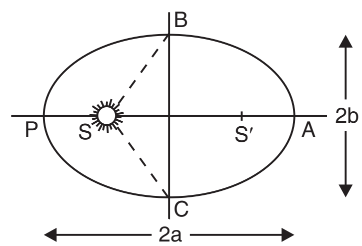
# 8 Gravitation

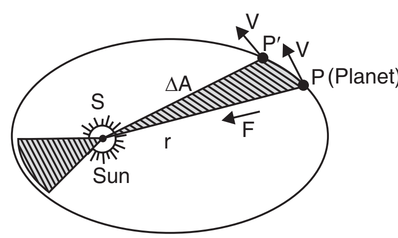
## Facts that Matter

- **Kepler's Laws of Planetary Motion**




Johannes Kepler formulated three laws which describe planetary motion. They are as follows:

(*i*) **Law of orbits.** Each planet revolves around the sun in an elliptical orbit with the sun at one of the foci of the ellipse.




(*ii*) **Law of areas.** The speed of planet varies in such a way that the radius, vector drawn from the sun to planet sweeps out equal areas in equal times.

(*iii*) **Law of periods.** The square of the time period of revolution is proportional to the cube of the semi-major axis of the elliptical orbit. *i.e.,* $T^2 \propto r^3$.

If $r_1$ and $r_2$ are the shortest and the longest distances of the planet from the sun, the semi-major axis is given by $\left(\frac{r_1 + r_2}{2}\right)$.

- **Newton's Law of Gravitation**

Newton's law of gravitation states that every particle in the universe attracts every other particle with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between them. The direction of the force is along the line joining the particles.

$$F \propto m_1 m_2 \quad \text{and} \quad F \propto \frac{1}{r^2}$$

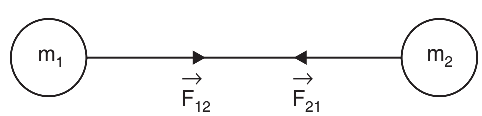
$$\therefore \qquad F = G\frac{m_1 m_2}{r^2}$$

where $G$ is a constant, called as the universal constant of gravitation.

- Vectorially the gravitation force is given as

$$\vec{F} = \frac{G\, m_1 m_2}{r^2}\hat{r}$$

$$\vec{F}_{12} + \vec{F}_{21} = 0$$



The gravitational force between two particles form an action-reaction pair.

- The value of '$G$' has been experimentally determined *i.e.,* $G = 6.67 \times 10^{-11}$ Nm$^2$ kg$^{-2}$. Dimensional formula of $G$ is $[M^{-1} L^3 T^{-2}]$.
- Universal constant of gravitation $G$ is numerically equal to the force of attraction between two particles of unit mass each separated by unit distance.

- **Important Characteristics of Gravitational Force**

(*i*) Gravitational force between two bodies is a central force *i.e.,* it acts along the line joining the centres of the two interacting bodies.

(*ii*) Gravitational force between two bodies is independent of the nature of the intervening medium.

(*iii*) Gravitational force between two bodies does not depend upon the presence of other bodies.

(*iv*) It is valid for point objects and spherically symmetrical objects.

(*v*) Magnitude of force is extremely small.

- **Principle of Superposition of Gravitation**

It states that the resultant gravitational force $\vec{F}$ acting on a particle due to number of point masses is equal to the vector sum of the forces exerted by the individual masses on the given particle *i.e.,*

$$\vec{F} = \vec{F}_{01} + \vec{F}_{02} + \ldots.. + \vec{F}_{0n} = \sum_{i=1}^{n} \vec{F}_{0i}$$

where $\vec{F}_{01}, \vec{F}_{02}, \ldots.. \vec{F}_{0n}$ are the gravitational forces on a particle of mass $m_0$ due to particles of masses $m_1$, $m_2$, ..... $m_n$ respectively.

- **Acceleration Due to Gravity**

The acceleration produced in a body on account of the force of gravity is known as acceleration due to gravity. It is usually denoted by '$g$'. It is always towards the centre of Earth.

If a body of mass '$m$' lying on the surface of the earth, the gravitational force acting on the body is given by

$$F = G\frac{M\,m}{R^2} \quad \text{where,} \quad M \rightarrow \text{mass of earth,} \quad R \rightarrow \text{radius of earth}$$

acceleration due to gravity, $g = \dfrac{GM}{R^2}$.

- **Mass and Mean Density of Earth**

Mass and Mean density of Earth is given in the following manner.

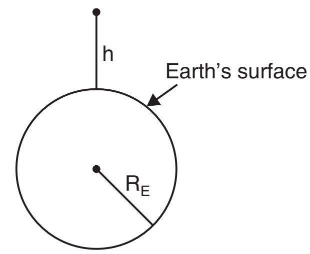
Mass of earth, $M = \dfrac{gR^2}{G}$

Mean density of earth, $\rho = \dfrac{3g}{4\pi GR}$.

- **Variation of Acceleration Due to Gravity**

The value of acceleration due to gravity changes with height (*i.e.,* altitude), depth, shape of the earth and rotation of earth about its own axis.

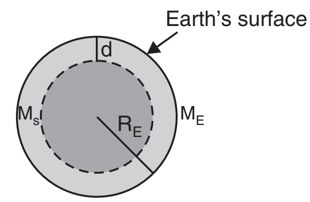
(*a*) **Effect of Altitude.** As one goes above the surface of Earth, value of acceleration due to gravity gradually goes on decreasing. If $g_h$ be the value of acceleration due to gravity at a height $h$ from the surface of Earth, then




$$g_h = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

If $h << R$, then

$$g_h = g\left(1-\frac{2h}{R}\right). \text{ (using Binomial Theorem)}$$




(*b*) **Effect of Depth.** The value of acceleration due to gravity decreases with depth. It is given as

$$g_d = g\left(1-\frac{d}{R}\right) \quad \text{where} \quad d \to \text{depth.}$$

At the centre of the earth, the value of acceleration due to gravity becomes zero.

(*c*) **Effect due to rotation of earth.** The acceleration due to gravity (*i*) decreases due to rotation of earth. (*ii*) increases with the increase in lattitude. It is given by

$$g_\theta = g\left(1-\frac{R\omega^2}{g}\cos^2\theta\right), \quad \text{where,} \quad \theta \to \text{latitude of the point.}$$

(*d*) **Effect due to shape.** The equatorial radius of earth is longer than its polar radius.
The value of $g$ increases from equator to pole. It is given as:

$$g_{\text{pole}} > g_{\text{equator}}$$

- **Gravitational Field**

The space around a body within which its gravitational force of attraction is experienced by other bodies is called gravitational field.

- **Intensity of Gravitational Field**

The intensity of the gravitational field of a body at a point in the field is defined as the force experienced by a body of unit mass placed at that point provided the presence of unit mass does not disturb the original gravitational field.
Intensity of gravitational field at a point $p$ is given by

$$I = \frac{GM}{x^2}$$

where $M$ creates a field and the point $p$ where the field is estimated is at a distance $r$.

In vector form, $$\vec{I} = -\frac{GM}{x^2}\hat{x}$$

Here *–ve* sign shows that the gravitational intensity is of attractive nature.

- Unit of intensity of gravitational field in S.I. is $Nkg^{-1}$ or $ms^{-2}$ and in cgs system, is dyne $g^{-1}$ or cm $s^{-2}$.

Dimensional formula of gravitational intensity is $[LT^{-2}]$.

- **Gravitational Potential**

The gravitational potential at a point in the gravitational field of a body is defined as the amount of work done in bringing a body of unit mass from infinity to that point.

Gravitational potential at a point situated at a distance $r$ from a body or particle of mass $M$ is given by

$$V = -\frac{GM}{r}$$

Its unit is joule/kg and it is a scalar quantity. Its dimensional formula is $[L^2 T^{-3}]$.

- **Gravitational Potential Energy**

The work done in carrying a mass '$m$' from infinity to a point at distance $r$ is called gravitational potential energy.

The gravitational potential energy of the system is given by

$$U = -\frac{GMm}{r}$$

*i.e.,* Gravitational potential energy = gravitational potential × mass of the body.

It is a scalar quantity and measured in joule.

- **Escape Velocity**

The minimum velocity required to project a body vertically upward from the surface of earth so that it comes out of the gravitational field of earth is called escape velocity.

It is given by $$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$ where $M \rightarrow$ Mass of Earth

$R \rightarrow$ radius of the Earth.

- **Satellite**

A satellite is a body which is revolving continuously in an orbit around a comparatively much larger body.

The orbit may be either circular or elliptical. A man-made object revolving in an orbit around a planet is called an artificial satellite.

- **Orbital Velocity**

Orbital velocity of a satellite is the minimum velocity required to put the satellite into a given orbit around earth.

The orbital velocity is given by

$$v_o = \sqrt{\frac{GM}{(R+h)}} = \sqrt{\frac{gR^2}{(R+h)}}$$

where, $h \rightarrow$ height of the satellite above the surface of earth

$M \rightarrow$ Mass of the earth

$R \rightarrow$ radius of the earth.

For a satellite orbiting just near earth (where $h << R$), we have

$$v_o = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

- Time taken by a satellite to complete one revolution around the earth is known as its time of revolution or time period $T$. It is given by

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}} = \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}} = \sqrt{\frac{3\pi(R+h)^3}{G\rho R^3}}$$

where ρ is the mean density of the earth.

- For satellite orbiting close to the surface of earth, $h << R$

$$T = \sqrt{\frac{3\pi}{G\rho}}.$$

- For satellite orbiting close to the surface of earth, the relation between orbital and escape velocity is given by

$$v_e = \sqrt{2}\, v_o .$$

- Height of satellite above the earth's surface is given by

$$h = \left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3} - R$$

- A satellite revolving around the earth possesses kinetic energy as well as potential energy. The P.E. of a satellite is

$$U = -\frac{GMm}{(R+h)}$$

The kinetic energy of satellite is

$$K = \frac{GMm}{2(R+h)}$$

Total mechanical energy of the satellite $= -\dfrac{GMm}{2(R+h)}$

- The energy required to remove the satellite from its orbit around the earth to infinity is called Binding energy of the satellite.

Binding energy of a satellite $= -E = \dfrac{GMm}{2(R+h)}$.

- **Geostationary Satellite**

The satellite having the same time period of revolution as that of the earth is called geostationary satellite. Such satellites should rotate in the equatorial plane from west to east.

The orbit of a geostationary satellite is called 'parking orbit'. These satellites are used for communication purposes.

A geostationary satellite revolves around the earth in a circular orbit at a height of about 36,000 km from the surface of earth.

