9 Ray Optics and Optical Instruments

Facts that Matter

The direction of propagation of light energy is called ray and the study of the behaviour of light ray is called ray optics or geometrical optics. When light falls on objects and they send it back to our eyes the objects becomes visible to us.

• Reflection of Light

When light falls on a smooth and rigid surface and comes back to the same medium, the phenomenon is called *reflection of light*. In reflection, the angle of incidence and the angle of reflection are always equal. The image formation is the consequence of reflection of light. If surface is not smooth and rigid, the reflected rays will not be parallel and image will not formed. This reflection is called irregular or diffused reflection. [Fig. 9.1]

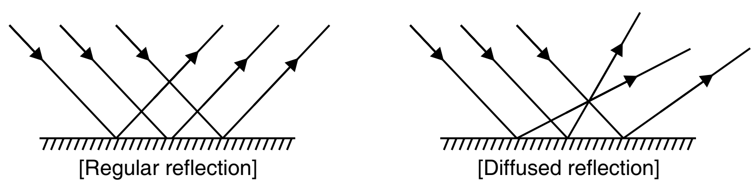


Fig. 9.1

• Reflection in Spherical Mirrors

There are two types of spherical mirrors. (*i*) convex and (*ii*) concave. The mid point of the mirror is called *pole*, *vertex* or *optical centre*. The radius of the curvature or point of intersection of normals draw at the surface of mirror is called *centre of curvature*. The line passing through centre of curvature and pole is called principal axis.

The point where incident ray coming parallel to the principal axis intersect the principal axis or appear to intersect the principal axis after reflection is called *focal point*. The distance of the focal point from the pole is called *focal length* of the mirror and distance of centre of curvature from the pole is called *radius of curvature*. [Fig 9.2]

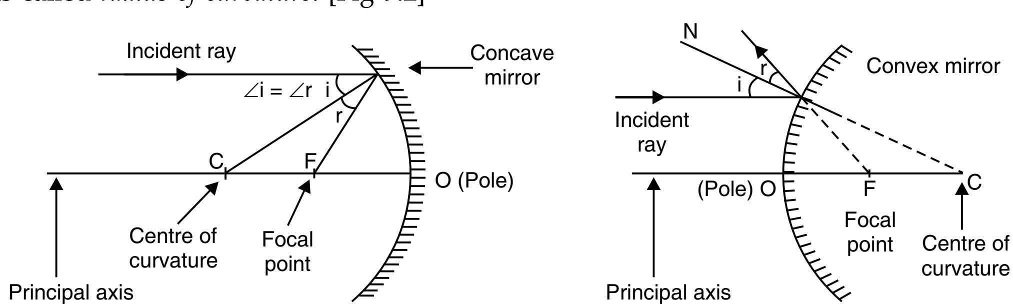


Fig. 9.2

- **Relation between Focal Length and Radius of Curvature**

Let an incident ray coming parallel to the principal axis of a concave mirror of focal length f and radius of curvature R passes through the focus F after reflection.

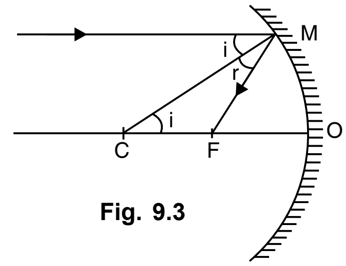


Fig. 9.3

$\because \quad \angle i = \angle r$

$\therefore$ In CMF,

$$CF = MF$$

For small aperture of the mirror,

$$MF = OF$$

or $\quad OC = CF + FO = 2FO$

or $\quad R = 2f$

This relation is applicable for all types of mirror (*i.e.* plane, convex)

- **Images in Mirrors**

If rays emanating from a point actually meet at another point after reflection the images of the point is formed at that point. It is called *real* image. If the rays do not actually meet, they appear to meet at the point of image formulation, it is called virtual image.

- **Sign Convention**

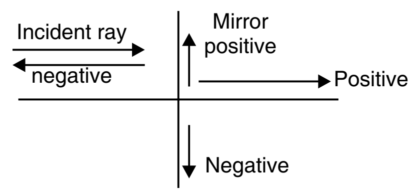


Fig. 9.4

  - All distances are measured from the pole.
  - Distances measured in the direction of incident ray are taken as positive.
  - Distance measured in the opposite direction of incident ray are taken as negative.
  - Distance above the principal axis are positive axis are positive and below the principal axis are negative. [Fig. 9.4]

- **Mirror Formula**

Let an incident ray coming parallel to the principal axis passes through focus F and the ray coming through the centre of curvature retraces its path. These two rays after reflection meet at point where image (I) of the object (O) is formed as shown in Fig. 9.5.

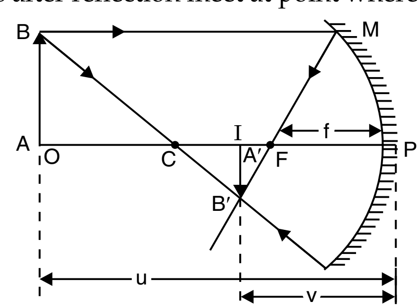


Fig. 9.5

In Fig. 9.5 ΔFMP and $\Delta A'B'F$ are similar.

$$\therefore \quad \frac{A'B'}{MP} = \frac{A'F}{FP}$$

$$\text{or} \quad \frac{A'B'}{AB} = \frac{A'F}{FP}$$

(For small aperture of mirror AB = MP) ...(*i*)

Similarly ΔABC and $\Delta A'B'C$ are similar

$$\therefore \quad \frac{A'B'}{AB} = \frac{A'C}{AC} \quad ...(ii)$$

From Eqs. (*i*) and (*ii*)

$$\frac{A'F}{FP} = \frac{A'C}{AC}$$

$$\text{or} \quad \frac{-(v-f)}{-f} = \frac{-(2f-v)}{-(u-2f)}$$

or $(v - f)(u - 2f) = 2f^2 - vf$

or $uv + 2f^2 - 2fv - uf = 2f^2 - vf$

or $uv = fv + fu$

or $$\boxed{\frac{1}{f} = \frac{1}{u} + \frac{1}{v}}$$

- Mirror formula will remain same for convex mirror. Only there will be a difference in sign convention.

• Magnification

It is the ratio of height of the image to the height of the object.

In Fig 9.5, magnification,

$$m = \frac{A'B'}{AB}$$

$$= \frac{v}{u} \qquad (\because \quad \Delta ABC \text{ and } \Delta A'B'C \text{ are similar})$$

$\because$ $$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

or $$\frac{v}{f} = \frac{v}{u} + 1$$

or $$\frac{v}{u} = m = \frac{v}{f} - 1$$

or $$\boxed{m = \frac{v - f}{f}}$$

- Magnification and position of image depend upon the position of object and its size.

• Refraction of Light

When light passes from one medium to another its speed changes and for oblique incidence it deviates from its path and the phenomenon is called refraction of light.

- During refraction, there is no change in frequency hence there is no change in colour.
- When light goes from denser to rarer medium its speed increases and it bends away from the normal.
- When light goes from rarer medium to denser medium its speed decreases and bends towards the normal.
- The ratio of sine of angle of incidence to the sine of angle of refraction is called refractive index. It is equal to the ratio of the speed of light in one medium to another.
- In Fig. 9.6, the refractive index at medium 2, with respect to medium 1,

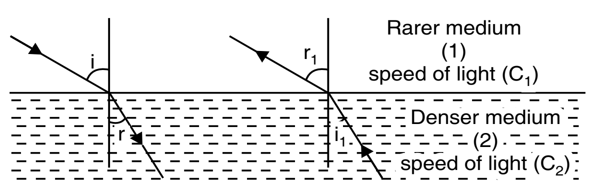


Fig. 9.6

$$n_{12} = \frac{C_1}{C_2} = \frac{\sin i}{\sin r} \quad ...(i)$$

The refractive index of medium 1 with respect to medium 2,

$$n_{21} = \frac{C_2}{C_1} = \frac{\sin i_1}{\sin r_1} \quad ...(ii)$$

From Eqs. (*i*) and (*ii*)

$$n_{21} = \frac{1}{n_{21}}$$

- When an object is placed below the transparent medium such as coin at the bottom of bucket of water or a glass slab on a mark, they appear to raised from their position.

  In Fig. 9.7 the object placed at C appear at B.

  In ΔACD,

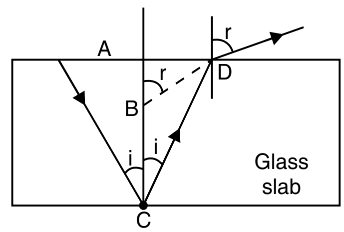


Fig. 9.7

$$\sin i = \frac{AD}{CD}$$

and in ΔABD

$$\sin r = \frac{AD}{BD}$$

∴ Refractive index of air with respect to glass

$$n_{ga} = \frac{\sin i}{\sin r} = \frac{AD/CD}{AD/BD} = \frac{BD}{CD}$$

⇒ Refractive index of glass with respect to air

$$n_{ag} = \frac{CD}{BD} = \frac{AC}{AB} \quad \text{(for small } i \text{ and } r\ CD = AC \text{ and } BD = AB)$$

or

$$n_{ag} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

• Critical Angle and Total Internal Reflection

When light passes obliquely, from denser medium to rarer medium the angle of incidence remains smaller than angle of refraction.

- The angle incidence for which angle of refraction becomes 90° is called critical angle.

  In Fig. 9.8

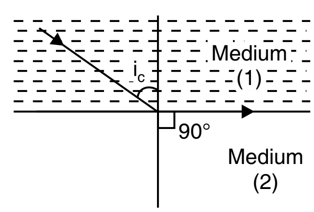


Fig. 9.8

$$n_{12} = \frac{\sin i_c}{\sin 90°} = \sin i_c$$

or critical angle, $i_c = \sin^{-1}(n_{12})$

or $i_c = \sin^{-1}\left(\frac{1}{n_{21}}\right)$

- When Light passes from denser to rarer medium and angle of incidence becomes greater than the critical angle, it comes back to the same medium and the phenomenon is called Total Internal Reflection.

• Mirage

It is the phenomenon of optical illusion. The atmosphere can be considered as made of different layers of air. Due to variation of temperature, different layers of air have different densities. The lower layers are rarer with respect to upper layers.

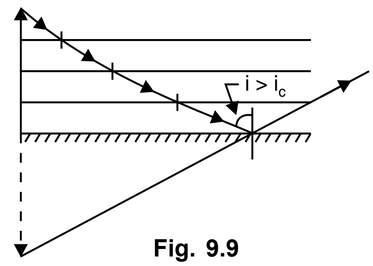


Fig. 9.9

When light passes through the different layers of air from top of an object the angle of incidence at each layer increases and when it becomes greater than the critical angle total internal reflection takes place and the image of the object is formed just below the object which given an illusion of presence of water.

- Mirage is very common in hot countries and deserts.
- In summer when sun is high, mirage can easily be observed at black dens roads.
- Similar to mirage in deserts, in polar regions *looming* takes place. Here the upper layer of air are rarer as compare to the lower layer of air. Hence the image of an object is formed in air as shown in Fig. 9.10.

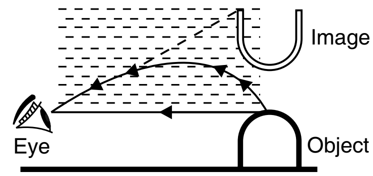


Fig. 9.10

- Total internal reflections take place in optical fibres.
- An air bubble in water shines due to total internal reflection at its outer surface. In air bubble when light goes from water to air *i.e.* from denser medium to rarer medium and angle of incidence becomes greater than critical angle total internal reflection takes place at surface of the bubble. [Fig. 9.11]

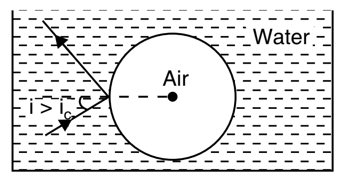


Fig. 9.11

- For a given thickness of a medium normal shift is independent of position of object and depends only on the thickness and nature of the medium.

  In Fig. 9.12, the normal shift

$$OI - OI' = n$$
$$= \left(1 - \frac{1}{n}\right)t$$
$$= \left(\frac{n-1}{n}\right)t$$

  where n is the refractive index of the medium of thickness t.

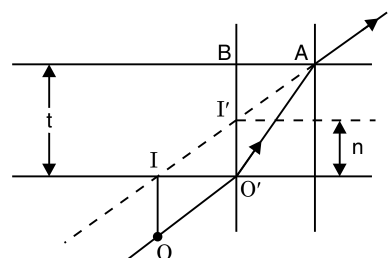


Fig. 9.12

- This is also why an introduction of a glass slab the point O of a conversing beam shifts by n as shown in Fig. 9.13.

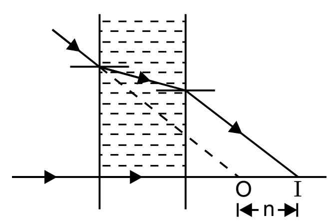


Fig. 9.13

• Refraction at Curved Surface

(*i*) At Concave Surface

Let the image of an object O is formed as I due to refraction at concave surface of radius of curvature R.

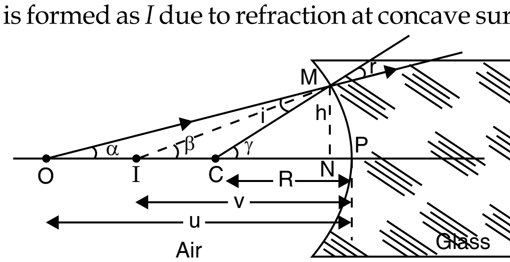


Fig. 9.14

In Fig. 9.14, ΔOMC,

$$\gamma = i + \alpha$$
$$i = \gamma - \alpha$$

And in ΔIMC,

$$\gamma = \beta + r$$

or $$r = \gamma - \beta$$

$\therefore$ The refractive index of glass with respect to air,

$$n_{ag} = \frac{\sin i}{\sin r} = \frac{i}{r} \quad \text{(for small angles)}$$

$$= \frac{(\gamma - \alpha)}{(\gamma - \beta)}$$

or $$(\gamma - \alpha) = n_{ag}(\gamma - \beta)$$

For angles, $$\gamma = \tan \gamma = -\frac{h}{R}$$

$$\beta = \tan \beta = -\frac{h}{v}$$

$$\alpha = \tan \alpha = -\frac{h}{u}$$

Thus, $$\left(\frac{-h}{R} + \frac{h}{u}\right) = n_{ag}\left(\frac{-h}{R} + \frac{h}{v}\right)$$

or $$-\frac{1}{R} + \frac{1}{u} = n_{ag}\left(\frac{1}{R} + \frac{1}{v}\right)$$

or $$\boxed{\frac{(n_{ag} - 1)}{R} = \frac{n_{ag}}{v} - \frac{1}{u}}$$

(*ii*) **At Convex Surface**

Similar to refraction at concave surface, in ΔOMC and ΔFMC

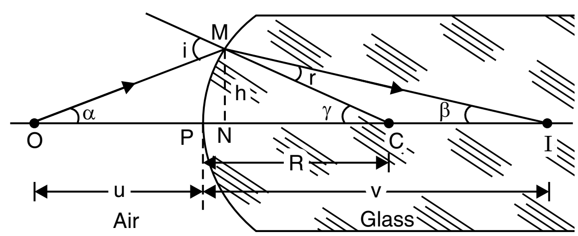


Fig. 9.15

$$i = \alpha + \gamma$$

and $$\gamma = \beta + r$$

or $$r = (\gamma - \beta)$$

$$n_{ag} = \frac{\sin i}{\sin r} = \frac{\alpha + \gamma}{\gamma - B} \quad \text{(for small angles)}$$

or $$(\gamma + \alpha) = n_{ag}(\gamma - \beta)$$

Also, $$\gamma = \tan \gamma = h/R$$

$$\beta = \tan \beta = h/v$$

and $$\alpha = \tan \alpha = -\frac{h}{u}$$

Thus, $$\left(\frac{h}{R} - \frac{h}{u}\right) = n_{ag}\left(\frac{h}{R} - \frac{h}{v}\right)$$

or $$\frac{1}{R} - \frac{1}{u} = \frac{n_{ag}}{R} - \frac{n_{ag}}{v}$$

or $$\boxed{\frac{n_{ag}}{v} - \frac{1}{v} = \frac{(n_{ag} - 1)}{R}}$$

• Refraction through Lenses

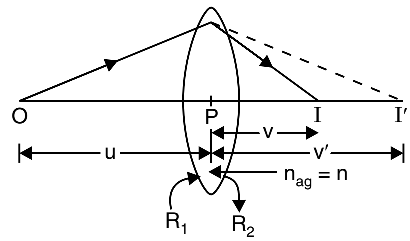


Fig. 9.16

Let a lens is made of two surfaces of radii of curvatures R_1 and R_2 having refractive index of its material

$$n \; (n_{ag} = n)$$

The image of an object O due surface of radius of curvature R_1 is formed at I' which acts as an object for surface of radius of curvature R_2 and the final image of O is formed at I as shown in Fig. 9.16.

For surface of radius of curvature R_1,

$$\frac{n}{v'} - \frac{1}{u} = \frac{(n-1)}{R_1} \quad ...(i)$$

and for surface of radius of curvature R_2,

$$\frac{1}{n/v} - \frac{1}{v'} = \frac{(1/n - 1)}{R_2}$$

$\left[\because \text{ Light is passing from glass to air } \therefore \; n_{ga} = 1/n\right]$

or $$\frac{1}{v} - \frac{n}{v'} = \frac{(1-n)}{R_2} \quad ...(ii)$$

Adding Eqs. (*i*) and (*ii*)

$$\frac{n}{v'} - \frac{1}{u} + \frac{1}{v} - \frac{n}{v'} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or $$\frac{1}{v} - \frac{1}{u} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad ...(iii)$$

If $$u = D, \; v = f$$

$\therefore$ $$\boxed{\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]} \quad ...(iv)$$

This is called lens maker formula from Eqs. (*iii*) and (*iv*)

$$\boxed{\frac{1}{f} = \frac{1}{v} - \frac{1}{u}.}$$ This is called lens formula.

• Power of the Lens

The ability of conversion or diversing the light is called power of the lens. In Fig. 9.17, the deviation due to lens δ can be given as

$$\delta = \tan\delta = \frac{h}{f} \text{(for small } \delta)$$

in power of the lens, $P \propto \delta$

or
$$P = \frac{1}{f(m)}$$

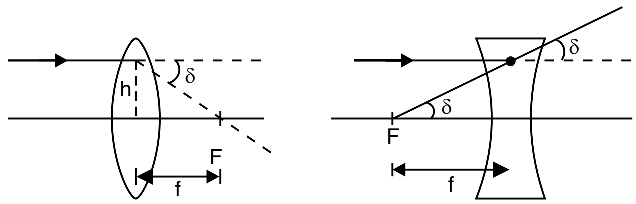


Fig. 9.17

The focal length is taken into metre and unit of the power of the lens in diopter.

- **Combination of Lenses**

When two lenses of focal lengths f_1 and f_2 are placed in contact the light coming parallel to the principal axis is to be focused at f_1 due to first lens, but due presence of second lens in contact it is finally focused at F.

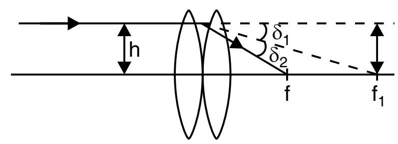


Fig. 9.18

∵ The deviation of light ray as shown in Fig. 9.18.

$$\delta = \delta_1 + \delta_2$$

where $\delta = \frac{h}{f}$ (f be the equivalent focal length of the combination)

and
$$\delta_1 = \frac{h}{f_1}$$

also
$$\delta_2 = \frac{h}{f_2}$$

∴
$$\frac{h}{f} = \frac{h}{f_1} + \frac{h}{f_2}$$

or
$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}}$$

- When the lenses are placed at certain separation 'd', then the light to be focused at f_1 by first lens will focus at f due to presence of second lens as shown in Fig. 9.19.

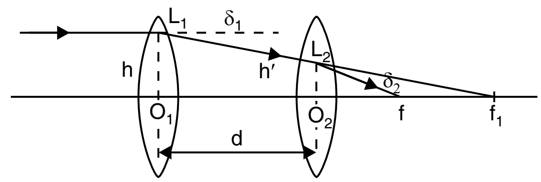


Fig. 9.19

The net deviation of light rays,

$$\delta = \delta_1 + \delta_2$$

We have, $\delta = \frac{h}{f}$ (f be the equivalent focal length of the combination)

and
$$\delta_1 = \frac{h}{f_1}$$

also
$$\delta_2 = \frac{h'}{f_2}$$

$$\therefore \qquad \frac{h}{f} = \frac{h}{f_1} + \frac{h'}{f_2} \qquad ...(i)$$

In $\Delta O_1L_1f_1$ and $\Delta O_2L_2f_1$

$$\frac{h}{f_1} = \frac{h'}{f_1 - d}$$

or

$$h' = \frac{h(f_1 - d)}{f_1} \qquad ...(ii)$$

Putting this value in Eq (*i*)

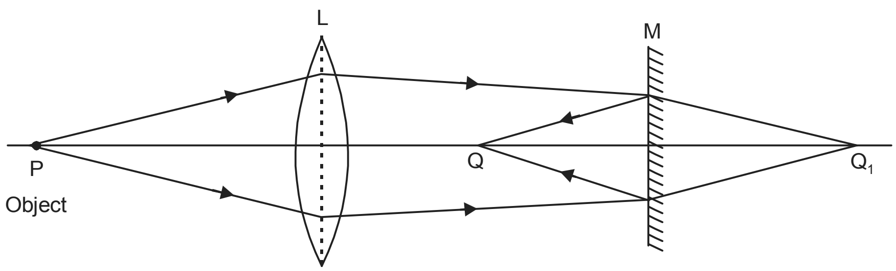
$$\frac{h}{f} = \frac{h}{f_1} + \frac{h(f_1 - d)}{f_1 f_2}$$

or

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}}$$

- **Combination of a Lens and a Mirror**

When a combination of a lens and a mirror is arranged co-axial and an object is placed in front of the lens, the incident rays first undergo refraction at lens and then incident at mirror. The image would have been formed behind the mirror if there had not been the mirror i.e. only lens. Now this image becomes the virtual object for the mirror and the real image is formed in front of the mirror at the same distance as shown in the figure.



