopiped.

7.6. *Find the components along the x, y, z-axes of the angular momentum $\vec{l}$ of a particle, whose position vector is $\vec{r}$ with components x, y, z and momentum is $\vec{p}$ with components p_x, p_y and p_z. Show that if the particle moves only in the x-y plane the angular momentum has only a z-component.*

Ans. We know that angular momentum $\vec{l}$ of a particle having position vector $\vec{r}$ and momentum $\vec{p}$ is given by

$$\vec{l} = \vec{r} \times \vec{p}$$

But $\vec{r} = [x\hat{i} + y\hat{j} + z\hat{k}]$, where x, y, z are the components of

$$\vec{r} \text{ and } \vec{p} = [p_x\hat{i} + p_y\hat{j} + p_z\hat{k}]$$

$$\therefore \quad \vec{l} = \vec{r} \times \vec{p} = [x\hat{i} + y\hat{j} + z\hat{k}] \times [p_x\hat{i} + p_y\hat{j} + p_z\hat{k}]$$

or
$$(l_x\hat{i} + l_y\hat{j} + l_z\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

$$= (yp_z - zp_y)\,\hat{i} + (zp_x - xp_z)\,\hat{j} + (xp_y - yp_x)\,\hat{k}$$

From this relation, we conclude that

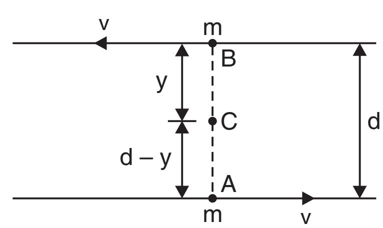
$$l_x = yp_z - zp_y, \quad l_y = zp_x - xp_z \quad \text{and} \quad l_z = xp_y - yp_x$$

If the given particle moves only in the $x - y$ plane, then $z = 0$ and $p_z = 0$ and hence, $\vec{l} = (xp_y - yp_x)\,\hat{k}$, which is only the z-comonent of $\vec{l}$.

It means that for a particle moving only in the $x - y$ plane, the angular momentum has only the z-component.

7.7. *Two particles, each of mass m and speed v, travel in opposite directions along parallel lines separated by a distance d. Show that the vector angular momentum of the two particle system is the same whatever be the point about which the angular momentum is taken.*

Ans. Angular momentum about A,



$$L_A = mv \times 0 + mv \times d$$
$$= mvd$$

Angular momentum about B,

$$L_B = mv \times d + mv \times 0$$
$$= mvd$$

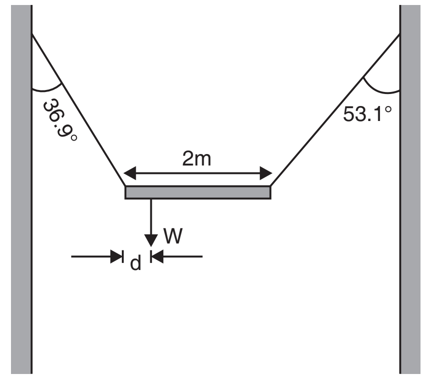
Angular momentum about C,

$$L_c = mv \times y + mv \times (d - y) = mvd$$

In all the three cases, the direction of angular momentum is the same.

$$\therefore \quad \vec{L}_A = \vec{L}_B = \vec{L}_C.$$

7.8. *A non-uniform bar of weight W is suspended at rest by two strings of negligible weight as shown in Fig. The angles made by the strings with the vertical are 36.9° and 53.1° respectively. The bar is 2 m long. Calculate the distance d of the centre of gravity of the bar from its left end.*



Ans. As it is clear from Fig.,

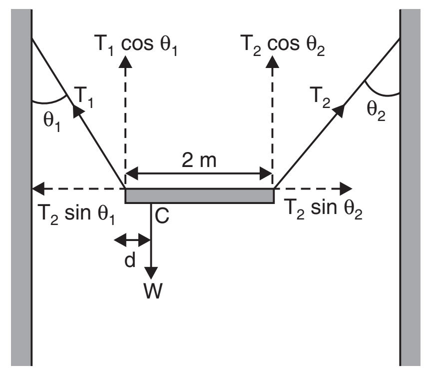
$$\theta_1 = 36.9°, \quad \theta_2 = 53.1°.$$

If T_1, T_2 are the tensions in the two strings, then for equilibrium along the horizontal,

$$T_1 \sin \theta_1 = T_2 \sin \theta_2$$

or $$\frac{T_1}{T_2} = \frac{\sin \theta_2}{\sin \theta_1} = \frac{\sin 53.1°}{\sin 36.9°}$$

$$= \frac{0.7404}{0.5477} = 1.3523$$



Let d be the distance of centre of gravity C of the bar from the left end.

For rotational equilibrium about C,

$$T_1 \cos \theta_1 \times d = T_2 \cos \theta_2 \, (2 - d)$$

$$T_1 \cos 36.9° \times d = T_2 \cos 53.1° \, (2 - d)$$

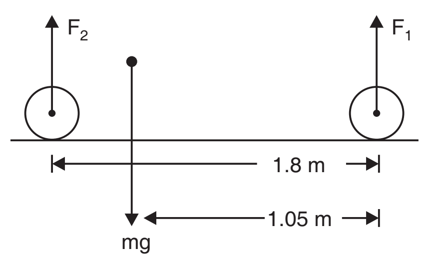
$$T_1 \times 0.8366 \, d = T_2 \times 0.6718 \, (2 - d)$$

Put $T_1 = 1.3523 \, T_2$ and solve to get

$$d = 0.745 \text{ m}$$

7.9. *A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05 m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.*

Ans. Let F_1 and F_2 be the forces exerted by the level ground on front wheels and back wheels respectively.



Considering rotational equilibrium about the front wheels,

$$F_2 \times 1.8 = mg \times 1.05$$

or $$F_2 = \frac{1.05}{1.8} \times 1800 \times 9.8 \text{ N}$$

$$= 10290 \text{ N}$$

Force on each back wheel is

$$\frac{10290}{2} \text{ N or } 5145 \text{ N.}$$

Considering rotational equilibrium about the back wheels.

$$F_1 \times 1.8 = mg \, (1.8 - 1.05) = 0.75 \times 1800 \times 9.8$$

or $$F_1 = \frac{0.75 \times 1800 \times 9.8}{1.8} = 7350 \text{ N.}$$

Force on each front wheel is $\frac{7350}{2}$ N or 3675 N.

7.10. *(a) Find the moment of inertia of a sphere about a tangent to the sphere, given the moment of inertia of the sphere about any of its diameters to be 2 $MR^2/5$, where M is the mass of the sphere and R is the radius of the sphere.*

(b) Given the moment of inertia of a disc of mass M and radius R about any of its diameters to be $\frac{1}{4}MR^2$, find the moment of inertia about an axis normal to the disc passing through a point on its edge.

Ans. (*a*) Moment of inertia of sphere about any diameter = $\frac{2}{5}MR^2$

Applying theorem of parallel axes,

Moment of inertia of sphere about a tangent to the sphere = $\frac{2}{5}MR^2 + M(R)^2 = \frac{7}{5}MR^2$.

(*b*) We are given, moment of inertia of the disc about any of its diameters = $\frac{1}{4}MR^2$

(*i*) Using theorem of perpendicular axes, moment of inertia of the disc about an axis passing through its centre and normal to the disc = $2 \times \frac{1}{4} MR^2 = \frac{1}{2}MR^2$.

(*ii*) Using theorem axes, moment of inertia of the disc passing through a point on its edge and normal to the dics = $\frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$.

7.11. *Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time?*

Ans. Let M be the mass and R the radius of the hollow cylinder, and also of the solid sphere. Their moments of inertia about the respective axes are

$$I_1 = MR^2 \quad \text{and} \quad I_2 = \frac{2}{5}MR^2$$

Let τ be the magnitude of the torque applied to the cylinder and the sphere, producing angular accelerations α_1 and α_2 respectively. Then

$$\tau = I_1\alpha_1 = I_2\alpha_2$$

or $$\frac{\alpha_1}{\alpha_2} = \frac{I_2}{I_1} = \frac{(2/5)MR^2}{MR^2} = \frac{2}{5} \quad \text{or} \quad \alpha_2 = \frac{5}{2}\alpha_1.$$

The angular acceleration α_2 produced in the sphere is larger. Hence, the sphere will acquire larger angular speed after a given time.

7.12. *A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad s^{-1}. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?*

Ans. $M = 20$ kg

Angular speed, $\omega = 100$ rad s^{-1}; $R = 0.25$ m

Moment of inertia of the cylinder about its axis

$$= \frac{1}{2}MR^2 = \frac{1}{2} \times 20 \times (0.25)^2 \text{ kg m}^2 = 0.625 \text{ kg m}^2$$

Rotational kinetic energy,

$$E_r = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.625 \times (100)^2 \text{ J} = 3125 \text{ J}$$

Angular momentum,

$$L = I\omega = 0.625 \times 100 \text{ Js} = 62.5 \text{ Js.}$$

7.13. *(a) A child stands at the centre of a turntable with his arms outstretched. The turntable is set rotating with an angular speed of 40 rev/min. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to 2/5 times the initial value? Assume that the turntable rotates without friction. (b) Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account of this increase in kinetic energy?*

Ans. (*a*) Suppose, initial moment of inertia of the child is I_1. Then final moment of inertia,

$$I_2 = \frac{2}{5}I_1$$

Also, $\nu_1 = 40$ rev min^{-1}

By using the principle of conservation of angular momentum, we get

$$I_1\omega_1 = I_2\omega_2 \quad \text{or} \quad I_1 (2\pi\nu_1) = I_2(2\pi\nu_2)$$

or $$\nu_2 = \frac{I_1\nu_1}{I_2} = \frac{I_1 \times 40}{\frac{2}{5} \times I_1} = 100 \text{ rev min}^{-1}$$

(*b*) $$\frac{\text{Final K.E. of rotation}}{\text{Initial K.E. of rotation}} = \frac{\frac{1}{2}I_2\omega_2^2}{\frac{1}{2}I_1\omega_1^2} = \frac{\frac{1}{2}I_2(2\pi\nu_2)^2}{\frac{1}{2}I_1(2\pi\nu_1)^2} = \frac{I_2\nu_2^2}{I_1\nu_1^2} = \frac{\frac{2}{5}I_1 \times (100)^2}{\frac{2}{5}I_1 \times (40)^2} = 2.5$$

Clearly, final $(\text{K.E.})_{rot}$ becomes more because the child uses his internal energy when he folds his hands to increase the kinetic energy.

7.14. *A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N? What is the linear acceleration of the rope? Assume that there is no slipping.*

Ans. Here, $M = 3$ kg, $R = 40$ cm $= 0.4$ m

Moment of inertia of the hollow cylinder about its axis.

$$I = MR^2 = 3(0.4)^2 = 0.48 \text{ kg m}^2$$

Force applied $F = 30$ N

∴ Torque, $\tau = F \times R = 30 \times 0.4 = 12$ N–m.

If α is angular acceleration produced, then from $\tau = I\alpha$

$$\alpha = \frac{\tau}{I} = \frac{12}{0.48} = 25 \text{ rad s}^{-2}$$

Linear acceleration, $a = R\alpha = 0.4 \times 25 = 10 \text{ ms}^{-2}$.

7.15. *To maintain a rotor at a uniform angular speed of 200 rad s^{-1}, an engine needs to transmit a torque of 180 Nm. What is the power required by the engine?*

Note: Uniform angular velocity in the absence of friction implies zero torque. In practice, applied torque is needed to counter fricitional torque). Assume that the engine is 100% efficient.

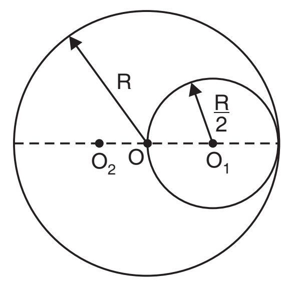
Ans. Here, $\omega = 200 \text{ rad s}^{-1}$; Torque, $\tau = 180$ N-m

Since, Power, P = Torque (τ) × angular speed (ω)

$= 180 \times 200 = 36000$ watt

$= 36$ KW.

7.16. *From a uniform disk of radius R, a circular hole of radius R/2 is cut out. The centre of the hole is at R/2 from the centre of the original disc. Locate the centre of gravity of the resulting flat body.*



Ans. Let from a bigger uniform disc of radius R with centre O a smaller circular hole of radius $\frac{R}{2}$ with its centre at O_1 (where $OO_1 = \frac{R}{2}$) is cut out. Let centre of gravity or the centre of mass of remaining flat body be at O_2, where $OO_2 = x$. If σ be mass per unit area, then mass of whole disc $M_1 = \pi R^2\sigma$ and mass of cut out part

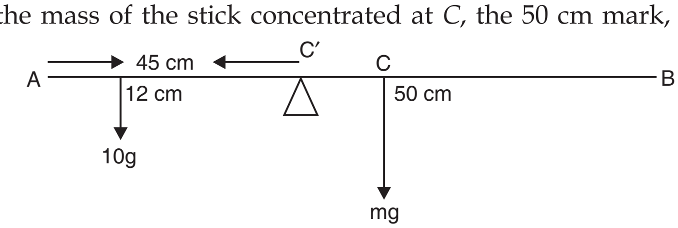
$$M_2 = \pi\left(\frac{R}{2}\right)^2 \sigma = \frac{1}{4}\pi R^2 \sigma = \frac{M_1}{4}$$

$$\therefore \quad x = \frac{M_1 \times (0) - M_2(OO_1)}{M_1 - M_2} = \frac{0 - \frac{M_1}{4} \times \frac{R}{2}}{M_1 - \frac{M_1}{4}} = -\frac{R}{6}$$

i.e., O_2 is at a distance $\frac{R}{6}$ from centre of disc on diametrically opposite side to centre of hole.

7.17. *A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of the metre stick?*

Ans. Let m be the mass of the stick concentrated at C, the 50 cm mark, see fig.



