9 Mechanical Properties of Solids

Facts that Matter

• Intermolecular Force

In a solid, atoms and molecules are arranged in such a way that each molecule is acted upon by the forces due to the neighbouring molecules. These forces are known as intermolecular forces.

• Elasticity

The property of the body to regain its original configuration (length, volume or shape) when the deforming forces are removed, is called elasticity.

• The change in the shape or size of a body when external forces act on it is determined by the forces between its atoms or molecules. These short range atomic forces are called elastic forces.

• Perfectly elastic body

A body which regains its original configuration immediately and completely after the removal of deforming force from it, is called perfectly elastic body. Quartz and phospher bronze are the examples of nearly perfectly elastic bodies.

• Plasticity

The inability of a body to return to its original size and shape even on removal of the deforming force is called plasticity and such a body is called a plastic body.

• Stress

Stress is defined as the ratio of the internal force *F*, produced when the substance is deformed, to the area A over which this force acts. In equilibrium, this force is equal in magnitude to the externally applied force. In other words,

$$\text{Stress} = \frac{F}{A}$$

The SI unit of stress is newton per square metre (Nm^{-2}). In CGS units, stress is measured in dyne cm^{-2}. Dimensional formula of stress is $[ML^{-1}T^{-2}]$.

• Stress is of two types:

(*i*) **Normal stress:** It is defined as the restoring force per unit area perpendicular to the surface of the body. Normal stress is of two types: tensile stress and compressive stress.

(*ii*) **Tangential stress:** When the elastic restoring force or deforming force acts parallel to the surface area, the stress is called tangential stress.

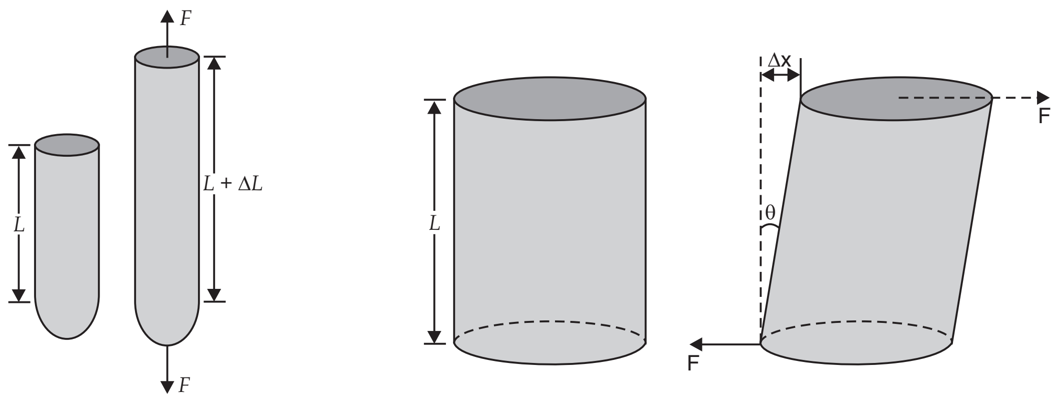
• Strain

It is defined as the ratio of the change in size or shape to the original size or shape. It has no dimensions, it is just a number.

Strain is of three types:

(*i*) **Longitudinal strain:** If the deforming force produces a change in length alone, the strain produced in the body is called longitudinal strain or tensile strain. It is given as:

$$\text{Longitudinal strain} = \frac{\text{Change in length } (\Delta l)}{\text{Original length } (l)}$$



(a) (b)

(*ii*) **Volumetric strain:** If the deforming force produces a change in volume alone, the strain produced in the body is called volumetric strain. It is given as:

$$\text{Volumetric strain} = \frac{\text{Change in volume } (\Delta V)}{\text{Original volume } (V)}$$

(*iii*) **Shear strain:** The angle tilt caused in the body due to tangential stress expressed is called shear strain. It is given as:

$$\text{Shear strain} = \theta = \frac{\Delta L}{L}.$$

• The maximum stress to which the body can regain its original status on the removal of the deforming force is called elastic limit.

• Hooke's Law

Hooke's law states that, within elastic limits, the ratio of stress to the corresponding strain produced is a constant. This constant is called the modulus of elasticity. Thus

$$\text{Modulus of elasticity} = \frac{\text{Stress}}{\text{Strain}}$$

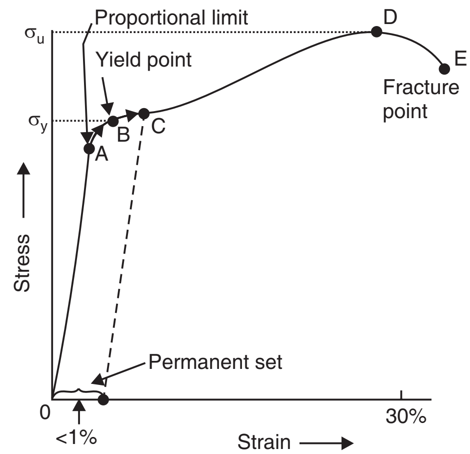
Since strain is a pure number, the units of this constant are the same as those of stress, *i.e.,* Nm^{-2}.

• Stress Strain Curve

Stress strain curves are useful to understand the tensile strength of a given material. The given figure shows a stress-strain curve of a given metal.

- The curve from O to A is linear. In this region Hooke's law is obeyed.
- In the region from A to B stress and strain are not proportional. Still, the body regains its original dimension, once the load is removed.

- Point B in the curve is yield point or elastic limit and the corresponding stress is known as yield strength of the material.
- The curve beyond B shows the region of plastic deformation.
- The point D on the curve shows the tensile strength of the material. Beyond this point, additional strain leads to fracture, in the given material.



• Young's Modulus

For a solid, in the form of a wire or a thin rod, Young's modulus of elasticity within elastic limit is defined as the ratio of longitudinal stress to longitudinal strain. It is given as:

$$\text{Young's modulus, } Y = \frac{F/A}{\Delta l/l} = \frac{Fl}{A.\Delta l} = \frac{mgl}{\pi r^2.\Delta l}$$

It has the unit of longitudinal stress and dimensions of $[ML^{-1}T^{-2}]$. Its unit is Pascal or N/m^2.

• Bulk Modulus

Within elastic limit the bulk modulus is defined as the ratio of longitudinal stress and volumetric strain. It is given as:

$$\text{Bulk modulus, } B = \frac{F/A}{\Delta V/V} = -\frac{P}{\Delta V/V}$$

– ve indicates that the volume variation and pressure variation always negate each other.

• Reciprocal of bulk modulus is commonly referred to as the "compressibility". It is defined as the fractional change in volume per unit change in pressure.

• Shear Modulus or Modulus of Rigidity

It is defined as the ratio of the tangential stress to the shear strain.

Modulus of rigidity is given by

$$\eta = \frac{\text{Tangential stress}}{\text{Shear strain}} = \frac{F/A}{\theta}.$$

• Poisson's Ratio

The ratio of change in diameter (ΔD) to the original diameter (D) is called lateral strain. The ratio of change in length (Δl) to the original length (l) is called longitudinal strain. The ratio of lateral strain to the longitudinal strain is called Poisson's ratio.

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = -\frac{\Delta D/D}{\Delta l/l}$$

For most of the substances, the value of σ lies between 0.2 to 0.4. When a body is perfectly incompressible, the value of σ is maximum and equals to 0.5.

• Elastic Fatigue

It is the property of an elastic body by virtue of which its behaviour becomes less elastic under the action of repeated alternating deforming forces.

• Relations between Elastic Moduli

For isotropic materials (*i.e.,* materials having the same properties in all directions), only two of the three elastic constants are independent. For example, Young's modulus can be expressed in terms of the bulk and shear moduli.

$$\frac{3}{Y} = \frac{1}{\eta}+\frac{1}{3B}$$

Also, $Y = 3B\ (1 - \sigma) = 2\eta\ (1 + \sigma)$

• Breaking Stress

The ultimate tensile strength of a material is the stress required to break a wire or a rod by pulling on it. The breaking stress of the material is the maximum stress which a material can withstand. Beyond this point breakage occurs.

• When a wire of original length L is stretched by a length l by the application of force F at one end, then

$$\text{Work done to stretch wire} = \frac{1}{2} \times \text{stretching force} \times \text{extension}$$

$$= \frac{1}{2}\frac{YAl^2}{L}$$

• Work done per unit volume of wire is given as:

$$W = \frac{1}{2}\text{Stress} \times \text{strain.}$$

According to the formula given by

$$Y = \frac{F \cdot L}{Al}$$

Where F is the force needed to stretch the wire of length L and area of cross-section A. l is the increase in the length of the wire.

$$\therefore \quad F = \frac{YAl}{L}$$

The work done by this force in stretching the wire is stored in the wire as potential energy.

$$\therefore \quad dW = F \times dl = \frac{YAl}{L} \cdot dl$$

Integrating both sides, we get

$$W = \frac{YA}{L}\int_0^l l \cdot dl$$

$$W = \frac{YA}{L}\left[\frac{1}{2}l^2\right]_0^l = \frac{YA}{L}\left[\frac{1}{2}l^2\right] = \frac{1}{2}\left(\frac{YAl}{L}\right) \cdot l = \frac{1}{2} \cdot F \cdot l$$

Which equal to the elastic potential energy U.

$$\therefore \quad U = \frac{1}{2}F \cdot l = \frac{1}{2} \times \text{Force} \times \text{extension}$$

Now the potential energy per unit volume is

$$\frac{1}{2}\frac{F\cdot l}{V} = \frac{1}{2}\left(\frac{YAl}{L}\right)\cdot\frac{l}{V}$$

$$\Rightarrow \quad \frac{1}{2}\times\frac{Fl}{AL} = \frac{1}{2}\left(\frac{YAl}{L}\right)\cdot\frac{l}{AL} \qquad [V = AL]$$

$$= \frac{1}{2}\left(\frac{Yl}{L}\right)\cdot\frac{l}{L} = \frac{1}{2}\left(\frac{F}{A}\right)\cdot\frac{l}{L} = \frac{1}{2}\times\text{Stress}\times\text{Strain}$$

Hence, the elastic potential energy of a wire (energy density) is equal to half the product of its stress and strain.

- **IMPORTANT TABLES**

TABLE 9.1 Young's moduli, elastic limit and tensile strengths of some materials.