- **IMPORTANT TABLES**

**TABLE 8.1 Data from measurement of planetary motions given below confirm Kepler's Law of Periods**

**$a \equiv$ Semi-major axis in units of $10^{10}$ m.**

**$T \equiv$ Time period of revolution of the planet in years($y$).**

**$Q \equiv$ The quotient ($T^3/a^3$) in units of $10^{-34}$ $y^2$ $m^{-3}$.)**

| Planet | $a$ | $T$ | $Q$ |
|---|---|---|---|
| Mercury | 5.79 | 0.24 | 2.95 |
| Venus | 10.8 | 0.615 | 3.00 |
| Earth | 15.0 | 1 | 2.96 |
| Mars | 22.8 | 1.88 | 2.98 |
| Jupiter | 77.8 | 11.9 | 3.01 |
| Saturn | 143 | 29.5 | 2.98 |
| Uranus | 287 | 84 | 2.98 |
| Neptune | 450 | 165 | 2.99 |
| Pluto | 590 | 248 | 2.99 |

**TABLE 8.2.**

| *Physical Quantity* | *Symbol* | *Dimensions* | *Unit* | *Remarks* |
|---|---|---|---|---|
| Gravitational Constant | $G$ | $[M^{-1} L^3 T^{-2}]$ | N m$^2$ kg$^{-2}$ | $6.67 \times 10^{-11}$ |
| Gravitational Potential Energy | $V(r)$ | $[M L^2 T^{-2}]$ | J | $-\frac{GMm}{r}$ (scalar) |
| Gravitational Potential | $U(r)$ | $[L^2 T^{-2}]$ | J kg$^{-1}$ | $-\frac{GM}{r}$ (scalar) |
| Gravitational Intensity | $\mathbf{E}$ or $\mathbf{g}$ | $[LT^{-2}]$ | m s$^{-2}$ | $\frac{GM}{r^2}\hat{r}$ (vector) |

# NCERT TEXTBOOK QUESTIONS SOLVED

**8.1.** *Answer the following:*

(a) *You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?*

(b) *An astronaut inside a small spaceship orbiting around the Earth cannot detect gravity. If the space station orbiting around the Earth has a large size, can he hope to detect gravity?*

(c) *If you compare the gravitational force on the Earth due to the Sun to that due to the Moon, you would find that the Sun's pull is greater than the Moon's pull. (You can check this yourself using the data available in the succeeding exercises). However, the tidal effect of the Moon's pull is greater than the tidal effect of Sun. Why?*

**Sol.** (*a*) No. Gravitational forces are independent of medium. A body cannot be shielded from the gravitational influence of nearby matter.

(*b*) Yes. If the size of the spaceship is extremely large, then the gravitational effect of the spaceship may become measurable. The variation in $g$ can also be detected.

(*c*) Tidal effect depends inversely on the cube of the distance, unlike force which depends inversely on the square of the distance. Since the distance of moon from the ocean water is very small as compared to the distance of sun from the ocean water on earth. Therefore, the tidal effect of Moon's pull is greater than the tidal effect of the sun.

**8.2.** *Choose the correct alternative:*

*(a) Acceleration due to gravity increases/decreases with increasing altitude.*

*(b) Acceleration due to gravity increases/decreases with increasing depth (assume the Earth to be a sphere of uniform density).*

*(c) Acceleration due to gravity is independent of the mass of the Earth/mass of the body.*

*(d) The formula* $-GMm\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$s *is more/less accurate than the formula* $mg\,(r_2 - r_1)$ *for the difference of potential energy between two points* $r_2$ *and* $r_1$ *distance away from the centre of the Earth.*

**Sol.** (*a*) decreases (*b*) decreases (*c*) mass of the body (*d*) more.

**8.3.** *Suppose there existed a planet that went around the Sun twice as fast as the Earth. What would be its orbital size as compared to that of the Earth?*

**Sol.** Here, $T_e$ = 1 year; $T_p = \frac{T_e}{2} = \frac{1}{2}$ year; $r_e$ = 1 A.U.

Using Kepler's third law, we have $r_p = r_e\left(\frac{T_p}{T_e}\right)^{2/3} \Rightarrow r_p = 1\left(\frac{1/2}{1}\right)^{2/3} = 0.63$ AU

**8.4.** *Io, one of the satellites of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is* $4.22 \times 10^8$ *m. Show that the mass of Jupiter is about one-thousandth that of the Sun.*

**Sol.** For a satellite of Jupiter, orbital period, $T_1$ = 1.769 days = 1.769 × 24 × 60 × 60 s

Radius of the orbit of satellite, $r_1 = 4.22 \times 10^8$ m

Mass of Jupiter, $M_1$ is given by $M_1 = \frac{4\pi^2 r_1^3}{GT_1^2} = \frac{4\pi^2 \times (4.22\times10^8)^3}{G\times(1.769\times24\times60\times60)^2}$ ...(*i*)

We know that the orbital period of Earth around the sun,

$T$ = 1 year = 365.25 × 24 × 60 × 60 s;

orbital radius, $r$ = 1 A.U. = $1.496 \times 10^{11}$ m.

Mass of sun is given by $M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2\times(1.496\times10^{11})^3}{G\times(365.25\times24\times60\times60)^2}$ ...(*ii*)

Dividing eqn. (*ii*) by (*i*), we get

$$\frac{M}{M_1} = \frac{4\pi^2\times(1.496\times10^{11})^3}{G\times(365.25\times24\times60\times60)^2}\times\frac{G\times(1.769\times24\times60\times60)^2}{4\pi^2\times(4.22\times10^8)^3} = 1046$$

or $$\frac{M_1}{M} = \frac{1}{1046} \simeq \frac{1}{1000} \quad \Rightarrow \quad M_1 = \frac{1}{1000} M.$$ **Proved**

**8.5.** *Let us assume that our galaxy consists of 2.5 × $10^{11}$ stars each of one solar mass. How long will a star at a distance of 50,000 ly from the galactic centre take to complete one revolution? Take the diameter of the Milky way to be $10^5$ ly.*

**Sol.** Here, $r = 50000 \text{ ly} = 50000 \times 9.46 \times 10^{15} \text{ m} = 4.73 \times 10^{20} \text{ m}$

$M = 2.5 \times 10^{11} \text{ solar mass} = 2.5 \times 10^{11} \times (2 \times 10^{30}) \text{ kg} = 5.0 \times 10^{41} \text{ kg}$

We know that

$$M = \frac{4\pi^2 r^3}{GT^2}$$

or $$T = \left(\frac{4\pi^2 r^3}{GM}\right)^{1/2} = \left[\frac{4 \times (22/7)^2 \times (4.73 \times 10^{20})^3}{(6.67 \times 10^{-11}) \times (5.0 \times 10^{41})}\right]^{1/2} = 1.12 \times 10^{16} \text{ s.}$$

**8.6.** *Choose the correct alternative:*

(*a*) *If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy.*

(*b*) *The energy required to launch an orbiting satellite out of Earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of Earth's influence.*

**Sol.** (*a*) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic energy.

(*b*) The energy required to launch an orbiting satellite out of Earth's gravitational influence is less than the energy required to project a stationary object at the same height (as the satellite) out of Earth's influence.

**8.7.** *Does the escape speed of a body from the Earth depend on (a) the mass of the body, (b) the location from where it is projected, (c) the direction of projection, (d) the height of the location from where the body is launched?*

**Sol.** The escape speed $v_{es} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$. Hence,

(*a*) The escape speed of a body from the Earth does not depend on the mass of the body.

(*b*) The escape speed does not depend on the location from where a body is projected.

(*c*) The escape speed does not depend on the direction of projection of a body.

(*d*) The escape speed of a body depends upon the height of the location from where the body is projected, because the escape velocity depends upon the gravitational potential at the point from which it is projected and this potential depends upon height also.

**8.8.** *A comet orbits the Sun in a highly elliptical orbit. Does the comet have a constant (a) linear speed (b) angular speed (c) angular momentum (d) kinetic energy (e) potential energy (f) total energy throughout its orbit? Neglect any mass loss of the comet when it comes very close to the Sun.*

**Sol.** (*a*) The linear speed of the comet is variable in accordance with Kepler's second law. When comet is near the sun, its speed is higher. When the comet is far away from the sun, its speed is very less.

(*b*) Angular speed also varies slightly.

(*c*) Comet has constant angular momentum.

(*d*) Kinetic energy does not remain constant.

(*e*) Potential energy varies along the path.

(*f*) Total energy throughout the orbit remains constant.

**8.9.** *Which of the following symptoms is likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientational problem.*

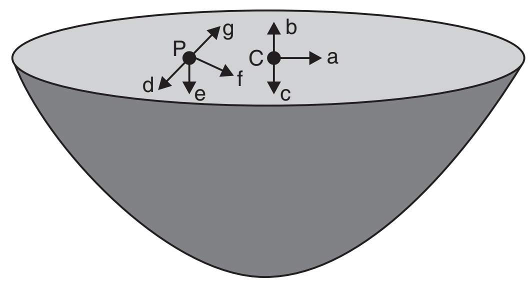
**Sol.** (*a*) The blood flow in feet would be lesser in zero gravity. So, the astronaut will not get swollen feet.

(*b*) In the conditions of weightlessness, the face of the astronaut is expected to get more supply. Due to it, the astronaut may develop swollen face.

(*c*) Due to more blood supply to face, the astronaut may get headache.

(*d*) Space also has orientation. We also have the frames of reference in space. Hence, orientational problem will affect the astronaut in space.

**8.10.** *In the following two exercises, choose the correct answer from among the given ones: The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see Fig.) (i) a, (ii) b, (iii) c, (iv) 0.*




**Sol.** At all points inside a hollow spherical shell, potential is same. So, gravitational intensity, which is negative of gravitational potential gradient, is zero. Due to zero gravitational intensity, the gravitational forces acting on any particle at any point inside a spherical shell will be symmetrically placed. It follows from here that if we remove the upper hemispherical shell, the net gravitational force acting on a particle at *P* will be downwards. Since gravitational intensity is gravitational force per unit mass therefore, the direction of gravitational intensity will be along *c*. So, option (*iii*) is correct.

**8.11.** *For the above problem, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow (i) d, (ii), e, (iii) f, (iv) g.*

**Sol.** Using the explanation given in the solution of the previous problem, the direction of the gravitational field intensity at *P* will be along *e*. So, option (*ii*) is correct.

**8.12.** *A rocket is fired from the earth towards the sun. At what distance from the earth's centre is the gravitational force on the rocket zero? Mass of the sun = $2 \times 10^{30}$ kg, mass of the earth = $6 \times 10^{24}$ kg. Neglect the effect of other planets etc. (orbital radius = $1.5 \times 10^{11}$ m).*

**Sol.** Mass of Sun, $M = 2 \times 10^{30}$ kg; Mass of Earth, $m = 6 \times 10^{24}$ kg

Distance between Sun and Earth, $r = 1.5 \times 10^{11}$ m

Let at the point *P*, the gravitational force on the rocket due to Earth

= gravitational force on the rocket due to Sun

Let $x$ = distance of the point *P* from the Earth

Then $$\frac{Gm}{x^2} = \frac{GM}{(r-x)^2}$$

$$\Rightarrow \quad \frac{(r-x)^2}{x^2} = \frac{M}{m} = \frac{2\times 10^{30}}{6\times 10^{24}} = \frac{10^6}{3}$$

or $$\frac{r-x}{x} = \frac{10^3}{\sqrt{3}} \quad \Rightarrow \quad \frac{r}{x} = \frac{10^3}{\sqrt{3}} + 1 \simeq \frac{10^3}{\sqrt{3}}$$

or $$x = \frac{\sqrt{3}r}{10^3} = \frac{1.732 \times 1.5 \times 10^{11}}{10^3} = 2.6 \times 10^8 \text{ m}.$$

**8.13.** *How will you 'weigh the sun', that is, estimate its mass? The mean orbital radius of the earth around the sun is 1.5 × 10⁸ km.*

**Sol.** The mean orbital radius of the Earth around the Sun

$$R = 1.5 \times 10^8 \text{ km} = 1.5 \times 10^{11} \text{ m}$$

Time period, $T = 365.25 \times 24 \times 60 \times 60$ s

Let the mass of the Sun be $M$ and that of Earth be $m$.