- **Refraction in Prism**

When light passes through a prism refraction takes place at both the surfaces of the prism.

In Fig. 9.20 *i* and *e* are the angle incidence and emergence respectively. Angles r_1 and r_2 are angle of refraction at both the surfaces of the prism. A is the angle of prism and δ be the angle of deviation.

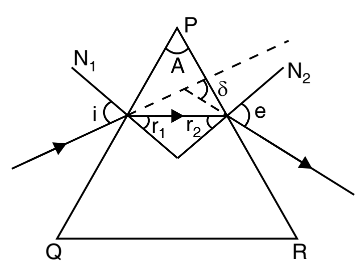


Fig. 9.20

In Fig 9.21, the angle of deviation

$$\delta = (i - r_1) + (e - r_2) \qquad ...(i)$$

In quadrilateral *APOQ*

$$\angle A + \angle O = 180^\circ$$

and in ΔPOQ

$$\angle r_1 + \angle r_2 + \angle O = 180^\circ$$

$$\Rightarrow \qquad \angle r_1 + \angle r_2 = \angle A$$

But for minimum deviation,

$$r_1 = r_2 = r \text{(say)}$$

$\Rightarrow$ $$r = A/2 \quad ...(ii)$$

Also $$i = e$$

$\therefore$ $$\delta_{min} = (i - r) + (e - r)$$

or $$\delta_{min} = i + e - 2r$$

or $$\delta_{min} + A = i + e \quad ...(iii)$$

Also $$i = \left(\frac{\delta_{min} + A}{2}\right) \quad (\because \; i = e)$$

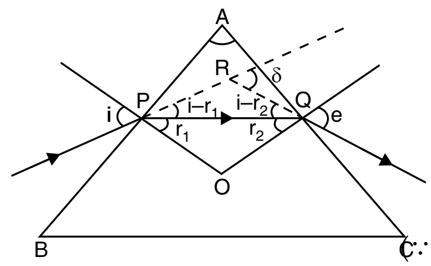


Fig. 9.21

Thus, no refractive index of the material of prism

$$n = \frac{\sin i}{\sin r}$$

or $$\boxed{n = \frac{\sin \frac{(A + \delta_{min})}{2}}{\sin A/2}}$$

For small angles

$$n = \frac{\delta_{min} + A}{A}$$

or $$nA = \delta_{min} + A$$

or $$\boxed{\delta_{min} = (n - 1)A}$$

- For maximum deviation in prism as shown in Fig. 9.22, angle of incidence must be maximum *i.e.*,

$$i = 90°$$

$\Rightarrow$ $$\delta_{max} = 90° + e - A$$

and $$\sin 90° = n \sin r_1$$

i.e., $$r_1 = \sin^{-1}\left(\frac{1}{n}\right)$$

or $$r_1 = i_c \text{ (critical angle)}$$

Thus, $$A = r_1 + r_2, \; r_2 = (A - i_c)$$

So at surface AC,

$$n \sin r_2 = \sin e$$

or $$\sin e = n \sin (A - i)$$

or $$e = \sin^{-1} [n \sin (A - i)]$$

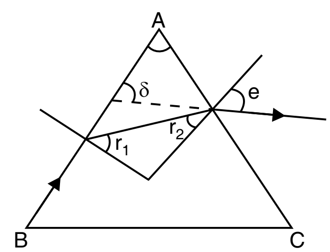


Fig. 9.22

- The deviation produced by a prism depends on angle of incidence, angle of prism, refractive index of the material of prism and wavelength of the light used.
- For a prism an angle of incidence increases, angle of deviation first decreases, reduces to minimum and then increases as shown in Fig. 9.23.

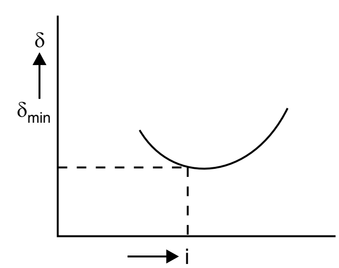


Fig. 9.23

• Dispersion

The splitting of white light into constituent colours is called Dispersion. A prism causes deviation as well as dispersion If δ_v, δ_r and δ_y are the deviations caused by prism in violet, red and yellow colours, the angular dispersion

$\phi = \delta_v - \delta_r = (\mu_v - \mu_r)$ A for small angled prism.

• Dispersion Power

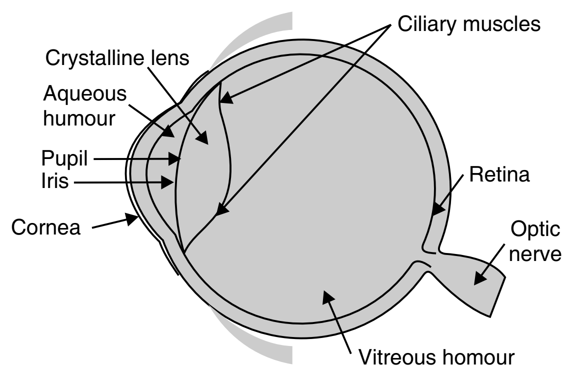
The ratio of (angular) dispersion to the deviation of the mean ray (yellow) is called the dispersion power of the prism. It is denoted by ω.

$$\omega = \frac{\theta}{\delta} = \frac{\delta_v - \delta_R}{\delta}$$

where δ is the deviation of the mean ray; δ_v and δ_R are the deviations of the violet and the red rays respectively.

• Human Eye

Light enters the eye through a curved surface called *Cornea*. Passing through aqueous humour, it passes through the pupil, having a hole in the middle called *Iris*. The size of pupil gets controlled by muscles called *ciliary muscles.* The light is further focused by the eye-lens on to the *retina.* The retina is a film made up of nerve fibres having *rod cells (responsible for brightness sensation)* and *cone cells (responsible for colour sensation).* These cells then transmit electrical signals via optical nerve to the brain. A normal human eye can see up to a minimum distance of 25 cm. This distance is known as *least distance of distinct vision or near point (D).*



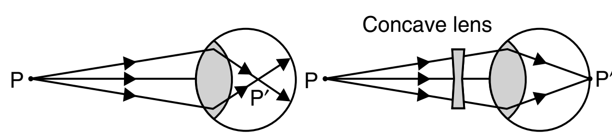
Human Eye

Fig. 9.24

Power of accommodation: The ability of human eye to adjust its focus depending upon the distance of the object is known as the power of accommodation.

Defects of Vision

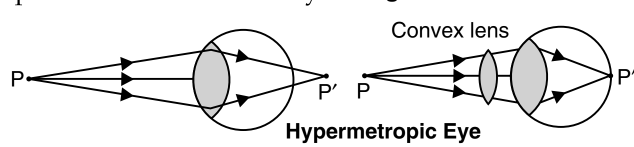
Myopia: Also called nearsightedness, visual defect in which the resting eye focuses the image of a distant object at a point in front of the retina. Myopic eyes are longer than normal from front to rear. Myopia is corrected by *concave lenses.* Near point 25 cm and far point less than at infinity.



Myopic Eye

Fig. 9.25

Hypermetropia: Also called *farsightedness.* It occurs when the image of the object is formed behind the retina. Hypermetropia frequently occurs when an eye is shorter than normal eyeball. A corrective lens for hypermetropia is *convex* on both its faces to supply the additional convexity needed for focusing. Near point more than 25 cm and for point at infinity.



Hypermetropic Eye

Fig. 9.26

Presbyopia: Loss of ability to focus the eye sharply on near objects as a result of the decreasing elasticity of the lens of the eye. Hence, the lenses of the eyes are left with little or no focusing ability. It is corrected by the use of convex lens for reading.

Astigmatism: It occurs when cornea is not spherical in shape. Due to this defect, the person is unable to focus on both the horizontal as well as vertical lines. This defect can be corrected by the use of cylindrical lens.

The devices which help human eye in observing highly magnified images of tiny objects, for detailed examination and in observing very far objects whether terrestrial or astronomical are called optical instruments.

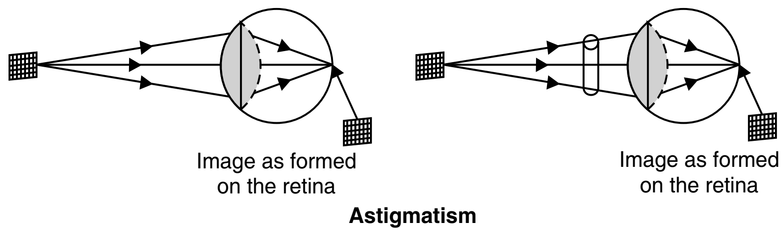


Fig. 9.27

• Microscope

In its simplest form, a compound microscope consists of two convergent lenses, one of very short focal length called the objective and the other of a longer focal length called the eyepiece. The lenses are mounted coaxially and the separation between them can be varied. The object AB (to be examined) is placed just outside the focus F_0 of the objective which forms its real image at $A'B'$ which serves as the object for the eyepiece. Image $A'B'$ lies within the focus F_e of the eyepiece which forms the final magnified virtual image at $A''B''$, as shown in the figure below.

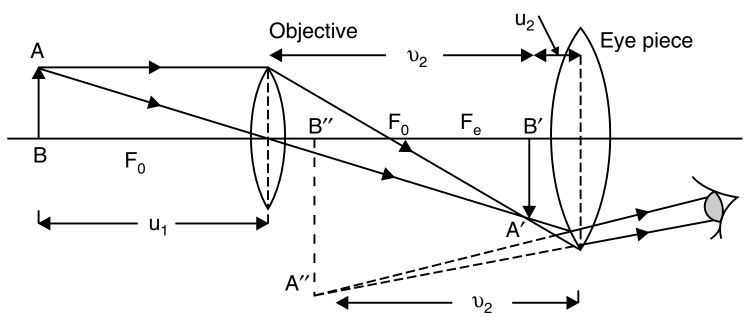


Fig. 9.28

Magnifying power of a compound microscope (M) = magnification by the objective × magnifying power of the eyepiece

i.e., $$(M)\ mp = \frac{v_1}{u_1} \times \left(1 - \frac{v_2}{f_e}\right)$$

where, u_1 = distance of the object from the objective

v_1 = distance (from the objective) of the image formed by the objective

f_e = focal length of the eyepiece

v_2 = distance of the final image from the eyepiece.

• Telescope

A telescope consists of two convergent lenses called the objective and the eyepiece. The objective has much longer focal length than the eyepiece.

(*i*) The magnifying power, M, of refracting telescope is given by

$$M = \frac{f_0}{f_e}$$

and $L = (f_0 + f_e)$; L = length of the telescope.

(*ii*) For the final image is formed at the least distance of distant vision,

$$M = \frac{f_0}{f_e}\left(1 + \frac{F_e}{D}\right)$$

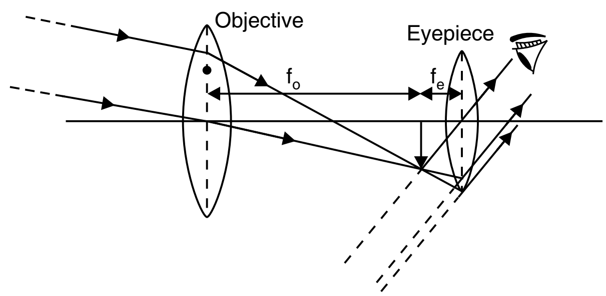
(*iii*) The resolving power of a telescope is given by

$$\theta = \frac{1.22\,\lambda}{d}$$

where, λ = wavelength of light

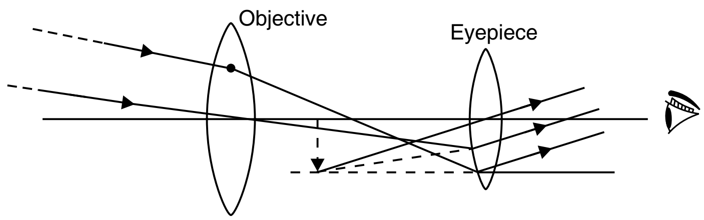
d = diameter of the objective of the telescope

θ = angle subtended by the point object on the objective.



[For normal adjustment image is formed at infinity]

Fig. 9.29



[Image is formed at the least distance of distinct vision]

Fig. 9.30

QUESTIONS FROM TEXTBOOK

9.1. *A small candle, 2.5 cm in size is placed at 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved?*

Sol. u = –27 cm, $f = \frac{R}{2} = \frac{-36}{2}$ = –18 cm

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{18} - \frac{1}{-27}$$

or $$\frac{1}{v} = \frac{1}{27} - \frac{1}{18}$$

On simplification, v = – 54 cm

The negative sign indicates that the image is formed in front of the mirror. Thus, the screen should be placed at a distance of 54 cm in front of the mirror.

Now, $$m = \frac{I}{O} = -\frac{v}{u}$$

$$\therefore \quad \frac{I}{2.5} = \frac{-54}{-27} \text{ or } I = -5 \text{ cm}$$

As size of image is (–)ive. So image is inverted and real.

When the candle is moved closer to mirror, the screen would have to be moved farther and farther. However, when the candle is closer than 18 cm from the mirror, the image would be virtual and therefore cannot be collected on the screen.

9.2. *A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror.*

Sol. Given,

$$u = -12 \text{ cm}, f = +15 \text{ cm}$$
$$O = 4.5 \text{ cm}$$

As, $$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{15} + \frac{1}{12} = \frac{4+5}{60} = \frac{9}{60}$$

$$v = 60/9 = 6.7 \text{ cm}$$

$\therefore$ Image formed at 6.7 cm at the back of the mirror.

As, $$m = \frac{I}{O} = -\frac{v}{u}$$

$$\therefore \quad \frac{I}{4.5} = -\frac{6.7}{-12} \text{ or } I = \frac{6.7 \times 4.5}{12} = 2.5 \text{ cm.}$$

$\therefore$ Image is erect, and of course virtual.

As needle is moved farther from the mirror, image moves away from the mirror (upto *F*) and goes on decreasing is size.

9.3. *A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the mircoscope have to be moved to focus on the needle agains?*

Sol. *Case I:* Given, real depth = 12.5 cm; apparent depth = 9.4 cm

As, $$\mu = \frac{\text{real depth}}{\text{apparent depth}}$$

or $$\mu = \frac{12.5}{9.4} = 1.33$$

Case II: $\mu = 1.63$, real depth = 12.5 cm

$$\text{apparent depth} = \frac{\text{real depth}}{\mu}$$

or $$\text{A.D.} = \frac{12.5}{1.63} = 7.67 \text{ cm}$$

∴ Distance through which microscope has to be moved downward.

$$= (9.4 - 7.67) \text{ cm} = 1.73 \text{ cm.}$$

9.4. *Figures 9.31 (a) and (b) show refraction of a ray in air incident at 60° with the normal to a glass-air and water-air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is 45° with the normal to a water-glass interface [Fig. 9.31 (c)].*

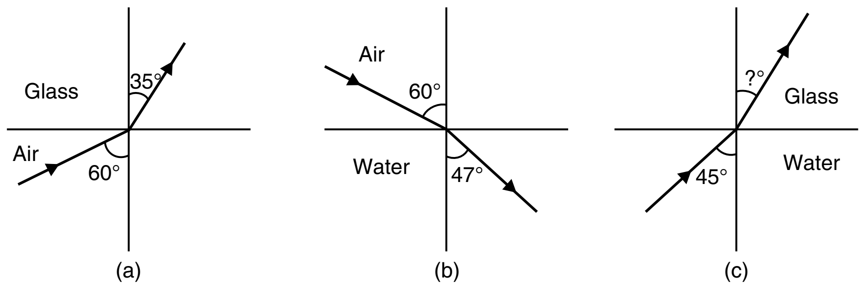


Fig. 9.31

Sol. In Fig. 9.31 (*a*)

$$i = 60°, \quad r = 35°$$

$$^{a}\mu_g = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 35°}$$

$$= \frac{0.8660}{0.5736} = 1.51$$

In Fig. 9.31 (*b*) $$i = 60°, \quad r = 41°$$

$$^{a}\mu_w = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 41°} = 1.32$$

In Fig. 9.31 (*c*) $$i = 45°$$

$$^{w}\mu_g = \frac{^{a}\mu_g}{^{a}\mu_w} = \frac{\sin i}{\sin r}$$

$$\frac{1.51}{1.32} = \frac{\sin 45°}{\sin r} = \frac{0.7071}{\sin r}$$

∴ $$\sin r = \frac{1.32 \times 0.7071}{1.51} = 0.6181$$

$$r = 38.2°.$$

9.5. *A small bulb is placed at the bottom of a tank containing water to a depth of 80 cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index of water is 1.33. (Consider the bulb to be a point source.)*

Sol. Let *O* be the bulb at the bottom of the tank.

$$OA = 80 \text{ cm}$$

The rays of light emitted from *O* will be refracted into air only if the angle of incidence is less than critical angle i_c. When the angle of incidence is equal to critical angle i_c, the light will not be refracted into air. Instead, it will graze the air-water interface. Thus, the light will appear to come out of a cone having vertex angle $2i_c$. [If the angle of incidence exceeds the critical of angle, then the rays of light will be totally reflected.]

We know that

$$\boxed{\sin i_c = \frac{1}{{}_a\mu_w}}$$

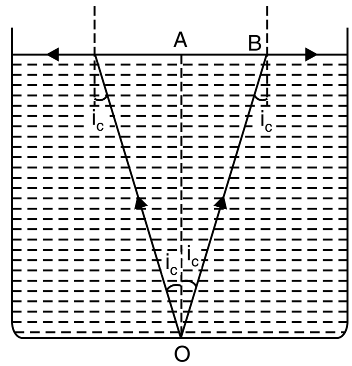


Fig. 9.32

or $$i_c = \sin^{-1}\left(\frac{1}{{}_a\mu_w}\right) = \sin^{-1}\left(\frac{1}{1.33}\right)$$

$$= \sin^{-1}(0.752) = 48.76°$$

Radius of patch of light = AB

Now, $$\tan i_c = \frac{AB}{OA} \text{ or } AB = OA \tan i_c$$

∴ Radius of patch of light

$$= 80 \tan 48.76° = 80 \times 1.14 \text{ cm} = 91.2 \text{ cm}$$

Area of patch of light $= \pi(91.2)^2 \text{ cm}^2 = 26140.5 \text{ cm}^2 = 2.61 \text{ m}^2$.

9.6. *A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. The angle of minimum deviation is measured to be 40°. What is the refractive index of the material of the prism? The refracting angle of the prism is 60°. If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light.*

Sol. $$A = 60°, \delta m = 40°$$

$$ {}_a\mu_g = \frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin A/2}$$

or $$ {}_a\mu_g = \frac{\sin 50°}{\sin 30°} = \frac{0.766}{0.54} = 1.532$$

After the prism is placed in water,

$$ {}_w\mu_g = \frac{\sin\left(\frac{A+\delta'_m}{2}\right)}{\sin A/2}$$

or, $$\frac{{}^a\mu_g}{{}^\alpha\mu_2} = \frac{\sin\left(\frac{60+\delta'_m}{2}\right)}{\sin 30°}$$

∴ $$\sin\left(30° + \frac{\delta'_m}{2}\right) = \frac{1}{2}\times\frac{1.532}{1.33} = 0.5759$$

or, $$30° + \frac{\delta'_m}{2} = 35°10'$$

on simplification, $\delta'_m = 10°20'$.

9.7. *Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length is to be 20 cm?*

Sol. Given, $\mu = 1.55$, $R_1 = R$ and $R_2 = -R$, $f = 20$ cm.

Since, $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

∴ $$\frac{1}{20} = (1.55 - 1)\left(\frac{1}{R} + \frac{1}{R}\right) = \frac{1.10}{R}$$

Hence, $R = 20 \times 1.1 = 22$ cm.

9.8. *A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is (a) a convex lens of a focal length 20 cm, and (b) a concave lens of focal length 16 cm?*

Sol. Here, the point P on the right of the lens acts as a virtual object,

$$u = 12 \text{ cm}$$

(*a*) $f = 20$ cm

Since, $$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

∴ $$\frac{1}{v} = \frac{1}{20} + \frac{1}{12} = \frac{3+5}{60} = \frac{8}{60}$$

or, $v = 60/8 = 7.5$ cm.

Image is at 7.5 cm to the right of the lens, where the beam converges.

(*b*) $f = -16$ cm, $u = 12$ cm,

∴ $$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = -\frac{1}{16} + \frac{1}{12} = \frac{-3+4}{48} = \frac{1}{48}$$

$$v = 48 \text{ cm}$$

Hence the image is at 48 cm to the right of the lens, where the beam would converge.

9.9. *An object of size 3.0 cm is placed 14 cm in front of a concave lens of focal length 21 cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?*

Sol. $O = 3.0$ cm, $u = -14$ cm, $f = -21$ cm

Since, $$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = -\frac{1}{21} - \frac{1}{14} = -\frac{35}{14 \times 21} \quad \text{or,} \quad v = -8.4 \text{ cm.}$$

The image is located 8.4 cm from the lens on the same side as the object.

As, $$m = \frac{I}{O} = \frac{v}{u}$$

∴ $$I = \frac{v}{u} \times O = \frac{-8.4}{-14} \times 3 = 1.8 \text{ cm}$$

As size of image is +ve. So, image is errect and virtual of smallar size.

As the object is moved away from the lens, the virtual image moves towards the focus of the lens but never beyond. The image progressively diminishes in size.

9.10. *What is the focal length of a convex lens of focal length 30 cm in contact with a concave lens of focal length 20 cm? Is the system a converging or a diverging lens? Ignore thickness of the lenses.*

Sol. Here, $f_1 = 30$ cm, $f_2 = -20$ cm

Since, $$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\therefore \quad \frac{1}{f} = \frac{1}{30} - \frac{1}{20} = \frac{2-3}{60} = \frac{-1}{60}$$

$$f = -60 \text{ cm}$$

∴ The combination of lenses behaves as a concave lens. The system is not converging.