Substance	*Young's modulus $10^9 N/m^2$* σ_y	*Elastic limit $10^7 N/m^2$* %	*Tensile strength $10^7 N/m^2$* σ_u
Aluminium	70	18	20
Copper	120	20	40
Iron (wrought)	190	17	33
Steel	200	30	50
Bone			
(Tensile)	16		12
(Compressive)	9		12

TABLE 9.2 Shear moduli (G) of some common materials

Material	*G(10^9 Nm^{-2} or GPa)*
Aluminium	25
Brass	36
Copper	42
Glass	23
Iron	70
Lead	5.6
Nickel	77
Steel	84
Tungsten	150
Wood	10

TABLE 9.3 Bulk moduli (B) of some common Materials

Material Solids	*B(10^9 Nm^{-2} or GPa)*
Aluminium	72
Brass	61
Copper	140
Glass	37
Iron	100
Nickel	260
Steel	160
Liquids	
Water	2.2
Ethanol	0.9
Carbon disulphide	1.56
Glycerine	4.76
Mercury	25
Gases	
Air (at STP)	1.0×10^{-4}

TABLE 9.4 Stress, strain and various elastic moduli

Type of Stress	*Stress*	*Strain*	*Change in*		*Elastic modulus*	*Name of modulus*	*State of Mater*
			shape	*volume*			
Tensile or Compressive	Two equal and opposite forces perpendicular to opposite faces ($\sigma = F/A$)	Elongation or compression parallel to force direction ($\Delta L/L$) (longitudinal strain)	Yes	No	$Y = (F \times L) / (A \times \Delta L)$	Young's modulus	Solid
Shearing	Two equal and opposite forces parallel to opposite surfaces [forces in each case such that total force and total torque on the body vanishes ($\sigma_s = F/A$)]	Pure shear, θ	Yes	No	$G = (F \times \theta) / A$	Shear modulus	Solid
Hydraulic	Forces perpendicular every where to the surface, force per unit area (pressure) same everywhere	Volume change (compression or elongation) ($\Delta V/V$)	No	Yes	$\frac{p}{\Delta V/V}$	Bulk modulus	Solid, liquid and gas

NCERT TEXTBOOK QUESTIONS SOLVED

9.1. *A steel wire of length 4.7 m and cross-sectional area 3.0×10^{-5} m² stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area of 4.0×10^{-5} m² under a given load. What is the ratio of the Young's modulus of steel to that of copper?*

Sol. For steel

$$l_1 = 4.7\text{m}, \quad A_1 = 3.0 \times 10^{-5} \text{ m}^2$$

If F newton is the stretching force and Δl metre the extension in each case, then

$$Y_1 = \frac{Fl_1}{A_1 \Delta l}$$

$$\Rightarrow \quad Y_1 = \frac{F \times 4.7}{3.0 \times 10^{-5} \times \Delta l} \quad \text{...}(i)$$

For copper

$$l_2 = 3.5\text{m}, \quad A_2 = 4.0 \times 10^{-5} \text{ m}^2$$

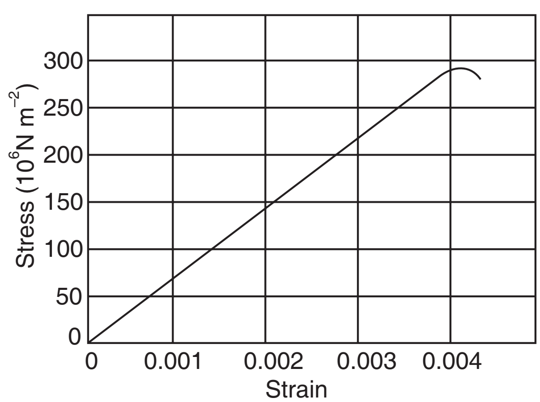
Now,
$$Y_2 = \frac{F \times 3.5}{4.0 \times 10^{-5} \times \Delta l} \quad \text{...}(ii)$$

Dividing (*i*) by (*ii*), we get

$$\frac{Y_1}{Y_2} = \frac{4.7}{3.0 \times 10^{-5}} \times \frac{4.0 \times 10^{-5}}{3.5} = \frac{4.7 \times 4.0}{3.0 \times 3.5} = 1.79.$$

9.2. *Figure shows the strain-stress curve for a given material. What are (a) Young's modulus and (b) approximate yield strength for this material?*

Sol. (*a*) Young's modulus of the material (Y) is given by



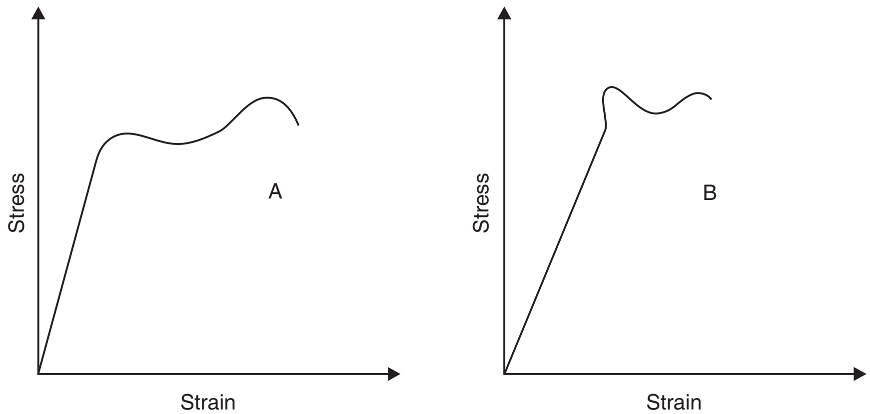
$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{150 \times 10^6}{0.002} = \frac{150 \times 10^6}{2 \times 10^{-3}} = 75 \times 10^9 \text{ Nm}^{-2} = 7.5 \times 10^{10} \text{ Nm}^{-2}$$

(*b*) Yield strength of a material is defined as the maximum stress it can sustain.

From graph, the approximate yield strength of the given material

$$= 300 \times 10^6 \text{ Nm}^{-2} = 3 \times 10^8 \text{ Nm}^{-2}.$$

9.3. *The stress-strain graphs for materials A and B are shown in figure.*



The graphs are drawn to the same scale.

(a) Which of the materials has the greater Young's modulus?

(b) Which of the two is the stronger material?

Sol. (*a*) From the two graphs we note that for a given strain, stress for *A* is more than that of *B*. Hence Young's modulus $\left(= \dfrac{\text{Stress}}{\text{Strain}}\right)$ is greater for *A* than that of *B*.

(*b*) Strength of a material is determined by the amount of stress required to cause fracture. This stress corresponds to the point of fracture. The stress corresponding to the point of fracture in *A* is more than for *B*. So, material *A* is stronger than material *B*.

9.4. *Read the following two statements below carefully and state, with reasons, if it is true or false.*

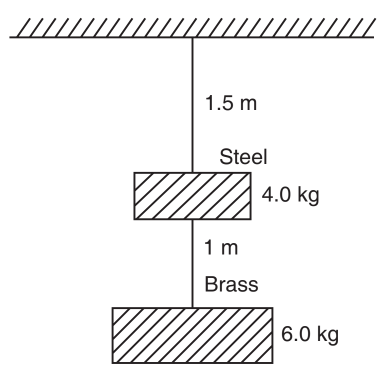
(a) The Young's modulus of rubber is greater than that of steel;

(b) The stretching of a coil is determined by its shear modulus.

Sol. (*a*) **False.** The Young's modulus is defined as the ratio of stress to the strain within elastic limit. For a given stretching force elongation is more in rubber and quite less in steel. Hence, rubber is less elastic than steel.

(*b*) **True.** Stretching of a coil is determined by its shear modulus. When equal and opposite forces are applied at opposite ends of a coil, the distance as well as shape of helicals of the coil change and it involves shear modulus.

9.5. *Two wires of diameter 0.25 cm, one made of steel and other made of brass are loaded as shown in figure. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Young's modulus of steel is 2.0 × 10^{11} Pa. Compute the elongations of steel and brass wires. (1 Pa = 1 Nm^{-2}).*



Sol. For steel wire; total force on steel wire;

$$F_1 = 4 + 6 = 10 \text{ kg } f = 10 \times 9.8 \ N;$$

$$l_1 = 1.5 \text{ m}, \quad \Delta l_1 = ?; \quad 2r_1 = 0.25 \text{ cm}$$

or $$r_1 = \left(\frac{0.25}{2}\right)\text{cm} = 0.125 \times 10^{-2} \text{ m}$$

$$Y_1 = 2.0 \times 10^{11} \text{ Pa}$$

For brass wire, $F_2 = 6.0 \text{ kg } f = 6 \times 9.8 \text{ N};$

$2r_2 = 0.25 \text{ cm}$

or $r_2 = \left(\frac{0.25}{2}\right) = 0.125 \times 10^{-2} \text{ m};$

$Y_2 = 0.91 \times 10^{11} \text{ Pa}, \quad l_2 = 1.0 \text{ m}, \quad \Delta l_2 = ?$

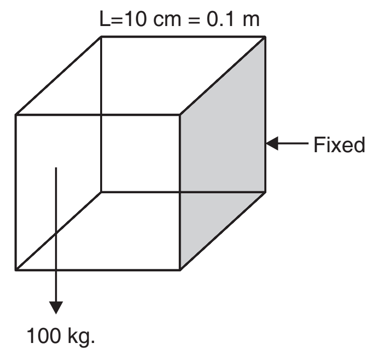
Since, $Y_1 = \frac{F_1 \times l_1}{A_1 \times \Delta l_1} = \frac{F_1 \times l_1}{\pi r_1^2 \times \Delta l_1} \Rightarrow \Delta l_1 = \frac{F_1 \times l_1}{\pi r_1^2 \times Y_1}$

or $\Delta l_1 = \frac{(10 \times 9.8) \times 1.5 \times 7}{22 \times (0.125 \times 10^{-2})^2 \times 2 \times 10^{11}} = 1.49 \times 10^{-4} \text{ m.}$

And $\Delta l_2 = \frac{F_2 \times l_2}{\pi r_2^2 \times Y_2} = \frac{(6 \times 9.8) \times 1 \times 7}{22 \times (0.125 \times 10^{-2})^2 \times (0.91 \times 10^{11})} = 1.3 \times 10^{-4} \text{ m.}$

9.6. *The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face?*

Sol. Here, side of cube, $L = 10 \text{ cm} = \frac{10}{100} = 0.1 \text{ m}$



$\therefore$ Area of each face, $A = (0.1)^2 = 0.01 \text{ m}^2$

Tangential force acting on the face,

$F = 100 \text{ kg} = 100 \times 9.8 = 980 \text{ N}$

Shear modulus, $\eta = 25 \text{ GPa} = 25 \times 10^9 \text{ Nm}^{-2}$

Since shear modulus is given as:

$$\eta = \frac{\text{Tangential stress}}{\text{Shearing strain}}$$

$$\therefore \text{Shearing strain} = \frac{\text{Tangential stress}}{\text{Shear modulus}} = \frac{F}{A\eta} = \frac{980}{0.01 \times 25 \times 10^9} = 3.92 \times 10^{-6}$$

Now, $\frac{\text{Lateral Strain}}{\text{Side of cube}} = \text{Shearing strain}$

$\therefore$ Lateral Strain = Shearing strain × Side of the cube

$= 3.92 \times 10^{-6} \times 0.1 = 3.92 \times 10^{-7} \text{ m} \approx 4 \times 10^{-7} \text{ m.}$

9.7. *Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 cm and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column. Young's modulus, $Y = 2.0 \times 10^{11}$ Pa.*

Sol. Here total mass to be supported, M = 50,000 kg

$\therefore$ Total weight of the structure to be supported = Mg

$= 50{,}000 \times 9.8 \text{ N}$

Since this weight is to be supported by 4 columns,

∴ Compressional force on each column (F) is given by

$$F = \frac{Mg}{4} = \frac{50{,}000 \times 9.8}{4} \text{ N}$$

Inner radius of a column, r_1 = 30 cm = 0.3 m

Outer radius of a column, r_2 = 60 cm = 0.6 m.

∴ Area of cross-section of each column is given by

$$A = \pi(r_2^2 - r_1^2)$$
$$= \pi[(0.6)^2 - (0.3)^2] = 0.27\,\pi \text{ m}^2$$

Young's modulus, $Y = 2 \times 10^{11}$ Pa

Compressional strain of each column =?

$$\therefore \quad Y = \frac{\text{Compressional force / area}}{\text{Compressional Strain}} = \frac{F/A}{\text{Compressional Strain}}$$

or Compressional strain of each column

$$= \frac{F}{AY} = \frac{50{,}000 \times 9.8 \times 7}{4 \times 0.27 \times 22 \times 2 \times 10^{11}} = 0.722 \times 10^{-6}$$

∴ Compressional strain of all columns is given by

$$= 0.722 \times 10^{-6} \times 4 = 2.88 \times 10^{-6}.$$

9.8. *A piece of copper having a rectangular cross-section of 15.2 mm × 19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain? Shear modulus of elasticity of copper is 42 × 10^9 N/m^2.*

Sol. Here, $A = 15.2 \times 19.2 \times 10^{-6} \text{ m}^2$; $F = 44500$ N; $\eta = 42 \times 10^9 \text{ Nm}^{-2}$

$$\text{Strain} = \frac{\text{Stress}}{\text{modulus of elasticity}} = \frac{F/A}{\eta}$$

$$= \frac{F}{A\eta} = \frac{44500}{(15.2 \times 19.2 \times 10^{-6}) \times 42 \times 10^9} = 3.65 \times 10^{-3}.$$

9.9. *A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8 Nm^{-2}, what is the maximum load the cable can support?*

Sol. Maximum load = Maximum stress × Cross-sectional area

$$= 10^8 \text{ Nm}^{-2} \times \frac{22}{7} \times (1.5 \times 10^{-2} \text{ m})^2 = 7.07 \times 10^4 \text{ N}.$$

9.10. *A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.*

Sol. Since each wire is to have same tension therefore, each wire has same extension. Moreover, each wire has the same initial length.