According to law of gravitation

$$F = G\frac{Mm}{R^2} \quad ...(i)$$

Centripetal force, $F = \frac{mv^2}{R} = m.R.\omega^2$ ...(*ii*)

From eqn. (*i*) and (*ii*), we have

$$\frac{GMm}{R^2} = m.R.\omega^2 = \frac{mR.4\pi^2}{T^2} \qquad \left[\because \omega = \frac{2\pi}{T}\right]$$

$$\therefore \quad M = \frac{4\pi^2 R^3}{G.T^2} = \frac{4 \times (3.14)^2 \times (1.5 \times 10^{11})^3}{6.67 \times 10^{-11} \times (365.25 \times 24 \times 60 \times 60)^2}$$

$$= 2.009 \times 10^{30} \text{ kg} = 2.0 \times 10^{30} \text{ kg}.$$

**8.14.** *A Saturn year is 29.5 times the Earth year. How far is the Saturn from the Sun if the Earth is 1.50 × 10⁸ km away from the Sun?*

**Sol.** We know that $T^2 \propto R^3$

$$\therefore \quad \frac{T_s^2}{T_e^2} = \frac{R_s^3}{R_e^3}$$

where subscripts $s$ and $e$ refer to the Saturn and Earth respectively.

Now $\frac{T_s}{T_e} = 29.5$ [given]; $R_e = 1.50 \times 10^8$ km

$$\left(\frac{R_s}{R_e}\right)^3 = \left(\frac{T_s}{T_e}\right)^2$$

$$R_s = R_e \times [(29.5)^2]^{1/3} = 1.50 \times 10^8 \times (870.25)^{1/3} \text{ km}$$

$$= 1.43 \times 10^9 \text{ km} = 1.43 \times 10^{12} \text{ m}$$

∴ Distance of Saturn from Sun = **$1.43 \times 10^{12}$ m.**

**8.15.** *A body weighs 63 N on the surface of the Earth. What is the gravitational force on it due to the Earth at a height equal to half the radius of the Earth?*

**Sol.** Let $g_h$ be the acceleration due to gravity at a height equal to half the radius of the Earth ($h = R/2$) and $g$ its value on Earth's surface. Let the body have mass $m$.

We know that

$$\frac{g_h}{g} = \left(\frac{R}{R+h}\right)^2 \quad \text{or} \quad \frac{g_h}{g} = \left(\frac{R}{R+\frac{R}{2}}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Let $W$ be the weight of body on the surface of Earth and $W_h$ the weight of the body at height $h$.

Then, $$\frac{W_h}{W} = \frac{mg_h}{mg} = \frac{g_h}{g} = \frac{4}{9} \quad \text{or} \quad W_h = \frac{4}{9}\text{W} = \frac{4}{9}\times 63\text{ N} = \mathbf{28\ N}.$$

**8.16.** *Assuming the earth to be a sphere of uniform mass density, how much would a body weigh half way down to the centre of the earth if it weighed 250 N on the surface?*

**Sol.** As $$g_d = g\left(1-\frac{d}{R}\right) \Rightarrow mg_d = mg\left(1-\frac{d}{R}\right)$$

Here $$d = \frac{R}{2}$$

$$\therefore \quad mg_d = (250)\times\left(1-\frac{R/2}{R}\right) = 250\times\frac{1}{2} = 125\text{ N}.$$

**8.17.** *A rocket is fired vertically with a speed of 5 km $s^{-1}$ from the earth's surface. How far from the earth does the rocket go before returning to the earth? Mass of the earth = 6.0 × $10^{24}$ kg; mean radius of the earth = 6.4 × $10^6$ m; G = 6.67 × $10^{-11}$ N $m^2$ $kg^{-2}$.*

**Sol.** Initial kinetic energy of rocket = $\frac{1}{2}mv^2 = \frac{1}{2} \times m \times (5000)^2 = 1.25 \times 10^7\ m$ J

At distance $r$ from centre of earth, kinetic energy becomes zero

$\therefore$ Change in kinetic energy = $1.25 \times 10^7 - 0 = 1.25 \times 10^7\ m$ J

This energy changes into potential energy.

Initial potential energy at the surface of earth = $\frac{GM_e m}{r}$

$$= \frac{-\left(6.67\times10^{-11}\right)\times\left(6\times10^{24}\right)m}{6.4\times10^6} = -6.25\ m \times 10^7\text{ J}$$

Final potential energy at distance, $r = -\frac{GM_e m}{r}$

$$= \frac{-\left(6.67\times10^{-11}\right)\times\left(6\times10^{24}\right)m}{r} = -4\times10^{14}\frac{m}{r}\text{ J}$$

$\therefore$ Change in potential energy = $6.25 \times 10^7\ m - 4 \times 10^{14}\ \frac{m}{r}$

Using law of conservation of energy,

$$6.25\times10^7\ m - \frac{4\times10^{14}m}{r} = 1.25 \times 10^7\text{ m} \quad i.e., \quad r = \frac{4\times10^{14}}{5\times10^7}m = 8\times10^{16}m.$$

**8.18.** *The escape speed of a projectile on the Earth's surface is 11.2 km $s^{-1}$. A body is projected out with thrice this speed. What is the speed of the body far away from the Earth? Ignore the presence of the Sun and other planets.*

**Sol.** Let $v_{es}$ be the escape speed from surface of Earth having a value $v_{es}$ = 11.2 kg s$^{-1}$ = 11.2 × 10$^3$ m s$^{-1}$. By definition

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R^2} \qquad ...(i)$$

When a body is projected with a speed $v_i = 3v_{es}$ = 3 × 11.2 × 10$^3$ m/s, then it will have a final speed $v_f$ such that

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 - \frac{GMm}{R^2} = \frac{1}{2}mv_i^2 - \frac{1}{2}mv_e^2$$

$$\Rightarrow \qquad v_f = \sqrt{v_i^2 - v_e^2} = \sqrt{\left(3\times 11.2\times 10^3\right)^2 - \left(11.2\times 10^3\right)^2}$$

$$= 11.2\times 10^3 \times \sqrt{8} = 31.7 \times 10^3 \text{ ms}^{-1} \text{ or } 31.7 \text{ km s}^{-1}.$$

**8.19.** *A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence? Mass of the satellite = 200 kg; mass of the earth = 6.0 × 10$^{24}$ kg; radius of the earth = 6.4 × 10$^6$ m; G = 6.67 × 10$^{-11}$ N m$^2$ kg$^{-2}$.*

**Sol.** Total energy of orbiting satellite at a height $h$

$$= -\frac{GMm}{(R+h)} + \frac{1}{2}mv^2 = -\frac{GMm}{(R+h)} + \frac{1}{2}m\frac{GM}{(R+h)} = -\frac{GMm}{2(R+h)}$$

Energy expended to rocket the satellite out of the earth's gravitational field

= – (total energy of the orbiting satellite)

$$= \frac{GMm}{2(R+h)} = \frac{\left(6.67\times 10^{-11}\right)\times\left(6\times 10^{24}\right)\times 200}{2\times\left(6.4\times 10^6 + 4\times 10^5\right)} = 5.9 \times 10^9 \text{ J}.$$

**8.20.** *Two stars each of one solar mass (= 2 × 10$^{30}$ kg) are approaching each other for a head on collision. When they are at a distance 10$^9$ km, their speeds are negligible. What is the speed with which they collide? The radius of each star is 10$^4$ km. Assume the stars to remain undistorted until they collide. (Use the known value of G).*

**Sol.** Here, mass of each star, $M$ = 2 × 10$^{30}$ kg

Initial potential between two stars, $r$ = 10$^9$ km = 10$^{12}$ m.

Initial potential energy of the system = $-\dfrac{GMm}{r}$

Total K.E. of the stars = $\dfrac{1}{2}Mv^2 + \dfrac{1}{2}Mv^2 = Mv^2$

where $v$ is the speed of stars with which they collide. When the stars are about to collide, the distance between their centres,

$$r' = 2R.$$

∴ Final potential energy of two stars = $-\dfrac{GMM}{2R}$

Since gain in K.E. is at the cost of loss in P.E.

$$\therefore \qquad Mv^2 = -\frac{GMM}{r} - \left(-\frac{GMM}{2R}\right) = -\frac{GMM}{r} + \frac{GMM}{2R}$$

or $$2 \times 10^{30} v^2 = -\frac{6.67 \times 10^{-11} \times (2 \times 10^{30})^2}{10^{12}} + \frac{6.67 \times 10^{-11} \times (2 \times 10^{30})^2}{2 \times 10^7}$$

$$= -2.668 \times 10^{38} + 1.334 \times 10^{43} = 1.334 \times 10^{43} \text{ J}$$

$$\therefore \quad v = \sqrt{\frac{1.334 \times 10^{43}}{2 \times 10^{30}}} = 2.583 \times 10^6 \text{ ms}^{-1}.$$

**8.21.** *Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational field and potential at the mid point of the line joining the centres of the spheres? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable?*

**Sol.** Here $G = 6.67 \times 10^{-11}$ Nm$^2$ kg$^{-2}$; $M = 100$ kg; $R = 0.1$ m, distance between the two spheres, $d = 1.0$ m

Suppose that the distance of either sphere from the mid-point of the line joining their centre is $r$. Then $r = \frac{d}{2} = 0.5$ m. The gravitational field at the mid-point due to two spheres will be equal and opposite.

Hence, the resultant gravitational field at the mid point = 0

The gravitational potential at the mid point = $\left(-\frac{GM}{r}\right) \times 2$

$$= -\frac{6.67 \times 10^{-11} \times 100 \times 2}{0.5} = -2.668 \times 10^{-8} \text{ J kg}^{-1}.$$

The object placed at that point would be in stable equilibrium.

**8.22.** *As you have learnt in the text, a geostationary satellite orbits the Earth at a height of nearly 36,000 km from the surface of the Earth. What is the potential due to Earth's gravity at the site of this satellite? (Take the potential energy at infinity to be zero). Mass of the Earth = $6.0 \times 10^{24}$ kg, radius = 6400 km.*

**Sol.** Distance of satellite from the centre of earth = $R = r + x$

$$= 6400 + 36000 = 42400 \text{ km} = 4.24 \times 10^7 \text{ m}$$

Using potential, $V = -\frac{GM}{R}$, we get

$$V = -\frac{(6.67 \times 10^{-11}) \times (6 \times 10^{24})}{(4.24 \times 10^7)} = -9.44 \times 10^6 \text{ J kg}^{-1}$$

**8.23.** *A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.5 rev. per second. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects called pulsars belong to this category). Will an object placed on its equator remain stuck to its surface due to gravity? (mass of the sun = $2 \times 10^{30}$ kg).*

**Sol.** Acceleration due to gravity of the star, $g = \frac{GM}{R^2}$ ...(*i*)

Here $M$ is the mass and $R$ is the radius of the star.

The outward centrifugal force acting on a body of mass $m$ at the equator of the star

$$= \frac{mv^2}{R} = mR\,\omega^2 \qquad ...(ii)$$

From equation ($i$), the acceleration due to the gravity of the star

$$= \frac{6.67\times10^{-11}\times2.5\times2\times10^{30}}{\left(12\times10^{3}\right)^{2}} = 2.316 \times 10^{12}\ \text{m/s}^2$$

$\therefore$ Inward force due to gravity on a body of mass $m$

$$= m \times 2.316 \times 10^{12}\ \text{N}$$

From equation ($ii$), the outward centrifugal force = $mR\omega^2$

$$= m \times (12 \times 10^3) \times \left(\frac{2\pi\times1.5}{-1}\right)^2 = m \times 1.06 \times 10^6\ \text{N}$$

Since the inward force due to gravity on a body at the equator of the star is about 2.2 million times more than the outward centrifugal force, the body will remain stuck to the surface of the star.