For equilibrium about C', the 45 cm mark,

$$10g\,(45 - 12) = mg\,(50 - 45)$$

$$10g \times 33 = mg \times 5$$

$$\Rightarrow \qquad m = \frac{10 \times 33}{5} \quad \text{or} \quad m = 66 \text{ grams.}$$

7.18. *A solid sphere rolls down two different inclined planes of the same heights but different angles of inclination. (a) Will it reach the bottom with the same speed in each case? (b) Will it take longer to roll down one plane than the other? (c) If so, which one and why?*

Ans. (*a*) Using law of conservation of energy,

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh \quad \text{or} \quad \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\frac{v^2}{R^2} = mgh$$

$$\text{or} \qquad \frac{7}{10}v^2 = gh \quad \text{or} \quad v = \sqrt{\frac{10gh}{7}}$$

Since h is same for both the inclined planes therefore v **is the same.**

$$(b) \qquad l = \frac{1}{2}\left(\frac{g \sin\theta}{1+\frac{K^2}{R^2}}\right)s\; t^2 = \frac{g\sin\theta}{2\left(1+\frac{2}{5}\right)}t^2 = \frac{5g\sin\theta}{14}t^2$$

$$\text{or} \qquad t = \sqrt{\frac{14\,l}{5g\sin\theta}}$$

$$\text{Now,} \qquad \sin\theta = \frac{h}{l} \quad \text{or} \quad l = \frac{h}{\sin\theta}$$

$$\therefore \qquad t = \frac{1}{\sin\theta}\sqrt{\frac{14\,h}{5g}}$$

Lesser the value of θ, more will be t.

(*c*) Clearly, the solid sphere will take longer to roll down the plane with smaller inclination.

7.19. *A hoop of radius 2 m weighs 100 kg. It rolls along a horizontal floor so that its centre of mass has a speed of 20 cm/s. How much work has to be done to stop it?*

Ans. Here, $R = 2$ m, $M = 100$ kg

$v = 20$ cm/s $= 0.2$ m/s

$$\text{Total energy of the hoop} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{2}(MR^2)\,\omega^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = Mv^2$$

Work required to stop the hoop = total energy of the hoop

$$W = Mv^2 = 100\,(0.2)^2 = 4 \text{ Joule.}$$

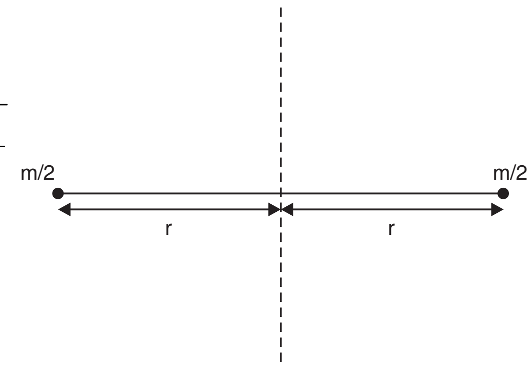
7.20. *The oxygen molecule has a mass of 5.30 × 10^{-26} kg and a moment of inertia of 1.94 × 10^{-45} kg m^2 about an axis through its centre perpendicular to the lines joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500 m/s and that its kinetic energy of rotation is two thirds of its kinetic energy of translation. Find the average angular velocity of the molecule.*

Ans. Here, $m = 5.30 \times 10^{-26}$ kg

$I = 1.94 \times 10^{-46}$ kg m^2

$v = 500$ m/s

If $\frac{m}{2}$ is mass of each atom of oxygen and $2r$ is distance between the two atoms as shown in Fig, then



$$I = \frac{m}{2}r^2 + \frac{m}{2}r^2 = mr^2$$

$$r = \sqrt{\frac{I}{m}} = \sqrt{\frac{1.94 \times 10^{-46}}{5.30 \times 10^{-26}}}$$

$$= 0.61 \times 10^{-10} \text{ m}$$

As K.E. of rotation $= \frac{2}{3}$ K.E. of translation

$$\therefore \quad \frac{1}{2}I\omega^2 = \frac{2}{3} \times \frac{1}{2}m\omega^2$$

$$\frac{1}{2}(mr^2)\,\omega^2 = \frac{1}{2}mv^2$$

$$\omega = \sqrt{\frac{2}{3}}\frac{v}{r} = \sqrt{\frac{2}{3}} \times \frac{500}{0.61 \times 10^{-10}} = 6.7 \times 10^{12} \text{ rad/s.}$$

7.21. *A solid cylinder rolls up an inclined plane of angle of inclination 30°. At the bottom of the inclined plane the centre of mass of the cylinder has a speed of 5 m/s.*

(a) How far will the cylinder go up the plane?

(b) How long will it take to return to the bottom?

Ans. Here, $\theta = 30°$, $v = 5$ m/s

Let the cylinder go up the plane upto a height h.

From $$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2 = mgh$$

$$\frac{3}{4}mv^2 = mgh$$

$$h = \frac{3v^2}{4g} = \frac{3 \times 5^2}{4 \times 9.8} = 1.913 \text{ m}$$

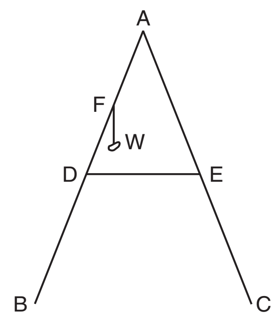
If s is the distance up the inclined plane, then as

$$\sin\theta = \frac{h}{s}, \quad s = \frac{h}{\sin\theta} = \frac{1.913}{\sin 30°} \; 3.856 \text{ m}$$

Time taken to return to the bottom

$$t = \sqrt{\frac{2s\left(1+\frac{k^2}{r^2}\right)}{g \sin \theta}} = \sqrt{\frac{2 \times 3.826\left(1+\frac{1}{2}\right)}{9.8 \sin 30°}} = 1.53\text{s}.$$

7.22. *As shown in Fig. the two sides of a step ladder BA and CA are 1.6 m long and hinged at A. A rope DE, 0.5 m is tied half way up. A weight 40 kg is suspended from a point F, 1.2 m from B along the ladder BA. Assuming the floor to be frictionless and neglecting the weight of the ladder, find the tension in the rope and forces exerted by the floor on the ladder. (Take g = 9.8 m s*^{2}*)*



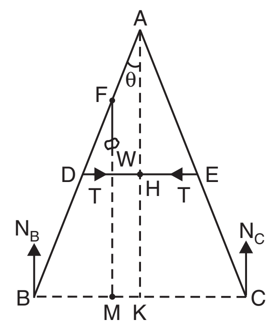
*(**Hint:** Consider the equilibrium of each side of the ladder separately.)*

Ans. The forces acting on the ladder are shown in Fig. 7.14.

Here, W = 40 kg = 40 × 9.8 N = 392 N, $AB = AC$ = 1.6 m, $BD = \frac{1}{2} \times 1.6$ m = 0.8 m,

BF = 1.2 m and DE = 0.5 m,

In the Fig. ΔADE and ΔABC are similar triangles, hence



$$BC = DE \times \frac{AB}{AD} = \frac{0.5 \times 1.6}{0.8} = 1.0 \text{ m}$$

Now, considering equilibrium at point B, $\sum \tau = 0$

$\therefore \quad W \times (MB) = N_C \times (CB)$ ...(*i*)

But $\quad MB = \frac{KB \times BF}{BA} = \frac{0.5 \times 1.2}{1.6} = 0.375$ m

Substituting this value in (*i*), we get

$$\therefore \quad N_C = \frac{W \times (MB)}{(CB)} = \frac{392 \times 0.375}{1} = 147 \text{ N}$$

Again considering equilibrium at point C in similar manner, we have

$$W \times (MC) = N_B \times (BC)$$

$$\therefore \quad N_B = \frac{W \times (MC)}{(BC)} = \frac{W \times (BC - BM)}{(BC)}$$

$$= \frac{392 \times (1 - 0.375)}{1} = 245 \text{ N}$$

Now, it can be easily shown that tension in the string $T = N_B - N_C$ = 245 – 147 = 98 N.

7.23. *A man stands on a rotating platform, with his arms stretched horizontally holding a 5 kg weight in each hand. The angular speed of the platform is 30 revolutions per minutes. The man then brings his arms close to his body with the distance of each weight from the axis changing from 90 cm to 20 cm. The moment of inertia of the man together with the platform may be taken to be constant and equal to 7.6 kg m*^{2}*.*

(a) What is his new angular speed? (Neglect friction)

(b) Is kinetic energy conserved in the process? If not, from where does the change come about?

Ans. Here, $\quad I_1 = 7.6 + 2 \times 5\, (0.9)^2 = 15.7$ kg m^2

ω_1 = 30 rpm

I_2 = 7.6 + 2 × 5 $(0.2)^2$ = 8.0 kg m^2

ω_2 = ?

According to the principle of conservation of angular momentum,

$$I_2\omega_2 = I_1\omega_1$$

$$\omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{15.7 \times 30}{8.0} = 58.88 \text{ rpm}$$

No, kinetic energy is not conserved in the process. In fact, as moment of inertia decreases, K.E. of rotation increases. This change comes about as work is done by the man in bringing his arms closer to his body.

7.24. *A bullet of mass 10 g and speed 500 m/s is fired into a door and gets embedded exactly at the centre of the door. The door is 1.0 m wide and weighs 12 kg. It is hinged at one end and rotates about a vertical axis practically without friction. Find the angular speed of the door just after the bullet embeds into it.*

*(**Hint:** The moment of inertia of the door about the vertical axis at one end is $ML^2/3$.)*

Ans. Angular momentum imparted by the bullet, $L = mv \times r$

$$= (10 \times 10^{-3}) \times 500 \times \frac{1}{2} = 2.5$$

Also, $$I = \frac{ML^2}{3} = \frac{12 \times (1.0)^2}{3} = 4 \text{ kg m}^2$$

Since $$L = I\omega$$

∴ $$\omega = \frac{L}{I} = \frac{2.5}{4} = 0.625 \text{ rad/s.}$$

7.25. *Two discs of moments of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre), and rotating with angular speed ω_1 and ω_2 are brought into contact face to face with their axes of rotation coincident. (a) What is the angular speed of the two-disc system? (b) Show that the kinetic energy of the combined system is less than the sum of the initial kinetic energies of the two discs. How do you account for this loss in energy? Take $\omega_1 \neq \omega_2$.*

Ans. (*a*) Let I_1 and I_2 be the moments of inertia of two discs having angular speeds ω_1 and ω_2 respectively. When they are brought in contact, the moment of inertia of the two-disc system will be $I_1 + I_2$. Let the system now have an angular speed ω. From the law of conservation of angular momentum, we know that

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\,\omega$$

∴ The angular speed of the two-disc system,

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

(*b*) The sum of kinetic energies of the two discs before coming in contact,

$$k_1 = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$$

The final kinetic energy of the two-disc system,

$$k_2 = \frac{1}{2}(I_1 + I_2)\,\omega^2 = \frac{1}{2}(I_1 + I_2) \times \left(\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right)^2 = \frac{1}{2}\frac{(I_1\omega_1 + I_2\omega_2)^2}{I_1 + I_2}$$

Now, $$k_1 - k_2 = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 - \frac{1}{2}\frac{(I_1\omega_1 + I_2\omega_2)^2}{I_1 + I_2}$$

$$= \frac{1}{2(I_1 + I_2)} \times \left[(I_1\omega_1^2 + I_2\omega_2^2)(I_1 + I_2) - (I_1^2\omega_1^2 + I_2^2\omega_2^2 + 2I_1I_2\,\omega_1\omega_2)\right]$$

$$= \frac{1}{2(I_1 + I_2)} \times [I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_1I_2\omega_1^2 + I_1I_2\,\omega_2^2$$

$$-I_1^2\omega_1^2 - I_2^2\omega_2^2 - 2I_1I_2\omega_1\omega_2]$$

$$= \frac{1}{2(I_1 + I_2)}[I_1I_2(\omega_1^2 + \omega_2^2 - 2\omega_1\omega_2)] = \frac{I_1I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2$$

Now, $(\omega_1 - \omega_2)^2$ will be positive whether ω_1 is greater or smaller than ω_2.

Also, $I_1I_2/2(I_1 + I_2)$ is also positive because I_1 and I_2 are positive.

Thus, $k_1 - k_2$ is a positive quantity.

$\therefore$ $k_1 = k_2$ + a positive quantity or $k_1 > k_2$

$\therefore$ The kinetic energy of the combined system (k_2) is less than the sum of the kinetic energies of the two dics.

The loss of energy on combining the two discs is due to the energy being used up because of the frictional forces between the surfaces of the two discs. These forces, in fact, bring about a common angular speed of the two discs on combining.

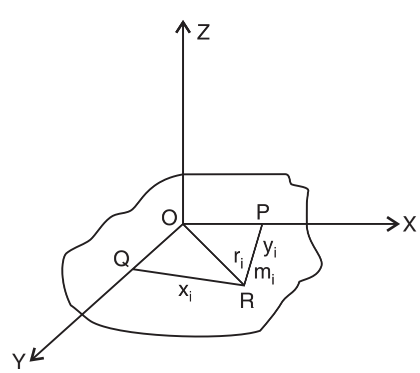
7.26. *(a) Prove the theorem of perpendicular axes.*

Hint: *Square of the distance of a point (x, y) in the x – y plane from an axis through the origin perpendicular to the plane is $x^2 + y^2$]*

(b) Prove the theorem of parallel axes.

Hint: *If the centre of mass of chosen the origin $\sum m_i r_i = 0$]*

Ans. (*a*) **The theorem of perpendicular axes:** According to this theorem, the moment of inertia of a plane lamina (*i.e.*, a two dimensional body of any shape/size) about any axis *OZ* perpendicular to the plane of the lamina is equal to sum of the moments of inertia of the lamina about any two mutually perpendicular axes *OX* and *OY* in the plane of lamina, meeting at a point where the given axis *OZ* passes through the lamina.

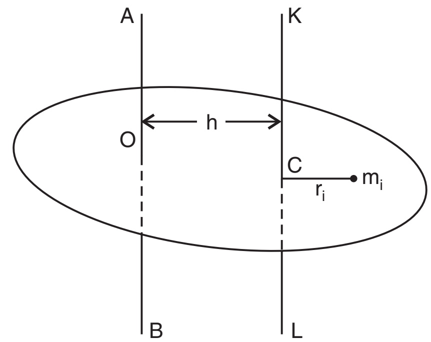


Suppose at the point '*R*' m_i particle is situated moment of inertia about *Z* axis of lamina

$$I_z = \sum m_i r_i^2 = \sum m_i(x_i^2 + y_i^2) = \sum m_i x_i^2 + \sum m_i y_i^2$$

or	$I_z = I_x + I_y$	
where	I_x = moment of inertia of body about x-axis	
and	I_y = moment of inertia of body about y-axis.	(**Proved**)

(*b*) **Theorem of parallel axes:** According to this theorem, moment of inertia of a rigid body about any axis AB is equal to moment of inertia of the body about another axis KL passing through centre of mass C of the body in a direction parallel to AB, plus the product of total mass M of the body and square of the perpendicular distance between the two parallel axes.