So, strain is same for each wire.

Now, $$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/(\pi D^2/4)}{\text{Strain}} \quad \text{or} \quad Y \propto \frac{1}{D^2} \Rightarrow D \propto \frac{1}{\sqrt{Y}}$$

$$\frac{D_{copper}}{D_{iron}} = \sqrt{\frac{Y_{iron}}{Y_{copper}}} = \sqrt{\frac{190 \times 10^9}{110 \times 10^9}} = \sqrt{\frac{19}{11}} = 1.314$$

9.11. *A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1 m, is whirled in a vertical circle with an angular velocity of 2 rev./s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm². Calculate the elongation of the wire when the mass is at the lowest point of its path.* $Y_{steel} = 2 \times 10^{11}\ Nm^{-2}$.

Sol. Here, m = 14.5 kg; $l = r$ = 1 m; ν = 2 rps; A = 0.065×10^{-4} m²

Total pulling force on mass, when it is at the lowest position of the vertical circle is

$$F = mg + mr\,\omega^2 = mg + mr\,4\,\pi^2\,\nu^2$$
$$= 14.5 \times 9.8 + 14.5 \times 1 \times 4 \times (22/7)^2 \times 2^2$$
$$= 142.1 + 2291.6 = 2433.9 \text{ N}$$

$$Y = \frac{F}{A} \times \frac{l}{\Delta l}$$

or $$\Delta l = \frac{Fl}{AY} = \frac{2433.7 \times 1}{(0.065 \times 10^{-4}) \times (2 \times 10^{11})} = 1.87 \times 10^{-3} \text{ m.}$$

9.12. *Compute the bulk modulus of water from the following data: Initial volume = 100.0 litre, Pressure increase = 100.0 atm (1 atm = 1.013 × 10⁵ Pa), Final volume = 100.5 litre. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large.*

Sol. Here P = 100 atmosphere

$= 100 \times 1.013 \times 10^5$ Pa $\quad (\because 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa})$

Initial volume, V_1 = 100 litre = 100×10^{-3} m³

Final volume, V_2 = 100.5 litre = 100.5×10^{-3} m³

$\therefore$ Change in volume = $\Delta V = V_2 - V_1$

$= (100.5 - 100) \times 10^{-3}$ m³

$= 0.5 \times 10^{-3}$ m³

Using formula of bulk modulus,

$$B = \frac{P}{\frac{\Delta V}{V}} = \frac{PV}{\Delta V} = \frac{100 \times 1.013 \times 10^5 \times 100 \times 10^{-3}}{0.5 \times 10^{-3}}$$

$$B = 2.026 \times 10^9 \text{ Pa}$$

Also we know that the bulk modulus of air = 1.0×10^5 Pa

Now, $$\frac{\text{Bulk modulus of water}}{\text{Bulk modulus of air}} = \frac{2.026 \times 10^9}{1.0 \times 10^5} = 2.026 \times 10^4 = 20260$$

The ratio is too large. This is due to the fact that the strain for air is much larger than for water at the same temperature. In other words, the intermolecular distances in case of liquids are very small as compared to the corresponding distances in the case of gases. Hence there are larger interatomic forces in liquids than in gases.

9.13. *What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is 1.03 × 10³ kg m⁻³?*

Sol. Compressibility of water,

$$k = \frac{1}{B} = 45.8 \times 10^{-11} \text{ Pa}^{-1}$$

Change in pressure,

$$\Delta p = 80 \text{ atm} - 1 \text{ atm} = 79 \text{ atm} = 79 \times 1.013 \times 10^5 \text{ Pa}$$

Density of water at the surface,

$$\rho = 1.03 \times 10^3 \text{ kg m}^{-3}$$

As $$B = \frac{\Delta p . V}{\Delta V} \quad \text{or} \quad \frac{\Delta V}{V} = \frac{\Delta p}{B} = \Delta p \times \frac{1}{B} = \Delta p \times k$$

or $$\frac{\Delta V}{V} = 79 \times 1.013 \times 10^5 \times 45.8 \times 10^{-11} = 3.665 \times 10^{-3}$$

Now $$\frac{\Delta V}{V} = \frac{(M/\rho)-(M/\rho')}{(M/\rho)} = 1 - \frac{\rho}{\rho'}$$

or $$\frac{\rho}{\rho'} = 1 - \frac{\Delta V}{V} \quad \text{or} \quad \rho' = \frac{\rho}{1-(\Delta V/V)}$$

or $$\rho' = \frac{1.03\times10^3}{1-3.665\times10^{-3}} = \frac{1.03\times10^3}{0.996} = 1.034\times10^3 \text{ kg/m}^3.$$

9.14. *Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.*

Sol. Here, $P = 10 \text{ atm} = 10 \times 1.013 \times 10^5 \text{ Pa}$; $k = 37 \times 10^9 \text{ Nm}^{-2}$

$$\text{Volumetric strain} = \frac{\Delta V}{V} = \frac{P}{K} = \frac{10\times1.013\times10^5}{37\times10^9} = 2.74 \times 10^{-5}$$

$\therefore$ Fractional change in volume = $\dfrac{\Delta V}{V} = 2.74 \times 10^{-5}$.

9.15. *Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7.0 × 10^6 Pa.*

Sol. Here a side of copper cube a = 10 cm, hence volume $V = a^3 = 10^{-3}$ m^3, hydraulic pressure applied $p = 7.0 \times 10^6$ Pa and from table we find that bulk modulus of copper $B = 140$ G Pa = 140×10^9 Pa.

Using the relation $B = -\dfrac{p}{\frac{\Delta V}{V}}$, we have decrease in

$$\text{volume } \Delta V = \frac{pV}{B}$$

$$\therefore \quad \Delta V = \frac{7.0\times10^6\times10^{-3}}{140\times10^9} = 5 \times 10^{-8} \text{ m}^3 = 5 \times 10^{-2} \text{ cm}^3.$$

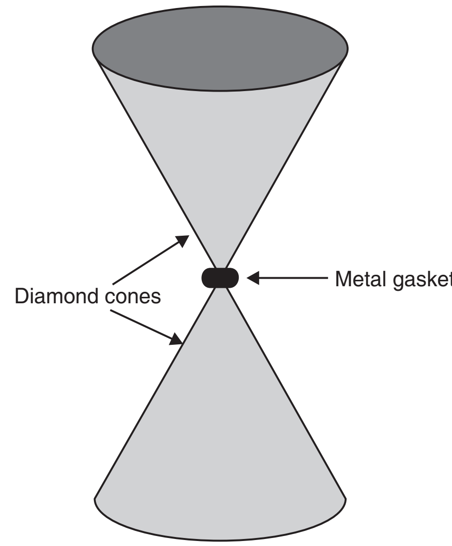
9.16. *How much should be pressure the a litre of water be changed to compress it by 0.10 %? Bulk modulus of elasticity of water = 2.2 × 10^9 Nm^{-2}.*

Sol. Here, $V = 1 \text{ litre} = 10^{-3} \text{ m}^3$; $\Delta V/V = 0.10/100 = 10^{-3}$

$$K = \frac{pV}{\Delta V}$$

or $$p = K\frac{\Delta V}{V} = (2.2 \times 10^9) \times 10^{-3} = 2.2 \times 10^6 \text{ Pa}.$$

9.17. *Anvils made of single crystals of diamond, with the shape as shown in figure are used to investigate behaviour of materials under very high pressures. Flat faces at the narrow end of the anvil have a diameter of 0.50 mm, and the wide ends are subjected to a compressional force of 50,000 N. What is the pressure at the tip of the anvil?*



Sol. Diameter of the corner end of the anvil,

$$d = 0.50 \text{ mm} = 0.50 \times 10^{-3} \text{ m}$$

Area of cross-section of tip,

$$A = \frac{\pi d^2}{4}$$

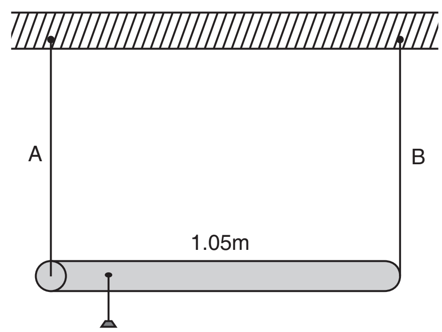
$$= \frac{22 \times (0.50 \times 10^{-3})^2}{7 \times 4} \text{m}^2$$

Stress (= pressure at the tip of the anvil)

$$= \frac{F}{A} = \frac{50{,}000 \times 4 \times 7}{22 \times (0.50)^2 \times 10^{-6}} \text{Nm}^{-2}$$

$$= 2.54 \times 10^{11} \text{ Nm}^{-2} \text{ (or Pa)}.$$

9.18. *A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in figure. The cross-sectional areas of wires A and B are 1.0 mm² and 2.0 mm², respectively. At what point along the rod should a mass m be suspended in order to produce (a) equal stresses and (b) equal strains in both steel and aluminium wires.*



Sol. For steel wire A, $l_1 = l$; $A_z = 1 \text{ mm}^2$; $Y_1 = 2 \times 10^{11} \text{ Nm}^{-2}$

For aluminium wire B, $l_2 = l$; $A_2 = 2\text{mm}^2$; $Y_2 = 7 \times 10^{10} \text{ Nm}^{-2}$

(*a*) Let mass m be suspended from the rod at distance x from the end where wire A is connected. Let F_1 and F_2 be the tensions in two wires and there is equal stress in two wires, then

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow \frac{F_1}{F_2} = \frac{A_1}{A_2} = \frac{1}{2} \qquad ...(i)$$

Taking moment of forces about the point of suspension of mass from the rod, we have

$$F_1 x = F_2 (1.05 - x) \quad \text{or} \quad \frac{1.05 - x}{x} = \frac{F_1}{F_2} = \frac{1}{2}$$

or $\quad 2.10 - 2x = x \Rightarrow x = 0.70 \text{ m} = 70 \text{ cm}$

(*b*) Let mass m be suspended from the rod at distance x from the end where wire A is connected. Let F_1 and F_2 be the tension in the wires and there is equal strain in the two wires *i.e.,*

$$\frac{F_1}{A_1 Y_1} = \frac{F_2}{A_2 Y_2} \Rightarrow \frac{F_1}{F_2} = \frac{A_1 Y_1}{A_2 Y_2} = \frac{1}{2} \times \frac{2 \times 10^{11}}{7 \times 10^{10}} = \frac{10}{7}$$

As the rod is stationary, so $F_1 x = F_2 (1.05 - x)$ or $\frac{1.05 - x}{x} = \frac{F_1}{F_2} = \frac{10}{7}$

$\Rightarrow \quad 10x = 7.35 - 7x$ or $x = 0.4324$ m = 43.2 cm.