**8.24.** *A space-ship is stationed on Mars. How much energy must be expended on the spaceship to rocket it out of the solar system? Mass of the spaceship = 1000 kg, Mass of the Sun = 2 × $10^{30}$ kg. Mass of the Mars = 6.4 × $10^{23}$ kg, Radius of Mars = 3395 km. Radius of the orbit of Mars = 2.28 × $10^{11}$ m, G = 6.67 × $10^{-11}$ N m$^2$ kg$^{-2}$.*

**Sol.** Let $R$ be the radius of orbit of Mars and $R'$ be the radius of the Mars. $M$ be the mass of the Sun and $M'$ be the mass of Mars. If $m$ is the mass of the space-ship, then

Potential energy of space-ship due to gravitational attraction of the Sun = $-GM\,m/R$

Potential energy of space-ship due to gravitational attraction of Mars = $-G\,M'\,m/R'$

Since the K.E. of space ship is zero, therefore,

$$\text{total energy of spaceship} = -\frac{GM\,m}{R} - \frac{GM'\,m}{R'} = -G\,m\left(\frac{M}{R}+\frac{M'}{R'}\right)$$

$\therefore$ Energy required to rocket out the spaceship from the solar system = – (total energy of spaceship)

$$= -\left[-G\,m\left(\frac{M}{R}+\frac{M'}{R'}\right)\right] = G\,m\left[\frac{M}{R}+\frac{M'}{R'}\right]$$

$$= 6.67 \times 10^{-11} \times 1000 \times \left[\frac{2\times10^{30}}{2.28\times10^{11}}+\frac{6.4\times10^{23}}{3395\times10^{3}}\right]$$

$$= 6.67 \times 10^{-8}\left[\frac{20}{2.28}+\frac{6.4}{33.95}\right] \times 10^{18}\ \text{J} = 5.98 \times 10^{11}\ \text{J}.$$

**8.25.** *A rocket is fired 'vertically' from the surface of Mars with a speed of 2 km s$^{-1}$. If 20% of its initial energy is lost due to Martian atmospheric resistance, how far will the rocket go from the surface of mars before returning to it? Mass of Mars = 6.4 × $10^{23}$ kg; radius of Mars = 3395 km; G = 6.67 × $10^{-11}$ N m$^2$ kg$^{-2}$.*

**Sol.** Initial K.E. = $\frac{1}{2}\,mv^2$; Initial P.E. = $-\frac{GMm}{R}$

where $m$ = Mass of rocket, $M$ = Mass of Mars, $R$ = Radius of Mars

$\therefore$ Total initial energy $= \frac{1}{2}mv^2 - \frac{GMm}{R}$

Since 20% of K.E. is lost, only 80% remains to reach the height.

$\therefore$ Total initial energy available

$$= \frac{4}{5}\times\frac{1}{2}mv^2 - \frac{GMm}{R} = 0.4\,mv^2 - \frac{GMm}{R}$$

When the rocket reaches the highest point, at a height $h$ above the surface, its K.E. is zero and P.E. $= -\frac{GMm}{R+h}$.

Using principle of conservation of energy.

$$0.4\,mv^2 - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

or $$\frac{GMm}{R+h} = \frac{GMm}{R} - 0.4\,mv^2 \Rightarrow \frac{GM}{R+h} = \frac{GM}{R} - 0.4\,v^2$$

or $$\frac{GM}{R+h} = \frac{1}{R}[GM - 0.4\,Rv^2] \Rightarrow \frac{R+h}{R} = \frac{GM}{GM - 0.4\,Rv^2}$$

or $$\frac{h}{R} = \frac{GM}{GM - 0.4\,Rv^2} - 1$$

or $$\frac{h}{R} = \frac{0.4\,Rv^2}{GM - 0.4\,Rv^2} \Rightarrow h = \frac{0.4\,R^2v^2}{GM - 0.4\,Rv^2}$$

or $$h = \frac{0.4\times\left(2\times10^3\right)^2\times\left(3.395\times10^6\right)^2}{6.67\times10^{-11}\times6.4\times10^{23} - 0.4\times\left(2\times10^3\right)^2\times3.395\times10^6}\text{ m}$$

$$= 495\text{ km.}$$

# ADDTIONAL QUESTIONS SOLVED

## I. VERY SHORT ANSWER TYPE QUESTIONS

**Q. 1.** *What is the gravitational potential at infinity?*

**Ans.** Zero.

**Q. 2.** *The masses of two bodies are doubled and the distance is halved, how will the gravitational force change?*

**Ans.** Since $F \propto \frac{m_1 m_2}{r^2}$

Thus gravitational force will be 8 times more.

**Q. 3.** *A satellite revolving around earth loses height. How will its time period be changed?*

**Ans.** Time period of satellite is given by $T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$

Therefore, $T$ will decrease if $h$ decreases.

**Q. 4.** *What is the amount of work done in bringing a mass from the surface of Earth on one side to a point diametrically opposite to the other side?*

**Ans.** Since gravitational potential difference is zero therefore the work done is zero.

**Q. 5.** *Give two uses of polar satellites.*

**Ans.** They are used for (*i*) ground water survey, (*ii*) detecting the areas under forest.

**Q. 6.** *Distance between two bodies is increased to three times its original value. What is the effect on the gravitational force between them?*

**Ans.** Since $F \propto \frac{1}{r^2}$, hence the force will remain only $\frac{1}{9}$th part of its original value when the distance between two bodies is increased to three times.

**Q. 7.** *From where does a satellite revolving around a planet get the required centripetal force ?*

**Ans.** From gravitational attraction of planet exerted on satellite.

**Q. 8.** *Is gravitational potential a scalar or a vector? Why?*

**Ans.** Gravitational potential at a point is a scalar quantity because it is defined as work done per unit mass and the work done is a scalar quantity.

**Q. 9.** *Two planets are at distances $R_1$ and $R_2$ from the Sun. What will be the ratio of the squares of their periods?*

**Ans.** $\frac{R_1^3}{R_2^3}$.

**Q. 10.** *Where does a body weigh more at the surface of the earth or in a mine?*

**Ans.** Since value of $g$ in a mine is lesser than at the surface of the Earth, so weight of body in a mine is lesser than the weight of the body on the surface of the earth.

**Q. 11.** *Two artificial satellites, one of mass 400 kg and another of mass 2500 kg, are set in the same orbit around a planet. What is the ratio of their (i) orbital velocities, (ii) time periods?*

**Ans.** $\frac{v_1}{v_2} = \frac{T_1}{T_2} = 1$, because both satellites are revolving in same orbit and for a given orbit the orbital velocity as well as time period is independent of the mass of satellite.

**Q. 12.** *Can we determine the gravitational mass of a body inside on artificial satellite?*

**Ans.** No, artificial satellite is like a freely falling body and the weight of the body inside the satellite is zero.

**Q. 13.** *Why are space rockets usually launched from west to east in the equatorial plane?*

**Ans.** Space rockets are usually launched from west to east in the equatorial plane, so that linear velocity of Earth's rotation about its own axis is added with the launching velocity of rocket and linear velocity of Earth's rotation is maximum in the equatorial plane.

**Q. 14.** *A mass M is broken into two parts: m and (M – m). How is m related to M so that the gravitational force between two parts is maximum?*

**Ans.** The gravitational force will be maximum when $m\,(M - m)$ is maximum *i.e.,* when

$$m = M - m \quad \text{or} \quad 2m = M \quad \Rightarrow \quad m = \frac{M}{2}.$$

**Q. 15.** *The gravitational potential energy of a body at a distance r from the centre of earth is U. What is the weight of the body at that point?*

**Ans.** $U = \frac{GMm}{r} = \left(\frac{GM}{r^2}\right) rm = grm \Rightarrow U = mgr$ or $mg = \frac{U}{r}$ (weight of the body).

**Q. 16.** *A mass thrown up returns to the surface of earth. What is the nature of total energy possessed?*

**Ans.** Total energy has to be negative, since potential energy dominates over kinetic energy.

**Q. 17.** *Should the value of escape velocity be less or more on the surface of Moon as compared to Earth?*

**Ans.** Since, escape velocity, $v_e = \sqrt{2gR}$

The value of '$g$' and '$R$' is less on Moon than the corresponding values on Earth, therefore, value of escape velocity at Moon surface is less.

**Q. 18.** *What is the relation between the orbital and escape velocity?*

**Ans.** If $v_e$ is escape and $v_0$ is orbital velocity, then

$$v_e = \sqrt{2}\, v_o$$

**Q. 19.** *What would happen to an artificial satellite, if its orbital velocity is slightly decreased due to some defects in it?*

**Ans.** It will fall onto the Earth.

**Q. 20.** *What is the time period of revolution of polar satellite of Earth?*

**Ans.** About 100 minutes.

**Q. 21.** *What do you mean by a parking orbit of a satellite?*

**Ans.** The orbit of a satellite which is concentric and coplanar with the equatorial plane of Earth and having a revolution period of 24 hours is called a parking orbit.

**Q. 22.** *Two satellites of masses 3m and m orbit the earth in circular orbits of radii r and 3r respectively. What is the ratio of their orbital speeds?*

**Ans.** Since $$v = \sqrt{\frac{GM}{r}} \quad i.e., \quad v \propto \frac{1}{\sqrt{r}}$$

Thus $$\frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{3r}{r}} = \sqrt{3}$$

**Q. 23.** *The gravitational force between two spheres is x when the distance between their centres is y. What will be the new force if the separation is made 3y?*

**Ans.** Since $F \propto \frac{1}{r^2}$. Therefore, if $r$ is increased by a factor of 3. $F$ will be reduced by a factor of 9. Thus, the new force will be $\frac{x}{9}$.

**Q. 24.** *Name one factor on which the period of revolution of a planet around the Sun depends.*

**Ans.** Period of revolution of a planet around the Sun depends upon the distance of the planet from the Sun (*i.e.,* orbital radius of the planet).

**Q. 25.** *Give the S.I. units of 'g' and 'G'.*

**Ans.** The S.I. unit of '$g$' is $ms^{-2}$ and the S.I. unit of '$G$' is $Nm^2/kg^2$.

**Q. 26.** *Give the dimensional formula of 'g' and 'G'.*

**Ans.** Dimensional formula of '$g$' $\rightarrow [LT^{-2}]$

Dimensional formula of '$G$' $\rightarrow [M^{-1} L^3 T^{-2}]$

**Q. 27.** *A satellite is orbiting around the Earth with a speed v. To make the satellite escape, what is the minimum percentage increase in its speed?*

**Ans.** Percentage increase in speed = $\frac{(v_e - v_0)}{v_0} \times 100 = \left(\frac{v_e}{v_o} - 1\right) \times 100$

$$= \left(\sqrt{2} - 1\right) \times 100 = 41.4\%.$$

**Q. 28.** *Is it possible to put a satellite into its orbit by firing it from a huge sized gun?*

**Ans.** Theoretically, it is possible but practically it is not feasible on account of large air resistance and other technical difficulties.

**Q. 29.** *If a satellite is revolving around a planet of density ρ, show that the entity ρ $T^2$ is a universal constant. When a satellite is orbiting close to earth, then*

**Ans.** $\frac{GM\,m}{R^2} = m\,R\,\omega^2$ or $\frac{G\frac{4}{3}\pi R^3 \rho\, m}{R^2} = m\,R\,\frac{4\pi^2}{T^2}$ or $\rho\, T^2 = \frac{3\pi}{G} = a$ constant.