If h is perpendicular distance between the axes AB and KL, then

Suppose rigid body is made up of n particles $m_1, m_2, ... m_i, ... m_n$ at perpendicular distances $r_1, r_2, ... r_i .... r_n$. respectively from the axis KL passing through centre of mass C of the body.

If r_i is the perpendicular distance of the particle of mass m_i from KL, then

$$I_{KL} = \sum_i m_i r_i^2 \qquad ...(i)$$

The perpendicular distance of i^{th} particle from the axis

$$AB = (r_i + h)$$

or
$$I_{AB} = \sum_i m_i (r_i + h)^2 = \sum_i m_i (r_i^2 + h^2 + 2r_i h)$$

$$= \sum_i m_i r_i^2 + \sum_i m_i h^2 + 2h \sum_i m_i r_i \qquad ...(ii)$$

As the body is balanced about the centre of mass, the algebraic sum of the moments of the weights of all particles about an axis passing through C must be zero.

$$\sum_i (m_i g) r_i = 0 \quad \text{or} \quad g \sum_i m_i r_i \quad \text{or} \quad \sum_i m_i r_i = 0 \qquad ...(iii)$$

From equation (*ii*), we have

$$I_{AB} = \sum_i m_i r_i^2 + \left(\sum m_i\right) h^2 + 0 \quad \text{or} \quad I_{AB} = I_{KL} + Mh^2$$

where
$$I_{KL} = \sum_i m_i r_i^2 \quad \text{and} \quad M = \sum m_i$$

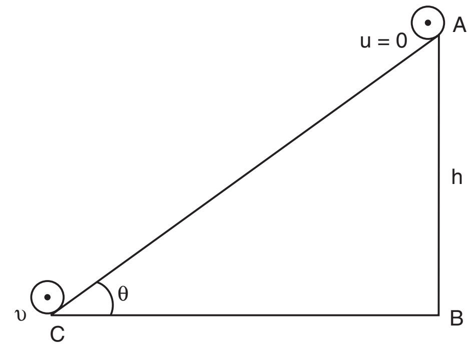
7.27. *Prove the result that the velocity v of translation of a rolling body (like a ring, disc, cylinder or sphere) at the bottom of an inclined plane of a height h is given by,*

$$v^2 = \frac{2gh}{1 + k^2/R^2}$$

using dynamical consideration (i.e., by consideration of forces and torques). Note k is the radius of gyration of the body about its symmetry axis, and R is the radius of the body. The body starts from rest at the top of the plane.

Ans. Let a rolling body ($I = mk^2$) rolls down an inclined plane with an initial velocity $u = 0$; When it reaches the bottom of inclined plane, let its linear velocity be v. Then from conservation of mechanical energy, we have

Loss in *P.E.* = Gain in translational K.E. + Gain in rotational K.E.

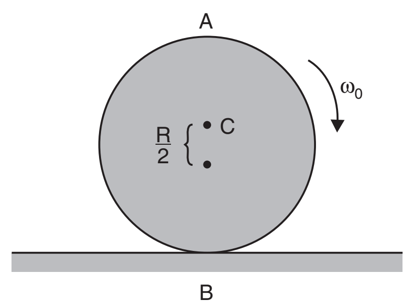


$$\therefore \quad mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}(mk^2)\left(\frac{v^2}{R^2}\right)$$

$$\therefore \quad mgh = \frac{1}{2}mv^2\left(1 + \frac{k^2}{R^2}\right) \Rightarrow v^2 = \frac{2gh}{\left(1 + \frac{k^2}{R^2}\right)}.$$

7.28. *A disc rotating about its axis with angular speed ω_0 is placed lightly (without any translational push) on a perfectly frictionless table. The radius of the disc is R. What are the linear velocities of the points A, B and C on the disc shown in Fig.? Will the disc roll in the direction indicated?*



Ans. Since $v = r\omega$,

For point, *A*, $v_A = R\omega_0$ in the direction of arrow.

For point, *B*, $v_B = R\omega_0$ in the opposite direction of arrow.

For point, *C*, $v_C = \frac{R}{2}\omega_0$ in the direction of arrow.

The disc will not roll in the given direction because friction is necessary for the same.

7.29. *Explain why friction is necessary to make the disc roll (refer to Q. 28) in the direction indicated.*

(a) Give the direction of frictional force at B, and the sense of frictional torque, before perfect rolling begins.

(b) What is the force of friction after perfect rolling begins?

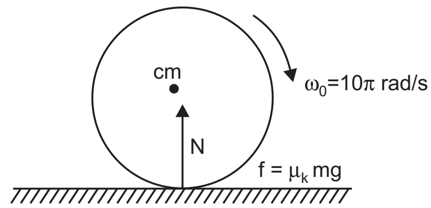
Ans. To roll a disc, we require a torque, which can be provided only by a tangential force. As force of friction is the only tangential force in this case, it is necessary.

(*a*) As frictional force at *B* opposes the velocity of point *B*, which is to the left, the frictional force must be to the right. The sense of frictional torque will be perpendicular to the plane of the disc and outwards.

(*b*) As frictional force at *B* decreases the velocity of the point of contact *B* with the surface, the perfect rolling begins only when velocity of point *B* becomes zero. Also, force of friction would become zero at this stage.

7.30. *A solid disc and a ring, both of radius 10 cm are placed on a horizontal table simultaneously, with initial angular speed equal to 10π rad/s. Which of two will start to roll earlier? The coefficient of kinetic friction is μ_k = 0.2.*

Ans. When a disc or ring starts rotatory motion on a horizontal surface, initial translational velocity of centre of mass is zero.



The frictional force causes the centre of mass to accelerate linearly but frictional torque causes angular retardation. As force of normal reaction $N = mg$, hence frictional force $f = \mu_k N = \mu_k\, mg$.

For linear motion $\quad f = \mu_k \cdot mg = ma$ ...(*i*)

and for rotational motion, $\tau = f \cdot R = \mu_k\, mg \cdot R = -I\alpha$ ...(*ii*)

Let perfect rolling motion starts at time t, when $v = R\omega$

From (*i*) $\quad a = \mu_k \cdot g$

$\therefore \quad v = u + at = 0 + \mu_k \cdot g \cdot t$ ...(*iii*)

From (*ii*) $$\alpha = -\frac{\mu_k \cdot mgR}{I} = -\frac{\mu_k \cdot mgR}{mK^2} = -\frac{\mu_k \cdot gR}{K^2}$$

$\therefore$ $$\omega = \omega_0 + \alpha t = \omega_0 - \frac{\mu_k \cdot gR}{K^2}t \quad ...(iv)$$

Since $\quad v = R\omega$, hence $\mu_k \cdot g \cdot t = R\left[\omega_0 - \mu_k \cdot \frac{gR}{K^2}t\right]$

$$\Rightarrow \quad t^2 = \frac{R\omega_0}{\mu_k \cdot g\left(1 + \frac{R^2}{K^2}\right)}$$

For disc, $\quad K^2 = \frac{R^2}{2}$, hence $t = \frac{\omega_0 R}{\mu_k \cdot g\left(1 + \frac{R^2}{R^2/2}\right)} = \frac{\omega_0 R}{3\mu_k \cdot g}$

For ring, $\quad K^2 = R^2$, hence $t = \frac{\omega_0 R}{\mu_k \cdot g\left(1 + \frac{R^2}{R^2}\right)} = \frac{\omega_0 R}{2\mu_k \cdot g}$

Thus, it is clear that disc will start to roll earlier. The actual time at which disc starts rolling will be

$$t = \frac{\omega_0 R}{2\mu_k \cdot g} = \frac{(10\pi) \times (0.1)}{3 \times (0.2) \times 9.8} = 0.538s.$$

7.31. *A cylinder of mass 10 kg and radius 15 cm is rolling perfectly on a plane of inclination 30°. The coefficient of static friction* $\mu_s = 0.25$.

(a) How much is the force of friction acting on the cylinder?

(b) What is the work done against friction during rolling?

(c) If the inclination θ of the plane is increased, at what value of θ does the cylinder begin to skid, and not roll perfectly?

Ans. (*a*) $$f = \frac{1}{3} mg \sin\theta$$

$$= \frac{1}{3} \times 10 \times 9.8 \times \sin 30° \text{ N} = \mathbf{16.3\ N.}$$

(*b*) **No work is done** against friction during rolling.

(c) $\mu = \frac{1}{3} \tan\theta$ or $\tan\theta = 3\mu$

$\tan\theta = 3 \times 0.25 = 0.75$

$\theta = \tan^{-1}(0.75) = 36.87° =$ **37°.**

7.32. *Read each statement below carefully, and state, with reasons, if it is true or false:*

(a) During rolling, the force of friction acts in the same direction as the direction of motion of the CM of the body.

(b) The instantaneous speed of the point of contact during rolling is zero.

(c) The instantaneous acceleration of the point of contact during rolloing is zero.

(d) For perfect rolling motion, work done against friction is zero.

(e) A wheel moving down a perfectly frictionless inclined plane will undergo slipping (not rolling) motion.

Ans. (*a*) **True.** When a body rolls without slipping, the force of friction acts in the same direction as the direction of motion of the centre of mass of rolling body.

(*b*) **True.** This is because rolling body can be imagined to be rotating about an axis passing through the point of contact of the body with the ground. Hence its instantaneous speed is zero.

(*c*) **False.** This is because when the body is rotating, its instantaneous acceleration is not zero.

(*d*) **True.** For perfect rolling motion as there is no relative motion at the point of contact, hence work done against friction is zero.

(*e*) **True.** This is because rolling occurs only on account of friction which is a tangential force capable of providing torque. When the inclined plane is perfectly smooth, it will simply slip under the effect of its own weight.

7.33. *Separation of Motion of a system of particles into motion of the centre of mass and motion about the centre of mass:*

(a) Show $\vec{p_i} = \vec{p_i}' + m_i \vec{V}$

where p_i is the momentum of the i^{th} particle (of mass m_i) and $\vec{p_i}' + m_i \overrightarrow{v'_i}$. Note $\overrightarrow{v'_i}$ is the veloity of i^{th} particle relative to the centre of mass.

Also, prove using the definition of the centre of mass $\sum \vec{p_i} = 0$

(b) Show $K = K' + \frac{1}{2}MV^2$

where K is the total kinteic energy of the system of particles, K′ is the total kinetic energy of the system when the particle velocities are taken with respect to the centre of mass and MV^2/2 is the kinetic energy of the translation of the system as a whole (i.e., of the centre of mass motion of the system). The result has been used in Sec. 7.14.

(c) Show $\vec{L} = m_i \overrightarrow{L'} + M\vec{R} \times \vec{V}$

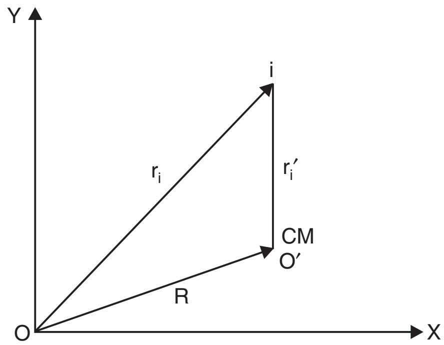
where $\vec{L} = \sum \vec{r'_i} \times \vec{p'_i}$ is the angular momentum of the system about the centre of mass with velocities taken relative to the centre of mass. Remember $\vec{r'_i} = \vec{r_i} - \vec{R}$; rest of the notation is the standard notation used in the chapter. Note $\vec{L'}$ and $\overrightarrow{MR} \times \vec{V}$ can be said to be angular momenta respectively, about and of the centre of mass of the system of particles.

(d) *Show* $$\frac{d\vec{L}'}{dt} = \sum \vec{r'}_i \times \frac{d\vec{p}'}{dt}$$

Further, show that:

$$\frac{d\vec{L}'}{dt} = \overrightarrow{\tau'_{ext}}$$

where $\overrightarrow{\tau'_{ext}}$ *is the sum of all external torques acting on the system about the centre of mass. [Hint: use the definition of centre of mass and Newton's Third Law. Assume the internal forces between any two particles act along the line joining the particles.]*



Ans. Here $\vec{r_i} = \vec{r'}_i + \vec{R} + R$ and $\vec{V_i} = \overrightarrow{V'_i} + \vec{V}$

where $\vec{r'_i}$ and $\vec{v'_i}$ denote the radius vector and velocity of the i^{th} particle referred to centre of mass O' as the new origin and $\vec{V}$ is the velocity of centre of mass relative to O.

(*a*) Momentum of i^{th} particle

$$\vec{P} = m_i \vec{V_i}$$

$$= m_i (\vec{V_i'} + \vec{V}) \text{ (since } \vec{V_i} = \vec{V_i'} + \vec{V})$$

or $$\vec{P} = m_i \vec{V_i'} + m_i \vec{V}$$

$$\vec{P} = \vec{P_i} + m_i \vec{V}$$

(*b*) Kinetic energy of the system of particles.

$$K = \frac{1}{2}\sum m_i V_i^2 = \frac{1}{2}\sum m_i \vec{V_i}\cdot\vec{V_i}$$

$$= \frac{1}{2}\sum m_i \left(\overrightarrow{V_i'} + \vec{V}\right)\cdot\left(\overrightarrow{V_i'} + \vec{V}\right)$$

$$= \frac{1}{2}\sum m_i \left(V_i'^2 + V^2 + 2\,\overrightarrow{V_i'}\cdot\vec{V}\right)$$

$$= \frac{1}{2}\sum m_i V_i'^2 + \frac{1}{2}\sum m_i V_i'^2 + \sum m_i \vec{V_i}\cdot\vec{V} = \frac{1}{2}MV^2 + K'$$

where $$M = \sum m_i = \text{total mass of the system}$$

$$K' = \frac{1}{2}\sum_i m_i V_i'^2$$

= kinetic energy of motion about the centre of mass

or $$\frac{1}{2}Mv^2 = \text{kinetic energy of motion of centre of mass.}$$ **(Proved)**

since $$\sum_i m_i \overrightarrow{V'_i}\cdot\vec{V} = \sum m_i \frac{d\vec{r_i}}{dt}\cdot\vec{V}$$

$$= \frac{d}{dt}\left(\sum m_i \overrightarrow{r'_i}\right)\cdot \vec{V} = \frac{d}{dt}\left(\text{M}\vec{\text{R}}\,.\,\vec{V}\right) = 0$$

(*c*) Total angular momentum of the system of particles.