9.19. *A mild steel wire of length 1.0 m and cross-sectional area 0.50×10^{-2} cm² is stretched, well within its elastic limit, horizontally between two pillars. A mass of 100g is suspended from the mid-point of the wire. Calculate the depression at the mid-point.*

Sol. Let AB be a mild steel wire of length $2L$ = 1m and its cross-section area A = 0.50×10^{-2} cm². A mass m = 100 g = 0.1 kg is suspended at mid-point C of wire as shown in figure. Let x be the depression at mid-point *i.e.*, CD = x

$$\therefore \quad AD = DB = \sqrt{AC^2 + CD^2} = \sqrt{L^2 + x^2}$$

$$\therefore \text{Increase in length } \Delta L = (AD + DB) - AB = 2\sqrt{L^2 + x^2} - 2L$$

$$= 2L\left[\left(1 + \frac{x^2}{L^2}\right)^{\frac{1}{2}} - 1\right] = 2L.\frac{x^2}{2L^2} = \frac{x^2}{L}$$

$$\therefore \quad \text{Longitudinal strain} = \frac{\Delta L}{2L} = \frac{x^2}{2L^2}.$$

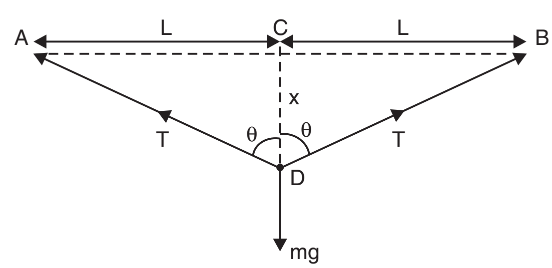
If T be the tension in the wire as shown in Fig., then in equilibrium

$$2T \cos \theta = mg$$

or
$$T = \frac{mg}{2\cos\theta} = \frac{mg}{2\frac{x}{\sqrt{x^2 + L^2}}} = \frac{mg\sqrt{x^2 + L^2}}{2x} = \frac{mgL}{2x}$$

[Since $x << L$]

$$\therefore \quad \text{Stress} = \frac{T}{A} = \frac{mgL}{2\,x\,A}$$



As Young's modulus $Y = \frac{\text{stress}}{\text{strain}}$

$$= \frac{\left(\frac{mgL}{2\,x\,A}\right)}{\left(\frac{x^2}{2L^2}\right)} = \frac{mgL}{2\,x\,A} \times \frac{2L^2}{x^2} = \frac{mgL^3}{A\,x^3}$$

$$\Rightarrow \quad x = \left[\frac{mgL^3}{YA}\right]^{\frac{1}{3}} = L\left[\frac{mg}{YA}\right]^{\frac{1}{3}} = \frac{1}{2}\left[\frac{0.1 \times 9.8}{2 \times 10^{11} \times 0.50 \times 10^{-2} \times 10^{-4}}\right]^{\frac{1}{3}}$$

$$= 1.074 \times 10^{-2} \text{ m} = 1.074 \text{ cm} \simeq 1.07 \text{ cm} \quad \text{or} \quad 0.01 \text{ m}.$$

9.20. *Two strips of metal are riveted together at their ends by four rivets, each of diameter 6.0 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed 6.9×10^7 Pa? Assume that each rivet is to carry one quarter of the load.*

Sol. Diameter = 6mm; Radius, $r = 3 \times 10^{-3}$ m;

Maximum stress = 6.9×10^7 Pa

Maximum load on a rivet

$$= \text{Maximum stress} \times \text{cross-sectional area}$$

$$= 6.9 \times 10^7 \times \frac{22}{7}(3 \times 10^{-3})^2 \text{ N} = 1952 \text{ N}$$

$$\text{Maximum tension} = 4 \times 1951.7 \text{ N} = 7.8 \times 10^3 \text{ N.}$$

9.21. *The Marina trench is located in the Pacific Ocean, and at one place it is nearly eleven km beneath the surface of water. The water pressure at the bottom of the trench is about 1.1×10^8 Pa. A steel ball of initial volume $0.32\ m^3$ is dropped into the ocean and falls to the bottom of the trench. What is the change in the volume of the ball when it reaches to the bottom? [$k = 1.6 \times 10^{11}\ Nm^{-2}$]*

Sol. Given, $P = 1.1 \times 10^8$ Pa, $V = 0.32\ \text{m}^3$, $K = 1.6 \times 10^{11}\ \text{Nm}^{-2}$

$$\text{Bulk modulus for steel} = 1.6 \times 10^{11}\ \text{Nm}^{-2}$$

Using relation, $$K = \frac{P}{\frac{\Delta V}{V}} = \frac{PV}{\Delta V} \quad \text{or,} \quad \Delta V = \frac{PV}{K}$$

$$\Rightarrow \quad \Delta V = \frac{1.1 \times 10^8 \times 0.32}{1.6 \times 10^{11}}\ \text{m}^3 = 2.2 \times 10^{-4}\ \text{m}^3.$$

QUESTIONS BASED ON SUPPLEMENTARY CONTENTS

Poisson's Ratio

Q. 1. *10 kg mass is attached to one end of a copper wire 3 m long and 1 mm in diameter. Calculate the lateral compression produced in it (Poisson's ratio is 0.25 and Young's modulus of the material of the wire is $12.5 \times 10^{10}\ N/m^2$)*

Sol. Let D*l* be the increase in length of the wire

$$\therefore \quad Y = \frac{\frac{F}{A}}{\frac{\Delta l}{l}}$$

$$Y = \frac{F \cdot l}{A \cdot \Delta l}$$

$$Y = \frac{F \cdot l}{\pi r^2 \cdot \Delta l}$$

$$\therefore \quad \Delta l = \frac{F \cdot l}{\pi r^2 \cdot Y} = \frac{10 \times 9.8 \times 3}{3.14 \times (0.5 \times 10^{-3})^2 \times 12.5 \times 10^{10}}$$

$$= 0.2993 \times 10^{-2}\ \text{m}$$

Now the Poisson's ratio $$\sigma = -\frac{\Delta D}{D} \times \frac{l}{\Delta l}$$

$$\therefore \quad \Delta D = -\frac{\sigma \cdot D \cdot \Delta l}{l} = \frac{-0.2993 \times 10^{-2} \times 10^{-3} \times 0.25}{3}$$

$$= -2.5 \times 10^{-7} = -0.25 \times 10^{-6} = -0.25\ \mu\text{m}$$

Here the lateral compression $= 0.25\ \mu$m (approx.)

Q. 2. *One end of a nylon rope of length 4.5 m and diameter 12 mm is fixed to a free limb. A monkey, weighing 100 N, jumps to catch the free end and stays there. Find the alongation of the rope and the corresponding change in the diameter. Given Young's modulus of Nylone = 4.8 × 10^{11} N/m^2 and Poision's ratio of nylon = 0.2.*

Sol. Here

$$l = 4.5 \text{ m}$$

$$r = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$$

$$mg = 100 \text{ N}$$

$$\therefore \quad Y = \frac{mg\,l}{\pi r^2 \times \Delta l}$$

$$\Delta l = \frac{mg\,l}{\pi r^2 \times Y} = \frac{100 \times 4.5}{3.14 \times (6 \times 10^{-3})^2 \times 4.8 \times 10^{11}}$$

$$= 8.2 \times 10^{-6} \text{ m}$$

Now

$$\Delta D = -\frac{\sigma D \cdot \Delta l}{l} = \frac{-0.2 \times 6 \times 10^{-3} \times 8.2 \times 10^{-6}}{4.5}$$

$$= -2.18 \times 10^{-9} = 2.18 \times 10^{-9} \text{ m}$$

Hence, the elongation in rope is 8.2 × 10^{-6} m and the change in the diameter is 2.18 × 10^{-9} m.

Q. 3. *Determine the poission's ratio of the material of wire whose volume remains constant under an external normal stress.*

Sol. Let volume of the wire before the expansion

$$V_1 = \pi r^2 l$$

Volume of the wire after expansion

$$V_2 = \pi (r - \Delta r)^2 \times (l + \Delta l)$$

But volume remains same during the expansion

$$\therefore \quad \pi r^2 l = \pi (r - \Delta r)^2 \times (l + \Delta l)$$

$$r^2 l = (r^2 + \Delta r^2 - 2r \times \Delta r) \times (l \times \Delta l)$$

$$= r^2(l + \Delta l) + \Delta r^2(l + \Delta l) - 2r \times \Delta r(l + \Delta l)$$

$$= r^2 l + r^2 \times \Delta l + \Delta r^2 l + \Delta r^2 \times \Delta l - 2r\Delta r l - 2r\Delta r \times \Delta l$$

$$\Rightarrow \quad 2r\Delta r l = r^2 \Delta l \quad \text{[Rejecting the higher power of } \Delta r \text{ and } \Delta r \times \Delta l]$$

$$\Rightarrow \quad \frac{r \cdot \Delta r \cdot l}{r^2 \cdot \Delta l} = \frac{1}{2} \quad \Rightarrow \quad \frac{\frac{\Delta r}{r}}{\frac{\Delta l}{l}} = \frac{1}{2}$$

$$\sigma = \frac{1}{2} = 0.5$$

Therefore, if the volume of the wire does not change, the value of the poission ratio σ is maximum equal to 0.5.

Q. 4. *A material has poission's ratio 0.5. If a uniform rod of it undergoes a longitudinal strain of 2×10^{-3}. What is the percentage increases in its volume.*

Sol. The poission's ratio of the material is 0.5 which is maximum and for maximum value of the Poission's ratio the volume of the material remains unchanged. Hence, the increase in volume = zero%.

Q. 5. *When a load on a wire is increased from 3 kg wt to 5 kg wt., the elongation increases from 0.61 mm to 1.02 mm. How much work is done during the extension of the wire?*

Sol.

$$W_1 = \frac{1}{2}\cdot F \times l = \frac{1}{2}\times 3\times 9.8\times 0.61\times 10^{-3} \text{ J}$$

$$W_2 = \frac{1}{2}\cdot F \times l = \frac{1}{2}\times 5\times 9.8\times 1.02\times 10^{-3} \text{ J}$$

$\therefore$ Net work done during the extensions

$$W = W_2 - W_1 = \left(\frac{1}{2}\times 5\times 9.8\times 1.02\times 10^{-3}\right) - \left(\frac{1}{2}\times 3\times 9.8\times 0.61\times 10^{-3}\right)$$

$$= \frac{1}{2}\times 9.8\times 10^{-3}[5\times 1.02 - 3\times 0.61]$$

$$= \frac{1}{2}\times 9.8\times 10^{-3}[5.10 - 1.83] = \frac{1}{2}\times 9.8\times 10^{-3}\times 3.27 = 16.023 \times 10^{-3} \text{ J}$$

Q. 6. *A steel wire of length 4 m is stretched through 2 mm. The cross-section area of the wire is 2.0 mm^2. If Young's modulus of steel is 2.0×10^{11} N/m^2, find (i) the energy density of the wire and (ii) the elastic potential energy stored in the wire.*

Sol. (*i*) Energy density

$$= \frac{1}{2}\left(\frac{Yl}{L}\right)\cdot\frac{l}{L} = \frac{1}{2}\left[\frac{2\times 10^{11}\times 2\times 10^{-3}}{4}\right]\times\left[\frac{2\times 10^{-3}}{4}\right] = \frac{1}{2}\times 10^{8}\times\frac{1}{2}\times 10^{-3}$$

$$= 0.25 \times 10^{5} = 2.5 \times 10^{4} \text{ J/m}^3$$

(*ii*) Potential energy stored in the wire

$$U = \frac{1}{2}\left(\frac{YAl}{L}\right)\cdot l = \frac{1}{2}\left[\frac{2\times 10^{11}\times 2\times 10^{-6}\times 2\times 10^{-3}}{4}\right]\times 2\times 10^{-3}$$

$$= 10^{2} \times 2 \times 10^{-3} = 0.2 \text{ J}$$

Q. 7. *A load of 31.4 kg is suspended from a wire of radius 10^{-3} m and density 9×10^{3} kg/m^3. Calculate the change in temperature of the wire if 75% of the work done is converted into heat. The Young's modulus and the specific heat capacity of the material of the wire are 9.8×10^{10} N/m^2 and 490 J/kg/k respectively.*

Sol. Volume of wire $V = \pi r^2 L$

$\therefore$ Density $= \dfrac{\text{mass}}{V}$

$$9 \times 10^3 = \frac{31.4}{\pi r^2 \times L}$$

$$\therefore \quad L = \frac{31.4}{\pi r^2 \times 9 \times 10^3}$$

$$L = \frac{31.4}{3.14 \times 10^{-6} \times 9 \times 10^3}$$

$$\therefore \quad L = \frac{10}{9} \times 10^3 \text{ m} = \frac{10^4}{9} \text{ m}$$

Now $$Y = \frac{mg\,L}{\pi r^2 l}$$

$$\therefore \quad l = \frac{mg\,L}{\pi r^2 . Y} = \frac{31.4 \times 9.8 \times 10^4}{3.14 \times 10^{-6} \times 9 \times 9.8 \times 10^{10}} = \frac{10}{9} \text{ m}$$

Now the work done $$= \frac{1}{2} F \cdot l = \frac{1}{2} \times 3.14 \times 9.8 \times \frac{10}{9}$$

75% of the work done is converted into heat energy.