## II. SHORT ANSWER TYPE QUESTIONS

**Q. 1.** *Calculate the acceleration due to gravity at the surface of Mars if its diameter is 6760 km and mass one tenth that of the earth. The diameter of earth is 12742 km and acceleration due to gravity on the earth is 9.8 m/s$^2$.*

**Ans.** As we know

$$g = \frac{GM}{R^2}$$

Let $g_M$ and $g_e$ be the acceleration due to gravity at Mars and Earth respectively.

$$\therefore \quad \frac{g_M}{g_e} = \left(\frac{M_M}{M_e}\right)\left(\frac{R_e}{R_M}\right)^2 = \left(\frac{1}{10}\right)\left(\frac{12742}{6760}\right)^2 \quad \text{or} \quad \frac{g_M}{g_e} = 0.35$$

$$\therefore \quad g_M = 0.35 \times g_e = 0.35 \times 9.8 = 3.48\ \text{ms}^{-2}.$$

**Q. 2.** *Under what circumstances would your weight become zero?*

**Ans.** The weight will become zero under the following circumstances :

(*i*) during free fall

(*ii*) at the centre of the earth

(*iii*) in an artificial satellite

(*iv*) at a point where gravitational pull of earth is equal to the gravitational pull of the Moon.

**Q. 3.** *Two planets have masses in the ratio 1 : 10 and radii in the ratio 2 : 5. Compare*

*(a) their densities*

*(b) the acceleration due to gravity on their surface*

*(c) escape velocities from their surfaces, and*

*(d) the periods of revolutions of satellites near to their surfaces.*

**Ans.** Let $M_1$, $M_2$ by the masses and $R_1$, $R_2$ be the radii of the planets.

$$\Rightarrow \quad \frac{M_1}{M_2} = \frac{1}{10} \quad \text{and} \quad \frac{R_1}{R_2} = \frac{2}{5}$$

(*a*) Ratio of densities = $\frac{d_1}{d_2}$ or, $\frac{d_1}{d_2} = \left[\frac{M_1}{\frac{4}{3}\pi R_1^3}\right]\left[\frac{\frac{4}{3}\pi R_2^3}{M_2}\right]$

or, $\frac{d_1}{d_2} = \frac{M_1}{M_2}\left[\frac{R_2}{R_1}\right]^3 \Rightarrow \frac{d_1}{d_2} = \left[\frac{1}{10}\right]\left[\frac{5}{2}\right]^3 = \frac{25}{16}$

(*b*) Acceleration due to gravity at the surface = $g = \frac{GM}{R^2}$

$$\therefore \quad \frac{g_1}{g_2} = \frac{M_1}{M_2}\left[\frac{R_2}{R_1}\right]^2 = \frac{1}{10}\left[\frac{5}{2}\right]^2 = \frac{5}{8}$$

(*c*) Escape velocity = $\sqrt{\frac{2GM}{R}} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{M_1}{M_2}}\sqrt{\frac{R_2}{R_1}} = \sqrt{\frac{1}{10}\times\frac{5}{2}} = \mathbf{\frac{1}{2}}$

(*d*) Time period of a satellite near the surface (orbit radius = $R$) = $\frac{2\pi}{\sqrt{GM}} R\sqrt{R}$

$$\Rightarrow \quad \frac{T_1}{T_2} = \sqrt{\frac{M_2}{M_1}}\left[\frac{R_1}{R_2}\right]\left[\sqrt{\frac{R_1}{R_2}}\right] = \sqrt{\frac{10}{1}}\left[\frac{2}{5}\right]\left[\sqrt{\frac{2}{5}}\right] = \frac{4}{5}$$

**Q. 4.** *What is the height at which the value of g is the same as at a depth of* $\frac{R}{2}$ *?*

**Ans.** At depth = $\frac{R}{2}$, value of acceleration due to gravity,

$$g' = g\left(1 - \frac{R}{2R}\right) = \frac{g}{2}$$

At height $x$,

$$g' = g\left(1 - \frac{2x}{R}\right)$$

$$\therefore \quad g\left(1 - \frac{2x}{R}\right) = \frac{g}{2}$$

$$\frac{1}{2} = \frac{2x}{R} \Rightarrow x = \frac{R}{4}.$$

**Q. 5.** *Compute the mass of a planet that has a satellite whose time period is T and orbital radius is r.*

**Ans.** Suppose that a satellite of mass $m$ described a circular orbit around a planet of Mass $M$. The force of attraction between the planet and its satellite is

$$F = -G\frac{Mm}{r^2}$$

This force must be mass times the centripetal acceleration, *i.e.*,

$$\frac{v^2}{r} = \omega^2 r$$

Thus $m\omega^2 r = \dfrac{4\pi^2 mr}{T^2} = G\dfrac{mM}{r^2}$ or $M = \dfrac{4\pi^2 r^3}{GT^2}$.

**Q. 6.** *An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth.*

*(a) Determine the height of the satellite above the earth's surface.*

*(b) If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, find the speed with which it hits the surface of the earth. [g = 9.8 m/s² and $R_e$ = 6400 km]*

**Ans.** (*a*) We know that for satellite motion,

$$v_o = \sqrt{\frac{GM}{r}} = R\sqrt{\frac{g}{(R+h)}} \quad \left[\text{as } g = \frac{GM}{R^2} \text{ and } r = R+h\right]$$

In this problem,

$$v_o = \frac{1}{2}v_e = \frac{1}{2}\sqrt{2gR}$$

So, $\dfrac{R^2 g}{R+h} = \dfrac{1}{2}gR$ *i.e.*, $h = R = 6400$ km

(*b*) By conservation of *ME*

$$0 + \left(-\frac{GMm}{r}\right) = \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right)$$

or $$v^2 = 2GM\left[\frac{1}{R} - \frac{1}{2R}\right] \qquad [\text{as } r = R + h = R + R = 2R]$$

or $$v = \sqrt{\frac{GM}{R}} = \sqrt{gR} = 8 \text{ km/s}$$

**Q. 7.** *Find an expression for the weight of a body at the centre of the Earth.*

**Ans.** We know that value of acceleration due to gravity at a depth '*d*' below the surface of Earth is given by

$$g_d = g\left(1 - \frac{d}{R}\right)$$

At the centre of Earth $d = R$ and hence,

$$g_{centre} = g\left(1 - \frac{R}{R}\right) = g\,(1 - 1) = 0$$

∴ Weight of a body at the centre of Earth = $mg_{centre} = m \times 0 = 0$

It means that at the centre of Earth a body will be weightless.

**Q. 8.** *Calculate the minimum energy required to launch a 250 kg satellite from earth's surface at an altitude of 2R when R is the radius of the earth and is equal to 6400 km.*

**Ans.** The total energy of a satellite of mass *m* in a circular orbit of radius *r* is

$$\frac{1}{2}mv^2 - G\frac{Mm}{r}$$

where *r* is measured from the centre of the earth. Total mechanical energy in the orbit is

$$E = G\frac{mM}{2r} - G\frac{Mm}{r} = -G\frac{mM}{2r}$$

Here, $r = 2R + R = 3R$

$$E = -G\frac{mM}{6R}$$

The potential energy on the surface of the earth $= -G\dfrac{Mm}{R}$

$$\text{Minimum energy required} = -\frac{1}{6}G\frac{mM}{R} - \left(-G\frac{Mm}{R}\right) = \frac{5}{6}G\frac{mM}{R}$$

$$= \frac{5}{6}mgR = \frac{5}{6} \times 250 \times 9.8 \times 6.4 \times 10^6 \text{ J}$$

$$= 1.3 \times 10^{10} \text{ J.}$$

**Q. 9.** *A satellite is revolving around the earth, close to the surface of earth with a kinetic energy E. How much kinetic energy should be given to it so that it escapes from the surface of earth?*

**Ans.** Let $v_o$, $v_e$ be the orbital and escape speeds of the satellite, then $v_e = \sqrt{2}\, v_o$.

Energy in the given orbit, $E_1 = \dfrac{1}{2}m v_o^2 = E$ ...(*i*)

Energy for the escape speed, $E_2 = \dfrac{1}{2}m v_e^2 = \dfrac{1}{2}m\left(\sqrt{2}\, v_o\right)^2 = 2\text{ E}$

$\therefore$ Energy required to be supplied $= E_2 - E_1 = \mathbf{E.}$

**Q. 10.** *Taking the moon's orbit around earth to be r and mass of earth 81 times the mass of the moon. Find the position of the point from the earth, where the net gravitational field is zero.*

**Ans.** Let $x$ be the distance of a point from the earth where resultant gravitational field intensity is zero. So

$$\frac{GM_e}{x^2} = \frac{GM_e}{(r-x)^2} \quad \text{or} \quad \frac{81\, M_m}{x^2} = \frac{M_m}{(r-x)^2} \quad \text{or} \quad \frac{9}{x} = \frac{1}{(r-x)}$$

or $9\,r = 10x$ or $x = 9r/10 = \mathbf{0.9\ r}$

**Q. 11.** *Why do different planets have different escape velocities?*

**Ans.** Escape velocity, $v = \sqrt{2gR} = \sqrt{\dfrac{2GM}{R}}$.

Thus escape velocity of a planet depends upon (*i*) its mass ($M$) and (*ii*) its size ($R$). As different planets have different masses and sizes, so they have different escape velocities.

**Q. 12.** *The distance between earth and moon is 3.8 × 10⁵ km and the mass of earth is 81 times the mass of moon. Deduce the position of a point on the line joining the centres of earth and moon, where the gravitational field is zero. What would be the value of gravitational field there due to earth and moon separately?*

**Ans.** Let $x$ be the distance of the point of no net field from earth. The distance of this point from moon is $(r - x)$, where $r = 3.8 \times 10^5$ km.

The gravitational field due to earth $= \dfrac{GM_e}{x^2}$ and that due to moon $= G\dfrac{M_m}{(r-x)^2}$.

For the net field to be zero these are equal and opposite.

$$\frac{GM_e}{x^2} = \frac{GM_m}{(r-x)^2} \quad \text{or} \quad \frac{M_e}{M_m} = \frac{x^2}{(r-x)^2}$$

Given, $\frac{M_e}{M_m} = 81$, Thus $81 = \frac{x^2}{(r-x)^2}$

$\Rightarrow$ $\frac{x}{r-x} = 9 \Rightarrow 9r - 9x = x$ or $x = \frac{9}{10}r$

or $x = \frac{9}{10} \times 3.8 \times 10^5 = 3.42 \times 10^5$ km $= 3.42 \times 10^8$ m

The intensity of the field $= \frac{GM_e}{x^2} = \frac{R_e^2 g}{x^2} = \frac{(6.4\times10^6)^2 \times 9.8}{(3.42\times10^8)^2}$ N/kg

$= 3.43 \times 10^{-3}$ N/kg.

**Q. 13.** *A particle is projected vertically upwards from the surface of Earth of radius R with a kinetic energy equal to half of the minimum value needed for it to escape. Find the height to which it rises above the surface of Earth.*

**Ans.** We know that escape velocity from the surface of Earth is given by

$$v_{es} = \sqrt{\frac{2GM}{R}} \text{ and corresponding K.E. of a body } K_{es} = \frac{1}{2}mv_{es}^2 = \frac{GMm}{R}$$

As in present problem, the body is projected from the surface of Earth with a kinetic energy half of that needed to escape from Earth's surface, hence

Initial kinetic energy of body $K = \frac{K_{es}}{2} = \frac{GMm}{2R}$

and its potential energy at surface of Earth $U = -\frac{GMm}{R}$

$\therefore$ Total initial energy of body $K + U = \frac{GMm}{2R} - \frac{GMm}{R} = -\frac{GMm}{2R}$

Let the body goes up to a maximum height $h$ from surface of Earth, where its final K.E. = 0 and P.E. = $-\frac{GMm}{(R+h)}$

$\therefore$ Total energy now $= 0 - \frac{GMm}{(R+h)} = -\frac{GMm}{(R+h)}$

From conservation law of mechanical energy, we have

$$-\frac{GMm}{2R} = -\frac{GMm}{(R+h)}$$

On simplification it leads to the result $h = R$.