$$\vec{L} = \vec{r_i} \times \vec{p} = (\vec{r_i} + \vec{R}) \times \sum_i m_i \vec{V_i} = (\vec{r}'_i + \vec{R}) \times \sum_i m_i (\vec{V_i}' + \vec{V})$$

$$= \sum_i \left(\vec{R}\times m_i \vec{V}\right) + \sum_i \vec{r}'_i \times m_i \vec{V}'_i + \left(\sum_i m_i \vec{r}'_i\right) \times \vec{V} + \vec{R} \times \sum_i m_i \vec{V}_i$$

$$= \sum_i \left(\vec{R}\times m_i \vec{V}\right) + \sum_i \vec{r}_i' \times m_i \vec{V}_i + \left(\sum_i m_i \vec{r}_i'\right) \times \vec{V} + \vec{R} \times \frac{d}{dt}\left(\sum_i m_i \vec{r}_i'\right)$$

The last two terms vanish for both contain the factor $\sum m_i \vec{r}'_i$ which is equal to

$$\sum_i m_i \vec{r}_i = \sum_i m_i \left(\vec{r}'_i - \vec{R}\right) = M\vec{R} - M\vec{R} = \vec{0}$$

from the definition of centre of mass. Also

$$\sum_i \left(\vec{R}\times m_i \vec{V}\right) = \vec{R} \times M\vec{V}$$

so that $\quad \vec{L} = \vec{R} \times M\vec{V} + \sum_i \vec{r}'_i \times \vec{P}_i \quad$ or $\quad \vec{L} = \vec{R} \times M\vec{V} + \vec{L}'$

where $\quad \vec{L}' = \sum \vec{r}'_i \times \vec{P}_i$

(*d*) From previous solution

$$\vec{L}' = \sum \vec{r}'_i \times \vec{P}_i$$

$$\frac{d\vec{L}'}{dt} = \sum \vec{r}_i' \times \frac{d\vec{P}_i}{dt} + \sum \frac{d\vec{r}'_i}{dt} \times \vec{P}_i = \sum \vec{r}'_i \times \frac{d\vec{P}_i}{dt}$$

$$= \sum \vec{r}_i \times \vec{F}_i^{ext} = \vec{\tau}'_{\text{ext}}$$

Since $\quad \sum \frac{d\vec{r}'_i}{dt} \times \vec{P}_i = \sum \frac{d\vec{r}'_i}{dt} \times m\vec{v}_i = 0$

Total torque $\quad \vec{\tau} = \sum \vec{r}_i \times \vec{F}_i^{ext} = \sum \left(\vec{r}_i + \vec{R}\right) \times \vec{F}_i^{ext}$

$$= \sum \vec{r}'_i \times \vec{F}_i^{ext} + \vec{R} \times \sum_i \vec{F}_i^{ext} = \vec{\tau}'_{ext} + \vec{\tau}_0^{(ext)}$$

where $\vec{\tau}_{ext}$ is the total torque about the centre of mass as origin and $\vec{\tau}_0^{ext}$, that about the origin *O*.

$$\vec{\tau}'_{ext} = \sum \vec{r}'_i \times \vec{F}_i^{ext} = \sum \vec{r}_i \times \frac{d\vec{P}'_i}{dt} = \frac{d}{dt}\sum_i \left(\vec{r}_i \times \vec{P}_i\right) = \frac{d\vec{L}'}{dt}$$

ADDITIONAL QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *Does the radius of gyration depend on the angular velocity of the body?*

Ans. No. $K = \sqrt{\frac{I}{M}}$.

Q. 2. *If a string of a rotating stone breaks, in which direction will the stone move?*

Ans. The stone will move along the tangent at the point of breaking.

Q. 3. *Which component of a force does not contribute towards torque?*

Ans. The radial component of a force does not contribute towards torque.

Q. 4. *Two satellites of equal masses are orbiting at different heights. Will their moments of inertia be the same or different?*

Ans. The moments of inertia will be different on account of different distances.

Q. 5. *What is moment of inertia of a solid sphere about its diameter?*

Ans. $I = \frac{2}{5}MR^2$, where M is the mass and R is radius of the solid sphere.

Q. 6. *What is moment of inertia of a hollow sphere about an axis passing through its centre?*

Ans. $I = \frac{2}{3}MR^2$, where M is the mass and R is radius of the hollow sphere.

Q. 7. *What type of motion is produced by couple?*

Ans. Only rotational motion.

Q. 8. *Can a body in translatory motion have angular momentum?*

Ans. Yes, a particle in translatory motion always has an angular momentum, unless the point (about which angular momentum is calculated) lies on the line of motion.

Q. 9. *Why is the head of screw made wide?*

Ans. The head of a screw is made wide so as to have a greater torque for the applied force.

Q. 10. *What is the direction of the torque of a force?*

Ans. Direction of torque of a force is a direction perpendicular to the plane of rotation given by $\vec{\tau} = \vec{r} \times \vec{F}$.

Q. 11. *Two solid spheres of the same mass are made of metals of different densities. Which of them has a larger moment of inertia about the diameter?*

Ans. Sphere of smaller density. This is because the distribution of mass is farther from the axis.

Q. 12. *What is rotational analogue of mass of a body?*

Ans. Rotational analogue of mass is moment of inertia.

Q. 13. *What are the two theorems of moment of inertia?*

Ans. Two theorems of moment of inertia are theorem of parallel axes and theorem of perpendicular axes.

Q. 14. *Is radius of gyration of a body a constant quantity?*

Ans. Radius of gyration of a given body depends upon the choice of rotation axis. However, for a given axis of rotation, the radius of gyration of a body has a fixed value.

Q. 15. *What are the factors on which moment of inertia of a body depend?*

Ans. Moment of inertia of a body depends upon

(*i*) mass of the body,

(*ii*) shape and size of body,

(*iii*) position of the axis of rotation, and

(*iv*) distribution of mass in the body about the axis of rotation.

Q. 16. *A labourer standing near the top of an old wooden step-ladder feels unstable. Why?*

Ans. The point of contact of the ladder with the ground is the point about which the ladder can rotate. When the labourer is at the top of the ladder, the lever arm of force is large. So, the turning effect can be large.

Q. 17. *Why does a girl have to lean towards right when carrying a bag in her left hand?*

Ans. When the girl carries a bag in her left hand, the centre of gravity of the system is shifted to the left. In order to bring it in the middle, the girl has to lean towards right.

Q. 18. *When do we call a body rigid?*

Ans. When the separation between any two masses consistiting does not vary, the body is said to be **rigid**.

Q. 19. *Is it possible to open a pen cap with one finger? Why?*

Ans. No, since torque cannot be applied.

Q. 20. *Why do we place handles at maximum possible distance from the hinges in a door?*

Ans. To develop torque with less force being applied.

Q. 21. *When the earth shrinks, without reducing its mass, what change will be there in the duration of a day?*

Ans. L is conserved. If the earth shrinks, duration of the day decreases.

Q. 22. *Name the quantity which can bring rolling without slipping.*

Ans. Friction with the surface.

Q. 23. *Can centre of mass of a body coincide with geometrical centre of the body?*

Ans. Yes, when the body has a uniform mass density.

Q. 24. *What are the units and dimensions of moment of inertia? Is it a vector quantity?*

Ans. The units of moment of inertia are kg m^2 and its dimensional formula is $[M^1L^2T^0]$. No, it is not a vector quantity.

Q. 25. *State the condition for translational equilibrium of a body.*

Ans. For translational equilibrium of a body net force acting on it *i.e.*, the vector sum of all the forces acting on the body must be zero.

Q. 26. *State the condition for rotational equilibrium of a body.*

Ans. For rotational equilibrium of a body the vector sum of torques of all the forces acting on the body about the reference point must be zero.

Q. 27. *Should the centre of mass of a body necessarily lie inside the body? Explain.*

Ans. No, it may lie outside the body. In case of semicircular ring, it is at the centre which is outside the ring.

Q. 28. *What is rotational analogue of force?*

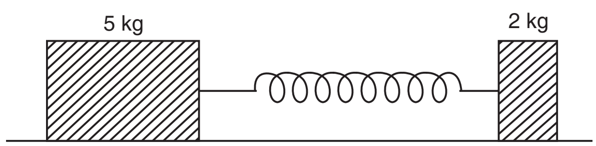
Ans. Rotational analogue of force is torque.

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *Fig. shows two blocks of masses 5kg and 2kg placed on a frictionless surface and connected by a spring. An external kick gives a velocity 14 m/s to the heavier block in the direction of lighter one. Deduce (a) the velocity gained by the centre of mass and (b) the separate velocities of the two blocks in the centre of mass coordinates just after the kick.*

Ans. (*a*) The velocity of centre of mass v_{cm} is given by the expression

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$



Frictionless surface

Here $m_1 = 5$ kg, $m_2 = 2$ kg, $v_1 = 14$ m/s, $v_2 = 0$

Substituting these values, we get

$$v_{cm} = \frac{5 \times 14 + 2 \times 0}{5 + 2} = 10 \text{ m/s}$$

(*b*) We know that the centre of mass coordinate reference frame is one in which centre of mass is at rest. So the velocity of heavier block in this frame just after the kick is

$$v'_1 = v_2 - v_{cm} = 14 - 10 = 4 \text{ m/s}$$

and that of lighter block is

$$v'_2 = v_1 - v_{cm} = 0 - 10 = -10 \text{ m/s.}$$

Q. 2. *If earth contracts to half its radius, what would be the duration of the day?*

Ans. According to the law of conservation of angular momentum,

$$I_1\omega_1 = I_2\omega_2 \Rightarrow \frac{I_1}{T_1} = \frac{I_2}{T_2} \quad \text{or} \quad T_2 = \frac{I_2}{I_1} T_1$$

$$I_1 = \frac{2}{5}MR^2, I_2 = \frac{2}{5}M\left(\frac{R}{2}\right)^2$$

$$\therefore \quad T_1 = \frac{1}{4} \times 24 = 6 \text{ hr.}$$

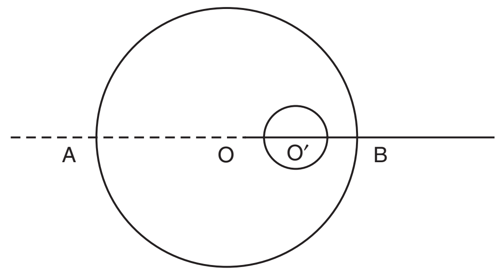
Q3. *Angular momentum of a system is conserved if its M.I. is changed. Is its rotational K.E. also conserved?*

Ans. Kinetic energy of rotation $= \frac{1}{2}I\omega^2 = \frac{1}{2}(I\omega)\,\omega = \frac{1}{2}L\omega$

$L = I\omega$ is constant if moment of inertia (I) of the system changes. It means as I changes, then ω also changes to keep $I\omega$ = constant. Hence K.E. of rotation also changes with the change in I. In other words, rotation K.E. is not conserved.

Q4. *A circular hole of radius 1 m is cut off from a disc of radius 6 m. The centre of the hole is 3 m from the centre of the disc. Find the centre of mass of the remaining disc.*

Ans. Let O be the centre of the disc and O' that of the hole. (see Fig.)



To find the centre of mass, we use the fact that a body balances at this point, *i.e.*, the algebraic sum of the moments of the weights about the centre of gravity is zero. The weight W_1 of the disc acts at point O. The hole can be regarded as a negative weight W_2 acting at O'.

If X is the distance of the centre of gravity of the combination from point O, then

$$X = \frac{W_1 \times O + (-W_2) \times 3}{W_1 + (-W_2)}$$

Also $\quad W_1 = \rho\pi \times (6)^2 = 36\ \rho\pi\ ;\ W_2 = \rho\pi \times (1)^2 = \rho\pi$

where ρ is the mass per unit area of the disc.

Substituting the values of W_1 and W_2, we get

$$X = \frac{-\rho\pi \times 3}{36\ \rho\pi - \rho\pi}\, m = \frac{-3}{35}\,\text{m}$$

The negative sign indicates that the centre of gravity is to the left of the point O.

Q5. *How much fraction of the kinetic energy of rolling is purely:*
(a) translational, (b) rotational.

Ans. (*a*) Fraction of translational kinetic energy

$$= \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2} = \frac{1}{1 + \frac{k^2}{r^2}} = \frac{r^2}{(k^2 + r^2)}$$

(*b*) Fraction of rotational kinetic energy

$$= \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2} = \frac{\frac{k^2}{r^2}}{1 + \frac{k^2}{r^2}} = \frac{k^2}{r^2 + k^2}.$$

Q. 6. *Establish the relation* $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$*, where the letters have their usual meanings.*

Ans. We know that angular velocity is defined as $\omega = \frac{d\theta}{dt}$.

$\therefore\ d\theta = \omega\, dt = (\omega_0 + \alpha t)\, dt$, where ω_0 is the value of initial angular velocity at time $t = 0$ and α = uniform angular acceleration.

On integrating, we get

$$\int_0^\theta d\theta = \int_0^t (\omega_0 + \alpha t)\, dt$$

$$\therefore \quad [\theta]_0^\theta = \left[\omega_0 t + \frac{1}{2}\alpha t^2\right]_0^t \quad \text{or} \quad \theta - 0 = \omega_0 (t - 0) + \frac{1}{2}\alpha (t^2 - 0)$$

$$\Rightarrow \quad \theta = \omega_0 t + \frac{1}{2}\alpha t^2, \text{ which is the requisite relation.}$$

Q. 7. *A solid sphere of mass 0.1 kg and radius 2.5 cm rolls without sliding with a uniform velocity of 0.1 ms^{-1} along a straight line on a smooth horizontal table. Find its total energy.*

Ans. Energy of a body rolling on the table is

$$E = \text{Translational K.E.} + \text{Rotational K.E.}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Now, $$I = \frac{2}{5}mr^2, \quad \omega = \frac{v}{r}$$

$$\therefore \quad E = \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{2}{5}mr^2\times\frac{v^2}{r^2} = \frac{1}{2}mv^2 + \frac{1}{5}mv^2$$

$$= \frac{7}{10}mv^2 = \frac{7}{10}\times 0.1\times(0.1)^2 = 7\times 10^{-4}\text{ J.}$$

Q. 8. *A particle performs uniform circular motion with an angular momentum L. If the frequency of particle's motion is doubled and its K.E. is halved, what happens to its angular momentum?*

Ans. $$L = m\,v\,r \quad \text{and} \quad v = r\omega = r\,(2\,\pi\,n)$$

$$r = \frac{v}{2\,\pi\,n}$$

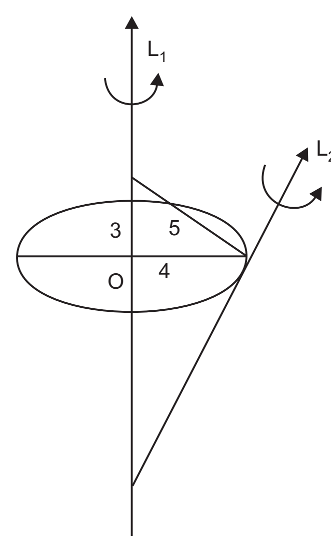
$$\therefore \quad L = mv\left(\frac{v}{2\,\pi\,n}\right) = \frac{mv^2}{2\,\pi\,n}$$

As $$\text{K.E.} = \frac{1}{2}mv^2, \quad \text{therefore,} \quad L = \frac{K.E.}{\pi\,n}$$

When K.E. is halved and frequency (n) is doubled, then,

$$L' = \frac{K.E.'}{\pi\,n'} = \frac{K.E./2}{\pi\,(2\,n)} = \frac{K.E.}{4\,\pi\,n} = \frac{L}{4}$$

i.e., angular momentum becomes one fourth.