9.11. *A compound microscope consists of an objective lens of focal length 2.0 cm and an eyepiece of focal length 6.25 cm separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image at (a) the least distance of distinct vision (25 cm), and (b) at infinity? What is the magnifying power of the microscope in each case?*

Sol. f_o = 2 cm, f_e = 6.25 cm, u_o =?

(*a*) For eyepiece, v_e = –25 cm, f_e = 6.25 cm, u_e =?

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e} \text{ or } \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$$

$$\therefore \quad \frac{1}{u_e} = \frac{1}{-25} - \frac{1}{6.25} = -\frac{1}{5}$$

or, $\quad u_e = -5$ cm

Now, $v_o = 15 - |u_e| = 10$ cm

For objective, $\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$ or $\frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0}$

or, $\frac{1}{u_0} = \frac{1}{10} - \frac{1}{2} = -\frac{2}{5}$ or $u_0 = \frac{-5}{2}$ cm = –2.5 cm

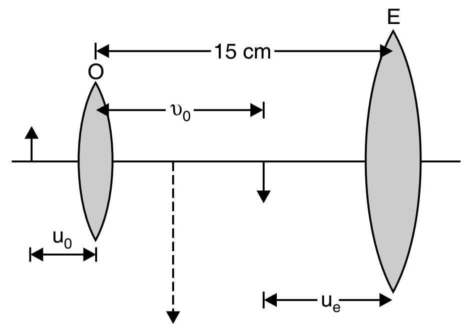


Fig. 9.33

$$M = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$$

Magnifying power, M = $\frac{10}{-2.5}\left(1 + \frac{25}{6.25}\right) = -20$

(*b*) The final image will be formed at infinity only if the image formed by the objective is in the focal plane of the eyepiece.

$\therefore \quad |u_e| = f_e = 6.25$ cm;

$|v_0| = (15 - 6.25)$ cm = 8.75 cm

Now, $\frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{8.75} - \frac{1}{2} = \frac{-6.75}{8.75 \times 2}$

$\therefore \quad u_0 = -\frac{8.75 \times 2}{6.75}$ cm or $u_0 = -2.59$ cm

Magnifying power, *M*

$$= \frac{8.75}{-2.59} \times \frac{25}{6.25} = -13.5.$$

9.12. *A person with a normal near point (25 cm) using a compound microscope with objective of focal length 8.0 mm and an eye piece of focal length 2.5 cm can bring an object placed 9.0 mm from the objective in sharp focus. What is the separation between the two lenses? Calculate the magnifying power of the microscope?*

Sol. Here, $u_0 = -0.9$ cm, $f_o = 0.8$ cm

As, $$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$$

$$\therefore \quad \frac{1}{v_0} = \frac{1}{f_0} + \frac{1}{u_0} = \frac{1}{0.8} - \frac{1}{0.9} = \frac{1}{7.2}$$

or, $$v_o = 7.2 \text{ cm}$$

Now for the eyepiece, we have

$f_e = 2.5$ cm, $v_e = -D$, u_e =?

$$\therefore \quad \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{2.5} = -\frac{11}{25}$$

or, $$u_e = -\frac{25}{11} = -2.27 \text{ cm}$$

Separation between the two lenses

$$= v_0 + |u_e|$$

$$= 7.2 + 2.27 = 9.47 \text{ cm}$$

Magnifying power, $M = M_0 \times M_e$

$$M = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right) = \frac{7.2}{0.9}\left(1 + \frac{25}{25}\right)$$

$$= 8 \times 11 = 32.$$

9.13. *A small telescope has an objective lens of focal length 144 cm and an eyepiece of focal length 6.0 cm. What is the magnifying power of the telescope? What is the separation between the objective and the eyepiece?*

Sol. Here, $f_0 = 144$ cm, $f_e = 6.0$ cm

As, $$\boxed{m = \frac{-f_0}{f_e}} = \frac{-144}{6.0} = -24$$

and $$\boxed{L = f_0 + f_e} = 144 + 6.0 = 150 \text{ cm.}$$

9.14. (*a*) *A giant refracting telescope at an observatory has an objective lens of focal length 15 m. If an eyepiece of focal length 1.0 cm is used, what is the angular magnification of telescope ?*

(*b*) *If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is 3.48 × 10^6 m and radius of lunar orbit is 3.8 × 10^8 m.*

Sol. Given, $f_o = 15$ m, $f_e = 1.0$ cm $= 10^{-2}$ m

(*a*) Angular magnification $= \dfrac{f_o}{f_e} = \dfrac{15}{10^{-2}} = 1500$

(*b*) If d is diameter of the image, then angle subtended by diameter of moon

$$= \frac{3.48 \times 10^6}{3.8 \times 10^8}$$

and, angle subtended by image

$$= \frac{d}{f_o} = \frac{d}{15}$$

$$\therefore \qquad \frac{d}{15} = \frac{3.48\times10^6}{3.8\times10^8}$$

or, $$d = \frac{3.48\times15\times10^{-2}}{3.8}$$

$$=13.73 \times 10^{-2} \text{ m} = 13.73 \text{ cm}.$$

9.15. *Use the mirror equation to deduce that:*

(a) an object placed between f and 2f of a concave mirror produces a real image beyond 2f.

(b) a convex mirror always produces a virtual image independent of the location of the object.

(c) the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.

(d) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.

*[***Note:** *This exercise helps you deduce algebraically properties of images that one obtains from explicit ray diagrams.]*

Sol. (*a*) The mirror formula is

$$\frac{1}{u}+\frac{1}{v} = \frac{1}{f}$$

$$\therefore \qquad \frac{1}{v} = \frac{1}{f}-\frac{1}{u}$$

Now for a concave mirror, $f < 0$ and for an object on the left $u < 0$.

$$2f < u < f$$

or, $$\frac{1}{2f} > \frac{1}{u} > \frac{1}{f}$$

or, $$-\frac{1}{2f} < -\frac{1}{u} < -\frac{1}{f}$$

or, $$\frac{1}{f}-\frac{1}{2f} < \frac{1}{f}-\frac{1}{u} < \frac{1}{f}-\frac{1}{f}$$

or, $$\frac{1}{2f} < \frac{1}{v} < 0$$

This implies that $v < 0$ so that real image is formed on left. Also the above inequality implies that

$$2f > v$$

or, $$|2f| > |v| \qquad [\because 2f \text{ and } v \text{ are } -ve]$$

i.e., real image is formed beyond $2f$.

(*b*) Now, for convex mirror, $f > 0$ and for an object of left, $u < 0$.

From mirror formula

$$\frac{1}{v} = \frac{1}{f}-\frac{1}{u}$$

$\Rightarrow \qquad \frac{1}{v} > 0$ or $v > 0$

This shows that whatever be the value of u, a convex mirror form a virtual image on the right.

(*c*) For convex mirror $f > 0$ and for an object on left $u < 0$, so from mirror formula

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} \qquad [\because \; v \text{ is +ive and } u \text{ is –ive}]$$

$$\Rightarrow \qquad \frac{1}{v} > \frac{1}{f} \text{ or } v < f \qquad (\therefore -\frac{1}{u} \text{ is } a \text{ '+ve' quantity})$$

This shows that the image is located between the pole and the focus of the mirror. Also from the mirror formula

$$\frac{1}{v} > -\frac{1}{u} \qquad \left(\because \frac{1}{f} > 0\right)$$

Multiply v to both side

$$\therefore \qquad \frac{v}{v} > -\frac{u}{v} \qquad [\because \; v \text{ is +ive}]$$

$$1 > m \qquad (\because u < 0)$$

$\therefore$ Magnitude of magnification, $m = \frac{v}{|u|} < 1$.

So the image is diminished in size.

(*d*) From the mirror formula, for a concave mirror, $f < 0$ and for an object located between the pole and focus of a concave mirror,

$$f < u < 0$$

$$\therefore \qquad \frac{1}{f} > \frac{1}{u}$$

$$\text{or,} \qquad \frac{1}{f} - \frac{1}{u} > 0$$

$$\text{or,} \qquad \frac{1}{v} > 0 \text{ or } v > 0 \qquad \left(\because \frac{1}{v} = \frac{1}{f} - \frac{1}{u}\right)$$

i.e., a virtual image is formed on the right.

$$\text{Also,} \qquad \frac{1}{v} < \frac{1}{|u|} \quad \text{or} \quad v > |u|$$

$$\therefore \qquad |m| = \frac{v}{|u|} > 1.$$

9.16. *A small pin fixed on a table top is viewed from above from a distance of 50 cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15 cm thick glass slab held parallel to the table? Refractive index of glass = 1.5. Does the answer depend on the location of the slab?*

Sol. Image of pin appear through glass slab

$$\mu = \frac{\text{real depth}}{\text{apparent depth}}$$

$$\text{apparent depth} = \frac{\text{real depth (thickness of glass slab)}}{\mu}$$

$$= \frac{15}{1.5} = 10 \text{ cm}$$

$\therefore$ Image left up by = 15 – 10 = 5 cm.

Location of slab will not affect the answer is any way.

9.17. *(a) Figure shows a cross-section of a 'light pipe' made of a glass fibre of refractive index 1.68. The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of the angles of the incident rays with the axis of the pipe for which total reflection inside the pipe take place, as shown in the figure.*

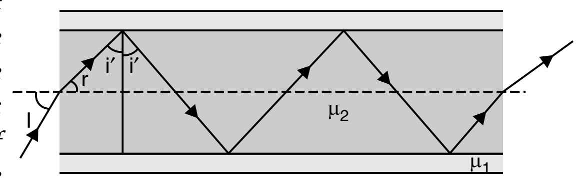


Fig. 9.34

(b) What is the answer if there is no outer covering of the pipe?

Sol. (*a*) $\mu_2 = 1.68$, $\mu_1 = 1.44$

$$\mu = \frac{\mu_2}{\mu_1} = \frac{i}{\sin i_c}$$

$\therefore$ Critical angle i_c' is given by

$$\sin i_c' = \frac{\mu_1}{\mu_2} = \frac{1.44}{1.68} = 0.8571$$

or, $$i_c' = 59°$$

Total internal reflection will occur if $i' > i_c'$ *i.e.,* if $i' > 59°$ or when $r > r_{max}$, where $r_{max} = 90° - 59° = 31°$. By Snell's law

$$\frac{\sin i_{max}}{\sin r_{max}} = 1.68$$

or, $$\sin i_{max} = 1.68 \times \sin r_{max}$$

$$= 1.68 \times \sin 31°$$

$$= 1.68 \times 0.5150 = 0.8662$$

$\therefore$ $$i_{max} = 60°.$$

So, all incident rays which make angles in the range $0 < i < 60°$ with the axis of the pipe will suffer total internal reflection in the pipe.

For the finite length of the pipe, the lower limit on i is determined by the ratio of the diameter to the length of the pipe.

(*b*) If there is no outer covering of the pipe

$$\mu_2 = 1.68, \mu_1 = 1$$

$$\sin i_c' = \frac{\mu_1}{\mu_2} = \frac{1}{1.68} = 0.5952$$

or, $$i_c' = 36.5°$$

Now for $i = 36.5°$, $r = 36.5°$ and $i' = 90 - 36.5° = 53.5°$ which is greater than i_c'. Thus all incident rays of angle in the range $0 < i < 90°$ will suffer total internal reflection.

9.18. *Answer the following questions:*

(a) *You have learnt that plane and convex mirrors produce virtual images of objects. Can they produce real images under some circumstances? Explain.*

(b) *A virtual image, we always say, cannot be caught on a screen. Yet when we 'see' a virtual image, we are obviously bringing it on to the 'screen' (i.e., the retina) of our eye. Is there a contradiction?*

(c) *A diver under water, looks obliquely at a fisherman standing on the bank of a lake. Would the fisherman look taller or shorter to the diver than what he actually is?*

(d) *Does the apparent depth of a tank of water change if viewed obliquely? If so, does the apparent depth increase or decrease?*

(e) *The refractive index of diamond is much greater than that of ordinary glass. Is this fact of some use to a diamond cutter?*

Sol. (*a*) Rays converging to a point 'behind' a plane or convex mirror are reflected to a point in front of the mirror on a screen. In other words, a plane or convex mirror can produce a real image if the object is virtual.

(*b*) When the reflected or refracted rays are divergent, the image is virtual. The divergent rays can be converged on to a screen with the help of a suitable converging lens. The convex lens of the eye performs this function precisely. In this case, the 'virtual image' serves as the 'virtual object' for the lens to produce a real image. It may be noted here that the screen is not located at the position of the virtual image. There is no contradiction.

(*c*) Taller. When the object is in rarer medium and the observer is in denser medium, then the "apparent depth" is greater than "real depth". In figure, *AB* represents a fisherman standing on the bank of the lake. The rays of light from the head (*A*) of the fisherman suffer refraction and bend towards the normals. The refracted rays appear to come from *A′*. The fisherman appears as *A′B* (>*AB*) *i.e.,* taller than what he actually is.

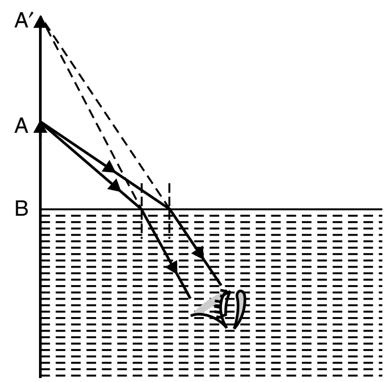


Fig. 9.35

(*d*) The apparent depth of oblique viewing decreases from its value for near-normal viewing.

(*e*) Refractive index of diamond is nearly 2.42. It is much larger than that of ordinary glass (nearly 1.5). The critical angle for diamond is nearly 24°. This is much less than that of glass. A skilled diamond cutter exploits the large range of angles of incidence (in the diamond), 24° to 90°, to ensure that light entering the diamond is totally reflected from many faces before getting out. This produces brilliance *i.e.,* sparkling effect in the diamond.

9.19. *The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 3 m away by means of a large convex lens. What is the maximum possible focal length of the lens required for the purpose?*

Sol. For a real image (on wall), minimum distance between the object and image should be 4*f*.

$$u = v = uf$$

$$\therefore \quad 4f = 3 \text{ m}$$

$$\therefore \quad f = \frac{3}{4} \text{ m} = 0.75 \text{ m.}$$

9.20. *A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. Determine the focal length of the lens.*

Sol. Here, distance between object and screen D = 90 cm

Distance between two locations of convex lens d = 20 cm

Since, $$f = \frac{D^2 - d^2}{4D}$$

$$\therefore \quad f = \frac{(90)^2 - (20)^2}{4 \times 90} = \frac{(90+20)(90-20)}{360} = \frac{110 \times 7}{36}$$

or, f = 21.4 cm.

9.21. *(a)* *Determine the 'effective focal length' of the combination of the two lenses having focal lenghs 30 cm and – 20 cm; if they are placed 8.0 cm apart with their principal axes coincident. Does the answer depend on which side of the combination a beam of parallel light is incident? Is the notion of effective focal length of this system useful at all?*

(b) *An object 1.5 cm in size is placed on the side of the convex lens in the arrangement (a) above. The distance between the object and the convex lens is 40 cm. Determine the magnification produced by the two-lens system, and the size of the image.*

Sol. (*a*) (*i*) Here, f_1 = 30 cm, f_2 = –20 cm, d = 8.0 cm

Let a parallel beam be incident on the convex lens first. If second lens were absent, then

$\therefore \quad u_1 = \infty$ and f_1 = 30 cm

As $$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$\therefore \quad \frac{1}{v_1} - \frac{1}{\infty} = \frac{1}{30}$$

or, v_1 = 30 cm

This image would now act as a virtual object for second lens.

$\therefore \quad u_2$ = + (30 – 8) = + 22 cm

f_2 = – 20 cm

Since, $$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} \quad \therefore \quad \frac{1}{v_2} = \frac{1}{-20} + \frac{1}{22}$$

$$= \frac{-11+10}{220} = \frac{-1}{220}$$

v_2 = – 220 cm.

$\therefore$ Parallel incident beam would appear to diverge from a point 220 – 4 = 216 cm from the centre of the two lens system.

(*ii*) Assume that a parallel beam of light from the left is incident first on the concave lens.

$\therefore \quad u_1 = -\infty$, f_1 = –20 cm

As $$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$\therefore \quad \frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{-20} + \frac{1}{-\infty} = -\frac{1}{20}$$

v_1 = – 20 cm

This image acts as a real object for the second lens

$$u_2 = -(20 + 8) = -28 \text{ cm}, f_2 = 30 \text{ cm}$$

Since, $$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$$\therefore \quad \frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = \frac{1}{30} - \frac{1}{28} = \frac{14-15}{420}$$

$$v_2 = -420 \text{ cm}$$

∴ The parallel beam appears to diverge from a point 420 – 4 = 416 cm, on the left of the centre of the two lens system.

We finally conclude that the answer depends on the side of the lens system where the parallel beam is incident. Therefore, the notion of effective focal length does not seem to be meaningful here.

(*b*) For convex lens

$u = -40$ cm, $f = 30$ cm, $O = 1.5$ cm

Using lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

We get, $$\frac{1}{v} - \frac{1}{-40} = \frac{1}{30}$$

or, $$\frac{1}{v} = \frac{1}{30} - \frac{1}{40} = \frac{1}{120}$$

$$v = 120 \text{ cm (for real object)}$$

From relation,

$$m = -\frac{v}{u}, \text{ we get}$$

$$m = -\frac{120}{-40} = +3$$

The image formed by the convex lens becomes object for concave lens at a distance of 120 – 8 = 112 cm on the other side.

For concave lens, $f = -20$ cm, $u = +112$ cm (on the other side)

$$v = ?$$

Using lens formula, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Now, $$\frac{1}{v} - \frac{1}{112} = \frac{1}{-20}$$

or, $$\frac{1}{v} = -\frac{1}{20} + \frac{1}{112} = -\frac{23}{560}$$

$$v = -\frac{560}{23} \text{ cm (for virtual object)}$$

From relation, $m = \frac{v}{u}$, we get

$$m = -\frac{560/23}{-112} = -\frac{560}{23}\times\frac{1}{112} = -\frac{5}{23}$$

$$\text{Net magnification} = 3\times\left(\frac{-5}{23}\right) = -\frac{15}{23} = 0.652 \text{ (negative due to virtual image)}$$

As $$m = \frac{I}{0}$$

$$I = m \times 0 = 0.652 \times 1.5 = 0.98 \text{ cm (size of final image).}$$

9.22. *At what angle should a ray of light be incident on the face of a prism of refracting angle 60° so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524.*

Sol. The refracted ray in the prism is incident on the second face at critical angle i_c.

Now, $60° + 90° - r + 90° - i_c = 180°$

$$\boxed{r_1 + r_2 = \angle A}$$

or, $$r = 60° - i_c$$

Now, $$\sin i_c = \frac{1}{\mu} = \frac{1}{1.524}$$

or, $$i_c = \sin^{-1}\left(\frac{1}{1.524}\right)$$

or, $$i_c \simeq 41°$$

$\therefore$ $$r = 60° - 41° = 19°$$

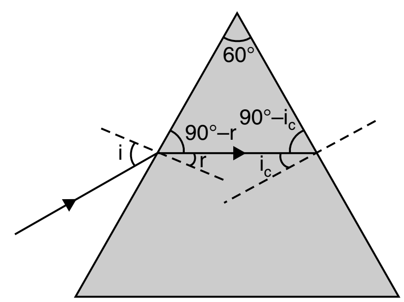


Fig. 9.36

Using Snell's law,

$$\sin i = \sin 19° \times 1.524 = 0.4962$$

$$i = \sin^{-1}(0.4962) = 29.75°.$$

9.23. *You are given prisms made of crown glass and flint glass with a wide variety of angles. Suggest a combination of prisms which will*

(i) deviate a pencil of white light without much dispersion.

(ii) disperse (and displace) a pencil of white light without much deviation.

Sol. (*i*) For no dispersion, angular dispersion produced by two prisms should be zero.

Angular dispersion by crown glass prism + angular dispersion by flint glass prism = 0

i.e., $(\mu_b - \mu_r)A + (\mu_b' - \mu_r')A' = 0$.

Since $(\mu_b' - \mu_r')$ for flint glass is more than that for crown glass, therefore, $A' < A$ *i.e.,* flint glass prism of smaller angle has to be suitably combined with crown glass prism of larger angle.

(*ii*) For almost no deviation, $(\mu_y - 1)A' + (\mu_y' - 1)A' = 0$

Taking crown glass prism of certain angle, we go on increasing angle of flint glass prism till this condition is met. In the final combination however, angle of flint glass prism will be smaller than the angle of crown glass prism as μ_y' for flint glass is more than μ_y for crown glass.

9.24. *For a normal eye, the far point is at infinity and the near point of distinct vision is about 25 cm in front of the eye. The cornea of the eye provides a converging power of about 40 dioptres, and the least converging power of the eye-lens behind the cornea is about 20 dioptres. From this rough data estimate the range of accommodation (i.e., the range of converging power of the eye-lens) of a normal eye.*

Sol. To see objects at infinity, the eye uses its least converging power = 40 + 20 = 60 dioptres.

This gives the rough idea of the distance between the retina and cornea eye-lens.

To focus at the near point (u = –25 cm)

$$v = \text{focal length of eye-lens} = \frac{100}{P} = \frac{10}{6} = \frac{5}{3} \text{ cm}$$

$$\therefore \quad v = -\frac{5}{3} \text{ cm} \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= \frac{3}{5} + \frac{1}{25} = \frac{16}{25}$$

$f = \frac{25}{16}$, corresponding to a converging power,

$$P = \frac{100}{\frac{25}{16}} = 64 \text{ dioptre.}$$

The power of the eye-lens = 64 – 40 = 24 dioptre.

The range of accommodation of the eye-lens is roughly 20 to 24 dioptre.

9.25. *Does short-sightedness (myopia) or longsightedness (hypermetropia) imply necessarily that the eye has partially lost its ability of accommodation? If not, what might cause these defects of vision?*

Sol. No, a person may have normal ability of accommodation and yet he may be myopic or hypermetropic.

In fact, myopia arises when length of eye ball (from front to back) gets elongated and hypermetropic arises when length of eye ball gets shortened.

However, when eye ball has normal length, but the eye-lens losses partially its power of accommodation, the defect is called presbiopia.