**Q. 14.** *Light from a massive star suffers 'gravitational red shift' i.e., its wavelength changes towards the red end due to the gravitational attraction of the star. Obtain the formula for this gravitational red shift using the simple consideration that a photon of frequency ν has energy hν (h = planck's constant) and mass hν/c². Estimate the magnitude of the red-shift for light of wavelength 5000 Å from a star of mass $10^{32}$ kg and radius $10^6$ km. G = $6.67 \times 10^{-11}$ Nm² kg⁻² and c = $3 \times 10^8$ m s⁻¹.*

**Ans.** $h\nu' = h\nu - \frac{GM\,h\nu}{R\,c^2}$ *i.e.,* $\nu' = \nu\left(1 - \frac{GM}{c^2 R}\right)$

where ν′ is the shifted frequency.

Now, $\lambda' = \lambda\left(1 + \frac{GM}{c^2 R}\right)$ if $\frac{GM}{Rc^2} < 1$

*i.e.,* $\lambda' - \lambda = \frac{\lambda GM}{c^2 R} = 0.371\ \text{Å}$

**Q. 15.** *A geostationary satellite is orbiting the earth at a height of 6R above the surface of the earth; R being the radius of the earth. What will be the time period of another satellite at a height 2.5 R from the surface of the earth?*

**Ans.** As $T^2 = kr^3$ or $T \propto r^{3/2}$

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} \Rightarrow T_2 = T_1\left(\frac{r_2}{r_1}\right)^{3/2}$$

$$= 24\left(\frac{2.5R + R}{6R + R}\right)^{3/2} = 24\left(\frac{1}{2}\right)^{3/2} = 6\sqrt{2} \text{ hour .}$$

**Q. 16.** *The planet Saturn has a mass 95 times that of the earth, and its radius is 9.5 times the earth's radius. Calculate the escape speed of a body from Saturn's surface, if the escape speed from the earth's surface is 11.2 kms$^{-1}$.*

**Ans.** Escape speed from the earth's surface is

$$v_e = \left[\frac{2\,GM}{R}\right]^{1/2} \text{ or } \left[\frac{2\,GM}{R}\right]^{1/2} = 11.2 \quad \text{...(1)}$$

Escape speed from Saturn's surface will be,

$$v = \left[\frac{2\,GM'}{R'}\right]^{1/2}$$

Now, $M' = 95$ M, $R' = 9.5$ R

$$v = \left[\frac{2 \times 95\,GM}{9.5\,R}\right]^{1/2} = 3.16 \times \sqrt{2\frac{GM}{R}}$$

But $\left[\frac{2GM}{R}\right]^{1/2} = 11.2$ km s$^{-1}$

$\therefore$ $v = 3.16 \times 11.2 = 35.4$ km s$^{-1}$.

**Q. 17.** *Find an expression for the orbital velocity of a satellite revolving around the earth in a circular orbit at a height h above the surface of earth.*

**Ans.** Consider a satellite of mass $m$ revolving around the earth at a height $h$ from its surface so that radius of its orbit $r = R + h$. If $v_o$ be the orbital velocity of satellite then centripetal force needed by it for its uniform circular motion is

$$F = \frac{mv_o^2}{r}$$

This value of centripetal force is provided by the gravitational pull of the earth acting on the satellite *i.e.,*

$$F = \frac{GMm}{r^2}$$

For equilibrium,

$$\frac{mv_o^2}{r} = \frac{GMm}{r^2} \Rightarrow v_o = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R+h)}}$$

But $g = \frac{GM}{R^2}$, hence $GM = gR^2$

$$\therefore \quad v_o = \sqrt{\frac{gR^2}{(R+h)}} = R\sqrt{\frac{g}{(R+h)}}.$$

**Q. 18.** *Two satellites A and B go around a planet P in circular orbits having radius 4R and R respectively. If the speed of the satellite A is 3 v, find the speed of the satellite B.*

**Ans.** As, $v_o = \sqrt{\frac{GM}{R}}$; so, $3v = \sqrt{\frac{GM}{4R}}$ and $v' = \sqrt{\frac{GM}{R}}$

$$\therefore \quad \frac{v'}{3v} = 2 \quad \text{or} \quad v' = 6\,v$$

**Q. 19.** *If the earth has a mass 9 times and radius twice of the planet Mars, calculate the minimum speed required by a rocket to pull out of the gravitational force of Mars. Escape speed on the surface of the earth is 11.2 km* $s^{-1}$.

**Ans.** Escape speed on the surface of earth is

$$v_e = \sqrt{\frac{2GM}{R}} = 11.2 \text{ km s}^{-1}$$

Now, Mass of Mars $= \frac{M}{9}$

Radius of Mars $= \frac{R}{2}$

$\therefore$ Escape speed on the surface of Mars is

$$v_m = \sqrt{\frac{2G(M/9)}{R/2}} = \sqrt{\frac{4}{9}\frac{GM}{R}} = \frac{\sqrt{2}}{3} \times v_e$$

or $$v_m = \frac{1.414}{3} \times 11.2 \text{ km s}^{-1} = 5.279 \text{ km s}^{-1}.$$

**Q. 20.** *A satellite is revolving just near the Earth's surface. Compute its orbital velocity. Given that radius of Earth R = 6400 km and g = 9.8 m* $s^{-2}$.

**Ans.** For a satellite revolving just near the Earth's surface, the orbital velocity has a magnitude given by

$$v_{orb} = \sqrt{gR}$$

$$\therefore \quad v_{orb} = \sqrt{9.8 \times 6400 \times 1000} = 7.92 \times 10^3 \text{ m s}^{-1} \text{ or } 7.92 \text{ km s}^{-1}.$$

**Q. 21.** *An astronaut, by mistake, drops his food packet from an artificial satellite orbiting around the Earth. Will it reach the surface of Earth? Why?*

**Ans.** The food packet will not fall on the Earth. As the satellite as well as astronaut were in a state of weightlessness, hence the food packet, when dropped by mistake, will also start moving with the same velocity as that of satellite and will continue to move along with the satellite in the same orbit.

**Q. 22.** *The planet Mars has two moons, phobos and Deimos.*

(*i*) *Phobos has a period 7 hours, 39 minutes and an orbital radius of 9.4 × 10³ km. Calculate the mass of mars.*

(*ii*) *Assume that earth and mars move in circular orbits around the sun, with the Martian orbit being 1.52 times the orbital radius of the earth. What is the length of the Martian year in days?*

**Ans.** (*i*) The Sun's mass replaced by the martian mass $M_m$.

$$T^2 = \frac{4\pi^2}{GM_m} R^3$$

$$M_m = \frac{4\pi^2}{G} \times \frac{R^3}{T^2}$$

or $$M_m = \frac{4 \times (3.14)^2 \times (9.4)^3 \times 10^{18}}{6.67 \times 10^{-11} \times (459 \times 60)^2} = \frac{4 \times (3.14)^2 \times (9.4)^3 \times 10^{18}}{6.67 \times (4.59 \times 6)^2 \times 10^{-5}}$$

$$= 6.48 \times 10^{23} \text{ kg}$$

(*ii*) Using Kepler's third law,

$$\frac{T_M^2}{T_E^2} = \frac{R_{MS}^3}{R_{ES}^3}$$

where $R_{MS}$ ($R_{ES}$) is the Mars (Earth) – Sun distance.

$$T_M = \left(\frac{R_{MS}}{R_{ES}}\right)^{3/2} \times T_E = (1.52)^{3/2} \times 365 = 684 \text{ days.}$$

**Q. 23.** *The magnitude of gravitational field at distances $r_1$ and $r_2$ from the centre of a uniform sphere of radius R and mass M are $I_1$ and $I_2$ respectively. Find the ratio of $(I_1/I_2)$ if $r_1 > R$ and $r_2 < R$.*

**Ans.** When $r_1 > R$, the point lies outside the sphere. Then sphere can be considered to be a point mass body whose whole mass can be supposed to be concentrated at its centre. Then gravitational intensity at a point distance $r_1$ from the centre of sphere will be,

$$I_1 = GM/r_1^2 \qquad ...(i)$$

When $r_2 < R$, the point $P$ lies inside the sphere. The unit mass body placed at $P$, will experience gravitational pull due to sphere of radius $r_2$, whose mass is

$$M' = \frac{M \times \frac{4}{3}\pi r_2^3}{\frac{4}{3}\pi R^3} = \frac{M r_2^3}{R^3}.$$

Therefore the gravitational intensity at $P$ will be $I_2 = \dfrac{GM r_2^3}{R^3} \times \dfrac{1}{r_2^2} = \dfrac{GM r_2}{R^3}$ ...(*ii*)

So $$\frac{I_1}{I_2} = \frac{GM}{r_1^2} \times \frac{R^3}{GM\, r_2} = \frac{R^3}{r_1^2\, r_2}.$$

**Q. 24.** *An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from earth. Determine*

*(a) the height of satellite above earth's surface.*

*(b) if the satellite is suddenly stopped, find the speed with which the satellite will hit the earth's surface after falling down.*

**Ans.** Escape velocity = $\sqrt{2gR}$, where $g$ is acceleration due to gravity on surface of earth and $R$ the radius of earth.

$$\text{Orbital velocity} = \frac{1}{2}v_e = \frac{1}{2}\sqrt{2gR} = \sqrt{\frac{gR}{2}} \qquad ...(i)$$

(*a*) If $h$ is the height of satellite above earth

$$\frac{mv_o^2}{R+h} = \frac{GMm}{(R+h)^2}$$

$$v_o^2 = \frac{GM}{R+h} = \frac{gR^2}{(R+h)}$$

$$\therefore \quad \left(\frac{1}{2}v_e\right)^2 = \frac{gR^2}{R+h} \quad \text{from eqn. } (i).$$

Now, $R + h = 2R$

$h = R.$

(*b*) If the satellite is stopped in orbit, the kinetic energy is zero and its potential energy is $-\dfrac{GMm}{2R}$.

Total energy = $-\dfrac{GMm}{2R}$

Let $v$ be its velocity when it reaches the earth.