$\therefore$ Heat energy $$= \frac{1}{2} \times 31.4 \times 9.8 \times \frac{10}{9} \times \frac{75}{100}$$

But heat energy = mass × S.P. heat × temp difference

= $31.4 \times 490 \times t$

$$\therefore \quad 31.4 \times 490 \times t = \frac{1}{2} \times 31.4 \times 9.8 \times \frac{10}{9} \times \frac{75}{100}$$

$$\therefore \quad t = \frac{\frac{1}{2} \times 31.4 \times 9.8 \times \frac{10}{9} \times \frac{75}{100}}{31.4 \times 490}$$

$$t = \frac{1}{120} \text{K} \quad \text{or} \quad 0.0083°\text{C}$$

ADDITIONAL QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *What is Poisson's ratio?*

Ans. The ratio of lateral strain to the longitudinal strain is called Poisson's ratio.

Q. 2. *How are the atoms and molecules arranged in a crystalline solid?*

Ans. They are arranged in a definite and regular manner throughout the body of the crystal.

Q. 3. *What will happen to the potential energy if a wire is (a) compressed, (b) stretched?*

Ans. In both the cases, the potential energy of the wire increases as work has to be done on the wire.

Q. 4. *What is the order of strain within elastic limit?*

Ans. 10^{-3}/cm.

Q. 5. *The breaking force for a wire is F. What will be the breaking force for (i) two parallel wires of this size and (ii) for a single wire of double thickness?*

Ans. (*i*) 2F (*ii*) 4F.

Q. 6. *Name one material which is famous for a large elastic after effect.*

Ans. Glass.

Q. 7. *What is the shape of stress-strain graph within elastic limit?*

Ans. A straight line.

Q. 8. *A wire of length L and cross-sectional area A is made of material of Young's modulus Y. What is the work done in stretching the wire by an amount x?*

Ans.

$$\text{Work done} = \text{elastic potential energy of stretched wire}$$

$$= \frac{1}{2} \times Y \times (\text{strain})^2 \times \text{volume} = \frac{1}{2} \times Y \times \left(\frac{x}{L}\right)^2 \times (A \times L)$$

$$= \frac{YAx^2}{2L}$$

Q. 9. *What is the bulk modulus of a perfectly rigid body?*

Ans. Infinity.

Q. 10. *Within elastic limit, what is the slope of stress-strain curve?*

Ans. Within elastic limit, the slope of stress-strain curve gives the value of modulus of elasticity of the given material.

Q. 11. *What are ductile materials?*

Ans. The material whose plastic range is comparatively large.

Q. 12. *How does Young's modulus change with rise in temperature?*

Ans. Young's modulus of a material decreases with rise in temperature.

Q. 13. *Give an example of pure shear.*

Ans. Twisting of cylinder produces pure shear.

Q. 14. *Write copper, steel, glass and rubber in the order of increasing coefficient of elasticity.*

Ans. Rubber, glass, copper and steel.

Q. 15. *How is shear modulus related to Young's modulus?*

Ans. For any material shear modulus is less than Young's modulus. For most materials shear modulus = $\frac{1}{3}$ × Young's modulus

Q. 16. *What is the value of modulus of rigidity for a liquid?*

Ans. Zero.

Q. 17. *How is compressibility related to bulk modulus?*

Ans. Compressibility is defined as the reciprocal of bulk modulus (B) *i.e.,*

$$K = \frac{1}{B}.$$

Q. 18. *What is the value of bulk modulus for an incompressible liquid?*

Ans. Infinite.

Q. 19. *Why strain has no units?*

Ans. Since strain is the ratio of two similar quantities, therefore it has no units.

Q. 20. *Why do spring balance shows wrong readings after they have been used for a long time?*

Ans. Because of elastic fatigue.

Q. 21. *Name three physical properties which can have different values in different directions.*

Ans. Thermal conductivity, electrical conductivity and compressibility.

Q. 22. *Which type of elasticity is possessed by liquids and gases?*

Ans. Liquids and gases possessed only the bulk modulus (*i.e.,* the volume elasticity).

Q. 23. *A wire is suspended from a roof but no weight is attached to the wire. Is the wire under stress?*

Ans. Yes. The weight of the wire itself acts as the deforming force.

Q. 24. *Which of the three Young's modulus of elasticity, Bulk modulus and shear modulus is possible in all the three states of matter (solid, liquid and gas)?*

Ans. Bulk modulus of elasticity only.

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *A wire cable 10 m long consists of 40 strands of steel each 5×10^{-6} m^2 in cross-section. By how much does the cable stretch when it is used to lift a crate weighing 4000 N? Young's modulus of steel = 20×10^{10} Nm^{-2}.*

Ans.

$$\text{Stress} = \frac{4000}{5\times10^{-6}} = 8.0 \times 10^{8} \text{ Nm}^{-2}$$

$$\text{Young's modulus } Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{Strain} = \frac{\text{Stress}}{Y} = \frac{8.0\times10^{8}}{20\times10^{10}} = 4.0\times10^{-3}$$

Now, $\text{Strain} = \frac{\Delta l}{l}\times 40$, where Δl is the increase in length in each strand.

Therefore, $\Delta l = \text{strain} \times \frac{l}{40} = \frac{10\times4.0\times10^{-3}}{40} = 1 \times 10^{-3} \text{ m} = 1 \text{ mm}$

Since each strand stretches by 1 mm, the cable is stretched by 1 mm. Alternatively, one could divide the stress by 40 to obtain the stress for each strand, since the load must distribute equally over the entire cross-section of the cable.

Q. 2. *Explain why steel is more elastic than rubber.*

Ans. Consider two pieces of wires, one of steel and the other of rubber. Suppose both are of equal length (L) and of equal area of cross-section (a).

Let each be stretched by equal forces, each being equal to F. We find that the change in length of the rubber wire (l_r) is more than that of the steel (l_s) *i.e.* $l_r < l_s$. If Y_s and Y_r the Young's moduli of steel and rubber respectively, then from the definition of Young's modulus,

$$Y_s = \frac{F.L}{a.l_s} \text{ and } Y_r = \frac{F.L}{a.l_r}.$$

$$\therefore \quad \frac{Y_s}{Y_r} = \frac{l_r}{l_s}.$$

As $l_r > l_s$ $\therefore$ $\frac{Y_s}{Y_r} > 1$ or $Y_s > Y_r$

i.e., the Young's modulus of steel is more than that of rubber. Hence steel is more elastic than rubber.

Or

Any material which offers more opposition to the deforming force to change its configuration is more elastic.

Q. 3. *Why are the springs made of steel and not of copper?*

Ans. A spring will be better one if a large restoring force is set up in it on being deformed, which in turn depends upon the elasticity of the material of the spring. Since the Young's modulus of elasticity of steel is more than that of copper, hence steel is preferred in making the springs.

Q. 4. *A wire stretches by a certain amount under a load. If the load and radius both are increased to four times, find the stretch caused in the wire.*

Ans.

$$Y = \frac{Fl}{A\Delta l} \Rightarrow \Delta l = \frac{Fl}{\pi r^2 Y}$$

$$\Delta l' = \frac{4Fl}{\pi (4r)^2 Y} = \frac{Fl}{4\pi r^2 Y} = \frac{\Delta l}{4}.$$

Q. 5. *Find the greatest length of steel wire that can hang vertically without breaking. Breaking stress of steel = 8.0 × 10^8 Nm^{-2}, density of steel = 8.0 × 10^3 kg m^{-3} and g = 10ms^{-2}.*

Ans. Let L be the maximum length of steel wire which can hang vertically without breaking. In such a case, the stretching force is equal to the own weight of wire. If A be the cross-section area of wire and ρ its density, then mass of wire $M = ALP$ and stretching force $F = Mg = ALP$g.

$$\therefore \text{ Maximum stress, } \sigma_{max} = \frac{\text{weight}}{A} = \frac{ALPg}{A} = LPg$$

$$\Rightarrow \quad L = \frac{\sigma_{max}}{\rho g} = \frac{8.0 \times 10^8}{8.0 \times 10^3 \times 10} = 10^4 \text{ m} \quad \text{or} \quad 10 \text{ km.}$$

Q. 6. *A solid sphere of radius 10 cm is subjected to a uniform pressure equal to 5 × 10^8 Nm^{-2}. Calculate the change in volume. Bulk modulus of the material of the sphere is 3.14 × 10^{11} Nm^{-2}.*

Ans. We know,

$$K = \frac{PV}{\Delta V}$$

$$\therefore \quad \Delta V = \frac{PV}{K}$$

Now

$$P = 5 \times 10^8 \text{ Nm}^{-2};$$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (0.1)^3 \text{ m}^3 = 4.19 \times 10^{-3} \text{ m}^3$$

[$\because$ r = 10 cm = 0.1 m]

$$K = 3.14 \times 10^{11} \text{ Nm}^{-2}$$

$$\therefore \quad \Delta V = \frac{5 \times 10^8 \times 4.19 \times 10^{-3}}{3.14 \times 10^{11}} = 6.67 \times 10^{-6} \text{ m}^3.$$

Q. 7. *The length of a metal wire is l_1 when the tension in it is T_1 and is l_2 when the tension is T_2. Find the original length of the wire.*

Ans. Let l and A be the original length and area of cross-section of the metal wire.