9.26. *A myopic person has been using spectacles of power – 1.0 dioptre for distant vision. During old age, he also needs to use separate reading glass of power +2.0 dioptres. Explain what may have happened.*

Sol. As the person is using spectacles of power –1.0 dioptre (*i.e.*, focal length – 100 cm), the far point of the person is at 100 cm. Near point of the eye might have been normal (*i.e.*, 25 cm).

The objects at infinity produce virtual images at 100 cm (using spectacles). To see objects between 25 cm to 100 cm, the person uses the ability of accommodation of his eye-lens. This ability is partially lost in old age. The near point of the eye may recede to 50 cm. He has, therefore, to use glasses of suitable power for reading.

Here, u = –25 cm, v = –50 cm

Since, $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = -\frac{1}{50} + \frac{1}{25}$$

or, $$f = 50 \text{ cm}$$

As $$P = \frac{100}{f} = \frac{100}{50} = +\ 2 \text{ dioptre.}$$

9.27. *A person looking at a person wearing a shirt with a pattern comprising vertical and horizontal lines is able to see the vertical lines more distinctly than the horizontal ones. What is this defect due to? How is such a defect of vision corrected?*

Sol. This defect is called Astigmatism. It arises because curvature of the cornea plus eye-lens refracting system is not the same in different planes. As vertical lines are seen distinctly, the curvature in the vertical plane is enough, but in the horizontal plane, curvature is insufficient.

This defect is removed by using a cylindrical lens with its axis along the vertical.

9.28. *A man with normal near point (25 cm) reads a book with small print using a magnifying glass: a thin convex lens of focal length 5 cm.*

(a) What are the closest and farthest distance at which he should keep the lens from the page so that he can read the book, when viewing through the magnifying glass?

(b) What is the maximum and the minimum angular magnification (magnifying power) possible using the above simple microscope?

Sol. (*a*) For the closest distance

$$v = -25 \text{ cm}, f = 5 \text{ cm}$$

As $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

∴ $$\frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

$$= \frac{1}{-25} - \frac{1}{5} = \frac{-1-5}{25} = \frac{-6}{25}$$

$$u = -\frac{25}{6} = -4.2 \text{ cm}$$

This is the closest distance at which the man can read the book.

For the farthest image

$$v = \infty, f = 5 \text{ cm}$$

Since, $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

or, $$\frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

$$= \frac{1}{\infty} - \frac{1}{5} = -\frac{1}{5}$$

or, $$u = -5 \text{ cm}$$

This is the farthest distance at which the man can read the book.

(*b*) Maximum angular magnification is

$$\frac{D}{U_{\min}} = \frac{25}{25/6} = 6$$

Minimum angular magnification is

$$\frac{D}{U_{\max}} = \frac{25}{5} = 5.$$

9.29. *A card sheet divided into squares each of size 1 mm*^{2} *is being viewed at a distance of 9 cm through a magnifying glass (a converging lens of focal length 10 cm) held close to the eye.*

(a) What is the magnification produced by the lens? How much is the area of each square in the virtual image?

(b) What is the angular magnification (magnifying power) of the lens?

(c) Is the magnification in (a) equal to the magnifying power in (b)? Explain.

Sol. (*a*) $u = -9$ cm, $f = 10$ cm

Now, $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

or, $$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

or, $$\frac{1}{v} = \frac{1}{10} + \frac{1}{-9} = \frac{-9+10}{-90} = -\frac{1}{90}$$

or, $$v = -90 \text{ cm}$$

Magnification $= \dfrac{v}{u} = \dfrac{-90}{-9} = 10$

As magnification is 10 therefore, the area of the each square in the virtual image is $(10 \times 10 \times 1)$ mm^2 *i.e.*, 100 mm^2 = 1 cm^2

(*b*) Angular magnification $= \dfrac{D}{u} = \dfrac{-25}{-9} = 2.78$

(*c*) No. Magnification of an image by a lens and angular magnification (or magnifying power) of an optical instrument are two separate things. The latter is the ratio of the angular size of the object (which is equal to the angular size of the image even if the image is magnified) to the angular size of the object if placed at the near point (25 cm). Thus magnification magnitude is $\left|\dfrac{v}{u}\right|$ and magnifying power is $\dfrac{25}{|u|}$.

Only when the image is located at the near point ($|v|$ = 25 cm), are the two quantities equal.

9.30. (*a*) *At what distance should the lens be held from the figure in Question 9.29 in order to view the squares distinctly with the maximum possible magnifying power?*

(*b*) *What is the magnification in this case?*

(*c*) *Is the magnification equal to the magnifying power in this case? Explain.*

Sol. (*a*) Maximum magnifying power is obtained when the image is at the near point (25 cm). Thus,

$$v = -25 \text{ cm}, f = +10 \text{ cm}, u = ?$$

As, $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$\therefore$ $$\frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

$$= \frac{1}{-25} - \frac{1}{10} = \frac{-2-5}{50} = \frac{-7}{50}$$

or, $$u = -\frac{50}{7} = -7.14 \text{ cm}$$

So the lens should be held 7.14 cm away from the figure.

(*b*) Magnitude of magnification is

$$m = \frac{v}{u} = \frac{25}{50/7} = 3.5$$

(*c*) Magnifying power, $M = \frac{D}{u} = \frac{25}{50/7} = 3.5$

Yes, the magnifying power is equal to the magnitude of magnification because image is formed at the least distance of distinct vision.

9.31. *What should be the distance between the object (in Question 9.30) and the magnifying glass if the virtual image of each square in the figure is to have an area of 6.25 mm^2. Would you be able to see the squares distinctly with your eyes very close to the magnifier?*

Note: Questions 9.29 to 9.31 will help you clearly understand the difference between magnification in absolute size and the angular magnification (or magnifying power of an instrument).

Sol. Here, magnification in area = 6.25

$\therefore$ Linear magnification

$$m = \sqrt{6.25} = 2.5$$

As $m = \frac{v}{u}$ or $v = mu = 2.5$ u

Using formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\therefore \quad \frac{1}{2.5\,u} - \frac{1}{u} = \frac{1}{10}$$

$$\frac{1-2.5}{2.5\,u} = \frac{1}{10}$$

$$2.5\,u = -15$$

or, $u = -6$ cm

$\therefore v = 2.5\,u = 2.5\,(-6) = -15$ cm

As the virtual image is at 15 cm; whereas distance of distinct vision is 25 cm, therefore, the image cannot be seen distinctly by the eye.

9.32. *Answer the following questions:*

(*a*) *The angle subtended at the eye by an object is equal to the angle subtended at the eye by the virtual image produced by a magnifying glass. In what sense then does a magnifying glass provide angular magnification?*

(*b*) *In viewing through a magnifying glass, one usually positions one's eyes very close to the lens. Does angular magnification change if the eye is moved back?*

(*c*) *Magnifying power of a simple microscope is inversely proportional to the focal length of the lens. What then stops us from using a convex lens of smaller and smaller focal length and achieving greater and greater magnifying power?*

(*d*) *Why must both the objective and eyepiece of a compound microscope have short focal lengths?*

(*e*) *When viewing through a compound microscope, our eyes should be positioned not on the eyepiece but a short distance away from it for best viewing. Why? How much should be that short distance between the eye and eyepiece?*

Sol. (*a*) Even though the absolute image size is bigger than object size, the angular size of the image is equal to the angular size of the object. The magnifier helps in the following way: without it the object would be placed no closer than 25 cm; with it, the object can be placed much closer. The closer object has larger angular size than the same object at 25 cm. It is in this sense that angular magnification is achieved.

(*b*) Yes, the angular magnification decreases a little because the angle subtended at the eye by the image is then slightly less than the angle subtended by the image at the lens. The angle subtended by the object at the eye is also less than that subtended by the object at the lens. However, this decrease is very small as compared to the case of image.

Also, when the eye is separated from the lens, the angles subtended at the eye by the object and its image are not equal.

It may further be noted that if the image is a very large distance away, then the effect, on magnification, of moving the eye shall be negligible.

(*c*) First, grinding lenses of very small focal lengths is not easy. More important, if you decrease focal length, aberrations (both spherical and chromatic) become more pronounced. So, in practice, you can't get a magnifying power of more than 3 or so with a simple convex lens. However, using an aberration-corrected lens system, one can increase this limit by a factor of 10 or so.

(*d*) Angular magnification of eyepiece is $\frac{25}{f_e}+1$ (f_e in cm) which increase if f_e is smaller.

Further, magnification of the objective is given by

$$\frac{v_0}{|u_0|} = \frac{1}{\frac{|u_0|}{f}-1}$$

which is large when $|u_0|$ is slightly greater than f_0. Now the microscope is used for viewing very close objects. So $|u_0|$ is small, and so is f_0.

(*e*) The image of the objective lens in the eyepiece is known as the 'eye-ring'. All the rays from the object refracted by the objective go through the eye-ring. Therefore, it is an ideal position for our eyes for viewing. If we place our eyes too close to the eyepiece, we shall not collect much of the light and also reduce our field of view. If we position our eyes on the eye-ring and the area of the pupil of our eye is greater or equal to the area of the eye-ring, our eyes will collect all the light refracted by the objective. The precise location of the eye-ring naturally depends on the separation between the objective and the eyepiece and the focal length of the eyepiece. When we view through a microscope by placing our eyes on one end, the ideal distance between the eye and the eyepiece is usually built in the design of the instrument.

9.33. *An angular magnification (magnifying power) of 30X is desired using an objective of focal length 1.25 cm and an eyepiece of focal length 5 cm. How will you set up the compound microscope?*

Sol. In normal adjustment, image is formed at least distance of distinct vision,

$$d = 25 \text{ cm}$$

Angular magnification of eyepiece $= \left(1+\frac{D}{f_e}\right)$

$$= \left(1+\frac{25}{5}\right) = 6$$

Since the total magnification is 30, magnification of objective lens,

$$m = \frac{30}{6} = 5$$

$\therefore$ $$m = -\frac{v_0}{u_0} = 5 \text{ or, } v_0 = -5u_0$$

As $\quad \dfrac{1}{v_0} - \dfrac{1}{u_0} = \dfrac{1}{f_0}$

$\therefore \quad \dfrac{1}{-5u_0} - \dfrac{1}{u_0} = \dfrac{1}{1.25}$

$$-\frac{6}{5u_0} = \frac{1}{1.25}$$

$$u_0 = -\frac{6\times1.25}{5} = -1.5 \text{ cm}$$

i.e., object should be held at 1.5 cm in front of objective lens.

As $\quad v_0 = -5\ u_0$

$\therefore \quad v_0 = -5(-1.5) = 7.5$ cm

From $\quad \dfrac{1}{v_e} - \dfrac{1}{u_e} = \dfrac{1}{f_e}$

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-25} - \frac{1}{5} = -\frac{6}{25}$$

$$u_e = -\frac{25}{6} = -\ 4.17 \text{ cm}$$

$\therefore$ Separation between the objective lens and eyepiece $= |u_e| + |v_0|$
$= 4.17 + 7.5 = 11.67$ cm.

9.34. *A small telescope has an objective lens of focal length 140 cm and an eyepiece of focal length 5.0 cm. What is the magnifying power of the telescope for viewing distant objects when*

(a) the telescope is in normal adjustment (i.e., when the final image is at infinity)?

(b) the final image is formed at the least distance of distinct vision (25 cm)?

Sol. Given, $f_0 = 140$ cm and $f_e = 5$ cm

(a) Magnifying power $= \dfrac{f_0}{f_e} = -\dfrac{140}{5}$ [For normal or final image at infinity]

$= -\ 28$

(*b*) Magnifying power $= -\dfrac{f_0}{f_e}\left(1+\dfrac{f_e}{D}\right)$ [For final image at least distinct of vision]

$$= -\frac{140}{5}\left(1+\frac{5}{25}\right) = -\ 33.6.$$

9.35. *(a) For the telescope describe in Question 9.34 (a), what is the separation between the objective lens and the eyepiece?*

(b) If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens?

(c) What is the height of the final image of the tower if it is formed at 25 cm?

Sol. (*a*) Separation between objective and eye-lens

$$\boxed{L = f_o + f_e}$$

$= 140 + 5 = 145$ cm.

(*b*) Angle subtended by 100 m tall tower at 3 km,

$$\theta = \frac{100}{3\times1000} = \frac{1}{30} \text{ radian}$$

If y be the height of the image formed by the objective,

then, $$\theta = \frac{y}{140}$$

Now, $$\frac{1}{30} = \frac{y}{140} \text{ or } y = 4.7 \text{ cm.}$$

(*c*) Magnification produced by eyepiece

$$= 1+\frac{D}{f_e} = 1+\frac{25}{5} = 6$$

Height of final image = 4.7 × 6 = 28.2 cm.

9.36. *A Cassegrain telescope uses two mirrors as shown in figure below. Such a telescope is built with the mirrors 20 mm apart. If the radius of curvature of the large mirror is 220 mm and the small mirror is 140 mm, where will the final image of an object at infinity be?*

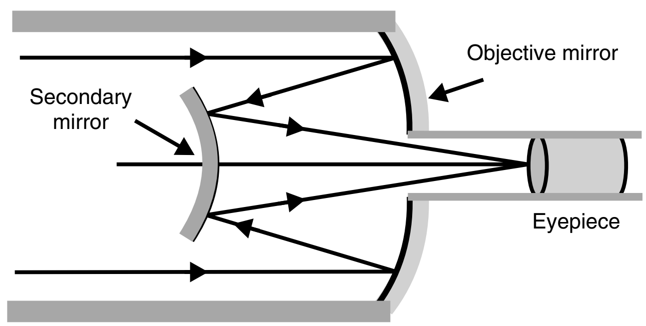


Fig. 9.37 *Schematic diagram of a reflecting telescope (Cassegrain)*

Sol. Here, radius of curvature of objective mirror

$$R_1 = 220 \text{ mm}$$

radius of curvature of secondary mirror

$$R_2 = 140 \text{ mm}; f_2 = \frac{R_2}{2} = \frac{140}{2} = 70 \text{ mm}$$

Distance between the two mirror, d = 20 mm

When object is at infinity, parallel rays falling on objective mirror, on reflection, would collect at its focus at $f_1 = \frac{R_1}{2} = \frac{220}{2} = 110$ mm.

Instead, they fall on secondary mirror at 20 mm from objective mirror.

∴ For secondary mirror, $u = f_1 - d = 110 - 20 = 90$ mm

Using formula, $$\frac{1}{v}+\frac{1}{u} = \frac{1}{f_2}$$

$$\frac{1}{v} = \frac{1}{f_2}-\frac{1}{u} = \frac{1}{70}-\frac{1}{90} = \frac{9-7}{630} = \frac{2}{630}$$

$$v = \frac{630}{2}$$ = 315 mm = 31.5 cm to the right of secondary mirror.

9.37. *Light incident normally on a plane mirror attached to a galvanometer coil retraces backwards as shown in figure A current in the coil produces a deflection of 3.5° of the mirror. What is the displacement of the reflected spot of light on a screen placed 1.5 m away?*

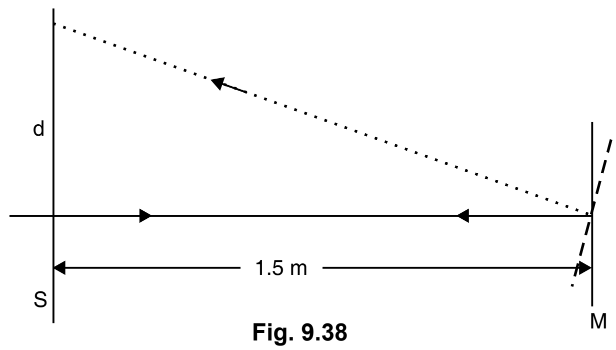


Fig. 9.38

Sol. We know that if the mirror is turned by an angle θ, the reflected ray turns by an angle 2θ. The current in the coil has produced a deflection of 3.5° in the mirror, therefore, the reflected ray will be deflected by an angle 2 × 3.5° = 7°.

From the figure, we have

$$\frac{d}{1.5} = \tan 7° \qquad \left[\tan\theta = \frac{\text{perpendicular}}{\text{base}}\right]$$

or, $$d = 1.5 \tan 7°$$
$$= 1.5 \times 0.123 = 0.184 \text{ m} = 18.4 \text{ cm}.$$

9.38. *The following figure shows an equiconvex lens (of refractive index 1.50) in contact with a liquid layer on top of a plane mirror. A small needle with its tip on the principal axis is moved along the axis until its inverted image is found at the position of the needle. The distance of the needle from the lens is measured to be 45.0 cm. The liquid is removed and the experiment is repeated. The new distance is measured to be 30.0 cm. What is the refractive index of the liquid?*

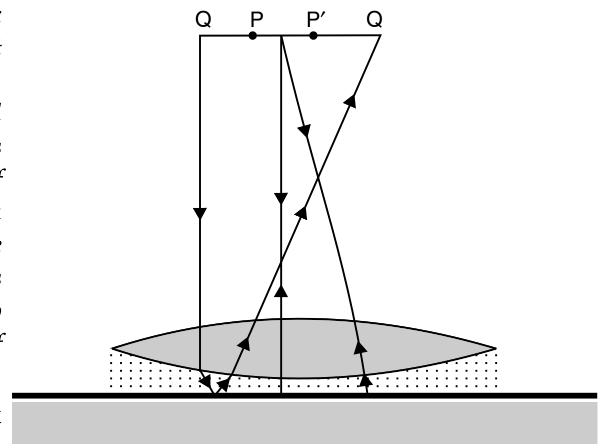


Fig. 9.39

Sol. Consider that the focal length of convex lens of glass = f_1 = 30 cm

and focal length of plane concave lens of liquid = f_2 combined focal length, F = 45.0 cm

As $$\frac{1}{f_1}+\frac{1}{f_2} = \frac{1}{F}$$

∴ $$\frac{1}{f_2} = \frac{1}{F}-\frac{1}{f_1} = \frac{1}{45}-\frac{1}{30} = -\frac{1}{90}$$

$$f_2 = -90 \text{ cm}$$

For glass lens, let $R_1 = R$, $R_2 = -R$

Since, $$\frac{1}{f_1} = (\mu - 1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

∴ $$\frac{1}{30} = \left(\frac{3}{2}-1\right)\left(\frac{1}{R}+\frac{1}{R}\right)$$

$$= \frac{1}{2} \times \frac{2}{R} = \frac{1}{R}$$

Thus, $R = 30$ cm

For liquid lens,

$$R_1 = -R = -30.0 \text{ cm}$$

$$R_2 = \infty$$

$$\frac{1}{f_2} = (\mu_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (\mu_l - 1)\left(\frac{1}{-30} - \frac{1}{\infty}\right)$$

or, $$\frac{1}{-90} = (\mu_l - 1) \times \frac{1}{-30}$$

or, $$(\mu_l - 1) = \frac{30}{90} = \frac{1}{3}$$

$$\mu_l = 1 + \frac{1}{3} = \frac{4}{3}.$$

9.39. *A convex lens of focal length 20 cm, has a point object placed on its principal axis at a distance of 40 cm from it.A plane mirror is placed 30 cm behind the convex lens. Locate the position of image formed by this combination.*

Sol.

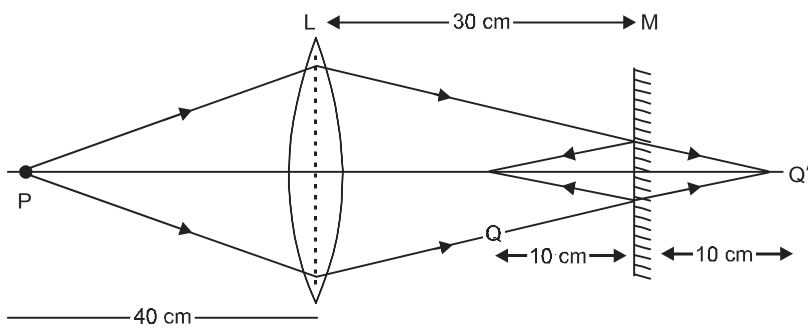


Fig. 9.40

Let us consider for lens only

$$u = -40 \text{ cm}$$

and $f = +20$ cm

∴ From lens formula, we get

$$\frac{1}{v} - \frac{1}{(-40)} = \frac{1}{20}$$

∴ $$v = +40 \text{ cm}$$

If the mirror was not there the image would have been at Q' i.e. 40 cm away from the lens or 40 – 30 = 10 cm from the mirror. This image will become the vertical object for the mirror which gives the real image at Q at the same distance i.e. 10 cm.

9.40. *A convex lens and a convex mirror of radius of curvature 20 cm are placed co-axially with the convex mirror placed at a distance of 30 cm from the lens. For a point object, at a distance of*

20 cm from the lens, the final image due to this combination concides with the object itself. What is the focal length of the convex lens?

Sol.

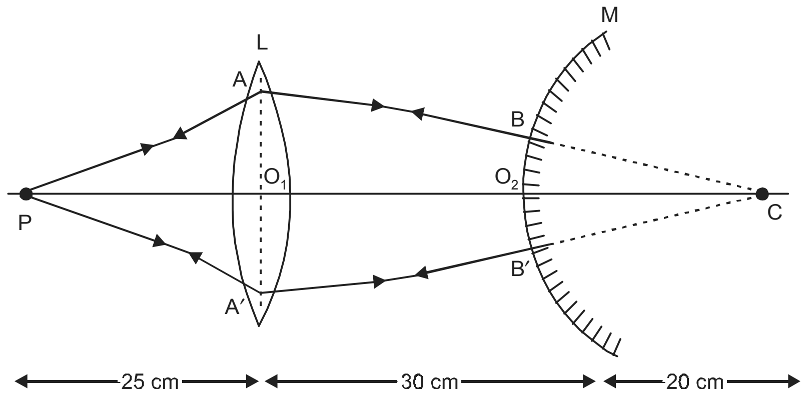


Fig. 9.41

Here O_2C = 20 cm (the radius of curvature of the mirror)

u = –25 cm

v = +(30 + 20) = +50 cm

If f be the focal length of the lens, then

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= \frac{1}{+50} - \frac{1}{(-25)} = \frac{1}{50} + \frac{1}{25}$$

$$\frac{1}{f} = \frac{3}{50}$$

$$\therefore \quad f = \frac{50}{3} = 16.67 \text{ cm}$$

9.41. *A convex lens of focal length 20 cm, is placed co-axially with a convex mirror of radius of convature 20 cm. The two are kept 15 cm apart from each other. A point object is placed 60 cm in front of the convex lens. Find the position of the image formed by this combinations.*

Sol.