Hence the kinetic energy = $\dfrac{1}{2}mv^2$

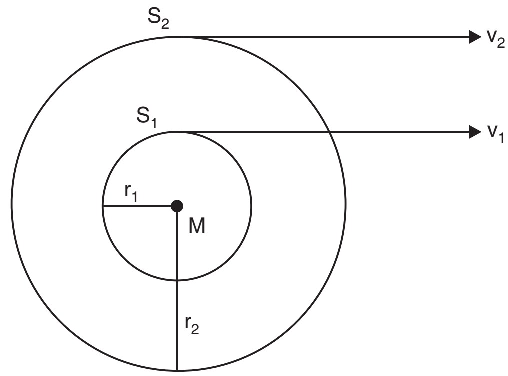
Potential energy = $-\dfrac{GMm}{R}$

$$\therefore \quad \frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{2R}$$

$$v^2 = 2GM\left(\frac{1}{R} - \frac{1}{2R}\right) = \frac{2gR^2}{2R}$$

$$\Rightarrow \quad v^2 = gR \quad \text{or} \quad v = \sqrt{gR}.$$

## III. LONG ANSWER TYPE QUESTIONS




**Q. 1.** *Two satellites $S_1$ and $S_2$ revolve round a planet in coplanar circular orbit in the same sense. Their periods of revolution are one hour and 8 hours respectively. The radius of the orbit of $S_1$ is $10^4$ km. When $S_2$ is close to $S_1$ find*

*(i) the speed of $S_2$ relative to $S_1$*

*(ii) the angular speed of $S_2$ as actually observed by an astronaut in $S_1$*

**Ans.** The centripetal force required by a satellite of mass $m$ revolving in a circular orbit of radius $r$ with a speed $v$ is supplied by the gravitational force extended by the planet of mass $M$ on the satellite. Thus

$$\frac{mv^2}{r} = G\frac{Mm}{r^2} \quad \text{or} \quad v = \sqrt{\frac{GM}{r}}$$

The period of revolution of the satellite is

$$T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r^3}{GM}}$$

For satellite $S_1$ let

$$T = T_1, \quad r = r_1, \quad v = v_1$$

Then $$T_1^2 = \frac{4\pi^2 r_1^3}{GM}$$

For satellite $S_2$, $T_2^2 = \dfrac{4\pi^2 r_2^2}{GM}$. Therefore $\dfrac{T_1^2}{T_2^2} = \dfrac{r_1^3}{r_2^3}$

or $$r_2 = r_1\left(\frac{T_2}{T_1}\right)^{2/3} = 10^4\left(\frac{8}{1}\right)^{2/3} = 4 \times 10^4 \text{ km}$$

$$v_1 = \frac{2\pi r_1}{T_1} = \frac{2\pi \times 10^4}{1} = 2\pi \times 10^4 \text{ km/hr}$$

$$v_2 = \frac{2\pi r_2}{T_2} = \frac{2\pi \times 4 \times 10^4}{8} = \pi \times 10^4 \text{ km/hour}$$

Velocity of $S_2$ relative to

$$S_1 = v_2 - v_1 = \vartheta_r \text{ (say)}$$

$$v_r = (\pi \times 10^4 - 2\pi \times 10^4) \text{ km/hr} = -\pi \times 10^4 \text{ km/hour}$$

Let $r_2 - r_1 = r$

The angular velocity of $S_2$ relative to $S_1$ is given by

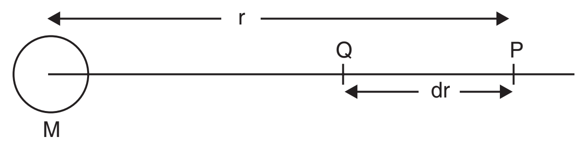
$$\omega = \frac{v_r}{r} = \frac{\pi \times 10^4}{(4-1)10^4} \text{ rad/hour}$$

$$\omega = \frac{\pi}{3} \text{ rad/hour}$$

**Q. 2.** *What is gravitational potential? Find the expression for gravitational potential energy.*

*A 400 kg satellite is in a circular orbit of radius $2R_E$ about the earth. Calculate the kinetic energy, potential energy and total energy of the satellite. Given that radius of Earth $R_E = 6.4 \times 10^6$ m and mass of Earth $M = 6 \times 10^{24}$ kg.*

**Ans.** The gravitational potential at a point in the gravitational field of a body is defined as the amount of work done in displacing a body of unit mass from infinity to that point in the field.




Consider a body of unit mass placed at a distance $r$ from the centre of a mass $M$. Then the gravitational force acting on the unit mass is given by

$$F = \frac{GM \times 1}{r^2} = \frac{GM}{r^2} \qquad \text{...}(i)$$

The direction of this force is towards the centre of the body of mass $M$. Let the unit mass be displaced from point $P$ to $Q$, through a distance $dr$ towards mass $M$, then the work done is given by

$$dW = \vec{F} \cdot \vec{dr} = F \cdot dr \cos \theta = F\, dr \cos 0$$

$$\Rightarrow \quad dW = F\, dr = \frac{GM}{r^2} dr \quad \text{(From eqn. (}i\text{))} \qquad \text{...}(ii)$$

Total work done in displacing the unit mass from infinity ($r = \infty$) to the point $P$ whose distance from mass $M$ is $r$ can be calculated by integrating eqn. ($ii$) between limits $r = \infty$ to $r = r$

$$\therefore \quad \int dW = \int_\infty^r \frac{GM}{r^2} dr \text{ or } W = \int_\infty^r \frac{GM}{r^2} dr \Rightarrow W = GM \int_\infty^r r^{-2} dr = GM \left[\frac{r^{-1}}{-1}\right]_\infty^r$$

$$\text{or} \quad W = -GM \left[\frac{1}{r}\right]_\infty^r = -GM \left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$\Rightarrow \quad W = -\frac{GM}{r} \qquad \left[\because \frac{1}{\infty} = 0\right]$$

This work done is equal to the gravitational potential

Gravitational potential, $V = -\dfrac{GM}{r}$.

**Numerical.** Here $M = 6 \times 10^{24}$ kg, $m = 400$ kg, $R_E = 6.4 \times 10^6$ m,

hence $r = 2R_E = 12.8 \times 10^6$ m and $G = 6.67 \times 10^{-11}$ Nm$^2$ kg$^{-2}$

$$\therefore \text{ K.E. of satellite, } K = \frac{GMm}{2r} = \frac{(6.67 \times 10^{-11}) \times (6 \times 10^{24}) \times 400}{2 \times (12.8 \times 10^6)} = 6.25 \times 10^9 \text{ J}$$

$$\text{P.E. of satellite, } U = -\frac{2GMm}{2r} = -2K = -2 \times 6.25 \times 10^9 \text{ J} = -12.5 \times 10^9 \text{ J}$$

and Total energy of satellite, $E = K + U$

$$\Rightarrow \quad E = (6.25 \times 10^9 - 12.5 \times 10^9) \text{ J} = -6.25 \times 10^9 \text{ J}$$

**Q. 3.** *State Kepler's laws of planetary motion.*

*What would be the speed of rotation of the earth in order that a body on the equator has no weight? Determine the apparent weights of the bodies situated at a latitude of 60° and at the poles. The radius of the earth = 6400 km and g = 9.8 $ms^{-1}$.*

**Ans.** **For Kepler's laws of planetary motion, please see facts that matter on page 345 of this book.**

The body will become weightless if the gravitational force *mg* on it is entirely used up in providing the centripetal acceleration for the rotation of the earth,

Then $$m\,g = \frac{m\,\vartheta^2}{R} = m\,\omega^2 R$$

$$\omega^2 = \frac{g}{R} = \frac{9.8}{6400 \times 10^3}$$

$$\omega = 1.237 \times 10^{-3} \text{ rad s}^{-1}$$

If the earth rotates at this speed, the bodies on the equator will have no weight. At a latitude $\phi$ the apparent weight $W_A$ is given by

$$W_A = m\,g\left(1 - \frac{\omega^2 R}{g}\cos^2\phi\right)$$

Here, however, $g = \omega^2 R$

Therefore, $W_A = m\,g\,(1 - \cos^2\phi)$

When $\phi = 60°$, $\cos\phi = 1/2$

$$W_A = m\,g\left(1 - \frac{1}{4}\right) = \frac{3}{4} \times \text{true weight}$$

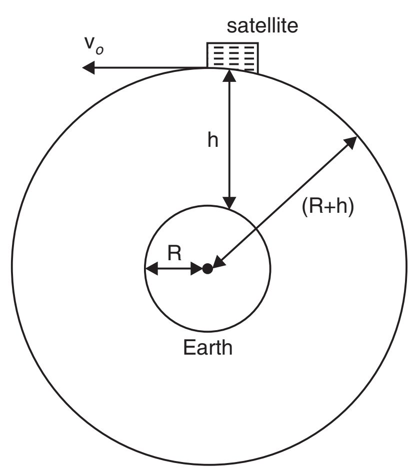
At poles, $\phi = \dfrac{\pi}{2}$

$$W_A = mg = \text{true weight}$$

Thus, a body situated on the poles remains unaffected, whatever the speed of rotation of the earth.

**Q. 4.** *What do you mean by orbital velocity? Find the expression for orbital velocity.*

**Ans.** The velocity required to put a satellite into its orbit around the earth is called orbital velocity. A satellite moves around the earth in its orbit with orbital velocity.




Consider a satellite of mass *m* revolving around the earth in an orbit of radius *R* + *h*, where *R* is the radius of earth and *h* is the height of satellite from the surface of earth.

Let $v_o$ be the orbital velocity of the satellite.

The gravitational force between the satellite and the earth provides the necessary centripetal force to the satellite to move in a circular path around the earth.

*i.e.,* Gravitational force = Centripetal force

$$\frac{GMm}{(R+h)^2} = \frac{mv_o^2}{(R+h)} \Rightarrow v_o^2 = \frac{GM}{(R+h)} \text{ or } v_o = \left[\frac{GM}{(R+h)}\right]^{1/2}$$

Since $\frac{GM}{R^2} = g$ or $GM = gR^2$

$$\therefore \quad v_o = \left[\frac{gR^2}{(R+h)}\right]^{1/2}$$

If satellite is very close to the surface of earth, then $(R + h) = R$

Hence $v_o = \sqrt{gR}$.

## IV. MULTIPLE CHOICE QUESTIONS

**1.** *A satellite is orbiting the earth. If its distance from the earth is increased, its*

(*a*) angular velocity would increase (*b*) linear velocity would increase
(*c*) angular velocity would decrease (*d*) time period would increase

**2.** *If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the earth's surface to a height equal to the radius R of the earth, is*

(*a*) 1/2 $m\ g\ R$ (*b*) 2 $m\ g\ R$ (*c*) $m\ g\ R$ (*d*) 1/4 $m\ g\ R$

**3.** *If three uniform spheres, each having mass M and radius r, are kept in such a way that each touches the other two, the magnitude of the gravitational force on any sphere due to the other two is*

(*a*) $\frac{GM^2}{4r^2}$ (*b*) $\frac{2GM^2}{r^2}$ (*c*) $\frac{2GM^2}{4r^2}$ (*d*) $\frac{\sqrt{3}GM^2}{4r^2}$

**4.** *A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is*

(*a*) $gx$ (*b*) $\frac{gR}{R-x}$ (*c*) $\frac{gR^2}{R+x}$ (*d*) $\left(\frac{gR^2}{R+x}\right)^{1/2}$

**5.** *Mars has about 1/10th as much mass as the earth and half as great a diameter. The acceleration of a falling body on Mars is about*

(*a*) 9.8 m s$^{-2}$ (*b*) 1.96 m s$^{-2}$ (*c*) 3.92 m s$^{-2}$ (*d*) 4.9 m s$^{-2}$

**6.** *If a particle is fired vertically upwards from the surface of earth and reaches a height of 6400 km, the initial velocity of the particle is (assume R = 6400 km and g = 10 ms$^{-2}$)*

(*a*) 4 km/sec (*b*) 2 km/sec (*c*) 8 km/sec (*d*) 16 km/sec

**7.** *If M is the mass of the earth and R its radius, the ratio of the gravitational acceleration and the gravitational constant is*

(*a*) $\frac{R^2}{M}$ (*b*) $\frac{M}{R^2}$ (*c*) $M\ R^2$ (*d*) $\frac{M}{R}$

**8.** *A satellite is launched into a circular orbit of radius R around the earth. A second satellite launched into an orbit of radius 1.01 R. The time period of the second satellite is larger than that of the first one by approximately*

(*a*) 0.5% (*b*) 1.5% (*c*) 1% (*d*) 3.0%

**Ans.** **1.**—(*c*) and (*d*) **2.**—(*a*) **3.**—(*d*) **4.**—(*d*) **5.**—(*c*)
**6.**—(*c*) **7.**—(*b*) **8.**—(*b*)

## V. QUESTIONS ON HIGH ORDER THINKING SKILLS (HOTS)

**Q. 1.** *A body is projected vertically from the surface of the earth with a velocity equal to half the escape velocity. What is the maximum height reached by the body?*

**Ans.** Initial kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{v_e}{2}\right)^2$

But $$v_e = \sqrt{\frac{2MG}{R}}$$

$\therefore$ $$\text{Initial K.E.} = \frac{1}{2}m\cdot\frac{2MG}{4R} = \frac{GMm}{4R}$$

$$\text{Initial P.E.} = -\frac{GMm}{R}$$

Total initial energy $= \frac{GMm}{4R} - \frac{GMm}{R} = -\frac{3GMm}{4R}$

If the body comes to rest at a distance $R$ from the centre of the earth, its final energy will be

$$= -\frac{GMm}{r}$$

$\therefore$ $$-\frac{3GMm}{4R} = -\frac{GMm}{r} \Rightarrow r = \frac{4}{3}R$$

Maximum height $= r - R = \frac{4}{3}R - R = \frac{R}{3}$.