Change in length in the first case = $(l_1 - l)$

Change in length in the second case = $(l_2 - l)$

$$\therefore \qquad Y = \frac{T_1}{A} \times \frac{l}{(l_1 - l)} = \frac{T_2}{A} \times \frac{l}{(l_2 - l)}$$

$$\text{or} \qquad T_1 l_2 - T_1 l = T_2 l_1 - T_2 l$$

$$\text{or} \qquad l\,(T_2 - T_1) = T_2 l_1 - T_1 l_2 \Rightarrow l = \frac{T_2 l_1 - T_1 l_2}{(T_2 - T_1)}.$$

Q. 8. *Prove that the elastic potential energy density of a stretched wire is equal to half the product of stress and strain.*

Ans. Let the length of a wire be increased by l by applying a force F. Average internal force

$$= \frac{F}{2}.$$

$$\text{Work done} = \left(\frac{F}{2}\right) l$$

This work done is stored as potential energy U

$$\therefore \qquad U = \frac{Fl}{2} = \frac{1}{2}\left(\frac{F}{A}\right)\left(\frac{l}{L}\right)(AL),$$

where A = cross-sectional area and L = length.

or $U = \frac{1}{2}$ stress × strain × volume of the wire, energy density

$$u = \frac{U}{\text{volume}} = \frac{1}{2} \text{stress} \times \text{strain}$$

Q. 9. *The elastic limit of a steel cable is 3.0 × 10^8 N/m^2 and the cross-section area is 4 cm and the maximum upward acceleration that can be given to a 900 kg elevator supported by the cable if the stress is not to exceed one-third of the elastic limit.*

Ans. Here mass of elevator M = 900 kg, elastic limit of steel cable = 3.0 × 10^8 N/m^2 and cross-section area A = 4 cm^2 = 4 × 10^{-4} m^2

Let elevator is going upward with an acceleration 'a' so that tension in cable

$$T = M\,(g + a)$$

$$\therefore \qquad \text{Stress } \sigma = \frac{T}{A} = \frac{M\,(g+a)}{A} = \frac{900\,(9.8 + a)}{4 \times 10^{-4}} \text{ N/m}^2$$

As stress should not exceed one-third of elastic limit of steel cable, hence in limiting case, we have

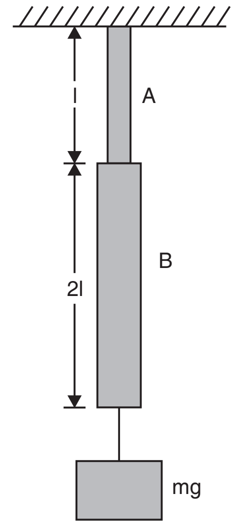
$$\sigma = \frac{3.0 \times 10^8}{3} = \frac{900\,(9.8 + a)}{4 \times 10^{-4}}$$

$$\Rightarrow \qquad a = \frac{10^8 \times 4 \times 10^{-4}}{900} - 9.8 = 44.4 - 9.8 = 34.6 \text{ ms}^{-2}.$$

Q. 10. *Elasticity is said to be internal property of matter. Explain.*

Ans. When a deforming force acts on a body, the atoms of the substance get displaced from their original positions. Due to this, the configuration of the body (substance) changes. The moment, the deforming force is removed, the atoms return to their original positions and hence the substance or body regains its original configuration. That is why, elasticity is said to be internal property of matter.

Q. 11. *Two wires A and B of length l, radius r and length 2l, radius 2r having same Young's modulus Y are hung with a weight mg, see fig. What is the net elongation in the two wires?*



Ans. Here, the pulling force F (= mg) is same on both the wires. Let Δl_1, Δl_2 be the elongations in the two wires.

As, $$Y = \frac{Fl}{\pi r^2 \Delta l} \quad \text{or} \quad \Delta l = \frac{Fl}{Y\pi r^2}$$

For wire 'A', $$\Delta l_1 = \frac{mgl}{Y\pi r^2}$$

For wire 'B', $$\Delta l_2 = \frac{mg\,(2l)}{Y\pi (2r)^2} = \frac{mgl}{2Y\pi r^2}$$

Total elongation $$= \Delta l_1 + \Delta l_2$$
$$= \frac{mgl}{Y\pi r^2} + \frac{1}{2}\frac{mgl}{Y\pi r^2} = \frac{3}{2}\frac{mgl}{Y\pi r^2}.$$

Q. 12. *A steel wire and a copper wire of equal length and equal cross-sectional area are joined end to end and the combination is subjected to a tension. Find the ratio of*

(a) the stresses developed in the two wires, (b) the strains developed in two wires. Given that Y of steel = 2.0 × 10^{11} N/m^2 and Y of copper = 1.1 × 10^{11} N/m^2.

Ans. Here $L_1 = L_2$, $A_1 = A_2$ and F is same.

(*a*) Stress $\sigma = \frac{F}{A}$ and F and A are common, hence stress in steel wire and in copper wire are equal.

(*b*) $$\text{Strain } \varepsilon = \frac{\Delta L}{L} = \frac{\text{stress}}{Y} \text{ and stress is equal}$$

$$\therefore \quad \frac{\varepsilon_{steel}}{\varepsilon_{copper}} = \frac{Y_{copper}}{Y_{steel}} = \frac{1.1\times 10^{11}}{2.0\times 10^{11}} = \frac{11}{20} = 0.55 : 1.$$

Q. 13. *Calculate the % increase in the length of a wire of diameter 2.5 mm stretched by a force of 100 kg f. Y for the wire = 12.5 × 10^{11} dyne cm^{-2}*

Ans. Here, $Y = 12.5 \times 10^{11}$ dyne cm^{-2} = 12.5×10^{10} Nm^{-2}

Diameter, $D = 2.5$ mm $= 2.5 \times 10^{-3}$m.

$F = 100$ kg $f = 100 \times 9.8$ N $= 980$ N.

$\frac{\Delta L}{L} \times 100 = ?$

$A = \pi r^2 = \pi(1.25 \times 10^{-3})^2$ m^2.

Using the relation,

$$Y = \frac{FL}{A\Delta L}, \text{ we get}$$

$$\% \text{ increase in length} = \frac{\Delta L}{L} \times 100 = \frac{F}{AY} \times 100 = \frac{F}{\pi r^2 Y} \times 100$$
$$= \frac{980}{3.142 \times (1.25\times 10^{-3})^2 \times 12.5 \times 10^{10}} \times 100$$
$$= 15.96 \times 10^{-2} = 0.16\ \%$$

Q. 14. *A sphere contracts in volume by 0.01% when taken to the bottom of sea 1 km deep. Find the bulk modulus of the material of the sphere.*

Ans. Here $\Delta V = 0.01\%$ of $V = \frac{0.01}{100} \times V = 10^{-4}$ V

$$P = h\rho g = 10^3 \times 10^3 \times 9.8 = 9.8 \times 10^6 \text{ Nm}^{-2}$$

Now, $K = \frac{PV}{\Delta V} = \frac{9.8 \times 10^6 \times V}{10^{-4} \times V} = 9.8 \times 10^{10} \text{ Nm}^{-2}$

Q. 15. *The breaking stress of aluminium is 7.5 × 10⁸ dyne cm⁻². Find the greatest length of aluminium wire that can hang vertically without breaking. Density of aluminium is 2.7 g cm⁻³. Given: g = 980 cm s⁻².*

Ans. Let l be the greatest length of the wire that can hang vertically without breaking.

Mass of wire, m = cross-sectional area (a) × length (l) × density (ρ)

Weight of wire $= mg = al\,\rho g$

This is equal to the maximum force that the wire can withstand.

$\therefore$ Breaking stress $= \frac{la\rho g}{a} = l\rho g$ or $7.5 \times 10^8 = l \times 2.7 \times 980$

or $l = \frac{7.5 \times 10^8}{2.7 \times 980}$ cm $= 2.834 \times 10^5$ cm $=$ **2.834 km**

Q. 16. *Artificial diamond crystals have been manufactured by subjecting carbon in the form of graphite to a pressure of 1.55 × 10¹⁰ Nm⁻² at a high temperature. Assuming that natural dimonds were formed at similar high pressures within the earth, what must have been the original volume of the Kohinoor diamond, whose mass before cutting was about 175g? The density of the diamond = 3.5g cm⁻³ and its bulk modulus = 62 × 10¹⁰ Nm⁻².*

Ans. Mass of the diamond =175g

Density = 3.5 gcm^{-3}

$$\text{Volume} = \frac{175}{3.5} = 50 \text{ cm}^3$$

If the original volume of the diamond were V, then

$$V = 50 + \Delta V$$

where ΔV is the increase in volume under the pressure during its formation.

$$\text{Bulk modulus} = B = -\frac{PV}{\Delta V}$$

Substituting (V – 50) for ΔV and the values of P and B, we have

$$\frac{B}{P} = \frac{62 \times 10^{10}}{1.55 \times 10^{10}} = \frac{40\,V}{V - 50} \quad \text{or} \quad V = 40V - 2000$$

$$39\,V = 2000 \Rightarrow V = 51.28 \text{ cm}^3$$

Alternatively, one could calculate ($-\Delta V$) from the equation

$$(-\Delta V) = \frac{PV}{B} = \frac{1.55 \times 10^{10} \times 50}{62 \times 10^{10}} = 1.25 \text{ cm}^3$$

and add this value to the present value giving $V = 51.25\ cm^3$. The difference is only in the second decimal place, *i.e.,* less than 0.06%. Hence the original volume of the diamond must have been equal to $51.3\ cm^3$.

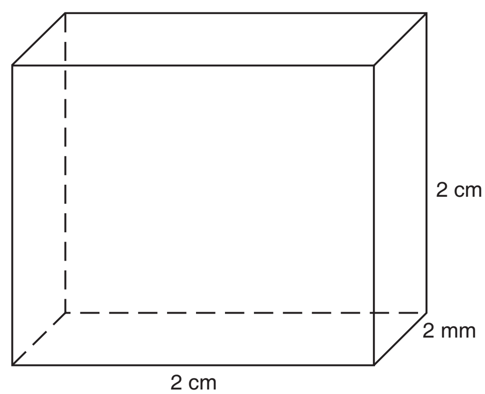
Q. 17. *A 4 m long aluminium wire whose diameter is 3 mm is used to support a mass of 50 kg. What will be the elongation of the wire? Y for aluminium is $7 \times 10^{10}\ Nm^{-2}$. Given: $g = 9.8\ ms^{-2}$.*

Ans.
$$l = 4\ \text{m},\ r = 1.5\ \text{mm} = 1.5 \times 10^{-3}\ \text{m},$$
$$M = 50\ \text{kg},$$
$$Y = 7 \times 10^{10}\ \text{Nm}^{-2},\quad F = 50 \times 9.8\ \text{N} = 490\ \text{N},\quad \Delta l = ?$$

$\therefore$
$$Y = \frac{F \times l}{A \times \Delta l}$$

or
$$\Delta l = \frac{F \times l}{\pi r^2 \times Y} = \frac{490 \times 4 \times 7}{22 \times (1.5 \times 10^{-3})^2 \times 7 \times 10^{10}}\ \text{m}$$
$$= 39.6 \times 10^{-4}\ \text{m} = 39.6 \times 10^{-4} \times 10^3\ \text{mm} = \mathbf{3.96\ mm}$$

Q. 18. *Calculate the force required to punch a hole 2 cm square in a steel sheet 2 mm thick whose shearing strength is $3.5 \times 10^8\ Nm^{-2}$.*



Ans. The shearing stress is exerted on the rectangular surface (2.0 cm × 2.0 cm × 0.2 cm) that is the boundary of the hole. The area of this surface is (see Fig)

$$A = 2.0 \times 10^{-2} \times 4 \times 0.2 \times 10^{-2}\ \text{m}^2$$
$$= 1.6 \times 10^{-4}\ \text{m}^2$$

Since the minimum shearing stress to rupture the steel is

$$\left(\frac{F}{A}\right)_{\min} = 3.5 \times 10^8\ \text{Nm}^{-2}$$

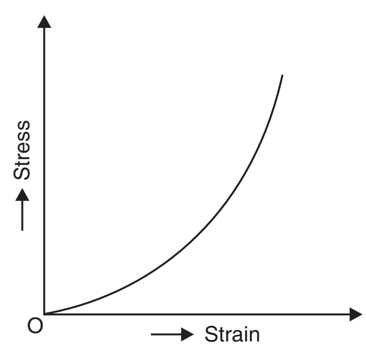
The force required is given by

$$F = 3.5 \times 10^8 \times 1.6 \times 10^{-4}\ \text{N} = 5.6 \times 10^4\ \text{N}$$

Q. 19. *What is an elastomer? What are their special features?*

Ans. Elastomers are those substances which can be stretched to cause large strains. Substances like tissue of aorta, rubber etc., are elastomers.