Q. 9. *A particle of 10 kg mass is moving in a circle of 4 m radius with a constant speed of 5 m/sec. What is its angular momentum about (i) the centre of circle (ii) a point on the axis of the circle and 3 m distant from its centre?*

Ans. The situation is shown in Fig.

(*a*) We know that $$\vec{L} = \vec{r} \times m\vec{v}$$

$$L = m\,v\,r\sin\theta$$

Here, $m = 10$ kg, $r = 4$ m,

$v = 5$ m/sec and $\theta = 90°$

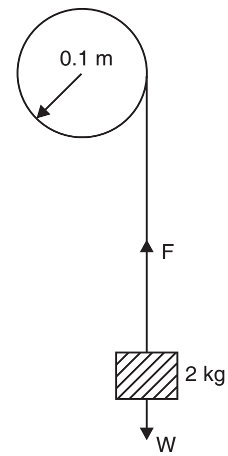
$$\therefore \quad L = 10\times 5\times 4\times\sin 90°$$

$$= 200 \text{ kg-m}^2/\text{sec.}$$

(*b*) In this case, $r = \sqrt{4^2 + 3^2} = 5$ m

$\therefore$ $L = 10 \times 5 \times 5 \sin 90° = 250$ kg m^2/sec.

Q. 10. *The moment of inertia of a solid flywheel about its axis is 0.1 kg-m^2. A tangential force of 2 kgwt is applied round the circumference of the flywheel with the help of a string and mass arrangement as shown in Fig. If the radius of the wheel is 0.1 m, find the acceleration of the mass.*



Ans. Let a be the linear acceleration of the mass and T the tension in the spring. It is clear that

$$mg - T = ma \qquad ...(1)$$

Let the angular acceleration of the flywheel be α. The couple applied to the flywheel is

$$I\alpha = TR \qquad ...(2)$$

The linear acceleration a and angular acceleration are related to each other as

$$a = R\alpha \qquad ...(3)$$

Combining Eqs (1), (2) and (3), we get

$$mg - \frac{I\alpha}{R} = m R \alpha$$

$$\alpha = \frac{m g R}{(I + m R)^2} \qquad ...(4)$$

It is given that $m = 2$ kg, $R = 0.1$ m and $I = 0.1$ kgm^2. Substituting these values, we get

$$\alpha = \frac{2 \times 9.8 \times 0.1}{(0.1 + 2 \times 0.1^2)} \text{ rad s}^{-2} = 16.7 \text{ rad s}^{-2}$$

Q. 11. *A ring, a disc and a shpere, all of the same radius and mass roll down an inclined plane from the same height h. Which of the three reaches the bottom (i) earliest (ii) latest?*

Ans. Linear acceleration of a body rolling down an inclined plane,

$$a = \frac{g \sin\theta}{1 + \frac{I}{Mr^2}}$$

Linear acceleartion of ring $= \dfrac{g \sin\theta}{1 + \frac{Mr^2}{Mr^2}} = \dfrac{g \sin\theta}{2}$ $[\because I = Mr^2]$

Linear acceleration of disc $= \dfrac{g \sin\theta}{1 + \frac{Mr^2}{2\,Mr^2}} = \dfrac{g \sin\theta}{1.5}$ $[\because I = \frac{1}{2}Mr^2]$

Linear acceleration of shpere $= \dfrac{g \sin\theta}{1 + \frac{2}{5}\frac{Mr^2}{Mr^2}} = \dfrac{g \sin\theta}{1.4}$ $[\because I = \frac{2}{5}Mr^2]$

Therefore, the sphere reaches first and ring the last.

Q. 12. *Mathematically establish the third equation of rotational motion $\omega^2 - \omega_0^2 = 2\alpha\theta$.*

Ans. We know that angular acceleration is defined as $\alpha = \dfrac{d\omega}{dt}$.

$$\therefore \quad \alpha = \frac{d\omega}{d\theta}\cdot\frac{d\theta}{dt} = \omega\,\frac{d\omega}{d\theta} \qquad \left[\because \omega = \frac{d\theta}{dt}\right]$$

or $\quad \alpha\, d\theta = \omega d\omega$

On integrating, we have

$$\int_0^\theta \alpha\, d\theta = \int_{\omega_0}^{\omega} \omega d\omega$$

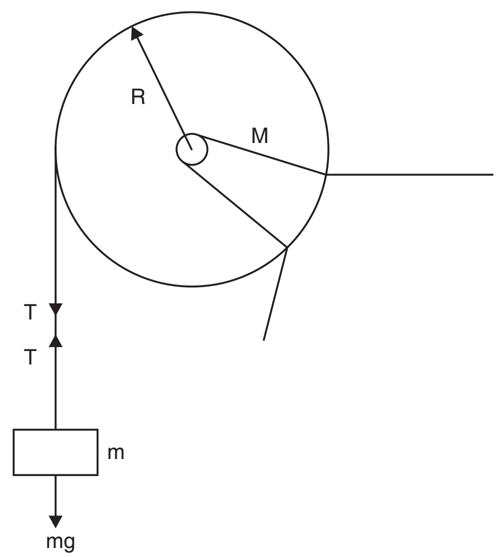
$$\therefore \quad [\alpha\,\theta]_0^\theta = \left[\frac{\omega^2}{2}\right]_{\omega_0}^{\omega} \quad \text{or} \quad \alpha(\theta - 0) = \frac{\omega^2}{2} - \frac{\omega_0^2}{2}$$

$\Rightarrow \quad \omega^2 - \omega_0^2 = 2\alpha\theta$, which is the requisite relation.

Q. 13. *A uniform disc of radius R and mass M is mounted on an axis supported in fixed frictionless bearing. A light chord is wrapped around the rim of the wheel and suppose that we hang a body of mass m from the chord. Find the angular acceleration of the disc and tangential acceleration of point on the rim.*

Ans. The situation is shown in the fig. Let T be the tension in the chord. Now,

$$mg - T = ma, \qquad ...(i)$$



where a is the tangential acceleration of a point on the rim of the disc

We know that $\quad \tau = I\alpha$.

But the resultant torque on the disc = TR and the rotational inertia.

$$I = \frac{1}{2}MR^2$$

$$\therefore \quad TR = \frac{1}{2}MR^2\left(\frac{a}{R}\right) \qquad \left(\because \alpha = \frac{a}{R}\right)$$

or $\quad 2TR = Ma \quad$ or $\quad a = \dfrac{2T}{M} \qquad ...(ii)$

From equations (*i*) and (*ii*), we get $mg - \left(\dfrac{Ma}{2}\right) = ma$

or $\quad a = \left(\dfrac{2\,\text{m}}{M + 2M}\right)g \qquad ...(iii)$

Again, $\quad mg - T = m \times \left(\dfrac{2T}{M}\right) \quad$ or $\quad T = \left(\dfrac{mM}{M + 2\,\text{m}}\right)g \qquad ...(iv)$

Q. 14. *Equal torques are applied on a cylinder and a sphere. Both have same mass and radius. The cylinder rotates about its axis and the sphere rotates about one of its diameters. Which will acquire greater speed? Explain why.*

Ans. We know, $\tau = I\alpha$ or $\alpha = \frac{\tau}{I}$

$\therefore$ Angular acceleration produced in the cylinder is $\alpha_c = \frac{\tau}{I_c}$

Similarly, acceleration produced in the sphere is

$$\alpha_s = \frac{\tau}{I_S}$$

$$\therefore \quad \frac{\alpha_c}{\alpha_s} = \frac{I_s}{I_c}$$

Now $I_s = \frac{2}{3}MR^2$ and $I_C = \frac{1}{2}MR^2$

$$\therefore \quad \frac{\alpha_c}{\alpha_s} = \frac{4}{3} \quad \text{or} \quad \alpha_c = \frac{4}{3}\alpha_s \quad \text{or} \quad \alpha_c > \alpha_s.$$

Thus cylinder will acquire greater speed than that of the sphere.

Q. 15. *A constant torque is acting on a wheel. If starting from rest, the wheel makes n rotations in t seconds, show that the angular acceleration is given by* $\alpha = \frac{4\pi n}{t^2}$ *rad* s^{-2}.

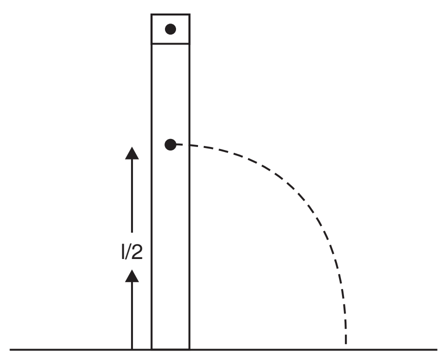
Ans. Here, $\omega_1 = 0, \quad \theta = 2\pi n$ radian,

$t = t, \quad \alpha = ?$

Since $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$

$$\therefore \quad 2\pi n = 0 + \frac{1}{2}\alpha t^2 \quad \text{or} \quad \alpha = \frac{4\pi n}{t^2} \text{ rad / s}^2$$

Q. 16. *A rod of length l and mass M held vertically is let go down, without slipping at the point of contact. What is the velocity of the top end at the time of touching the ground?*



Ans. Loss in potential energy = Gain in rotational kinetic energy

$$Mg\frac{l}{2} = \frac{1}{2}\frac{Ml^2}{3}.\omega^2$$

$$\omega = \sqrt{\frac{3gl}{l^2}} = \sqrt{\frac{3g}{l}}$$

$$\therefore \quad v = l\omega = \sqrt{3gl}$$

Q. 17. *A comet revolves around the sun in a highly elliptical orbit having a minimum distance of* 7×10^{10} *m and a maximum distance of* 1.4×10^{13} *m. If its speed while nearest to the Sun is 60 km* s^{-1}*, find its linear speed when situated farthest from the Sun.*

Ans. Let mass of comet be M and its angular speed be ω when situated at a distance r from the Sun, then its angular momentum $L = I\,\omega = Mr^2\,\omega$

If v be the linear speed, then $L = Mr^2\,\omega = Mr\,v$

In accordance with conservation law of angular momentum, we can write that

$$Mr_1v_1 = Mr_2v_2$$

$$\therefore \quad v_2 = \frac{r_1v_1}{r_2} = \frac{7\times10^{10}\,\text{m}\times60\text{ km/s}}{1.4\times10^{13}\,\text{m}} = 0.3\text{ km/s} \quad \text{or} \quad 300\text{ m/s.}$$

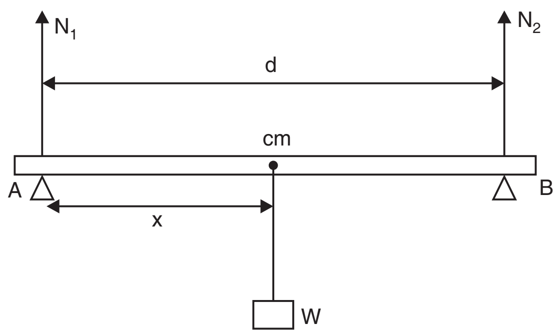
Q. 18. *A rod of weight W is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The distance between the knife edges is d and the centre of mass of the rod is at a distance x from A. Find the value of normal reactions at the knife edges A and B.*

Ans. The situation is shown in Fig. Let reactions on two knife edges be N_1 and N_2 respectively. Then

$$N_1 + N_2 = W \qquad ...(i)$$

and from principle of moments, taking moments at point A, we have

$$N_2 \times d = W \times x \qquad ...(ii)$$



Equation (*ii*) leads

$$N_2 = \frac{Wx}{d}$$

and substituting this value in (*i*), we get $N_1 = W - N_2 = W - \frac{Wx}{d} = W\left(1-\frac{x}{d}\right)$.

Q. 19. *The moment of inertia of a body about a given axis is 1.2 kg m*^{2}*. Initially the body is at rest. In order to produce a rotational K.E. of 1500 J, for how much duration, an acceleration of 25 rad/s*^{2} *must be applied about that axis?*

Ans. Here, $\frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2 = 1500$

Since $\omega_0 = 0$

$$\therefore \quad \frac{1}{2}I\omega^2 = 1500 \quad \text{or} \quad \omega = \sqrt{\frac{3000}{I}} = \sqrt{\frac{3000}{1.2}} = 50\text{ rad/s}$$

Now, $\omega = \omega_0 + \alpha t$ or $t = \frac{\omega-\omega_0}{\alpha} = \frac{50-0}{25} = 2\text{s}.$

Q. 20. *If angular momentum is conserved in a system whose moment of inertia is decreased, will its rotational kinetic energy be also conserved? Explain.*

Ans. Here, $L = I\omega$ = constant

K.E. of rotation, $K = \frac{1}{2}I\omega^2$

$$K = \frac{1}{2I}I^2\omega^2 = \frac{L^2}{2I} \qquad [\because L = I\omega]$$

As L is constant, $\therefore$ $K \propto \frac{1}{I}$

When moment of inertia (I) decreases, K.E. of rotation (K) increases. Thus K.E. of rotation is not conserved.