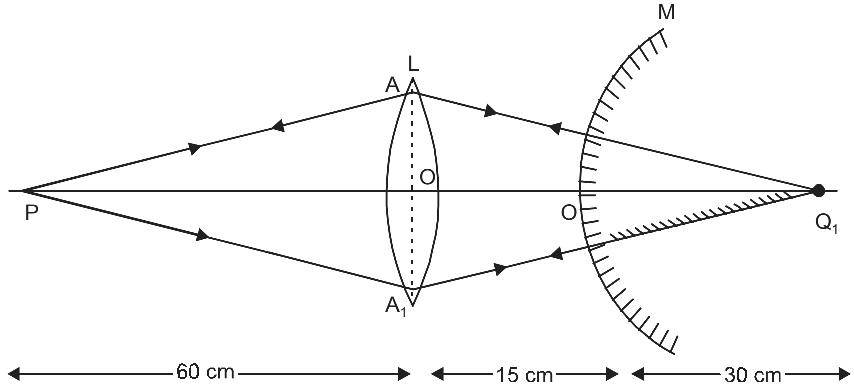


Fig. 9.42

For the convex lens, we have

$$u_1 = -60 \text{ cm} \quad \text{and} \quad f = +20 \text{ cm}$$

∴ Using lens formula, we get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{+20} = \frac{1}{v_1} - \frac{1}{-60}$$

∴ $$\frac{1}{v_1} = \frac{1}{20} - \frac{1}{60}$$

∴ $$v_1 = 30 \text{ cm}$$

Had there been only the lens L, the image of P would have been formed at Q_1 which acts as a virtual object for the convex mirror.

∴ $$OQ_1 = (30 - 15) = 15 \text{ cm}$$

Here for the convex mirror

$$u_2 = +15 \text{ cm and } R = +20 \text{ cm}$$

∴ From mirror formula, we get

$$\frac{1}{v_2} + \frac{1}{u_2} = \frac{2}{R}$$

$$\frac{1}{v_2} + \frac{1}{15} = \frac{2}{20}$$

$$\frac{1}{v_2} = \frac{1}{10} - \frac{1}{15}$$

∴ $$v_2 = +30 \text{ cm}$$

9.42. *A convex lens of focal length 20 cm and a concave mirror of focal length 10 cm, are placed co-axially 50 cm apart from each other. An incident beam parallel to its principal axis, is incident on the convex lens. Locate the position of the final image formed due to this combination.*

Sol.

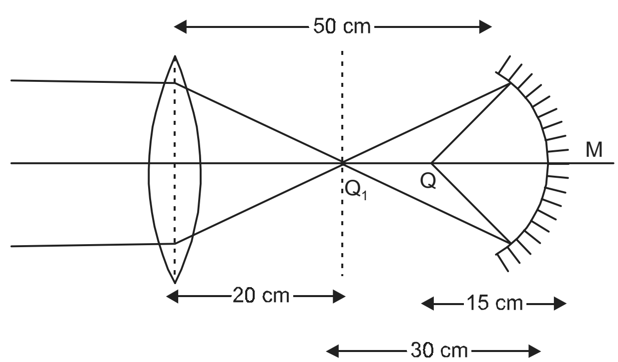


Fig. 9.43

Here $$u = -30 \text{ cm} \quad \text{and} \quad f = 10 \text{ cm}$$

Hence using mirror formula

$$\frac{1}{v} + \frac{1}{-30} = \frac{1}{-10}$$

$$\frac{1}{v} = \frac{1}{30} - \frac{1}{10}$$

$$v = -15$$

Therefore lens-mirror combination forms a real image Q at a distance of 15 cm from the mirror.

9.43. *A point object is placed 60 cm in front of a convex lens of focal length 30 cm. A plane mirror is placed 10 cm behind the convex lens. Where is the image formed by this system?*

Sol.

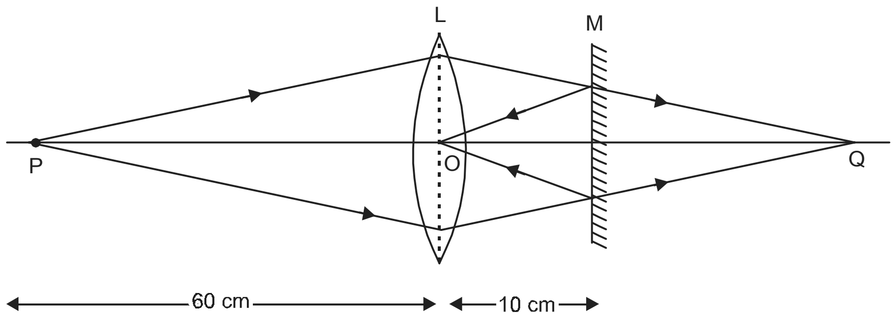


Fig. 9.44

Here $u = -60$ cm

$f = 30$ cm

∴ From lens formula

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{30} = \frac{1}{v} - \frac{1}{-60}$$

$$\frac{1}{v} = \frac{1}{30} - \frac{1}{60}$$

$$\frac{1}{v} = \frac{1}{60}$$

∴ $v = 60$ cm

The real image of the object will be formed at a distance of 60 cm i.e. on radius of convature which has become the virtual object for plane mirror. So the real image will form at the optical centre.

9.44. *A convex lens of focal length 15 cm, and a concave mirror of radius of convature 20 cm are placed coaxially 10 cm apart. An object is placed in front of the convex lens so that there is no parallex between the object and its image formed by the combination. Find the position of the object.*

Sol.

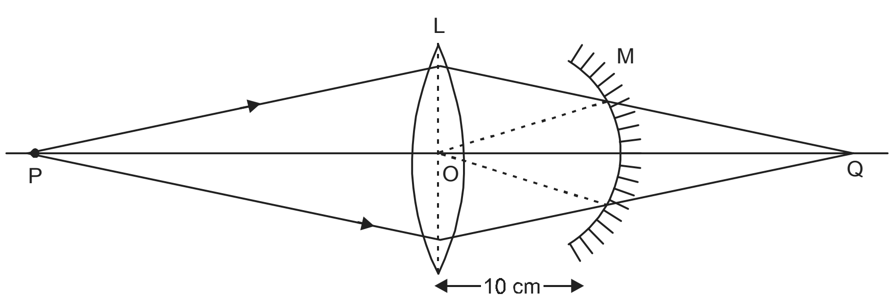


Fig. 9.45

Here $u = ?$

as there is no parallex between the object and the image

$\therefore \quad v = u$

$\therefore$ From Convex lens formula, we get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{+15} = \frac{1}{30} - \frac{1}{u}$$

$$\frac{1}{u} = \frac{1}{30} - \frac{1}{15}$$

$$\frac{1}{u} = -\frac{1}{30}$$

$\therefore \quad u = -30$ cm

Now the image will form at Q which is the virtual object for concave mirror. The final image will form at the optical centre. Hence the distance of the object is 30 cm from the mirror.

MORE QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *Two thin lenses of power +4D and –2D are in contact. What is the focal length of the combination?*

Ans. $\boxed{P = P_1 + P_2} = 4 - 2 = +2D$

Since focal length $\boxed{f = \frac{1}{P}}$

$\therefore \quad f = \frac{1}{2} = 0.5$ m = 50 cm.

Q. 2. *Here three lenses have been given. Which two lenses will you use as an eyepiece and as an objective to construct an astronomical telescope?*

Lenses	*Power (P)*	*Aperture (A)*
L_1	*3D*	*8 cm*
L_2	*6D*	*1 cm*
L_3	*10D*	*1 cm*

Ans. Since the aperture of lens L_1 is largest, it is used as objective for a telescope. The lens L_3 is used as eyepiece since its focal length is smaller.

Q. 3. *What are the laws of reflection?*

Ans. Laws of reflection:

(*i*) The incident rays, the reflected ray and the normal lie in the same plane.

(*ii*) The angle of incident (*i*) is equal to the angle of reflection (*r*), *i.e.*, $\angle i = \angle r$.

Q. 4. *What is presbyopia? How can it be corrected?*

Ans. Refer to Page 490.

Q. 5. *A glass lens of refractive index 1.5 is placed in a trough of liquid. What must be the refractive index of the liquid in order to mark the lens disappear?*

Ans. The refractive index of liquid must be greater than or equal to 1.5.

Q. 6. *A converging lens of refractive index 1.5 is kept in a liquid medium having same refractive index. What would be the focal length of the lens in this medium?*

Ans. The lens in the liquid will act like a plane sheet of glass.

∴ Its focal length will be infinite (∞)

By using the formula of lens maker

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or, $$\boxed{\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

as, $$\mu_1 = \mu_2$$

∴ $$\frac{1}{f} = 0$$

Thus, $$f = \infty.$$

Q. 7. *How does the power of a convex lens vary, if the incident red light replaced by violet light?*

Ans. By the lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

∴ Power of lens $$\boxed{P = \frac{1}{f}} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

∵ $\mu_{violet} > \mu_{red}$

∴ Power of the lens will be increased.

Q. 8. *How does the angle of minimum deviation of a glass prism vary, if the incident violet light is replaced with red light?*

Ans. ∵ $\delta_{violet} > \delta_{red}$

When the incident violet light is replaced with red light, the angle of minimum deviation of a glass decreases.

Q. 9. *How does the angle of minimum deviation of a glass prism of refractive index 1.5 change, if it is immersed in a liquid of refractive index 1.3?*

Ans. Given, ${}^a\mu_g = 1.5$ and ${}^a\mu_w = 1.3$

As, $$\boxed{\delta = (\mu - 1)A}$$

For deviation in air, $$\mu = \frac{\mu_g}{\mu_a} = \frac{1.5}{1} = 1.50$$

∴ $$\delta = (1.5 - 1) \times 60° = 30°$$

For deviation in water, $\mu = \frac{\mu_g}{\mu_w} = \frac{1.5}{1.3} = 1.15$

$\therefore \qquad \delta = (1.15 - 1) \times 60° = 9°$

Therefore, the angle of deviation is decreased.

Q. 10. *Write the necessary conditions, for the phenomenon of total internal reflection to take place.*

Ans. Necessary conditions for the total internal reflection are:

(*i*) Light must travel from denser medium to rarer medium.

(*ii*) The angle of incidence (in the denser medium) must be greater than the critical angle i_c where

$$\sin i_c = \frac{1}{\mu}.$$

Q. 11. *An object is placed at the focus of concave lens. Where will its image be formed?*

Ans. Image is formed at infinity.

Q. 12. *A lens of glass is immersed in water. What will be its effect on the power of the lens?*

Ans. Power increases.

Q. 13. *Why is convex mirror used as driver's mirror?*

Ans. Since the field of view of a convex mirror is large thus it is used as driver's mirror and image formed virtual, errect and near to mirror with in focal length of mirror.

Q. 14. *A ray of light falls on a mirror normally. What are the values of angle of incidence and the angle of reflection.*

Ans. Both angle of incidence and the angle of reflection is zero.

Q. 15. *What is the number of images of an object held between two parallel plane mirrors?*

Ans. Given θ = 0°

$$\therefore \text{ Number of images, } n = \frac{360°}{\theta} - 1$$

$$= \frac{360°}{0°} - 1 = \text{infinite.}$$

Q. 16. *What is the length of a telescope in a normal adjustment?*

Ans. $L = f_0 + f_e$.

Q. 17. *If a telescope is inverted, will it be able to work as a microscope?*

Ans. No.

Q. 18. *What is the magnification produced by a single convex lens used as a simple microscope in normal use?*

Ans. $M = 1 + \frac{D}{f}$.

Q. 19. *Which of the two main parts of an optical fibre has a higher value of refractive index?*

Ans. The refractive index of the material of optical fibre is more than the refractive index of the coating material.

Q. 20. *A lens when immersed in a transparent liquid becomes invisible. Under what condition does it happens?*

Ans. When refractive index of liquid is equal to refractive index of glass.

Q. 21. *What is the physical principle on which the working of optical fibre is based?*

Ans. Total internal reflection.

Q. 22. *Write the relation for the refractive index μ of the prism in terms of the angle of minimum deviation δm and the angle A of prism.*

Ans. $\mu = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin A/2}$.

Q. 23. *Refractive indices of glass for blue, red and yellow colours are μ_b, μ_y and μ_r respectively. Write these symbols in decreasing order of values.*

Ans. μ_b, μ_y, μ_r.

Q. 24. *In a simple microscope, why the focal length of the lens should be small?*

Ans. This is because the angular magnification is inversely proportional to the focal length.

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *Define refractive index of a transparent medium. A ray of light passes through a triangular prism. Plot a graph showing the variation of the angle of deviation with the angle of incidence.*

Ans. Ratio of the speed of light in air (vacuum) to the speed of light in the medium is called the refractive index of the medium.

Graph: The graph of angle of deviation (δ) versus angle of incidence (i) for a triangular prism is given as follow:

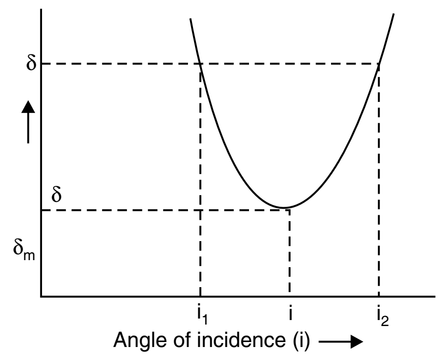


Fig. 9.46

Q. 2. *Draw a labelled ray diagram of a reflecting type telescope. Write its any one advantage over refracting type telescope.*

Ans. The ray diagram of a reflecting type telescope is shown below:

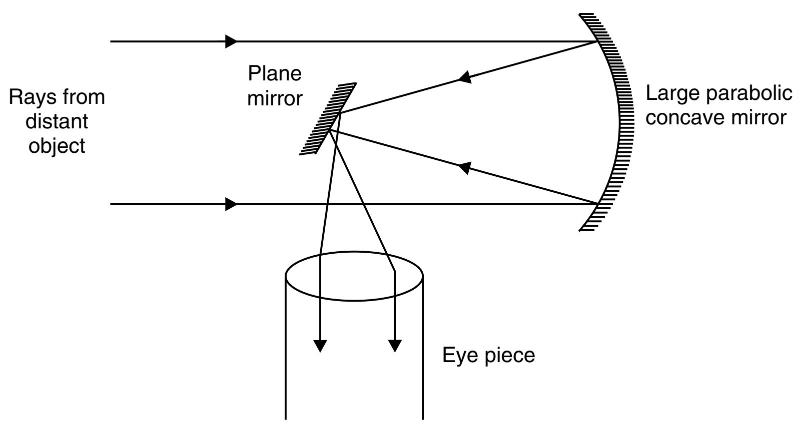


Fig. 9.47

Since a reflecting telescope has mirror objective, the image formed is free from chromatic aberration.

Q. 3. *A convex lens of refractive index 1.5 has a focal length of 18 cm in air. Calculate the change in its focal length when it is immersed in water of refractive index 4/3.*

Ans. Given ${}_a\mu_g = 1.5$, ${}_a\mu_w = 4/3$

$$f_a = 18$$

For the lens in air,

$$\frac{1}{f_a} = ({}_a\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or, $$\frac{1}{18} = (1.5 - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or, $$\frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{4}$$

When the lens is immersed in water

$$\frac{1}{f_w} = \left(\frac{{}^a\mu_g}{{}_a\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left(\frac{1.5}{4/3} - 1\right) \times \frac{1}{4}$$

$$= \frac{1}{8} \times \frac{1}{4} = \frac{1}{32}$$

Thus, $f_w = 32$ cm.

Q. 4. *A convex mirror always produces a virtual image independent of the location of the object. Use mirror equation to prove it.*

Ans. Using mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

For convex mirror,

object distance u is negative and the focal length f is positive.

$$\therefore \quad \frac{1}{v} = \frac{1}{+f} - \frac{1}{-u} = \frac{1}{f} + \frac{1}{u} \Rightarrow \text{a positive quantity.}$$

Therefore, the image distance is always positive in case of convex mirror.

The image is always formed on the other side of the object *i.e.,* behind the convex mirror.

$\because \quad m = \dfrac{-v}{u} \quad \dfrac{1}{v}$ is +ive *i.e.,* v is +ive but u is always (–) so m will be +ive *i.e.,* an errect image which is virtual formed is convex mirror.

Hence, the convex mirror always form a virtual image.

Q. 5. *Draw a ray diagram of a compound microscope. Write the expression for its magnifying power.*

Ans. Ray diagram of a compound microscope.

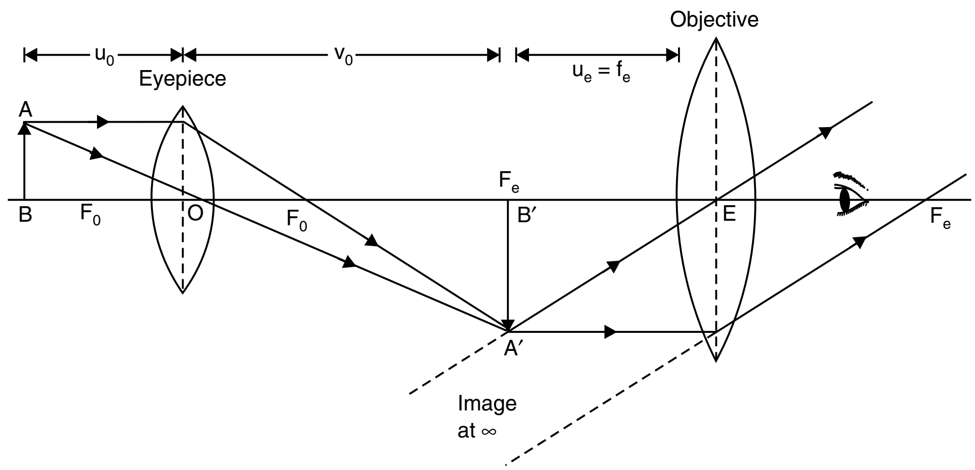


Fig. 9.48

When the final image is formed at the least distance of distinct vision,

$$m = -\frac{v_0}{u_0}\left(1+\frac{D}{f_e}\right)$$

For the image formed at infinity, $u_e = f_e$

and $$m = -\frac{v_0}{u_0}.\frac{D}{f_e}$$

By making focal length of the objective small, the magnifying power can be increased.

Q. 6. *Draw a ray diagram of an astronomical telescope in the near point adjustment. Write down expression for its magnifying power.*

Ans. Least distance of distinct vision (Refer to Page 492).

Magnifying power, $m = \frac{F_0}{F_e}\left(1+\frac{F_e}{D}\right)$

Q. 7. *An object 0.5 cm high is placed 30 cm from a convex mirror whose focal length is 20 cm. Find the position, size and nature of the image.*

Ans. We have,

$$u = -30 \text{ cm}; v = ?, f = +20 \text{ cm}$$

We know,

$$\frac{1}{v}+\frac{1}{u} = \frac{1}{f}$$

or, $v = \frac{uf}{u-f} = \frac{(-30)(+20)}{-30-20} = +12$ cm

The image is formed 12 cm behind the mirror. It is virtual and erect.

Now, $m = \frac{I}{O} = -\frac{v}{u} = -\frac{12}{-30}$

or, $I = \frac{2}{5}\times O = \frac{2}{5}\times 0.5 = +.2$cm

Hence, the height of the image = +.2 cm.

The positive sign indicates that the image is erect.

Q. 8. *An object 0.2 cm high is placed 15 cm from a concave mirror length 5 cm. Find the position and size of the image.*

Ans. We have,

$$u = -15 \text{ cm}, f = -5 \text{ cm}, \ v = ?$$

Using formula,

$$\frac{1}{v}+\frac{1}{u} = \frac{1}{f}$$

or, $$v = \frac{uf}{u-f} = \frac{(-15)(-5)}{-15+5} = -7.5 \text{ cm}$$

The image is formed at a distance of 7.5 cm in front of mirror.

Now, $$m = \frac{I}{O} = -\frac{v}{u}$$

or $$\frac{I}{O} = -\frac{(-7.5)}{(-15)} \quad \text{or,} \quad I = -0.1 \text{ cm}$$

The negative sign indicates that the image is inverted.

Q. 9. *A fish rising vertically to the surface of water in a lake uniformly at the rate of 3 m/s observes a king fisher bird diving vertically towards water at the rate 9 m/s vertically above it. If the refractive index of water is 4/3, find the actual velocity of the dive of the bird.*

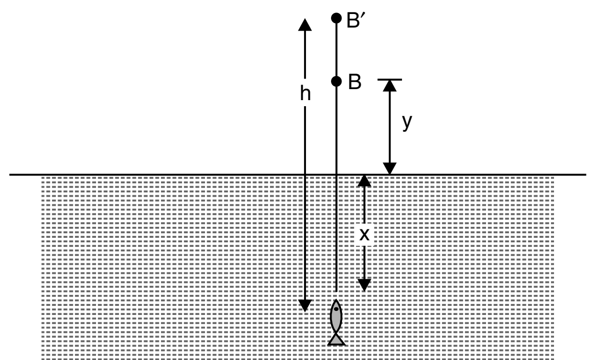


Fig. 9.49

Ans. If at any instant, the fish is at a depth 'x' below water surface while the bird at a height y above the surface, then the apparent height of the bird from the surface as seen by the fish will be given by

$$\mu = \frac{\text{Apparent height}}{\text{Real height}}$$

or, Apparent height = μy

So, the total apparent distance of the bird as seen by the fish in water will be $h = x + \mu y$

or, $$\frac{dh}{dt} = \frac{dx}{dt}+\mu\frac{dy}{dt}$$

or, $$9 = 3+\mu\left(\frac{dy}{dt}\right)$$

or, $$\frac{dy}{dt} = \frac{6}{(4/3)} = 4.5 \text{ m/s.}$$

Q. 10. *A compound microscope with an objective of 1.0 cm focal length and an eyepiece of 2.0 cm focal length has a tube length of 20 cm. Calculate the magnifying power of microscope, if the final image is formed at the near point of the eye.*

Ans. Here, l = 20 cm, D = 25 cm, f_0 = 1 cm and f_e = 2

$$m_0 = \frac{l}{f_0} = \frac{20}{1} = 20$$

$$m_e = \frac{D}{f_0} = \frac{25}{2}$$

$$m = m_0 m_e = 20 \times \frac{25}{2} = 250.$$

Q. 11. *A ray of light incident at 49° on the face of an equilateral prism passes symmetrically. Calculate the refractive index of the material of the prism.*

Ans. As the prism is an equilateral one, $A = 60°$. Since the ray of light passes symmetrically, the prism is in the position of **minimum deviation.**

So, $$r = A/2 = \frac{60°}{2} = 30°$$

Also, $$i = 49°$$

∴ $$\mu = \frac{\sin i}{\sin r} = \frac{\sin 49°}{\sin 30°} = \frac{0.7547}{0.5}$$

or, $$\mu = 1.5.$$

Q. 12. *The magnifying power of an astronomical telescope in the normal adjustment position is 100. The distance between the objective and the eyepiece is 101 cm. Calculate the focal lengths of the objective and of the eyepiece.*

Ans. Here, $$m = \frac{f_0}{f_e} = 100$$

or, $$f_0 = 100\, f_e$$

Since, $$f_0 + f_e = 101$$

Then, $$100 f_e + f_e = 101$$

∴ $$f_e = 1 \text{ cm and } f_o = 100 \text{ cm}.$$

Q. 13. *A prism of refractive index of* $\sqrt{2}$ *has a refracting angle of 60°. At what angle must a ray be incident on it so that it suffers a minimum deviation?*

Ans. For minimum deviation $i = e$.