**Q. 2.** *A planet of mass m moves along an ellipse around the sun so that its maximum and minimum distances from the sun are $r_1$ and $r_2$. Find the angular momentum of the planet relative to centre of sun. (Mass of sun = M)*

**Ans.** The angular momentum of planet is constant

*i.e.,* $$mv_1 r_1 = mv_2 r_2 \quad \text{or} \quad v_1 r_1 = v_2 r_2$$

Total energy of planet is constant

*i.e.,* $$\frac{-GMm}{r_1} + \frac{1}{2}mv_1^2 = \frac{-GMm}{r_2} + \frac{1}{2}mv_2^2$$

*i.e.,* $$GM\left\{\frac{1}{r_2} - \frac{1}{r_1}\right\} = \frac{v_2^2 - v_1^2}{2} = \frac{v_2^2}{2} - \frac{v_1^2}{2}$$

or $$GM\left\{\frac{r_1 - r_2}{r_1 r_2}\right\} = \frac{\left(\frac{v_1 r_1}{r_2}\right)^2}{2} - \frac{v_1^2}{2} = \frac{v_1^2}{2}\left\{\frac{r_1^2}{r_2^2} - 1\right\}$$

*i.e.,* $$GM\left\{\frac{r_1 - r_2}{r_1 r_2}\right\} = \frac{v_1^2}{2}\frac{\left(r_1^2 - r_2^2\right)}{r_2^2}$$

or $$v_1^2 = \frac{2GM\,(r_1 - r_2)\,r_2^2}{(r_1^2 - r_2^2)\,r_1 r_2} = \frac{2GM \times r_2}{r_1\,(r_1 + r_2)}$$

$$v_1 = \sqrt{\frac{2GMr_2}{r_1\,(r_1 + r_2)}}$$

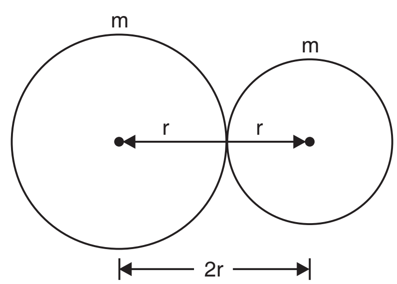
Angular momentum of the planet = $mv_1r_1 = m\sqrt{\dfrac{2GMr_1r_2}{r_1 + r_2}}$.

**Q. 3.** *The binding energy of a particle of mass m attached with the earth of mass M and radius R is $\frac{GMm}{R}$. How much energy must be supplied to the particle so that it may escape the gravitational field of the earth? In what form this energy is supplied to the particle?*

**Ans.** The particle will escape the gravitational field of the earth if the energy supplied to it is equal to its binding energy. Thus, energy supplied to the particle = $\frac{GMm}{R}$. This energy is supplied in the form of the kinetic energy of the particle.

**Q. 4.** *The gravitational force between two bodies each of mass m and separated by a distance r varies as inversely proportional to the square of the distance r. However, the gravitational force between two solid spheres of same density and same radius r placed in contact with each other is directly proportional to the fourth power of the radius r. Prove it.*

**Ans.** $$F = \frac{Gm \times m}{(2r)^2} = \frac{Gm^2}{4r^2}$$




But, $m$ = volume × density

$$= \frac{4}{3}\pi r^3 \rho$$

$$\therefore \quad F = \frac{G \times \frac{16}{9}\pi^2 r^6 \rho^2}{4r^2} = \frac{4}{9}\pi^2 G\rho^2 r^4$$

Since $\frac{4}{9}\pi^2 G\rho^2$ = constant

$\therefore \quad F \propto r^4$.

**Q. 5.** *A projectile is fired vertically upward from the surface of earth with a speed $kv_e$ where $v_e$ is the escape speed and $k < 1$. Neglecting air resistance show that the maximum height to which it will rise measured from the centre of earth is $R/(1 - k^2)$, where R is the radius of the earth.*

**Ans.** Let a body of mass $m$ be projected from the surface of earth with speed $v$ and it reaches to a height $h$. Using law of conservation of energy (relative to surface of earth) we have

$$\frac{1}{2}mv^2 = \frac{mgh}{1 + h/R}$$

In this problem, $v = kv_e = k\sqrt{2gR}$ and $h = r - R$

So, $\frac{1}{2}mk^2\,2gR = \frac{mg(r-R)}{[1+(r-R)/R]}$ or $k^2 = \frac{r-R}{r} = 1-\frac{R}{r} \Rightarrow r = \frac{R}{1-k^2}$.

**Q. 6.** *If the radius of the Earth were increased by a factor of 3, by what factor would its density have to be changed to keep 'g' the same?*

**Ans.** As $g = \frac{GM}{R^2}$

Let ρ be the density of earth

$$\rho = \frac{M}{\text{Volume of earth}} = \frac{M}{\frac{4}{3}\pi R^3} \Rightarrow M = \frac{4}{3}\pi R^3\rho$$

$$\therefore \quad g = \frac{G}{R^2}\times\frac{4}{3}\pi R^3\rho \quad \text{or} \quad g = \frac{4}{3}\pi G\rho R$$

$\frac{4}{3}$, π, $G$ are constants.

For no change in value of $g$, $R \propto \frac{1}{\rho}$.

Thus, if $R$ is made $3R$, ρ must become $\frac{\rho}{3}$.

**Q. 7.** *Two bodies of masses $M_1$ and $M_2$ are placed at a distance d apart. What is the potential at the position where the gravitational field due to them is zero?*

**Ans.** Let the field be zero at a point at distance $x$ from $M_1$.

$$\therefore \quad \frac{GM_1}{x^2} = \frac{GM_2}{(d-x)^2}$$

$$\therefore \quad \frac{x}{d-x} = \sqrt{\frac{M_1}{M_2}} \Rightarrow x\sqrt{M_2} = \sqrt{M_1}\cdot d - x\sqrt{M_1}$$

$$x\left[\sqrt{M_1}+\sqrt{M_2}\right] = \sqrt{M_1}\cdot d$$

$$x = \frac{d\sqrt{M_1}}{\sqrt{M_1}+\sqrt{M_2}}$$

$$d - x = \frac{d\sqrt{M_2}}{\sqrt{M_1}+\sqrt{M_2}}$$

Potential at this point due to both the masses will be

$$= -\frac{GM_1}{x} - \frac{GM_2}{(d-x)} = -G\left[\frac{M_1\left(\sqrt{M_1}+\sqrt{M_2}\right)}{d\sqrt{M_1}} + \frac{M_2\left(\sqrt{M_1}+\sqrt{M_2}\right)}{d\sqrt{M_2}}\right]$$

$$= -\frac{G}{d}\left(\sqrt{M_1}+\sqrt{M_2}\right)^2 = \frac{G}{d}\left(M_1+M_2+2\sqrt{M_1}\sqrt{M_2}\right).$$

## VI. VALUE-BASED QUESTIONS

**Q. 1.** *Arun went on a picnic tour to a hill station with his friends and teacher. Before going to hill station, he was weighed by weighing machine in his school. His weight was 40 kg. When he reached the hill station alongwith his friend and teacher, he saw a weighing machine installed infront of a shoping mall. He wanted to get his weight again on hill station. He took out a coin from his pocket and put it into the machine and stood on the platform of the machine. He got a ticket coming out from the machine on which his weight was quoted lesser than 40 kg i.e. 38 kg. He was surprised and asked his teacher the reason behind it. The teacher explained him about the acceleration due to gravity whose value is decreasing on hill station. So his weight was shown lesser than on the ground.*

*(i) What values of Arun are displayed here?*

*(ii) If a body is taken to a height equal to the radius of earth from its surface, how much the weight of a body will decrease?*

**Ans.** (*i*) The displayed values are : awareness, sharp mind, caring and intelligence.

(*ii*) On the surface of the earth,

$$g = G\cdot\frac{M}{R^2} \qquad (1)$$

and on the height where $h = R$

$$g' = G\cdot\frac{M}{(R+R)^2} = G\cdot\frac{M}{4R^2} \qquad (2)$$

Dividing (2) by (1) we get

$$\frac{g'}{g} = \frac{G\cdot\frac{M}{4R^2}}{G\cdot\frac{M}{R^2}}$$

$$\therefore \qquad \frac{g'}{g} = \frac{1}{4} \Rightarrow g' = \frac{g}{4}$$

Hence the weight of the body will reduce to one-forth of its original weight on the surface of the earth.

**Q. 2.** *Raja's physics teacher was teaching is his class that if a body is projected vertically upwards with escope velocity, it will just crosses the gravitational field and will never comeback to the earth. All the students got surprised to hear it. Raja' stood up in the class and asked his teacher about the escape velocity and how it is possible. The teacher explained him about this velocity very well.*

*(i) What values Raja showed here?*

*(ii) Define Escape velocity and find its value.*

**Ans.** (*i*) The values are : courageous, curious, sharp mind and intelligence.

(*ii*) The velocity with which a body is escaped vertically upwards and it just cross the gravitational field is called escape velocity of the body. Its value is

$$V_e = \sqrt{2gR_e} = 11.2 \text{ km/s}$$

**Q. 3.** Anurag who is a student of class XI Science could not attend the class on the day when the topic of satellites was taught is the class. Next day Anurag asked about it to his best friend Ram but Ram could not explain very well. Both of them decided to go to their teacher and requested to explain about the natural and artificial satellites. They were happy to know about our natural satellite moon and other artificial satellites.

*(i) What values of Anurag are displayed here?*

*(ii) A body has a sense of weightlessness in a satellite revolving around the earth, why?*

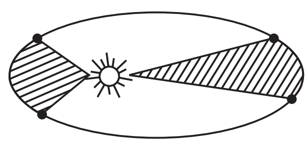
**Ans.** (*i*) Curiosity, awareness, sharp mind and keen observation.

(*ii*) The astronauts and the satellite require the centripetal force to revolve around the earth. Their weight is used up in providing the necessary centripetal force. Hence an astronaut feels weightlessness in the space.

# TEST YOUR SKILLS

**1.** What do you understand by superposition of gravitational forces acting on a body ?

**2.** Following diagram shows a planet moving around the sun. What is the relationship between shaded area, if areas are swept in equal time intervals ?



**3.** A trader purchased gold in a gold mine, which was about 50 gm when weighed in the mine. He then went to sell the gold at Mt. Everest, which is 8,000 m high. Will he find some difference in weight ? Give reasons for your answer:

**4.** What is the difference between geostationary and polar satellites ?

**5.** A rocket is fired vertically from earth's surface, with a speed of 10 $kms^{-1}$. Calculate the maximum height attained by the rocket.

(mass of Earth = $6.0 \times 10^{24}$ kg ; Radius of Earth = $6.4 \times 10^{6}$ m ; G = $6.67 \times 10^{-11}$ $Nm^2 kg^{-2}$)

**6.** Why does a person feel weight lessness in an artificial satellite and not on a natural satellite ?

**7.** Why do we need the help of communication satellites to transmit signals for television or mobile phones?

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