Q. 21. *Calculate the angular momentum and rotational kinetic energy of earth about its own axis. How long could this amount of energy supply one kilowatt power to each of the 3.5 × 10^9 persons on earth? (Mass of earth = 6.0 × 10^{24} kg and radius = 6.4 × 10^3 km).*

Ans. Here we assume the earth to be a solid sphere. We know that the moment of inertia of a solid sphere about its axis is

$$I = \frac{2}{5}MR^2 = \frac{2}{5}\times(6.0\times10^{24}\,\text{kg})\times(6.4\times10^6\ \text{m})^2$$
$$= 9.8 \times 10^{37}\ \text{kg-m}^2$$

In one day (= 24 × 60 × 60 sec.) the earth completes one revolution. *i.e.,* traces an angle of 2π radian. Hence its angular velocity is given by

$$\omega = \frac{2\pi}{24\times60\times60} = 7.27 \times 10^{-5}\ \text{rad./sec.}$$

$\therefore$ Angular momentum,

$$I\,\omega = (98 \times 10^{37}\ \text{kg-m}^2)\ (7.27 \times 10^{-5}\ \text{s}^{-1}) = \mathbf{7.1 \times 10^{33}\ kg\text{-}m^2/sec.}$$

The rotational energy,

$$\frac{1}{2}I\omega^2 = \frac{1}{2}(9.8 \times 10^{37}\ \text{kg-m}^2)\ (7.27 \times 10^{-5}\ \text{s}^{-1})^2 = \mathbf{2.6 \times 10^{29}\ joule.}$$

Power supplied by this energy,

$$P = \frac{\text{Energy}}{\text{Time}} = \frac{2.6\times10^{29}}{t}\ \text{watt} = \frac{2.6\times10^{29}}{10^3\ t}\text{kilowatt.}$$

Power required by 3.5 × 10^9 persons = 3.5 × 10^9 × 1 kilowatt.

$$\therefore \qquad \frac{2.6\times10^{29}}{10^3\ t} = 3.5 \times 10^9$$

or

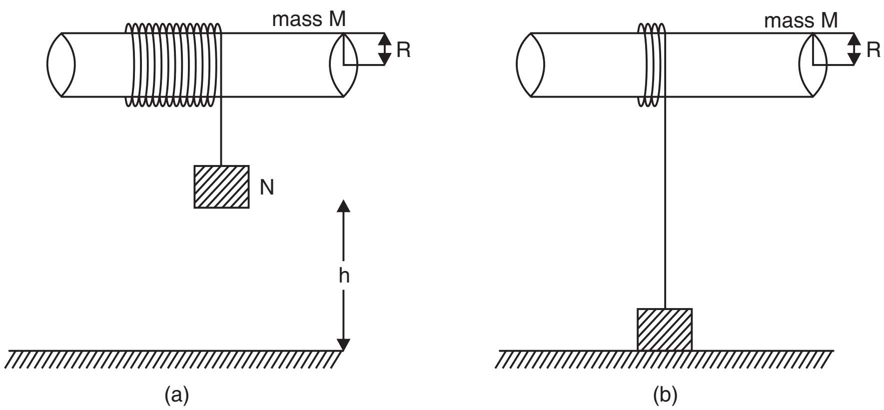
$$t = \frac{2.6\times10^{29}}{10^3\times3.5\times10^9}\text{sec}$$
$$= \frac{2.6\times10^{29}}{10^3\times3.5\times10^9\times365\times24\times60\times60}\ \text{year}$$
$$= \mathbf{2.35 \times 10^9\ years.}$$

Q. 22. *A solid cylinder of mass M and radius R has a light flexible rope wound around it. The rope carries a mass m at its free end. The mass is rest at a height h above the floor. Find the angular velocity of the cylinder at the instant the mass m, after release, strikes the floor. Assume friction to be absent and the cylinder to rotate about its own axis.*

Ans. As the mass m descends, its initial potential energy gets converted into the kinetic energy of the falling mass M itself and kinetic energy of the cylinder set into rotation. We thus have

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Now I, the moment of inertia of the cylinder about its own axis is $\frac{1}{2}MR^2$. Also $\omega = \frac{v}{R}$.



We thus have

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}MR^2 \cdot \frac{v^2}{R^2}$$

$$\Rightarrow \qquad v^2 = \frac{2\,mgh}{m + \left(\frac{M}{2}\right)} = \left[\frac{2\,gh}{1 + \left(\frac{M}{2\,m}\right)}\right]$$

Thus the required angular velocity,

$$\omega = \frac{v}{R} = \frac{1}{R}\sqrt{\frac{2\,gh}{1 + \left(\frac{M}{2\,m}\right)}}$$

Q. 23. *A star of mass twice the solar mass and radius 10^6 km rotates about its axis with an angular speed of 10^{-6} rad per sec. What is the angular speed of the star when it collapses (due to inward gravitationl forces) to a radius of 10^4 km? Solar mass = 1.99×10^{23} kg.*

Ans. According to the law of conservation of angular momentum,

$$I_1\omega_1 = I_2\omega_2 \qquad ...(1)$$

Since sun is a sphere, so

$$I_1 = \frac{2}{5}MR_1^2, \quad R_1 = \text{Radius of sun}$$

M.I. of star $\qquad I_2 = \frac{2}{5}2M\,R_2^2,$

R_2 = radius of star

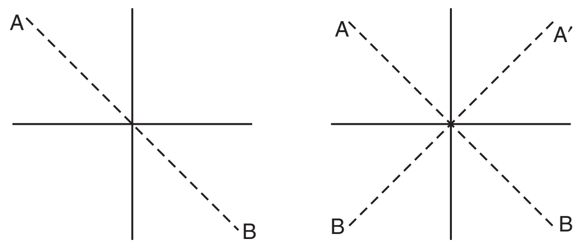
$$\therefore \quad \frac{2}{5} M R_1^2 \omega_1 = \frac{4}{5} M R_2^2 \omega_2 \quad \text{or} \quad \omega_2 = \frac{R_1^2}{2R_2^2} \omega_1$$

Since $R_1 = 10^6$ km; $R_2 = 10^4$ km;

$\omega_1 = 10^{-6}$ per sec

$$\therefore \quad \omega_2 = \frac{(10^6)^2}{2\,(10^4)^2} \times 10^{-6} = 5 \times 10^{-3} \text{ rad per sec.}$$

Q. 24. *Two uniform thin identical rods, each of mass M and length L, are joined so as to form a cross as shown in Fig. What is the moment of inertia of the system about the bisector line AB?*



Ans. Take a bisector line $A'B'$ perpendicular to bisector line AB.

Moment of inertia about an axis perpendicular to the plane and passing through the point of intersection is

$$2 \times \frac{ML^2}{12} \quad \text{or} \quad \frac{ML^2}{6}$$

Applying theorem of perpendicular axis, we get

$$\frac{ML^2}{6} = I_{AB} + I_{A'B'} \quad \text{or} \quad 2I = \frac{ML^2}{12} \quad \text{or} \quad I = \frac{\mathbf{ML^2}}{\mathbf{12}}$$

Q. 25. *Prove that for an earth satellite, the ratio of its velocity at apogee (when farthest from earth) to its velocity at perigee (when nearest to earth) is equal to the inverse ratio of its distances from apogee and perigee.*

Ans. Let mass of satellite = m ; distance of apogee from earth = r_a

Distance of perigee from earth = r_p ;

Velocity of satellite at apogee = v_a

Velocity of satellite at perigee = v_p

Now, angular momentum of satellite at apogee = $mv_a r_a$

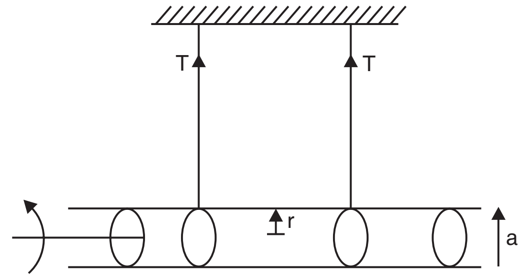
Angular momentum of the satellite at perigee = $mv_p r_p$

According to the law of conservation of angular momentum $mv_a r_a = mv_p r_p$

$$\therefore \quad \boxed{\frac{v_a}{v_p} = \frac{r_p}{r_a}}$$

Q. 26. *A cylinder is suspended by two strings wrapped around the cylinder near each end, the free ends of the string being attached to hooks on the ceiling, such that the length of the cylinder is horizontal. From the position of rest, the cylinder is allowed to roll down as suspension strings unwind. Calculate : (i) the downward linear acceleration of the cylinder and (ii) tension in the strings. Mass of cylinder is 12 kg.*

Ans. Let the downward linear acceleration of the cylinder be a. If M be the mass of the cylinder, then



$$Mg - 2T = Ma$$

or $$T = \frac{1}{2} m (g - a) \quad ...(i)$$

Now,

$$\text{Torque} = \text{Moment of inertia} \times \text{angular acceleration}$$

i.e., $$2Tr = I\alpha \qquad (\text{where } \alpha = \text{linear acceleration}/r)$$

or $$2Tr = \frac{1}{2}mr^2 \times \left(\frac{a}{r}\right) \quad \text{or} \quad T = \frac{ma}{4} \qquad ...(ii)$$

From eqns (i) and (ii), we get

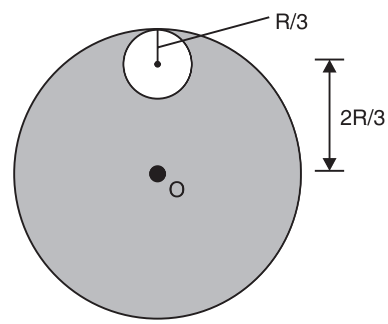
$$m\,\frac{a}{4} = \frac{1}{2}m\,(g - a) \qquad ...(iii)$$

(i) Solving eqns (iii), we get $a = \left(\frac{2}{3}\right)g$

(ii) Substituting the value of a in eq. (ii), we get

$$T = \frac{m\left(\frac{2}{3}\right)g}{4} = \frac{m \times 2g}{12} = \frac{12 \times 2g}{12} = 2 \text{ kgf.}$$

Q. 27. *From a circular disc of radius R and mass 9 M, a small disc of radius $\frac{R}{3}$ is removed as shown in Fig. Find the moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through the point O.*



Ans. Moment of inertia of the complete disc about the given axis $= \frac{1}{2}(9M)\,R^2$

Mass of removed disc $= \frac{9M}{\pi R^2}\pi\left(\frac{R}{3}\right)^2 = M$

Moment of inertia of this disc about the given axis

$$= \frac{1}{2}M\left(\frac{R}{3}\right)^2 + M\left(\frac{2R}{3}\right)^2 = \frac{1}{2}MR^2$$

$\therefore$ Moment of inertia of the remaining disc about the given axis

$$= \frac{9}{2}MR^2 - \frac{1}{2}MR^2 = 4\,MR^2.$$

Q. 28. *Show that for an isolated system the centre of mass moves with a uniform velocity along a straight line path.*

Ans. Let M be the total mass of a system supposed to be concentrated at the centre of mass whose position vector is $\vec{r}$, then in the presence of an external force $\vec{F}$, we have

$$\vec{F} = M\frac{d^2\vec{r}}{dt^2} = M\frac{d}{dt}\left(\frac{\overrightarrow{dr}}{dt}\right) = M\frac{d}{dt}(\vec{v}_{cm})$$

However for an isolated system, force $\vec{F} = 0$ and hence, we have

$$M\frac{d}{dt}(\vec{v}_{cm}) = 0 \quad \text{or} \quad \frac{d}{dt}(\vec{v}_{cm}) = 0 \quad \text{or} \quad \vec{v}_{cm} = \text{a constant}$$

It means that for an isolated system the centre of mass moves with a uniform velocity along a straight line path.

Q. 29. *A grindstone has a moment of inertia of 6 kg m^2. A constant torque is applied and the grindstone is found to have a speed of 150 rpm, 10 secs. after starting from rest. Calculate the torque.*

Ans. Here, Moment of inertia of grindstone, I = 6 kg m^2

Initial angular velocity, $\omega_1 = 0$

Final angular velocity,

$$\omega_2 = 2\pi n = 2\pi \times \frac{150}{60} = 5\pi \text{ rad./sec}$$

Time for which torque acts,

$$t = 10 \text{ sec.}$$

$$\therefore \text{ Angular acceleration } (\alpha) = \frac{\omega_2 - \omega_1}{t} = \frac{5\pi - 0}{10} = \frac{\pi}{2}\text{ rad/sec}^2$$

As $\quad \tau = I\alpha$

$$\therefore \quad \tau = 6 \times \frac{\pi}{2} = 3\pi \; Ns$$

Q. 30. *Two equal and opposite forces act on a rigid body. Under what conditions will the body (i) rotate (ii) not rotate?*

Ans. Two equal and opposite forces acting on a rigid body such that their lines of action do not coincide, constitute a couple. This couple produces the turning effect on the body. Hence the rigid body will rotate. If two equal and opposite forces act in such a way that their lines of action coincide, then these forces cancel out the effect of each other. Hence the body will not rotate.

III. LONG ANSWER TYPE QUESTIONS

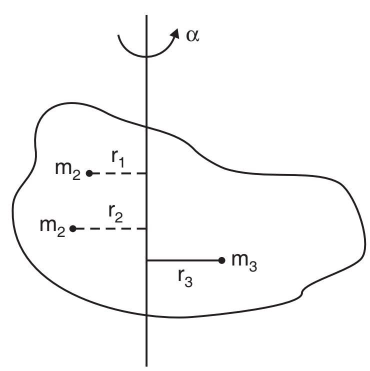
Q. 1. *(i) Define Moment of Inertia. Write the parallel and perpendicular axis theorem.*

(ii) Establish the relationship between Torque and Moment of Inertia.

Ans. (*i*) For definition of Moment of Inertia, parallel and perpendicular axis theorem, see text.

(*ii*) Consider a rigid body rotating about a given axis with a uniform angular acceleration α, under the action of a torque.

Let the body consist of particles of masses m_1, m_2, m_3 ... m_n at perpendicular distances r_1, r_2, r_3 ... r_n respectively from the axis of rotation, (see Fig.).



As the body is rigid, angular acceleration α of all the particles of the body is the same. However, their linear accelerations are different because of different distances of the particles from the axis.

If $a_1, a_2, a_3 \ldots a_n$ are the respective linear accelerations of the particles, then

$$a_1 = r_1 \alpha, \quad a_2 = r_2 \alpha, \quad a_3 = r_3 \alpha$$

Force on particle of mass m_1 is

$$f_1 = m_1 a_1 = m_1 r_1 \alpha$$

Moment of this force about the axis of rotation

$$= f_1 r_1 = (m_1 r_1 \alpha) \times r_1 = m_1 r_1^2 \alpha$$

Similarly, moments of forces on other particles about the axis of rotation are $m_2 r_2^2 \alpha$, $m_3 r_3^2 \alpha, \ldots m_n r_n{}^2 \alpha$.