Therefore, $$A + \delta_m = i + r$$

or, $$i = \frac{A + \delta_m}{2} \quad \text{for } \delta \text{ minimin } i = r$$

Now, $$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin A/2} = \frac{\sin i}{\sin A/2}$$

or, $$\sin i = \mu \sin A/2$$

$$= \mu \sin\left(\frac{60°}{2}\right) = \sqrt{2} \sin 30° = \frac{1}{\sqrt{2}}$$

Hence, $$i = 45°.$$

Q. 14. *The image of a candle is formed by a convex lens on a screen. The lower half of the lens is painted black to make it completely opaque.*

Draw the ray diagram to show the image formation. How will this image be different from the one obtained when the lens is not painted black?

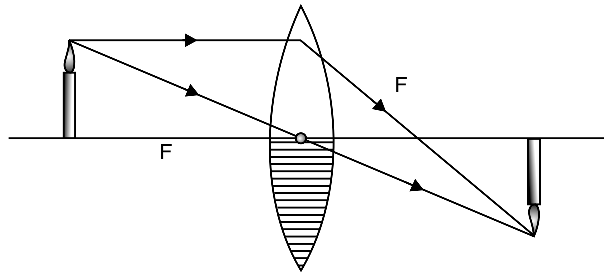


Fig. 9.50

Ans. The image formed will still be of full size but the intensity of the image will be lesser, when the lower half of the lens is painted black.

Q. 15. *The refracting angle of the prism is 60° and the refractive index of the material of the prism is 1.632. Calculate the angle of minimum deviation.*

Ans. Here, $A = 60°; \quad \mu = 1.632$

Now, $$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

or, $$1.632 = \frac{\sin\left(\frac{60°+\delta_m}{2}\right)}{\sin\frac{60°}{2}} = \frac{\sin\left(\frac{60+\delta_m}{2}\right)}{\sin 30°}$$

or, $$\sin\left(\frac{60°+\delta_m}{2}\right) = 1.632 \sin 30° = 1.632 \times .5$$

or, $$\sin\left(\frac{60°+\delta_m}{2}\right) = 0.816 \quad \text{or,} \quad \frac{60°+\delta_m}{2} = 54°42' \quad \delta_m = 49°24'.$$

Q. 16. *A vessel 20 cm deep is half-filled with oil of refractive index 1.37 and the other half is filled with water of refractive index 1.33. Find the apparent depth of the vessel when viewed from above.*

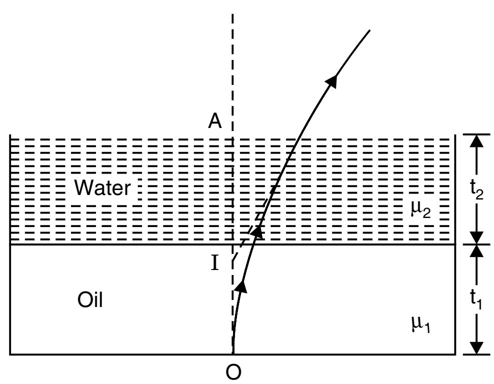


Fig. 9.51

Ans. Referring to figure given below, the apparent depth of the vessel is

$$\boxed{AI = \frac{t_1}{\mu_1} + \frac{t_2}{\mu_2}} = \frac{t}{2}\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right) = \frac{t(\mu_1+\mu_2)}{2\mu_2\mu_2}$$

where t = 20 cm, μ_1 = 1.37 and μ_2 = 1.33,

Substituting for t, μ_1 and μ_2, we get

$$AI = 15.37 \text{ cm.}$$

Q. 17. *Show that the least possible distance between an object and its real image in a convex lens is 4f where f is the focal length of the lens.*

Ans. Suppose I is the real image of an object O. (See figure below). Let d be the distance between them. If the image distance is x, the object distance will be $(d - x)$.

Thus, $u = -(d - x)$ and $v = +x$

Sustituting in the lens formula we have

$$\frac{1}{x} - \frac{1}{-(d-x)} = \frac{1}{f}$$

or, $$\frac{1}{x} + \frac{1}{(d-x)} = \frac{1}{f}$$

or, $$x^2 - xd - fd = 0$$

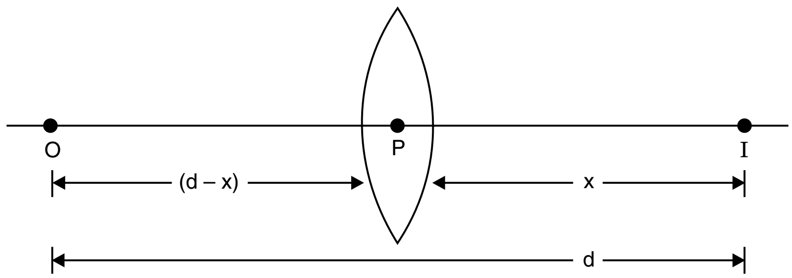


Fig. 9.52

For a real image, the value of x must be real, *i.e.*, the roots of the above equation must be real. This is possible if

$$d^2 \geq 4fd$$

or, $$d \geq 4f$$

Hence, $4f$ is the minimum distance between the object and its real image formed by a convex lens.

Q. 18. (*i*) *What is the relation between critical angle and refractive index of a material?*

(*ii*) *Does critical angle depend on the colour of light? Explain.*

Ans. (*i*) $\mu = \sin i_c$ or $n_{21} = \sin i_c$

where n_{21} is the refractive index of rarer medium 1 with respect to denser medium 2.

(*ii*) As μ depends on wavelength, therefore, critical angle for the same pair of media in contact will be different for different colours.

Q. 19. *A ray of light falls on one side of a prism whose refracting angle is 60°. Find the angle of incidence in order that the emergent ray may just graze the other side.* $\left(\mu = \frac{3}{2}\right)$

Ans. Given, $A = 60°$, $e = 90°$

$\therefore$ $r_2 = C$, the critical angle of the prism.

Now, $${}_a\mu_g = \frac{1}{\sin C}$$

or, $$\sin C = \frac{1}{\mu} = \frac{2}{3}$$

$\Rightarrow$ $$C = 41° \ 49'$$

Again, $$\boxed{A = r_1 + r_2}$$

$\Rightarrow$ $$r_1 = A - r_2 = 60° - 41°49' = 18°11'$$

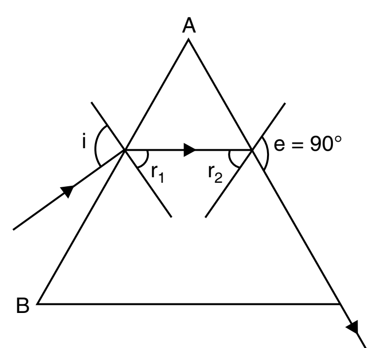


Fig. 9.53

For the refraction at the surface AB, we have

$$\mu = \frac{\sin i}{\sin r_1} \quad \text{(Snell's law)}$$

or, $\sin i = \mu \sin r_1$

$= 1.5 \times \sin 18°11'$

$= 1.5 \times 0.3121$

$= 0.46815$

$\Rightarrow \quad i = 27°55'$.

Q. 20. *Two Plano–concave lens of glass of refractive index 1.5 have radii of curvature 20 cm and 30 cm. They are placed in contact with curved surfaces towards each other and the space between them is filled with a liquid of refractive index 4/3. Find the focal length of the system.*

Ans. As shown in the figure, the system is equivalent to combination of three lenses in contact,

i.e.,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

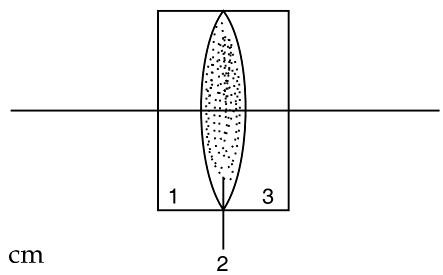


Fig. 9.54

By lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Now,

$$\frac{1}{f_1} = \left(\frac{3}{2} - 1\right)\left[\frac{1}{\infty} - \frac{1}{20}\right] = \frac{1}{40} \text{ cm}$$

$$\frac{1}{f_2} = \left(\frac{4}{3} - 1\right)\left[\frac{1}{20} - \frac{1}{-30}\right] = \frac{5}{180} \text{ cm}$$

$$\frac{1}{f_3} = \left(\frac{3}{2} - 1\right)\left[\frac{1}{-30} - \frac{1}{\infty}\right] = -\frac{1}{60} \text{ cm}$$

$\therefore$

$$\frac{1}{F} = -\frac{1}{40} + \frac{5}{180} - \frac{1}{60}$$

$\Rightarrow \quad F = -72$ cm.

Thus, the system will behave as a concave lens of focal length 72 cm.

Q. 21. *A convex lens of crown glass ($\mu_g = 1.5$) has a focal length of 15 cm. The lens is placed in (a) water ($\mu_w = 1.33$) and (b) carbon bisulphide ($\mu_c = 1.65$). Determine in each case, whether the lens behaves as a converging or diverging lens and determine its focal length.*

Ans. For lens in glass

$$\frac{1}{15} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= 0.5\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad ...(i)$$

For lens in water

$$\frac{1}{f_w} = \left(\frac{\mu_g - \mu_w}{\mu_w}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left(\frac{1.5 - 1.33}{1.33}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \frac{0.17}{1.33}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad ...(ii)$$

For lens in carbon bisulphide

$$\frac{1}{f_c} = \left(\frac{\mu_g - \mu_c}{\mu_c}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left(\frac{1.5 - 1.65}{1.65}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= -\frac{0.15}{1.65}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad ...(iii)$$

Dividing (*i*) by (*ii*), we get

$$f_w = -58.7 \text{ cm}$$

The positive sign indicates that the lens is converging.

Dividing (*i*) by (*iii*), we get

$$f_e = -82.5 \text{ cm}$$

The negative sign indicates that the lens behaves as a diverging lens when it is immersed in carbon bisulphide.

Q. 22. *Two thin lenses, both of 10 cm focal length – one convex and other concave, are placed 5 cm apart. An object is placed 20 cm in front of the convex lens. Find the nature and position of the final image.*

Ans.

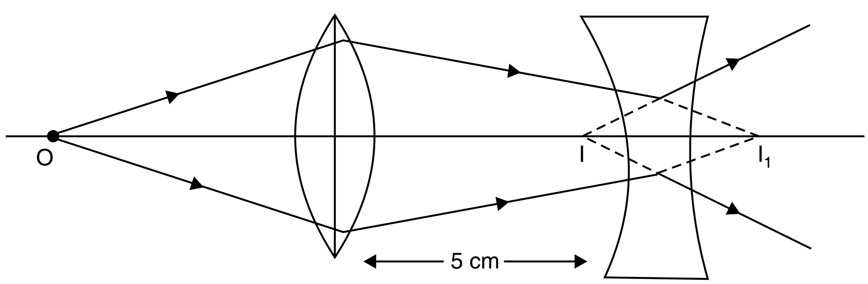


Fig. 9.55

For refraction at the convex lens, we have

$$u = -20 \text{ cm};\ f_1 = 10 \text{ cm};\ v = v_1 = ?$$

Using lens formula, we have

$$\frac{1}{v_1} - \frac{1}{(-20)} = \frac{1}{10}$$

$$\Rightarrow \qquad v_1 = 0 + 20 \text{ cm}$$

The convex lens produces converging rays trying to meet at I_1, 20 cm from the convex lens, *i.e.*, 15 cm behind the concave lens.

I_1 will serve as a virtual object for the concave lens.

For refraction at the concave lens, we have

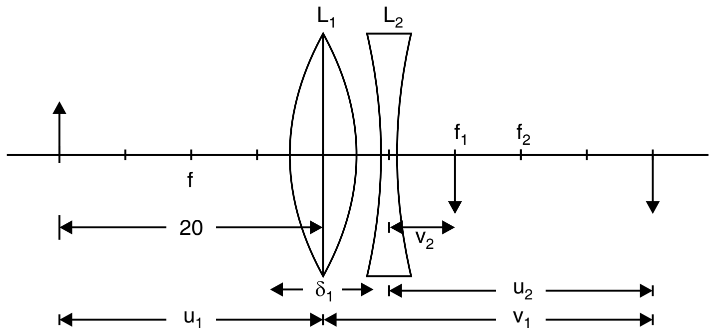


Fig. 9.56

For concave lens

$$u = 20 - 5 = 15 \text{ cm}$$

$$f = -10 \text{ cm}$$

As per sign convention

$$u = -15$$

$$f = -10$$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

$$= -\frac{1}{10} - \frac{1}{15}$$

$$= \frac{-3-2}{30} = \frac{-6}{30}$$

$$\frac{1}{v} = -\frac{1}{5}$$

$$v = -5 \text{ cm}$$

i.e., This image is in the side of object 5 cm Right to concave and 10 cm (5 + 5) from convex

$$u = +15 \text{ cm}; v = ?, f = -10\text{cm}$$

Using lens formula, we have

$$\frac{1}{v} - \frac{1}{15} = -\frac{1}{10}$$

$\Rightarrow$ $$v = -30 \text{ cm}$$

Hence, the final image is virtual and is located at 30 cm to the left of the concave lens.

Q. 23. *An astronomical telescope consists of two thin lenses set 36 cm apart and has a magnifying power of 8 in normal adjustment. Calculate the focal lengths of lenses.*

Ans. In the normal adjustment, the final image is formed at infinity.

$\therefore$ $$f_0 + f_e = 36 \quad ...(i)$$

and $$-\frac{f_0}{f_e} = -8$$

$\therefore \qquad f_0 = 8\,f_e \qquad ...(ii)$

$\therefore$ From equations (*i*) and (*ii*), we have

$$8f_e + f_e = 36$$
$$9f_e = 36$$
$$f_e = 4 \text{ cm}$$

and $\qquad f_0 = 8 \times 4 = 32$ cm.

III. LONG ANSWER TYPE QUESTIONS

Q. 1. *An equi-convex lens of refractive index μ_g = 1.5 and focal length 10cm is placed on the surface of water (μ_w = 4/3) such that its lower surface is immersed in water but its upper surface is in contact with air outside. (a) At what distance from the lens will a beam parallel to its principal axis come to focus? (b) How is the position of the focus altered if the lens is wholly immersed in water?*

Ans. Let the radius of curvature of each face of the lens be x cm. Then $R_1 = +x$ cm and $R_2 = -x$ cm. Also, μ_g = 1.5 and f = + 10 cm.

Substituting in formula,

$$\frac{1}{f} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

We have,

$$\frac{1}{10} = (1.5 - 1)\left(\frac{1}{x} - \frac{1}{-x}\right)$$

Which gives,

$$x = 10 \text{ cm}$$

Thus, R_1 = + 10 cm and R_2 = –10 cm.

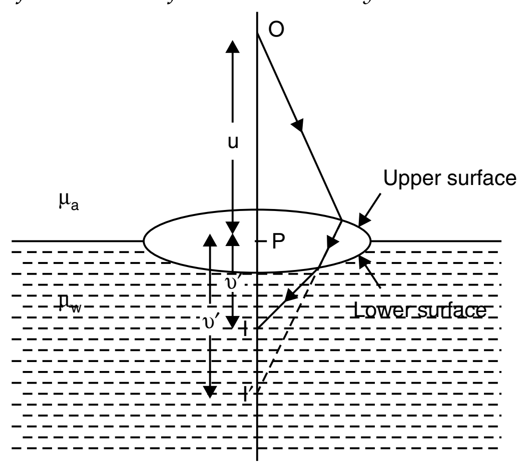


Fig. 9.57

(*a*) As shown in the figure *I* is the image of *O* due to refraction at the upper face. Since the incident ray is in air of refractive index μ_a and the refracted ray in glass of refractive index μ_g, we have for refraction at this face (whose radius of curvature is R_1)

$$\frac{\mu_g}{v'} - \frac{\mu_a}{u} = \frac{\mu_g - \mu_a}{R_1} \qquad ...(i)$$

Where, $v' = PI'$

The image *I′* serves as the virtual object for refraction at the lower surface. For refraction at this surface, the incident ray is in glass and the refracted ray in water. *I* is the final image. Thus, for refraction at the lower surface (whose radius of curvature is R_2) we have

$$\frac{\mu_w}{v} - \frac{\mu_g}{v'} = \frac{\mu_w - \mu_g}{R_2} \qquad ...(ii)$$

Adding (*i*) and (*ii*) we get

$$\frac{\mu_w}{v} - \frac{\mu_a}{u} = \frac{\mu_g - \mu_a}{R_1} + \frac{\mu_w - \mu_g}{R_2}$$

Putting $\qquad u = -\infty$; R_1 = + 10 cm, R_2 = –10 cm

μ_w = 4/3, μ_g = 1.5 and μ_a = 1, we get

v = 20 cm

(*b*) If the lens is wholly immersed in water, the formula is

$$\frac{1}{v}-\frac{1}{u} = \left(\frac{\mu_g-\mu_w}{\mu_2}\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Which (for $u = -\infty$) gives $v = 40$ cm.

Q. 2. *A thin equi-convex lens (radius of curvature of either face being 33 cm) is placed on a horizontal plane mirror and a pin held 20 cm vertically above the lens coincides in position with its own image. The space between the lower surface of the lens and the mirror is filled with a liquid and then, to coincide with the image as before, the pin has to be raised to a distance of 25 cm from the lens. Find the refractive index of the liquid.*

Ans. In the first case, the image will coincide with the pin if the rays from the pin, after refraction through the lens, fall normally on the mirror and retrace their path, as shown in Fig. 9.52 (*a*). This means that the focal length of the convex lens is 20 cm.

$$f = 20 \text{ cm}$$

In the second case, the focal length F of the combination of the convex lens and the plano-concave liquid lens is 25 cm [see Fig. 9.52 (*b*)] *i.e.,*

$$F = 25 \text{ cm}$$

Let f_2 be the focal length of the liquid lens, then

$$\boxed{\frac{1}{F} = \frac{1}{f_1}+\frac{1}{f_2}}$$

$$\frac{1}{f_2} = \frac{1}{F}-\frac{1}{f_1}=\frac{1}{25}-\frac{1}{20}$$

or, $$f_2 = -100 \text{ cm}$$

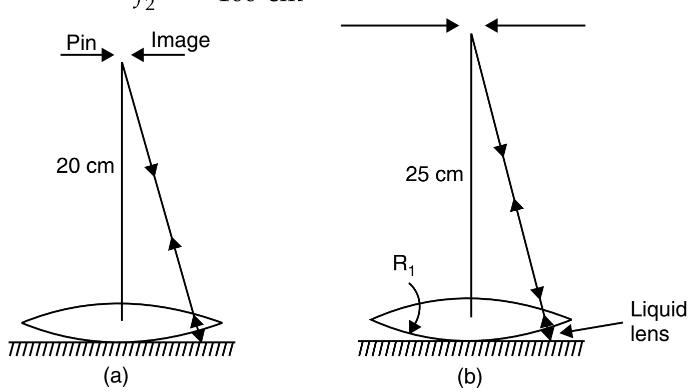


Fig. 9.58

For the liquid lens, $R_1 = -33$ cm, the radius of curvature of the common surface and $R_2 = \infty$. If μ is refractive index of the liquid.

$$\frac{1}{f_2} = (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

or, $$-\frac{1}{100} = (\mu-1)\left(-\frac{1}{33}+\frac{1}{\infty}\right)$$

or, $$\mu - 1 = \frac{33}{100} = 0.33$$

or, $$\mu = 1.33.$$

Q. 3. *(i) With the help of suitable ray diagram, derive the mirror formula for a concave mirror?*

(ii) The near point of a hypermetropic person is 50 cm from the eye. What is the power of the lens required to enable the person to read clearly a book held at 25 cm from the eye?

Ans. Mirror formula for concave mirror:

(*b*) Given, $u = -25$ cm, $v = -50$ cm, $f = ?$

Using lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-50} - \frac{1}{-25} = -\frac{1}{50} + \frac{1}{25}$$

$$= \frac{-1+2}{50} = \frac{1}{50}$$

$\therefore$ $f = 50$ cm.

$$P = \frac{1}{f(\text{in m})} = \frac{100}{f(\text{in cm})}$$

$\Rightarrow$ $P = \frac{100}{50} = +2D$

Hence, the corrective lens is convex.