The stress-strain curve for an elastomer is as shown in figure below. Although elastic region is very large but the material does not obey Hooke's law over most of the region. Moreover, there is no well defined plastic region.



Q. 20. *Determine the force required to double the length of a steel wire of area of cross-section $5 \times 10^{-5} m^2$. Young's modulus of steel = $2 \times 10^{11}\ Nm^{-2}$.*

Ans. Here, Young's modulus, $Y = 2 \times 10^{11}\ \text{Nm}^{-2}$

Area of cross-section, $A = 5 \times 10^{-5}\ \text{m}^2$

Let the initial length of wire be L. Then, increase in length of wire,

$$\Delta L = L$$

Now, $$Y = \frac{F \times L}{A \times \Delta L}$$

$\therefore$ $$F = \frac{Y \times A \times \Delta L}{L} \Rightarrow F = \frac{2 \times 10^{11} \times 5 \times 10^{-5} \times L}{L}$$

or $$F = 10^7 \text{ N}.$$

Q. 21. *Four identical cylindrical columns of steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 cm and 40 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column. The Young's modulus of steel is 2.0×10^{11} Pa.*

Ans. Here, $M = 50{,}000$ kg; $r_1 = 0.30$ m

and $r_2 = 0.40$ m; $Y = 2.0 \times 10^{11}$ Pa.

Area of cross section of each column;

$$a = \pi (r_2^2 - r_1^2) = \pi [(0.4)^2 - (0.3)^2] = \pi \times 0.07 \text{ m}^2$$

Whole weight of the structure

$$= Mg = 50000 \times 9.8 \text{ N}$$

This weight is equally shared by four columns,

$\therefore$ Compressional force on one column,

$$F = \frac{50000 \times 9.8}{4} \text{N}$$

Now, $$Y = \frac{F/a}{\text{compressional strain}}$$

$$\therefore \text{Compressional strain} = \frac{F}{aY} = \frac{50000 \times 9.8/4}{(\pi \times 0.07) \times 2.0 \times 10^{11}} = \mathbf{2.785 \times 10^{-6}}$$

Q. 22. *What do you mean by compressibility? Why are solids least compressible and gases most compressible?*

Ans. Compressibility of the material of a body is defined as the reciprocal of its bulk modulus. It is, thus, defined as the fractional change in volume per unit increase in pressure.

$$\text{Compressibility, } K = \frac{1}{B} = -\left(\frac{\Delta V}{V}\right) \times \frac{1}{P}$$

The solids are least compressible whereas gases are most compressible. It is on account of the fact that in solids neighbouring atoms are tightly coupled but molecules in gases are very poorly coupled to their neighbours.

Q. 23. *What is elastic hysteresis?*

Ans. We know that some materials take appreciable time to recover their original condition completely. In other words, the strain persists even when the stress is removed. This lagging behind of strain is called elastic hysteresis.

Q. 24. *A uniform pressure P is exerted on all sides of a solid cube. It is heated through t°C in order to bring its volume back to the value it had before the application of pressure. Find the value of t.*

Ans. Let γ = coefficient of cubical expansion of the cube.

Let K be bulk modulus of elasticity of its material.

V = initial volume, P = pressure applied

ΔV = Decrease in its volume

$\therefore$ By definition, $$K = \frac{P}{\frac{\Delta V}{V}} \quad \text{or} \quad \Delta V = \frac{PV}{K} \qquad ...(i)$$

Also $$\Delta V \propto V$$
$$\propto t \quad \text{or} \quad \Delta V = \gamma\, V \times t = \gamma\, V\, t \qquad ...(ii)$$

where t = rise in its temperature so as to increase the volume by ΔV *s.t.* it is brought back to its initial volume.

$\therefore$ From (*i*) and (*ii*), we get

$$\frac{PV}{K} = \gamma\, V\, t \quad \text{or} \quad t = \frac{PV}{K\, \gamma\, V} = \frac{P}{\gamma K}.$$

III. LONG ANSWER TYPE QUESTIONS

Q. 1. Show that work done by a stretching force to produce certain extension in the wire is given by $W = \frac{1}{2}$ stretching force × extension. A wire that obeys Hooke's law is of length l_1 when it is in equilibrium under a tension F_1. Its length becomes l_2 when the tension is increased to F_2. Calculate the energy stored in the wire during this process.

Ans. Consider a wire of length L and area of cross-section A. Let a force F be applied to stretch the wire (Fig. *a*). If l be the length through which the wire is stretched, then

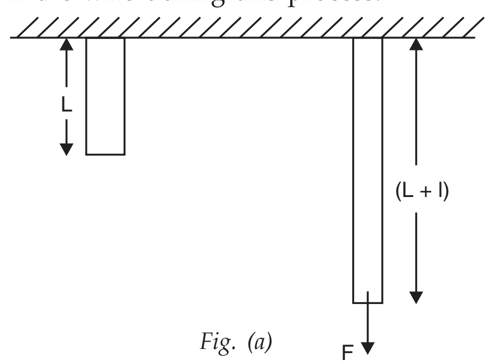


Fig. (a)

$$\text{Longitudinal strain} = \frac{l}{L}$$

$$\text{and tensile stress} = \frac{F}{A}$$

$\therefore$ Young's modulus of elasticity,

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{l/L} = \frac{FL}{Al} \Rightarrow F = \frac{YAl}{L} \qquad ...(i)$$

If the wire is stretched through a length dl, then work done is given by

$$dW = Fdl = \frac{YAl}{L}dl \qquad ...(ii)$$

$\therefore$ Total work done to stretch the wire through length l can be calculated by integrating eqn. (*ii*) between the limits $l = 0$ to $l = l$

i.e., $$\int dW = \int_0^l \frac{YA}{L}\, l\, dl \Rightarrow W = \frac{YA}{L}\frac{l^2}{2} = \frac{1}{2}\left(\frac{YA\,l}{L}\right) \times l$$

or $$\text{W} = \frac{1}{2}F \times l \qquad \left[F = \frac{YA\,l}{L}, \text{ from eqn. } (i)\right]$$

Numerical:

Hence, work done $= \frac{1}{2}$ stretching force × extension

$$W_1 = \frac{1}{2} F_1 (l_1 - l) \quad \text{and} \quad W_2 = \frac{1}{2} F_2 (l_2 - l)$$

$$U = W_2 - W_1 = \frac{1}{2} F_2 (l_2 - l) - \frac{1}{2} F_1 (l_1 - l) = \frac{1}{2} [F_2 l_2 - F_1 l_1 + (F_1 - F_2) l] \quad ...(i)$$

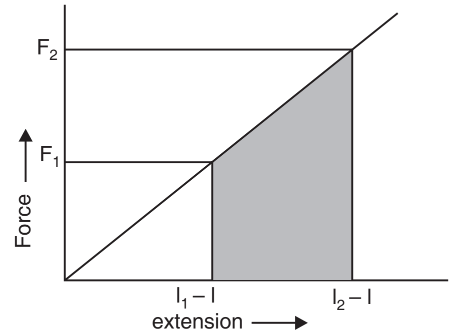


Fig. (b)

Now, $\frac{F_1}{F_2} = \frac{l_1 - l}{l_2 - l}$ or $F_1 l_2 - F_1 l = F_2 l_1 - F_2 l$

or $(F_2 - F_1) l = F_2 l_1 - F_1 l_2$

$\Rightarrow$ $l = \frac{F_2 l_1 - F_1 l_2}{F_2 - F_1}$

From eqn. (*i*), $U = \frac{1}{2}\left[F_2 l_2 - F_1 l_1 + (F_1 - F_2)\frac{F_2 l_1 - F_1 l_2}{F_2 - F_1}\right]$

$$= \frac{1}{2}[F_2 l_2 - F_1 l_1 - F_2 l_1 + F_1 l_2] = \frac{1}{2}[(F_2 + F_1) l_2 - (F_2 + F_1) l_1]$$

or $U = \frac{1}{2}(F_2 + F_1)(l_2 - l_1)$.

Q. 2. *What do you understand by Poisson's ratio? Find the value of Poisson's ratio at which the volume of a wire does not change when the wire is subjected to a tension.*

Ans. For Poisson's ratio, see text.

$$\text{Poisson's ratio } \sigma = \frac{\Delta r / r}{\Delta l / l}$$

where r is the radius of the wire and l its length.

Volume of the wire before expansion $V_1 = \pi r^2 l$

Volume of the wire after expansion

$$V_2 = \pi (r - \Delta r)^2 (l + \Delta l)$$

If the volume is to remain unchanged during expansion, we require that

$$V_1 = V_2, \text{ i.e.,}$$

$$\pi r^2 l = \pi [r^2 - 2r \Delta r + (\Delta r)^2] (l + \Delta l)$$

$$= \pi r^2 (l + \Delta l) - 2 \pi r \Delta r l - 2 r \Delta r \Delta l \pi + (\Delta r)^2 (l + \Delta l) \pi$$

or $2 \pi r \Delta r l = \pi r^2 \Delta l$ + terms containing the product of Δr and Δl, which can be neglected.

Hence $$\frac{\Delta r/r}{\Delta l/l} = \frac{1}{2} \quad \text{or} \quad \sigma = 0.5$$

Thus the volume of the wire does not change if the Poisson's ratio of the material of the wire is 0.5.

Q. 3. *State Hooke's law. Define Young's modulus of elasticity. A wire loaded by a weight of density 7.6 g cm^{-3} is found to measure 90 cm. On immersing the weight in water, the length decreased by 0.18 cm. Find the original length of wire.*

Ans. For Hooke's law and Young's modulus of elasticity, see fact that matter on pages 364–365 of this text book.

Let L be the original length of the wire, A be its area of cross-section and W be the load attached to the wire. Then, Young's modulus of the wire is given by

$$Y = \frac{W \times L}{A \times \Delta L} \qquad (\because F = W)$$

Since $\Delta L = 90 - L$

$$\therefore \quad Y = \frac{W \times L}{A(90 - L)} \qquad \text{...(1)}$$

Volume of weight attached

$$= \frac{W}{\text{density of weight}} = \frac{W}{7.6}\text{cm}^3$$

Weight of water displaced

$$= \frac{W}{7.6} \times \text{density of water} = \frac{W}{7.6} \times 1 = \frac{W}{7.6}$$

∴ Net weight after immersing in water is

$$W' = W - \frac{W}{7.6} = \frac{6.6\,W}{7.6}$$

Length of wire after immersing in water

$$= (90 - 0.18) = 89.82 \text{ cm}$$

∴ Change in length on immersing in water,

$$\Delta L' = (89.82 - L) \text{ cm}$$

$$\therefore \quad Y = \frac{W'L}{A\Delta L'} = \frac{6.6\,W \times L}{7.6 \times A \times (89.82 - L)} \qquad \text{...(2)}$$

Comparing eqns. (1) and (2), we get

$$\frac{W \times L}{A(90 - L)} = \frac{6.6\,W \times L}{7.6 \times A \times (89.82 - L)}$$

$$7.6 \times (89.82 - L) = 6.6\,(90 - L)$$

$$682.632 - 7.6L = 594 - 6.6L$$

$$\therefore \quad \mathbf{L} = 88.632 \text{ cm}$$

Q. 4. *Define the following terms:*

(i) elastic limit *(ii) elastic fatigue* *(iii) breaking stress*

A steel wire of cross-sectional area 0.5 mm² is held between two fixed supports. If the tension in the wire is negligible and it is just taut at a temperature of 20°C, determine the tension when the temperature falls to 0°C. Young's modulus of steel is 21×10^{11} dyne cm^{-2} and the coefficient of linear expansion of steel is 12×10^{-6} per °C. Assume that the distance between the supports remains unchanged.

Ans. For definitions, see the chapter in NCERT Textbook.