$\therefore$ Torque acting on the body, τ

$$= m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + m_3 r_3^2 \alpha + \ldots m_n r_n^2 \alpha$$

$$= (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots m_n r_n^2) \alpha$$

$$\tau = \left(\sum_{i=1}^{i=n} m_i r_i^2 \right) \alpha \quad \text{or} \quad \tau = I \alpha \quad \text{or} \quad \vec{\tau} = I \vec{\alpha}$$

Q. 2. *What do you mean by the term "equilibrium"? What are equilibrium of rest and equilibrium of motion? State the conditions for complete equilibrium of a body.*

Ans. **Equilibrium:** A body or a system of particles is said to be in a state of equilibrium if inspite of a number of forces or torques acting on it, the body or the system of particles remains in its original state of rest or of uniform motion (translational or rotational or both). Thus, equilibrium state means that acceleration (both linear as well as angular) of the body/system must be zero. Hence, equilibrium is of two types:

(*i*) **Equilibrium of rest:** If a given system remains in a state of rest and does not change its position inspite of number of forces acting on it, it is said to be in an equilibrium of rest *e.g.,* our house, our school etc.

(*ii*) **Equilibrium of motion:** If a given system maintains its state of uniform motion, translational or rotational or combined, under the action of a number of forces then it is said to be in an "equilibrium of motion" *e.g.,* our Earth, the planetary system, electrons revolving around the nucleus of an atom. For a state of equilibrium of motion, the value of linear momentum and/or angular momentum of the system should have a finite and constant value.

Conditions for complete equilibrium: For complete equilibrium condition for translational equilibrium and condition for rotational equilibrium both must be fulfilled. The conditions are:

(*a*) **For transtational motion,** we know that $\dfrac{d\vec{p}}{dt} = \sum \vec{F}_{ext}$.

For equilibrium $\vec{p} = a$ constant or $\dfrac{d\vec{p}}{dt} = 0$ or $\sum \vec{F}_{ext} = 0$

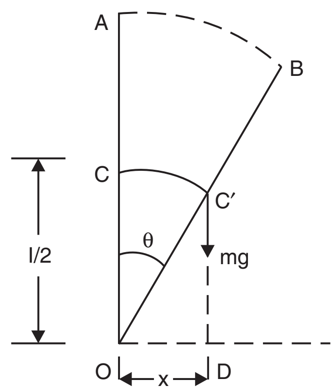
Hence for translational equilibrium, the vector sum of all the external forces acting on the system/ body under discussion must be zero.

(*b*) **For rotational motion,** we that $\frac{d\vec{L}}{dt} = \sum \vec{\tau}_{ext}$

For equilibrium $\vec{L}$ = a constant or $\frac{d\vec{L}}{dt} = 0$ or $\sum \vec{\tau}_{ext} = 0$

Hence for rotational equilibrium, the vector sum of all the external torques acting on the system/body must be zero.

Q. 3. *A meter stick held vertically, with one end on the ground, is then allowed to fall. What is the value of the radial and tangential acceleration of the top end of the stick when the stick has turned through on angle θ? What is the speed with which the top end of the stick hits the ground? Assume that the end of the stick in contact with the ground does not slip.*



Ans. Let *OA* be the initial vertical position of the stick of length *l* with the end *O* in contact with ground. As the stick falls, it revolves around point *O*. *OB* represents the position of the stick at any time *t* = *t* when it has turned thorough an angle θ. *C* and *C′* are the centre of mass of the stick at *t* = 0 and *t* = *t* respectively. The torque of the weight *mg* of the stick at *t* = *t* about *O* is

$$= mg \times x = mg\ \frac{l}{2} \sin\theta \qquad ...(1)$$

If *I* and α denote the moment of inertia, and angular acceleration of stick, we have

$$\tau = I\alpha \quad \text{or} \quad \alpha = \tau/I = \frac{m\,g\,l}{2I}\sin\theta \qquad ...(2)$$

The moment of inertia, *I* of stick about axis passing through one end *O* is

$$I = ml^2/3$$

$$\therefore \qquad \alpha = \frac{3}{2}\frac{g}{l}\sin\theta \qquad ...(3)$$

Now, by definition $\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta}\cdot\frac{d\theta}{dt} = \omega\frac{d\omega}{d\theta}$ ...(4)

From eqns. (3) and (4)

$$\omega d\omega = \frac{3}{2}\frac{g}{l}\sin\theta\ d\theta \qquad ...(5)$$

Intergrating, we get

$$\frac{\omega^2}{2} = -\frac{3}{2}\frac{g}{l}\cos\theta + C_1 \qquad ...(6)$$

where C_1 is a constant of integration. Its value is found from the initial condition, *i.e.*, at $t = 0;\ \theta = 0$ and $\omega = 0$

$$\therefore \qquad 0 = -\frac{3}{2}\frac{g}{l} + C_1 \quad \text{or} \quad C_1 = \frac{3}{2}\frac{g}{l} \qquad ...(7)$$

Combining Eqns. (6) and (7), we get

$$\omega^2 = \frac{3g}{l}(1 - \cos\theta) \quad ...(8)$$

The radial acceleration a_r is

$$a_r = \omega^2 l = 3g(1 - \cos\theta)$$

The tangential acceleration a_t is

$$a_t = l \,.\, \alpha = \frac{3}{2} g \sin\theta \quad ...(10)$$

Let ω_0 be the angular speed of the end A of stick as it this the ground. (The stick has turned through $\theta = 90°$). Then

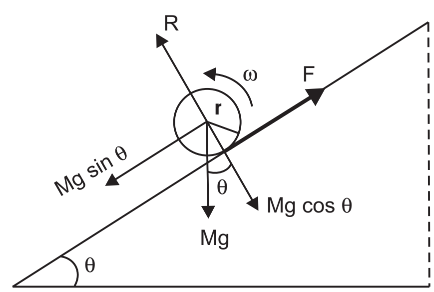
$$\omega_0 = 3\; g \quad ...(11)$$

The linear speed v_0 of the end A of stick as it hits are ground is

$$v_0 = l\omega_0 = 3gl \quad ...(12)$$

Q. 4. *Discuss the rolling motion of a cylinder on an inclined plane.*

Ans. Consider a spherical body of mass M and radius r, rolling down on an inclined plane without slipping. Let the angle of inclination of the plane is θ with the horizontal (See fig.)



It ω be the angular velocity of the body, then the linear velocity of the body is,

$$v = r\omega \quad ...(i)$$

The various forces acting on the body are:

(*a*) The weight (Mg) of the body in the vertically downward direction

(*b*) Normal reaction (R) of the surface of the plane on the body which acts vertically upward.

(*c*) The force of friction (F) which acts opposite to the direction of motion of the body.

Resolve Mg into two components:

→ $Mg \cos\theta$ is the horizontal component, which is equal and opposite to the normal reaction

i.e., $$R = Mg\cos\theta \quad ...(ii)$$

→ Mg $\sin\theta$ is the vertical component, which acts in the direction of the motion of the body

∴ Net force acting in the direction of motion of the body = $Mg \sin\theta - F$

Let a = acceleration of the body

Therefore, equation of motion of the body is given by,

$$Ma = Mg\sin\theta - F \quad ...(iii)$$

The external torque acting on the body is produced by the force of friction (F) and its lever arm is r (the radius of the spherical body) [The lines of action of Mg and R pass through the centre of mass of body and hence do not contribute anything to the torque about the centre of mass].

$$\therefore \qquad \tau = Fr \qquad ...(iv)$$

If α be the angular acceleration produced in the body whose moment of inertia is I, then

$$\tau = I\alpha \qquad ...(v)$$

From eqns (*iv*) and (*v*), we get

$$Fr = I\alpha \quad \text{or} \quad F = I\,\frac{\alpha}{r} \qquad ...(vi)$$

Using eqn. (*vi*) in eqn. (*iii*), we get

$$\therefore \qquad Ma = Mg\sin\theta - I\frac{\alpha}{r}$$

But

$$a = r\alpha \quad \text{or} \quad \alpha = \frac{a}{r}$$

$$\therefore \qquad Ma = Mg\sin\theta - I\,\frac{a}{r^2} \quad \Rightarrow \quad a = g\sin\theta - \frac{Ia}{Mr^2}$$

or

$$a\left(1+\frac{I}{Mr^2}\right) = g\sin\theta \quad \text{or} \quad a = \frac{g\sin\theta}{1+\left(\frac{I}{Mr^2}\right)}$$

Now

$$\alpha = \frac{a}{r} = \frac{g\sin\theta}{r\left[1+\frac{I}{Mr^2}\right]}$$

Substituting this value in eqn. (*vi*), we get

$$F = \frac{I}{r}\cdot\frac{g\sin\theta}{r\left[1+\frac{I}{Mr^2}\right]} = \frac{Ig\sin\theta}{r^2\left[1+\frac{I}{Mr^2}\right]}$$

This is the force of friction, required by the body to roll down an inclined plane without slipping.

Q. 5. *Define radius of gyration and give the physical significance of moment of inertia.*

Ans. The radius of gyration of a body about the axis of rotation of a body is the point at which the weighed mass of the body acts. It is also equal to the square root of moment of inertia of all particles of the body about the axis of rotation divided by the total mass of the body *i.e.,*

$$k = \sqrt{\frac{m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots + m_nr_n^2}{m_1 + m_2 + m_3 + \ldots + m_n}} = \sqrt{\frac{\Sigma mR^2}{M}} = \sqrt{\frac{I}{M}}$$

Its dimensions are those of length and its is measured in metre in SI units.

The moment of a body is a quantity which comes in rotational motion and plays same role in rotational motion as does mas in translational motion. Thus a body continues of rotate or be at rest in the absence of any external torque. This is similar to the law of inertia in translational motion. This aspect is used in over creasing the dead points in the engines and crankshafts. Similarly, the kinetic energy of rotation is dependent on the moment of inertia of the body in the same manner the kinetic energy of translation of motion depends, on the mass of the body. For a given angular velocity (ω) kinetic energy of rotation ∝ I. If equal torques are applied I_1 and I_2 the their angular acceleration are inversely proportional to the moments of inertia of the bodies.

$$\frac{I_1\alpha_1}{I_2\alpha_2} = 1 \quad \text{or} \quad \frac{\alpha_1}{\alpha_2} = \frac{I_2}{I_1} \quad \text{or} \quad \alpha \propto \frac{1}{I}$$

Similarly if two bodies have same angular acceleration, then the moments of inertia are directly proportional to the torque applied on then.

$$\frac{\tau_1}{\tau_2} = \frac{I_1}{I_2} \quad \text{or} \quad \alpha_1 = \alpha_2$$

The linear momentum of a body depends on its mass and velocity. If two bodies have same velocity, then their moments are proportional to their masses *i.e.,*

$$\frac{\rho_1}{\rho_2} = \frac{m_1}{m_2}$$

Similarly for angular momentum

$$\frac{L_1}{L_2} = \frac{I_1}{I_2}$$

Thus, moment of inertia of a body plays same role in the rotational motion as does mass in translational motion.

The moment of inertia determines the amount of torque to be applied to produce desired angular acceleration.

IV. MULTIPLE CHOICE QUESTIONS

1. *A couple produces a:*

(*a*) pure linear motion (*b*) pure rotational motion

(*c*) none of the above. (*d*) both linear and rotational motion

2. *A cylindrical solid of mass M has raidus R and length L. Its moment of inertia about a generator is:*

(*a*) $N\left(\frac{L}{R} + \frac{R^2}{4}\right)$ (*b*) $\frac{1}{2}MR^2$ (*c*) $\frac{3}{2}MR^2$ (*d*) $M\left(\frac{L^2}{3} + \frac{R^2}{4}\right)$

3. *A man of mass M is standing at the centre of a rotating turn table rotating with an angular velocity ω. The man holds two 'dumb bells' of mass M/4 each in each of his two hands. If he stretches his arms to a horizontal position, the turn table acquires a new angular velocity ω′ where*

(*a*) $\omega' = 2\omega$ (*b*) $\omega' = \omega/2$ (*c*) $\omega' > \omega$ (*d*) $\omega' < \omega$

4. *A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is*

(*a*) $4L$ (*b*) $\frac{L}{2}$ (*c*) $\frac{L}{4}$ (*d*) $2L$.

5. *A loaded spring gun of mass M fires a 'shot' of mass m with a velocity ϑ at an angle of elevation θ. The gun is initially at rest on a horizontal frictionless surface. After firing, the centre of mass of the gun-shot system*

(*a*) moves with a velocity ϑ m/M

(*b*) moves with velocity $\frac{\vartheta m}{M}$ cos θ in the horizontal direction

(*c*) remains at rest

(*d*) moves with a velocity $\frac{\vartheta(M-m)}{(M+m)}$ in the horizontal direction.

6. *One end of a thin uniform rod of length L and mass M_1 is riveted to the centre of a uniform circular disc of radius r and mass M_2 so that both are coplanar. The centre of mass of the combination from the centre of the disc is (assume that the point of attachment is at the origin).*

(*a*) $\frac{L(M_1+M_2)}{2M_1}$ (*b*) $\frac{LM_1}{2(M_1+M_2)}$ (*c*) $\frac{2(M_1+M_2)}{LM_1}$ (*d*) $\frac{2LM_1}{(M_1+M_2)}$.