Q. 4. *An object is placed 40 cm in front of the curved surface of a thin plano-convex lens whose plane surface is silvered. Due to refraction at the curved surface and reflection at the silvered surface, the real image of the object is 60 cm from the lens on the same side as the object. Find the focal length of the lens.*

Ans. The incident ray OA is refracted along AB due to refraction at the curved surface (see Fig. 9.53). This ray AB appears to come from I_1, the virtual image point due to refraction at the curved surface. For this refraction we have

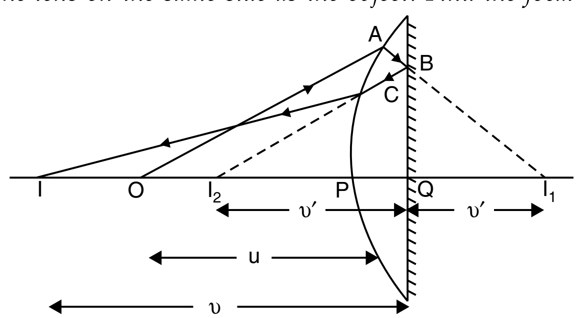


Fig. 9.59

$$\frac{\mu_g}{v'} - \frac{\mu_a}{u} = \frac{\mu_g - \mu_a}{R}$$

Since the incident rays are from left to right, $v' = +v'$, $u = -u$ and $R = +R$.

Therefore, we have

$$\frac{\mu_g}{v'} + \frac{\mu_a}{u} = \frac{\mu_g - \mu_a}{R} \qquad ...(i)$$

where $u = +40$ cm (given).

The ray AB is reflected (by the plane surface) along BC. The reflected ray appears to come from I_2 such that $QI_1 = QI_2 = v'$.

The ray BC suffers refraction at the curved surface and the final image is formed at I. Thus I is the image of the virtual object I_2 due to refraction at the curved surface. For this refraction, since the incident ray travels from glass to air, we have

$$\frac{\mu_a}{v} - \frac{\mu_g}{v'} = \frac{\mu_a - \mu_g}{R}$$

From our sign convention, $v = -v$, $v' = QI_2 = PI_2$ (since lens is thin) $= -v'$ and $R = +R$. Therefore, we have

$$-\frac{\mu_a}{v}+\frac{\mu_g}{v'} = \frac{\mu_a-\mu_g}{R} \qquad ...(ii)$$

where, $v = +60$ cm (given).

Subtracting (*ii*) from (*i*), we get

$$\frac{\mu_a}{u}+\frac{\mu_a}{v} = \frac{2}{R}(\mu_g-\mu_a)$$

or,
$$\frac{1}{u}+\frac{1}{v} = \frac{1}{\mu_a}\frac{2}{R}(\mu_g-\mu_a)$$

But
$$\left(\frac{\mu_g-\mu_a}{\mu_a}\right)\frac{1}{R} = \frac{1}{f_1}$$

where f_1, the focal length of the lens. Hence

$$\frac{2}{f_1} = \frac{1}{u}+\frac{1}{v} = \frac{1}{40}+\frac{1}{60} = \frac{1}{24}$$

or,
$$f_1 = 48 \text{ cm.}$$

Q. 5. *Derive the expression for the refractive index of the material of the prism in terms of the angle of the prism and angle of minimum deviation.*

Use this formula to calculate the angle of minimum deviation for an equilateral triangular prism of refractive index $\sqrt{3}$.

Ans.

$\Rightarrow$ Given, $\mu = \sqrt{3}$, $A = 60°$

$$\therefore \quad \sqrt{3} = \frac{\sin\frac{\delta_m+60°}{2}}{\sin 60°/2} \qquad \left[\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin A/2}\right]$$

or,
$$\sqrt{3}\times\sin 30° = \sin\left(\frac{\delta_m+60°}{2}\right)$$

or,
$$\sin\left(\frac{\delta_m+60°}{2}\right) = \sqrt{3}\times\frac{1}{2} = \sin 60°$$

or,
$$\frac{\delta_m+60°}{2} = 60°$$

$$\Rightarrow \quad \delta_m + 60° = 120°$$

$$\therefore \quad \delta_m = 60°.$$

Q. 6. *A parallel beam of light travelling in water (refractive index = 4/3) is refracted by a spherical air bubble of radius 2 mm situated in water. Assuming the light rays to be paraxial, (a) find the position of the image due to refraction at the first surface and the position of the final image and (b) draw a ray diagram showing the positions of both the images.*

Ans. Refer to Fig. 9.60 below

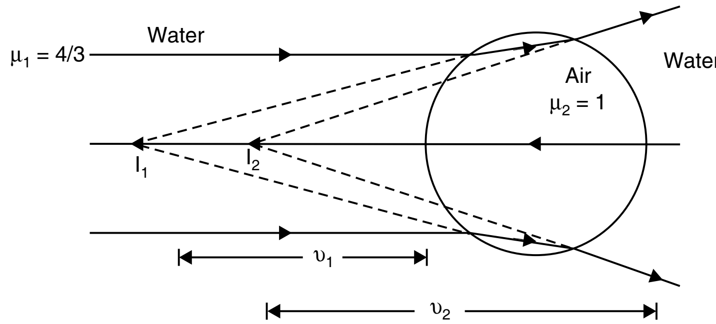


Fig. 9.60

(*a*) For refraction at the first surface, we use

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

where μ_1 = 4/3, μ_2 = 1 and $u = \infty$ and R = 2 mm. Thus

$$\frac{1}{v_1} - \frac{4/3}{\infty} = \frac{1 - 4/3}{2}$$

which gives v_1 = –6 mm, the negative sign indicates that the image I_1 is virtual and is on the same side as the object at a distance of 6 mm from the first surface.

For refraction at the second surface, the image I_1 serves as the virtual object which is at a distance of 6 mm + 4 mm = 10 mm from the second surface. For this refraction, we use

$$\frac{\mu_1}{v_2} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

where u = –10 mm and R = –2 mm. Thus

$$\frac{4/3}{v_2} - \frac{1}{-10} = \frac{4/3 - 1}{(-2)}$$

which given v_2 = –5 mm. The final image I_2 is virtual and is formed at a distance of 5 mm from the second surface to the left of the second surface, *i.e.,* the final image is formed at a distance of 1 mm from the first surface.

(*b*) Figure given above shows the ray diagram.

Q. 7. *With the help of ray diagram, show the formation of image of a point object by refraction of light at a spherical surface separating two media of refractive indices n_1 and n_2 ($n_2 > n_1$) respectively. Using this diagram, derive the relation.*

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

Write the sign conventions used. What happens to the focal length of convex lens when it is immersed in water?

Ans. (*i*) *AMB* is a convex surface separating two media of refractive indices n_1 and n_2 ($n_2 > n_1$). Consider a point object *O* placed on the principal axis. A ray *ON* is incident at *N* and refracts along *NI*. The ray along *ON* goes straight and meets the previous ray at *I*. Thus *I* is the real image of *O*.

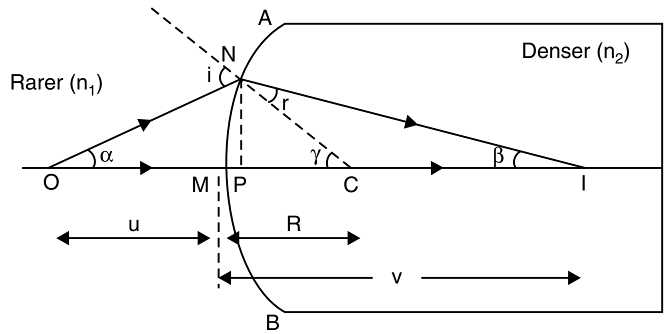


Fig. 9.61

From Snell's law, $n_2 = \frac{\sin i}{\sin r}$

$$n_1 \sin i = n_2 \sin r$$

$$\frac{n_2}{n_1} = \frac{\sin i}{\sin r}$$

or, $n_1 i = n_2 r$ [$\because$ sin θ ≃ θ as θ is very small]

From Δ *NOC*, $i = \alpha + \gamma$

From Δ *NIC*, $\gamma = r - \beta$

or, $r = \gamma - \beta$

$\therefore$ $n_1 (\alpha + \gamma) = n_2(\gamma - \beta)$

or, $n_1\alpha + n_2\beta = (n_2 - n_1)\gamma$

But $\alpha \cong \tan \alpha = \frac{NP}{OP} = \frac{NP}{OM}$ [*P* is close to *M*]

$$\beta \cong \tan \beta = \frac{NP}{PI} = \frac{NP}{MI}$$

$$\gamma \cong \tan \gamma = \frac{NP}{PC} = \frac{NP}{MC}$$

$\therefore$ $n_1 \cdot \frac{NP}{OM} + n_2 \cdot \frac{NP}{MI} = (n_2 - n_1)\frac{NP}{MC}$

or, $\frac{n_1}{OM} + \frac{n_2}{MI} = \frac{n_2 - n_1}{MC}$

Using Cartesian sign convention,

$$OM = -u,\ MI = +v,\ MC = +R$$

$\therefore$ $\frac{n_1}{-u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$

or, $\boxed{\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}}$

Sign convention: Refer to Page 476.

Q. 8. *Two concave glass refracting surfaces, each with radius of curvature R = 35 cm and refractive index μ = 1.5, are placed facing each other in air as shown in Fig. A point object O is placed at a distance of R/2 from one of the surfaces as shown. Find the separation between the images of O formed by each refracting surface.*

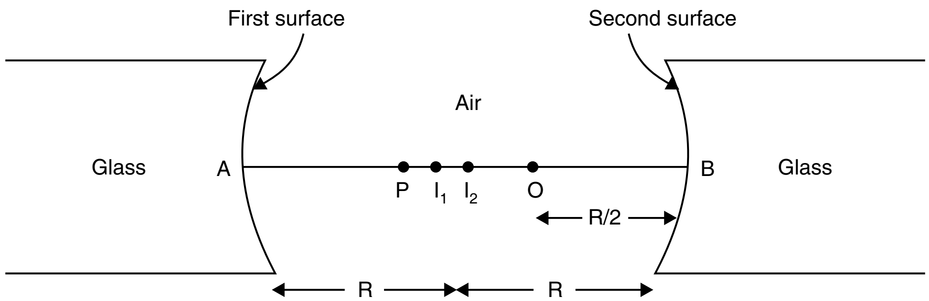


Fig. 9.62

Ans. For refraction at the first surface, we have

$$u = AO = -\frac{3R}{2} = -\frac{105}{2}\text{cm}$$

$$\mu_2 = 1.5$$

$$\mu_1 = 1$$

$$R = -35 \text{ cm}$$

The distance v_1 of the image I_1 from A is given by the relation

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

or, $$\frac{1.5}{v_1} - \frac{1}{-105/2} = \frac{1.5-1}{-35}$$

or, $v_1 = -45$ cm or $AI_1 = 45$ cm.

Since the radius of curvature is 35 cm, this image is 10 cm to the right of P, *i.e.*, $PI_1 = 10$ cm.

For refraction at second surface we have

$$u = BO = -\frac{R}{2} = -\frac{35}{2}\text{cm}$$

The distance v_2 of the image I_2 from the second surface is given by

$$\frac{\mu_2}{v_2} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

which gives $v_2 = -21$ cm or $BI_2 = 21$ cm. Thus, the image is 14 cm to the right of P *i.e.*, $PI_2 = 14$ cm. Hence, the separation between the two images $(I, I_2) = 14 - 10 = 4$ cm.

Q. 9. *Two convex lenses A and B of focal lengths 20 cm and 10 cm are placed coaxially 10 cm apart. An object is placed on the common axis at a distance of 10 cm from lens A. Find the position and magnification of the final image.*

Ans. From figure below, we have, for lens A

$$f_1 = +20 \text{ cm and } u_1 = -10 \text{ cm}$$

The image distance v_1 is given by

$$\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{20} + \frac{1}{(-10)} = \frac{1}{20} - \frac{1}{10}$$

which gives $v_1 = -20$ cm

Thus, a virtual image is formed at I_1 at a distance of 20 cm from lens A, if the lens B were absent. This image acts as a virtual object for lens B which forms the final image at I_2 at a distance v_2 from lens B. For lens B we have,

x = 10 cm, $u_2 = -(20 + 10) = -30$ cm

$f_2 = +10$ cm.

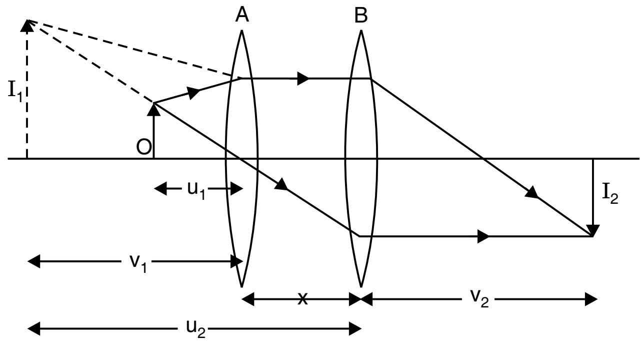


Fig. 9.63

The image distance v_2 is given by

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = \frac{1}{10} - \frac{1}{30} = \frac{1}{15}$$

Which gives $v_2 = +15$ cm

Thus, a real image I_2 is formed at a distance of 15 cm from lens B.

Magnification due to A $(m_1) = \frac{v_1}{u_1} = \frac{-20}{-10} = +2$

Magnification due to B $(m_2) = \frac{v_2}{u_2} = \frac{15}{-30} = -\frac{1}{2}$

Magnification of the final image is

$$m = m_1 \times m_2 = +2 \times (-1/2) = -1$$

This shows that the final image is inverted and is of the same size as the object.

Q. 10. *A prism is found to give a minimum deviation of 51°. The same prism gives a deviation of 62°48′ for two values of the angles of incidence, namely, 46°6′ and 82°42′. Determine the refractive angle of the prism and the refractive index of its material.*

Ans. The incident ray is deviated through $\delta = 62°48'$ wen angle $i = 40°6'$. From the principle of reversibility of light, it is clear from the figure that the emergent ray (for which angle $e = 82°42'$) is also deviated through the same angle δ. Now,

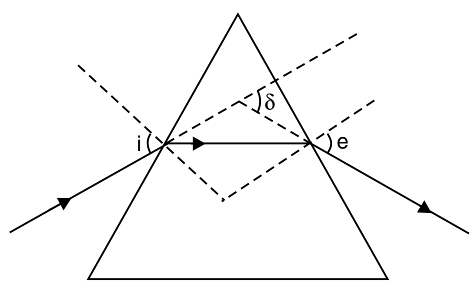


Fig. 9.64

$$\delta = (i + e) - A$$

or, $A = (i + e) - \delta$

$= 40°6' + 82°42' - 62°48'$

or, $A = 60°$

which is the refracting angle of the prism.

For minimum deviation, $i = e$.

Hence, $\delta_{min} = 2i - A$

or, $i = \left(\frac{\delta_{min} + A}{2}\right)$

$= \frac{(51° + 60°)}{2} = 55°30'$

Which is the angle of incidence at minimum deviation? The refractive index of the material of the prism is given by

$$\mu = \frac{\sin\frac{(\delta_{min}+A)}{2}}{\sin\frac{A}{2}}$$

or, $$\mu = \frac{\sin\left(\frac{51° + 60°}{2}\right)}{\sin\frac{60°}{2}}$$

or, $$\mu = 1.648.$$

QUESTIONS ON HIGH ORDER THINKING SKILLS (HOTS)

Q. 1. *A magnifying lens has a focal length of 10 cm. (i) Where should the object be placed if the image is to be 30 cm from the lens? (ii) What will be the magnification?*

Ans. (*i*) In case of magnifying lens, the lens is convergent and the image is erect, virtual and enlarged and between infinity and the object on the same side of the lens as shown in the above diagram.

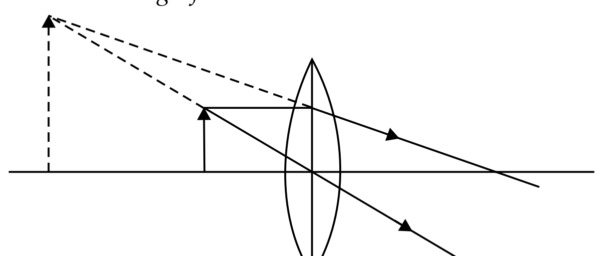


Fig. 9.65

So, here

$f = +10$ cm, $v = -30$ cm

Let 'x' be the object distance using lens formula

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

We have,

$$\frac{1}{-30} - \frac{1}{(-x)} = \frac{1}{10}$$

$$\Rightarrow \quad x = 7.5 \text{ cm}$$

(*ii*) $$m = \frac{I}{O} = \frac{v}{u} = \frac{-30}{-8.5} = +4$$

Thus, the image is erect and virtual and four times of the object.

Q. 2. *The focal length of objective and eyepiece of an astronomical telescope are 25 cm and 2.5 cm respectively. The telescope is focused on an object 1.5 m from objective, the final image is at 25 cm from the eye of the observer. Calculate the length of the telescope.*

Ans. Here, $f_0 = 25$ cm, $f_e = 2.5$ cm

$u_0 = -1.5$ m $= -150$ cm, $v_e = 25$ cm

From $$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$$

$$\frac{1}{v_0} = \frac{1}{f_0} + \frac{1}{u_0} = \frac{1}{25} - \frac{1}{150} = \frac{5}{150} = \frac{1}{30}$$

$$v_0 = 30 \text{ cm.}$$

From $$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

$$-\frac{1}{u_e} = \frac{1}{f_e} - \frac{1}{v_e}$$

$\Rightarrow$ $$-\frac{1}{u_e} = \frac{1}{2.5} - \frac{1}{25} = \frac{9}{25}$$

or, $$u_e = -\frac{25}{9} \text{ cm}$$

$$L = v_0 + |u_e| = 30 + \frac{25}{9} = \frac{295}{9} = 32.8 \text{ cm.}$$

Q. 3. *A slide projector has to project a 35 mm slide (35 mm × 35 mm) on a 2m × 2m screen at a distance of 10 m from the lens. What should be the focal length of the lens in the projector?*

Ans. $$m = \frac{-2 \text{ m}}{35 \text{ mm}} = \frac{-2 \times 10^3}{35}$$

$$\frac{v}{u} = \frac{-2000}{35}$$

$$u = \frac{-35}{2000} v = -\frac{35}{2000} \times 10 \times 10^3 \text{ mm}$$

$$= -175 \text{ mm} = -0.175 \text{ m}$$

Now, $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

or, $$\frac{1}{10} + \frac{1}{0.175} = \frac{1}{f}$$

or, $$\frac{1}{f} = \frac{10.175}{1.75}$$

$\Rightarrow$ $$f = \frac{1.75}{10.175} \text{ cm} = 0.172 \text{ m.}$$

Q. 4. *You are given 3 lenses having powers $P_1 = 6D$, $P_2 = 3D$ and $P_3 = 12D$. Which two of these lenses will you select to construct a microscope?*

Ans. Magnifying power of a microscope,

$$m \propto \frac{1}{f_0} \cdot \frac{1}{f_e} \propto P_0 P_e$$

$\therefore$ $P_2 = 3D$ is out. We use $P_1 = 6D$ and $P_3 = 12D$ for constructing a microscope.

As $P_0 > P_e$

$\therefore$ $P_3 = 12D$ should serve as objective lens and $P_1 = 6D$ should serve as eye-lens.

Q. 5. *A point object is placed at a distance of 12 cm on the axis of a convex lens of focal length 10 cm. On the other side of the lens, a convex mirror is placed at the distance of 10 cm from the lens such that the image formed by the combination coincides with the object itself. What is the focal length of the mirror?*

Ans. For the refraction at convex lens, we have

$u = -12$ cm; v =? $f = +10$ cm

Using lens formula, we have

$$\frac{1}{v} - \frac{1}{(-12)} = \frac{1}{10}$$

or, $v = +60$ cm

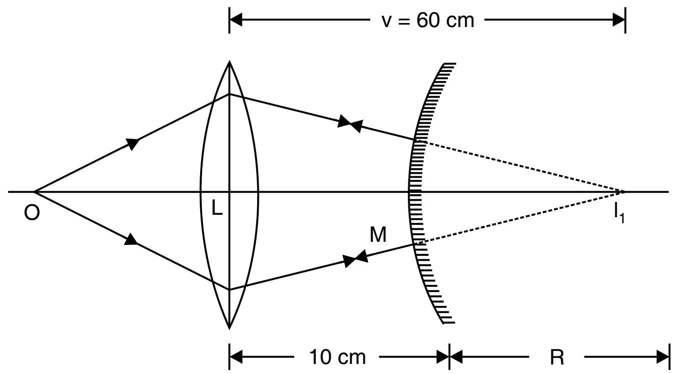


Fig. 9.66

Thus, in the absence of the convex mirror, convex lens will form the image I_1, at a distance of 60 cm behind the lens. As the mirror is at a distance of 10 cm from the lens, I_1 will be at a distance of (60 – 10) = 50 cm from the mirror, *i.e.,* $MI_1 = 50$ cm.

Now, as the final image I_2 is formed at the object itself, the rays after reflection from the mirror retraces its path, *i.e.,* the rays on the mirror are incident normally, *i.e.,* I_1 is the centre of the mirror so that

$$R = MI_1 = +50 \text{ cm}$$

and $$f = \frac{R}{2} = \frac{50}{2} = 25 \text{ cm.}$$

Q. 6. *The plane surface of a plano-convex lens of focal length 60 cm is silvered. A point object is placed at a distance 20 cm from the lens. Find the position and nature of the final image formed.*

Ans. Let f be the focal length of the equivalent spherical mirror.

We have,

$$\boxed{\frac{1}{f} = \frac{1}{f_l} + \frac{1}{f_m} + \frac{1}{f_l}}$$

or, $$\frac{1}{f} = \frac{2}{f_l} + \frac{1}{f_m}$$

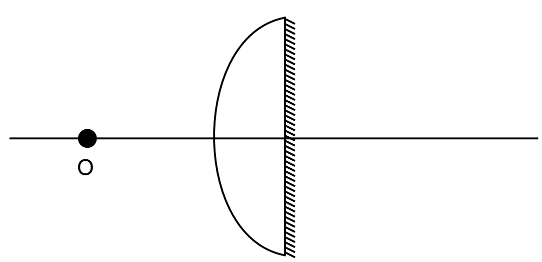


Fig. 9.67

Here, $f_l = +60$ cm

$f_m = \infty$

$\therefore$ $$\frac{1}{f} = \frac{2}{60} + \frac{1}{\infty} = \frac{1}{30}$$

or, $f = +30$ cm

The problem is reduced to a simple case where a point object is placed in front of a concave mirror.