Numerical: Let l be the length of the wire at 20°C and l_0, the length at 0°C. Then

$$l - l_0 = \alpha l_0 \, \Delta T = 20 \, \alpha . \, l_0$$

$$\text{Compressive strain} = \frac{l - l_0}{l_0} = 20\alpha = 20 \times 12 \times 10^{-6} = 2.4 \times 10^{-4}$$

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{Stress} = Y \times \text{strain} = \frac{F}{A}$$

Hence,

$$\text{tension } T = YA \times \text{Strain} = 21 \times 10^{11} \times 0.5 \times 10^{-2} \times 2.4 \times 10^{-4}$$

$$= 2.52 \times 10^{6} \text{ dyne} = 25.2 \text{ N}$$

This is the tension in the wire when the temperature falls to 0°C.

IV. MULTIPLE CHOICE QUESTIONS

1. *Dimensional formula of stress is same as that of*

(*a*) impulse (*b*) strain (*c*) force (*d*) pressure

2. *Young's modulus of a material has the same unit as*

(*a*) stress (*b*) energy (*c*) compressibility (*d*) pressure

3. *Elastic limit is equal to*

(*a*) Young's modulus (*b*) Modulus of rigidity

(*c*) stress (*d*) strain

4. *Which of the following is not a unit of Young's modulus?*

(*a*) Nm^{-2} (*b*) Mega Pascal (MPa)

(*c*) dyne cm^{-2} (*d*) Nm^{-1}

5. *A wire suspended vertically from one end, is stretched by attaching a weight 200 N to the lower end. The weight stretches the wire by 1 mm. The energy gained by the wire is*

(*a*) 0.1 J (*b*) 0.2 J (*c*) 0.4 J (*d*) 10 J

6. *A spring of force constant k is cut into two equal parts. The force constant of each part is*

(*a*) $k/2$ (*b*) k (*c*) $2\,k$ (*d*) $4\,k$

7. *Young's modulus of a wire depends on*

(*a*) its material (*b*) its length

(*c*) its area of cross-section (*d*) both (*b*) and (*c*)

Ans. **1.**—(*d*) **2.**—(*a*) and (*d*) **3.**—(*c*) **4.**—(*d*) **5.**—(*a*)
6.—(*c*) **7.**—(*a*)

V. QUESTIONS ON HIGH ORDER THINKING SKILLS (HOTS)

Q. 1. *When a weight W is hung from one end of a wire of length L (other end being fixed), the length of the wire increases by l (Fig. a). If the wire is passed over a pulley and two weights W each are hung at the two ends (Fig. b), What will be the total elongation in the wire?*

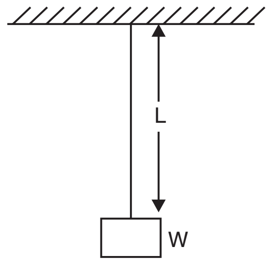


Fig. (*a*)

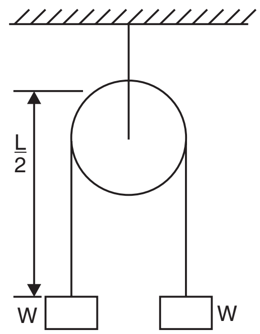


Fig. (*b*)

Ans. (*a*) Let Y = Young's modulus of the material of wire.
If a be its area of cross-section, then

$$Y = \frac{F/A}{l/L} = \frac{WL}{Al} \qquad (\because F = W)$$

or
$$l = \frac{WL}{AY} \qquad ...(i)$$

(*b*) When the wire is passed over a pully, let l' be the increase in length of the each segment. Since $\frac{L}{2}$ = length of each segment.

$$\therefore \qquad Y = \frac{W(L/2)}{Al'} \quad \text{or} \quad l' = \frac{1}{2}\frac{WL}{AY} = \frac{l}{2} \text{ [From eqn. } (i)] \qquad ...(ii)$$

$\therefore$ Total increase in the length of the wire

$$= l' + l' = 2l' = 2 \times \frac{l}{2} = l.$$

Q. 2. *A steel wire is suspended vertically from a rigid support. When loaded with weight in air, it extends by x_1. When the weight is completely inside the water, the extension becomes x_2. Find the relative density of the material of the weight.*

Ans. Let V be the volume of the load attached to lower end of steel wire of Young's modulus Y, area of cross-section A and length of wire L. Let ρ be the relative density of rod.

Then,
$$Y = \frac{FL}{Ax_1} = \frac{V\rho g L}{Ax_1} \qquad ...(i)$$

When load is immersed in liquid, then
$$Y = \frac{(V\rho g - V \times 1 \times g)\, L}{Ax_2} \qquad ...(ii)$$

From (*i*) and (*ii*),
$$\frac{\rho}{x_1} = \frac{\rho - 1}{x_2} \quad \text{or} \quad \rho x_2 = \rho x_1 - x_1$$

or
$$x_1 = \rho\,(x_1 - x_2) \quad \text{or} \quad \rho = \frac{x_1}{x_1 - x_2}$$

Q. 3. *Why are the bridges declared unsafe after long use?*

Ans. Due to the repeated stress and strain, the material used in the bridges loses elastic strength and ultimately may collapse. Hence, bridges are declared unsafe after long use.

Q. 4. *Why a hollow shaft is stronger than a solid shaft made from the same and equal amounts of material?*

Ans. The torque required to produce a unit twist in a solid shaft of radius r is given by

$$\tau = \frac{\pi\eta r^4}{2l}, \qquad ...(1)$$

where η is the modulus of rigidity of the material and l is the length of the shaft.

The torque required to produce a unit twist in a hollow shaft of inner and outer radii r_i and r_0 is given by

$$\tau' = \frac{\pi\eta(r_0^4 - r_i^4)}{2l} = \frac{\pi\eta(r_0^2 - r_i^2)(r_0^2 + r_i^2)}{2l} \qquad ...(2)$$

Dividing (2) by (1), we get

$$\frac{\tau'}{\tau} = \frac{(r_0^2 - r_i^2)(r_0^2 + r_i^2)}{r^4} \qquad ...(3)$$

Since the two shafts are made of the same material and the amounts of material are equal,

$$\therefore \qquad \pi r^2 l = \pi(r_0^2 - r_i^2)\,l \quad \text{or} \quad r^2 = r_0^2 - r_i^2$$

From (3), $$\frac{\tau'}{\tau} = \frac{r_0^2 + r_i^2}{r^2} \quad \text{or} \quad \frac{\tau'}{\tau} > 1 \quad \text{or} \quad \tau' > \tau$$

The torque required to twist a hollow shaft is clearly more than the torque required to twist a solid shaft. Thus, hollow shaft is stronger than a solid shaft.

Q. 5. *A boy's catapult is made of a rubber cord 42 cm long and 6 mm in diameter. The boy stretches the cord by 20 cm. Find the Young's modulus of the rubber if a stone weighing 0.02 kg when catapulated flies with a velocity of 20 ms^{-1}. Disregard the change in the cross-section of the cord in stretching.*

Ans. Due to extension produced in the cord, energy is stored in it which is converted into kinetic energy when the stone flies away. Assuming that there is no loss of energy in this process, the kinetic energy of the stone is given by

$$W = \frac{1}{2}mv^2 = 4J$$

This must be equal to the work done in stretching the cord.

Using the equation.

$$W = \frac{1}{2}F\Delta l = 4\text{J}$$

Where F is the stretching force. Since

$$\Delta l = 20 \text{ cm} = 0.2 \text{ m}$$

$$F = \frac{4 \times 2}{0.2} = 40\text{N}$$

$$\text{Stress} = \frac{F}{A} = \frac{40}{\pi r^2}$$

Now, $r = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$

Hence, $$\text{Stress} = \frac{40}{\pi (3\times10^{-3})^2} = 1.415 \times 10^6 \text{ Nm}^{-2}$$

But $$\text{Strain} = \frac{20}{42} = 0.476$$

$$\text{Young's modulus} = Y = \frac{1.415\times10^6}{0.476} = 2.97 \times 10^6 \text{ Nm}^{-2}.$$

Q. 6. *A force of 10^6 Nm^{-2} is required for breaking a material. If the density of the material is 3×10^3 kg m^{-3}, then what should be the length of the wire made of material so that it breaks by its own weight?*

Ans. Let L = length of the wire;

A = Area of cross section;

σ = density of the substance

Now, weight of wire $= Mg = F$

F = Volume × density × g $= A\, L\, \sigma\, g$

$$\text{stress} = \frac{F}{A} = \frac{AL\sigma g}{A} = L\,\sigma\, g$$

= Breaking stress

$$\therefore \quad L\,\sigma\, g = 10^6 \quad \text{or} \quad L = \frac{10^6}{\sigma g} = \frac{10^6}{3\times10^3\times9.8} = 34\text{m}.$$

Q. 7. *A steel ring of radius r and cross-section area A is fitted onto a wooden disc of radius R (R > r). If Young's modulus be E, find the force with which the steel ring is expanded.*

Ans. Original length, $l = 2\,\pi\, r$; Extension, $\Delta l = 2\,\pi\, R - 2\,\pi r = 2\,\pi(R - r)$

$$\text{Strain} = \frac{\Delta l}{l} = \frac{2\pi\,(R-r)}{2\,\pi\, r} = \frac{R-r}{r}$$

$$\text{Young's modulus, } E = \frac{F/A}{(R-r)/r} \quad \text{or} \quad F = EA\left(\frac{R-r}{r}\right)$$

Q. 8. *The bulk modulus of water is 2.3×10^9 Nm^{-2}. Find its compressibility. How much pressure in atmosphere is needed to compress a sample of water by 0.1%. Take 1 atmosphere pressure = 1.01×10^5 Nm^{-2}.*

Ans. Here $B = 2.3 \times 10^9 \text{ Nm}^{-2} = \dfrac{2.3\times10^9}{1.01\times10^5}$ atmosphere $= 2.27 \times 10^4$ atmosphere

(*a*) $\therefore$ Compressibility $k = \dfrac{1}{B} = \dfrac{1}{2.27\times10^4} = 4.4 \times 10^{-4} \text{ atm}^{-1}$

(*b*) As $\frac{\Delta V}{V} = -0.1\% = -\frac{0.1}{100} = -0.001$

$\therefore$ Increase in pressure $p = B\left(-\frac{\Delta V}{V}\right) = 2.27 \times 10^4 \times 0.001 = 22.7$ atm.

Q. 9. *Two cylinders A and B of radii r and 2r are soldered co-axially. The free end of A is clamped and the free end of B is twisted by an angle ϕ. Find twist at the junction taking the material of two cylinders to be same and of equal length.*

Ans. Let τ be the torque applied at the free end and ϕ' be the angle of twist at the junction. Then

$$\tau = \frac{\pi\eta r^4(\phi' - 0)}{2l} = \frac{\pi\eta (2r)^4(\phi - \phi')}{2l}$$

$$\Rightarrow \quad \phi' = 16(\phi - \phi') \quad \text{or} \quad 17\phi' = 16\phi \quad \text{or} \quad \phi' = \frac{16}{17}\phi.$$

TEST YOUR SKILLS

1. What do you understand by elastomers ? How does the stress-strain curve of elastomers behave ?
2. What is the modulus of elasticity ? What is the significance of modulus of elasticity. In construction industry ?
3. Why is steel a preferred material, while making bridges ? Why don't we use other metals, like aluminium or copper, for this ?
4. Why is RCC (Reinforced cement concrete) stronger than concrete ?
5. A steel rod as radius of 5 mm and length 50 cm. A force of 50 kN is applied to stretch this rod. What is the strain, stress and elongation experienced by the rod ?
   (Young's Modulus of steel = 2.0×10^{11} Nm^{-2})
6. How would you determine the Young's Modulus of a given metallic wire ?
7. What do you understand by Shear Modulus of a given metallic wire?
8. Why is it easier to compress gases, compared to solids ?

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