7. *The radius of gyration of a uniform rod of length L about an axis passing through its centre of mass is:*

(*a*) $\frac{L}{\sqrt{12}}$ (*b*) $\frac{L}{\sqrt{2}}$ (*c*) $\frac{L^2}{12}$ (*d*) $\frac{L^2}{\sqrt{3}}$

Ans. **1.**—(*b*) **2.**—(*c*) **3.**—(*d*) **4.**—(*c*) **5.**—(*c*)
6.—(*b*) **7.**—(*a*)

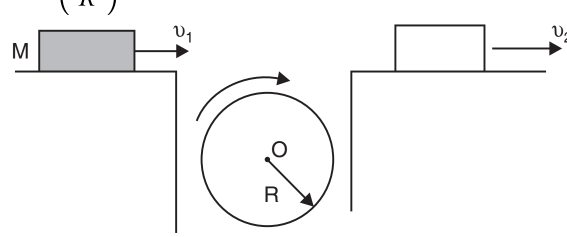
V. QUESTIONS ON HIGH ORDER THINKING SKILLS

Q. 1. *A block of mass M is moving with a velocity v_1 on a frictionless surface as shown in fig. It passes over to a cylinder of radius R and moment of inertia I which has fixed axis and is initially at rest. When it first makes contact with the cylinder, it slips on the cylinder, but the friction is large enough so that slipping ceases before it losses contact with the cylinder. Finally it goes to the dotted position with velocity v_2 compute v_2 in terms of v_1, M, I and R.*

Ans. When the mass M makes first contact with the cylinder, its angular momentum about O.

$$L_1 = \text{moment of inertia about } O \times \text{angular velocity}$$

$$= MR^2 \times \alpha = MR^2\left(\frac{v_1}{R}\right) = MRv_1$$



When the mass M loses the contact with the cylinder, its final angular momentum,

$$L_2 = MR\ v_2$$

$\therefore$ Loss in angular momentum

$$\Delta L = L_1 - L_2 = MR\ (v_1 - v_2) \qquad ...(i)$$

This provides an angular impulse to the cylinder about O which is given by $\tau\ \Delta t$. Now,

Loss in angular momentum of mass = angular impulse to the cylinder

$$\therefore \qquad MR\,(v_1 - v_2) = \tau\Delta t \qquad ...(ii)$$

$$\therefore \qquad \tau = I\alpha = I\,\frac{\Delta\omega}{\Delta t}$$

Initially cylinder is at rest and finally it moves with a velocity v_2, hence,

$$\Delta\omega = \frac{v_2}{R}$$

$$\therefore \qquad \tau = I.\,\frac{v_2}{R\,.\,\Delta t} \qquad ...(iii)$$

Substituting the value of τ from eqn. (*iii*) in eqn. (*ii*), we get,

$$MR\,(v_1 - v_2) = \frac{I\,v_2\,.\,\Delta t}{R\,.\,\Delta t} = I\,\frac{v_2}{R}$$

or

$$(v_1 - v_2) = \frac{I}{MR^2}\,.\,v_2 \quad \text{or} \quad v_1 = v_2\left[1 + \left(\frac{I}{MR^2}\right)\right]$$

$$v_2 = \frac{v_1}{\left[1 + \left(\frac{I}{MR^2}\right)\right]}$$

Q. 2. *An isolated particle of mass m is moving in a horizontal plane (x – y), along the x-axis at a certain height about the ground. It explodes suddenly into two fragments of masses m/4 and 3 m/4. An instant later, the smaller fragment is at y = +15 cm. What is the position of larger fragment at this instant?*

Ans. As isolated particle is moving along *x*-axis at a certain height above the ground, there is no motion along Y-axis. Further, the explosion is under internal forces only. Therefore, centre of mass remains stationary along Y-axis after collision. Let the co-ordinates of centre of mass be $(x_{cm}, 0)$.

Now,

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = 0$$

$$\therefore \qquad m_1 y_1 + m_2 y_2 = 0 \quad \text{or} \quad y_2 = -\frac{m_1 y_1}{m_2} = -\frac{m/4}{3\,m/4} \times 15 = -5\text{cm}$$

$\therefore$ Larger fragment will be at $y = -5$ cm; along *x*-axis.

Q. 3. *A solid sphere of mass 2 kg is rolling down an inclined plane of angle of inclination 30°. What is the agent which supplies the external torque for the rotational motion of the sphere? Calculate its value. Also find the value of the torque if radius of sphere is 40 cm.*

Ans. When a solid sphere rolls down an inclined plane then the force of friction between the sphere and the inclined plane supplies the external torque.

Force of friction for a body rolling down an inclined plane is given by

$$F = \frac{Ig\sin\theta}{R^2\left[1 + \frac{I}{MR^2}\right]}$$

For solid sphere,

$$I = \frac{2}{5}MR^2$$

$\therefore \quad F = \frac{2}{7}Mg \times \sin\theta = \frac{2}{7} \times 2 \times 10 \times \sin 30° = \frac{20}{7} = 2.86 \text{ N}$

Torque $= FR = 2.86 \times 0.4 = 1.44$ Nm

Q. 4. *Two masses M_1 and M_2 are separated by a distance r. Find the moment of inertia of this arrangement about an axis passing through the centre of mass and perpendicular to the line joining them.*

Ans. If COM is origin, $M_1 r_1 = M_2 r_2$.

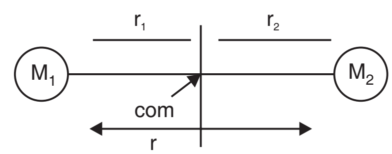
Also, $r_1 + r_2 = r$

Using the equations, $r_1 = \frac{M_2 r}{M_1 + M_2}$ and $r_2 = \frac{M_1 r}{M_1 + M_2}$

$$I_{cm} = M_1 r_1^2 + M_2 r_2^2 = \frac{1}{(M_1 + M_2)^2}[M_1 M_2^2 r^2 + M_2 M_1^2 r^2]$$

$$= \frac{M_1 M_2 r^2 [M_2 + M_1]}{(M_1 + M_2)^2}$$

$$= \frac{M_1 M_2 r^2}{(M_1 + M_2)}$$



Q. 5. *A threaded rod with 12 turns/cm and diameter 1.18 cm is mounted horizontally. A bar with a threaded hole to match the rod is screwed onto the rod. The bar spins at 216 rev/min. How long will it take for the bar to move 1.50 cm along the rod?*

Ans. Here, distance between two consecutive threads = pitch = $\frac{1}{12}$ cm.

Total distance to be moved = 1.5 m

$\therefore$ No. of rotations $= \frac{1.5}{1/12} = 18$

Total angle of turning, $\theta = 18 \times 2\pi = 36\pi$ radian

angular speed, $\omega = 2\pi n = 2\pi \times \frac{216}{60} = 7.2\pi$ rad/s

$\therefore$ time taken, $t = \frac{\theta}{\omega} = \frac{36\pi}{7.2\pi} = 5$s

Q. 6. *Locate the centre of mass of a system of particles of mass $m_1 = 1$ kg, $m_2 = 2$ kg and $m_3 = 3$ kg, situated at the corners of an equilateral triangle of side 1.0 metre.*

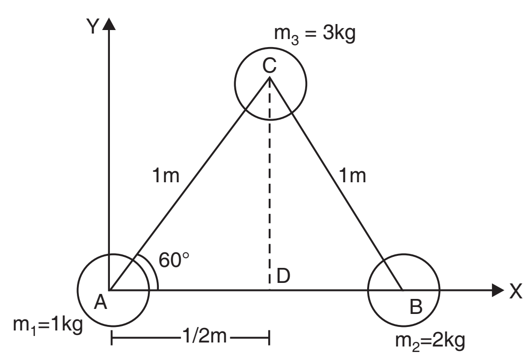
Ans. Consider an equilateral triangle of side 1 m as shown in fig. Take *X* and *Y* axes as shown in fig.

By the definition of centre of mass, we have

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

and

$$\bar{y} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$



Here, $m_1 = 1$ kg, $m_2 = 2$ kg and $m_3 = 3$ kg

$[x_1 = 0, \; y_1 = 0]$, $[x_2 = 1, y_2 = 0]$ and $[x_3 = 0.5, y_3 = \frac{\sqrt{3}}{2}]$

Here, $y_3 = CD = AC \sin 60° = \frac{\sqrt{3}}{2}$.

$$\bar{x} = \frac{1\times 0 + 2\times 1 + 3\times 0.5}{1+2+3} = \frac{3.5}{6}\text{m}$$

and
$$\bar{y} = \frac{1\times 0 + 2\times 0 + 3\times\left(\frac{\sqrt{3}}{2}\right)}{1+2+3} = \frac{\sqrt{3}}{4}\text{m}$$

The co-ordinates of centre of mass are $\left(\frac{3.5}{6}, \frac{\sqrt{3}}{4}\right)$.

Q. 7. *If the Earth were to suddenly contract to $\frac{1}{n}$th of its present radius, without any change in its mass, then what will be the effect on the duration of the day?*

Ans. Applying conservation of angular momentum,

$$I\omega = \text{constant}; \quad \frac{2}{5}MR^2 \times \frac{2\pi}{T} = \text{constant} \;\Rightarrow\; T \propto R^2$$

If R becomes $\frac{1}{n}R$, then T (duration of the day) would become $\frac{1}{n^2}T$. So, the new duration of the day will be $\frac{24}{n^2}$ hours.

Q. 8. *The moments of inertia of two rotating bodies A and B are I_A and I_B. ($I_A > I_B$) and their angular momenta are equal. Which one has greater K.E.?*

Ans. We know, angular momentum $L = I\omega$

and K.E. of rotation, $K = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{I^2\omega^2}{I} = \frac{1}{2I}(L^2)$

When L is constant, $K \propto 1/I$

As $I_A > I_B$

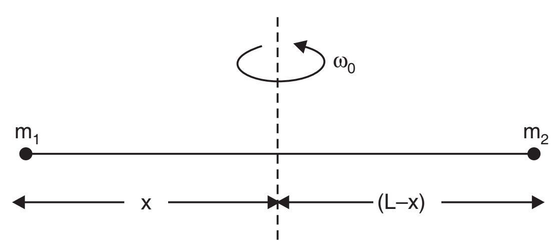
$\therefore$ $K_A < K_B$

or $K_B > K_A$ *i.e.*, the body B has greater K.E. of rotation than the body A.

Q. 9. *Point masses m_1 and m_2 are placed at the opposite ends of a rigid rod of length L, and negligible mass. The rod is to be set in rotation about an axis perpendicular to it. Find the position on this rod through which the axis should pass in order that the work required to set the rod in rotation with angular velocity ω_0 should be minimum.*

Ans. Let the axis of rotation be at a distance x from mass m_1. Therefore, the distance of the axis of rotation from mass m_2 is $(L - x)$.

When the rod is set into rotation, the increase in the rotational K.E. is given by



$$\text{K.E.} = \frac{1}{2}I_1\omega_0^2 + \frac{1}{2}I_2\omega_0^2 = \frac{1}{2}m_1x^2\omega_0^2 + \frac{1}{2}m_2(L-x)^2\omega_0^2$$

According to work-energy theorem

Work done, W = increase in rotational K.E.

i.e., $$W = \frac{1}{2}m_1x^2\omega_0^2 + \frac{1}{2}m_2(L-x)^2\omega_0^2$$

Work will be minimum if $\frac{dW}{dx} = 0$

or $$\frac{d}{dx}\left[\frac{1}{2}m_1x^2\omega_0^2 + \frac{1}{2}m_2(L-x)^2\omega_0^2\right] = 0$$

$$m_1x\,\omega_0^2 - m_2(L-x)\,\omega_0^2 = 0$$

$$\omega_0^2\,[m_1x - m_2(L-x)] = 0$$

Since $\omega_0^2 \neq 0$ $\therefore$ $m_1x - m_2(L-x) = 0$ or $(m_1 + m_2)\,x = m_2 L$

$\therefore$ $x = m_2L/(m_1 + m_2)$

Q. 10. *Two discs of same mass and thickness are made of materials having different densities. Which one of them will have larger M.I.?*

Ans. Since mass of both the discs is same

$\therefore$ $\pi r_1^2 t \rho_1 = \pi r_2^2 t \rho_2$

[Mass = Volume × Density = Area × thickness × Density]

$\therefore$ $$\frac{\rho_1}{\rho_2} = \frac{r_2^2}{r_1^2}$$

$\therefore$ $$\frac{I_1}{I_2} = \frac{\frac{1}{2}Mr_1^2}{\frac{1}{2}Mr_2^2} = \frac{r_1^2}{r_2^2} = \frac{\rho_2}{\rho_1} \quad i.e., \quad I \propto \frac{I}{\rho}$$

This shows that M.I. of the disc having less density will be larger.

V. VALUE-BASED QUESTIONS

Q. 1. *A 8 years old boy was sitting stationary at one end of a long trolly. The trolly is moving with a definite speed on a smooth horizontal floor. The boy was enjoying it very much. After sometime he stood up and ran about on the trolly in any manner. The boy surprised that the trolley did not become unbalanced. He asked this question to his teacher immediately. The teacher explained him about the centre of mass of the trolley and child together.*

(i) What values of the boy are shown here?

(ii) How did his teacher explained the reason?

Ans. (*i*) Values are : Curiosity, presence of mind, high thinking and sincerity.

(*ii*) The teacher explained the reason as the body and the trolley constitute one single system and the forces involved are purely internal forces. As there is no external force, there is no change in momentum of the system and velocity remains unchanged.

Q. 2. *Raja, a student of class V was playing with 'LATTU' with his elder brother who was studying in XI science. Raja asked his brother, why this lattu is rotating so fast about a sharp nail and did not fall even in tilted position. His elder brother explained Raja about the centre of mass and centre of gravity of the body.*

(i) What values are displayed by Raja?

(*ii*) *A constant torque of 1000 Nm turns a wheel of moment of inertia 200 kg m^2 about an axis through its centre. What is its angular velocity after 3 seconds?*

Ans. (*i*) The value displayed are : curiosity, wants to learn something, sharp mind and sensible.

(*ii*) Angular acceleration $\alpha = \frac{\tau}{I} = \frac{1000}{200} = 5 \text{ rad s}^{-2}$

$\therefore$ Angular velocity after 3 sec.

$$\omega = \omega_0 + \alpha t = 0 + 5 \times 3 = 15 \text{ rad s}^{-1}$$

Q. 3. *Deepak went to a fair with his father. There a show was going on. He saw that a person sits near the edge of a circular platform revolving with a uniform angular speed. After sometime the person gets up and starts moving from the edge towards the centre of the platform. Deepak observed that the angular velocity of the moving plateform increases. He wants this answer from his father. His father explained him that when moment of inertia of body decreases, the angular velocity increases.*

(*i*) *What are the values of Deepak displayed here?*

(*ii*) *What is the relation between angular momentum, moment of inertia and angular velocity.*

Ans. (*i*) Values are : Sharp mind, keen observation, curiosity and intelligence.

(*ii*) If L = Angular momentum

I = Moment of Inertia

ω = Angular velocity

$\therefore L = I\omega$

TEST YOUR SKILLS

Q. 1. Establish a relation between torque and moment of inertia of a rigid body.

Q. 2. If angular momentum is conserved in a system whole moment of inertia is decreased, will its rotational kinetic energy be also conserved? Explain.

Q. 3. State and prove the "Theorem of Perpendicular axes".

Q. 4. What is radius of gyration? Give its mathematical expression.

Q. 5. Derive the following equations of rotational motion.

(*i*) $\omega = \omega_0 + \alpha t$ (*ii*) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ (*iii*) $\omega^2 - \omega_0^2 = 2\alpha\theta$

Q. 6. State and explain the principle of conservation of angular momentum.

Q. 7. Derive an expression for moment of inertia of a uniform circular disc about any diameter.

Q. 8. Obtain an expression of kinetic energy of rotation of a body. What is the total energy if the body has translational motion as well?

Q. 9. A ring, a disc and a sphere having same radius and same mass, roll down an inclined plane from the same height '*h*'. Which of these reaches the bottom (*i*) earliest (*ii*) Latest?

Q. 10. A planet revolves around a massive star in a highly elliptical orbit. Is its angular momentum constant over the entire orbit?

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