Now, using mirror formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}, \text{ we have}$$

$$m = \frac{-v}{u} = \frac{-60}{-20} = +3$$

$$\frac{1}{-20}+\frac{1}{v} = \frac{1}{-30}$$

$\Rightarrow$ $v = 60$ cm

The image is erect and virtual and 3 lines of object.

Q. 7. *An equi-convex lens with radii of curvature of magnitude r each, is put over a liquid layer poured on top of a plane mirror. A small needle with its tip on the principal axis of the lens is moved along the axis until its inverted real image coincides with the needle itself. The distance of needle from lens is measured to be a. On removing the liquid layer and repeating the experiment, the distance is found to b. Given that two values of distances measured represent the real length values in the two cases, obtain a formula for refractive index of the liquid.*

Ans. Here, combined focal length of glass lens and liquid lens, $F = a$, and

Focal length of convex lens, $f_1 = b$.

If f_2 is focal length of liquid lens, then

$$\frac{1}{f_1}+\frac{1}{f_2} = \frac{1}{F}$$

$$\frac{1}{f_2} = \frac{1}{F}-\frac{1}{f_1} = \frac{1}{a}-\frac{1}{b}$$

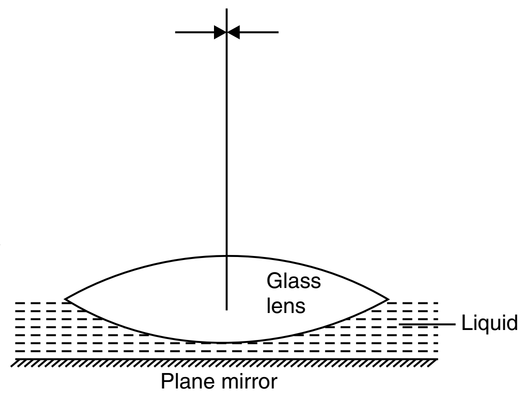


Fig. 9.68

The liquid lens is plano-concave lens for which

$$R_1 = -r,\ R_2 = \infty$$

From
$$\frac{1}{f_2} = (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{a}-\frac{1}{b} = (\mu-1)\left(\frac{1}{-r}-\frac{1}{\infty}\right)$$

$\therefore$
$$(\mu-1) = \frac{r}{b}-\frac{r}{a}$$

$$\mu = 1+\frac{r}{b}-\frac{r}{a}.$$

Q. 8. *A short-sighted person can see objects most distinctly at a distance of 16 cm. If he wears spectacles at a distance of 1 cm from the eye, what focal length should they have so as to enable him to see distinctly at a distance of 26 cm?*

Ans. The person is suffering from myopia (shortsightedness). Since the person wears spectacles at a distance of 1 cm from the eyes,

$\therefore$ $v = -(16-1)$ cm $= -15$ cm

Also, $u = -(26-1)$ cm $= -25$ cm

Now,
$$\frac{1}{f} = \frac{1}{v}-\frac{1}{u} = \frac{1}{-15}-\frac{1}{-25}$$

$$= \frac{1}{25}-\frac{1}{15} = -\frac{2}{75}$$

$$f = -\frac{75}{2}\text{ cm} = -37.5\text{ cm}$$

So, a concave lens of focal length 37.5 cm is to be used.

Q. 9. *In Fig. 9.69 the direct image formed by the lens (f = 10 cm) of an object placed at O, and that formed after reflection from the spherical mirror are formed at same point O. What is the radius of curvature of the mirror?*

Ans. Here, $f = 10$ cm, $u = -15$ cm.

Let v be the distance of image I formed by the lens L.

From
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$
$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$
$$= \frac{1}{10} - \frac{1}{15}$$
$$= \frac{1}{30}$$
$$v = LI = 30 \text{ cm}$$

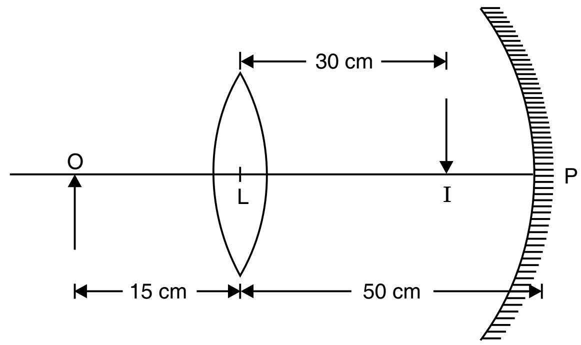


Fig. 9.69

The mirror will form the image at I itself provided I coincides with centre of curvature C of the mirror.

$\therefore$ $R = PC = PI = PL - LI = 50 - 30 = 20$ cm.

Q. 10. *The least distance of distinct vision of a long sighted man is 50 cm. He reduces this distance to 10 cm by wearing spectacles.*

What is the power and type of lenses used by him?

Ans. For such a person, $u = -10$ cm and $v = -50$ cm.

Using formula
$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$
$$\Rightarrow \quad \frac{1}{f} = \frac{1}{-50} + \frac{1}{10} = +\frac{4}{50}$$
$$\text{or,} \quad f = +\frac{50}{4} = +12.5 \text{ cm}$$
$$P = \frac{100}{f}$$
$$\Rightarrow \quad P = \frac{100}{12.5} = 8D$$

$\therefore$ The type of lenses used is convex.

Q. 11. *If you sit in a parked car, you glance in the rear view mirror R = 2 m and notice a jogger approaching. If the jogger is running at a speed of 5 ms^{-1}, how fast is the image of the jogger moving when the jogger is (a) 39 m (b) 29 m (c) 19 m and (d) 9 m away ?*

Ans. Using mirror equation,
$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\Rightarrow \qquad \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\text{or,} \qquad v = \frac{fu}{u-f}$$

(*a*) For convex mirror,

$$u = -39 \text{ m}, f = 1 \text{ m},$$

$$v = \frac{(-39)\times 1}{-39-1} = \frac{39}{40} \text{ m}$$

Since the jogger moves at a constant speed of 5 ms^{-1}, after 1s the position of the image v for $u = -39 + 5 = -34$ m is $\frac{34}{35}$ m.

Difference in the position of image is $\frac{1}{280}$ ms^{-1}.

For u = –29 m, –19 m and –9 m, the speed of image is $\frac{1}{150}$ ms^{-1}, $\frac{1}{60}$ ms^{-1} and $\frac{1}{10}$ ms^{-1} respectively.

The speed becomes very high as the jogger approaches the car. The change in speed can be experienced by anybody while travelling in a bus or a car.

Q. 12. *A ray of light, incident on an equilateral glass prism ($\mu_{medium} = \sqrt{3}$) moves parallel to the base of the prism inside it. What is the angle of incidence for this ray?*

Sol. When light ray moves parallel to the base of the prism inside it then $r_1 = r_2$ so, the minimum deviation takes place.

For minimum deviation $r = A/2 = 30°$

$$\therefore \qquad \sqrt{3} = \frac{\sin i}{\sin 30°} = \frac{\sin i}{1/2}$$

$$\text{or} \qquad \sin i = \frac{\sqrt{3}}{2}$$

$$\text{or} \qquad i = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60°.$$

Q. 13. *An object is placed at a distance of 15 cm from a convex lens of focal length 10 cm. On the other side of the lens, a convex mirror is placed such that its distance, from the lens, equals the focal length of the lens. The image formed by this combination is observed to coincide with the object itself. Find the focal length of the convex mirror.*

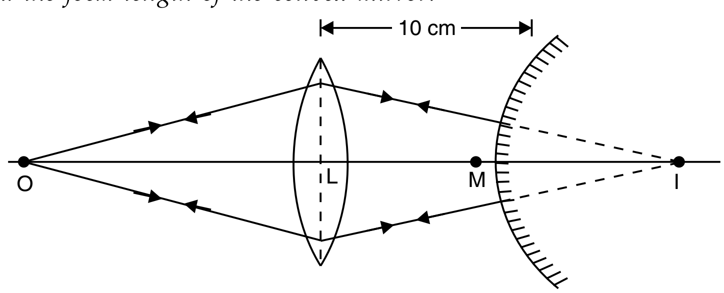


Fig. 9.70

Sol. For lens,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

or $$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

$$= \frac{1}{10} + \frac{1}{-15}$$

$$\frac{3-2}{30} = \frac{1}{30}$$

$$v = 30 \text{ cm}$$

For mirror, $$u = 30 - 10 = 20 \text{ cm}$$

and $$v = 10 + 15 = 25 \text{ cm}$$

$\therefore$ $$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$= \frac{1}{25} + \frac{1}{-20}$$

$$= \frac{4-5}{100} = \frac{-1}{100}$$

or $$f = -100 \text{ cm.}$$

Q. 14. *A 5 cm long needle is placed 10 cm from a convex mirror of focal length 40 cm. Find the position, nature and size of image of the needle. What happens to the size of the image when the needle is moved further away from the mirror?*

Sol. For mirror

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$= \frac{1}{40} - \frac{1}{-10} = \frac{1+4}{40} = \frac{5}{40}$$

or $$v = \frac{40}{5} = 8 \text{ cm.}$$

magnification $$m = -\frac{v}{u}$$

When needle moves further away from the convex mirror, the image moves further behind the mirror towards the focus and size decreases. When it is far off, it appears almost as a point image of the object.

Q. 15. *A luminescent object is placed at a depth of 'd' in a (optically) denser medium of refractive index 'μ'. Prove that radius r of the base of the cone of light, from the object, that can emerge out from the surface is* $r = \dfrac{d}{\sqrt{\mu^2 - 1}}$.

Sol. Cone of light is formed due to total internal reflection.

$\therefore$ For *TIR*,

$$\sin i_c = \frac{1}{\mu}$$

also from the figure $\tan i_c = \frac{r}{d}$

$$\therefore \quad \frac{\sin i_c}{\sqrt{1 - \sin^2 i_c}} = \frac{r}{d}$$

$$\Rightarrow \quad r = \frac{d}{\sqrt{\mu^2 - 1}}.$$

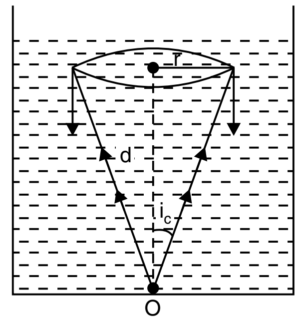


Fig. 9.71

Q. 16. *A plot between the angle of deviation (δ) and angle of incidence (i) for triangular prism is shown below. Explain any given value of 'δ' corresponds to two values of angle of incidence? State the significance of point P on the graph. Use this information to derive an expression for refractive index of the material of the prism.*

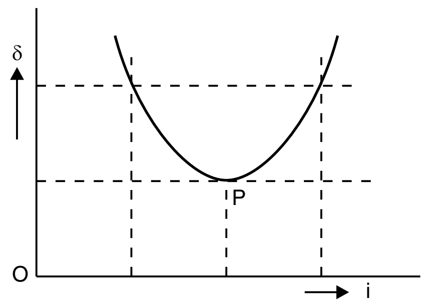


Fig. 9.72

Sol. In general, any given value of δ, except for $i = e$ corresponds to two values i and e. This, in fact, is expected from the symmetry of i and e as $\delta = i + e - A$, *i.e.*, δ remains the same if i and e are interchanged. Point P is the point of minimum deviation. This is related to the fact that the path of the ray as shown in Fig. 9.67 can be traced back resulting in the same angle of deviation. At the minimum deviation D_m, the refracted ray inside the prism becomes parallel to the base

For $\delta = D_m$,

$i = e$

and $r_1 = r_2 = r$

or $2r = A$

or $r = A/2$

In the same way

$$D_m = 2i - A$$

or $$i = \frac{A + D_m}{2}$$

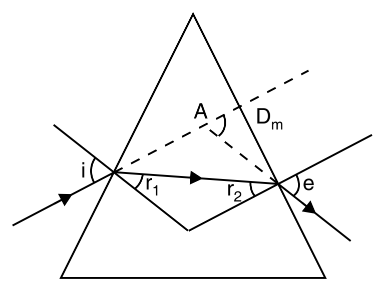


Fig. 9.73

$\therefore$ The refractive index of the prism is

$$\mu = \frac{\sin \frac{(A + D_m)}{2}}{\sin A/2}.$$

Q. 17. *A thin lens, made of material of refractive index μ has a focal length f. If the lens is placed in a transparent medium of refractive index 'n' (n < μ), obtain an expression for the change in focal length of the lens. Use the result to show that the focal length of a lens of the glass (μ = μ_g) becomes $\frac{\mu_w(\mu_g - 1)}{(\mu_g - \mu_w)}$ times its focal length in air, when it is placed in water (μ = μ_w).*

Sol. The focal length of the lens having radii of its curvature R_1 and R_2 and refractive index of its material with respect to air μ, f is given by

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad ...(i)$$

When the lens is put in a transparent medium, the refractive index of the material of the lens with respect to the medium = $\left(\frac{\mu}{n}\right)$. Thus focal length of the lens in the medium f' is given by

$$\frac{1}{f'} = \left(\frac{\mu}{n} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad ...(ii)$$

From Eqns. (*i*) and (*ii*),

$$\frac{f}{f'} = \frac{(\mu - 1)}{\left(\frac{\mu}{n} - 1\right)}$$

or

$$\boxed{f' = \frac{(\mu - 1)n}{(\mu - n)} \cdot f} \qquad ...(iii)$$

Thus, change in focal length

$$\Delta f = f' - f = \left[\frac{(\mu - 1)n}{(\mu - n)} f - f\right]$$

$$= f\left[\frac{(\mu - 1)n - (\mu - n)}{(\mu - n)}\right]$$

$$= f\left[\frac{\mu n - n - \mu + n}{(\mu - n)}\right]$$

$$= \frac{1}{(\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}\left[\frac{\mu n - \mu}{(\mu - n)}\right]$$

$$= \frac{R_1 R_2}{(R_2 - R_1)(\mu - 1)}\left[\frac{\mu(n - 1)}{(\mu - n)}\right]$$

$$\Delta f = \frac{R_1 R_2}{(R_2 - R_1)}\left[\frac{n}{(\mu - n)} - \frac{1}{(\mu - 1)}\right] \qquad ...(iv)$$

From Eq (*iii*),

$$f' = \frac{(\mu - 1)n}{(\mu - n)} \cdot f$$

Putting $\mu = \mu_g$ and $n = \mu_w$

$$f' = \frac{(\mu_g - 1)\mu_w}{(\mu_g - \mu_w)} \cdot f$$

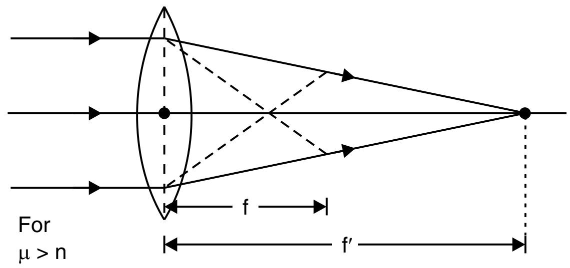


Fig. 9.74

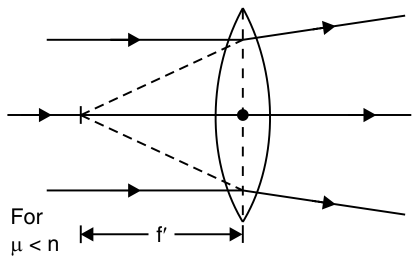


Fig. 9.75

MULTIPLE CHOICE QUESTIONS

1. A person uses spectacles of power +2*D*. He is suffering from
(*a*) short sightedness or myopia (*b*) long sightedness or hypermetropia
(*c*) Presbyopia (*d*) Astigmatism

2. The principal behind optical fibre is
(*a*) total external reflection (*b*) total internal reflection
(*c*) Both (*a*) and (*b*) (*d*) Diffraction

3. When light is passed through a prism, the colour which deviates least is
(*a*) Red (*b*) Violet (*c*) Blue (*d*) Green

4. Blue colour of sky is due to phenomenon of
(*a*) Reflection (*b*) Refraction (*c*) Scattering (*d*) Dispersion

5. Which of the following is used in optical fibre?
(*a*) Total internal reflection (*b*) Scattering
(*c*) Reflection (*d*) Interference

6. To get three images of a single object, one should have two plain mirrors at an angle of
(*a*) 60° (*b*) 90° (*c*) 120° (*d*) 30°

7. The image formed by an objective of a compound microscope is
(*a*) Virtual and diminished (*b*) Real and diminished
(*c*) Real and enlarged (*d*) Virtual and enlarged

8. A convex lens is dipped in a liquid whose refractive index is equal to the refractive index of the lens. Then its focal length will
(*a*) become zero (*b*) become infinite (*c*) reduce (*d*) increase

9. A ray of light strikes a silvered surface inclined to another one at an angle of 90°. Then the reflected ray will turn through

(*a*) 0° (*b*) 45° (*c*) 90° (*d*) 180°

10. In a compound microscope, the objective lens of f_o and eyepiece of f_e are placed at distance L such that L equals

(*a*) $f_o + f_e$ (*b*) $f_o - f_e$
(*c*) much greater than f_o or f_e (*d*) much less than f_o or f_e

11. In a compound microscope, the intermediate image is

(*a*) Virtual, errect and magnified (*b*) Real, errect and magnified
(*c*) Real, inverted and magnified (*d*) Virtual, inverted and magnified

12. The focal length (f) of spherical mirror of radius curvature R is

(*a*) $R/2$ (*b*) R (*c*) $3/2R$ (*d*) $2R$

13. A fish at a depth of 12 cm in water is viewed by an observer on the bank of a lake. Through what height is the image of fish raised? ($u = 4/3$)

(*a*) 9 cm (*b*) 12 cm (*c*) 3 cm (*d*) 3.8 cm

14. The refractive index of glass is 1.520 for red light and 1.525 for blue light. Let D_1 and D_2 be angles of minimum deviation for red and blue light respectively in a prism of this glass. Then

(*a*) $D_1 = D_2$ (*b*) $D_1 > D_2$
(*c*) $D_1 < D_2$ (*d*) Depends on the angle of prism

15. A convex lens and concave lens, each having same focal length of 25 cm are put in correct to form a combination of lenses. The power of the combination is

(*a*) zero (*b*) 25 (*c*) 50 (*d*) infinite

Answers

1. (*a*)	**2.** (*b*)	**3.** (*a*)	**4.** (*c*)	**5.** (*a*)
6. (*b*)	**7.** (*c*)	**8.** (*b*)	**9.** (*d*)	**10.** (*a*)
11. (*c*)	**12.** (*a*)	**13.** (*c*)	**14.** (*c*)	**15.** (*a*).

TEST YOUR SKILLS

1. Draw a ray diagram of a reflecting type telescope. State two advantages of this telescope over a refracting telescope.
2. A ray of light passing through an equilateral triangular glass prism form air undergoes minimum deviation when angle of incidence is 3/4th of the angle of prism. Calculate the speed of light in the prism.
3. How does the power of a convex lens vary, if the incident red light is replaced by violet light?
4. Draw a ray diagram of a compound microscope. Write the expression for its magnifying power.
5. Write the necessary conditions for the phenomenon of total internal reflection to take place.
6. A ray of light falls on a triangular glass prism in such a way that the deviation of the emergent ray is minimum for that prism. Draw the ray diagram for this case and write the relation between the angle of incidence and angle of emergence.
7. An equiconvex lens, with radii of a curvature of magnitude 30 cm each, is put over a liquid layer poured on top of a plane mirror. A small needle, with its tip on the principal axis of the lens, is moved along the axis until its inverted real image coincides with the needle itself. The distance of the needle, from the lens, is measured to be 45 cm.

On removing the liquid layer, and repeating the experiment, the distance is measured to be 30 cm.

Given that the two values of the distance measured represent the focal length values in the two cases, calculate the refractive index of the liquid.

8. Derive the lens formula, $1/f = 1/v - 1/u$ for a concave lens, using the necessary ray diagram.

Two lens of power 10 *D* and – 5 *D* are placed in contact with each other forming a combination lens.

(*i*) What is the focal length of this combination?

(*ii*) Where should an object be held from the lens, so as to obtain a virtual image of magnification 2?

9. How does the angle of minimum deviation of a glass prism of refractive index 1.15 change, if it is immersed in a liquid of refractive index 1.3?

10. How does the fringe width of interference fringes change, when the whole apparatus of Young's experiment is kept in a liquid of refractive index 1.13?

11. How is a wavelength defined? Using Huygen's construction draw a figure showing the propagation of a plane wave refracting at a plane surface separating two media. Hence verify Snell's law of refraction.

12. (*a*) What is polarised light? Two polaroids are placed at 90° to each other and the transmitted intensity is zero. What happens when one more polaroid is placed between these two, bisecting the angle between them? How will the intensity of transmitted light vary on further rotating the third polaroid?

13. For a ray of light travelling from a denser, medium of refractive index n_1 to a rarer medium of refractive index n_2. Prove that $n_2/n_1 = \sin i_c$ where i_c is the critical angle of incidence for the media.

14. Two thin lenses of focal length + 10 cm and – 5 cm are kept in contact. What is the (*i*) focal length and (*ii*) power of the combination?

15. The convex lenses of same focal length but of apertures 5 cm and 10 cm are used as objective lenses in two astronomical telescopes.

(*i*) What will be the ratio of their resolving power?

(*ii*) Compare the intensity of image formed in two power.

16. You are given three lenses having power *P* and apertures *A* as follows

$$P_1 = 6\ D,\ A_1 = 3\ \text{cm};$$
$$P_2 = 3\ D,\ A_2 = 15\ \text{cm}$$
$$P_3 = 12\ D,\ A_3 = 1.5\ \text{cm}.$$

Which two of these will you select to construct (*i*) to telescope and (*ii*) a microscope? State the basis for your answer in each case.

17. Two convex lenses of focal length 0.3 m and 0.05 m are used to make a telescope. Find the distance kept between the two in order to obtain an image at infinity.

The refractive indices of glass and water with respect to air are 3/2 and 4/3 respectively. What will be the refractive index of glass with respect to water?

❑❑❑